

**CAREER POINT**

JEE Main Online Exam 2026

Memory Based
Questions & Solution
24th January 2026 | Morning

PHYSICS

1. Electric potential at a point is $V = Ar^3 + B$. Find charge enclosed in a sphere of radius 1 m, centered at $r = 0$

(1) $-4\epsilon_0 \text{ A}$

(2) $-8\epsilon_0 \text{ A}$

(3) $-12\epsilon_0 \text{ A}$

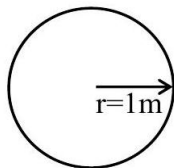
(4) $-16\epsilon_0 \text{ A}$

Ans. [3]

Sol. $E = -\frac{dv}{dr}$

$$E = -3Ar^2$$

Charge enclosed in 1 m radius is



Applying guass law

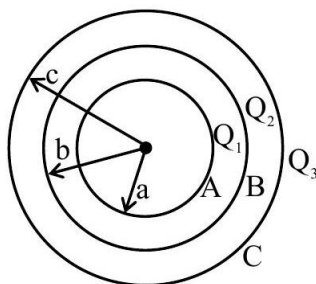
$$\oint \epsilon \cdot ds = \frac{q_{in}}{\epsilon_0}$$

$$E \cdot S = \frac{q_{in}}{\epsilon_0}$$

$$q_{in} = \epsilon_0 ES = -\epsilon_0 \cdot (3Ar^2) (4\pi r^2)$$

$$q_{in} \big|_{r=1 \text{ m}} = -12\epsilon_0 \text{ A}$$

2. Three uniformly charged concentric shells are kept as shown in the diagram. Charges on individual shells are as shown. Find the final potential on each shell :



$$(1) V_A = \frac{KQ_1}{a} + \frac{KQ_2}{b} + \frac{KQ_3}{c}$$

$$V_B = \frac{K(Q_1 + Q_2 + Q_3)}{c}$$

$$V_C = \frac{KQ_1}{b} + \frac{KQ_2}{b} + \frac{KQ_3}{c}$$

$$(3) V_A = \frac{K(Q_1 + Q_2 + Q_3)}{c}$$

$$V_B = \frac{KQ_1}{b} + \frac{KQ_2}{b} + \frac{KQ_3}{c}$$

$$V_C = \frac{KQ_1}{a} + \frac{KQ_2}{b} + \frac{KQ_3}{c}$$

$$(2) V_A = \frac{KQ_1}{b} + \frac{KQ_2}{b} + \frac{KQ_3}{c}$$

$$V_B = \frac{KQ_1}{a} + \frac{KQ_2}{b} + \frac{KQ_3}{c}$$

$$V_C = \frac{K(Q_1 + Q_2 + Q_3)}{c}$$

$$(4) V_A = \frac{KQ_1}{a} + \frac{KQ_2}{b} + \frac{KQ_3}{c}$$

$$V_B = \frac{KQ_1}{b} + \frac{KQ_2}{b} + \frac{KQ_3}{c}$$

$$V_C = \frac{K(Q_1 + Q_2 + Q_3)}{c}$$

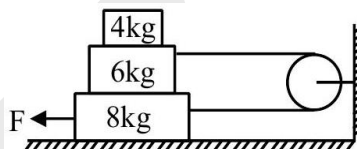
Ans. [4]

Sol. $V_A = \frac{KQ_1}{a} + \frac{KQ_2}{b} + \frac{KQ_3}{c}$

$$V_B = \frac{KQ_1}{b} + \frac{KQ_2}{b} + \frac{KQ_3}{c}$$

$$V_C = \frac{KQ_1}{c} + \frac{KQ_2}{c} + \frac{KQ_3}{c} = \frac{K(Q_1 + Q_2 + Q_3)}{c}$$

3. Figure shows three block with masses 8 kg, 6 kg and 4 kg. Friction coefficient between each surface is $\frac{1}{2}$. The maximum value of force 'F' such that 8 kg block moves with constant velocity will be :



(1) 210 N

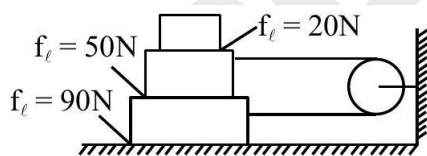
(2) 400 N

(3) 110 N

(4) 300 N

Ans. [1]

Sol. For 8 kg to move with constant velocity $F_{\text{net}} = 0$.



$$\therefore F = 90 + T + 50 \text{ (for 8 kg block)}$$

$$T = 20 + 50 \text{ (for 6 kg block)}$$

$$\therefore F = 210 \text{ N.}$$

4. **Statement-I:** Greater is the mass of nucleus, more will be its binding energy.

Statement - II : Nucleus with less $\frac{BE}{A}$ (Binding energy/nucleon) breaks into nucleus with higher $\frac{BE}{A}$.

Choose the correct option :

(1) Statement I is true & statement II is false

(2) Statement I is false & statement II is true

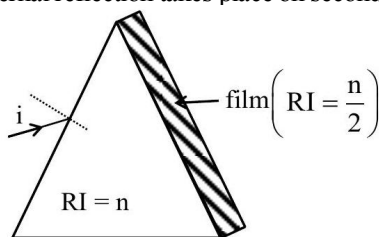
(3) Both are true

(4) Both are false

Ans. [3]

Sol. On increasing number of nucleon, BE increase but stability of nucleus depends on BE / A .

5. Light is incident at such an angle so that minimum deviation takes place. Now a film of refractive index ($RI = \frac{n}{2}$) is stick on other face such that total internal reflection takes place on second surface. Find angle of prism :



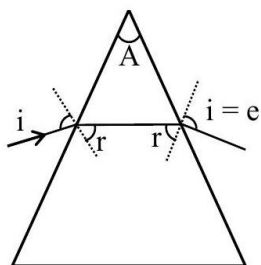
- (1) 60° (2) 50° (3) 90° (4) 30°

Ans.

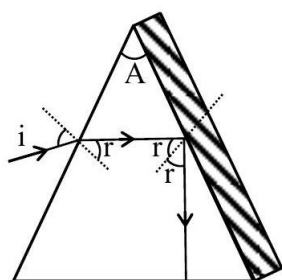
[1]

Sol.

$i = e$ & $r = A/2$ for minimum deviation



For TIR ; $r > \theta_c$



$$\sin r > \sin \theta_c$$

$$\sin r > \frac{n/2}{n}$$

$$\sin r > \frac{1}{2}$$

$$\sin \frac{A}{2} > \sin 30^\circ$$

$$\frac{A}{2} > 30^\circ$$

$$A > 60^\circ$$

6. There is a compound microscope of lenses having focal lengths 2 cm and 5 cm and tube length 10 cm . Find magnifying power in normal adjustment. If your answer is 5^α , find ' α ' :

Ans.

[2]

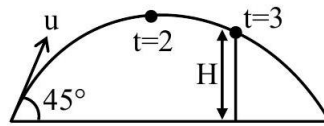
Sol.

$$f_o = 2 \text{ cm, } f_e = 5 \text{ cm}$$

$$\ell = 10 \text{ cm}$$

$$M = \frac{\ell}{f_o} \cdot \frac{D}{f_e} = 25$$

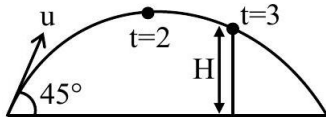
7. A projectile is projected with certain speed at an angle of 45° with horizontal as shown. At $t = 2$ s, projectile is at maximum height and at $t = 3$ s, it just touches a wall at a height H above horizontal. Find H in meters :



- (1) 20 m (2) 10 m (3) 15 m (4) 25 m

Ans. [3]

Sol. $T = \frac{2u_y}{g} = 4$



$$\Rightarrow u_y = \frac{40}{2} = 20 \text{ m/s}$$

$$u_x = 20 \text{ m/s}$$

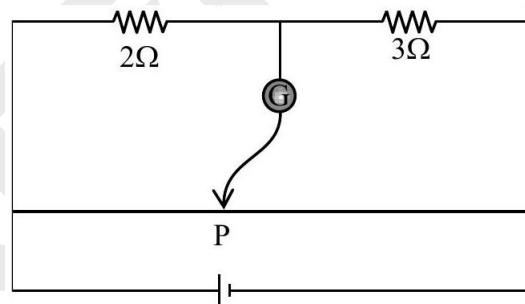
$$\Delta y = u_y \Delta t - \frac{1}{2} g (\Delta t)^2$$

$$\Rightarrow H = 20 \times 3 - 5 \times 9$$

$$= 60 - 45$$

$$= 15 \text{ m}$$

8. Figure shows a meter-bridge. Initially null point was achieved at point P as shown in the figure.



When an unknown resistance " R " is connected in parallel with 3Ω the null point was shifted by 22.5 cm. Then the value of unknown resistance is :

- (1) 2Ω (2) 3Ω (3) 2.5Ω (4) 5Ω

Ans. [1]

Sol. Initially, $\frac{2}{3} = \frac{40}{60}$

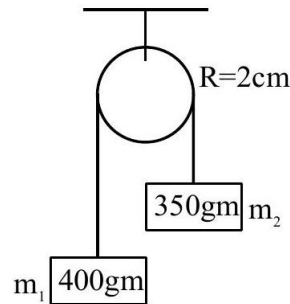
Now when ' R ' connected in parallel

$$\frac{2}{3R} = \frac{40 + 22.5}{60 - 22.5} = \frac{62.5}{37.5}$$

$$3 + R$$

$$\therefore R = 2\Omega$$

9. After release, the blocks moves 81 cm in 9 seconds. Find moment of inertia of the pulley :



- (1) $97 \times 10^{-4} \text{ Kg} - \text{m}^2$ (2) $100 \times 10^{-4} \text{ Kg} - \text{m}^2$ (3) $21 \times 10^{-4} \text{ Kg} - \text{m}^2$ (4) $87 \times 10^{-4} \text{ Kg} - \text{m}^2$

Ans. [1]

Sol.
$$a = \frac{(m_1 - m_2)}{m_1 + m_2 + \frac{I}{R^2}} \cdot g$$

$$S = ut + \frac{1}{2}at^2$$

$$\frac{81}{100} = \frac{1}{2} \left(\frac{m_1 - m_2}{m_1 + m_2 + \frac{I}{R^2}} \right) g \times (81)$$

$$500(m_1 - m_2) = (m_1 + m_2) + \frac{I}{R^2}$$

$$500 \left(\frac{50}{1000} \right) = \left(\frac{750}{1000} \right) + \frac{I}{R^2}$$

$$I = 97 \times 10^{-4} \text{ Kg} - \text{m}^2$$

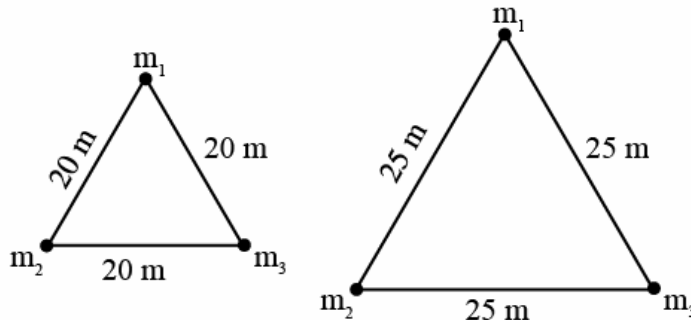
10. Following are two lists, list-I contains the types of electromagnetic waves and list-II contains theirs source. Match the entries from list-I to appropriate entries from list-II.

| | List-I | | List-II |
|-----|---------------|-----|--|
| (a) | x-rays | (p) | Hot bodies and molecules |
| (b) | Infrared rays | (q) | Oscillatory current in antennas |
| (c) | Microwaves | (r) | Magnetron |
| (d) | Radio waves | (s) | Fast moving electrons striking a metal plate |

- (1) (a) → (r), (b) → (q), (c) → (s), (d) → (q)
 (2) (a) → (p), (b) → (s), (c) → (r), (d) → (q)
 (3) (a) → (s), (b) → (p), (c) → (q), (d) → (r)
 (4) (a) → (s), (b) → (p), (c) → (r), (d) → (q)

Ans. [4]

11. Three masses $m_1 = 200 \text{ kg}$, $m_2 = 300 \text{ kg}$ and $m_3 = 400 \text{ kg}$ are kept at the vertices of an equilateral triangle of side 20 m . If the masses are shifted to new configuration such that they are at the vertices of an equilateral triangle of 25 m now. Find the work done in this process :



- (1) $1.735 \times 10^{-7} \text{ J}$ (2) $17.35 \times 10^{-7} \text{ J}$ (3) $173.5 \times 10^{-7} \text{ J}$ (4) $1735 \times 10^{-7} \text{ J}$

Ans.

[1]

Sol.

Work done by external agent :

$$W_{\text{ext}} = \Delta U$$

$$U_i = \frac{Gm_1 m_2}{r_i} + \frac{Gm_2 m_3}{r_i} + \frac{Gm_1 m_3}{r_i} : r_i = 20 \text{ m}$$

$$U_f = \frac{Gm_1 m_2}{r_f} + \frac{Gm_2 m_3}{r_f} + \frac{Gm_1 m_3}{r_f} : r_f = 25 \text{ m}$$

$$U_i = \frac{-6.67 \times 10^{-11}}{20} [200 \times 300 + 300 \times 400 + 200 \times 400]$$

$$= \frac{-6.67 \times 10^{-11}}{20} \times 26 \times 10^4 = -86.71 \times 10^{-8} \text{ J}$$

$$U_f = \frac{-6.67 \times 10^{-11}}{0.25} [200 \times 300 + 300 \times 400 + 200 \times 400]$$

$$= \frac{-6.67 \times 10^{-11}}{0.25} \times 26 \times 10^4 = -693.68 \times 10^{-9}$$

$$= -69.36 \times 10^{-8} \text{ J} \quad W = +\Delta U = 17.35 \times 10^{-8}$$

$$= 1.735 \times 10^{-7} \text{ J}$$

12. A cylindrical body of mass m and cross section A is floating in a liquid of density ρ_L such that its axis is vertical. If body is displaced by a small displacement 'x' vertically, find the time period of oscillation of the body:

- (1) $2\pi \sqrt{\frac{m}{\rho_L A g}}$ (2) $3\pi \sqrt{\frac{m}{\rho_L A g}}$ (3) $4\pi \sqrt{\frac{m}{\rho_L A g}}$ (4) $5\pi \sqrt{\frac{m}{\rho_L A g}}$

Ans.

[1]

Sol.

$$\rho_L A \times hg = mg$$

After displacing by x ,

$$F = \rho_L A (h + x)g - mg$$

$$F = \rho_L A hg + \rho_L A xg - mg$$

$$F = \rho_L A xg$$

$$a = \left(\frac{\rho_L A g}{m} \right) x$$

comparing,

$$a = \omega^2 x$$

$$\omega = \sqrt{\frac{\rho_L A g}{m}}$$

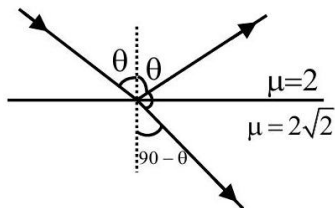
$$T = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{m}{\rho_L A g}}$$

13. Light wave are incident from a medium of refractive index 2 making an angle θ with normal on to a medium of refractive index $2\sqrt{3}$. What should be the value of θ for which reflected wave and refracted wave will be perpendicular to each other.

- (1) 60° (2) 30° (3) 53° (4) 45°

Ans. [1]

Sol.



$$2\sin\theta = 2\sqrt{3}\sin(90 - \theta)$$

$$\tan\theta = \sqrt{3}$$

$$\theta = 60^\circ$$

14. A brass rod is fixed rigidly at two ends at 27°C . If it is cooled to temperature -43°C , tension in rod becomes T_0 . Find temperature (in $^\circ\text{C}$) at which tension will be $1.4 T_0$:

Ans. -71°C

Sol. Thermal stress causes tension

$$T = \alpha y A \Delta T$$

$$-43^\circ\text{C } T_0 = \alpha y A (43 + 27) \quad \dots (i)$$

$$-t^\circ\text{C } T_0 = \alpha y A (t + 27) \quad \dots (ii)$$

$$(ii)/(i)$$

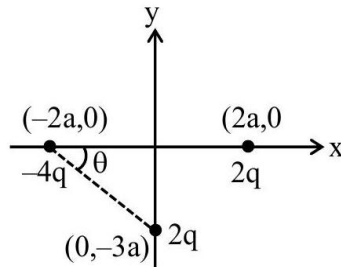
$$1.4 = \frac{t + 27}{70}$$

$$t + 27 = 98$$

$$t = 71^\circ$$

$$\therefore \text{temp } (-71^\circ\text{C})$$

15. In the following configuration of charges. Find the net dipole moment of the system :



(1) $\sqrt{180}qa$

(2) $\sqrt{150}qa$

(3) $\sqrt{200}qa$

(4) $\sqrt{140}qa$

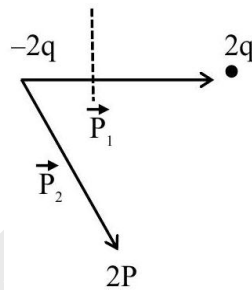
Ans. [1]

Sol. $\vec{P}_1 = (2q)(4a)\hat{i} = 8qa\hat{i}$

$$\vec{P}_2 = (2q)(\sqrt{13}a)(\cos\theta\hat{i} - \sin\theta\hat{j})$$

$$= (3q)(\sqrt{3}a)(\cos\theta\hat{i} - \sin\theta\hat{j})$$

$$= (3q)(\sqrt{3}a)\left(\frac{2}{\sqrt{13}}\hat{i} - \frac{3}{\sqrt{3}}\hat{j}\right)$$



$$= 2qa(2\hat{i} - 3\hat{j}) \quad \cos\theta = \frac{2}{\sqrt{13}}$$

$$= 4qa\hat{i} - 6qa\hat{j} \quad \sin\theta = \frac{3}{\sqrt{13}}$$

$$\vec{P}_{\text{net}} = \vec{P}_1 + \vec{P}_2 =$$

$$= 12qa\hat{i} - 6qa\hat{j}$$

$$|\vec{P}_{\text{net}}| = \sqrt{180}qa$$

16. A spring of spring constant $K = 15 \text{ N/m}$ is cut into two parts of ratio of length 3 : 1. Find the spring constant of spring with smaller length (in N/m).

(1) 60

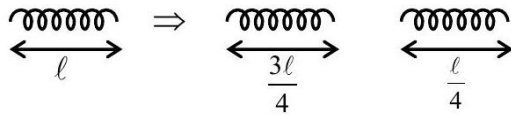
(2) 40

(3) 30

(4) 70

Ans. [1]

Sol.



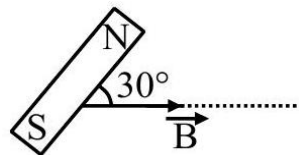
$$Kl = \text{constant}$$

$$Kl = K' \left(\frac{l}{4} \right)$$

$$K' = 4K$$

$$K' = 60 \text{ N/m}$$

- 17.** A bar magnet is kept such that it is making an angle of 30° with the magnetic field. The torque acting on the magnet is 0.016 N-m . Find the amount of work done by external agent in rotating the magnet from most stable position to most unstable position.



(1) 0.064 J

(2) 0.020 J

(3) 0.034 J

(4) 0.055 J

Ans. [1]

Sol. $\tau = \mu B \sin \theta \Rightarrow 0.016 = \mu \times B \times \frac{1}{2}$

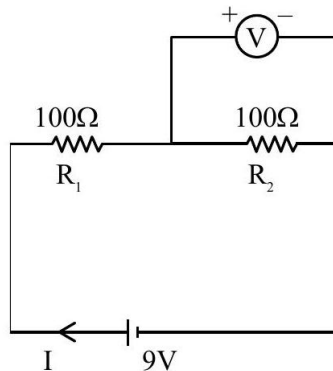
$$\Rightarrow \mu = \frac{0.032}{B}$$

$$W_{\text{ext}} = U_f - U_i = \mu B - (\mu B) = 2\mu B$$

$$= 2 \times \frac{0.032}{B} \times B$$

$$= 0.064 \text{ J}$$

- 18.** Two resistors of resistances $R_1 = 100\Omega$ and $R_2 = 100\Omega$ are connected in series. A voltmeter of resistance 400Ω is connected in parallel to one of the resistance. Find the reading of voltmeter. The emf of battery is 9 V .



(1) 3 V

(2) 4 V

(3) 2 V

(4) 5 V

Ans. [2]

Sol. Current in circuit.

$$I = \frac{E}{R_{eq}}$$

$$R_{eq} = 100 + \frac{400 \times 100}{500} = 180\Omega$$

$$\therefore I = \frac{9}{180} = \frac{1}{20} \text{ A}$$

$$\text{Reading of voltmeter} = V = I \times 80 = \frac{1}{20} \times 80 = 4 \text{ V}$$

19. An ideal gas in a closed rigid container is at 50°C and pressure 3.23 kPa . If temperature is doubled, find new pressure in Pa :

(1) 3730 Pa

(2) 3230 Pa

(3) 6460 Pa

(4) 6430 Pa

Ans. [1]

Sol. Closed rigid container

$V = \text{constant}$

$P \propto T$

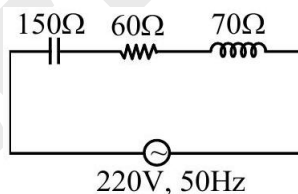
$$T_1 = 50^\circ\text{C} = 323 \text{ K}$$

$$T_f = 2 \times 50^\circ\text{C} = 100^\circ\text{C} = 373 \text{ K}$$

$$\frac{P_1}{P_2} = \frac{T_1}{T_2} \Rightarrow \frac{3.23}{P_2} = \frac{323}{373}$$

$$\therefore P_2 = 3730 \text{ Pa}$$

20. Figure shows a circuit consisting capacitor, inductor and a resistor connected in series with an AC source. Find the power factor of the circuit.



(1) 0.2

(2) 0.4

(3) 0.6

(4) 0.8

Ans. (3)

Sol. Power factor $= \frac{R}{Z}$

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$= \sqrt{60^2 + (150 - 70)^2} = 100\Omega$$

$$\therefore \text{Power factor} = \frac{60}{100} = 0.6$$

21. In a H-like ion, ratio of speed of electron in two orbit is $3 : 2$, then ratio of energies in these orbits should be :

(1) $\frac{3}{5}$

(2) $\frac{9}{4}$

(3) $\frac{1}{4}$

(4) $\frac{3}{4}$

Ans. [2]

Sol. $v = v \cdot \frac{Z}{n}$

$$\frac{v_1}{v_2} = \frac{z_1}{z_2} \cdot \frac{n_2}{n_1} = \frac{3}{2}$$

$$E = -E_0 \frac{z^2}{n^2}$$

$$\frac{E_1}{E_2} = \frac{\left(\frac{z_1}{n_1}\right)^2}{\left(\frac{z_2}{n_2}\right)^2} = \frac{9}{4}$$

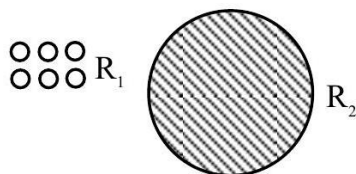
- 22.** Terminal velocity of drop of radius 1 cm is 10 cm/sec. 64 such balls are combined to make a large drop. Find terminal velocity of this larger drop. :

(1) 160 cm/sec (2) 140 cm/sec (3) 180 cm/sec (4) 150 cm/sec

Ans. [1]

Sol. $V_T = \frac{2r^2 g}{9\eta} [\sigma - \rho]$

$$V_T \propto r^2$$



64 drop

$$64 \left(\frac{4}{3} \pi R_1^3 \right) = \frac{4}{3} \pi R_2^3$$

$$R_2 = 4R_1$$

$$\frac{(V_T)_1}{(V_T)_2} = \left(\frac{R_1}{R_2} \right)^2 = \left(\frac{1}{4} \right)^2$$

$$\frac{10}{(V_T)_2} = \frac{1}{16}$$

$$(V_T)_2 = 160 \text{ cm/sec}$$

- 23.** Column-I gives physical quantities and Column-II represent their dimensions. Choose the option representing correct matching.

| Column-I | | Column-II | |
|----------|--------------------------|-----------|----------------------|
| (I) | Magnetic field intensity | (P) | $MLT^{-2} A^{-2}$ |
| (II) | Magnetic flux | (Q) | $ML^2 T^{-2} A^{-2}$ |
| (III) | Magnetic permeability | (R) | $ML^2 T^{-2} A^{-1}$ |
| (IV) | Magnetic inductance | (S) | $MT^{-2} A^{-1}$ |

(1) I-S, II-R, III-P, IV-Q

(2) I-Q, II-R, III-P, IV-S

(3) I-R, II-S, III-P, IV-Q

(4) I-S, II-P, III-R, IV-Q

Ans. [1]

Sol. Magnetic field intensity, $B = [MT^{-2} A^{-1}] - S$

Magnetic Flux, $\phi = [ML^2 T^{-2} A^{-1}] - R$

Magnetic Permeability, $\mu = [MLT^{-2} A^{-2}] - P$

Magnetic inductance, $L = [ML^2 T^{-2} A^{-1}] - Q$

- 24.** Density of water at $4^\circ C$ is 1000 kg/m^3 and at $20^\circ C$ it is 998 kg/m^3 . If 4 kg of water is heated from $4^\circ C$ to $20^\circ C$, the change in internal energy of water is : (Given : specific heat capacity of water = 4200 J/kg).

(1) 268799.2 J (2) 268800.8 J (3) 268800.0 J (4) 267765.2 J

Ans. [1]

Sol. $Q = m\Delta T = 4 \times 4200 \times 16 \text{ J} = 268800 \text{ J}$

$$W = P\Delta V$$

$$\Delta V = \left(\frac{m}{\rho_f} - \frac{m}{\rho_i} \right) = 4 \left[\frac{1}{998} - \frac{1}{1000} \right]$$

$$P = 10^5 \text{ Pa .}$$

$$\therefore W = 10^5 \times 4 \times \left[\frac{1}{998} - \frac{1}{1000} \right] = \frac{8 \times 10^5}{10^3 \times 998} \approx 0.8 \text{ J}$$

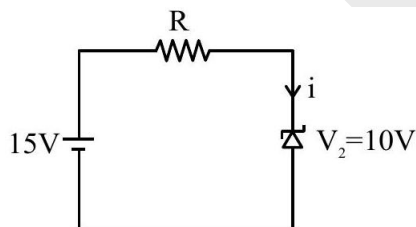
$$\Delta U = Q - W = 268799.2 \text{ J}$$

- 25.** A zener diode of breakdown voltage 10 V is connected to an external voltage of 15 V and a resistance R in series. If power of zener diode is 0.4 W. Find value of unknown resistance R :

(1) 125Ω (2) 105Ω (3) 130Ω (4) 115Ω

Ans. [1]

Sol.



$$P_D = 0.4 \text{ W} = 10i$$

$$i = 0.04 \text{ A}$$

$$R = \frac{15 - 10}{0.04} = \frac{5}{0.04} = 125\Omega$$

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CHEMISTRY

1. **Statement-I** : $K > Mg > Al > B$ metallic character order.

Statement-II : Ionic radius of any element is less than its atomic radius.

In the light of above statements, choose the most appropriate answer from the options given below :

- (1) Both statements are true
- (2) Statement I is false but statement II is true.
- (3) Both statements are False.
- (4) Statement I is true but statement II is false.

Ans. [4]

Sol.

Statement-I : Correct

$EN \uparrow$ metallic character \downarrow

Metallic character : $K > Mg > Al > B$

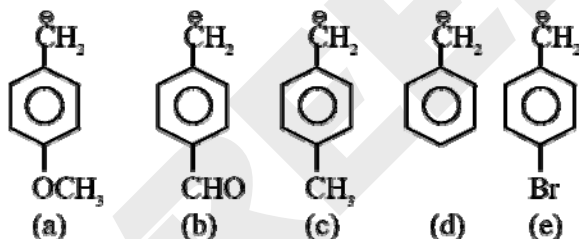
$EN : 0.8 < 1.2 < 1.5 < 2.0$

Statement-II : Incorrect

Ionic size $M^+ < M < M^-$

Anionic radius $>$ Atomic radius.

2. The correct order of stability of given carbanions is



- (1) $a > b > c > d > e$
- (2) $b > e > d > a > c$
- (3) $a > c > d > e > b$
- (4) $b > e > d > c > a$

Ans. [4]

Sol.

Electron withdrawing group increase the stability of carbanions.

3. **Statement-I** : $[\text{Co}(\text{CO}_3)_3]^{3-}$ has magnetic moment of 4.9 BM & hybridization is $sp^3 d^2$

Statement-II : $[\text{Ni}(\text{CN})_4]^{2-}$, $[\text{Cr}(\text{H}_2\text{O})_6]^{2+}$ and $[\text{MnF}_6]^{4-}$ have square planar, octahedral and octahedral geometry respectively and $dsp^2, sp^3 d^2, d^2 sp^3$ hybridization respectively and $\mu = 0, 4.9 \text{ BM}, 5.9 \text{ BM}$ respectively.

- (1) Both statements are correct
- (2) Statement-I is correct & statement-II is incorrect
- (3) Statement-I is incorrect & statement-II is correct
- (4) Both statements are incorrect.

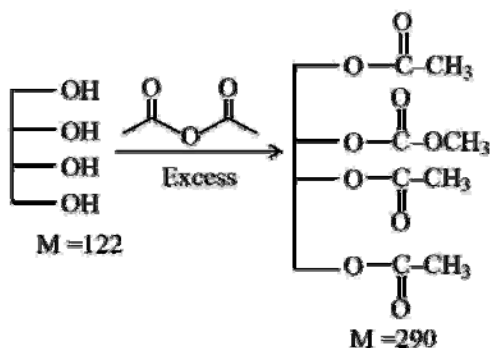
Ans. [4]

Sol. $[\text{Co}(\text{CO}_3)_3]^{3-}$ is d^2sp^3 hybridized $[\text{MnF}_6]^{4-}$ is $sp^3 d^2$ hybridized.

4. Hydroxy compound (A) with molecular mass = 122 react with excess of acetic anhydride and gives compound (X) with molecular mass = 290, then find the no. of hydroxy groups in given compound (A).

Ans. [42]

Sol.



$$\text{No. of OH groups} = \frac{290 - 122}{4} = 42$$

5. 4 kg of water is heated from 4°C to 20°C at constant pressure 10^5 Pa so that density changes from 1000 kg/m^3 to 998 kg/m^3 . Then find ΔU (in Joules) given C_s of $\text{H}_2\text{O} = 4.2 \text{ Joule/gm.K}$:

(1) 268799.2 Joule (2) 368900 Joule (3) 168400 Joule (4) 578876.8 Joule

Ans. [1]

Sol. $q = mc_s \Delta T$

$$q = 4000 \times 4.2 (20 - 4)$$

$$q = 268800$$

$$w = -P_{\text{ext}} (V_2 - V_1)$$

$$w = -10^5 \left(\frac{4}{998} - \frac{4}{1000} \right)$$

$$w = -0.8 \text{ Joule}$$

$$\Delta U = q + w$$

$$\Delta U = 268800 - 0.8$$

$$\Delta U = 268799.2 \text{ Joule}$$

6. For a reaction at 300 K, on addition of catalyst, activation energy of reaction lowered by 10 kJ. Then

calculate the value of $\log \frac{K_{\text{catalysed}}}{K_{\text{uncatalysed}}}$

(1) 1.74 (2) 0.174 (3) 17.4 (4) 3.48

Ans. [1]

Sol. $\frac{K_{\text{catalyst}}}{K_{\text{uncatalyst}}} = e^{\frac{\Delta E_a}{RT}}$

$$\ln \frac{K_{\text{catalyst}}}{K_{\text{uncatalyst}}} = \frac{\Delta E_a}{RT}$$

$$\log \frac{K_{\text{catalyst}}}{K_{\text{uncatalyst}}} = \frac{\Delta E_a}{2.303RT}$$

$$= \frac{10 \times 1000}{2.303 \times 8.314 \times 300}$$

$$\log \frac{K_{\text{catalyst}}}{K_{\text{uncatalyst}}} = 1.74$$

7.

| List-I (Isothermal Process) | | List-I (work done) ($V_f > V_i$) | |
|--------------------------------|--------------------------|---------------------------------------|-----------------------------------|
| P. | Reversible expansion | 1. | $w = 0$ |
| Q. | Free expansion | 2. | $w = -nRT \ln \frac{V_f}{V_i}$ |
| R. | Irreversible expansion | 3. | $w = -P_{\text{ext}} (V_f - V_i)$ |
| S. | Irreversible Compression | 4. | $w = -P_{\text{ext}} (V_i - V_f)$ |

Select the correct match

| P | Q | R | S |
|-------|---|---|---|
| (A) 4 | 3 | 2 | 1 |
| (B) 2 | 1 | 3 | 4 |
| (C) 1 | 2 | 3 | 4 |
| (D) 3 | 4 | 1 | 2 |

Ans. [2]

Sol. Theoretical

8. W gm of non-volatile electrolyte solute is added in 100 ml pure water ($P^\circ = 640$ mm Hg) showing vapour pressure of solution 600 mm Hg. This solution have b.p. of 375 K.

Given K_b of $H_2O = 0.52 \frac{K \cdot kg}{mol}$,

Molar mass of solute = M

Select the correct option about mole fraction of solute (X_{solute}).

(1) $\frac{1.3}{8} \left(\frac{W}{M} \right)$ (2) $\frac{8}{1.3} \left(\frac{W}{M} \right)$ (3) $\frac{2.6}{16} \left(\frac{M}{W} \right)$ (4) $\frac{1.3}{8} \left(\frac{M}{W} \right)$

Ans. [1]

Sol. $\Delta T_b = i \times K_b \times m$

$$2 = i \times 0.52 \times \frac{\frac{W}{M}}{\frac{100}{1000}}$$

$$i \times \frac{W}{M} = 2 \times \frac{100}{1000} \times \frac{1}{.52}$$

$$i = \frac{1}{2.6} \times \frac{M}{W}$$

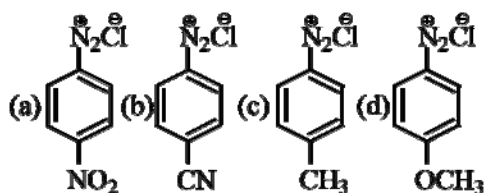
$$RLVP = \frac{P^\circ - P_s}{P^\circ} = i \times X_{\text{solute}}$$

$$\frac{640 - 600}{640} = i \times X_{\text{solute}}$$

$$\frac{1}{16} = \frac{1}{2.6} \times \frac{M}{W} \times X_{\text{solute}}$$

$$X_{\text{solute}} = \frac{1.3}{8} \times \frac{W}{M}$$

9. Correct order of stability is :



(1) $a > b > c > d$

(2) $d > c > b > a$

(3) $b > a > c > d$

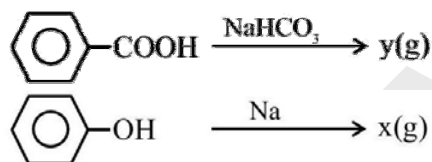
(4) $d > b > c > a$

Ans. [2]

Sol. +M group or +I group increases stability (i.e. $-\text{OCH}_3 - \text{CH}_3$)

– M decreases stability (i.e. $-\text{NO}_2$ and $-\text{CN}$)

10.



Sum of molar mass of gas (x) & (y) is

(1) 44

(2) 88

(3) 46

(4) 160

Ans. [3]

Sol. $x = \text{H}_2$ (gas), $y = \text{CO}_2$ (gas)

Sum of molar mass = $2 + 44 = 46$

11. **Statement-I** : Among V_2O_5 , $[\text{TiF}_6]^{3-}$, $[\text{Fe}(\text{CN})_6]^{3-}$, $[\text{CoF}_6]^{3-}$ paramagnetic species are three in number.

Statement-II : Increasing number of unpaired electrons in the following.



(1) Both statements are correct

(2) Statement-I is correct ; statement-II is incorrect

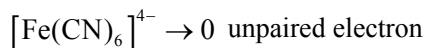
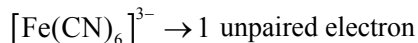
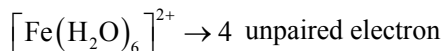
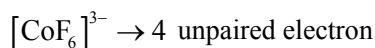
(3) Statement-I is incorrect statement-II is correct

(4) Both statements are incorrect

Ans. [1]

Sol. $[\text{TiF}_6]^{3-} \rightarrow 1$ unpaired electron

$[\text{Fe}(\text{CN})_6]^{3-} \rightarrow 1$ unpaired electron



12.

| | List-I Species | List-II Hybridization | List-III Shape |
|-----|-------------------|---------------------------|-------------------|
| (A) | IF_3 | sp^3 | T-shape |
| (B) | IF_7 | $\text{sp}^3 \text{ d}^3$ | P.B.P |
| (C) | IF_5 | $\text{sp}^3 \text{ d}^2$ | square pyramidal |
| (D) | ClO_4^- | $\text{sp}^2 \text{ d}$ | square planar |

Select the correct match

(1) A, B, C

(2) A, B, C, D

(3) B, C, D

(4) A, B, D

Ans. [1]

Sol. $\text{ClO}_4^- \rightarrow \text{sp}^3 \rightarrow$ tetrahedral, so (D) is incorrect, all others are correct.

13. Two solutes, 0.3 gm of A (Mw = 60gm / mol) & 0.9 gm of B (Mw = 180gm / mol) are dissolved in 100 ml solution. Find osmotic pressure of solution at 300 K (in atm) ($R = 0.082 \text{ atm} \cdot \text{L} / \text{mol} \cdot \text{K}$)

(1) 1.23

(2) 2.46

(3) 4.92

(4) 3.69

Ans. [2]

Sol. $\pi = (C_1 + C_2)$

$$= \left(\frac{0.3 \times 1000}{60 \times 100} + \frac{0.9 \times 1000}{180 \times 100} \right) \times 0.082 \times 300$$

$$= 2.46 \text{ atm}$$

14. Select correct statements(s)

(A) NF_3 has more dipole moment than NH_3

(B) O_2^{2-} and F_2 both have same bond order

(C) In O_3 central oxygen atom has -1 formal charge

(D) In NO_2 all the atoms follow octet rule, so it is stable.

(E) BeH_2 is planar

(1) B, C

(2) A, B, C

(3) C, D, E

(4) B, E

Ans. [4]

Sol. (1) dipole moment $\text{NF}_3 < \text{NH}_3$

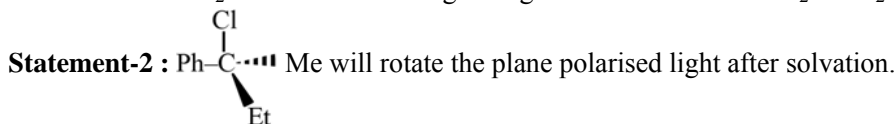
(2) O_2^{2-} , F_2 both have $B \cdot O = 1$

(3) In O_3 central oxygen atom has +1 formal charge

(4) In NO_2 , octet of 'N' atom is not complete

(5) BeH_2 is linear, so planar

15. **Statement-1 :** $\text{CH}_2 = \text{CH} - \text{Cl}$ is having stronger C – Cl bond then $\text{CH}_2 - \text{CH}_2 - \text{Cl}$.

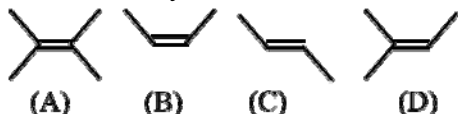


- (1) Both statements-I and II are correct
 (2) Both statements-I and II are incorrect
 (3) Statement-I is correct and statement-II is incorrect
 (4) Statement-I is incorrect and statement-II is correct.

Ans. [1]

Sol. Theory based

16. Correct stability order of alkene :

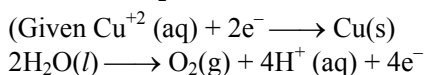


- (1) $A > D > C > B$ (2) $D > A > B > C$ (3) $A > D > B > C$ (4) $B > C > D > A$

Ans. [1]

Sol. Hyperconjugation (+H) and inductive group (+I) increases the stability of alkenes.

17. Electrolysis of aqueous solution of CuSO_4 is carried out, where 300 mg of copper is deposited (atomic mass of $\text{Cu} = 63.54$). After this 600 milli amp. current is further passed for 28 minutes. Calculate total volume of O_2 released (in ml).



Ans. [111]

Sol. Eq of $\text{Cu} = \text{Eq of } \text{O}_2$

$$\frac{300 \times 10^{-3} \times 2}{63.54} = n_{\text{O}_2} \times 4$$

$$2.36 \times 10^{-3} = n_{\text{O}_2}$$

When current is further passed

$$n_{\text{O}_2} \times 4 = \frac{600 \times 28 \times 60}{96500 \times 1000}$$

$$n_{\text{O}_2} = 2.611 \times 10^{-3}$$

Total O_2 released

$$= [10^{-3} \times (2.36 + 2.611)] \times 22400 \text{ ml}$$

$$= 111.35 \text{ ml}$$

18. Salt (X) is soluble in water.
 Salt (Y) is sparingly soluble in water.
 Salt (Z) is soluble only in hot water.
 X, Y, Z respectively are.

- (1) $\text{AgCl}, \text{Hg}_2\text{Cl}_2, \text{PbCl}_2$ (2) $\text{AlCl}_3, \text{AgCl}, \text{PbCl}_2$
 (3) $\text{BaCl}_2, \text{PbCl}_2, \text{Hg}_2\text{Cl}_2$ (4) $\text{MgCl}_2, \text{Hg}_2\text{Cl}_2, \text{CaCl}_2$

Ans. [2]

Sol. Theory based.

19. Match the List-I and List-II

| | |
|-----|----------------------|
| (1) | (I) Vinyl chloride |
| (2) | (II) Allyl chloride |
| (3) | (III) Aryl chloride |
| (4) | (IV) Benzyl chloride |

- (1) A → I, B → II, C → III, D → IV
 (2) A → I, B → II, C → IV, D → III
 (3) A → III, B → II, C → I, D → IV
 (4) A → III, B → II, C → IV, D → II

Ans. [2]

Sol. Common Names

20. Line corresponding to lyman series are $L_1, L_2, L_3, L_4, \dots$, among these L_1 line corresponds to lowest energy. Similarly lines corresponding to balmer series are $B_1, B_2, B_3, B_4, \dots$, among these B_1 line corresponds to lowest energy

ΔE_L = Energy of 1st line of lyman series

ΔE_B = Energy of 1st line of balmer series

If $\Delta E_L = x \cdot \Delta E_B$

Calculate $(x \times 10^{-1})$

Ans. [54]

Sol.
$$\Delta E_L = 13.6 \times Z^2 \left(\frac{1}{1^2} - \frac{1}{2^2} \right) = 13.6 Z^2 \times \frac{3}{4}$$

$$\Delta E_B = 13.6 \times Z^2 \times \left(\frac{1}{2^2} - \frac{1}{3^2} \right) = 13.6 \times Z^2 \times \frac{5}{4 \times 9}$$

$$\frac{\Delta E_L}{\Delta E_B} = \frac{3}{5} \times 9 = \frac{27}{5} = x$$

$$= \left(\frac{27}{5} \times 10 \right) \times 10^{-1}$$

21. Select correct statements.

(I) Hybridisation of ClO_4^- is dsp^2

(II) $[\text{Ni}(\text{CN})_4]^{2-}$ is tetrahedral

(III) $[\text{Co}(\text{H}_2\text{O})_6]^{2+}$ has $\text{sp}^3 \text{d}^2$ hybridisation

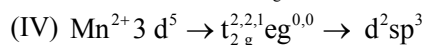
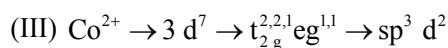
(IV) $[\text{Mn}(\text{CN})_6]^{4-}$ has $\text{sp}^3 \text{d}^2$ hybridisation

(1) II and III and (2) III only (3) II, III and IV only (4) I, II, III and IV

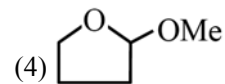
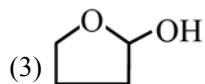
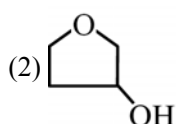
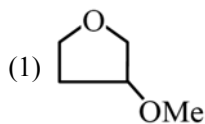
Ans. [2]

Sol. (I) $\text{ClO}_4^- \rightarrow \text{sp}^3$

(II) $\text{Ni}^{2+} \rightarrow 3 \text{d}^8 \rightarrow \text{dsp}^2$ (square planar)

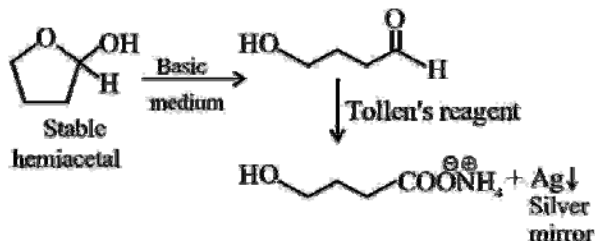


22. Which of the following gives positive tollen's test ?



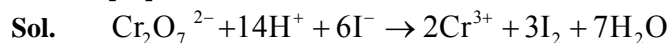
Ans. [3]

Sol. In basic medium cyclic hemiacetal isomers to open hydroxyl aldehyde compound which easily gives positive tollen's test.

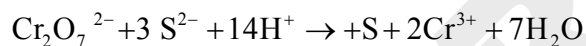


23. x & y are the number of moles of electrons involved respectively during oxidation of I^- to I_2 & S^{2-} to S by acidified $\text{K}_2\text{Cr}_2\text{O}_7$. The value of $x + y$ is?

Ans. [12]



no. of moles e^- involved = $x = 6$



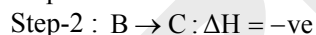
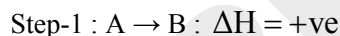
No. of moles e^- involved = $y = 6$

Sum of $x + y = 6 + 6 = 12$

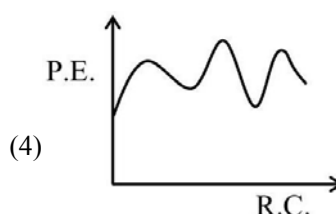
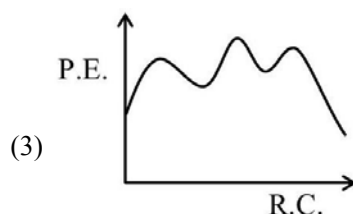
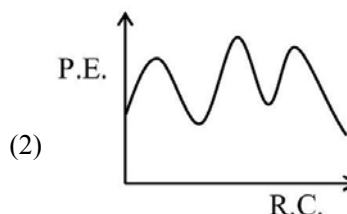
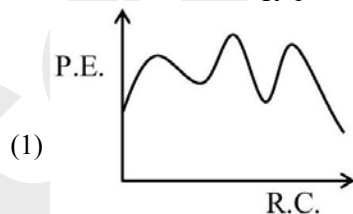
24. For a chemical reaction :



Mechanism is



Select the correct energy plot





Ans. [1]

Sol. $\Delta H = E_{\text{Product}} - E_{\text{Reactant}}$

25. 0.5 gm of unknown organic compound undergo Duma's method for estimation of nitrogen. Percentage of nitrogen gas collected over water at $P = 715$ mm and 27°C has volume = 70 ml. Calculate % N in the unknown organic compound. (aq. Tension = 15 mm)

Ans. [14.65]

Sol. $P_{\text{N}_2} = (715 - 15) \text{ mm} = \frac{700}{760} \text{ atm}$

$$V_{\text{N}_2} = 70 \text{ ml} = \frac{70}{1000} \text{ l}$$

$$n_{\text{N}_2} = \frac{PV}{RT} = \frac{\left(\frac{700}{760}\right) \times \left(\frac{70}{1000}\right)}{0.821 \times 300}$$

$$W_{\text{N}_2} = \frac{700}{760} \times \frac{\frac{70}{1000}}{0.821 \times 300} \times 28$$

$$\begin{aligned} \% \text{ N} &= \frac{W_{\text{N}_2}}{0.5} \times 100 = \frac{700}{760} \times \frac{\frac{70/1000}{0.821 \times 300} \times 28}{0.5} \times 100 \\ &= 14.65\% \end{aligned}$$

**CAREER POINT**

JEE Main Online Exam 2026

Memory Based
Questions & Solution
24th January 2026 | Morning

MATHEMATICS

1. If $A = \{1, 2, 3, 4\}$. A relation from set A to $A(a, b)R(c, d)$ such that $2a + 3b = 3c + 4d$, then find the number of element(s) in relation :

(1) 9 (2) 10 (3) 11 (4) 12

Ans. [3]

Sol. (a, b) (c, d)

(1, 1) x

(1, 2) x

(1, 3) (1, 2)

(1, 4) (2, 2)

(2, 1) (1, 1)

(2, 2) (2, 1)

(2, 3) (3, 1)

(2, 4) (4, 1)

(3, 1) x

(3, 2) x

(3, 3) (1, 3)

(3, 4) (2, 3)

(4, 1) (1, 2)

(4, 2) (2, 2)

(4, 3) x

(4, 4) (4, 2)

2. If the domain of $f(x) = \log_{(10x^2 - 17x + 7)}(18x^2 - 11x + 1)$ is $(-\infty, a) \cup (b, c) \cup (d, \infty) - \{e\}$, then find $90(a + b + c + d + e)$:

(1) 316 (2) 320 (3) 163 (4) 631

Ans. [1]

Sol. $18x^2 - 11x + 1 > 0$

$$(9x-1)(2x-1) > 0 \Rightarrow x > \frac{1}{2} \text{ or } x < \frac{1}{9} \dots(1)$$

$$10x^2 - 17x + 7 > 0$$

$$(10x-7)(x-1) > 0$$

$$x > 1 \text{ or } x < \frac{7}{10} \dots(2)$$

$$10x^2 - 17x + 7 > 0$$

$$10x^2 - 17x + 6 \neq 0$$

$$(5x-6)(2x-1) \neq 0$$

$$x \neq \frac{6}{5}, \frac{1}{2} \dots(3)$$

Eq. (1) & (2) & (3)

$$x \in \left(-\infty, \frac{1}{9}\right) \cup \left(\frac{1}{2}, \frac{7}{10}\right) \cup (1, \infty) - \left\{\frac{6}{5}\right\}$$

$$a = \frac{1}{9}, b = \frac{1}{2}, c = \frac{7}{10}, d = e, l, e = \frac{6}{5}$$

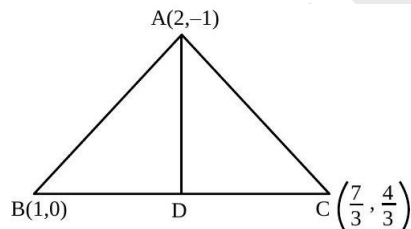
$$90(9 + b + c + d + e)$$

$$= 316$$

3. Let the vertices of the triangle are $(1,0), (2,-1), \left(\frac{7}{3}, \frac{4}{3}\right)$. If the equation of internal angle bisector through $(2,-1)$ is $\alpha x + \beta y = 5$ then value of $(\alpha^2 + \beta^2)$ is:

Ans. [10]

Sol.



$$\frac{BD}{DC} = \frac{AB}{AC} = \frac{\sqrt{2} \times 3}{5\sqrt{2}} = \frac{3}{5}$$

$$D = \left(\frac{12}{8}, \frac{4}{8}\right) = \left(\frac{3}{2}, \frac{1}{2}\right)$$

$$\text{Slope of AD} = \frac{-3/2}{1/2} = -3$$

$$3x + y = 5$$

$$\alpha = 3, \beta = 1; \alpha^2 + \beta^2 = 10$$

4. Let mean & variance of 10 numbers are 10 & 2 respectively. If one number α is replaced by another number β , then new mean & variance are 10.1 & 1.99. Find $(\alpha + \beta)$

(1) 20 (2) 19 (3) 18 (4) 17

Ans. [1]

Sol. Let first 10 numbers are $x_1, x_2, \dots, x_9, \alpha$

$$\Rightarrow \alpha + \sum_{i=1}^9 x_i = 100 \Rightarrow \sum_{i=1}^9 x_i = 100 - \alpha$$

$$\text{Variance} = \left(\frac{\sum x_i^2}{n} \right) - \left(\frac{\sum x_i}{n} \right)^2$$

$$\Rightarrow \frac{\sum x_i^2}{n} = 98$$

$$\Rightarrow x_1^2 + x_2^2 + \dots + x_9^2 + \alpha^2 = 1020 \Rightarrow \sum x_i^2 = 1020 - \alpha^2$$

In second case, let number are

$$x_1, x_2, \dots, x_9, \beta$$

$$100 - \alpha + \beta = 101\alpha - \beta + 1 = 0$$

$$\frac{\sum x_i^2 + \beta^2}{10} - (10.1)^2 = 1.99$$

$$\beta^2 - \alpha^2 = 20$$

$$\alpha = \frac{19}{2}$$

$$\beta = \frac{21}{2}$$

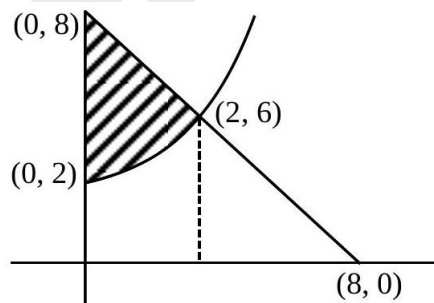
$$\alpha + \beta = \frac{19 + 21}{2} = 20$$

5. A_1 is the area bounded by $y = x^2 + 2, x + y = 8, y$ -axis in the 1st quadrant and A_2 is the area bounded by $y = x^2 + 2, y^2 = x, x = 0$ and $x = 2$ in the 1st quadrant find $(A_1 - A_2)$:

(1) $\frac{2}{3} + \frac{4\sqrt{2}}{3}$ (2) $\frac{3}{2} + \frac{4\sqrt{2}}{3}$ (3) $\frac{3}{5} + \frac{4\sqrt{2}}{3}$ (4) None of these

Ans. [1]

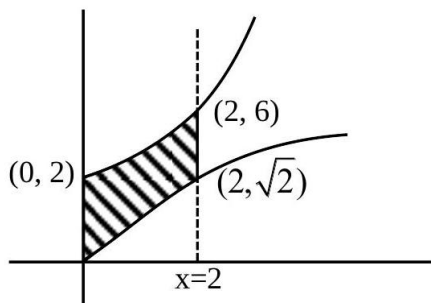
Sol.



$$A_1 = \int_0^2 ((8 - x) - (x^2 + 2)) dx$$

$$= A_1 = \int_0^2 (6 - x - x^2) dx$$

$$A_1 \left(6x - \frac{x^2}{2} - \frac{x^3}{3} \right)_0^2 = 12 - 2 - \frac{8}{3} = 10 - \frac{8}{3} = \frac{22}{3}$$



$$A_2 = \int_0^2 (x^2 + 2) dx - \frac{2}{3} (2\sqrt{2})$$

$$A_2 = \left(\frac{x^3}{3} + 2x \right)_0^2 - \frac{4\sqrt{2}}{3}$$

$$A_2 = \frac{8}{3} + 4 - \frac{4\sqrt{2}}{3} = \frac{20}{3} - \frac{4\sqrt{2}}{3}$$

$$A_1 - A_2 = \frac{2}{3} + \frac{4\sqrt{2}}{3}$$

6. $\lim_{x \rightarrow 0} \frac{e^x (e^{\tan x - x} - 1) + \ln(\sec x + \tan x) - x}{(\tan x - x)}$

(1) $\frac{3}{2}$

(2) $\frac{3}{2}e$

(3) $\frac{5}{2}e$

(4) $\frac{5}{2}$

Ans. [1]

Sol. $\lim_{x \rightarrow 0} \frac{e^{\tan x} - e^x + \ln(\sec x + \tan x) - x}{\tan x - x}$

Applying L'hospital rule

$$\Rightarrow \lim_{x \rightarrow 0} \frac{e^{\tan x} \cdot \sec^2 x - e^x + \sec x - 1}{\sec^2 x - 1}$$

$$\lim_{x \rightarrow 0} \frac{e^{\tan x} (\sec^2 x - 1) + (e^{\tan x} - e^x) + \sec x - 1}{\tan^2 x}$$

$$\lim_{x \rightarrow 0} \left(e^{\tan x} + \frac{e^x (e^{\tan x - x} - 1)}{\tan^2 x} + \frac{1}{\sec x + 1} \right)$$

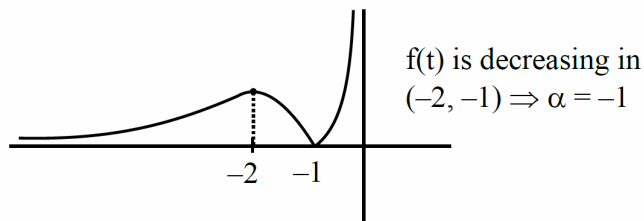
$$\Rightarrow 1 + 0 + \frac{1}{2} = \frac{3}{2}$$

7. Given $f(t) = \left| \frac{t+1}{t^2} \right|$; ($t < 0$) is strictly decreasing in the interval $(2\alpha, \alpha)$ then maximum value of

$$g(x) = 2\log_e(x-2) + \alpha x^2 + 4x - \alpha \text{ is}$$

Ans. [4]

Sol. Drawing graph of $f(t)$ for $t < 0$



$$g(x) = \log_e(x-2) - x^2 + 4x + 1; x > 2$$

$$g'(x) = \frac{2}{x-2} - (2(x-2)); x > 2$$

$$g'(x) = \frac{1 - (x-2)^2}{(x-2)} = \frac{-(x-3)(x-1)}{(x-2)}$$

$$\begin{array}{c} + \quad - \\ \hline 2 \quad 3 \end{array}$$

as $x > 2$

maxima occur at $x = 3$

$$g(3) = 2\log_e 1 - 9 + 12 + 1 = 4$$

8. If $\cot x = \frac{5}{12}$ for some $x \in \left(\pi, \frac{3\pi}{2}\right)$ then $\sin 7x \left(\cos \frac{13x}{2} + \sin \frac{13x}{2} \right) + \cos 7x \left(\cos \frac{13x}{2} - \sin \frac{13x}{2} \right)$ is equal to :

(1) $\frac{1}{\sqrt{13}}$

(2) $\frac{5}{\sqrt{13}}$

(3) $-\frac{1}{\sqrt{13}}$

(4) $\frac{8}{\sqrt{13}}$

Ans. [1]

Sol. $\cot x = \frac{5}{12} \Rightarrow \cos x = \frac{-5}{13} = 2\cos^2 \frac{x}{2} - 1$

$$\cos\left(\frac{x}{2}\right) = -\frac{2}{\sqrt{13}} \text{ or } \frac{2}{\sqrt{13}} \text{ (rejected)}$$

$$\left\{ \because \frac{x}{2} \in \left(\frac{\pi}{2}, \frac{3\pi}{4}\right) \right\}$$

$$\left(\sin 7x \frac{\sin 13x}{2} + \cos 7x \frac{\cos 13x}{2} \right) + \left(\sin 7x \frac{\cos 13x}{2} - \cos 7x \frac{\sin 13x}{2} \right)$$

$$\cos\left(7x - \frac{13x}{2}\right) + \sin\left(7x - \frac{13x}{2}\right)$$

$$\cos \frac{x}{2} + \sin\left(\frac{x}{2}\right)$$

$$\frac{3}{\sqrt{13}} - \frac{2}{\sqrt{13}} = \frac{1}{\sqrt{13}}$$

9. Let $f(x) = \int \frac{1 - \sin(\ln t)}{1 - \cos(\ln t)} dt$ and $f(e^{\pi/2}) = -e^{\frac{\pi}{2}}$ then $f\left(e^{\frac{\pi}{4}}\right)$ is :

- (1) $e^{-\frac{\pi}{4}}(\sqrt{2} + 1)$ (2) $-e^{\frac{\pi}{4}}(\sqrt{2} + 1)$ (3) $-e^{\frac{\pi}{4}}(\sqrt{2} - 1)$ (4) $e^{\frac{\pi}{4}}(\sqrt{2} - 1)$

Ans. [2]

Sol. $f(t) = \int \frac{1 - \sin(\ln t)}{1 - \cos(\ln t)} dt$

Let $\ln t = x \Rightarrow t = e^x \Rightarrow dt = e^x dx$

$= \frac{1}{2} \int \left(\operatorname{cosec}^2 \frac{x}{2} - 2 \cot \frac{x}{2} \right) e^x dx - t \cot \left(\frac{\ln t}{2} \right) + C$

$\left(\therefore \int (f(x) + f'(x)) e^x dx = f(x) \cdot e^x + C \right)$

Now $f(e^{\pi/2}) = -e^{\pi/2} \cot \left(\frac{\pi}{4} \right) + C = -e^{\frac{\pi}{2}}$ (given)

$C = 0$

Now $f(e^{\pi/4}) = -e^{\pi/4} \cot \left(\frac{\pi}{8} \right) + C = -e^{\frac{\pi}{4}}(\sqrt{2} + 1)$

10. Consider a geometric sequence 729, 81, 9, 1, If P_n denotes the product of $I^{st} n$ terms of G.P. such that

$\sum_{n=1}^{40} (P_n)^{\frac{1}{n}} = \frac{3^\alpha - 1}{2 \times 3^\beta}$, then value of $(\alpha + \beta)$ is :

- (1) 72 (2) 74 (3) 73 (4) 75

Ans. [3]

Sol. $P_n = 729.81.9..... (n \text{ terms})$

$= 3^6.3^4.3^2.....3^{-2n+8}$

$P_n = 3^{6+4+2+.....+(-2n+8)} = 3^{n(7-n)}$

$P_n^{1/n} = 3^{7-n}$

$\Rightarrow \sum_{n=1}^{40} (P_n)^{\frac{1}{n}} = 3^6 + 3^5 + + (40 \text{ terms})$

$= 3^6 \left[\frac{1 - \left(\frac{1}{3}\right)^{40}}{1 - \frac{1}{3}} \right]$

$= \frac{3^6 [3^{40} - 1] \times 3^1}{3^{40} \times 2}$

$\sum (P_n)^{\frac{1}{n}} = \frac{(3^{40} - 1)}{2 \times 3^{33}}, \alpha = 40$

$\beta = 33$

$\alpha + \beta = 73$

11. If $\int_0^{36} f\left(\frac{tx}{36}\right) dt = 4\alpha f(x)$, $y = f(x)$ is standard parabola passing through $(2,1)$ and $(-4,\beta)$. Then value of β^α is :

Ans. [64]

Sol. $\int_0^{36} f\left(\frac{tx}{36}\right) dt = 4\alpha f(x)$, Put $\frac{tx}{36} = y$

$$\frac{dy}{dt} = \frac{x}{36}$$

$$\int_0^x \frac{f(y)36dy}{x} = 4\alpha f(x)$$

$$\int_0^x f(y)dy = \frac{\alpha f(x)x}{9}$$

$$f(x) = \frac{\alpha}{9}(f(x) + xf'(x))$$

$$\left(1 - \frac{\alpha}{9}\right)f(x) = \frac{\alpha x}{9}f'(x) \Rightarrow (9 - \alpha)f(x) = \alpha xf'(x)$$

$$\frac{f'(x)}{f(x)} = \left(\frac{9}{\alpha} - 1\right)\frac{1}{x}$$

$$\log_e f(x) = \left(\frac{9}{\alpha} - 1\right)\log_e x + \log_e c$$

$$f(x) = cx^{\left(\frac{9}{\alpha} - 1\right)} \text{ for standard parabola}$$

$$\frac{9}{\alpha} - 1 = 2$$

$$\alpha = 3$$

$$f(x) = cx^2$$

passing through $(2,1)$

$$1 = 4c \Rightarrow c = 1/4$$

$$y = \frac{x^2}{4} \text{ passing through } (-4,\beta)$$

$$\beta = 4$$

$$\beta^\alpha = 4^3 = 64$$

12. Let the lines $L_1 : \vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 4\hat{k})$, $\lambda \in \mathbb{R}$ and $L_2 : \vec{r} = (4\hat{i} + \hat{j}) + \mu(5\hat{i} + 2\hat{j} + \hat{k})$, $\mu \in \mathbb{R}$ intersect at the point R. Let P and Q be the points lying on the line L_1 & L_2 respectively. Such that $|PR| = \sqrt{29}$ and $|PQ| = \sqrt{\frac{47}{3}}$. If the point P lies in the first octant then $27(QR)^2$ is :

(1) 340

(2) 360

(3) 320

(4) 348

Ans. [2]

Sol. For POI

$$2\lambda + 1 = 5\mu + 4; 3\lambda + 2 = 2\mu + 1; 4\lambda + 3 = \mu$$

$$\Rightarrow \lambda = \mu = -1$$

$$R(-1, -1, -1) \quad P(2\lambda + 1, 3\lambda + 2, 4\lambda + 3)$$

$$PR^2 = 29 \Rightarrow (2\lambda + 2)^2 + (3\lambda + 3)^2 + (4\lambda + 4)^2 = 29$$

$$\Rightarrow \lambda = 0 \text{ or } \lambda = -2 \text{ (Reject)}$$

$$\Rightarrow P(1, 2, 3)$$

$$Q(5\mu + 4, 2\mu + 1, \mu)$$

$$|PQ| = \sqrt{\frac{47}{3}} \Rightarrow PQ^2 = \frac{47}{3}$$

$$\Rightarrow (5\mu + 3)^2 + (2\mu - 1)^2 + (\mu - 3)^2 = \frac{47}{3}$$

$$\Rightarrow \mu = -\frac{1}{3}$$

$$Q = \left(\frac{7}{3}, \frac{1}{3}, -\frac{1}{3}\right)$$

$$(QR)^2 = \left(\frac{7}{3} + 1\right)^2 + \left(\frac{1}{3} + 1\right)^2 + \left(-\frac{1}{3} + 1\right)^2$$

$$= \frac{100 + 16 + 4}{9} = \frac{120}{9}$$

$$\Rightarrow 27 \times (QR)^2 = 27 \times \frac{120}{9} = 360$$

13. The value of $\frac{\sqrt{3}\operatorname{cosec}20^\circ - \sec20^\circ}{\cos20^\circ \cos40^\circ \cos60^\circ \cos80^\circ}$ is :

(1) 64

(2) 48

(3) 46

(4) 40

Ans. [1]

Sol.
$$E = \frac{\frac{\sqrt{3}}{\sin20^\circ} - \frac{1}{\cos20^\circ}}{\frac{1}{2} \cdot \frac{1}{4} \cdot \cos60^\circ}$$

$$= \frac{(\sqrt{3}\cos20^\circ - \sin20^\circ)}{\cos20^\circ \cdot \sin20^\circ} \cdot 16$$

$$= \frac{\left(\frac{\sqrt{3}}{2}\cos20^\circ - \frac{1}{2}\sin20^\circ\right) 32 \times 2}{2\cos20^\circ \cdot \sin20^\circ}$$

$$= \frac{\sin40^\circ}{\sin40^\circ} \times 64 = 64$$

14. z is a complex number satisfying $\left|\frac{z-6i}{z-2i}\right| = 1$ and $\left|\frac{z-8+2i}{z+2i}\right| = \frac{3}{5}$ then $\sum |z|^2$ is

(1) 225

(2) 321

(3) 284

(4) 385

Ans. [4]

Sol. Solving $\left| \frac{z-6i}{z-2i} \right| = 1 \Rightarrow y = 4 \dots (1)$

(where $z = x + iy$)

Now solving $\left| \frac{z-8+2i}{z+2i} \right| = \frac{3}{5}$

$$\Rightarrow x^2 + y^2 - 25x + 4y + 104 = 0 \dots (2)$$

Solving (1) & (2) $\Rightarrow z = 17 + 4i$ & $8 + 4i$

$$\Rightarrow \sum |z|^2 = (17)^2 + (4)^2 + (8)^2 + (4)^2 = 385$$

15. Let $5000 < N < 9000$ and N has digits from $\{0, 1, 2, 5, 9\}$ and digits can be repeated then find the number of N divisible by 3.

Ans. [84]

Sol. [84]

$$\underline{5} \ \underline{9} \ \underline{9} \ \underline{9} \ \underline{\times} \\ \underline{01} = 3! = 6$$

$$\underline{22} = \frac{3!}{2!} = 3$$

$$\underline{55} = \frac{3!}{2!} = 3$$

$$\underline{52} = 3! = 6$$

$$\underline{5} \ \underline{5} \ \underline{0} \ \underline{2} = 3! = 6$$

$$\underline{05} = \frac{3!}{2!} = 3$$

$$\underline{92} = 3! = 6$$

$$\underline{95} = \frac{3!}{2!} = 3$$

$$\underline{5} \ \underline{2} \ \underline{0} \ \underline{2} = 3$$

$$\underline{05} = 6$$

$$\underline{92} = 3$$

$$\underline{95} = 6$$

$$\underline{5} \ \underline{1} \ \underline{0} \ \underline{0} = 3$$

$$\underline{21} = 3$$

$$\underline{51} = 3$$

$$\underline{90} = 6$$

$$\underline{5} \ \underline{0} \ \underline{0} \ \underline{1} = 3$$

$$\underline{22} = 3$$

$$\underline{52} = 3! = 6$$

$$\underline{55} = 3$$

$$\text{Total} = 8 \times 6 + 12 \times 3$$

$$= 48 + 36$$

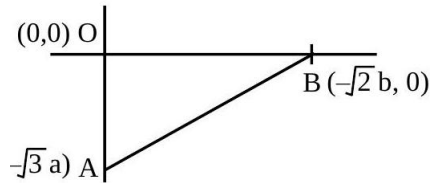
$$= 84$$

16. Given triangle OAB where O is the origin, $A = (0, -\sqrt{3}a)$ & $B = (-\sqrt{2}b, 0)$ let the circumradius of ΔOAB is 4 units. If the locus of the centroid of ΔOAB is a circle then its radius is :

- (1) $\frac{8}{3}$ (2) $\frac{7}{3}$ (3) $\frac{11}{3}$ (4) $\frac{5}{3}$

Ans. [1]

Sol.



$$4 = \frac{\sqrt{2b^2 + 3a^2}}{2}$$

$$2b^2 + 3a^2 = 8^2$$

$$(h, k) = \left(\frac{-\sqrt{2}b}{3}, \frac{-\sqrt{3}a}{3} \right)$$

$$b = \frac{3h}{-12}, a = \frac{3k}{-\sqrt{3}}$$

$$2b^2 + 3a^2 = 64$$

$$9h^2 + 9k^2 = 64 \Rightarrow x^2 + y^2 = \left(\frac{8}{3}\right)^2$$

$$r = \frac{8}{3}$$

17. Given that $\vec{a} = 2\hat{i} + \hat{j} - \hat{k}$, $\vec{b} = \hat{i} + \hat{j}$, $\vec{c} = \vec{a} \times \vec{b}$ $|\vec{d} \times \vec{c}| = 3$ & $\vec{d} \wedge \vec{c} = \frac{\pi}{4}$ & $|\vec{a} - \vec{d}| = \sqrt{11}$ find $\vec{a} \cdot \vec{d}$

- (1) 2 (2) $\frac{3}{2}$ (3) $\frac{1}{2}$ (4) $-\frac{1}{4}$

Ans. [3]

Sol. $\vec{c} = \vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -1 \\ 1 & 1 & 0 \end{vmatrix} = \hat{i} - \hat{j} + \hat{k}$ (1)

$$|\vec{d} \times \vec{c}| = |\vec{d}| |\vec{c}| \sin \theta = |\vec{d}| \sqrt{3} \cdot \frac{1}{\sqrt{2}} = 3 \text{ (given)}$$

$$\Rightarrow |\vec{d}| = \sqrt{6} \quad \dots(2)$$

Now

$$|\vec{a} - \vec{d}| = \sqrt{11} \Rightarrow |\vec{a}|^2 + |\vec{d}|^2 - 2\vec{a} \cdot \vec{d} = 11$$

$$\Rightarrow 6 + 6 - 2\vec{a} \cdot \vec{d} = 11 \Rightarrow \vec{a} \cdot \vec{d} = \frac{1}{2}$$

18. Find the number of real solutions of $x|x-3|+|x-1|+3=0$:

(1) 1

(2) 2

(3) 3

(4) 4

Ans. [1]

Sol.

 A horizontal number line with points 1 and 3 marked. The region to the left of 1 is labeled III, the region between 1 and 3 is labeled II, and the region to the right of 3 is labeled I.

(I) if $x \geq 3$

$$\therefore x^2 - 3x + x - 1 + 3 = 0$$

$$\therefore x^2 - 2x + 2 = 0$$

No real solution

(ii) If $1 < x < 3$

$$\therefore 3x - x^2 + x - 1 + 3 = 0$$

$$\Rightarrow x^2 - 4x - 2 = 0$$

$$\rightarrow x = \frac{4 \pm \sqrt{16+8}}{2} = \frac{4 \pm \sqrt{24}}{2} = 2 \pm \sqrt{6}$$

(III) If $x \leq 1$

$$\therefore 3x - x^2 + 1 - x + 3 = 0$$

$$\Rightarrow x^2 - 2x - 4 = 0 \rightarrow x = \frac{2 \pm \sqrt{4+16}}{2} = 1 \pm \sqrt{5}$$

So, total 1 real solution

19. Consider an ellipse $E_1 : \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 (a > b)$ and $E_2 : \frac{x^2}{A^2} + \frac{y^2}{B^2} = 1 (B > A)$ where $e = \frac{4}{5}$ for both the curves and ℓ_1 is the length of latus rectum of E_1 and ℓ_2 is the length of latus rectum of E_2 . Let the distance between the foci of the first curve is 8. Find the distance between the foci of the second curve (Given $(2\ell_1^2 = 9\ell_2)$):

(1) $\frac{64}{5}$

(2) $\frac{8}{5}$

(3) $\frac{32}{5}$

(4) $\frac{16}{5}$

Ans. [3]

Sol. $2ae = 8 \Rightarrow a = 5$

$$b^2 = a^2(1 - e^2)$$

$$b^2 = a^2 \times \frac{9}{25} \quad b^2 = 9$$

$$E_1 : \frac{x^2}{25} + \frac{y^2}{9} = 1$$

$$\ell_1 : \frac{2b^2}{a} = \frac{2 \times 9}{5} = \frac{18}{5}$$

$$A^2 = B^2(1 - e^2) \Rightarrow A^2 = \frac{9}{25}B^2 \Rightarrow A = \frac{3}{5}B$$

$$2\ell^2 = 9\ell_2 \Rightarrow 2\left(\frac{18}{5}\right)^2 = 9\ell_2 \Rightarrow \ell_2 = \frac{4 \times 18}{25}$$

$$\frac{2A^2}{B} = \frac{72}{25} \Rightarrow A^2 \frac{36}{25} B$$

$$\frac{9}{25} B^2 = \frac{36B}{25} \Rightarrow B = 4,$$

$$\text{Distance between foci } 2Be = 2 \times \frac{4}{5} \times 4 = \frac{32}{5}$$

20. Evaluate the series $\frac{1}{25!} + \frac{1}{3!23!} + \frac{1}{5!21!} + \dots +$ upto 13 terms :

(1) $\frac{2^{26}}{26!}$

(2) $\frac{2^{25}}{26!}$

(3) $\frac{2^{26}}{25!}$

(4) $\frac{2^{25}}{25!}$

Ans. [2]

Sol. $\frac{1}{26!} \left(\frac{26!}{25!!} + \frac{26!}{3!23!} + \frac{26!}{5!21!} + \dots + 13 \text{ terms} \right)$

$$\frac{1}{26!} \left({}^{26}C_1 + {}^{26}C_3 + {}^{26}C_5 + \dots + 13 \text{ terms} \right)$$

$$\frac{1}{26!} \left({}^{26}C_1 + {}^{26}C_3 + \dots + {}^{26}C_{25} \right)$$

$$\frac{1}{26!} \times 2^{25}$$

21. Find number of matrices A whose order is 3×2 has elements from the set $\{\pm 2, \pm 1, 0\}$ if $\text{Tr}(A^T A) = 5$:

(1) 310

(2) 312

(3) 320

(4) 325

Ans. [2]

Sol. $\begin{pmatrix} a_1 & b_1 \\ a_2 & b_2 \\ a_3 & b_3 \end{pmatrix}_{3 \times 2}$

$$A^T A = \begin{pmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{pmatrix}_{2 \times 3} \begin{pmatrix} a_1 & b_1 \\ a_2 & b_2 \\ a_3 & b_3 \end{pmatrix}_{3 \times 2}$$

$$= \begin{pmatrix} a_1^2 + a_2^2 + a_3^2 & - \\ - & b_1^2 + b_2^2 + b_3^2 \end{pmatrix}$$

$$\text{Tr}(A^T A) = a_1^2 + a_2^2 + a_3^2 + b_1^2 + b_2^2 + b_3^2 = 5$$

$$\{2, 1, 0, 0, 0, 0\}$$

$$\{2, -1, 0, 0, 0, 0\}$$

$$\{-2, 1, 0, 0, 0, 0\}$$

$$\{-2, -1, 0, 0, 0, 0\}$$

$$\{1, 1, 1, 1, 0, 0\}$$

$$\text{No. of ways} = \frac{6!}{4!} \times 4 + 2 \times \frac{6!}{5!} + 2 \times \frac{6!}{4!} + 2 \times \frac{6!}{3!2!}$$

$$= \frac{6!}{3!} + 2 \times 6 + 2 \times 15 \times 2 \times \frac{6!}{3!}$$

$$= 120 + 120 + 12 + 60 = 312$$

22. Consider an A.P. a_1, a_2, \dots, a_n ; $a_1 > 0$, $a_2 - a_1 = \frac{-3}{4}$, $a_n = \frac{a_1}{4}$, $\sum_{i=1}^n a_i = \frac{525}{2}$ then $\sum_{i=1}^{17} a_i$ is equal to:

(1) 231

(2) 234

(3) 236

(4) 238

Ans. [4]

Sol. $S_n = \frac{n}{2}[a_1 + a_n] = \frac{525}{2}$, $d = \frac{-3}{4}$

$$\frac{n}{2}\left[a_1 + \frac{a_1}{4}\right] = \frac{525}{2}$$

$$\frac{5a_1 n}{4} = 525$$

$$a_1 n = 420$$

$$a_n = a_1 + (n-1)\left(\frac{-3}{4}\right)$$

$$\Rightarrow \frac{-3}{4}a_1 = \left(\frac{-3}{4}\right)(n-1) \Rightarrow a_1 = n-1$$

$$n(n-1) = 420$$

$$n^2 - n - 420 = 0$$

$$(n-21)(n+20) = 0$$

$$n = 21, a_1 = 20$$

$$\sum_{i=1}^{17} a_i = \frac{17}{2}[2a_1 + 16d]$$

$$= \frac{17}{2}\left[40 + 16\left(\frac{-3}{4}\right)\right]$$

$$= \frac{17}{2}[40 - 12]$$

$$= 17 \times 14 = 238$$

23. There are 10 defective & 90 non-defective balls in a bag. 8 balls are taken one by one with replacement then probability that at least 7 defective balls are selected.

(1) $\left(\frac{73}{10^8}\right)$

(2) $\left(\frac{37}{10^8}\right)$

(3) $\left(\frac{105}{10^8}\right)$

(4) $\left(\frac{11}{10^8}\right)$

Ans. [1]

Sol. 10 defective & 90 non-defective

Req. probability = (7 def 1 fair) or (8 defective)

$$\text{Req. probability} = \frac{(10^7 \times 90) \times 8 + 10^8}{100^8}$$

$$= \frac{72 \times 10^8 + 10^8}{100^8} = \frac{73}{10^8}$$