JEE Main Online Exam 2025

Questions & Solution

08[nd] April 2025 | evening

MATHEMATICS

Section-A: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct..

Q.1 Let the ellipse $3x^2 + py^2 = 4$ pass through the centre C of the circle $x^2 + y^2 - 2x - 4y - 11 = 0$ of radius r. Let f_1, f_2 be the focal distances of the point C on the ellipse. Then $6f_1f_2 - r$ is equal to

(1) 68 (2) 74 (3) 70 (4) 78

Ans. **[3]**

Sol. $x^2 + y^2 - 2x - 4y - 11 = 0$

Centre C(1, 2)

radius $= \sqrt{1+4+11}$

$= 4$

Ellipse $3x^2 + py^2 = 4$ passes through (1, 2) $3(1) + p(4) = 4$

$4p = 1$

$p = \frac{1}{4}$

$E : 3x^2 + \frac{y^2}{4} = 4$

or $\frac{x^2}{\frac{4}{3}} + \frac{y^2}{16} = 1$

$e = \sqrt{1 - \frac{\frac{4}{3}}{16}}$

$e = \sqrt{1 - \frac{1}{12}}$

$e = \sqrt{\frac{11}{12}}$

Focus $= \left(0, \pm 2\sqrt{\frac{11}{3}}\right)$

$f_1 = \sqrt{1 + \left(2 - 2\sqrt{\frac{11}{3}}\right)^2}$ and $f_2 = \sqrt{1 + \left(2 + 2\sqrt{\frac{11}{3}}\right)^2}$

$f_1f_2 = \frac{37}{3}$

$\Rightarrow 6f_1f_2 - r = 74 - 7$

$= 70$

Q.2 The integral $\int_{-1}^{\frac{3}{2}} \left(\left|\pi^2 x \sin(\pi x)\right|\right) dx$ is equal to :

(1) $3 + 2\pi$ (2) $2 + 3\pi$

(3) $4 + \pi$ (4) $1 + 3\pi$

Ans. **[4]**

Sol. $I = \int_{-1}^{\frac{3}{2}} |\pi^2 x \sin(\pi x)|\, dx$

$= \int_{-1}^{1} |\pi^2 x \sin(\pi x)|\, dx + \int_{1}^{\frac{3}{2}} |\pi^2 x \sin(\pi x)|\, dx$

$= 2\int_{0}^{1} |\pi^2 x \sin(\pi x)|\, dx - \pi^2 \int_{1}^{3/2} |x \sin(\pi x)|\, dx$

$= 2\pi^2 \int_{0}^{1} |x \sin(\pi x)|\, dx - \pi^2 \int_{1}^{3/2} |x \sin(\pi x)|\, dx$

$\because \int x \sin(\pi x)\, dx = x\left(\frac{-\cos \pi x}{\pi}\right) - \int \frac{-\cos \pi x}{\pi} dx$

$= -\frac{x}{\pi}\cos\pi x + \frac{1}{\pi^2}\sin \pi x + C$

$\therefore I = 2\pi^2\left(\frac{1}{\pi}\right) - \pi^2\left(-\frac{1}{\pi^2} - \frac{1}{\pi}\right)$

$= 2\pi + 1 + \pi$

$= 3\pi + 1$

Q.3 The value of

$\cot^{-1}\left(\frac{\sqrt{1+\tan^2(2)} - 1}{\tan(2)}\right) - \cot^{-1}\left(\frac{\sqrt{1+\tan^2\left(\frac{1}{2}\right)} + 1}{\tan\left(\frac{1}{2}\right)}\right)$

is equal to

(1) $\pi - \frac{3}{2}$ (2) $\pi - \frac{5}{4}$

(3) $\pi + \frac{3}{2}$ (4) $\pi + \frac{5}{2}$

Ans. [2]

Sol. $\cot^{-1}\left(\frac{\sqrt{1+\tan^2(2)}-1}{\tan(2)}\right) - \cot^{-1}\left(\frac{\sqrt{1+\tan^2\left(\frac{1}{2}\right)}+1}{\tan\left(\frac{1}{2}\right)}\right)$

$= \cot^{-1}\left(\frac{|\sec 2|-1}{\tan 2}\right) - \cot^{-1}\left(\frac{\left|\sec\left(\frac{1}{2}\right)\right|+1}{\tan\frac{1}{2}}\right)$

$= \cot^{-1}\left(\frac{-\sec 2-1}{\tan 2}\right) - \cot^{-1}\left(\frac{\sec\frac{1}{2}+1}{\tan\frac{1}{2}}\right)$

$= \pi - \cot^{-1}\left(\frac{1+\cos 2}{\sin 2}\right) - \cot^{-1}\left(\frac{1+\cos\frac{1}{2}}{\sin\frac{1}{2}}\right)$

$= \pi - \cot^{-1}\left(\frac{2\cos^2 1}{2\sin 1\cdot\cos 1}\right) - \cot^{-1}\left(\frac{2\cos^2\frac{1}{4}}{2\sin\frac{1}{4}\cdot\cos\frac{1}{4}}\right)$

$= \pi - \cot^{-1}(\cot 1) - \cot^{-1}\left(\cot\frac{1}{4}\right)$

$= \pi - 1 - \frac{1}{4}$

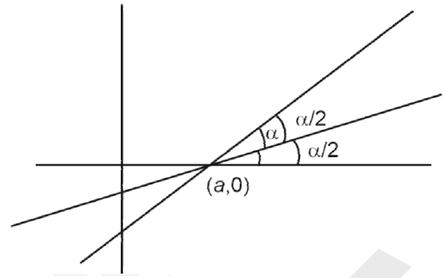
$= \pi - \frac{5}{4}$

Q.4 A line passing through the point P(a, 0) makes an acute angle α with the positive x-axis. Let this line be rotated about the point P through an angle $\frac{\alpha}{2}$ in the clock-wise direction. If in the new position, the slope of the line is $2 - \sqrt{3}$ and its distance from the origin is $\frac{1}{\sqrt{2}}$, then the value of $3a^2\tan^2\alpha - 2\sqrt{3}$ is

(1) 8 (2) 6 (3) 5 (4) 4

Ans. [4]

Sol.



$\tan\frac{\alpha}{2} = 2 - \sqrt{3}$

$\Rightarrow \tan\alpha = \frac{1}{\sqrt{3}}$

Equation of new line : $(y - 0) = (2 - \sqrt{3})(x - a)$

$y = (2 - \sqrt{3})x - (2 - \sqrt{3})a$

Distance from origin $= \frac{1}{\sqrt{2}}$

$\left|\frac{-(2-\sqrt{3})a}{4+3-4\sqrt{3}+1}\right| = \frac{1}{\sqrt{2}}$

$|a| = \frac{\sqrt{8-4\sqrt{3}}}{\sqrt{2}(2-\sqrt{3})}$

$|a| = \frac{2\sqrt{2-\sqrt{3}}}{\sqrt{2}(2-\sqrt{3})}$

$|a| = \frac{\sqrt{2}}{\sqrt{2-\sqrt{3}}}$

$a^2 = \frac{2}{2-\sqrt{3}} = 2(2+\sqrt{3})$

$3a^2\tan^2\alpha - 2\sqrt{3} = 3(4 + 2\sqrt{3}) \times \frac{1}{3} - 2\sqrt{3}$

$= 4$

Q.5 Let $A = \begin{bmatrix} 2 & 2+p & 2+p+q \\ 4 & 6+2p & 8+3p+2q \\ 6 & 12+3p & 20+6p+3q \end{bmatrix}$

If $\det(\text{adj}(\text{adj}(3A))) = 2^m \cdot 3^n$, $m, n \in N$, then $m + n$ is equal to

(1) 20 (2) 26 (3) 22 (4) 24

Ans. [4]

Sol. $\begin{vmatrix} 2 & 2+p & 2+p+q \\ 4 & 6+2p & 8+3p+2q \\ 6 & 12+3p & 20+6p+3q \end{vmatrix}$

$= \begin{vmatrix} 2 & 2 & 2+p+q \\ 4 & 6 & 8+3p+2q \\ 6 & 12 & 20+6p+3q \end{vmatrix} + \underbrace{\begin{vmatrix} 2 & p & 2+p+q \\ 4 & 2p & 8+3p+2q \\ 6 & 3p & 20+6p+3q \end{vmatrix}}_{=0}$

$$= 2 \times 2 \begin{vmatrix} 1 & 1 & 2+p+q \\ 2 & 3 & 8+3p+2q \\ 3 & 6 & 20+6p+3q \end{vmatrix}$$

$C_3 \to C_3 \to pC_2$

$$= 4\begin{vmatrix} 1 & 1 & 2+p+q \\ 2 & 3 & 8+2q \\ 3 & 6 & 20+3q \end{vmatrix} = 4\begin{vmatrix} 1 & 1 & 2 \\ 2 & 3 & 8 \\ 3 & 6 & 20 \end{vmatrix} + 0$$

$$= 4\begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 4 \\ 3 & 6 & 10 \end{vmatrix}$$

$= 8(1(6) - 1(8) + 1(3))$

$= 8$

$|adj(adj(3A))| = (|3A|)^{2^2} = |3A|^4$

$= (3^3|A|)^4 = 3^{12} \cdot |A|^4$

$= 3^{12} \cdot (2^3)^4$

$= 3^{12} \cdot 2^{12}$

Q.6 Given below are two statements.

Statement I: $\lim\limits_{x\to 0}\left(\dfrac{\tan^{-1}x + \log_e\sqrt{\dfrac{1+x}{1-x}} - 2x}{x^5}\right) = \dfrac{2}{5}$

Statement II : $\lim\limits_{x\to 1}\left(x^{\frac{2}{1-x}}\right) = \dfrac{1}{e^2}$

In the light of the above statements, choose the **correct** answer from the options given below

(1) Statement I is false but Statement II is true

(2) Both Statement I and Statement II are false

(3) Both Statement I and Statement II are true

(4) Statement I is true but Statement II is false

Ans. [3]

Sol. $\lim\limits_{x\to 0}\left(\dfrac{\tan^{-1}x + \ln\sqrt{\dfrac{1+x}{1-x}} - 2x}{x^5}\right)$

$$\lim_{x\to 0}\frac{\left(x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + ...\right) + \frac{1}{2}\left(+x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} + ...\right) - \frac{1}{2}\left(-x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} - \frac{x^5}{5} + ...\right) - 2x}{x^5}$$

$$= \lim_{x\to 0}\frac{\left(x + \frac{1}{2}(x+x) - 2x\right) + x^3\left(\frac{-1}{3} + \frac{1}{2}\times\frac{1}{3} + \frac{1}{2}\times\frac{1}{3}\right) + x^5\left(\frac{1}{5} + \frac{1}{2}\times\frac{1}{5} + \frac{1}{2}\times\frac{1}{5}\right) + ...}{x^5}$$

$$= \lim_{x\to 0}\frac{\left(\frac{1}{5} + \frac{1}{10} + \frac{1}{10}\right)x^5}{x^5} = \frac{2}{5}$$

$$\Rightarrow \lim_{x\to 1} x^{\left(\frac{2}{1-x}\right)} = \lim_{x\to 1}\left([1 + (x-1)]^{\frac{1}{x-1}}\right)^{\frac{(x-1)2}{(1-x)}}$$

$$= e^{-2} = \frac{1}{e^2}$$

Q.7 Let $\vec{a} = \hat{i} + 2\hat{j} + \hat{k}$ and $\vec{b} = 2\hat{i} + \hat{j} - \hat{k}$. Let $\hat{c}$ be a unit vector in the plane of the vector $\vec{a}$ and $\vec{b}$ and be perpendicular to $\vec{a}$. Then such a vector $\hat{c}$ is :

(1) $\dfrac{1}{\sqrt{3}}\left(\hat{i} - \hat{j} + \hat{k}\right)$ (2) $\dfrac{1}{\sqrt{5}}\left(\hat{j} - 2\hat{k}\right)$

(3) $\dfrac{1}{\sqrt{2}}\left(-\hat{i} + \hat{k}\right)$ (4) $\dfrac{1}{\sqrt{3}}\left(-\hat{i} + \hat{j} - \hat{k}\right)$

Ans. [3]

Sol. $\vec{c} = x\vec{a} + y\vec{b}$

$\vec{c} = x(\hat{i} + 2\hat{j} + \hat{k}) + y(2\hat{i} + \hat{j} - \hat{k})$

$\vec{a} \cdot \vec{c} = (\hat{i} + 2\hat{j} + \hat{k}) \cdot (x(\hat{i} + 2\hat{j} + \hat{k}) + y(2\hat{i} + \hat{j} - \hat{k})$

$(\hat{i} + 2\hat{j} + \hat{k}) \cdot (x\hat{i} + 2x\hat{j} + x\hat{k}) + (2y\hat{i} + y\hat{j} - y\hat{k}) = 0$

$\Rightarrow (x + 2y) + 2(x + 9) + (x - y) = 0$

$\Rightarrow y = -2x$

$\therefore \vec{c} = x(-3\hat{i} + 3\hat{k})$

$|\vec{c}| = |x| \sqrt{9+9} = 3|x| \sqrt{2}$

$\therefore |\vec{c}| = 1$

$3|x| \sqrt{2} = 1$

$|x| = \dfrac{1}{3\sqrt{2}}$

Let $x = \dfrac{1}{3\sqrt{2}}$

$\vec{c} = \dfrac{1}{3\sqrt{2}}(-3\hat{i} + 3\hat{k})$

or $\vec{c} = \dfrac{1}{\sqrt{2}}(-\hat{i} + \hat{k})$

Q.8 Let the value of λ for which the shortest distance between the lines $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ and $\frac{x-\lambda}{3}=\frac{y-4}{4}=\frac{z-5}{5}$ is $\frac{1}{\sqrt{6}}$ be λ_1 and λ_2. Then the radius of the circle passing through the points (0, 0), (λ_1, λ_2) and (λ_2, λ_1) is

(1) $\frac{\sqrt{2}}{3}$ (2) $\frac{5\sqrt{2}}{3}$ (3) 3 (4) 4

Ans. **[2]**

Sol. $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ .... (1)

$\frac{x-\lambda}{3}=\frac{y-4}{4}=\frac{z-5}{5}$ ..... (2)

$$\vec{n}_1\times\vec{n}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix}$$

$= \hat{i}(15-16) - \hat{j}(10-12) + \hat{k}(8-9)$

$= -\hat{i} + 2\hat{j} - \hat{k}$

L_1 passing through (1, 2, 3) and L_2 through (λ, 4, 5)

$d = \frac{1}{\sqrt{6}}$

$\Rightarrow \frac{|(\lambda-1)(-1)-2(-2)+2(-1)|}{\sqrt{1^2+4^2+1}} = \frac{1}{\sqrt{6}}$

$|-\lambda + 1 + 4 - 2| = 1$

$|-\lambda + 3| = 1$

$\lambda - 3 = \pm 1$

$\lambda = 4, 2$

Circle passing through (0, 0), (1, 4) (4, 1)

$$\therefore \text{Area} = \frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ 1 & 4 & 1 \\ 4 & 1 & 1 \end{vmatrix}$$

$= \left|\frac{1}{2}(1-16)\right| = \frac{15}{2}$

$\therefore r = \frac{abc}{4\Delta}$

$= \frac{5\sqrt{2}}{3}$

Q.9 Let A = {0, 1, 2, 3, 4, 5}. Let R be a relation on A defined by (x, y) $\in$ R if and only if max{x, y} $\in$ {3, 4}. Then among the statements

(S_1) : The number of elements in R is 18, and

(S_2) : The relation R is symmetric but neither reflexive nor transitive.

(1) only (S_2) is true (2) both are false

(3) only (S_1) is true (4) both are true

Ans. **[1]**

Sol. Let's write the pairs (x, y) in R

(0, 3), (1, 3), (2, 3), (3, 3), (4, 3), (5, 3), (3, 0), (3, 1),

(3, 2), (3, 4), (3, 5), (0, 4), (1, 4), (2, 4), (4, 4), (5, 4),

(4, 0), (4, 1), (4, 2), (4, 5)

There are total 20 pairs

If (x, y) $\in$ R, then max(x, y) $\in$ {3, 4}

This means, max(y, x) $\in$ {3, 4}. So, (y, x) $\in$ R.

Thus R is symmetric

Since max(5, 5) = 5 $\notin$ {3, 4}, (5, 5) $\notin$ R

Thus R is not reflexive

(3, 4) $\in$ R & (4, 2) $\in$ R, but (3, 2) $\notin$ R, 2 $\notin$ {3, 4}

Thus, R is not transitive

Therefore, only S_2 is true

Q.10 If $\frac{1}{1^4}+\frac{1}{2^4}+\frac{1}{3^4}+.......\infty=\frac{\pi^4}{90}$,

$\frac{1}{1^4}+\frac{1}{3^4}+\frac{1}{5^4}+.......\infty=\alpha$

$\frac{1}{2^4}+\frac{1}{4^4}+\frac{1}{6^4}+.......\infty=\beta$

Then $\frac{\alpha}{\beta}$ is equal to

(1) 14 (2) 15 (3) 18 (4) 23

Ans. **[2]**

Sol. $\alpha = \frac{1}{1^4}+\frac{1}{3^4}+\frac{1}{5^4}+......$

$\beta = \frac{1}{2^4}+\frac{1}{4^4}+\frac{1}{6^4}+......$

$= \frac{1}{2^4}\left[\frac{1}{1^4}+\frac{1}{2^4}+\frac{1}{3^4}+\frac{1}{4^4}+......\right]$

$\Rightarrow 16\beta = \left[\frac{1}{1^4}+\frac{1}{3^4}+\frac{1}{5^4}+......\right] + \left[\frac{1}{2^4}+\frac{1}{4^4}+\frac{1}{6^4}+......\right]$

$= \alpha + \beta \Rightarrow 15\beta = \alpha \Rightarrow \frac{\alpha}{\beta} = 15$

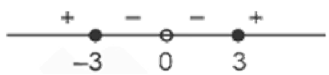
Q.11 Let the function $f(x) = \frac{x}{3} + \frac{3}{x} + 3,\ x \neq 0$ be strictly increasing in $(-\infty, \alpha_1) \cup (\alpha_2, \infty)$ and strictly decreasing in $(\alpha_3, \alpha_4) \cup (\alpha_4, \alpha_5)$. Then $\sum_{i=1}^{5} \alpha_i^2$ is equal to

(1) 48 (2) 40 (3) 36 (4) 28

Ans. [3]

Sol. $f(x) = \frac{x}{3} + \frac{3}{x} + 3,\ x \neq 0$

$$f'(x) = \frac{1}{3} - \frac{3}{x^2} = \left(\frac{x^2 - 9}{3x^2}\right) = \frac{(x-3)(x+3)}{3x^2}$$



$\Rightarrow f'(x) > 0\ \forall\ x \in (-\infty, -3) \cup (3, \infty)$

$f'(x) < 0\ \forall\ x \in (-3, 0) \cup (0, 3)$

$\Rightarrow \alpha_1 = -3, \alpha_2 = 3, \alpha_3 = -3, \alpha_4 = 0, \alpha_5 = 3$

$\Rightarrow \sum_{i=1}^{5} \alpha_i^2 = (-3)^2 + (3^2) + 0^2 + (-3)^2 + (3)^2$

$= 4(9) = 36$

Q.12 If A and B are two events such that P(A) = 0.7, P(B) = 0.4 and $P(A \cap \overline{B}) = 0.5$, where $\overline{B}$ denotes the complement of B, then $P(B|(A \cup \overline{B}))$ is equal to

(1) $\frac{1}{2}$ (2) $\frac{1}{4}$ (3) $\frac{1}{3}$ (4) $\frac{1}{6}$

Ans. [2]

Sol. P(A) = 0.7

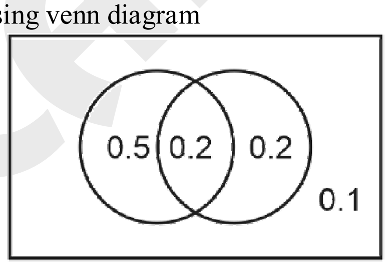
P(B) = 0.4

$P(A \cap B^C) = 0.5$

$$P\left(B / A \cup B^C\right) = \frac{P(B \cap (A \cup B^C))}{P(A \cup B^C)}$$

$$= \frac{P(A \cap B)}{P(A \cup B^C)}$$

Using venn diagram



$$\Rightarrow \frac{P(A \cap B)}{P(A \cup B^C)} = \frac{0.2}{0.5 + 0.2 + 0.1} = \frac{2}{8} = \frac{1}{4}$$

Q.13 Let f(x) = x – 1 and $g(x) = e^x$ for $x \in R$. If $\frac{dy}{dx} = \left(e^{-2\sqrt{x}} g(f(f(x))) - \frac{y}{\sqrt{x}}\right)$, y(0) = 0, then y(1) is

(1) $\frac{2e-1}{e^3}$ (2) $\frac{1-e^2}{e^4}$ (3) $\frac{1-e^3}{e^4}$ (4) $\frac{e-1}{e^4}$

Ans. [4]

Sol. f(x) = x – 1

f(f(x)) = (x – 1) – 1 = x – 2

$g(f(f(x))) = e^{x-2}$

$$\frac{dy}{dx} = e^{-2\sqrt{x}}(x-2) - \frac{y}{\sqrt{x}}$$

$$\Rightarrow \frac{dy}{dx} + \frac{y}{\sqrt{x}} = e^{-2\sqrt{x}} e^{x-2}$$

$$IF = e^{\int x^{-1/2} dx} = e^{2\sqrt{x}}$$

$$y.\ e^{2\sqrt{x}} = \int e^{-2\sqrt{x}} e^{x-2} e^{2\sqrt{x}}\, dx$$

$= y \cdot e^{2\sqrt{x}} = e^{x-2} + c$

$y(0) = 0 \Rightarrow c = e^{-2}$

$\therefore y \cdot e^{2\sqrt{x}} = e^{x-2} - e^{-2}$

$$y \cdot e^2 = e^{-1} - e^{-2} = \frac{1}{e} - \frac{1}{e^2} = \frac{e-1}{e^2}$$

$$y = \frac{e-1}{e^4}$$

Q.14 The sum of the squares of the roots of $|x - 2|^2 + |x - 2| - 2 = 0$ and the squares of the roots of $x^2 - 2|x - 3| - 5 = 0$, is

(1) 24 (2) 30 (3) 36 (4) 26

Ans. [3]

Sol. $|x - 2|^2 + |x - 2| - 2 = 0$

$\Rightarrow (|x - 2| + 2)(|x - 2| - 1) = 0$

$\Rightarrow |x - 2| = 1$

$\Rightarrow x = 3, 1$

$x^2 - 2|x - 3| - 5 = 0$

$x \geq 3$

$x^2 - 2(x - 3) - 5 = 0$

$\Rightarrow x^2 - 2x + 1 = 0$

$\Rightarrow (x - 1)^2 = 0$

$\Rightarrow x = 1$ (rejected)

$x < 3$

$x^2 + 2(x - 3) - 5 = 0$

$\Rightarrow x^2 + 2x - 11 = 0$

$x = -1 + 2\sqrt{3}$

Sum of squares of roots

$= 3^2 + 1^2 + \left(-1 + 2\sqrt{3}\right)^2 + \left(-1 - 2\sqrt{3}\right)^2$

$= 10 + (1 + 12)2$

$= 36$

Q.15 Let

$$A = \left\{\theta \in [0,2\pi] : 1 + 10\,\text{Re}\left(\frac{2\cos\theta + i\sin\theta}{\cos\theta - 3i\sin\theta}\right) = 0\right\}$$

Then $\sum_{\theta \in A} \theta^2$ is equal to

(1) $\frac{27}{4}\pi^2$ (2) $8\pi^2$ (3) $6\pi^2$ (4) $\frac{21}{4}\pi^2$

Ans. [4]

Sol. $\text{Re}\left(\frac{2\cos\theta + i\sin\theta}{\cos\theta - 3i\sin\theta}\right)$

$\text{Re}\left(\frac{(2\cos\theta + i\sin\theta)(\cos\theta + 3i\sin\theta)}{\cos^2\theta - 9\sin^2\theta}\right)$

$= \frac{2\cos^2\theta - 3\sin^2\theta}{1 + 8\sin^2\theta}$

Now, $1 + 10\left(\frac{2\cos^2\theta - 3\sin^2\theta}{1 + 8\sin^2\theta}\right) = 0$

$= 1 + 8\sin^2\theta + 20\cos^2\theta - 30\sin^2\theta = 0$

$= 1 - 22\sin^2\theta + 20\cos^2\theta = 0$

$= 1 + 20(\cos 2\theta) - 2\sin^2\theta = 0$

$= 20\cos 2\theta + \cos 2\theta = 0$

$= 21\cos 2\theta = 0$

$2\theta = (2n + 1)\frac{\pi}{2}, n \in I$

$2\theta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}$

$\Rightarrow \theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

$\sum_{\theta \in A} \theta^2 = \frac{\pi^2}{16} + \frac{9\pi^2}{16} + \frac{25\pi^2}{16} + \frac{49\pi^2}{16}$

$= \frac{84\pi^2}{16}$

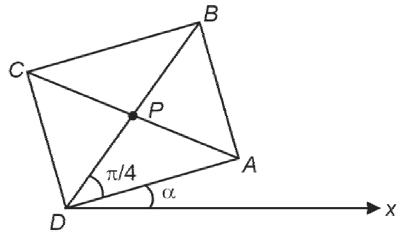
$= \frac{21\pi^2}{4}$

Q.16 Let a be the length of a side of a square OABC with O being the origin. its side OA makes an acute angle α with the positive x-axis and the equations of its diagonals are $(\sqrt{3}+1)x + (\sqrt{3}-1)y = 0$ and $(\sqrt{3}-1)x - (\sqrt{3}+1)y + 8\sqrt{3} = 0$. Then a^2 is equal to

(1) 24 (2) 32 (3) 16 (4) 48

Ans. [4]

Sol.



OB : $(\sqrt{3}+1)x + (\sqrt{3}-1)y = 0$

AC : $(\sqrt{3}-1)x - (\sqrt{3}+1)y + 8\sqrt{3} = 0.$

$\Rightarrow (x, y) = P(3 - \sqrt{3}, 3 + \sqrt{3})$

Let AB = a = OA

$\Rightarrow OA^2 + AB^2 = OB^2$

$2a^2 = 4\left[(3-\sqrt{3})^2 + (3-\sqrt{3})^2\right]$

$a^2 = 2 \times 24 = 48$

Q.17 Let f(x) be a positive function and $I_1 = \int_{-\frac{1}{2}}^{1} 2xf(2x(1-2x))dx$ and $I_2 = \int_{-1}^{2} f(x(1-x))dx$.

Then the value of $\frac{I_2}{I_1}$ is equal to ____ .

(1) 12 (2) 4 (3) 6 (4) 9

Ans. [2]

Sol. $I_1 = \int_{-\frac{1}{2}}^{1} 2xf(2x(1-2x))dx$

$I_1 = \int_{-\frac{1}{2}}^{1} 2\left(\frac{1}{2} - x\right) f\left(2\left(\frac{1}{2} - x\right)\left(1 - 2\left(\frac{1}{2} - x\right)\right)\right) dx$

$I_1 = \int_{-\frac{1}{2}}^{1} (1-2x) f((1-2x)(2x))\, dx$

$I_1 = \int_{-\frac{1}{2}}^{1} f((1-2x)(2x))\, dx - \underbrace{\int_{-\frac{1}{2}}^{1} 2xf((1-2x)(2x))dx}_{I_1}$

$2I_1 = \int_{-\frac{1}{2}}^{1} f((1-2x)(2x))\, dx$

Put 2x = t

2dx = dt

$dx = \frac{dt}{2}$

$2I_1 = \frac{1}{2}\int_{-1}^{2} f((1-t)(t))\, dt$

$$I_1 = \frac{1}{4}\int_{-1}^{2} f((1-x)(x))\,dx$$

$$I_1 = \frac{1}{4} I_2$$

$$4 \Rightarrow \frac{I_2}{I_1}$$

Q.18 The number of integral terms in the expansion of $\left(5^{\frac{1}{2}} + 7^{\frac{1}{8}}\right)^{1016}$ is

(1) 127 (2) 128 (3) 129 (4) 130

Ans. **[2]**

Sol. $T_{r+1} = 1016_{C_r} \cdot \left(7^{\frac{1}{8}}\right)^4 \cdot 5^{\frac{1}{2}(1016-r)}$

$\Rightarrow \frac{r}{8}$ and $\frac{1016-r}{2} \in$ integer

$\Rightarrow 8 / r \Rightarrow r = 0, 8, 16, ...1016$

$\Rightarrow r = 0 \times 8, 1 \times 8, ... 127 \times 8$

$\Rightarrow$ Total 128 r such that T_{r+1} is rational

Q.19 There are 12 points in a plane, no three of which are in the same straight line, except 5 points which are collinear. Then the total number of triangles that can be formed with the vertices at any three of these 12 points is

(1) 230 (2) 200 (3) 220 (4) 210

Ans. **[4]**

Sol.

Number of triangle = ${}^5C_1 \times {}^7C_2 + {}^7C_3 + {}^5C_2 + {}^7C_1 = 210$

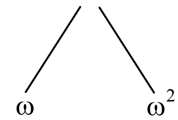
Q.20 Let α be a solution of $x^2 + x + 1 = 0$, and for some a and b in **R**, $[4\ \ a\ \ b]\begin{bmatrix} 1 & 16 & 13 \\ -1 & -1 & 2 \\ -2 & -14 & -8 \end{bmatrix} = [0\ \ 0\ \ 0]$.

If $\frac{4}{\alpha^4} + \frac{m}{\alpha^a} + \frac{n}{\alpha^b} = 3$, then m + n is equal to ______.

(1) 7 (2) 11 (3) 3 (4) 8

Ans. **[2]**

Sol. $x^2 + x + 1 = 0$



$$[4\ \ a\ \ b]\begin{bmatrix} 1 & 16 & 13 \\ -1 & -1 & 2 \\ -2 & -14 & -8 \end{bmatrix}_{3\times 3} = [0\ \ 0\ \ 0]$$

$\Rightarrow [4 - a - 2b, 64 - a - 14b, 52 + 2a - 8b] = [0\ 0\ 0]$

$a + 2b = 4$ …(i)

$a + 14b = 64$ …(ii)

Solving (i) and (ii)

We get a = – 6, b = 5

$\therefore \frac{4}{\alpha^4} + \frac{m}{\alpha^a} + \frac{n}{\alpha^b} = 3$

$\Rightarrow \frac{4}{\omega^4} + \frac{m}{\omega^{-6}} + \frac{n}{\omega^5} = 3$

$\Rightarrow 4\omega^2 + m + n\omega = 3$ .....(i)

for $\alpha = \omega^2$,

$\frac{4}{\omega^8} + \frac{m}{\omega^{-12}} + \frac{n}{\omega^{10}} = 3$

$\Rightarrow \frac{4}{\omega^2} + m + \frac{n}{\omega} = 3$

$\Rightarrow 4\omega + m + n\omega^2 = 3$ ....(ii)

Adding (i) & (ii)

$\Rightarrow 4(\omega^2 - \omega) + n(\omega - \omega^2) = 0$

$\Rightarrow (\omega^2 - \omega)(4 - n) = 0$

$\Rightarrow n = 4$

$\therefore 4\omega + m + 4\omega^2 = 3$

$\Rightarrow -4 + m = 3$

$\Rightarrow m = 7$

$\therefore m + n = 11$

Section-B: Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

Q.21 The product of the last two digits of $(1919)^{1919}$ is _____.

Ans. **[63]**

Sol. $(1919)^{1919} = (1920 - 1)^{1919}$

$= {}^{1919}C_0(1920)^{1919} - {}^{1919}C_1(1920)^{1918} + ... + {}^{1919}C_{1918}(1920) - {}^{1919}C_{1919}$

For last two digit $\Rightarrow {}^{1919}C_{1919}(1920) - 1$

= 3684479

$\therefore$ Product of last two digit = 63

Q.22 Let the domain of the function $f(x) = \cos^{-1}\left(\frac{4x+5}{3x-7}\right)$ be $[\alpha, \beta]$ and the domain of $g(x) = \log_2(2 - 6\log_{27}(2x+5))$ be (γ, δ). Then $|7(\alpha + \beta) + 4(\gamma + \delta)|$ is equal to ______.

Ans. **[96]**

Sol. $f(x) = \cos^{-1}\left(\frac{4x+5}{3x-7}\right)$

$-1 \le \frac{4x+5}{3x-7} \le 1$

$\frac{7x-2}{3x-7} \ge 0,\ \frac{x+12}{3x-7} \le 0$

$x \in \left(-\infty, \frac{2}{7}\right] \cup \left(\frac{7}{3}, \infty\right),\ x \in \left[-12, \frac{7}{3}\right)$

$\Rightarrow x \in \left[-12, \frac{2}{7}\right] \Rightarrow \alpha = -12,\ \beta = \frac{2}{7}$

$g(x) = \log_2(2 - 6\log_{27}(2x+5))$

$\Rightarrow 2 - 6\log_{27}(2x+5) > 0,\ \boxed{2x+5>0}$

$\log_{27}(2x+5) < \frac{1}{3} \Rightarrow (2x+5)^3 < 27$

$\Rightarrow 2x + 5 < 3 \Rightarrow \boxed{x < -1}$

$\Rightarrow x \in \left(-\frac{5}{2}, -1\right) \Rightarrow \gamma = -\frac{5}{2},\ \delta = -1$

$\Rightarrow |7(\alpha + \beta) + 4(\gamma + \delta)| = 96$

Q.23 Let r be the radius of the circle, which touches x-axis at point (a, 0), a < 0 and the parabola $y^2 = 9x$ at the point (4, 6). Then r is equal to ____.

Ans. **[30]**

Sol. Equation of tangent to $y^2 = gx$ at (4, 6) is $3x - 4y + 12 = 0$

Equation of circle is $(x-4)^2 + (y-6)^2 + \lambda(3x - 4y + 12) = 0$

$\Rightarrow x^2 + y^2 + (3\lambda - 8)x + (-12 - 4\lambda)y + 52 + 12\lambda = 0$

$\because 2\sqrt{g^2 - c} = 0 \Rightarrow g^2 = c$

$\Rightarrow \left(-\frac{3\lambda - 8}{2}\right)^2 = 52 + 12\lambda$

$= 9\lambda^2 + 64 - 48\lambda = 208 + 48\lambda \Rightarrow 9\lambda^2 - 96\lambda - 144 = 0$

$\Rightarrow \lambda = 12, -\frac{2}{3} \Rightarrow f = -30, -\frac{14}{3}$

$\Rightarrow r = \sqrt{g^2 + f^2 - c} = |f| = |-(2\lambda + 6)|$

$\because$ centre lies in 2^{nd} quadrant

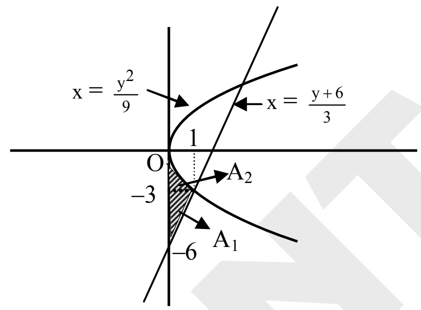
$\Rightarrow 3\lambda - 8 > 0 \Rightarrow \lambda > \frac{8}{3}$

$\Rightarrow \lambda = 12,\ f = -30,\ r = 30$

Q.24 Let the area of the bounded region $\{(x, y) : 0 \le 9x \le y^2,\ y \ge 3x - 6\}$ be A. Then 6A is equal to ______.

Ans. **[15]**

Sol.



$A = A_1 + A_2$

$= \frac{1}{2} \times 3 \times 1 + \int_{-3}^{0} \frac{y^2}{9}\,dx$

$= \frac{3}{2} + \left.\frac{y^3}{27}\right|_{-3}^{0}$

$= \frac{3}{2} + 1 = \frac{5}{2}$

$6A = 6 \times \frac{5}{2} = 15$

Q.25 Let the area of the triangle formed by the lines $x + 2 = y - 1 = z$, $\frac{x-3}{5} = \frac{y}{-1} = \frac{z-1}{1}$ and $\frac{x}{-3} = \frac{y-3}{3} = \frac{z-2}{1}$ be A. Then A^2 is equal to ______.

Ans. **[56]**

Sol. $L_1 = \frac{x+2}{1} = \frac{y-1}{1} = \frac{z}{1} = \lambda$, any point on it $(\lambda - 2, \lambda + 1, \lambda)$

$L_2 = \frac{x-3}{5} = \frac{y}{-1} = \frac{z-1}{1} = \mu$, any point on it $(5\mu + 3, -\mu, \mu + 1,)$

$L_3 = \frac{x}{-3} = \frac{y-3}{3} = \frac{z-2}{1} = k$, any point on it $(-3k, 3k + 3, k + 2)$

$P \equiv$ point of intersection of L_1 and $L_2 = (-2, 1, 0)$

$Q \equiv$ point of intersection of L_1 and $L_2 = (0, 3, 2)$

$R \equiv$ point of intersection of L_1 and $L_2 = (3, 0, 1)$

$\overline{PQ} = 2\hat{i} + 2\hat{j} + 2\hat{k}$

$\overline{PR} = 5\hat{i} - \hat{j} + \hat{k}$

$A = \frac{1}{2}|\overline{PQ} \times \overline{PR}| = \sqrt{56}$

$A^2 = 56$

PHYSICS

Section-A: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct..

Q.26 A rod of linear mass density 'λ' and length 'L' is bent to form a ring of radius 'R'. Moment of inertia of ring about any of its diameter is

(1) $\frac{\lambda L^3}{4\pi^2}$ (2) $\frac{\lambda L^3}{8\pi^2}$

(3) $\frac{\lambda L^3}{12}$ (4) $\frac{\lambda L^3}{16\pi^2}$

Ans. **[2]**

Sol. $2\pi R = L$ $\Rightarrow M = \lambda L$

$R = \frac{L}{2\pi}$

Moment of inertia about diameter

$$= \frac{MR^2}{2} = \frac{M}{2}\left(\frac{L}{2\pi}\right)^2 = \frac{\lambda L^3}{8\pi^2}$$

Q.27 In a young's double slit experiment, the source is white light. One of the slits is covered by red filter and another by a green filter. In this case

(1) There shall be alternate interference fringes of red and green.

(2) There shall be an interference pattern for red distinct from that for green.

(3) There shall be no interference fringes.

(4) There shall be an interference pattern, where each fringe's pattern center is green and outer edges is red.

Ans. **[3]**

Sol. Frequency of Green and frequency of Red will be different. No interference pattern is observed for two lights of different frequencies as phase difference does not remain constant.

Q.28 Two balls with same mass and initial velocity, are projected at different angles in such a way that maximum height reached by first ball is 8 times higher than that of the second ball. T_1 and T_2 are the total flying times of first and second ball, respectively, then the ratio of T_1 and T_2 is

(1) $2\sqrt{2}$: 1 (2) $\sqrt{2}$: 1

(3) 4 : 1 (4) 2 : 1

Ans. **[1]**

Sol. $H_1 = \frac{u^2 \sin^2\theta_1}{2g}$ $H_2 = \frac{u^2 \sin^2\theta_2}{2g}$

$T_1 = \frac{2u\sin\theta_1}{g}$ $T_2 = \frac{2u\sin\theta_2}{g}$

$\frac{H_1}{H_2} = \frac{\sin^2\theta_1}{\sin^2\theta_2}$ $\frac{T_1}{T_2} = \frac{\sin\theta_1}{\sin\theta_2}$

$8 = \left(\frac{T_1}{T_2}\right)^2$ $\Rightarrow \frac{T_1}{T_2} = 2\sqrt{2}$

$T_1 : T_2 = 2\sqrt{2} : 1$

Q.29 For a nucleus of mass number A and radius R, the mass density of nucleus can be represented as

(1) A^3 (2) $A^{\frac{2}{3}}$

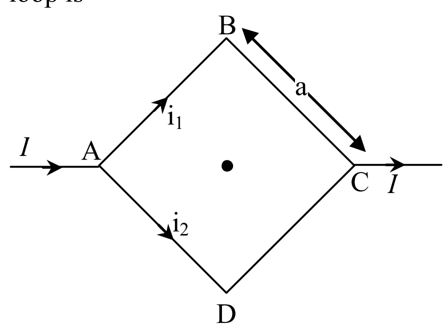
(3) $A^{\frac{1}{3}}$ (4) Independent of A

Ans. **[4]**

Sol. $R = R_0A^{1/3}$

$$\text{Density} = \frac{\text{Mass number}}{\text{Volume}} = \frac{A}{\frac{4}{3}\pi R_0^3 A} = \text{constant}$$

Q.30 Figure shows a current carrying squares loop ABCD of edge length is 'a' lying in a plane. If the resistance of the ABC part is r and that of ADC part is 2r, then the magnitude of the resultant magnetic field at centre of the square loop is

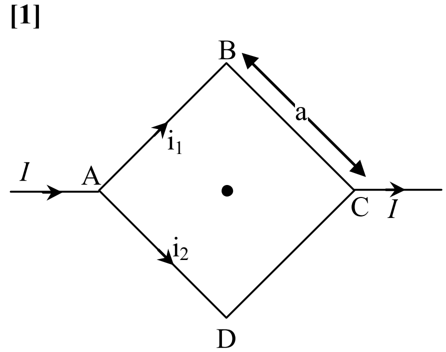


(1) $\frac{\sqrt{2}\mu_0 I}{3\pi a}$ (2) $\frac{\mu_0 I}{2\pi a}$

(3) $\frac{2\mu_0 I}{3\pi a}$ (4) $\frac{3\pi\mu_0 I}{\sqrt{2}a}$

Ans. **[1]**

Sol.



$R_{ABC} = r$ $\quad R_{ADC} = 2r$

$i_1 = \frac{2I}{3}$ $\quad i_2 = \frac{I}{3}$

$$B_{centre} = \frac{2(\mu_0\sqrt{2})}{4\pi\left(\frac{a}{2}\right)}\left[\frac{2I}{3} - \frac{I}{3}\right] = \sqrt{2}\frac{\mu_0 I}{3\pi a}$$

Q.31 A convex lens of focal length 30 cm is placed in contact with a concave lens of focal length 20 cm. An object is placed at 20 cm to the left of this lens system. The distance of the image from the lens in cm is ______.

(1) 30 (2) $\frac{60}{7}$ (3) 15 (4) 45

Ans. [3]

Sol. $\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{30} + \frac{1}{-20} = -\frac{1}{60}$

$v = \frac{Fu}{u+F} = \frac{-60 \times -20}{-20 + -60} = -15$ cm

Q.32 The amplitude and phase of a wave that is formed by the superposition of two harmonic travelling waves, $y_1(x, t) = 4\sin(kx - \omega t)$ and $y_2(x, t) = 2\sin\left(kx - \omega t + \frac{2\pi}{3}\right)$, are :

(Take the angular frequency of initial waves same as ω)

(1) $\left[2\sqrt{3}, \pi/6\right]$ (2) $\left[6, 2\pi/3\right]$

(3) $\left[6, \pi/3\right]$ (4) $\left[\sqrt{3}, \pi/6\right]$

Ans. [1]

Sol. $A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\phi}$

$\phi = \frac{2\pi}{3}$

$A = \sqrt{4^2 + 2^2 + 2 \times 4 \times 2\cos\frac{2\pi}{3}} = 2\sqrt{3}$

$\tan\alpha = \frac{2\sin\phi}{4 + 2\cos\phi} = \frac{1}{\sqrt{3}}$,

$\alpha = \pi/6$

Q.33 A body of mass 2 kg moving with velocity of $\vec{v}_{in} = 3\hat{i} + 4\hat{j}\,ms^{-1}$ enters into a constant force field of 6N directed along positive z-axis. If the body remains in the field for a period of $\frac{5}{3}$ seconds, then velocity of the body when it emerges from force field is :

(1) $3\hat{i} + 4\hat{j} + 5\hat{k}$ (2) $3\hat{i} + 4\hat{j} - 5\hat{k}$

(3) $4\hat{i} + 3\hat{j} + 5\hat{k}$ (4) $3\hat{i} + 4\hat{j} + \sqrt{5}\hat{k}$

Ans. [1]

Sol. $\vec{F} \cdot t = m(\vec{v}_2 - \vec{v}_1)$

$\frac{6}{2} \times \frac{5}{3}\hat{k} = \vec{v}_2 - \vec{v}_1$

$\vec{v}_2 = \vec{v}_1 + 5\hat{k} = 3\hat{i} + 4\hat{j} + 5\hat{k}$

Q.34 A quantity Q is formulated as $X^{-2}Y^{+\frac{3}{2}}Z^{-\frac{2}{5}}$. X, Y and Z are independent parameters which have fractional errors of 0.1, 0.2 and 0.5, respectively in measurement. The maximum fractional error of Q is

(1) 0.1 (2) 0.6 (3) 0.7 (4) 0.8

Ans. [3]

Sol. $\frac{\Delta Q}{Q} = -2\frac{(\pm\Delta x)}{x} + \frac{3}{2}\frac{(\pm\Delta y)}{y} - \frac{2}{5}\frac{(\pm\Delta z)}{z}$

For maximum fractional error

$\frac{\Delta Q}{Q} = \frac{2\Delta x}{x} + \frac{3}{2}\frac{\Delta y}{y} + \frac{2}{5}\frac{\Delta z}{z}$

$= 2 \times 0.1 + \frac{3}{2} \times 0.2 + \frac{2}{5} \times 0.5$

$= 0.7$

Q.35 A monoatomic gas having $\gamma = \frac{5}{3}$ is stored in a thermally insulated container and the gas is suddenly compressed to $\left(\frac{1}{8}\right)^{th}$ of its initial volume. The ratio of final pressure and initial pressure is (γ is the ratio of specific heats of the gas at constant pressure and at constant volume)

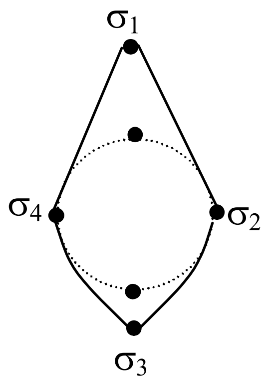
(1) 40 (2) 28 (3) 32 (4) 16

Ans. [3]

Sol. $P_1V_1^\gamma = P_2V_2^\gamma$ $\quad V_2 = \frac{V_1}{8}$

$\frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^\gamma = (8)^{\frac{5}{3}} = 32$

Q.36 Electric charge is transferred to an irregular metallic disk as shown in figure. If σ_1, σ_2, σ_3 and σ_4 are charge densities at given points then, choose the correct answer from the options given below:



(A) $\sigma_1 > \sigma_3$; $\sigma_2 = \sigma_4$
(B) $\sigma_1 > \sigma_2$; $\sigma_3 > \sigma_4$
(C) $\sigma_1 > \sigma_3 > \sigma_2 = \sigma_4$
(D) $\sigma_1 < \sigma_3 < \sigma_2 = \sigma_4$
(E) $\sigma_1 = \sigma_2 = \sigma_3 = \sigma_4$

(1) A and C Only (2) D and E Only
(3) A, B and C Only (4) B and C Only

Ans. [3]

Sol. $\sigma \propto \frac{1}{R}$ R → radius of curvature

$R_2 = R_4 > R_3 > R_1$

$\Rightarrow \sigma_1 > \sigma_3 > \sigma_2 = \sigma_4$

Q.37 Two strings with circular cross section and made of same material are stretched to have same amount of tension. A transverse wave is then made to pass through both the strings. The velocity of the wave in the first string having the radius of cross section R is v_1 and that in the other string having radius of cross section R/2 is v_2. Then $\frac{v_2}{v_1}$ =

(1) 8 (2) 2 (3) $\sqrt{2}$ (4) 4

Ans. [2]

Sol. $v = \sqrt{\frac{T}{\mu}}$ $\mu = \rho\pi R^2$

$\frac{v_1}{v_2} = \sqrt{\frac{R_2^2}{R_1^2}} = \frac{R_2}{R_1} = \frac{1}{2}$

$\frac{v_2}{v_1} = 2$

Q.38 An infinitely long wire has uniform linear charge density λ = 2nC/m. The net flux through a Gaussian cube of side length $\sqrt{3}$ cm, if the wire passes through any two corners of the cube, that are maximally displaced from each other, would be x Nm^2C^{-1}, where x is

[Neglect any edge effects and use $\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9$ SI units]

(1) 0.72 π (2) 2.16 π
(3) 1.44 π (4) 6.48 π

Ans. [2]

Sol. $\phi = \frac{q_{enclosed}}{\varepsilon_0}$; $\ell = \sqrt{3}a = 3$

$q = \lambda\ell$

$\phi = \frac{\lambda\ell}{\varepsilon_0} = 2 \times 10^{-9} \times 3 \times 10^{-2} \times 4\pi \times 9 \times 10^9$

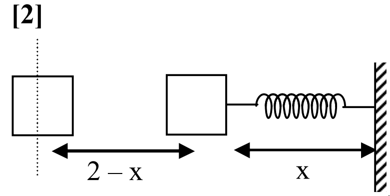
= 2.16 π

Q.39 A block of mass 2 kg is attached to one end of a massless spring whose other end is fixed at a wall. The spring-mass system moves on a frictionless horizontal table. The spring's natural length is 2m and spring constant is 200 N/m. The block is pushed such that the length of the spring becomes 1 m and then released. At distance x m (x < 2) from the wall, the speed of the block will be

(1) $10\left[1-(2-x)^2\right]^2$ m/s
(2) $10\left[1-(2-x)^2\right]^{1/2}$ m/s
(3) $10\left[1-(2-x)^2\right]^{3/2}$ m/s
(4) $10\left[1-(2-x)^2\right]$ m/s

Ans. [2]

Sol.



Energy conservation

$\frac{1}{2}k(1)^2 = \frac{1}{2}mv^2 + \frac{1}{2}k(2-x)^2$

Compression in the spring = (2 – x)

$\Rightarrow \frac{1}{2}k(1-(2-x)^2) = \frac{1}{2}mv^2$

$\Rightarrow v = [100(1-(2-x)^2)]^{1/2}$

$v = 10[1-(2-x)]^{1/2}$

Q.40 Given below are two statements: one is labelled as **Assertion A** and the other is labelled as **Reason R.**

Assertion A : Work done in moving a test charge between two points inside a uniformly charged spherical shell is zero, no matter which path is chosen.

Reason R : Electrostatic potential inside a uniformly charged spherical shell is constant and is same as that on the surface of the shell.

In the light of the above statements, choose the correct answer from the options given below

(1) A is false but R is true

(2) Both A and R are true and R is the correct explanation of A

(3) Both A and R are true but R is NOT the correct explanation of A

(4) A is true but R is false

Ans. [2]

Sol. Both A & R are correct and R is correct explanation

$\Delta w = q(\Delta V)$

$\Delta V = 0$

$\Rightarrow \Delta w = 0$

Q.41 Two metal spheres of radius R and 3R have same surface charge density σ. If they are brought in contact and then separated, the surface charge density on smaller and bigger sphere becomes σ_1 and σ_2, respectively. The ratio $\frac{\sigma_1}{\sigma_2}$ is

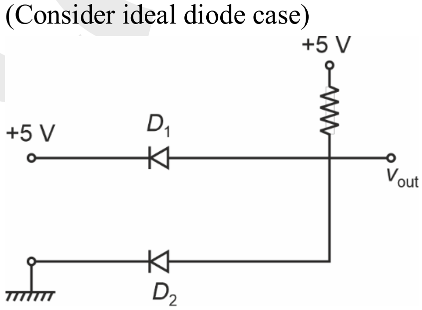
(1) 9 (2) 3 (3) $\frac{1}{9}$ (4) $\frac{1}{3}$

Ans. [2]

Sol. $V_1 = V_2 \Rightarrow \frac{Q_1}{R_1} = \frac{Q_2}{R_2} \Rightarrow \frac{\sigma_1 R_1^2}{R_1} = \frac{\sigma_2 R_2^2}{R_2}$

$\Rightarrow \frac{\sigma_1}{\sigma_2} = \frac{R_2}{R_1} = 3$

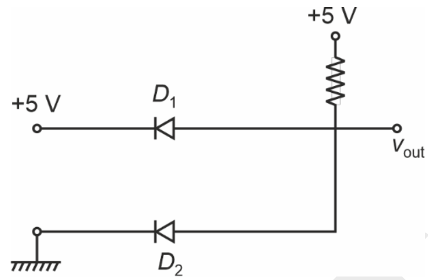
Q.42 The output voltage in the following circuit is (Consider ideal diode case)



(1) 10 V (2) 0 V (3) +5 V (4) –5V

Ans. [2]

Sol.



D_1 = Reverse biased

D_2 = Forward biased

$V_{(out)} = 0$ V

Q.43 Water falls from a height of 200 m into a pool. Calculate the rise in temperature of the water assuming no heat dissipation from the water in the pool.

(Take $g = 10$ m/s^2, specific heat of water = 4200 J/(kgK))

(1) 0.36 K (2) 0.23 K

(3) 0.48 K (4) 0.14 K

Ans. [3]

Sol. $mgh = ms\Delta T$

$\Delta T = \frac{gh}{S} = \frac{10 \times 200}{4200} = 0.48$ K

Q.44 A 3m long wire of radius 3 mm shows an extension of 0.1 mm when loaded vertically by a mass of 50 kg in an experiment to determine Young's modulus. The value of Young's Modulus of the wire as per this experiment is $P \times 10^{11}$ Nm^{-2}, where the value of P is:

(Take $g = 3\pi$ m/s^2)

(1) 2.5 (2) 25 (3) 5 (4) 10

Ans. [3]

Sol. $Y = \frac{F/A}{\Delta L/L} = \frac{FL}{A\Delta L} = \frac{(500N)(3)}{\pi \times 9 \times 10^{-6} \times 1 \times 10^{-4}}$

$= 5 \times 10^{11}$ N/m^2

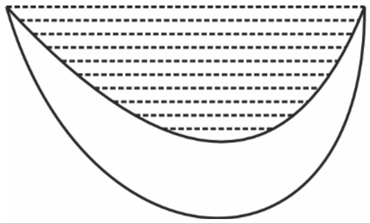
Q.45 A concave-convex lens of refractive index 1.5 and the radii of curvature of its surfaces are 30 cm and 20 cm, respectively. The concave surface is upwards and is filled with a liquid of refractive index 1.3. The focal length of the liquid-glass combination will be

(1) $\frac{600}{11}$ cm (2) $\frac{500}{11}$ cm

(3) $\frac{700}{11}$ cm (4) $\frac{800}{11}$ cm

Ans. [1]

Sol.



$$\frac{1}{F}=\frac{1}{F_1}+\frac{1}{F_2}$$

$$\frac{1}{F_1}=(1.3-1)\left(-\left(\frac{-1}{30}\right)\right)=\frac{0.3}{30}=\frac{1}{100}$$

$$\frac{1}{F_2}=(1.5-1)\left(\frac{-1}{30}-\frac{1}{-20}\right)$$

$$\frac{1}{F_2}=0.5\left(\frac{3}{60}-\frac{2}{60}\right)=\frac{1}{120}$$

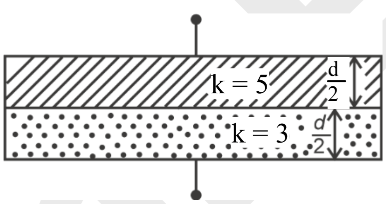
$$\frac{1}{F}=\frac{1}{120}+\frac{1}{100}=\frac{10+12}{1200}$$

$$\frac{1}{F}=\frac{22}{1200}$$

$$F=\frac{600}{11}$$

Section-B: Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

Q.46



Space between the plates of a parallel plate capacitor of plate area 4 cm^2 and separation of (d) 1.77 mm, is filled with uniform dielectric materials with dielectric constants (3 and 5)as shown in figure. Another capacitor of capacitance 7.5 pF is connected in parallel with it. The effective capacitance of this combination is _________ pF.

(Given $\varepsilon_0 = 8.85 \times 10^{-1}$ F/m)

Ans. [15]

Sol.

$$\frac{1}{C}=\frac{1}{C_1}+\frac{1}{C_2}=\frac{d/2}{Ak_1\varepsilon_0}+\frac{d/2}{Ak_2\varepsilon_0}$$

$$=\left(\frac{1}{k_1}+\frac{1}{k_2}\right)\frac{d}{2A\varepsilon_0}$$

$$=\left(\frac{1}{3}+\frac{1}{5}\right)\frac{d}{2A\varepsilon_0}$$

$$\frac{1}{C}=\frac{4}{15}\frac{d}{A\varepsilon_0}$$

$$C=\frac{15}{4}\times\frac{A\varepsilon_0}{d}=\frac{15}{4}\times\frac{4\times10^{-4}\times8.85\times10^{-12}}{1.77\times10^{-3}}$$

$= 7.5$ pF

$C + C_3 = (7.5 + 7.5)$pF $= 15$ pF

Q.47 An electron is released from rest near an infinite non-conducting sheet of uniform charge density '$-\sigma$'The rate of change of de-Broglie wave length associated with the electron varies inversely as n^{th} power of time. The numerical value of n is __________.

Ans. [2]

Sol.

$$F=qE=q\left(\frac{\sigma}{2\varepsilon_0}\right)=\frac{\Delta p}{\Delta t}$$

$$\lambda=\frac{h}{p}\quad\frac{d\lambda}{dt}=\frac{h}{p^2}\times\frac{\Delta p}{\Delta t}=\frac{h}{p^2}\times\frac{q\sigma}{2\varepsilon_0}$$

$$\frac{d\lambda}{dt}\propto\frac{1}{p^2}\propto\frac{1}{t^2}\propto\frac{1}{t^n}$$

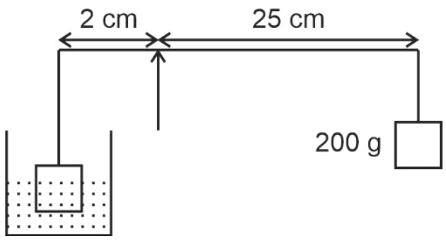
$$p=\frac{q\sigma}{2\varepsilon_0}t$$

$n = 2$

Q.48 A cube having a side of 10 cm with unknown mass and 200 gm mass were hung at two ends of an uniform rigid rod of 27 cm long. The rod along with masses was placed on a wedge keeping the distance between wedge point and 200 gm weight as 25 cm. Initially the masses were not at balance. A beaker is placed beneath the unknown mass and water is added slowly to it. At given point the masses were in balance and half volume of the unknown mass was inside the water. (Take the density of unknown mass is more than that of the water, the mass did not absorb water and water density is 1 gm/cm^3.) The unknown mass is _________ kg.

Ans. [3]

Sol.



$25 \times 0.2 \times g = 2 \times (m - \rho \times V)g$

$m - \rho V = 2.5$ kg

$\rho V = \dfrac{1\times10^{-3}\,kg}{cm^3} \times \dfrac{10^3\,cm^3}{2} = \dfrac{1}{2}$ kg

$m = 3$ kg

Q.49 A sample of a liquid is kept at 1 atm. It is compressed to 5 atm which leads to change of volume of 0.8 cm^3. If the bulk modulus of the liquid is 2 GPa, the initial volume of the liquid was _________ litre.
(Take 1 atm = 10^5 Pa)

Ans. [4]

Sol. $-\dfrac{\Delta VB}{V} = \Delta P$

$\Rightarrow 4 \times 10^5\ Pa = \dfrac{2\times10^9\,Pa\times10^{-6}\times0.8}{V}$

$V = \dfrac{2\times10^9\,Pa\times10^{-6}\times0.8}{4\times10^5} = 4 \times 10^{-3}\ m^3$

$\Rightarrow$ 4 litre

Q.50 A thin solid disk of 1 kg is rotating along its diameter axis at the speed of 1800 rpm. By applying an external torque of 25π Nm for 40s, the speed increases to 2100 rpm. The diameter of the disk is _________ m.

Ans. [40]

Sol. $\tau dt = I\Delta w$

$\Rightarrow 25\pi \times 40 = I(300) \times \dfrac{2\pi}{60}$

$I = \dfrac{25\times60\times40}{300\times2} = 100 = \dfrac{MR^2}{4}$

$R^2 = 400$

$R = 20$ m

$D = 40$ m

CHEMISTRY

Section-A: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct..

Q.51 Correct statements for an element with atomic number 9 are

A. There can be 5 electrons for which $m_s = +\dfrac{1}{2}$ and 4 electrons for which $m_s = -\dfrac{1}{2}$

B. There is only one electron in p_z orbital

C. The last electron goes to orbital with n = 2 and $\ell = 1$

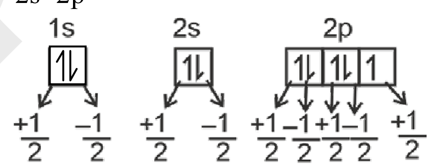
D. The sum of angular nodes of all the atomic orbitals is 1

Choose the correct answer from the options given below :

(1) A and B only (2) C and D only
(3) A and C only (4) A, C and D only

Ans. [3]

Sol. Electronic configuration $\Rightarrow$
$1s^2\ 2s^2\ 2p^5$



• There can be $5e^-$ for which $m_s = +\dfrac{1}{2}$

and $4e^-$ for which $m_s = -\dfrac{1}{2}$

• There can be $2e^-$ in p_z orbital

• Last e^- enters 2p (n = 2, $\ell = 1$)

• Angular nodes = ℓ

∴ Sum of angular nodes = 0 + 0 + 3 =3

Q.52 Match List-I with List-II.

	List-I		*List-II*
A.	$[Ni(CO)_4]$	I.	*Tetrahedral, 2.8 BM*
B.	$[Ni(CN)_4]^{2-}$	II.	*Square planar, 0 BM*
C.	$[NiCl_4]^{2-}$	III.	*Tetrahedral, 0 BM*
D.	$[MnBr_4]^{2-}$	*IV.*	*Tetrahedral, 5.9 BM*

Choose the correct answer from the options given below

(1) A(III), B(II), C(I), D(IV)

(2) A(I), B(II), C(III), D(IV)

(3) A(IV), B(I), C(III), D(II)

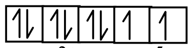
(4) A(III), B(IV), C(II), D(I)

Ans. **[1]**

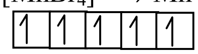
Sol. A) $Ni(CO)_4 \rightarrow Ni(0) \Rightarrow sp^3$, tetrahedral, 0 BM
$(3d^{10})$ (pairing)

B) $[Ni(CN)_4]^{2-} \rightarrow Ni^{2+} \Rightarrow dsp^2$, square plananr, 0 BM $(3d^{10})$ (pairing)

C) $[NiCl_4]^{2-} \rightarrow Ni^{2+}$(no pairing) $\Rightarrow sp^3$, tetrahedral 2.8 BM

$3d^8$ 

D) $[MnBr_4]^{2-} \Rightarrow Mn^{2+}, \Rightarrow 3d^5$ (no pairing)



μ = 5.9 BM

(A-III, B-II, C-I, D-IV)

Q.53 $HA(aq) \rightleftharpoons H^+(aq) + A^-(aq)$

The freezing point depression of a 0.1 m aqueous solution of a monobasic weak acid HA is 0.20°C the dissociation constant for the acid is

Given: $K_f(H_2O)$ = 1.8 K kg mol^{-1}, molality ≡ molarity

(1) 1.1×10^{-2} (2) 1.38×10^{-3}

(3) 1.90×10^{-3} (4) 1.89×10^{-1}

Ans. **[2]**

Sol. $\Delta T_f = iK_f m$

$$i = \frac{\Delta T_f}{K_f m}$$

$$i = \frac{0.20}{1.8 \times 0.1} = 1.11$$

$i = 1.11$

$$\alpha = \frac{i-1}{n-1} \text{ (for HA, n = 2)}$$

$$\alpha = \frac{1.11-1}{1} = 0.11$$

$$K_a = \frac{c\alpha^2}{1-\alpha} = \frac{0.1 \times (0.11)^2}{1-0.11} = 1.38 \times 10^{-3}$$

Q.54 Given below are two statements :

Statements-I : A homoleptic octahedral complex, formed using monodentate ligands, will not show stereoisomerism

Statements-II : cis and trans platin are heteroleptic complexes of Pd

In the light of the above statements, choose the correct answer from the options given below

(1) Both Statement I and Statement II are true

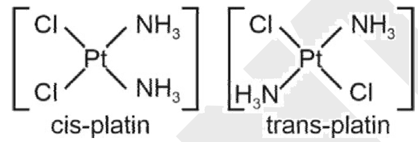
(2) Statement I is true but Statement II is false

(3) Both Statement I and Statement II are false

(4) Statement I is false but Statement II is true

Ans. **[2]**

Sol. $[Ma_6]$ type complex will not show stereoisomerism where a is monodentate ligand Cis and trans-platin are heterolytic complexes of Pt(Platinum), Formula is $[Pt(NH_3)_2)Cl_2]$



Q.55 Which of the following binary mixture does not show the behaviour of minimum boiling azeotropes?

(1) $C_6H_5OH + C_6H_5NH_2$

(2) $H_2O + CH_3COC_2H_5$

(3) $CH_3OH + CHCl_3$

(4) $CS_2 + CH_3COCH_3$

Ans. **[1]**

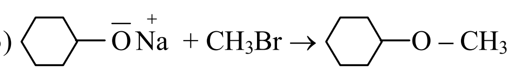
Sol. The solution showing positive deviation from Raoult's law will form minimum boiling azeotrope. Phenol + Aniline shows negative deviation, so they will not form minimum boiling azeotrope

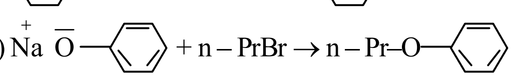
Q.56 Which one of the following reactions will not lead to the desired ether formation in major proportion?

(iso-Bu ⇒ isobutyl, sec-Bu ⇒ sec-butyl, nPr ⇒ n-propyl, ^{t}Bu ⇒ tert-butyl, Et ⇒ ethyl)

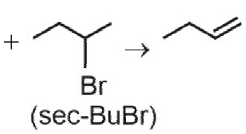
(1) iso-Bu$\bar{O}\overset{+}{Na}$ + sec-BuBr → sec-Bu–O–iso-Bu

(2) tBu$\bar{O}\overset{+}{Na}$ + EtBr → ^{t}Bu – O – Et

(3) 

(4) 

Ans. **[1]**

Sol. iso-BuO$^{\ominus}$Na$^+$ + 

For 2° RX ⇒ elimination would be more favourable than substitution

For williamson's synthesis of ether

$RO^{\ominus}Na^+ + RX \rightarrow ROR$ (undergo S_N2 reaction)
(1° alkyl halide)

Q.57 Match the List-I with List-II

	List-I		*List-II*
A.	Carbocation	I.	*Species that can supply a pair of electrons*
B.	C-Free radical	II.	*Species that can receive a pair of electrons*
C.	Nucleophile	III.	*sp^2 hybridized carbon with empty p-orbital*
D.	*Electrophile*	*IV.*	*sp^2/sp^3 hybridised carbon with one unpaired electron*

Choose the correct answer from the options given below :

(1) A(III), B(IV), C(II), D(I)

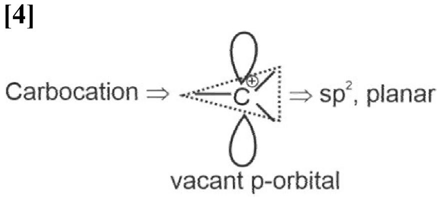
(2) A(II), B(III), C(I), D(IV)

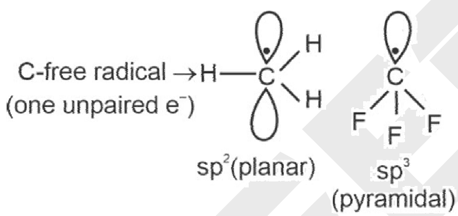
(3) A(IV), B(II), C(III), D(I)

(4) A(III), B(IV), C(I), D(II)

Ans. **[4]**

Sol.





Nucleophile ⇒ e^- rich species like anions etc.

⇒ can supply a pair of electrons

Electrophile ⇒ e^- deficient species ⇒ can receive a pair of electrons.

Q.58 The correct decreasing order of spin only magnetic moment values (BM) of Cu^+, Cu^{2+}, Cr^{2+} and Cr^{3+} ions is :

(1) $Cu^+ > Cu^{2+} > Cr^{3+} > Cr^{2+}$

(2) $Cu^{2+} > Cu^+ > Cr^{2+} > Cr^{3+}$

(3) $Cr^{3+} > Cr^{2+} > Cu^+ > Cu^{2+}$

(4) $Cr^{2+} > Cr^{3+} > Cu^{2+} > Cu^+$

Ans. **[4]**

Sol. $Cu^+ \Rightarrow 3d^{10} \Rightarrow$ [↑↓][↑↓][↑↓][↑↓][↑↓] ⇒ 0 BM

$Cu^{2+} \Rightarrow 3d^9 \Rightarrow$ [↑↓][↑↓][↑↓][↑↓][↑] ⇒ 1.73 BM

$Cr^{2+} \Rightarrow 3d^4 \Rightarrow$ [↑][↑][↑][↑][ ] ⇒ 4.89 BM

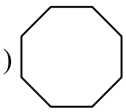
$Cr^{3+} \Rightarrow 3d^3 \Rightarrow$ [↑][↑][↑][ ][ ] ⇒ 3.87 BM

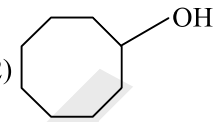
So order

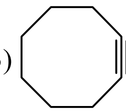
$Cr^{2+} > Cr^{3+} > Cu^{2+} > Cu^+$

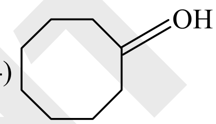
Q.59 1, 2-dibromocyclooctane $\xrightarrow[\text{(ii) } NaNH_2 \\ \text{(iii) } Hg^{2+}/H^+ \\ \text{(iv) Zn-Hg}/H^+]{\text{(i) KOH (alc.)}}$ P (major product)

P is

(1) 

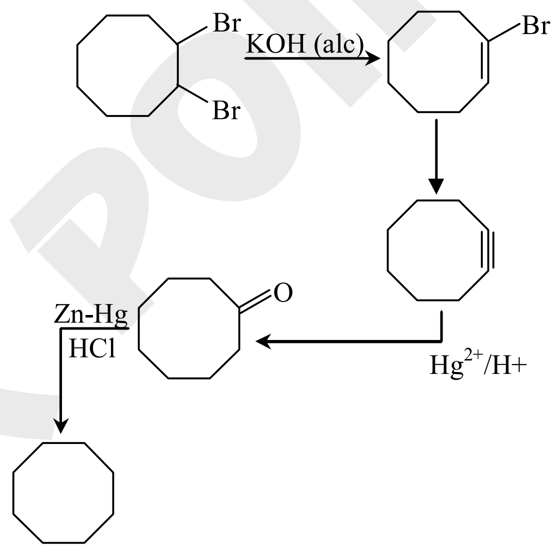
(2) 

(3) 

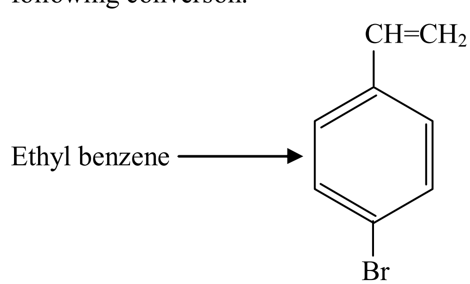
(4) 

Ans. **[1]**

Sol.



Q.60 Choose the correct set of reagents for the following converson.



(1) Cl_2/anhy.$AlCl_3$; Br_2/Fe ; alc. KOH

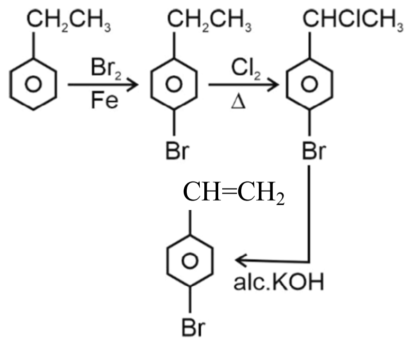
(2) Br_2/Fe ; Cl_2 Δ ; alc. KOH

(3) Br_2/anhy.$AlCl_3$; Cl_2, Δ ; aq. KOH

(4) Cl_2/Fe ; Br_2/anhy.$AlCl_3$; aq. KOH

Ans. **[2]**

Sol.



Q.61

$$\underset{\text{pH}=2\ \swarrow\quad\searrow\ \text{pH}=10}{\overset{\displaystyle CH(CH_3)_2}{\overset{|}{H_2N-CH-COOH}}}$$

A B

Choose the correct option for structures of A and B, respectively

(1) $H_2N-\underset{\displaystyle CH(CH_3)_2}{\underset{|}{CH}}-CO\bar{O}$ and $\overset{+}{H_3N}-\underset{\displaystyle CH(CH_3)_2}{\underset{|}{CH}}-COOH$

(2) $H_2N-\underset{\displaystyle CH(CH_3)_2}{\underset{|}{CH}}-CO\overset{\ominus}{O}$ and $\overset{+}{H_3N}-\underset{\displaystyle CH(CH_3)_2}{\underset{|}{CH}}-CO\bar{O}$

(3) $\overset{+}{H_3N}-\underset{\displaystyle CH(CH_3)_2}{\underset{|}{CH}}-CO\bar{O}$ and $\overset{+}{H_3N}-\underset{\displaystyle CH(CH_3)_2}{\underset{|}{CH}}-COOH$

(4) $\overset{+}{H_3N}-\underset{\displaystyle CH(CH_3)_2}{\underset{|}{CH}}-COOH$ and $H_2N-\underset{\displaystyle CH(CH_3)_2}{\underset{|}{CH}}-CO\bar{O}$

Ans. [4]

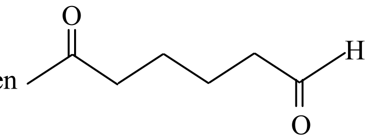
Sol. At pH = 2 ⇒ Medium is acidic so $-NH_2$ group will convert to $-NH_3^+$

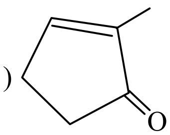
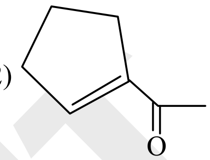
So A = $\overset{\oplus}{H_3N}-\overset{\displaystyle CH(CH_3)_2}{\overset{|}{CH}}-COOH$

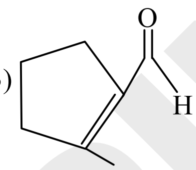
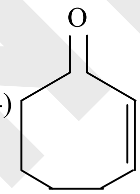
∴ At pH = 10 ⇒ medium is basic

So –COOH group will convert to $-COO^{\ominus}$

So B = $H_2N-\overset{\displaystyle CH(CH_3)_2}{\overset{|}{CH}}-CO\overset{\ominus}{O}$

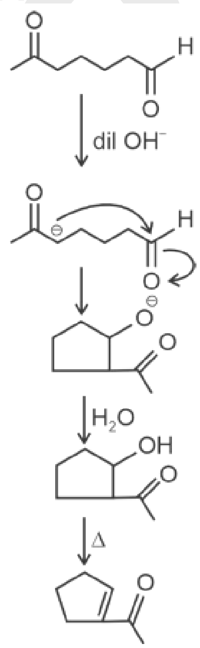
Q.62 When  undergoes intramolecular aldol condensation, the major product formed is :

(1)  (2) 

(3)  (4) 

Ans. [2]

Sol.



Q.63 What is the correct IUPAC name of

(1) 4-Ethylcyclopent-2-en-1-ol

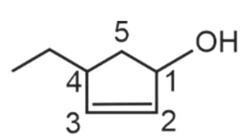
(2) 4-Ethyl-1-hydroxycyclopent-2-ene

(3) 1-Ethyl-3-hyderoxycyclopent-2-ene

(4) 1-Ethylcyclopent-2-en-3-ol

Ans. [1]

Sol.



4-Ethylcyclopent-2-en-1-ol

Q.64 Given below are two statements:
Statement I : H_2Se is more acidic than H_2 Te
Statement II : H_2Se has higher bond enthalpy for dissociation than H_2 Te
In the light of the above statement, choose the correct answer from the option given
(1) Both statement I and Statement II are false
(2) Both Statement I and Statement II are true
(3) Statement I is false but Statement II is true
(4) Statement I is true but Statement II is false

Ans. [3]

Sol. $\therefore$ H_2Te is more acidic than H_2 Se
because of lesser bond dissociation energy of H_2Te
It can release H^+ more easily
pK_{a_1} : $H_2Se(3.89) > H_2Te\ (2.6)$

Q.65 In a first order decomposition reaction, the time taken for the decomposition of reactant to one fourth and one eights of its initial concentration are t_1 and t_2(s), respectively. The ratio t_1/t_2 will be:

(1) $\frac{4}{3}$ (2) $\frac{3}{2}$ (3) $\frac{3}{4}$ (4) $\frac{2}{3}$

Ans. [4]

Sol. $t_1 = t_{\frac{1}{4}} = \frac{1}{k} \text{lm} \frac{A_0}{\frac{A_0}{4}} = \frac{1}{k} \ln 4$

$t_2 = t_{\frac{1}{8}} = \frac{1}{k} \text{lm} \frac{A_0}{\frac{A_0}{8}} = \frac{1}{k} \ln 8$

$\frac{t_1}{t_2} = \frac{\ln 4}{\ln 8} = \frac{2\ln 2}{3\ln 2} = \frac{2}{3}$

Q.66 On combustion of 0.210 g of an organic compound containing C, H and O gave 0.127 g H_2O and 0.307 g CO_2. The percentage of hydrogen and oxygen in the given organic compound respectively are:
(1) 53.41, 39.6 (2) 6.72, 39.87
(3) 6.72, 53.41 (4) 7.55, 43.85

Ans. [3]

Sol. Mass of organic compound = 0.210 g
Mass of water formed = 0.127 g
Mass of CO_2 = 0.307 g

Mass of hydrogen = $\frac{0.127 \times 2}{18}$ = 0.014 g

Percentage of hydrogen = $\frac{0.014 \times 100}{0.210}$ = 6.72%

Mass of carbon = $\frac{0.307 \times 12}{44}$ = 0.084 g

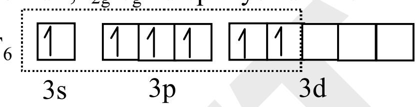
Percentage of carbon = $\frac{0.084 \times 100}{0.210}$ = 39.87%

Percentage of oxygen = 53.41%

Q.67 The number of species from the following that are involved in sp^3d^2 hybridization is $[Co(NH_3)_6]^{3+}$, SF_6, $[CrF_6]^{3-}$, $[Mn(CN)_6]^{3-}$ and $[MnCl_6]^{3-}$
(1) 4 (2) 6 (3) 5 (4) 3

Ans. [4]

Sol. (i) $[Co(NH_3)_6]^{3+}$
$Co^{3+} \Rightarrow 3d^6$; $t_{2g}{}^6 e_g 0$ d^2sp^3 hybridisation

(ii) SF_6 

sp^3d^2 hybridisation

(iii) $[CrF_6]^{3-}$
$Cr^{3+} \Rightarrow 3d^3$; $t_{2g}{}^3 eg^0$ d^2sp^3 hybridisation

(iv) $[CoF_6]^{3-}$
$Co^{3+} \Rightarrow 3d^6\ t_{2g}{}^4 e_g{}^2$ sp^3d^2 hybridisation

(v) $[Mn(CN)_6]^{3-}$
$Mn^{3+} \Rightarrow 3d^4 ; t_{2g}{}^4 e_g{}^0$ d^2sp^3 hybridisation

(vi) $[Mn(Cl)_6]^{3-}$
$Mn^{3+} \Rightarrow 3d^4 ; t_{2g}{}^3 e_g{}^1$ sp^3d^2 hybridisation

Q.68 The atomic number of the element from the following with lowest 1st ionisation enthalpy is
(1) 35 (2) 19 (3) 32 (4) 87

Ans. [4]

Sol.

Atomic No.	Period No.	*Group No.*
35	4	*17*
19	4	*1*
32	4	*14*
87	*6*	*1*

First ionisation energy of an element generally decreases down the group and increases from left to right along a period. Therefore, element having atomic number 87 has the lowest first ionisation energy.

Q.69 Match the List-I with List-II

List-I (Reagent)		*List-II (Functional Group detected)*	
A.	Sodium bicarbonate	I.	*Double bond/unsaturation*
B.	Neutral ferric chloride	II.	*Carboxylic acid*
C.	Ceric ammonium nitrate	III.	*Phenolic-OH*
D.	*Alkaline $KMnO_4$*	*IV.*	*Alcoholic-OH*

Choose the correct answer from the option given below:

(1) A-III, B-II, C-IV, D-I

(2) A-II, B-IV, C-III, D-I

(3) A-II, B-III, C-IV, D-I

(4) A-II, B-III, C-I, D-IV

Ans. **[3]**

Sol.

	Reagent		Functional group
(A)	Sodium bicarbonate	(II)	Carboxylic acid
(B)	Neutral ferric chloride	(III)	Phenolic-OH
(C)	Ceric Ammonium nitrate	(IV)	Alcoholic-OH
(D)	Alkaline $KMnO_4$	(I)	Double bond/Unsaturation

(A) Carboxylic acid gives effervescence with sodium bicarbonate

$R - COOH + NaHCO_3 \rightarrow RCOONa + CO_2 \uparrow + H_2O$

(B) Phenolic-OH gives characteristic colour with neutral $FeCl_3$

(C) Alcoholic-OH gives red colour with ceric ammonium nitrate

$(NH_4)_2[Ce(NO_3)_6] + 2ROH \rightarrow [Ce(ROH)_2(NO_3)_4] + 2NH_4NO_3$

(D) Purple colour of alkaline $KMnO_4$ is discharged by multiple bond of alkenes and alkynes

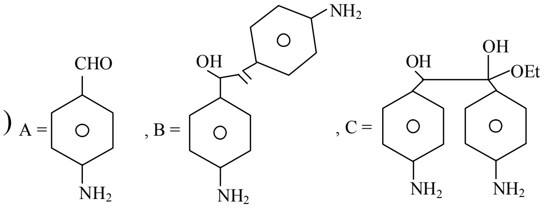
Q.70 $A \xrightarrow[\text{(ii) } H_3O^+]{\text{(i) NaOH}} B \xrightarrow[\text{(ii) } H_2SO_4,\ \Delta]{\text{(i) EtOH}} C$

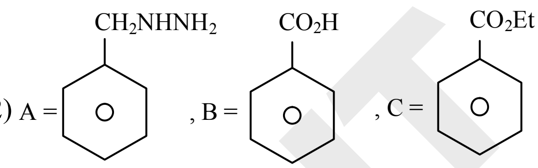
'A' shows positive Lassaigne's test for N and its molar mass is 121.

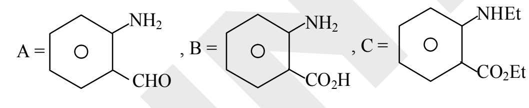
'B' gives effervescence with aq $NaHCO_3$

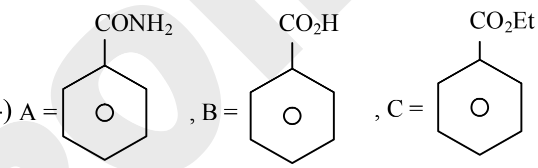
'C' gives fruity smell

Identify A, B and C from the following

(1) 

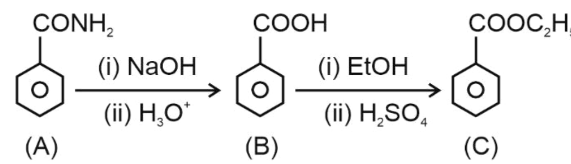
(2) 

(3) 

(4) 

Ans. **[4]**

Sol. Compound (A) is likely to be amide which gives carboxylic acid (B) after hydrolysis. Compound (B) reacts with alcohol to give ester



Section-B: Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

Q.71 Consider the following half cell reaction

$Cr_2O_7^{2-}(aq) + 6e^- + 14H^+(aq) \rightarrow 2Cr^{3+}(aq) + 7H_2O(\ell)$ The reaction was conducted with the ratio of $\frac{[Cr^{3+}]^2}{[Cr_2O_7^{2-}]} = 10^{-6}$. The pH value at which the EMF of the half cell will become zero is ______. (nearest integer value)

[Given : standard half cell reduction potential $E^o_{Cr_2O_7^{2-},H^+/Cr^{3+}} = 1.33V, \frac{2.303RT}{F} = 0.059V$]

Ans. **[10]**

Sol. $Cr_2O_7^{2-}(aq) + 6e^- + 14H^+(aq) \rightarrow 2Cr^{3+}(aq) + 7H_2O(\ell)$

Using Nernst equation

$$E_{Cr_2O_7^{2-},H^+/Cr^{3+}} = E^o_{Cr_2O_7^{2-},H^+/Cr^{3+}} = -\frac{2.303RT}{6F}\log\frac{[Cr^{3+}]^2}{[Cr_2O_7^{2-}][H^+]^{14}}$$

$$0 = E^o_{Cr_2O_7^{2-},H^+/Cr^{3+}} - \frac{0.059}{6}\log\left(10^{-6}[H^+]^{-14}\right)$$

$$0 = 1.33 + 0.059 - \frac{0.059\times14}{6}pH$$

$$pH = \frac{1.389\times6}{0.059\times14} = 10.10 \approx 10$$

Q.72 The equilibrium constant for decomposition of $H_2O(g) \rightleftharpoons H_2(g) + \frac{1}{2}O_2(g)$ (ΔG^o = 92.34 kJ mol^{-1}) is 8.0×10^{-3} at 2300 K and total pressure at equilibrium is 1 bar. Under this condition, the degree of dissociation (α) of water is _______ $\times 10^{-2}$ (nearest integer value) [Assume α is negligible with respect to 1]

Ans. **[5]**

Sol. $H_2O(g) \rightleftharpoons H_2(g) + \frac{1}{2}O_2(g)$

	$H_2O(g)$	$H_2(g)$	$\frac{1}{2}O_2(g)$
Initial mole	1	–	–
moles at equil.	$1-\alpha$	α	$\frac{\alpha}{2}$

Equilibrium pressure = 1 bar

$$K_p = \frac{\left(\frac{2\alpha}{2+\alpha}\right)\left(\frac{\alpha}{2+\alpha}\right)^{\frac{1}{2}}}{2\left(\frac{1-\alpha}{2+\alpha}\right)} = 8.0\times10^{-3}$$

$$\frac{\alpha^{\frac{3}{2}}}{\sqrt{2}} = 8.0\times10^{-3}$$

$$\alpha = \left(8\sqrt{2}\times10^{-3}\right)^{\frac{2}{3}}$$

$$= \left(2^{\frac{7}{2}}\times10^{-3}\right)^{\frac{2}{3}} = 2^{\frac{7}{3}}\times10^{-2} = 5\times10^{-2}$$

Q.73 20 mL of sodium iodide solution gave 4.74 g silver iodide when treated with excess of silver nitrate solution. The molarity of the sodium iodide solution is ____M. (Nearest Integer Value)

Given : Na = 23, I = 127, Ag = 108, N = 14, O = 16 g mol^{-1})

Ans. **[1]**

Sol. Let Molarity of NaI solution be x M

$NaI + AgNO_3 \rightarrow AgI + NaNo_3$

Moles of AgI formed = $\frac{4.74}{235} = 0.02$

Moles of NaI = $\frac{20\times x}{1000} = 0.02x$

$0.02x = 0.02$

$x = 1$

$\therefore$ Molarity of NaI solution = 1 M

Q.74 The energy of an electron in first Bohr orbit of H^- atom is –13.6 eV. The magnitude of energy value of electron in the first excited state of Be^{3+} is _______ eV (nearest integer value)

Ans. **[54]**

Sol. E_1 of H-atom = –13.6 eV

$$E_2 \text{ of } Be^{3+} = \frac{-13.6\times Z^2}{n^2}$$

$$= \frac{-13.6\times(4)^2}{(2)^2}$$

= –54.4 eV

$|E_2|$ of Be^{3+} = 54 eV

Q.75 Resonance in X_2Y_2 can be represented as

$$\overset{\ominus}{\ddot{\underset{..}{X}}} = \overset{\oplus}{X} = \ddot{\underset{..}{Y}} \leftrightarrow :X\equiv\overset{\oplus}{X} - \overset{\ominus}{\ddot{\underset{..}{Y}}}:$$

The enthalpy of formation of X_2Y $\left(x\equiv X(g) + \frac{1}{2}Y = Y(g) \rightarrow X_2Y(g)\right)$ is 80 kJ mol^{-1}. The magnitude of resonance energy of X_2Y is _______ kJ mol^{-1} (nearest integer value)

Given : Bond energies of X $\equiv$ X, X = X, Y = Y and X = Y are 940, 410, 500 and 602 kJ mol^{-1} respectively.

Valence X : 3, Y : 2

Ans. **[98]**

Sol. $X\equiv X(g) + \frac{1}{2}Y = Y(g) \rightarrow \overset{(-)}{X} = \overset{(+)}{X} = Y(g)$

$[\Delta H_f(X_2Y_2)]_{Actual} = 80$ kJ mol^{-1}

$[\Delta H_f(X_2Y)]_{Theoretical} = 940 + \frac{1}{2}(500) - (410 + 602)$

= 1190 – 1012

= 178 kJ mol^{-1}

Resonance energy = $(\Delta H_f)_{Actual} - (\Delta H_f)]_{Theoretical}$

= |80 – 178| = 98 kJ mol^{-1}