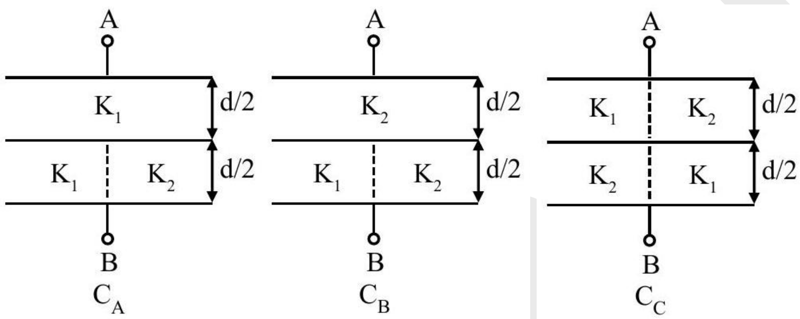
JEE Main Online Exam 2026

Memory Based
Questions & Solution
24th January 2026 | Evening

PHYSICS

1. Diagram shows three arrangement of di-electric in the capacitor.

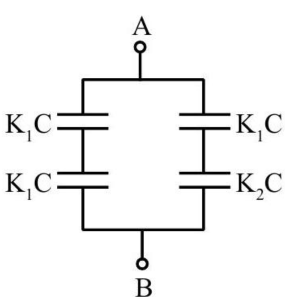


Arrange the capacitors in increasing order of capacitance between A & B if $K_1 > K_2$:

(1) $C_A < C_B < C_C$ (2) $C_A < C_C < C_B$ (3) $C_B < C_C < C_A$ (4) $C_B < C_A < C_C$

Ans. [3]

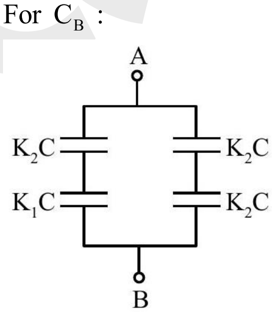
Sol. For C_A :



Let $\frac{\varepsilon_0 A}{d} = C$

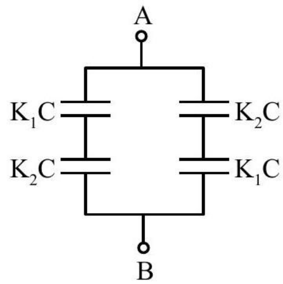
$$\therefore C_A = \frac{K_1C}{2} + \frac{K_1 K_2C}{K_1 + K_2} = K_1C\left[\frac{K_1 + 2K_2}{2(K_1 + K_2)}\right]$$

For C_B :



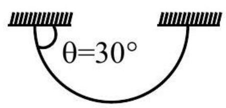
$$\frac{K_2C}{2}+\frac{K_1\,K_2C}{K_1+K_2}=K_2C\left[\frac{K_1+2\,K_2}{2\left(K_1+K_2\right)}\right]$$

For C_C :



$$C_C=\frac{2\,K_1\,K_2C}{\left(K_1+K_2\right)}$$

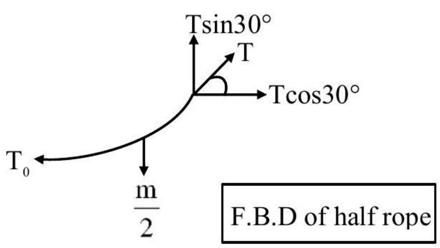
2. A flexible chain of mass m is hanging as shown. Find tension at the lowest point :



(1) $\frac{\sqrt{3}}{2}mg$ (2) $\frac{1}{2}mg$ (3) $\frac{\sqrt{2}}{3}mg$ (4) $\sqrt{2}mg$

Ans. **[1]**

Sol.



$$T\sin30^\circ=\frac{m}{2}g$$

$$T\cos30^\circ=T_0$$

$$\tan30^\circ=\frac{mg}{2\,T_0}$$

$$T_0=\frac{\sqrt{3}}{2}mg$$

3. In case of meter bridge experiment balance length for 2Ω and 3Ω is ℓ and for $x\Omega$ and 3Ω is $\left(\ell+10\right)$cm. Find x.

Ans. **[3]**

Sol. $\frac{2}{3}=\frac{\ell}{100-\ell}$

$\ell=40$ cm

$\frac{x}{3}=\frac{\ell+10}{90-\ell}=\frac{50}{50}$

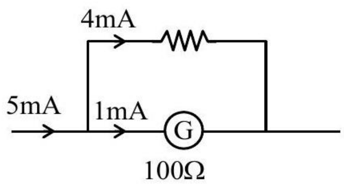
$x=3\Omega$

4. There is a galvanometer of resistance 100Ω and full scale current $Ig = 1\text{ mA}$. Find shunt resistance required to increase it's range to 5 mA :

(1) 25Ω (2) 0.25Ω (3) 0.5Ω (4) 1Ω

Ans. **[1]**

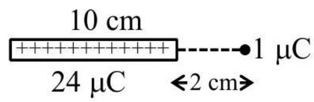
Sol.



$$4\times r_s = 1\times G$$

$$r_s = \frac{G}{4} = \frac{100}{4} = 25\Omega$$

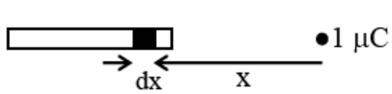
5. Rod has uniformly distributed charge $24\mu C$ and length 10 cm. Find force on $1\mu C$ particle ?



(1) 70 N (2) 10.5 N (3) 90 N (4) 25 N

Ans. **[3]**

Sol.



$$F = \int dF = \int_{2\text{ cm}}^{12\text{ cm}} \frac{kq\lambda dx}{x^2} = kq\lambda\left(\frac{1}{2\times10^{-2}} - \frac{1}{12\times10^{-2}}\right)$$

$$F = (9\times10^9)(10^{-6})\left(\frac{24\times10^{-6}}{10^{-1}}\right)\left(\frac{5}{12}\right)\times10^2$$

$$= 9\times24\times\frac{5}{12} = 90\text{ N}$$

6. 300 Joule of energy is given to a gas at constant volume which increases it's temperature from $20^\circ C$ to $50^\circ C$. If $R = 8.3$ SI units $\& C_P = \frac{7R}{2}$ then find mass of gas :

Ans. **[ ]**

Sol. $\Rightarrow$ For Isochoric process

$$Q = nC_v\Delta T$$

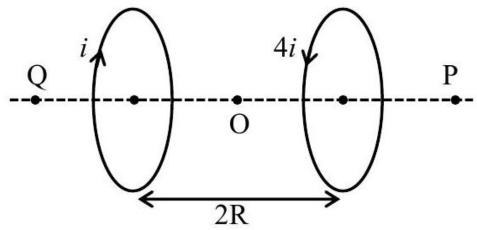
$$300 = n\cdot\frac{5R}{2}\cdot(50-20)$$

$$n = \left(\frac{4}{R}\right)\text{ mole}$$

mass of gas $= \left(\frac{4}{R}\right)$ (molecular weight)

NOTE : molecular weight of gas is unknown in question.

7. Find magnetic field at midpoint O. Rings have radius R and direction of current in opposite sense.



(1) $\frac{3\mu_0 i}{4\sqrt{2}R}$ Towards P

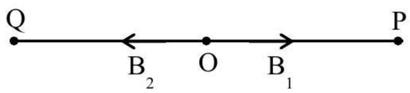
(2) $\frac{3\mu_0 i}{4\sqrt{2}R}$ Towards Q

(3) $\frac{3\mu_0 i}{2\sqrt{2}R}$ Towards P

(4) $\frac{3\mu_0 i}{2\sqrt{2}R}$ Towards Q

Ans. **[1]**

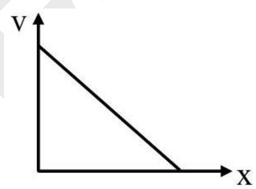
Sol. $B_{net} = B_1 - B_2$

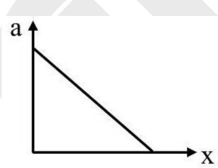


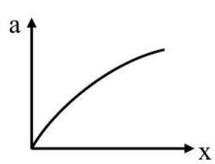
$$= \frac{4\mu_0 i R^2}{2\left(R^2 + R^2\right)^{3/2}} - \frac{\mu_0 i R^2}{2\left(R^2 + R^2\right)^{3/2}}$$

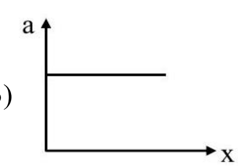
$$= \frac{3\mu_0 i}{4\sqrt{2}R}$$

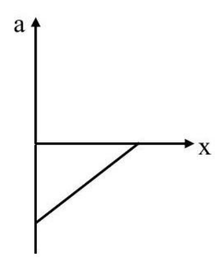
8. Velocity of particle varies with position as shown in figure. Find the correct variation of acceleration with position :



(1)


(2)


(3)


(4)


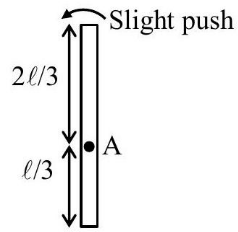
Ans. **[4]**

Sol. $v = -mx + c$

$$a = v\frac{dv}{dx} = (-mx + c)(-m)$$

$$a = m^2x - mc$$

9. When rod becomes horizontal find it's angular velocity. It is pivoted at point A as shown :



(1) $\sqrt{\frac{3g}{\ell}}$ (2) $\sqrt{\frac{2g}{\ell}}$ (3) $\sqrt{\frac{g}{\ell}}$ (4) $\sqrt{\frac{5g}{\ell}}$

Ans. **[1]**

Sol. $mg\frac{\ell}{6} = \frac{1}{2}I\omega^2$

Here $I = \frac{m\ell^2}{12} + \frac{m\ell^2}{36} = \frac{m\ell^2}{9}$

$mg\frac{\ell}{6} = \frac{m\ell^2}{18}\omega^2 \Rightarrow \omega^2 = \frac{3g}{\ell}$

$\omega = \sqrt{\frac{3g}{\ell}}$

10. 5[th] Harmonic of closed organ pipe frequency matches with 1[st] Harmonic of open organ pipe. Find ratio of their lengths.

(1) 5 (2) 2 (3) 5 / 2 (4) 2 / 5

Ans. **[3]**

Sol. $f_{5\text{ closed}} = f_{1\text{ open}}$

$\frac{5v}{4L_{closed}} = \frac{v}{2L_{open}}$

$\frac{L_{closed}}{L_{open}} = \frac{5}{2}$

11. In a vernier callipers 50 VSD are equal to 48 MSD. 1 MSD is equal to 0.05 mm. Find least count of this vernier callipers :

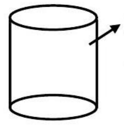
(1) 0.005 mm (2) 0.004 mm (3) 0.001 mm (4) 0.002 mm

Ans. **[4]**

Sol. $LC = 1MSD - 1MSD = 1MSD - \frac{48}{50}MSD$

$= \frac{2}{50}MSD = \frac{2}{50} \times .05\text{ mm} = 0.002\text{ mm}$

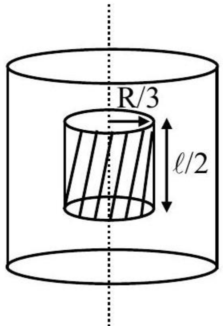
12. A solid cylinder of radius $\frac{R}{3}$ and length $\frac{L}{2}$ is removed along the central axis. Find ratio of Initial moment of inertia and moment of inertia of removed cylinder :



(1) 162 (2) 158 (3) 138 (4) 178

Ans. [1]

Sol.



Original mass (M)

The removed mass (m)

$$m = \rho \times \pi\left(\frac{R}{3}\right)^2 \times \frac{L}{2} = \frac{\rho \cdot \pi R^2 L}{18} = \frac{M}{18}$$

$$I' = \frac{1}{2}\cdot\frac{M}{18}\cdot\frac{R^2}{9} = \frac{1}{324}MR^2$$

$$\frac{I}{I'} = \frac{\frac{1}{2}MR^2}{\frac{1}{324}MR^2} = 162$$

13. A cylindrical object of density $600\ kg/m^3$ and height 8 cm is floating in a liquid of density 900 kg/m^3. Find height of cylinder inside liquid.

(1) $\frac{16}{3}$ cm (2) $\frac{20}{3}$ cm (3) $\frac{5}{3}$ cm (4) $\frac{25}{3}$ cm

Ans. [1]

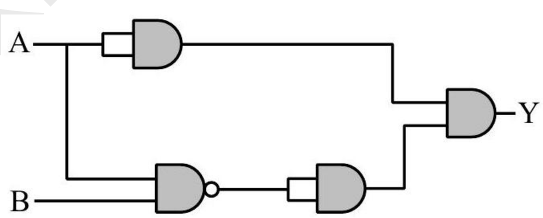
Sol. $Mg = F_b$

$dAHg = \rho Ahg$

$600 \times 8\ cm = 900 \times h$

$h = \frac{16}{3}$ cm

14. Select correct truth table?



(1)

A	B	Y
0	0	0
0	1	0
1	0	1
1	1	1

(2)

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	0

(3)

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	1

(4)

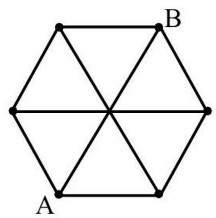
A	B	Y
0	0	0
0	1	0
1	0	1
1	1	0

Ans. **[4]**

Sol. $Y = \left(\overline{A \cdot B}\right) \cdot A = \left(\overline{A} + \overline{B}\right) \cdot A = 0 + A \cdot \overline{B}$

A	B	Y
0	0	0
0	1	0
1	0	1
1	1	0

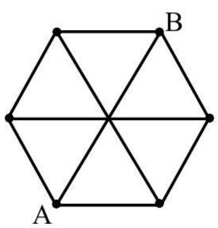
15. Resistance of each side is R. Find equivalent resistance between two opposite points as shown in figure.

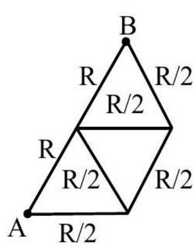


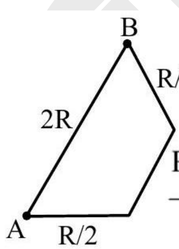
(1) $\frac{4}{5}R$ (2) $\frac{8}{5}R$ (3) $\frac{8}{10}R$ (4) $\frac{2}{5}R$

Ans. **[1]**

Sol.



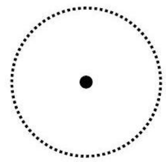




$$\frac{R \times \frac{R}{2}}{\frac{3R}{2}} = \frac{R}{3}$$

$$R_{eq} = \frac{2R \times \frac{4R}{3}}{2R + \frac{4R}{3}} = \frac{8R^2}{10R} = \frac{4}{5}R$$

16. The intensity at spherical surface due to a isotropic point source placed at its center is I_0. If it's volume is increased by 8 times, what will be intensity at the spherical surface :

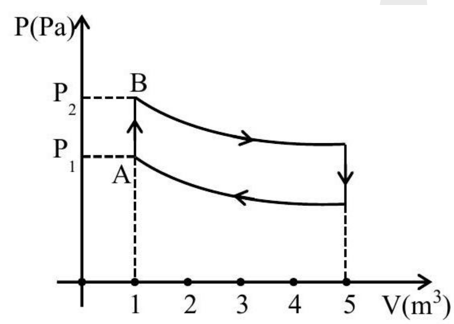


(1) Increase by 128 times (2) Increase by 8 times
(3) Decrease by 4 times (4) Decrease by 8 times

Ans. **[3]**

Sol. $V \rightarrow 8\,V \Rightarrow R \rightarrow 2R \Rightarrow A \rightarrow 4\,A \Rightarrow I \rightarrow \frac{I_0}{4}$

17. Find heat given to gas to take it from A to B . (Given : $C_v = 21$ S.I. units, $P_2 = 30$ Pa, $P_1 = 21.7$ Pa , $R = 8.3$ S.I.units, $n = 10$ moles)



(1) 30 J (2) 21 J (3) 42 J (4) 50 J

Ans. **[2]**

Sol. $Q = \Delta U + W = \Delta U = nC_v \Delta T$

$= \frac{f}{2}(P_2 - P_1)V$ … (i)

Here $C_v = 21 = \frac{f}{2}R$

$f = \frac{42}{R}$

So, from eq(1)

$Q = \frac{42}{R \times 2}(8.3) \times 1 = 21$ J

18. In YDSE, slits separation d is 2 mm, distance between slits and screen D is 10 m. Wave length of light is $\lambda = 6000$Å . If intensity of light through each slit is I_0 then find intensity at point directly in front of one of the slits

(1) $4I_0$ (2) Zero (3) I_0 (4) $2I_0$

Ans. **[3]**

Sol. Path difference $\Delta x = d\sin\theta = d \times \frac{y}{D} = d \times \frac{d}{D}$

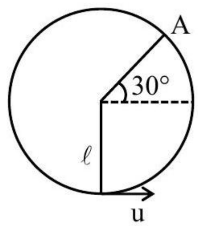
$$\Delta x = \frac{d^2}{D}$$

$$\Delta\phi = \frac{2\pi}{\lambda} \times \Delta x = \frac{2\pi\, d^2}{\lambda D}$$

$$\Delta\phi = \frac{2\pi \times 4 \times 10^{-6}}{6 \times 10^{-7} \times 10} = \frac{4\pi}{3}$$

$$I = I_0 + I_0 + 2I_0\cos\left(\frac{4\pi}{3}\right) = I_0$$

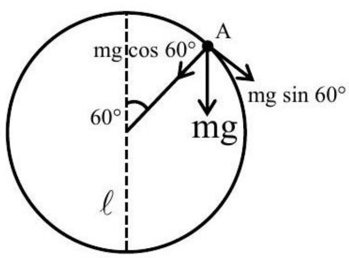
19. Find speed given to particle at lowest point so that tension in string at A point becomes zero :



(1) $\sqrt{\frac{7\,g\ell}{2}}$ (2) $\sqrt{3\,g\ell}$ (3) $\sqrt{\frac{9}{4}\,g\ell}$ (4) $\sqrt{\frac{g\ell}{2}}$

Ans. **[1]**

Sol.



$$T + mg\cos 60^\circ = \frac{mV^2}{\ell}$$

$$T = 0$$

$V^2 = \frac{g\ell}{2}$ here V is the speed at point A

M.E.C.

$$\frac{1}{2}mu^2 = mg\left(\ell + \ell\cos 60^\circ\right) + \frac{1}{2}mV^2$$

$$u^2 = 3\,g\ell + \frac{g\ell}{2}$$

$$u = \sqrt{\frac{7\,g\ell}{2}}$$

20. For given nuclear reactions

$_2He^3 + _0n^1 \rightarrow _2He^4 + 20MeV$

$_2He^4 + _0n^1 \rightarrow _2He^5 - 0.9MeV$

X_3 represents stability of $_2He^3, X_4$ represents stability of $_2He^4$ and X_5 represents stability of $_2He^5$. compare the stabilities.

(1) $X_4 > X_5 > X_3$ (2) $X_4 < X_3 > X_5$ (3) $X_3 > X_4 > X_5$ (4) $X_4 > X_3 > X_5$

Ans. [1]

Sol. $BE_{He^4} - BE_{He^3} = 20MeV$ … (1)

$BE_{He^5} - BE_{He^4} = -0.9MeV$ … (2)

From eq (1) & (2)

$BE_{He^4} > BE_{He^5} > BE_{He^3}$

21. A soap bubble of diameter 7 cm its diameter is increased to 14 cm. If change in its surface energy $(15000 - x)\mu J$. Find x

(Given surface tension is $0.04\ N/m$)

(1) 208 (2) 216 (3) 432 (4) 512

Ans. [2]

Sol. $\Delta E =$ (change in surface area) . (surface tension)

$\Delta E = 2\left[(4\pi)\left(r_2^2 - r_1^2\right)\right](T)$

$\Delta E = 8\pi\left(r_2^2 - r_1^2\right)T$

$= 8 \times \frac{22}{7}\left(\frac{14^2 - 7^2}{10^4}\right) \times 0.04$

$= 14784\mu\ J$

$15000 - x = 14784\mu\ J$

$x = 216\mu\ J$

22. An electron makes transition from higher energy orbit n_2 to lower energy orbit n_1 in Li^{+2} ion such that $n_1 + n_2 = 4\ \&\ n_2 - n_1 = 2$. Determine the wavelength of emitted photon in transition (in cm) :

(1) 1.14×10^{-6} cm (2) 3.28×10^{-6} cm (3) 5.76×10^{-6} cm (4) 8.23×10^{-6} cm

Ans. [1]

Sol. $n_1 + n_2 = 4$

$n_2 - n_1 = 2$

$n_2 = 3; n_1 = 1$

$E_3 - E_1 = +13.6 \times 9\left(\frac{1}{1} - \frac{1}{9}\right)eV$

$= 108.8eV$

$\lambda = \frac{12400}{108.8}\text{Å} \approx 114\text{Å} = 1.14 \times 10^{-6}$ cm

23. On a surface, if photon of λ wavelength is incident. The stopping potential is 3.2 V. If the wavelength incident is 2λ, stopping potential is 0.7 V. Find λ.

(1) 4.96×10^{-7} m (2) 3.62×10^{-7} m (3) 7.24×10^{-7} m (4) 2.48×10^{-7} m

Ans. [4]

Sol. $q.(3.2) = \frac{hc}{\lambda} - \phi$ … (1)

$q(0.7) = \frac{hc}{2\lambda} - \phi$ … (2)

Eq. (1) - Eq. (2)

$q \cdot (2.5) = \frac{hc}{2\lambda}$

$2.5 = \left(\frac{hc}{e}\right)\left(\frac{1}{2\lambda}\right)$

$2.5 = \frac{12400}{2(\lambda)}$

$\lambda = \frac{12400}{5}Å$

$\lambda = 2480Å$

$\lambda = 2.48 \times 10^{-7}$ m

24. Power of convex lens is 5D. Four students measure object and image distances as shown :

	u (in cm)	v (in cm)
A	35	37
B	30	60
C	60	30
D	25	100

(1) Students A & B are correct
(2) All are correct
(3) Student A is wrong
(4) Students C & D are wrong

Ans. **[3]**

Sol. $f = \frac{100}{5} = 20$ cm

For student A

$\frac{1}{f} = \frac{1}{37} + \frac{1}{35} \Rightarrow f \approx 18$ cm

For student B

$\frac{1}{f} = \frac{1}{60} + \frac{1}{30} \Rightarrow f = 20$ cm

For student C

$\frac{1}{f} = \frac{1}{30} + \frac{1}{60} \Rightarrow f = 20$ cm

For student D

$\frac{1}{f} = \frac{1}{25} + \frac{1}{100} \Rightarrow f = 20$ cm

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CHEMISTRY

1. If percentage of N_2 above a liquid solution is 80% at a total pressure of 10 atm then find the mole fraction of N_2 gas dissolved in solution. [Given that Henry's constant for N_2 is 7.6×10^7 mm Hg].

(1) 10^{-4} (2) 8×10^{-5} (3) 10^{-7} (4) 10^{-6}

Ans. **[2]**

Sol. $P_{N_2} = K_H \cdot X_{N_2}$

$P_{N_2} = 0.8 \times 10 = 8$ atm

$8 \times 760 = 7.6 \times 10^7 \times X_{N_2}$

$X_{N_2} = \dfrac{8 \times 760}{7.6 \times 10^7}$

$X_{N_2} = 8 \times 10^{-5}$.

2. The half-life of radio-active isotope Zn^{65} is 245 days. Find the time after which activity of Zn sample remains 75% of its initial value ?

Ans. **[102]**

Sol. $t_{1/2} = \dfrac{\ln 2}{K}$

$K = \dfrac{\ln 2}{245}$

$t = \dfrac{1}{K} \ell n \dfrac{a_0}{a_t}$

$t_{25\%} = \dfrac{1}{K} \ell n \dfrac{4}{3}$

$t_{25\%} = \dfrac{1}{\frac{\ln 2}{245}} \ln \dfrac{4}{3}$

$t_{25\%} = 245 \dfrac{\ln \frac{4}{3}}{\ell n 2} = 245 \left[\dfrac{2\log 2 - \log 3}{\log 2} \right]$

$= 245 \left[\dfrac{2 \times 0.3010 - 0.4771}{0.3010} \right] = 101.66$ day.

3. If for Li^{2+} ion, electron is in transition between energy levels such that sum of principal quantum numbers is 4 and difference is 2 then find the wavelength (in cm) emitted for transition between these energy levels.
[Given : $R_H = 1.1 \times 10^5\ cm^{-1}$]

(1) 114×10^{-8} cm (2) 1026×10^{-8} cm (3) 12.66×10^{-8} cm (4) 10^{-8} cm

Ans. **[1]**

Sol. $n_1 \rightarrow$ lower energy level

$n_2 \rightarrow$ higher energy level

$n_1 + n_2 = 4,\ n_2 = 3$

$n_2 - n_1 = 2, n_1 = 1$

Rydberg's formula :

$$\frac{1}{\lambda} = R_H Z^2 \left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

$$\frac{1}{\lambda} = R_H (3)^2 \left[\frac{1}{1^2} - \frac{1}{3^2}\right]$$

$$\frac{1}{\lambda} = 8R_H$$

$$\lambda = \frac{1}{8R_H}$$

$$\lambda = \frac{1}{8 \times 1.1 \times 10^5}$$

$$\lambda = \frac{1000}{8.8} \times 10^{-8}\ cm$$

$$\lambda = 113.63 \times 10^{-8}\ cm$$

$$\lambda \simeq 114 \times 10^{-8}\ cm$$

Mass % of ' O ' $= \frac{0.1833}{0.25} \times 100 = 73.32\%$.

4. For the reaction :

$Cl_2 + KOH \rightarrow KCl + KClO + H_2O$

1 mole of Cl_2 passed into 2 litre, 2 M solution of KOH. Determine the molarity of Cl^-, ClO^- and OH^- respectively.

(1) 1M, 0.5M, 0.5M (2) 0.5M, 0.5M, 1M (3) 1M, 1M, 0.5M (4) 0.5M, 1M, 0.5M

Ans. **[2]**

Sol. $Cl_2 + 2KOH \rightarrow KCl + KClO + H_2O$

	Cl_2	$2KOH$	KCl	$KClO$
t=0	1 mole	4 mole		
t_f	0	2mole	1 mole	1 mole

$$[OH^-] = 1M$$

$$[Cl^-] = \frac{1}{2}M$$

$$[ClO^-] = \frac{1}{2}M$$

5. Correct order of 2^{nd} ionisation energy is :

(1) O < C < N < F (2) C < N < O < F (3) C < N < F < O (4) C < O < N < F

Ans. **[3]**

Sol. $O \rightarrow O^{+}(2p^3)$

$N \rightarrow N^{+}(2p^2)$

$C \rightarrow C^{+}(2p^1)$

$F \rightarrow F^{-}(2p^4)$

Left to right IE increases but IE of $2p^3 > 2p^4$

6. Heat of atomisation of $CH_4(g)$ and $C_2H_6(g)$ are x kJ/mole and y kJ/mole. Find the maximum wavelength of photon required to dissociate (C–C) bond in C_2H_6 :

(1) $\frac{hcN_A}{\left[y-\frac{3x}{2}\right]}$ (2) $\frac{hcN_A}{\left[\frac{4x-6y}{4}\right]}$ (3) $\frac{hcN_A}{250\left[\frac{3x}{2}-y\right]}$ (4) $\frac{hcN_A}{500[2y-3x]}$

Ans. **[4]**

Sol. $CH_4(g) \rightarrow C(g) + 4H(g);\ \Delta_r H = x kJ / \text{ mole}$

$C_2H_6(g) \rightarrow 2C(g) + 6H(g); \Delta_r H = y kJ / \text{mole}$

$1000x = 4 \times \varepsilon_{C-H}$

$1000y = 1 \times \varepsilon_{C-C} + 6 \times \varepsilon_{C-H}$

$\varepsilon_{C-C} = \left[y - \frac{3x}{2}\right] \times 1000 = \frac{hc}{\lambda}.N_A$

('λ') wavelength of photon $_3 \frac{hcN_A}{\left[y-\frac{3x}{2}\right]\times 1000}$

7. Among the following species O_2^+, N_2^-, N_2^{2-} and O_2^- which have same bond order as well as paramagnetic in nature.

(1) N_2^-, N_2^{2-} (2) O_2^+, N_2^- (3) O_2^+, O_2^- (4) O_2^-, N_2^-

Ans. **[2]**

Sol.

Species	**B.O.**	n_e
O_2^+	2.5	1
N_2^-	2.5	1
N_2^{2-}	2	2
O_2^-	1.5	1

8. Consider the following complexes :

(A) $[Co(CN)_6]^{3-}$ (B) $\left[Co(NH_3)_5 H_2O\right]^{3+}$ (C) $\left[Co(H_2O)_6\right]^{3+}$ (D) $[CoF_6]^{3-}$

The wavelength absorbed by the above complexes are in the order of :

(1) A > B > C > D (2) A < B < C < D (3) B < A < C < D (4) C > A > B > D

Ans. **[2]**

Sol. Ligand strength : $CN^- > NH_3 > H_2O > F^-$

9. Element ' X ' is the lightest element of group 7 of periodic table which forms oxo-anion in +6 oxidation state. The colour of potassium salt of the oxo-anion is :

(1) Green (2) Orange (3) Yellow (4) Brown

Ans. **[1]**

Sol. K_2MnO_4 (Green in colour)

10. In general tests of Ba^{2+} and Ca^{2+} give the respective test as :

(1) Chromate, Sulphate (2) Sulphite, Sulphate

(3) Hydroxide, Carbonate (4) Carbonate, Carbonate

Ans. **[4]**

Sol. Group reagent is $(NH_4)_2 CO_3$ in presence of NH_4OH and NH_4Cl. Precipitate formed is $BaCO_3$ (white) and $CaCO_3$ (white).

11. Select incorrect option :

(1) Carbon can have negative oxidation state in its compounds.

(2) CO_2 is most acidic oxide among oxides of group-14 elements.

(3) Maximum covalency of carbon is four.

(4) Carbon has least catenation property in group 14.

Ans. **[4]**

Sol. Carbon has highest catenation property in group 14.

12. **Statement-I :** Two different aldehyde on cross aldol condensation always give four product :

Statement-II : Among benzaldehyde and acetophenone only acetophenone reacts with semicarbazide.

(1) Statement-I and Statement-II both are correct

(2) Statement-I is incorrect Statement-II is correct

(3) Statement-I is correct Statement-II is incorrect

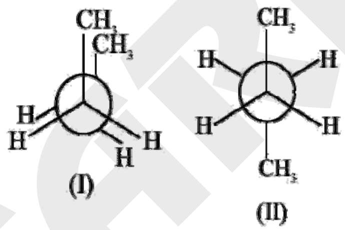
(4) Statement-I and Statement-II both incorrect

Ans. **[3]**

Sol. **Statement-I :** If both aldehydes have αH, then only on cross aldol condensation give four products if only one have αH then three products will form.

Statement-II : Benzaldehyde and Acetophenone both react with semicarbazide and form semicarbazone

13.



Statement-I : IInd is more stable than Ist

Statement-II : As dihedral angle increases stability decreases.

(1) Statement-1 is incorrect but Statement-2 is correct

(2) Statement-1 is correct but Statement-2 is incorrect

(3) Both statements are correct.

(4) Both statements are incorrect.

Ans. **[2]**

Sol. IInd compound is more stable because it has less steric and Torsional strain and statement II is incorrect.

14. 0.18 M HQ solution has molar conductivity $\frac{1}{30}$ times the molar conductivity of 0.02 M HZ solution. Find the value of $pK_a(HQ) - pK_a(HZ)$. [Given that α is very less than 1]
Assume that $\lambda_m^{\infty}(Q^-) = \lambda_m^{\infty}(Z^-)$

Ans. **[2]**

Sol. $K_a(HQ) = C_1\alpha_1{}^2 \qquad \alpha_1 = \frac{\lambda_m(HQ)}{\lambda_m^{\infty}(HQ)}$

$K_a(HZ) = C_2\alpha_2{}^2 \qquad \alpha_2 = \frac{\lambda_m(HZ)}{\lambda_m^{\infty}(HZ)}$

$$\frac{K_a(HQ)}{K_a(HZ)} = \frac{C_1}{C_2}\cdot\left(\frac{\alpha_1}{\alpha_2}\right)^2 = \frac{0.18}{0.02}\cdot\left[\frac{\lambda_m(HQ)}{\lambda_m(HZ)}\right]^2$$

$$\frac{K_a(HQ)}{K_a(HQ)} = 9\times\left(\frac{1}{30}\right)^2 = \frac{1}{100}$$

$pK_a(HQ) - pK_a(HZ) = 2$

15. **Statement-I :** RMgX react with CO_2 followed by acidification form product, which reacts with NH_3/Δ then reacts with NaOCl form product which further reacts with $CHCl_3$/NaOH and final product is $R-N \rightleftarrows C$.
Statement-II : $R-N \rightleftarrows C$ on hydrolysis gives RCOOH .
Which amongs the following statement is correct.
(1) Statement-I and Statement-II both are correct
(2) Statement-I is incorrect Statement-II is correct
(3) Statement-I is correct Statement-II is incorrect
(4) Statement-I and Statement-II both incorrect

Ans. **[3]**

Sol. **Statement-I**

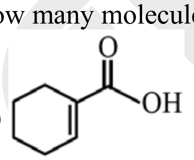
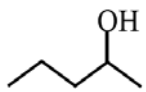
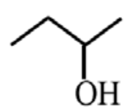
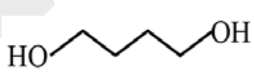
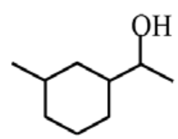
$$RMgX \xrightarrow[\substack{(ii)\,H^{\oplus} \\ (iii)\,NH_3/\Delta}]{(i)\,CO_2} R-CONH_2 \xrightarrow{NaOCl} R-NH_2$$

$$\xrightarrow[NaOH]{CHCl_3} R-N \overset{\rightarrow}{=} C \text{ (Alkyl isocyanide)}$$

Statement-II

$$R-N \overset{\rightarrow}{=} C \xrightarrow{H_3O^{\oplus}} R\text{–}NH_2 + HCOOH$$

16. How many molecules are secondary alcohol? O

(a) 
(b) 
(c) 
(d) 
(e) 

(1) 2 (2) 3 (3) 4 (4) 5

Ans. **[2]**

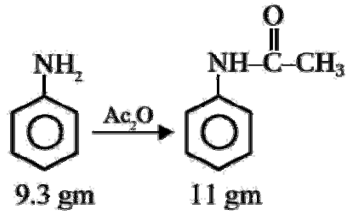
Sol. – OH group attached to secondary carbon is secondary alcohol
So compound b, c, e are secondary alcohol

17. Aniline (9.3 gm) on reaction with acetic anhydride forms ' X ' (11 gm), find the percentage yield of the reaction :

(1) 80 (2) 90 (3) 81.48 (4) 93.2

Ans. **[3]**

Sol.



$$n = \frac{9.3}{93} = 0.1 \ n = \frac{11}{135} = 0.08148$$

$$\% \text{ yield } = \frac{0.08148}{0.1} \times 100 = 81.48\%$$

18. How many tri peptides are possible when following three amino acid make tri peptide.
(No amino acid should repeat twice)

(A) Glycine (B) Alanine (C) Valine

(1) 4 (2) 6 (3) 8 (4) 9

Ans. **[2]**

Sol. Gly ala val
Gly val ala
Val gly ala
Val ala gly
Ala val gly
Ala gly val
Total tri peptides = 6

19. A complex $Cr(H_2O)_6.Cl_3$ show conductance similar to $1:2$ electrolyte in aq. Solution. 9.3 g of this complex is passed through a cation exchanger then excess $AgNO_3$ is added. Find mass of AgCl precipitated in gram? [Molar mass of $Cr = 52\ g/mol$.]

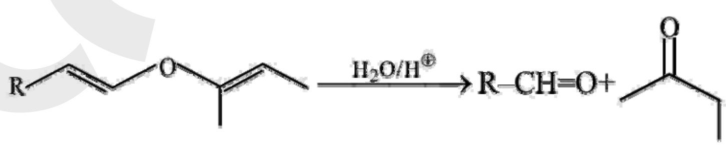
Ans. **[10]**

Sol. $\left[Cr(H_2O)_5 Cl\right]Cl_2 \cdot H_2O + AgNO_3 \rightarrow 2AgCl$

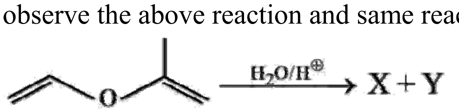
9.3/266.5 ----

$= 0.0349$ moles 0.0698 mole

Mass of $AgCl = 0.0698 \times 143.5 = 10.015gm$

20. 

observe the above reaction and same reaction is carried out with following compound.

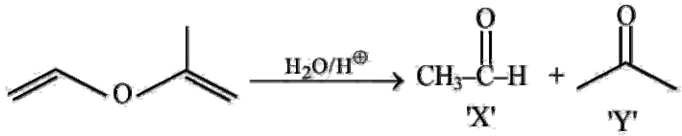


X and Y can be differentiated by.

(1) Fehling's test (2) $2,4-DNP$ (3) $NaHSO_3$ (4) Lucas test

Ans. [1]

Sol.



'X' and 'Y' can be differentiated by Fehling's test.
X gives positive Fehling test
Y gives negative Fehling test

21. **Statement (A) :** Resonating structure with more σ-bonds and charges far apart are more stable.
Statement (B) : Unsaturated hydrocarbon shows +I or –I depending upon the group they are attached with
Statement (C) : Carbanion with more % s character are more stable.
Statement (D) : In nucleophilic addition to carbonyl group-E (Electromeric) is responsible while in electrophilic additions +E is responsible.
Statement (E) : Alkene having more alkyl groups have more heat of hydrogenation.
How many statements are correct
(1) A,B,D,E (2) B,C,D,E (3) B,C,D (4) A,B,C,D,E

Ans. [3]

Sol. Statements B, C, D are correct.

22. If pure liquids A and B have a vapour pressure of 55 kPa and 15 kPa respectively. If in a solution of A and B, mole fraction of A in vapour is 0.8 , then find mole fraction of A in liquid phase?
(1) 0.813 (2) 0.5217 (3) 0.407 (4) 0.363

Ans. [2]

Sol. $\frac{Y_A}{Y_B} = \frac{P_A^o}{P_B^0} \cdot \frac{X_A}{X_B}$

$\frac{0.8}{0.2} = \frac{55}{15} \times \frac{X_A}{X_B}$

$\frac{X_A}{X_B} = \frac{60}{55} = \frac{12}{11}$

$X_A = \frac{12}{23} = 0.5217$

23. Two moles each of the gases P_2, Q_2 and PQ are present in a vessel at equilibrium. If 1 mole each of P_2 and Q_2 is added at equilibrium, then determine the composition (in mole) of each species at the new equilibrium?
(1) $n_{P_2} = 0.5, n_{Q_2} = 0.5, n_{PQ} = 1$ (2) $n_{P_2} = 1.33, n_{Q_2} = 1.33, n_{PQ} = 1.67$
(3) $n_{P_2} = 2.67, n_{Q_2} = 2.67, n_{PQ} = 2.33$ (4) $n_{P_2} = 2.67, n_{Q_2} = 2.67, n_{PQ} = 2.67$

Ans. [4]

Sol.

$$P_2(g) + Q_2(g) \rightleftharpoons 2PQ(g)$$

$t = t_{eq}$ 2 mole 2 mole 2mole

$K_{eq} = \frac{2^2}{2.2} = 1$

Now 1 mole of each P_2 and Q_2 is added

So reaction will move in forward direction

$$\begin{array}{lccccc} & P_2(g) & + & Q_2(g) & \rightleftharpoons & 2PQ(g) \\ t = t'_{eg} & 3 - x & & 3 - x & & 2 + 2x \end{array}$$

$$K_C = 1 = \frac{(2+2x)^2}{(3-x)(3-x)}$$

$$\frac{2+2x}{3-x} = 1$$

$$2 + 2x = 3 - x$$

$$x = \frac{1}{3}$$

At new equilibrium :

Moles of $P_2 = \frac{8}{3} = 2.67$

Moles of $Q_2 = \frac{8}{3} = 2.67$

Moles of $PQ = \frac{8}{3} = 2.67$

24. An organic compound contains ' C ', ' H ' and ' O '. 0.25 gm of organic compound. On combustion produces $CO_2(g)$ and $H_2O(\ell)$.. When residual gases passes through potash solution, it's mass increases by 0.18 g and when passed through anhydrous $CaCl_2$, the increase in mass of $CaCl_2$ is 0.15 g. Find mass % of oxygen in the organic compound.

Ans. **[73]**

Sol.

$$\begin{array}{lcccc} & C_xH_yO_z + O_2 & \longrightarrow & CO_2 & + \; H_2O \\ t = 0 & 0.25\text{ gm} & & & \\ t_{eq} & - & & 0.18\text{ gm} & 0.15\text{ gm} \end{array}$$

Mass of ' C ' $= \frac{0.18}{44} \times 12 = 0.049 \approx 0.05\text{gm}$

Mass of ' H ' $= \frac{0.15}{18} \times 2 = 0.016 \approx 0.017\text{gm}$.

Mass of ' O ' $= 0.25 - 0.05 - 0.017 = 0.1833\text{gm}$

JEE Main Online Exam 2026

Memory Based
Questions & Solution

24th January 2026 | Evening

MATHEMATICS

1. Let $f(x)$ be a differentiable function satisfying

$f(x) = e^x + \int_0^1 (y + xe^x) f(y) dy$. Find $f(0) + e$ (where $e =$ napiers constant) :

(1) 2 (2) 4 (3) 6 (4) 8

Ans. **[1]**

Sol. $f(x) = e^x + \int_0^1 yf(y) dy + xe^x \int_0^1 f(y) dy$

$f(x) = e^x + A + Bxe^x$

$A = \int_0^1 yf(y) dy = \int_0^1 y(A + e^y + Bye^y) dy$

$A = \frac{A}{2} + (0 - (-1)) + B(e - 1)$

$\frac{A}{2} + B(1 - e) = 1$

$B = \int_0^1 f(y) dy$

$B = \int_0^1 (e^y + A + Bye^y) dy$

$B = (e - 1) + A + B(0 - (-1))$

$B = e - 1 + A + B \Rightarrow A = 1 - e$

$f(x) = e^x + A + Bxe^x$

$f(0) = 1 + A = 1 - e + 1 = 2 - e$

$e + f(0) = 2$

2. Find number of solutions of the equation $\tan(x + 100^\circ) = \tan(x + 50^\circ) \tan x \tan(x - 50^\circ)$ where $x \in (o, \pi)$

(1) 3 (2) 4 (3) 5 (4) 6

Ans. **[2]**

Sol. $\frac{\tan(x + 100^\circ)}{\tan x} = \tan(x + 50^\circ) \tan(x - 50^\circ)$

$$\frac{\sin\left(x+100^\circ\right)\cos x}{\cos\left(x+100^\circ\right)\sin x}=\frac{\sin\left(x+50^\circ\right)\sin\left(x-50^\circ\right)}{\cos\left(x+50^\circ\right)\cos\left(x-50^\circ\right)}$$

Apply C & D

$$\frac{\sin\left(2x+100^\circ\right)}{\sin 100^\circ}=\frac{\cos 100^\circ}{-\cos 2x}$$

$$2\sin\left(2x+100^\circ\right)\cos 2x+\sin 200^\circ=0$$

$$\sin\left(4x+100^\circ\right)+\sin 100^\circ+\sin 200^\circ=0$$

$$\sin\left(4x+100^\circ\right)=-2\sin 150^\circ\sin 50^\circ$$

$$\sin\left(4x+100^\circ\right)=-\sin 50^\circ$$

$$\therefore 4x+100^\circ=n\pi+(-1)^n\cdot\left(-50^\circ\right)$$

$$4x=\frac{n\pi+(-)^{n+1}\left(50^\circ\right)-100^\circ}{4}$$

$$\therefore x=\frac{130^\circ}{4},\frac{210^\circ}{4},\frac{490^\circ}{4},\frac{570^\circ}{4}\ \text{ in } (0,\pi)$$

$\therefore$ no. of solutions $=4$

3. Let $z=(1+i)(1+2i)(1+3i)\ldots..(1+ni)$ and $\left|z^2\right|=44200$. Find the value of n.

(1) 5 (2) 6 (3) 8 (4) 7

Ans. **[1]**

Sol. $|z|^2=2^3.5^2.13.17$

$$\prod_{r=1}^{n}\left(1+r^2\right)=2^3\cdot5^2\cdot13\cdot17$$

$$=(2)\cdot(5)\cdot(2\cdot5)\cdot(17)\cdot(2\cdot13)=2\cdot5\cdot10\cdot17\cdot26$$

so $n=5$.

4. Let $f(x)=\left|\log_e x\right|-|x-1|+5$

Statement 1: $f(x)$ is differentiable for all $x\in(0,\infty)$

Statement 2 : $f(x)$ is increasing in $(1,\infty)$

Statement 3 : $f(x)$ is decreasing in $(0,1)$

Which of the following is correct?

(1) All the statements are correct

(2) Statement -1 & Statement -3 are correct

(3) Statement -1 & Statement -2 are correct

(4) Statement -2 & Statement -3 are correct

Ans. **[2]**

Sol. $$Rf'(1)=\lim_{h\to0^+}\frac{f(1+h)-f(1)}{h}=\lim_{h\to0^+}\frac{\left|\log_e(1+h)\right|-|h|+5-5}{h}$$

$$Rf'(1)=\lim_{h\to0^+}\frac{\log_e(1+h)-h}{h}=0$$

$$Lf'(1)=\lim_{h\to0^+}\frac{f(1-h)-f(1)}{-h}=\lim_{h\to0^+}\frac{|\log_e(1-h)|-|-h|+5-5}{-h}$$

$$Lf'(1)=\lim_{h\to0^+}\frac{|\log_e(1-h)|-h}{-h}=\lim_{h\to0^+}\frac{-\log_e(1-h)-h}{-h}=0$$

$f(x)$ is differentiable at $x=1$

$$f(x)=f(x)=\begin{cases}\log_e x-(x-1)+5 & x\ge1\\ -\log_e x+(x-1)+5 & x\in(0,1]\end{cases}$$

$$f'(x)=\begin{cases}\frac{1}{x}-1 & x\ge1\\ -\frac{1}{x}+1 & x\in(0,1]\end{cases}$$

$$f'(x)=\begin{cases}\frac{1-x}{x} & x\ge1\\ \frac{x-1}{x} & x\in(0,1]\end{cases}$$

$f'(x)\le0\,\forall x\in(1,\infty)\Rightarrow f(x)\downarrow$ in $x\in(1,\infty)$

$f'(x)<0\,\forall x\in(0,1)\Rightarrow f(x)\downarrow$ in $x\in(0,1)$

5. Let S has 5 elements and $P(S)$ is the power set of S . Let an ordered pair (A, B) is selected at random from $P(S)\times P(S)$. If the probability that $A\cap B=\phi$ is $\frac{3^m}{2^n}$, then value of $(m+n)$ is equal to

(1) 88 (2) 96 (3) 64 (4) 28

Ans. **[2]**

Sol. $S=\{a,b,c,d,e\}$

$$P=\frac{3^{32}}{4^{32}}\left(\frac{\text{fav}}{\text{total}}\right)$$

$$P=\frac{3^{32}}{2^{64}}=\frac{3^m}{2^n}$$

$m=32, n=64$

$m+n=32+64=96$

6. If equation $x^4-ax^2+9=0$ have four real & distinct roots then least possible integral value of a is

(1) 5 (2) 6 (3) 7 (4) 8

Ans. **[3]**

Sol. $x^4-ax^2+9=0$ .....(1)

let $x^2=t$

$t^2-at+9=0$ ....(2)

for roots of equation (1) to be real & distinct roots of equation (2) must be positive & distinct.

(i) $D>0\Rightarrow a^2-36>0\Rightarrow a\in(-\infty,-6)\cup(6,\infty)$

(ii) $\frac{-b}{2a}>0\Rightarrow\frac{a}{2}>0\Rightarrow a>0$

(iii) $f(0) > 0 \Rightarrow 9 > 0 \Rightarrow a \in R$

By (i) $\cap$ (ii) $\cap$ (iii)

$\therefore a \in (6, \infty)$

$\therefore$ least integral value of a is 7

7. If all the letters of the word 'UDAYPUR' are arranged in all possible permutations and these permutations are listed in dictionary order, then the rank of the word 'UDAYPUR' is :

(1) 1580 (2) 1579 (3) 1582 (4) 1580

Ans. **[1]**

Sol. ADIPRUU

$A \rightarrow \frac{6!}{2!} = 360$

$D \rightarrow \frac{6!}{2!} = 360$

$P \rightarrow \frac{6!}{2!} = 360$

$R \rightarrow \frac{6!}{2!} = 360$

UA $\rightarrow 5! = 120$

UDAP $\rightarrow 3! = 6$

UDAR $\rightarrow 3! = 6$

UDAU $\rightarrow 3! = 6$

UDAYPRU $\rightarrow 1$

UDAYPUR $\rightarrow 1$

Total $= 1580$

8. Let $E = \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ be an ellipse if eccentricity of this ellipse is equal to the greatest value of the function $f(t) = \frac{-3}{4} + 2t - t^2$ & Length of latus rectum is 30

then find $a^2 + b^2 = ?$

(1) 496 (2) 250 (3) 376 (4) 175

Ans. **[1]**

Sol. $f(t) = \frac{-3}{4} + 2t - t^2$

$f(t)\Big|_{maximum} = \frac{1}{4} = e \Rightarrow e^2 = \frac{1}{16} \Rightarrow \frac{a^2 - b^2}{a^2} = \frac{1}{16}$. .....(1)

$\because \frac{2b^2}{a} = 30 \Rightarrow b^2 = 15a$ ....(2)

By (1) & (2)

$16(a^2 - 15a) = a^2 \Rightarrow 15a^2 - 16 \times 15a = 0$

$a = 16$

$b^2 = 240$

$a^2 + b^2 = 256 + 240$

$= 496$

9. If $2(\vec{a}\times\vec{c})+3(\vec{b}\times\vec{c})=0$

where $\vec{a}=2i-5j+5k$ & $\vec{b}=i-j+3k$

and $(\vec{a}-\vec{b})\cdot\vec{c}=-97$ find $|\vec{c}\times k|^2$

(1) 218 (2) 207 (3) 165 (4) 210

Ans. **[1]**

Sol. $2(\vec{a}\times\vec{c})+3(\vec{b}\times\vec{c})=0$

$\Rightarrow(2\vec{a}+3\vec{d})\times\vec{c}=0\Rightarrow\vec{c}=\lambda(2\vec{a}+3\vec{d})$

$\Rightarrow\vec{c}=\lambda(7i-13j+19k)$

Now $(\vec{a}-\vec{b})\cdot\vec{c}-=\lambda(7+52+38)+97\lambda=-97$

$\Rightarrow\lambda=-1$

Now $\vec{c}=-7i+13j-19k$

$\Rightarrow\vec{c}\times\vec{k}-=7j+13i\Rightarrow|\vec{c}\times\hat{k}|^2=7^2+13^2=218$

10. Let $f(x)$ be a differentiable function satisfying the equations : $\lim\limits_{t\to x}\frac{t^2f(x)-x^2f(t)}{x-t}=3$ and $f(1)=2$. Find the value of $2f(2)$.

(1) 20 (2) 23 (3) 25 (4) 27

Ans. **[2]**

Sol. $\lim\limits_{t\to x}\frac{2tf(x)-x^2f'(t)}{-1}=3$

$x^2f'(x)-2xf(x)=3$

$\frac{dy}{dx}-\frac{2y}{x}=\frac{3}{x^2}$

I.F. $=e^{-\int\frac{2}{x}dx}=e^{-2\log_e x}=1/x^2$

$y\cdot\frac{1}{x^2}=\int\frac{3}{x^4}dx$

$\frac{y}{x^2}=-\frac{1}{x^3}+c\Rightarrow y=cx^2-\frac{1}{x}=f(x)$

$f(1)=2=c-1\Rightarrow c=3$

$f(x)=3x^2-\frac{1}{x}$

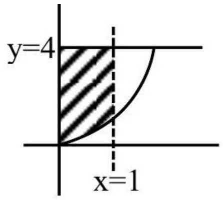
$f(2)=12-\frac{1}{2}\Rightarrow 2f(2)=23$

11. If $f(\alpha)$ is the area bounded in the first quadrant by $x=0, x=1, y=x^2, y=|\alpha x-5|-|1-\alpha x|+\alpha x^2$, then find $f(0)+f(1)$

(1) $\frac{11}{3}$ (2) $\frac{13}{3}$ (3) $\frac{17}{3}$ (4) $\frac{23}{3}$

Ans. **[4]**

Sol.

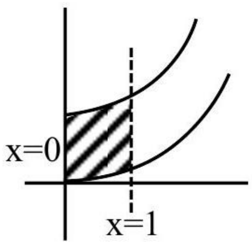


$$f(0)=\int_0^1\left(4-x^2\right)dx=\left(4x-\frac{x^3}{3}\right)_0^1=4-\frac{1}{3}=\frac{11}{3}$$

$f(1)=$ area bounded by $x=0, x=1, y=x^2$,

$y=|x-5|-|x-1|+x^2$

for $x\in(0,1)\ y=4+x^2$



$$f(1)=A\int_0^1\left(\left(4+x^2\right)-x^2\right)dx=4$$

$$f(0)+f(1)=\frac{11}{3}+4=\frac{23}{3}$$

12. If shortest distance between the lines

$$\frac{x+1}{\alpha}=\frac{y-2}{-2}=\frac{z-4}{-2\alpha}$$

and $\frac{x}{\alpha}=\frac{y-1}{1}=\frac{z-1}{\alpha}$ is $\sqrt{2}$

then find sum of all possible values of α

(1) –6 (2) 2 (3) –8 (4) 4

Ans. **[1]**

Sol.
$$\sqrt{2}=\frac{\begin{vmatrix}-1 & 1 & 3\\ \alpha & -2 & -2\alpha\\ \alpha & 1 & \alpha\end{vmatrix}}{\begin{vmatrix}\hat{i} & \hat{j} & k\\ \alpha & -2 & -2\alpha\\ \alpha & 1 & \alpha\end{vmatrix}}$$

$$=\frac{-1(0)-1\left(3\alpha^2\right)+3(3\alpha)}{\left|\hat{i}(0)-\hat{j}\left(3\alpha^2\right)+x(3\alpha)\right|}$$

$$\sqrt{2}=\frac{-3\alpha^2+9\alpha}{\sqrt{9\alpha^4+9\alpha^2}}$$

$2\left(9\alpha^4+9\alpha^2\right)=9\alpha^2(-\alpha+3)^2$

$2\left(\alpha^2+1\right)=\alpha^2-6\alpha+9$

$\alpha^2+6\alpha-7=0$

$(\alpha+7)(\alpha-1)=0 \Rightarrow \alpha=-7,1$

$\therefore$ Req. sum $=-6$

13. Let mirror image of parabola $x^2=4y$ in the line $x-y=1$ is $(y+a)^2=b(x-c)$ then value of $(a+b+c)$ is :

(1) 3 (2) 6 (3) 9 (4) 12

Ans. **[2]**

Sol. Parametric point P on $x^2=4y$ is $P\left(2t,t^2\right)$

$\therefore$ mirror image of P in $x-y=1$ is

$$Q\equiv\left(2t-\frac{2\cdot1\cdot\left(2t-t^2-1\right)}{2},t^2+\frac{2\cdot2(1)\cdot\left(2t-t^2-1\right)}{2}\right)$$

$Q\equiv\left(t^2+1,2t-1\right)\equiv(h,k)$

$\therefore$ locus of Q is $x=\frac{(y+1)^2}{4}+1$ which is the required parabola.

$\therefore (y+1)^2=4(x-1)$

$\therefore a=1,\ b=4, c=1$

$\therefore a+b+c=6$

14. If domain of $f(x)=\sin^{-1}\left(\frac{1}{x^2-2x-2}\right)$ is $(-\infty,\alpha]\cup[\beta,\gamma]\cup[\delta,\infty)$, then $(\alpha+\beta+\gamma+\delta)$ is

(1) 0 (2) 4 (3) 3 (4) 1

Ans. **[2]**

Sol. $-1\le\frac{2}{x^2-2x-2}\le1$

$\frac{1+x^2-2x-2}{x^2-2x-2}\ge0\Rightarrow\frac{(x-1)^2-2}{(x-1)^2-3}\ge0$

$\Rightarrow\frac{\left(x-1-\sqrt{2}\right)\left(x-1+\sqrt{2}\right)}{\left(x-1-\sqrt{3}\right)\left(x-1+\sqrt{3}\right)}\ge0$

$\Rightarrow x\in\left(-\infty,1-\sqrt{3}\right]\cup\left[1-\sqrt{2},1+\sqrt{2}\right]\cup\left[1+\sqrt{3},0\right)$ .....(1)

$1-\frac{1}{x^2-2x-2}\ge0\Rightarrow\frac{x^2-2x-3}{x^2-2x-2}$

$\Rightarrow\frac{(x+1)(x-3)}{\left(x-1+\sqrt{3}\right)\left(x-1-\sqrt{3}\right)}\ge0$

$x\in(-\infty,-1]\cup\left(1-\sqrt{3},\sqrt{3}+1\right)\cup[3,\infty)$ ....(2)

$(1)\cap(2)$

$\Rightarrow x\in\left(-\infty,-1\right]\cup\left[1-\sqrt{2},1+\sqrt{2}\right]\cup[3,\infty)$

$\therefore \alpha+\beta+\gamma+\delta=4$

15. Maximum value of n for which 40^n divides 60! is

(1) 10 (2) 14 (3) 20 (4) 27

Ans. **[2]**

Sol. $40^n = 2^{3n} \times 5^n$

$E_2(60!) = \left[\frac{60}{2}\right] + \left[\frac{60}{2^2}\right] + \left[\frac{60}{2^3}\right] + \left[\frac{60}{2^4}\right] + \left[\frac{60}{2^5}\right]$

$= 30 + 15 + 7 + 3 + 1 = 56$

$E_5(60!) = \left[\frac{60}{5}\right] + \left[\frac{60}{5^2}\right]$

$= 12 + 2 = 14$

$40^n = \left(2^3\right)^n \times 5^n = \left(2^3 \times 5\right)^n$

$60! = 2^{56} \times 5^{14} \ldots = 2^{14} \cdot \left(2^3 \cdot 5\right)^{14}$

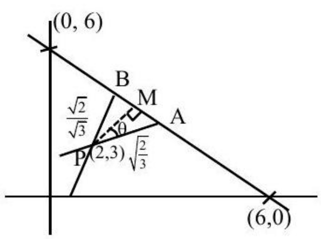
$\therefore$ Maximum value of n is 14 .

16. If two lines drawn from a point $P(2,3)$ intersecting the line $x + y = 6$ at a distance of $\sqrt{\frac{2}{3}}$, then angle between the lines is -

(1) $\frac{\pi}{6}$ (2) $\frac{\pi}{3}$ (3) $\frac{\pi}{12}$ (4) $\frac{5\pi}{12}$

Ans. **[2]**

Sol. $PM = \frac{1}{\sqrt{2}}$



In ΔAPM -

$\cos\theta = \frac{1}{\sqrt{2}} \cdot \sqrt{\frac{3}{2}} = \frac{\sqrt{3}}{2}$

$\theta = \frac{\pi}{6}$

$\therefore \angle APB = \frac{\pi}{3}$

17. If $P(h,k)$ is a variable point on $x^2 + y^2 = 4$ & $Q(2h+1, 3k+3)$ always lie on an ellipse if eccentricity of ellipse is e then $\frac{5}{e^2}$ is equal to:

(1) 9 (2) 5 (3) 3 (4) 6

Ans. **[1]**

Sol. Let $P \equiv (2\cos\theta, 2\sin\theta)$

$\therefore$ coordinates of $Q = (4\cos\theta + 1, 6\sin\theta + 3)$

$\therefore$ locus of Q is $\left(\frac{x-1}{4}\right)^2+\left(\frac{y-3}{6}\right)^2=1$

$\therefore e^2=1-\frac{16}{36}=\frac{5}{9} \Rightarrow \therefore \frac{5}{e^2}=9$

18. Let P and Q be any two 3×3 matrices
(where $P=\left[p_{ij}\right]_{3\times 3}, Q=\left[q_{ij}\right]_{3\times 3}$) such that $q_{ij}=2^{ij-1}p_{ij}$ where $|Q|=2^{10}$ then find $\left|adj(adj(P))\right|$

(1) 32 (2) 8 (3) 16 (4) 64

Ans. **[3]**

Sol. $\begin{vmatrix} 2p_{11} & 2^2p_{12} & 2^3p_{13} \\ 2^2p_{21} & 2^3p_{22} & 2^4p_{23} \\ 2^3p_{31} & 2^4p_{32} & 2^5p_{33} \end{vmatrix}=2^{10}$

$2^2\cdot 2\cdot 2^3\begin{vmatrix} p_{11} & p_{12} & p_{13} \\ 2p_{21} & 2p_{22} & 2p_{23} \\ 2^2p_{31} & 2^2p_{32} & 2^2p_{33} \end{vmatrix}=2^{10}$

$2^9\begin{vmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{vmatrix}=2^{10} \Rightarrow |P|=2$

$\left|adj(adj(P))\right|=\left|P\right|^{(n-1)^2}=\left|P\right|^4=2^4=16$

19. Let 4 integers a_1, a_2, a_3, a_4 are in A.P. with integral common difference ℓ such that $a_1+a_2+a_3+a_4=48$ & $a_1a_2a_3a_4+\ell^4=361$ then the greatest term in this A.P. is

(1) 24 (2) 23 (3) 27 (4) 21

Ans. **[3]**

Sol. a_1, a_2, a_3, a_4 as $a-3d, a-d, a+d, a+3d$ where $d=\frac{\ell}{2}$

$\because a_1+a_2+a_3+a_4=48 \Rightarrow 4a=48 \Rightarrow a=12$

& $a_1a_2a_3a_4+\ell^4=361 \Rightarrow \left(a^2-9d^2\right)\left(a^2-d^2\right)+16d^4=$
361

$\Rightarrow \left(144-9d^2\right)\left(144-d^2\right)+16d^4=361$

$\Rightarrow 25d^4-1440d^2+(144)^2=361$

$\left(5d^2-144\right)^2=19^2$

$\therefore 5d^2-144=19$ or -19

$d^2=\frac{163}{5}$ or $d^2=\frac{125}{5}=25$

$d=\sqrt{\frac{163}{5}}$ or $d=5$

$\therefore \ell=2\sqrt{\frac{163}{5}}$ or $\ell=10$

$\because$ common difference is an integer

$\therefore$ largest term $=12+15=27$

20. Let a function $f(x)$ satisfies $3f(x)+2f\left(\frac{m}{19x}\right)=5x$

Where $m=\sum_{i=1}^{9} i^2$. Find $f(5)+f(2)$.

(1) –1 (2) 0 (3) –5 (4) 6

Ans. **[2]**

Sol. $m=\frac{9\times 10\times 19}{6}=15\times 19$

$3f(x)+2f\left(\frac{15}{x}\right)=5x$

Replace x by $\frac{15}{x}$

$3f\left(\frac{15}{x}\right)+2f(x)=\frac{75}{x}$

$9f(x)-4f(x)=15x-\frac{150}{x}$

$5f(x)=15x-\frac{150}{x}$

$f(x)=3x-\frac{30}{x}$

$f(5)=15-\frac{30}{5}=9$

$f(2)=6-15=-9$

$f(5)+f(2)=0$

21. Evaluate : $\left(\frac{4}{7}+\frac{1}{3}\right)+\left(\left(\frac{4}{7}\right)^2+\left(\frac{4}{7}\right)\left(\frac{1}{3}\right)+\left(\frac{1}{3}\right)^2\right)+\left(\left(\frac{4}{7}\right)^3+\left(\frac{4}{7}\right)^2\cdot\frac{1}{3}+\left(\frac{4}{7}\right)\left(\frac{1}{3}\right)^2+\left(\frac{1}{3}\right)^3\right)+\ldots\infty$

(1) $\frac{5}{2}$ (2) 5 (3) $\frac{7}{2}$ (4) $\frac{8}{3}$

Ans. **[1]**

Sol. Let $a=\frac{4}{7},\ b=\frac{1}{3}$

Multiply N^r and D^r by $(a-b)=\frac{4}{7}-\frac{1}{3}=\frac{5}{21}$

$\frac{1}{a-b}\left[\left(a^2-b^2\right)+\left(a^3-b^3\right)+\left(a^4-b^4\right)+\ldots\infty\right]$

$\frac{1}{a-b}\left[\frac{a^2}{1-a}-\frac{b^2}{1-b}\right]=\frac{21}{5}\left[\frac{\frac{16}{49}}{1-\frac{4}{7}}-\frac{\frac{1}{9}}{1-\frac{1}{3}}\right]$

$=\frac{21}{5}\left[\frac{16}{21}-\frac{1}{6}\right]=\frac{21}{5}\left[\frac{96-21}{21.6}\right]$

$=\frac{75}{5.6}=\frac{15}{6}=\frac{5}{2}$

22. If dataset $A = \{1, 2, 3, \ldots, 19\}$

& dataset $B = \{ax_i + b; x_i \in A\}$

If mean of B is 30 & variance of B is 750 , then sum of possible values of b is

(1) 30 (2) 90 (3) 20 (4) 60

Ans. **[4]**

Sol. $A = \{1, 2, 3, \ldots, 19\}$

$\therefore$ mean of this data set $\bar{x} = 10$

& $\sigma^2 = \dfrac{19^2 - 1}{12} = 30$

now the dataset B is $ax_i + b$

$\therefore$ new mean $= a.10 + b = 30$ .....(1)

& new variance $= a^2.30 = 750$

$\Rightarrow a^2 = 25 \Rightarrow a = \pm 5$

by equation (1)

if $a = 5 \Rightarrow b = -20$

if $a = -5 \Rightarrow b = 80$

$\therefore$ sum of possible values of $b = 60$

23. Let

$$f(x) = \begin{cases} b^2 \sin\left(\dfrac{\pi}{2}\left[\dfrac{\pi}{2}(\sin x + \cos x) \cdot \cos x\right]\right) & ; x > 0 \\ \dfrac{\sin x - \dfrac{\sin 2x}{2}}{x^3} & ; x < 0 \\ a & ; x = 0 \end{cases}$$

be a continuous function at $x = 0$ then the value of $\left(a^2 + b^2\right)$ is equal to

(where [.] denotes greatest integer function)

(1) $\dfrac{1}{4}$ (2) $\dfrac{1}{2}$ (3) $\dfrac{3}{4}$ (4) $\dfrac{5}{4}$

Ans. **[3]**

Sol. $LHL = \lim\limits_{x \to 0} \dfrac{\sin x - \sin x \cos x}{x^3}$

$= \lim\limits_{x \to 0} \dfrac{\sin x}{x} \dfrac{(1 - \cos x)}{x^2} = \dfrac{1}{2}$

$f(0) = a$

$RHL = \lim\limits_{x \to 0^+} b^2 \sin\dfrac{\pi}{2}\left[\dfrac{\pi}{2}(\sin x + \cos x)\cos x\right] = b^2$

$b^2 = a = \dfrac{1}{2}$

$a^2 + b^2 = \dfrac{1}{4} + \dfrac{1}{2} = \dfrac{3}{4}$

24. Let $f(x)=\int\frac{(7x^{10}+9x^8)}{(1+x^2+2x^9)^2}dx$, and $f(1)=\frac{1}{4}$. Given that $A=\begin{bmatrix}0 & 0 & 1\\ 4 & f'(1) & 1\\ \alpha^2 & \frac{1}{4} & 1\end{bmatrix}$ and $B=\text{adj}(\text{adj}A), |B|=$ 81 . Find the value of α^2 (where $\alpha\in R$)

(1) 2 (2) 4 (3) 6 (4) 8

Ans. **[2]**

Sol. $f(x)=\frac{\int\left(\frac{7}{x^8}+\frac{9}{x^{10}}\right)}{\left(\frac{1}{x^9}+\frac{1}{x^7}+2\right)^2}dx$

Put $t=\frac{1}{x^9}+\frac{1}{x^7}+2\Rightarrow\frac{dt}{dx}=\frac{-9}{x^{10}}-\frac{7}{x^8}$

$f(x)=\int\frac{-dt}{t^2}=\frac{1}{t}+C$

$f(x)=\frac{1}{\frac{1}{x^9}+\frac{1}{x^7}+2}+C$

$=\frac{x^9}{1+x^2+2x^9}+C$

Given $f(1)=\frac{1}{4}=\frac{1}{4}+C\Rightarrow C=0$

$f(x)=\frac{x^9}{1+x^2+2x^9}$

$f'(x)=\frac{(1+x^2+2x^9)-9x^8-x^9(2x+18x^8)}{(1+x^2+2x^9)^2}$

$f'(x)=\frac{36-20}{16}=1$

$A=\begin{pmatrix}0 & 0 & 1\\ 4 & 1 & 1\\ \alpha^2 & \frac{1}{4} & 1\end{pmatrix}$

$|A|=|1-\alpha^2|=3$

$1-\alpha^2=3,-3\Rightarrow\alpha^2=-2,4$

Value of $\alpha^2=4$

$B=\text{adj}(\text{adj}A)$

$|B|=81=|A|^4\Rightarrow|A|=3$