CAREER POINT

JEE Main Online Exam 2026

Memory Based

Questions & Solution

22st January 2026 | Evening

PHYSICS

1. Find the dimension of the expression $\frac{\varepsilon_0 E}{T}$ where $\varepsilon_0, E \& T$ are permittivity, electric field and time :

(1) $\left[AL^{-2}\right]$ (2) $\left[AL^{-3}\right]$ (3) $\left[AL^{-3}\ T\right]$ (4) $\frac{\left[AL^{-3}\right]}{T}$

Ans. **[1]**

Sol. $$\frac{\varepsilon_0 E}{T} = \frac{\varepsilon_0}{T} \times \frac{1}{4\pi\varepsilon_0} \times \frac{q}{r^2}$$

$$= \frac{[AT]}{[T][L^2]}$$

$$= \left[AL^{-2}\right]$$

2. For H atom if shortest wavelength of Lyman series is 91 nm, then find difference of minimum wavelength of Balmer and Paschen series :

(1) 525 nm (2) 455 nm (3) 305 nm (4) 545 nm

Ans. **[2]**

Sol. For Lyman:

$$\frac{1}{\lambda} = R\left(\frac{1}{12} - \frac{1}{\infty^2}\right) \Rightarrow \lambda = \frac{1}{R} = 91 \text{ nm}$$

For Balmer shortest wavelength:

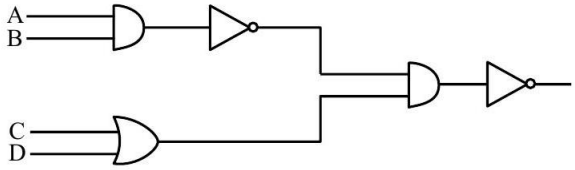
$$\frac{1}{\lambda_1} = R\left(\frac{1}{2^2} - \frac{1}{\infty^2}\right) \Rightarrow \lambda_1 = \frac{4}{R}$$

For shortest wavelength of Paschen series

$$\frac{1}{\lambda_2} = R\left(\frac{1}{3^2} - \frac{1}{\infty^2}\right) \Rightarrow \lambda_2 = \frac{9}{R}$$

So $\lambda_2 - \lambda_1 = \frac{5}{R} = 455 \text{ nm}$

3. For given logic gate circuit select correct output corresponding to each input :



A	B	C	D
1	1	1	1
1	0	1	0
0	1	0	1
0	0	1	1

(1) 1,0,0,0 (2) 1,0,1,0 (3) 0,1,0,1 (4) 1,0,0,1

Ans. **[1]**

Sol. On simplifying :

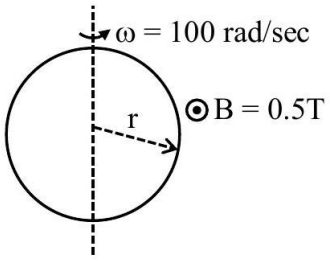
Output $Y = \overline{\left(\overline{A \cdot B}\right) \cdot \left(C + D\right)}$

$= \left(A \cdot B\right) + \left(\overline{C + D}\right)$

4. A ring of radius 'r' mm rotating with 100rad / sec about its diameter. It is present in a uniform magnetic field 0.5 T perpendicular to the plane of paper. If EMF produced in the ring when it has rotated by 30° is 15.4 mV, find ' r ':

Ans. **[14]**

Sol.



$\phi = \pi r^2 \, B\cos\omega t$

$\therefore -\dfrac{d\phi}{dt} = \pi r^2 \, B\omega\sin\omega t$

given $\left|\dfrac{d\phi}{dt}\right| = 15.4\times10^{-3}$ V

$\therefore 15.4\times10^{-3} = \pi r^2 \, B\omega\sin\left(\omega t\right)$

$\Rightarrow r = \sqrt{\dfrac{15.4\times10^{-3}}{3.14\times0.5\times100\times\dfrac{1}{2}}} = 14 \text{ mm}$

5. Wavelength of light in water is 540 nm. Refractive index of water is $4/3$. Find wavelength of light in glass $(\mu = 3/2)$:

(1) 480 nm (2) 240 nm (3) 360 nm (4) 630 nm

Ans. **[1]**

Sol. $\Rightarrow \lambda \propto \frac{1}{\mu}$

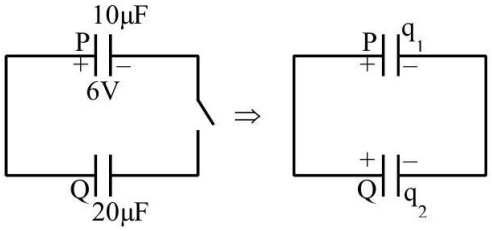
$\Rightarrow \frac{\lambda_1}{\lambda_2} = \frac{\mu_2}{\mu_1} = \frac{3/2}{4/3} = \frac{9}{8}$

$\lambda_2 = \frac{8}{9} \times 540 = 480 \text{ nm}$

6. A capacitor " P " of capacitance 10×10^{-6} F is charged to 6 Volts and is now connected to another capacitor Q of capacitance 20×10^{-6} F (Q has no initial charge). The final charge on Q is $\alpha \times 10^{-5}$ C. Find α.

Ans. **[4]**

Sol. Charge will be distributed in ratio of capacitors



$\frac{q_1}{q_2} = \frac{1}{2}$ and $q_1 + q_2 = 60$

$3q_1 = 60$

$q_1 = 20\mu C$

$q_2 = 40\mu C = 4 \times 10^{-5} C$

7. $I = 4 \times 10^{14}$ W / m^2 then find amplitude of magnetic field of laser Beam :

(1) 2.87 T (2) 1.83 T (3) 3.86 T (4) 4.78 T

Ans. **[2]**

Sol. $I = \left(\frac{B^2}{2\mu_0} . C \right) = 4 \times 10^{14}$

$B^2 = \frac{2\mu_0}{C} \times 4 \times 10^{14}$

$B^2 = \frac{2 \times 4\pi \times 10^{-7} \times 4 \times 10^{14}}{3 \times 10^8}$

$B = 1.83$ T

8. **Statement-1 :** Work done by $\vec{F}$ from $\vec{r}_1$ to $\vec{r}_2$ is given as $W = -\int_{r_1}^{r_2} \vec{F}.d\vec{r}$, if $\vec{F}$ is conservative.

Statement-2 : There are infinite ways through which we can go from $\vec{r}_1$ to $\vec{r}_2$ and work done for each case will be different for conservative force.

(1) Statement-1 is True, Statement-2 is True ; Statement-2 is correct explanation of Statement-1.

(2) Statement-1 is True, Statement-2 is True ; Statement-2 is NOT correct explanation of Statement-1.

(3) Statement-1 is True, Statement-2 is False.

(4) Statement-1 and Statement-2 both are False.

Ans. **[4]**

Sol. Theoretical

9. For non parallax in concave mirror, what should be the position of object from the pole :

(1) Beyond centre of curvature only (2) Between centre of curvature and focus only

(3) Between focus and pole only (4) Beyond focus

Ans. **[4]**

Sol. For non-parallax, image should be real.

$\therefore$ Object must be kept beyond focus.

10. 3 identical bubbles each have same charge q. They combine to form one bubble. Find $\frac{V_i}{V_f}$?

(1) $\frac{1}{3^{2/3}}$ (2) $\frac{3^{2/3}}{1}$ (3) $\frac{1}{2^{2/3}}$ (4) $3^{1/3}$

Ans. **[1]**

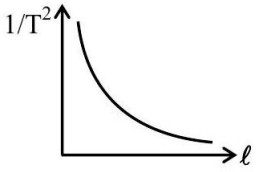
Sol. Using volume conservation

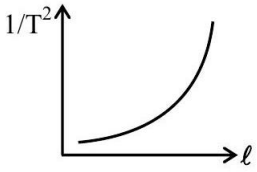
$$3\left(\frac{4}{3}\pi r^3\right) = \left(\frac{4}{3}\pi R^3\right)$$

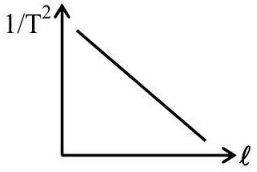
$$R = 3^{1/3} r$$

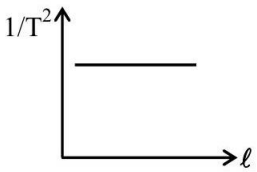
$$\frac{V_i}{V_f} = \frac{\frac{kq}{r}}{\frac{k3q}{R}} = \frac{R}{3r} = \frac{3^{1/3} r}{3r} = \frac{1}{3^{2/3}}$$

11. In simple pendulum experiment gravity (g) is determined by its time period (T). Which of the following graph is correct?

(1) 

(2) 

(3) 

(4) 

Ans. **[1]**

Sol. $T = 2\pi\sqrt{\frac{\ell}{g}}$

$$T^2 = \frac{4\pi^2\ell}{g}$$

$$\frac{1}{T^2} = \left(\frac{g}{4\pi^2\ell}\right)$$

12. An ideal gas at pressure 2×10^5 Pa, temperature $27^\circ C$ has volume $60\ cm^3$. If volume of same gas is $20\ cm^3$ & temperature is $77^\circ C$, find out pressure at this state. :

(1) 7×10^5 Pa (2) 6×10^5 Pa (3) 3×10^5 Pa (4) 5×10^5 Pa

Ans. **[1]**

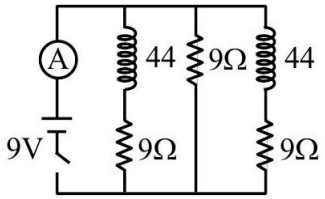
Sol. $PV = nRT$

$$\frac{P_1 V_1}{RT_1} = \frac{P_2 V_2}{RT_2}$$

$$\frac{2\times10^5\times60}{R\times300} = \frac{P_2\times20}{R\times350}$$

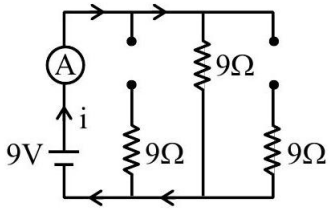
$$P_2 = 7\times10^5 \text{ Pa}$$

13. In the figure shown below switch S is closed at $t = 0$, find reading of ideal ammeter (in Amp.) just after S is closed.



Ans. **[1]**

Sol.



at $t = 0$ all inductors act as open circuit $i = \frac{9}{9} = 1$ A

14. Transmission line having resistance 2Ω and power delivered is 1000 W. When potential difference of 250 volts is applied, find efficiency of transmission line.

(1) 94% (2) 96.9% (3) 100% (4) 91%

Ans. **[2]**

Sol. Current in wire : $I=\frac{1000}{250}=4\text{Amp}$

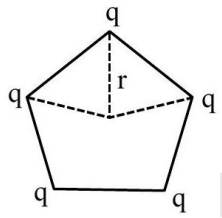
Heat loss $=I^2R=32\ \text{W}$

Power input $=1032\ \text{W}$

$$\%\eta=\frac{\text{Power output}}{\text{Power input}}$$

$$\%\eta=\frac{1000}{1032}=96.89\%$$

15. Five positive charges each having charge q are placed at the vertices of a pentagon as shown in the figure. The electric potential (V) & the electric field ($\vec{E}$) at the centre of the pentagon due to the 5 positive charges are :



(1) $V=\frac{5q}{4\pi\varepsilon_0 r},E=\frac{5q}{4\pi\varepsilon_0 r^2}$

(2) $V=\frac{5q}{4\pi\varepsilon_0 r},E=0$

(3) $V=0,E=\frac{5q}{4\pi\varepsilon_0 r^2}$

(4) $V=\frac{5q}{4\pi\varepsilon_0 r},E=\frac{q}{4\pi\varepsilon_0 r^2}$

Ans. **[2]**

Sol.

16. In case of capillary action if surface tension of liquid, radius of capillary and density of liquid decreases by 1% then percentage change in height of liquid level inside capillary :

(1) 1% increase

(2) 1% decrease

(3) 2% increase

(4) 2 % decrease

Ans. **[1]**

Sol. $h=\frac{2\ T\cos\theta}{\rho g r}$

$$\frac{dh}{h}=\frac{dT}{T}-\frac{d\rho}{\rho}-\frac{dr}{r}$$

$$\frac{dh}{h}\%=-1\%+1\%+1\%=1\%$$

1 % increase

17. In an open organ pipe 3^{rd} and 6^{th} harmonic frequency differ by 3200 Hz. Find the length of organ pipe. (Speed of sound $=320\ \text{m/s}$)

Ans. **[15]**

Sol. $f = n\frac{v_0}{2L}$

$$\frac{6v_0}{2L} - \frac{3v_0}{2L} = 3200$$

$$\frac{3\times 320}{2L} = 3200$$

$$L = \frac{3}{20}\ m$$

$$L = \frac{3}{20}\times 100\ cm = 15\ cm$$

18. A metal has work function $\phi = 110\times 10^{-20}$ J when exposed with monochromatic light maximum kinetic energy of photoelectrons is found to be zero. Find angular frequency of incident light. ($h = 6.63\times 10^{-34}$ Js)

(1) 1.04×10^{16} (2) 1.04×10^{18} (3) 1.66×10^{17} (4) 1.66×10^{18}

Ans. **[1]**

Sol. $\omega = 2\pi f$

$$hf - \phi = k_{max}$$

$$f = \frac{\phi}{h} = \frac{110\times 10^{-20}}{6.63\times 10^{-34}} = 1.66\times 10^{15}$$

$$\omega = 2\pi f = 1.04\times 10^{16}\ rad/sec$$

19. For single slit diffraction :

(A) If we increase λ , keeping slit width constant, width of central maxima increases.

(B) If we increase λ , keeping slit width constant, width of central maxima decreases.

(C) If we keep λ same and decreases slit width, the width of central maxima increases

(D) If we keep λ same and decreases slit width the width of central maxima decreases

(E) If we increases λ by keeping slit width same, intensity of central maxima increases

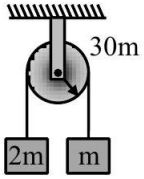
Choose the correct option:

(1) A, C only (2) B,D & E only (3) A, C & E only (4) A, D only

Ans. **[1]**

Sol. Width of central maxima $= \frac{2\lambda D}{a}$

20. A pulley has mass 30 m. There are two blocks of masses m and 2 m. Find speed of 2 m when it descends by distance 3.6 meter. ($g = 10\ m/s^2$)



(1) $-2\ m/s$ (2) $4\ m/s$ (3) $8\ m/s$ (4) $2\ m/s$

Ans. **[4]**

Sol. Using energy conservation

$$2mgh - mgh = \frac{1}{2}mv^2 + \frac{1}{2}2mv^2 + \frac{1}{2}\frac{30mR^2}{2} \times \frac{v^2}{R^2} mgh = 9mv^2$$

$$v = \sqrt{\frac{gh}{9}} = \sqrt{\frac{3.6 \times 10}{9}} = \sqrt{4} = 2 \text{ m/s}$$

21. A tube carries 1.6 A current has length 2 m & cross section area $= 0.2 \text{ mm}^2$. If potential difference of 2 V is applied and no. of electrons/volume is 5×10^{28}, mobility of electron is $\alpha \times 10^{-3}$ find α :

Ans. **[1]**

Sol. $i = neAV_d = neA\mu E$

$$\mu = \frac{i}{neAE} = \frac{1.6}{5 \times 10^{28} \times 1.6 \times 10^{-19} \times 0.2 \times 10^{-6} \times (2/2)}$$

$$= 1 \times 10^{-3}$$

22. **Statement-1 :** Time period of revolution of satellite around earth depends on density of earth.

Statement-2 : Time period of revolution of satellite just above the surface of earth is given by $2\pi\sqrt{\frac{Re}{g}}$ (Re = Radius of earth)

(1) Statement 1 & 2 both are correct & statement 1 is correct explanation

(2) Statement 1 & 2 both are correct & statement 1 is not correct explanation

(3) Statement is true Statement 2 is false

(4) Statement is false Statement 2 is true

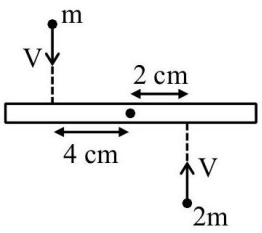
Ans. **[1]**

Sol. Both are correct & explanation

$$T = 2\pi\sqrt{\frac{Re}{g}} = 2\pi\sqrt{\frac{Re}{\frac{4}{3}\pi G\rho Re}} = 2\pi\sqrt{\frac{3}{4\pi G\rho}}$$

$$T \propto \frac{1}{\sqrt{\rho}}$$

23. Mass of rod is 20 m. If both particles stick with rod after collision than find V/ω ? Here ω is angular velocity of rod after collision. Length of rod is 12 cm :



(1) 64 (2) 66 (3) 33 (4) $\sqrt{88}$

Ans. **[3]**

Sol. Using angular momentum conservation about COM of rod :

$$L_i = L_f$$

$$m\times V\times 4 + 2m\times V\times 2 = \left(\frac{20m(12)^2}{12} + m\times 4^2 + 2m\times 2^2\right)\omega$$

$$8mV = (240m + 24\ m)\omega$$

$$8V = 264\ \omega$$

$$\frac{V}{\omega} = 33$$

24. There are 2 different gases in 2 different containers A and B. Gas A has temperature 'T' and pressure 'P' and number of molecules per unit volume in Gas A is N. Gas B has temperature 'T' and pressure 'P' and number of molecules per unit volume is N. Mass of gas A is 4 times of mass of gas 'B' and size of molecules of gas A is half the size of molecules of gas 'B'. If collision frequency of B is 32×10^8 / sec. Find collision frequency of A :

(1) 16×10^8 / sec (2) 4×10^8 / sec (3) 2×10^8 / sec (4) 8×10^8 / sec

Ans. **[2]**

Sol. Collision frequency $(z) = \sqrt{\frac{8RT}{\pi M}}\left(\sqrt{2}\pi d^2 N\right)$

$$\frac{Z_A}{Z_B} = \frac{\left(\frac{d}{2}\right)^2\frac{N}{\sqrt{4M}}}{\left(\frac{d^2N}{\sqrt{M}}\right)} \Rightarrow Z_A = Z_B\times\frac{1}{8}$$

$$= 32\times10^8\times\frac{1}{8}$$

$$= 4\times10^8\ /\ \text{sec}$$

25. **Statement-1:** Total KE of system can be written as sum of KE of individual particles from ground frame.
Statement-2: Total KE of system can be written as KE of center of mass +KE of individual particles in C.M. frame

(1) statement-1 is true statement-2 is false (2) statement-1 is false statement-2 is true
(3) Both statement are false. (4) Both statement are true.

Ans. **[4]**

Sol. $KE_{system} = KE_{cm} + KE_{inc\text{-}frame}$

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CHEMISTRY

1. Which of the following are correct ?

(A) Hydrated salt can be primary standard.

(B) Primary standard should not react with air.

(C) Primary standard should react instantaneously and stoichiometrically.

(D) Primary standard should not be water soluble.

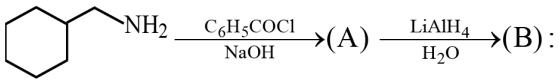
(E) Primary standard should not be lower mass.

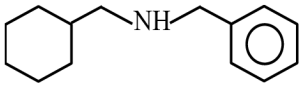
(1) A, B, C, E (2) C, D, E (3) A, B, E (4) A, B, C

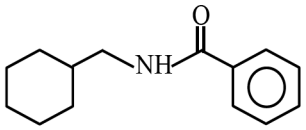
Ans. **[3]**

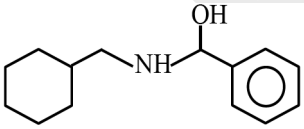
Sol. Refer theory.

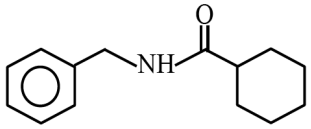
2. In the given reaction find final product (B)



(1) 

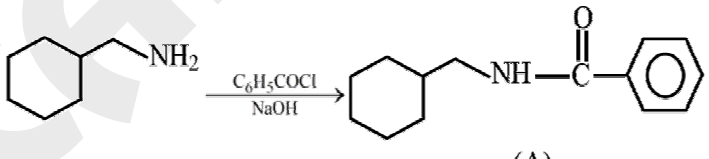
(2) 

(3) 

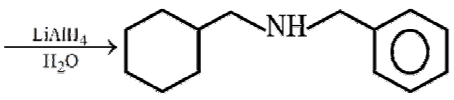
(4) 

Ans. **[1]**

Sol.



(A)



(B)

3. Which of the following buffer solution will have a pH of 9.25. (Given that pK_b of $NH_3 : 4.75$)

(1) $0.2M\ NH_4OH(0.4\ell) + 0.1M\ HCl(1\ell)$ (2) $0.4M\ NH_4OH(1\ell) + 0.1M\ HCl(1\ell)$

(3) $0.5M\ NH_4OH(0.5\ell) + 0.2M\ HCl(0.5\ell)$ (4) $0.2M\ NH_4OH(0.5\ell) + 0.1M\ HCl(0.5\ell)$

Ans. **[4]**

Sol. $pOH = pK_b + \log\frac{Salt}{Base}$

$4.75 = 4.75 + \log\frac{Salt}{Base}$

Milimoles of [Salt] = milimoles of [Base]

Option (4) :

$NH_4OH + HCl \rightarrow NH_4Cl + H_2O$

0.2M, 0.5L 0.1M, 0.5L

100 mmole 50 mmole

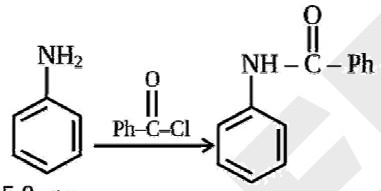
50 mmole – 50 mmole

Milimoles of NH_4OH = milimoles of NH_4Cl

4. Find the mass of product obtained by benzoylation of 5.8 gm aniline, if reaction yield is 82%.

Ans. **[10.05]**

Sol.



5.8 gm

$n = \frac{5.8}{93}$

n = 0.0623

$n = 0.0623 \times \frac{82}{100}$

mass = 0.051 × 197

mass = 10.047

5. How many of the following are mixed oxides.

$Cr_2O_3, Mn_3O_4, Fe_3O_4, Fe_2O_3, V_2O_4, Pb_3O_4$

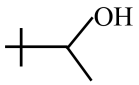
Ans. **[3]**

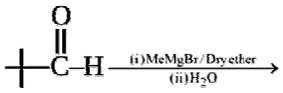
Sol. $Mn_3O_4 \rightarrow 2MnO + MnO_2$

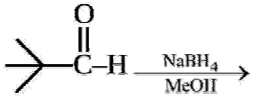
$Fe_3O_4 \rightarrow FeO + Fe_2O_3$

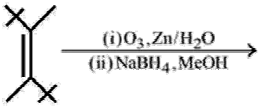
$Pb_3O_4 \rightarrow 2PbO + PbO_2$

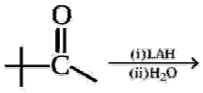
6. In which among the following reaction(s),

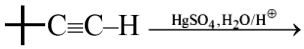

is not formed?

(a) 

(b) 

(c) 

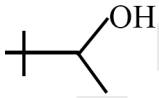
(d) 

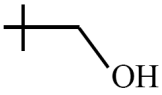
(e) 

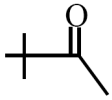
Set of incorrect reaction(s) is :

(1) b, c (2) b, e (3) b, c, e (4) Only b

Ans. **[2]**

Sol. Product of option(a), (c), (d) is 

Product of option (b) is 

Product of option (e) is 

7. Molecules with least dipole moment among the following species $CHCl_3, NF_3, H_2S, H_2O, NH_3$ has 'X' number of lone pair on central atom. Find 'X'.

(1) 0 (2) 1 (3) 2 (4) 3

Ans. **[2]**

Sol. NF_3 has least dipole moment. It has one lone pair of electron on central atom.

8. Energy of 1^{st} Balmer line of H -atom is x J . The energy (in J) of second Balmer line of H-atom is :

(1) x (2) $2x$ (3) 1.35 x (4) $\frac{x}{1.35}$

Ans. **[3]**

Sol. Transition of first Balmer line

$n_1 = 2; n_2 = 3$

$$\Delta E = x = 13.6(1)^2\left[\frac{1}{2^2} - \frac{1}{3^2}\right] \quad ...(i)$$

Transition of 2^{nd} Balmer line

$n_1 = 2; n_2 = 4$

$\Delta E = 13.6(1)^2\left[\frac{1}{2^2} - \frac{1}{4^2}\right]$ ...(ii)

Divide Eq. (ii) by eq. (i)

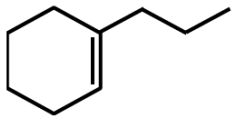
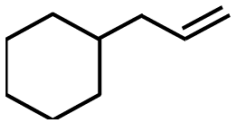
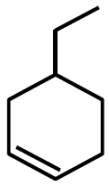
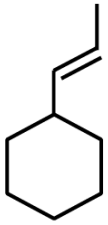
$$\frac{\Delta E}{x} = \frac{\frac{1}{4} - \frac{1}{16}}{\frac{1}{4} - \frac{1}{9}}$$

$$\frac{\Delta E}{x} = \frac{\frac{3}{16}}{\frac{5}{36}}$$

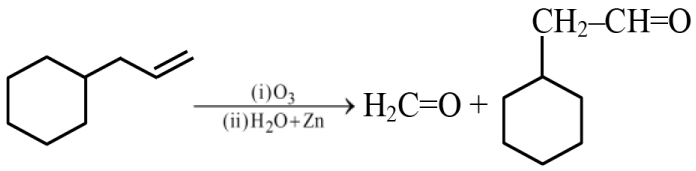
$$\frac{\Delta E}{x} = \frac{27}{20}$$

$\Delta E = 1.35x$.

9. An alkene on reductive ozonolysis gives methanal as one of the product it's structure is :

(1)  (2)  (3)  (4) 

Ans. **[2]**

Sol. 

10. Find magnitude of lattice energy of $LiF(inkJ/mol)$

Given :

Enthalpy of sublimation of $Li(s) = 161\ kJ/mol$

Ionisation enthalpy of $Li(g) = 520\ kJ/mol$

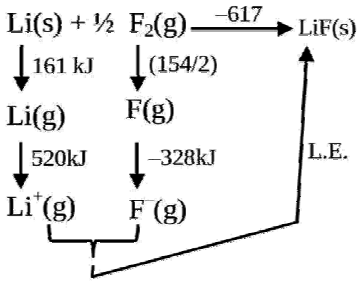
Bond Enthalpy of $F_2(g) = 154\ kJ/mol$

Electron gain enthalpy of $F(g) = -328\ kJ/mol$

Enthalpy of formation of $LiF(s) = -617\ kJ/mol$

Ans. **[1047]**

Sol.



$$-617 = 161 + 520 + \frac{154}{2} - 328 + (\text{L.E.})$$

$$\text{L.E.} = -1047 \text{ kJ/mol}$$

11. $A \rightarrow B; E_{a_1}$

$C \rightarrow D; E_{a_2}$

$\log_{10} K$ for first reaction $= 14.34 - \frac{1.5 \times 10^4}{T}$

E_{a_2} is $1/5^{th}$ of E_{a_1}. Then the value of E_{a_2} is (in kJ/mol) :

Ans. **[57 kJ]**

Sol. $$\frac{E_{a_1}}{2.303R} = 1.5 \times 10^4$$

$$E_{a_1} = 1.5 \times 10^4 \times 2.303 \times 8.314$$

$$E_{a_1} = 28.7207 \times 10^4 \text{ J}$$

$$E_{a_1} = 287.207 \text{ kJ}$$

$$E_{a_2} = \frac{E_{a_1}}{5} = \frac{287.207}{5} = 57.44 \text{ kJ}$$

12. **Statement-I :** BCl_3 and $AlCl_3$ are covalent compounds.

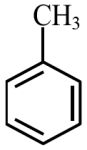
Statement-II : BCl_3 on reaction with water produces $[B(OH)_4]^{-1}$ ion and $[B(H_2O)_6]^{3+}$ ions.

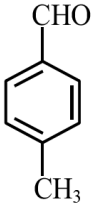
(1) Statement-I is correct and Statement-II is incorrect.
(2) Statement-I and Statement II both are correct.
(3) Statement-I is incorrect while Statement-II is correct.
(4) Both Statements are incorrect.

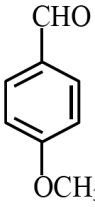
Ans. **[1]**

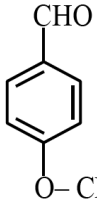
Sol. Both BCl_3 and $AlCl_3$ are covalent compounds. BCl_3 on reaction with water gives H_3BO_3 and HCl.

13. Organic compound A with molecular formula $C_8H_8O_2$ gives cross aldol condensation reaction with acetone and A also react with conc. NaOH and produce alcohol as one of the product, identify structure of A :

(1) 

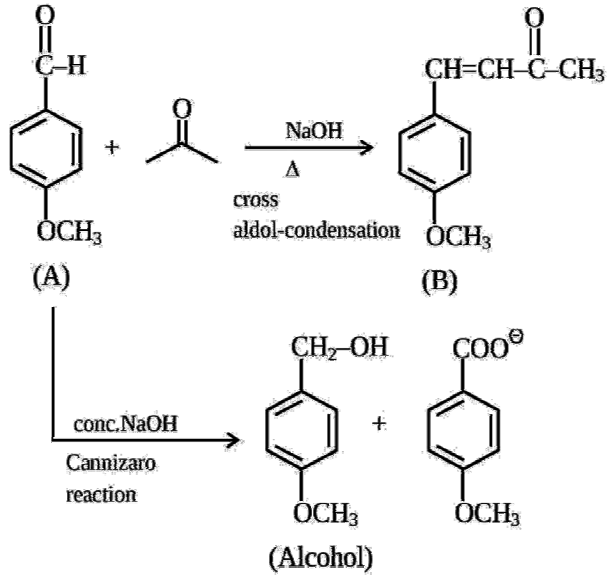
(2) 

(3) 

(4) 

Ans. **[3]**

Sol.



14. Consider the following statement(s) about Arrhenius equation:

(A) The fraction of particles having energy less than activation energy is $e^{-\frac{E_a}{RT}}$.

(B) Reaction with lower activation energy is faster.

(C) On increasing temperature by $10^{\circ}C$, rate of reaction doubles.

(D) Graph of $\log K \text{ v / s } \frac{1}{T}$ is a straight line with slope $\frac{-E_a}{R}$.

Select correct statement:

(1) A and B are correct (2) B and D are correct

(3) B and C are correct (4) C and D are correct

Ans. **[3]**

Sol. Fact based.

15. Correct statement for the paramagnetic complex $\left[Ni(PPh_3)_2Cl_2\right]$.

(1) CFSE value is $-0.2\Delta_0$

(2) It is white in colour

(3) It has magnetic moment 2.84 B.M.

(4) It has two geometrical isomers

Ans. **[3]**

Sol. $\left[Ni(PPh_3)_2Cl_2\right]$ is tetrahedral and paramagnetic in nature having two unpaired electrons.

16. 100 g of 98% by weight H_2SO_4 is mixed with 100 gm 49% by weight H_2SO_4. Mole fraction of H_2SO_4 in the final solution is :

(1) 0.9 (2) 0.1 (3) 0.67 (4) 0.33

Ans. **[4]**

Sol. 100 g, 98% by weight has

$98\ gH_2SO_4$ and $2\ gH_2O$ (water).

$100\ g, 49\%$ by weight has

$49\ gH_2SO_4$ and $51\ gH_2O$

Total wt. of $H_2SO_4 = 49 + 98 = 147gm$.

Moles of $H_2SO_4 = 1.5$ mole

Total mass of water $= 53gm$.

Moles of water $= 2.944$

Moles fraction of $H_2SO_4 = \dfrac{1.5}{1.5 + 2.944} = 0.33$.

17. When 1 gm of compound (X) is subjected to Kjeldahl's method for estimation of nitrogen, 15 mL, $1M\ H_2SO_4$ was neutralized by ammonia evolved. The % of nitrogen in compound (X) is :

(1) 21 (2) 0.21 (3) 42 (4) 0.42

Ans. **[3]**

Sol. Eq. of H_2SO_4 = eq. of Ammonia

$$\Rightarrow \frac{15 \times 1 \times 2}{1000} = \text{moles of ammonia} \times 1$$

$\Rightarrow$ Moles of ammonia = moles of 'N'

$$\Rightarrow \text{Weight of nitrogen} = \frac{15 \times 1 \times 2}{1000} \times 14 = 0.42$$

$$\%\ \text{weight of 'N'} = \frac{0.42}{1} \times 100 = 42\%$$

18. Find pH above which O_2 will be evolved at anode :

$E^{o}_{M^{+2}(aq)/M(s)} = 0.997$ V, $E^{o}_{O_2(g)/H_2O(\ell)} = +1.23$ V

$Pt(s)|O_2(g)|H^+(aq) \,||M^{+2}|M$

(Given that $2.303\dfrac{RT}{F} = 0.059$)

Ans. **[4]**

Sol. For spontaneity $E_{cell} > 0$

At limiting condition :

E_{Oxi} (anode) $= -E_{Red}$ (cathode)

$$H_2O \rightarrow 2H^+ + \frac{1}{2}O_2 + 2e^-$$

$$E = E^{\circ} - \frac{0.059}{2}\log\left[\frac{\left[H^+\right]^2 \times P_{O_2}^{1/2}}{1}\right]$$

$-0.997 = -1.23 + 0.059 \times pH$

$pH = 3.94$

$pH \simeq 4$

19. Find correct matching of reaction of Glucose with given reagent in column-I and product formed in column-II :

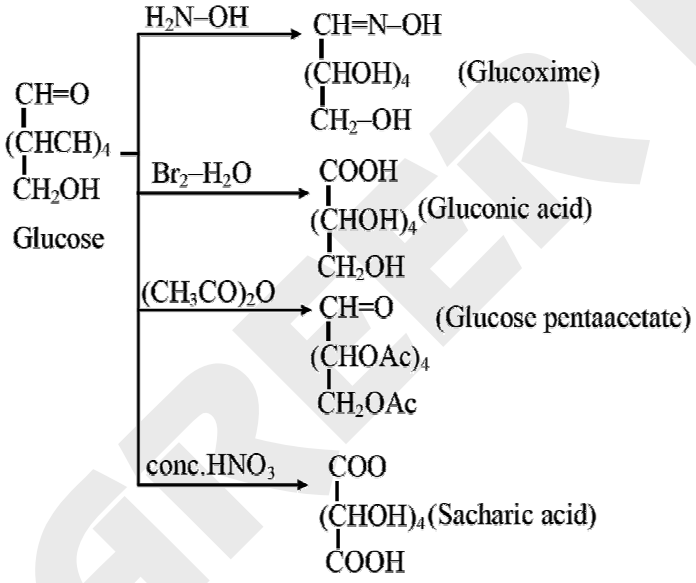
Column-I		Column-II	
(P)	Hydroxylamine	(1)	Gluconic acid
(Q)	Br_2 -water	(2)	Glucose pentaacetate
(R)	Excess of acetic anhydride	(3)	Sacharic acid
(S)	Conc. HNO_3	(4)	Glucose-oxime

(1) P → 4; Q → 1; R → 2; S → 3 (2) P → 4; Q → 3; R → 2; S → 1

(3) P → 1; Q → 3; R → 4; S → 2 (4) P → 3; Q → 1; R → 4; S → 2

Ans. **[1]**

Sol.



20. 36 g of A react with 56 g of B to form AB_2 according to given reaction :

$A + 2\ B \rightarrow AB_2$

Which of the following is correct :

(Given : Molar mass of $A = 60\ g/mol$, Molar mass of $B = 80\ g/mol$)

(1) Molecular weight of AB_2 is 140. (2) A is limiting reagent.

(3) 15 gm of A remains unreacted. (4) Weight of AB_2 is 132 gm.

Ans. **[3]**

Sol.

$$\underset{\text{Mole}}{\underset{\frac{36}{60}}{A} + \underset{\frac{56}{80}}{2B}} \longrightarrow AB_2$$

0.6 mole 0.7 mole

0.25 mole – 0.35 mole

(A) Molecular wt. of AB_2 is

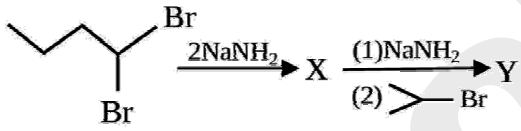
$60 + 2 \times 80 = 220$ g/mol

(B) LR is AB.

(C) Wt. of A remaining $= 0.25 \times 60 = 15$ g

(D) wt. of AB_2 formed $= 0.35 \times 220 = 77\text{gm}$.

21. Write correct IUPAC name in the final product formed in the given reaction :

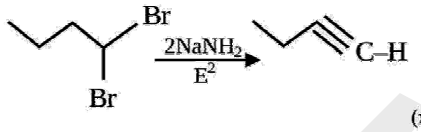


(1) Isopropyl but-1-yne (2) 2-methyl hex-2-yne

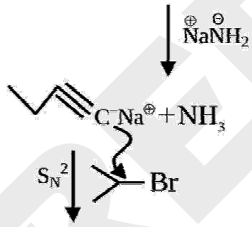
(3) 5-methyl hex-2-yne (4) 2-methyl hex-3-yne

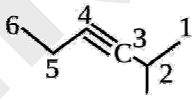
Ans. **[4]**

Sol.



(x)





2–methyl hex–3–yne

22. **Statement-I :** Correct order of ionization energy of C, N, O and F is $F > N > O > C$

Statement-II : The correct order of electron gain enthalpy (magnitude only) for group 16 elements is $S > Se > Te > O$:

(1) Statement-I is correct and Statement-II is incorrect.

(2) Statement-I and Statement II both are correct.

(3) Statement-I is incorrect while Statement-II is correct.

(4) Both Statements are incorrect.

Ans. **[2]**

Sol. $IE_1 : F > N > O > C$

Magnitude of electron gain enthalpy order is $S > Se > Te > O$

23. **Statement-I :** First ionization energy of Cr is lower than that of Mn.

Statement-II : 2^{nd} and 3^{rd} ionization energy of Cr is higher than 2^{nd} and 3^{rd} ionization energy of Mn.

(1) Statement I is correct and statement II is incorrect.
(2) Both statements I and II are correct.
(3) Statement I is incorrect and statement II is correct.
(4) Both statements I and II are incorrect.

Ans. **[1]**

Sol. $IE_1(Cr) = 653 \text{ kJ/mol}$

$IE_2(Cr) = 1592 \text{ kJ/mol}$

$IE_3(Cr) = 2990 \text{ kJ/mol}$

$IE_1(Mn) = 717 \text{ kJ/mol}$

$IE_2(Mn) = 1509 \text{ kJ/mol}$

$IE_3(Mn) = 3260 \text{ kJ/mol}$

24. 'X' is the most electronegative element and Y is the least electronegative element in the group 15 elements N, P, As and Sb

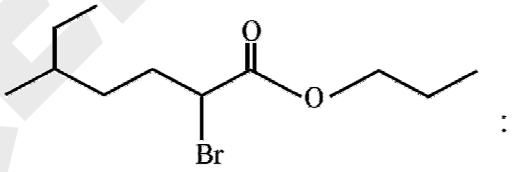
(1) X_2O_3 is acidic and Y_2O_3 is amphoteric
(2) X_2O_3 is basic and Y_2O_3 is amphoteric
(3) X_2O_3 is acidic and Y_2O_3 is basic
(4) X_2O_3 is amphoteric and Y_2O_3 is acidic

Ans. **[1]**

Sol. Electronegativity $N(3.0), P(2.1), As(2.0), Sb(1.9), Bi(1.9)$

Sb_2O_3 is amphoteric while N_2O_3 is acidic.

25. Correct IUPAC name of given compound is

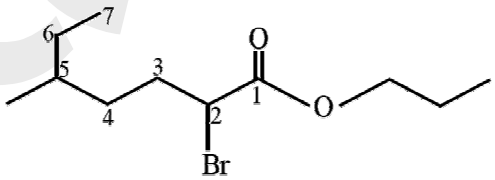


:

(1) 2-Bromo-5-Ethylhexanoate
(2) Propyl-2-Bromo-5-Methylheptanoate
(3) Propyl-2-Bromo-5-Ethylhexanoate
(4) 2-Bromo-5-Methylpropylheptanoate

Ans. **[2]**

Sol.



Propyl-2-Bromo-5-Methylheptanoate

CAREER POINT

JEE Main Online Exam 2026

Memory Based

Questions & Solution

22st January 2026 | Evening

MATHEMATICS

1. $x - ny + z = 6$

$x - (n-2)y + (n+1)z = 8$

$(n-1)y + z = 1$

Let n = numbers on the die when rolled then P (probability when system of equations has unique solution) $= \frac{k}{6}$, then the sum of values of k and all possible values of n is

(1) 20 (2) 21 (3) 22 (4) 24

Ans. **[4]**

Sol. $x - ny + z = 6$

$x - (n-2)y + (n+1)z = 8$

$(n-1)y + z = 1$

$$\begin{vmatrix} 1 & -n & 1 \\ 1 & -(n-2) & n+1 \\ 0 & n-1 & 1 \end{vmatrix} = 0$$

$\Rightarrow n = 2$ or $n = -1$ (rejected)

for unique solution $n = 1,3,4,5,6$

Now

P (probability when system of equations has unique solution) $= \frac{5}{6}$

So $k = 5$

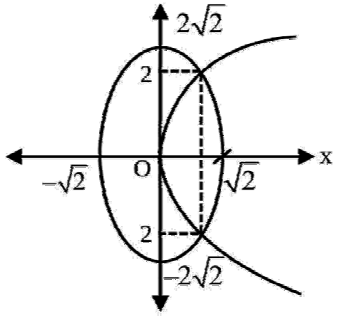
Now required sum $= 5 + (1+3+4+5+6) = 24$

2. Area enclosed by $4x^2 + y^2 \le 8$ and $y^2 \le 4x$ (in sq. units)

(1) $\left(\pi + \frac{4}{3}\right)$ (2) $\pi - \frac{4}{3}$ (3) $\pi + \frac{2}{3}$ (4) $\pi - \frac{2}{3}$

Ans. **[3]**

Sol.



$$A = \int_0^1 2\sqrt{x}\,dx + 2\int_1^{\sqrt{2}} \sqrt{8-4x^2}\,dx$$

$$= \frac{8}{3}\left(x^{\frac{3}{2}}\right)\Big|_0^1 + 4\int_1^{\sqrt{2}} \sqrt{2-x^2}\,dx$$

$$= \frac{8}{3} + 4\times\frac{1}{2}\left[x\sqrt{2-x^2} + 2\sin^{-1}\left(\frac{x}{\sqrt{2}}\right)\right]_1^{\sqrt{2}}$$

$$= \frac{8}{3} + 2\left[2\times\frac{\pi}{2} - 1 - 2\times\frac{\pi}{4}\right]$$

$$= \frac{8}{3} + 2\pi - 2 - \pi = \pi + \frac{2}{3}$$

Sq. unit

3. $\alpha = \int_0^{64}\left(x^{\frac{1}{3}} + \left[x^{\frac{1}{3}}\right]\right)dx$, where [.] denotes greatest integer function. Find $\frac{1}{\pi}\int_0^{\alpha\pi} \frac{\sin^2\theta}{\sin^6\theta+\cos^6\theta}d\theta$

(1) 160 (2) 174 (3) 180 (4) 183

Ans. **[2]**

Sol. $\because \int_0^{64} x^{\frac{1}{3}}dx = \frac{3}{4}\cdot\left[x^{\frac{4}{3}}\right]_0^{64} = 192$ &

$$\int_0^{64}\left[x^{1/3}\right]dx = \int_0^1\left[x^{1/3}\right]dx + \int_1^8\left[x^{1/3}\right]dx + \int_8^{27}\left[x^{1/3}\right]dx + \int_{27}^{64}\left[x^{1/3}\right]dx = 156$$

So $\alpha = 156 + 192 = 348$

Now $E = \frac{1}{\pi}\int_0^{348\pi} \frac{\sin^2\theta}{\sin^6\theta+\cos^6\theta}d\theta$

$$= \frac{348}{\pi}\int_0^{\pi} \frac{\sin^2\theta}{\sin^6\theta+\cos^6\theta}d\theta$$

$$\Rightarrow E = \frac{348\cdot 2}{\pi}\int_0^{\pi/2} \frac{d\theta}{\sin^6\theta+\cos^6\theta}$$

Let $J = \int_0^{\pi/2} \frac{\sin^2\theta}{\sin^6\theta+\cos^6\theta}\,d\theta$ .....(1)

Applying King

$$J = \int_0^{\pi/2} \frac{\cos^2\theta}{\sin^6\theta+\cos^6\theta}\,d\theta$$ .....(2)

Now $2J = \int_0^{\pi/2} \frac{1}{\sin^6\theta + \cos^6\theta}\, d\theta$ $\quad (\text{add}(1)\,\&\,(2))$

$= \int_0^{\pi/2} \frac{\sec^6\theta}{\tan^6\theta + 1} d\theta$

$= \int_0^{\infty} \frac{\left(1+\lambda^2\right)^2}{\lambda^4 - \lambda^2 + 1}\, d\lambda$

$= \int_0^{\infty} \frac{1 + \frac{1}{\lambda^2}}{\lambda^2 - 1 + \frac{1}{\lambda^2}} d\lambda$

$= \frac{\pi}{2}$

$\Rightarrow J = \frac{\pi}{4}$

$\Rightarrow E = \frac{348 \cdot 2}{\pi} \times J = 174$

4. Let the domain of function $f(x) = \log_3\left(\log_5\left(7 - \log_2\left(x^2 - 10x + 85\right)\right)\right) + \sin^{-1}\left(\left|\frac{3x-7}{17-x}\right|\right)$ be $(\alpha, \beta]$ then value of $\alpha + \beta$ is equal to :

Ans. **[9]**

Sol. Let $x^2 - 10x + 85 = \lambda$

$\therefore$ Domain for first term

$\lambda > 0$ ...(1)

$7 - \log_2\lambda > 0 \Rightarrow \lambda < 2^7$ ...(2)

$\log_5\left(7 - \log_2\lambda\right) > 0 \Rightarrow \lambda < 2^6$ ...(3)

$\therefore$ from (1), (2) & (3)

$0 < \lambda < 2^6$

$0 < x^2 - 10x + 85 < 64$

$\Rightarrow x \in (3,7)$ ...(A)

& domain for second term $-1 \le \frac{3x-7}{x-17} \le 1$

$\Rightarrow x \in [-5,6]$ ...(B)

From (A) & (B), domain of function will be (3,6]

$\Rightarrow \alpha = 3, \beta = 6$

$\Rightarrow \alpha + \beta = 9$

5. Let α, β be the roots of quadratic equation $12x^2 - 20x + 3\lambda = 0, \lambda \in Z$. If $\frac{1}{2} \le |\beta - \alpha| \le \frac{3}{2}$, then the sum of all the possible values of λ is :

(1) 2 (2) 3 (3) 6 (4) 5

Ans. **[2]**

Sol. $\frac{1}{2} \le |\alpha - \beta| \le \frac{3}{2}$

$\frac{1}{4} \le |\alpha - \beta|^2 \le \frac{9}{4}$

$\frac{1}{4} \le (\alpha + \beta)^2 - 4\alpha\beta \le \frac{9}{4}$

$\frac{1}{4} \le \frac{25}{9} - 4 \times \frac{\lambda}{4} \le \frac{9}{4}$

$-\frac{91}{36} \le -\lambda \le \frac{-19}{36}$

$\frac{19}{36} \le \lambda \le \frac{91}{36}$

$\lambda = 1, 2$

Sum $= 3$

6. If $\sin(\alpha - \beta) = \frac{3}{8}, \cos(\alpha + \beta) = \frac{-1}{10}$ and $\tan 2\alpha = \frac{3(1 - r\sqrt{5})}{\sqrt{11}(s + \sqrt{5})}$ then $r + s =$?

(Given: $0 < (\alpha - \beta) < \frac{\pi}{2}$ & $\frac{\pi}{2} < \alpha + \beta < \pi$)

(1) 10 (2) 14 (3) 18 (4) 20

Ans. **[4]**

Sol. $\tan 2\alpha = \tan\left[(\alpha + \beta) + (\alpha - \beta)\right]$

$\tan 2\alpha = \frac{\tan(\alpha + \beta) + \tan(\alpha - \beta)}{1 - \tan(\alpha + \beta) \cdot \tan(\alpha - \beta)}$

$\tan 2\alpha = \frac{\left(-\sqrt{99} + \frac{3}{\sqrt{55}}\right)}{1 - \left(\sqrt{99}\right)\left(\frac{3}{\sqrt{55}}\right)}$

$\tan 2\alpha = \frac{-3\sqrt{11} + \frac{3}{\sqrt{5} \times \sqrt{11}}}{1 + \frac{9\sqrt{11}}{\sqrt{5} \times \sqrt{11}}}$

$\tan 2\alpha = \frac{3(1 - 11\sqrt{5})}{\sqrt{11}(9 + \sqrt{5})}$

$r = 11,\ s = 9$

$r + s = 20$

7. If complex numbers $z_1, z_2, z_3, \ldots, z_n$ satisfying the equation $4z^2 + \bar{z} = 0$, then $\sum_{i=1}^{n} |z_i|^2 =$

(1) $\frac{3}{16}$ (2) $\frac{3}{64}$ (3) $\frac{9}{64}$ (4) $\frac{1}{16}$

Ans. **[1]**

Sol. $4z^2 + \bar{z} = 0$

Let $z = x + iy$

$4(x+iy)^2 + x - iy = 0$

$4x^2 - 4y^2 + 8xyi + x - iy = 0$

$4x^2 - 4y^2 + x = 0 \;\&\; y(8x-1) = 0$

$\Rightarrow y = 0$ or $x = \frac{1}{8}$

If $y = 0, 4x^2 + x = 0$

$x = 0, \frac{-1}{4}$

$\therefore z_1 = 0 + 0i \qquad |z_1|^2 = 0$

$z_2 = 0 - \frac{1}{4}i \qquad |z_2|^2 = \frac{1}{16}$

If $x = \frac{1}{8}$,

$4 \times \frac{1}{64} - 4y^2 + \frac{1}{8} = 0$

$\Rightarrow 4y^2 = \frac{3}{16} \Rightarrow y = \pm\frac{\sqrt{3}}{8}$

$\therefore z_3 = \frac{1}{8} + \frac{\sqrt{3}}{8}i \qquad |z_3|^2 = \frac{1}{64} + \frac{3}{64} = \frac{1}{16}$

$z_4 = \frac{1}{8} - \frac{\sqrt{3}}{8}i \qquad |z_4|^2 = \frac{1}{64} + \frac{3}{64} = \frac{1}{16}$

$\therefore \sum_{i=1}^{n} |z_i|^2 = 0 + \frac{1}{16} + \frac{1}{16} + \frac{1}{16} = \frac{3}{16}$

8. If $\frac{\cos y}{1-2\sin y}dy = \frac{dx}{16\sqrt{(9\sqrt{x}+x)4+\sqrt{9+\sqrt{x}}}}$ and $f(256) = \frac{\pi}{2}$ and $f(49) = \alpha$, the find $(2\sin\alpha)$

(1) $1 + \frac{3}{2\sqrt{2}}$ (2) $1 - \frac{3}{2\sqrt{2}}$ (3) $1 + \frac{3}{4\sqrt{2}}$ (4) $1 - \frac{3}{4\sqrt{2}}$

Ans. **[1]**

Sol. $\int \frac{\cos y}{1-2\sin y}dy = \int \frac{dx}{16\sqrt{(9\sqrt{x}+x)4+\sqrt{9+\sqrt{x}}}}$

$4 + \sqrt{9+\sqrt{x}} = t$

$\frac{1}{2\sqrt{9+\sqrt{x}}} \times \frac{dx}{2\sqrt{x}} = 1dx$

$-\frac{1}{2}\ell n|1-2\sin y| = \int \frac{4dt}{16t} + C$

$-\frac{1}{2}\ell n|1-2\sin y| = \frac{1}{4}\ell n\left|4+\sqrt{9+\sqrt{x}}\right| + C$

$$-\frac{1}{2}\ell n(2\sin y-1)=\frac{1}{4}\ell n\left|x+\sqrt{9+\sqrt{x}}\right|+C$$

$$\left(2\sqrt{6},\frac{\pi}{2}\right)$$

$$\frac{1}{4}\ell n9+C=0\ C=\frac{-1}{4}\ell n9$$

$$(49,\alpha)-\frac{1}{2}\ell n(2\sin\alpha-1)=\frac{1}{4}\ell n8-\frac{1}{4}\ell n9=\frac{1}{4}\ell n\frac{8}{9}$$

$$-\ell n(2\sin\alpha)=\frac{1}{2}\ell n\frac{8}{9}$$

$$\ell n(2\sin\alpha-1)=\ell n\frac{3}{2\sqrt{2}}$$

$$2\sin-1=\frac{3}{2\sqrt{2}}$$

$$2\sin\alpha=\frac{3+2\sqrt{2}}{2\sqrt{2}}$$

$$=1+\frac{3}{2\sqrt{2}}$$

9. If $x=\begin{bmatrix}x\\y\\z\end{bmatrix}$ is a solution of system of equation $AX=B$ where $A\text{Adj}A=\begin{pmatrix}4&2&2\\-5&0&5\\1&-2&3\end{pmatrix}$ & $B=\begin{pmatrix}4\\0\\2\end{pmatrix}$ then $|x+y+z|$ is

(1) 3 (2) 2 (3) $\frac{3}{2}$ (4) 1

Ans. **[2]**

Sol. $$X=A^{-1}\ B=\left(\frac{\text{adj}A}{|A|}\right)B$$

$$=\pm\frac{1}{10}\begin{pmatrix}4&2&2\\-5&0&5\\1&-2&3\end{pmatrix}\begin{pmatrix}4\\0\\2\end{pmatrix}$$

$$=\pm\frac{1}{10}\begin{pmatrix}20\\-10\\10\end{pmatrix}=\pm\begin{pmatrix}2\\-1\\1\end{pmatrix}$$

$$\therefore|x+y+z|=2$$

10. If $\lim\limits_{x\to0}\dfrac{e^{(a-1)x}-2\cos bx+e^{-x}(c-2)}{x\cos x-\ell n(1+x)}=2$, then find $a^2+b^2+c^2$

(1) 11 (2) 12 (3) 13 (4) 14

Ans. **[3]**

Sol. $$\lim_{x\to 0}\frac{\left(1+(a-1)x+\frac{(a-1)^2x^2}{2!}\right)-2\left(1-\frac{b^2x^2}{2!}\right)+(c-2)\left(1-x+\frac{x^2}{2!}\right)}{x\left(1-\frac{x^2}{2!}\right)-\left(x-\frac{x^2}{2}\ldots\right)}=2$$

$$\lim_{x\to 0}\frac{(1-2+c-2)+x(a-1-c+2)+x^2\left(\frac{(a-1)^2}{2}+b^2+\left(\frac{c-2}{2}\right)\right)}{\frac{x^2}{2}-\frac{x^3}{2!}+\ldots}=2$$

For which

$\because c-3=0\Rightarrow c=3$

$\because a-c=-1\Rightarrow a=2$

$\because \frac{(a-1)^2}{2}+b^2+\left(\frac{c-2}{2}\right)=1\Rightarrow b^2=0$

$a^2+b^2+c^2=4+0+9=13$

11. Let P,M,Q be the points on $\frac{x+1}{2}=\frac{y+1}{3}=\frac{z+3}{6}$ where M also lies on $\frac{x+1}{2}=\frac{y+1}{3}, z=9$ and $PM=MQ=7$, then find the sum of co-ordinates of P & Q :

(1) 34 (2) 6 (3) 28 (4) 55

Ans. **[1]**

Sol. M is the point of intersection of L_1 & L_2

$\Rightarrow 2\lambda-1=2\mu-1, 3\lambda-1=3\mu-1, 6\lambda-3=9$

$\Rightarrow \lambda=2=\mu$

$\Rightarrow M(3,5,9)$

Now let point P be ($2K-1, 3-1, 6K-3$) on L_2 such that $PM=7$

$\Rightarrow \sqrt{(2K-4)^2+(3K-6)^2+(6K-12)^2}=7$

$\Rightarrow 49K^2+196-196K=49$

$\Rightarrow K^2+4-4K=1$

$\Rightarrow K^2-4K+3=0$

$\Rightarrow K=1,3$

So points P & Q are $(1,2,3)$ & $(5,8,15)$

So sum of all co-ordinates of P & Q = 34

12. If $A=\{1,2,3\ldots\ldots\ldots 11\}$ let B is a subset of A such that $n(B)\geq 2$ & the product of all elements in B is even, then the number of such possible sets B is

Ans. **[1979]**

Sol. A $\{1,2,3\ldots..11\}$

$\therefore n(B)\geq 2$ & product of all elements in B is even

Case (i) $n(B)=2\Rightarrow {}^{11}C_2-{}^{6}C_2$

$n(B)=3\Rightarrow {}^{11}C_3-{}^{6}C_3$

$n(B)=4\Rightarrow {}^{11}C_4-{}^{6}C_4$

$n(B)=5 \Rightarrow {}^{11}C_5 - {}^{6}C_5$

$n(B)=6 \Rightarrow {}^{11}C_6 - {}^{6}C_6$

$n(B)=7 \Rightarrow {}^{11}C_7$

:

:

$n(B)=11 \Rightarrow {}^{11}C_{11}$

$\therefore$ number of set $B \Rightarrow \sum_{r=2}^{11} {}^{11}C_r - \sum_{r=2}^{6} {}^{6}C_r$

$= 2^{11} - (12) - (2^6 - 7)$

$= 2048 - 64 - 5$

$= 1979$

Alternate sol.

Total subsets $= 2^{11}$

No. of subsets having odd terms only $= 2^6$

No. of subsets having one term only & also having even terms $= 5$

Req. ways $= 2^{11} - 2^6 - 5 = 1979$

13. $f(x+y)=f(x)\cdot f(y), g(x+y)=g(xy), f(1)=7, g(1)=1$. Given $\sum_{x=1}^{n} \frac{f(x)}{g(x)} = 19607$. Then the find the value of n is

Ans. **[5]**

Sol. $f(x+y)=f(x)\cdot f(y) \Rightarrow f(x)=a^x$

$(\because f(1)=7 \Rightarrow = a^1 = 7)$

So $f(x)=7^x$

Now

$g(x+y)=g(xy)$ (put $y=1$)

$\Rightarrow g(x+1)=g(x)$

so $g(1)=g(2)=g(3)=\ldots=g(n)=1$

Given $\sum_{x=1}^{n} \frac{f(x)}{g(x)} = 19607$

$\sum_{x=1}^{n} \frac{7^x}{1} = 19607$

$\Rightarrow 7\left(\frac{7^n - 1}{7-1}\right) = 19607$

$7^n - 1 = \frac{6}{7} \times 19607$

$7^n = 16807 \Rightarrow n = 5$

14. A point $P\left(10,2\sqrt{15}\right)$ lies on hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ & length of its latus rectum = 8. If S_1 and S_2 are foci then find the square of the area of ΔPS_1S_2.

(1) 1800 (2) 900 (3) 2700 (4) 3000

Ans. **[3]**

Sol. $P\left(10,2\sqrt{15}\right)$ lies on $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

$\therefore \frac{100}{a^2}-\frac{60}{b^2}=1$ ...(1)

$\because$ length of latus rectum $=8$

$\frac{2\cdot b^2}{a}=8\Rightarrow \frac{b^2}{a}=4$ ...(2)

From (1) & (2)

$\frac{100}{a^2}-\frac{60}{4a}=1$

$400-60a=4a^2$

$4a^2+60a-400=0$

$a^2+15a-100=0$

$a=5\ \&\ -20$ (rejected)

$\Rightarrow b=\sqrt{20}$

$\therefore$ Hyperbola is $\frac{x^2}{25}-\frac{y^2}{20}=1$

$\therefore$ Focal length $S_1S_2=2ae=2.5\cdot\left(\sqrt{1+\frac{4}{5}}\right)=6\sqrt{5}$

$\therefore$ Area of $\Delta PS_1S_2=\frac{1}{2}\cdot 6\sqrt{5}\cdot 2\sqrt{15}=30\sqrt{3}=A$

$\therefore A^2=2700$

15. If the mean deviation about the median of the numbers $k,2k,3k,\ldots,1000k$ is 500, then k^2 is equal to

(1) 4 (2) 8 (3) 12 (4) 16

Ans. **[1]**

Sol. $\because$ median $=\frac{1001k}{2}=X_M$

$\therefore$ mean deviation about median $=\frac{\sum|X_i-X_M|}{n}$

$=\frac{2\left(\frac{k}{2}+\frac{3k}{2}+\frac{5k}{2}+\ldots 500\text{ terms}\right)}{1000}$

$=\frac{2\cdot\frac{k}{2}(500)^2}{1000}=\frac{500k}{2}=500$ (given)

$\therefore k=2$

$\therefore k^2=4$

16. Let $P(\alpha,\beta)$ be a point on the ellipse $\frac{x^2}{25}+\frac{y^2}{9}=1$ if $PS^2+PS'^2-PS.PS'=37$ where S,S' are focii of ellipse, then the value of $(\alpha^2+\beta^2)$ is

Ans. **[13]**

Sol. $\because$ P lies on ellipse $\Rightarrow \frac{\alpha^2}{25}+\frac{\beta^2}{9}=1$

$\because PS+PS'=2a \Rightarrow PS+PS'=10$

$\therefore (PS)^2+(PS')^2-PS'PS'=37$

$(PS+PS')^2-3PS.PS'=37$

$100-3\ PS.\ PS'=37$

$\underline{3PS.PS'}=63 \Rightarrow \underline{PS.PS'}=21$

$\because PS \& PS'$ are $\left(5\pm\frac{4}{5}\cdot\alpha\right)$

$\therefore \underline{PS.PS'}=25-\frac{16}{25}\alpha^2=21$

$\frac{16}{25}\alpha^2=4$

$\alpha=\frac{5}{2}\Rightarrow\alpha^2=\frac{25}{4}$

$\therefore \beta^2=\frac{27}{4}$

$\therefore \alpha^2+\beta^2=\frac{52}{4}=13$

17. If $P_n=\sum_{r=0}^{n}\frac{(-2)^r\cdot{}^nC_r}{r+1}$, then find the value of $\sum_{n=1}^{25}P_{2n}$

(1) 575 (2) 675 (3) 600 (4) 500

Ans. **[1]**

Sol. $P_n=\sum_{r=0}^{n}\frac{{}^nC_r(-2)^r}{r+1}=\sum_{r=0}^{n}\frac{1}{(n+1)}{}^{n+1}C_{r+1}(-2)^r$

$=\frac{-1}{2(n+1)}\sum_{r=0}^{n}{}^{n+1}C_{r+1}(-2)^{r+1}$

$=\frac{-1}{2(n+1)}\left[(1-2)^{n+1}-1\right]$

$P_n=\frac{1}{2(n+1)}\left[1-(-1)^{n+1}\right]$

$P_{2n}=\frac{1}{2(n+1)}\left[1-(-1)^{2n+1}\right]$

$P_{2n}=\frac{1}{2n+1}$

$$\sum_{n=0}\frac{1}{P_{2n}}=\sum_{n=1}^{25}(2n+1)$$
$$=3+5+\ldots+51$$
$$=\frac{25}{2}[51+3]$$
$$=25\times 27=575$$

18. If $f(x)=[x]^2-[x+3]-3,[\cdot]$ denotes G.I.F. then

(1) $f(x)=0$ has finitely many solutions

(2) $\int_0^2 f(x)dx=-6$

(3) $f(x)<0$ only in $[-1,3)$

(4) $f(x)>0$ only in $(-2,4)$

Ans. **[3]**

Sol. (A) $f(x)=0\Rightarrow[x]=3$ or $[x]=-2$

$\Rightarrow x\in[3,4)\cup[-2,-1)$

$\Rightarrow f(x)=0$ then infinite solutions

$f(x)=[x]^2-[x+3]-3$

$=[x]^2-[x]-6$

$=([x]-3)([x]+2)$

(B) $\int_0^2 f(x)dx=\int_0^2([x]^2-[x]-6)$

$\int_0^2 f(x)dx=\int_0^2([x]^2-[x]-6)dx$

$=\int_0^2([x]^2-[x])dx-12$

$=-12$

(C) $f(x)<0\Rightarrow([x]-3)([x]+2)<0$

$\Rightarrow -2<[x]<3$

$\Rightarrow -1\le x<3$

(D) $f(x)>0[x]<-2$ or $[x]>3$

$\Rightarrow x<-2$ or $x\ge 4$

$\Rightarrow x\in(-x,-2)0[4,\infty]$

19. Let $f(x)=\min\{\sqrt{2}x,x^2\}$ & $g(x)=|x|[x^2]$ & let S be the set such that $S=\{x,x\in(-2,2);x$ is the point of discontinuity of $g(x)\}$, then $\sum_{x\in S}f(x)$

(1) $1-\sqrt{2}$ (2) $1-\sqrt{3}$ (3) $1+\sqrt{2}$ (4) $1+\sqrt{3}$

Ans. **[1]**

Sol. $g(x)=|x|\left[x^2\right]$

points of discontinuity of $g(x)$ in $(-2,2)$ are $\left(\pm1,\pm\sqrt{2},\pm\sqrt{3}\right)$

$\therefore S=\left\{-1,1,-\sqrt{2},\sqrt{2},-\sqrt{3},\sqrt{3}\right\}$

$\because f(x)=\min\left\{\sqrt{2}x,x^2\right\}$

$\therefore \sum_{x\in S} f(x)=-\sqrt{2}+1-2+2-\sqrt{6}+\sqrt{6}$

$=1-\sqrt{2}$

20. $\vec{a}=\sqrt{2}\hat{i}+\hat{j}+\lambda\hat{k}$

$\vec{b}=-\lambda^2\hat{i}-4\sqrt{2}\hat{j}+4\sqrt{2}\hat{k}$, where $\vec{a}\wedge\vec{b}$ is obtuse and $\vec{a}\,\hat{\wedge}\,\hat{k}\in\left(\frac{\pi}{6},\frac{\pi}{2}\right)$ $\lambda\in(\alpha,\beta)-\{\gamma\}$, then the value of $\alpha+\beta+\gamma$ is ____

Ans. **[5]**

Sol. $\vec{a}\,\hat{\wedge}\,\hat{k}\in\left(\frac{\pi}{2},\frac{\pi}{2}\right)$

$\frac{\lambda}{\sqrt{(2+1+\lambda^2)}}\in\left(0,\frac{\sqrt{3}}{2}\right)\Rightarrow 0<\frac{\lambda}{\sqrt{3+\lambda^2}}<\frac{\sqrt{3}}{2}$

$\lambda>0\ \&\ \lambda\in(-3,3)\Rightarrow\lambda\in(0,3)\ldots(1)$

$\vec{a}\wedge\vec{b}$ is obtuse

$\vec{a}\cdot\vec{b}<0\Rightarrow-\sqrt{2}\lambda^2-4\sqrt{2}+4\sqrt{2}\lambda<0$

$\Rightarrow\lambda^2-4\lambda+4>0\Rightarrow(\lambda-2)^2>0$

$\Rightarrow\lambda\in R-\{2\}\ldots(2)$

(1) & (2) $=1\lambda\in(0,3)-\{2\}\Rightarrow\alpha+\beta+\gamma=5$

21. If a,b,c are in A.P. $(a>b>c)$, $a^2,2b^2,c^2$ are in G.P. and $a+b+c=1$ then find $9\left(a^2+b^2+c^2\right)=$?

(1) 9 (2) 18 (3) 27 (4) 12

Ans. **[1]**

Sol. $a=b+d,c=b-d,\Rightarrow b=\frac{1}{3}$

$\Rightarrow 4\,b^4=a^2c^2$

$4b^4=[(b-d)(b+d)]^2$

$\frac{4}{81}=\left(\frac{1}{9}-d^2\right)^2$

$\Rightarrow\frac{4}{81}=\frac{1}{81}-\frac{2d^2}{9}+d^4$

$\Rightarrow d^4-\frac{2d^2}{9}-\frac{1}{27}=0$

$\Rightarrow 27d^4-6d^2-1=0$

$d^2 = 1/3 \Rightarrow d = +\frac{1}{\sqrt{3}} (as\ a > b > c)$

$9\left(a^2 + b^2 + c^2\right) =$

$9\left[\left(\frac{1}{3} + \frac{1}{\sqrt{3}}^{\ 2}\right) + \left(\frac{1}{3}\right)^2 + \left(\frac{1}{3} - \frac{1}{\sqrt{3}}\right)^2\right]$

$= 9\left[\frac{1}{3} + \frac{2}{3}\right] = 3 + 6 = 9$

22. Let S be the locus of mid points of chords joining any point P on the parabola $y^2 = 4x$ to the origin. Now, R is the locus of a point dividing line segment OQ in the ratio 3 : 1 (internally) where Q is any point on the locus S , then equation of locus of R is:

(1) $3y^2 = 2x$ (2) $2y^2 = 3x$ (3) $9y^2 = 4x$ (4) $4y^2 = 3x$

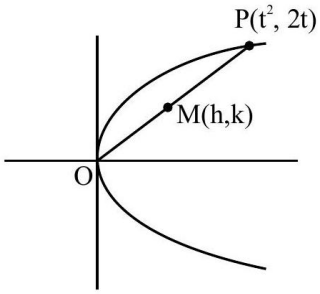
Ans. **[2]**

Sol. $y^2 = 4x$

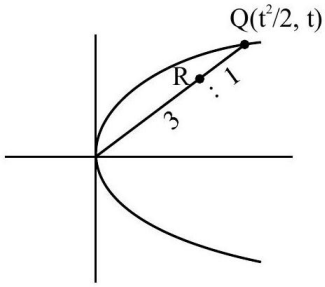
Locus of mid point of OP

$M(h,k) \Rightarrow h = \frac{t^2}{2}, k = t$

$\Rightarrow k^2 = 2\,h \Rightarrow y^2 = 2x$



$S : y^2 = 2x$



R (h, k)

$\Rightarrow h = \frac{\frac{3t^2}{2}}{4}, k = \frac{3t}{4}$

$t^2 = \frac{8\,h}{3}, t = \frac{4k}{3}$

$\Rightarrow \frac{16k^2}{9} = \frac{8\,h}{3} \Rightarrow 2k^2 = 3\,h$

Locus of R : $2y^2 = 3x$

23. **Statement 1 :** If two vertices of triangle ABC are $(-2,3)$ & $(5,-1)$ & orthocentre of triangle is $(0,0)$ then third vertex of triangle is $(-4,-7)$.

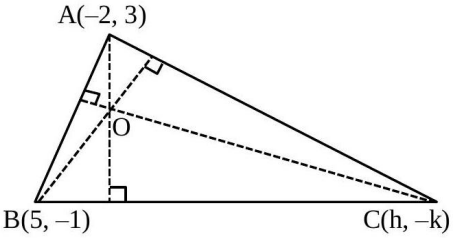
Statement 2 : If 2a, b, c are in A.P., then lines ax + by + c = 0 are concurrent at point $(2,-2)$.

(1) Both statement-1 and statement-2 are correct
(2) Both statement-1 and statement-2 are incorrect
(3) Statement-1 is correct and statement-2 is incorrect
(4) Statement-1 is incorrect and statement-2 is correct

Ans. **[1]**

Sol. Solution of statement-1

$m_{AO} \cdot m_{BC} = -1$



$\Rightarrow 2h - 3k = 13 \quad \ldots(1)$

& $m_{AB} \cdot m_{OC} = -1$

$\Rightarrow 4k = 7h \quad \ldots (2)$

$\Rightarrow$ third vertex is $(-4,-7)$

Solution of statement-2

$2a, b, c \rightarrow$ A.P.

$b = \dfrac{2a+c}{2}$

$\Rightarrow 2a - 2b + c = 0$

$\because$ lines $ax + by + c = 0$ are concurrent then $\dfrac{x}{2} = \dfrac{y}{-2} = \dfrac{1}{1}$

$x = 2$ and $y = -2$

$\therefore$ Pt. of concurrency is $(2,-2)$

$\therefore$ Statement 2 is correct.

24. The number of elements in the relation $R = \{(x,y) : 4x^2 + y^2 < 52, x, y, \in \mathbb{Z}\}$ is

(1) 85 (2) 77 (3) 104 (4) 95

Ans. **[2]**

Sol. $4x^2 + y^2 < 52,\ x, y \in z$

$\downarrow \quad \downarrow$

$0 \quad 0, \pm1, \pm2, \pm3, \pm4, \pm5, \pm6, \pm7 \rightarrow 1 \times 15 = 15$

$\pm1 \quad 0, \pm1, \pm2, \pm3 \ldots\ldots\ldots \pm 6 \rightarrow 2 \times 13 = 26$

$\pm2 \quad 0, \pm1, \pm2, \pm3 \ldots\ldots\ldots \pm 5 \rightarrow 2 \times 11 = 22$

$\pm3 \quad 0, \pm1, \pm2 \pm 3 \rightarrow 2 \times 7 = 14$

Number of elements = 77