JEE Main Online Exam 2025

Questions & Solution

07th April 2025 | Morning

MATHEMATICS

Section-A: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct..

Q.1 The mean and standard deviation of 100 observations are 40 and 5.1, respectively. By mistake one observation is taken as 50 instead of 40. If the correct mean and the correct standard deviation are μ and σ respectively, then $10(\mu+\sigma)$ is equal to

(1) 447 (2) 445 (3) 449 (4) 451

Ans. [3]

Sol. Let the observations be $x_1, x_2, ..., X_{99}, 50$

$$\text{Mean} = \frac{x_1 + x_2 + ... + x_9 + 50}{100} = 40$$

$\Rightarrow x_1 + x_2 + \ldots + x_{99} = 4000 - 50$

$\Rightarrow x_1 + x_2 + \ldots + x_{99} = 3950$

$$\text{Current Mean} = \frac{3950 + 40}{100}$$

$$\mu = \frac{399}{10} = 39.9$$

$$(\text{S.D})^2 = \sum_{i=1}^{99} \frac{(x_i)^2 + 2500}{100} - (40)^2$$

$$\sum_{i=1}^{99} x_i^2 = 160101$$

$$(\text{Correct S.D})^2 = \frac{160101 + 1600}{100} - \left(\frac{399}{10}\right)^2$$

$\sigma = 5$

$10(\mu + \sigma) = 10(39.9 + 5) = 449$

Q.2 Let the set of all values $p \in R$, for which both the roots of the equation $x^2 - (p + 2)x + (2p + 9) = 0$ are negative real numbers, be the interval $(\alpha, \beta]$. Then $\beta - 2\alpha$ is equal to

(1) 9 (2) 5 (3) 20 (4) 0

Ans. [2]

Sol. $x^2 - (p + 2)x + (2p + 9) = 0$

$D \geq 0$

$(p + 2)^2 - 4(2p + 9) \geq 0$

$p^2 + 4p + 4 - 8p - 36 \geq 0$

$p^2 - 4p - 32 \geq 0$

$(p - 8)(p + 4) \geq 0$

$p \in (-\infty, -4] \cup [8, \infty)$ …(1)

Sum: $p + 2 < 0$

$p < -2$ …(2)

Product > 0

$2p + 9 > 0$

$P > \frac{-9}{2}$ .....(3)

From (1), (2) and (3)

$P \in \left(\frac{-9}{2}, -4\right]$

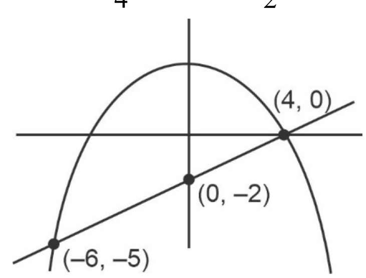
$\beta - 2a = -4 - 2\left(\frac{-9}{2}\right) = 5$

Q.3 If the area of the region bounded by the curves $y = 4 - \frac{x^2}{4}$ and $y = \frac{x-4}{2}$ is equal to α, then 6α equals

(1) 250 (2) 210 (3) 220 (4) 240

Ans. [1]

Sol. $y = 4 - \frac{x^2}{4}$ and $y = \frac{x-4}{2}$



$$\text{Area} = \int_{-6}^{4}\left(4 - \frac{x^2}{4} - \frac{x}{2} + 2\right)dx$$

$$= \left[6x - \frac{x^3}{12} - \frac{x^2}{4}\right]_{-6}^{4}$$

$$= 6\,(4+6) - \left(\frac{64}{12} + \frac{216}{12}\right) - \left(\frac{16}{4} - \frac{36}{4}\right)$$

$$= 60 - \frac{70}{3} + 5$$

$$\alpha = \frac{125}{3}$$

$$6\alpha = 6 \times \frac{125}{3} = 250$$

Q.4 Let x = –1 and x = 2 be the critical points of the function $f(x) = x^3 + ax^2 + b\log_e|x| + 1, x \neq 0$. Let m and M respectively be the absolute minimum and the absolute maximum values of f in the interval $\left[-2, -\frac{1}{2}\right]$. Then |M + m| is equal to (Take log2 = 0.7) :

(1) 21.1 (2) 20.9 (3) 19.8 (4) 22.1

Ans. [1]

Sol. $f'(x) = 3x^2 + 2ax + \frac{b}{x} = 0$ → –1, 2

$$\left.\begin{aligned} 3 - 2a - b &= 0 \\ 12 + 4a + \frac{b}{2} &= 0 \end{aligned}\right\} \begin{aligned} a &= \frac{-9}{2} \\ b &= 12 \end{aligned}$$

$$\therefore f(x) = x^3 - \frac{9}{2}x^2 + 12 \ln|x| + 1$$

$$f(-1) = -1 - \frac{9}{2} + 1 = -\frac{9}{2} = = -4.5$$

$$f(-2) = -8 - 18 + 12\ln 2 + 1$$

$$= -25 + 12\ln 2 = -16.6$$

$$f\left(-\frac{1}{2}\right) = -\frac{1}{8} - \frac{9}{8} + 12\ln\left(\frac{1}{2}\right) + 1 = -8.5$$

$$|M + m| = |\ -16.6 - 4.5| = 21.1$$

Q.5 Let y = y(x) be the solution curve of the differential equation $x(x^2 + e^x)dy + (e^x(x-2)y - x^3)dx = 0$, x > 0, passing through the point (1, 0). Then y (2) is equal to

(1) $\frac{2}{2+e^2}$ (2) $\frac{4}{4-e^2}$ (3) $\frac{2}{2-e^2}$ (4) $\frac{4}{4+e^2}$

Ans. [4]

Sol. $x(x^2 + e^x)dy + (e^x(x-2)y - x^3)dx = 0$

$$\frac{dy}{dx} + \frac{e^x(x-2)}{x(x^2+e^x)}y = \frac{x^3}{x(x^2+e^x)}$$

$$\text{I.F.} = e^{\int \frac{e^x(x-2)}{x(x^2+e^x)}dx}$$

$$= e^{\int \frac{e^x+2x}{x^x+x^2}dx - \int \frac{2}{x}dx}$$

$$= e^{\ln|e^x+x^2| - 2\ln x}$$

$$= \frac{e^x + x^2}{x^2}$$

$$\therefore y\left(\frac{e^2+x^2}{x^2}\right) = \int dx + c$$

$$\Rightarrow y\left(\frac{e^x+x^2}{x^2}\right) = x + c$$

Also, y(1) = 0

$\Rightarrow$ c = –1

$$\therefore y\left(\frac{e^2+x^2}{x^2}\right) = x - 1$$

Hence, $y(2) = \frac{4}{e^2+4}$

Q.6 Let A be a 3 × 3 matrix such that |adj(adj(adjA))| = 81. If $S = \left\{n \in Z : (|\text{adj}(\text{adj}A)|)^{\frac{(n-1)^2}{2}} = |A|^{(3n^2-5n-4)}\right\}$, then $\sum_{n\in S} |A^{(n^2+n)}|$ is equal to

(1) 750 (2) 866 (3) 732 (4) 820

Ans. [3]

Sol. |adj(adj(adjA))| = 81

$$= |A|^{(n-1)^3} = (3)^4 \Rightarrow |A|^8 = 3^4 \Rightarrow |A| = 3^{1/2}$$

$$|\text{adj}(\text{abj}A)|^{\frac{(n-1)^2}{2}} = |A|^{(3n^2-5n-4)}$$

$$\left[|A|^{2(n-1)^2}\right]^{\frac{(n-1)^2}{2}} = |A|^{3n^2-5n-4}$$

$$|A|^{2(n-1)^2} = |A|^{3n^2-5n-4}$$

$$\Rightarrow 2(n-1)^2 = 3n^2 - 5n - 4$$

$$n^2 - n - 6 = 0$$

$$\Rightarrow n = -2, 3$$

$$\sum_{x \leftarrow S} |A^{n^2+n}| = |A^2| + |A^{12}|$$

$$= 3 + 3^6 = 732$$

Q.7 Let ABC be the triangle such that the equations of lines AB and AC be $3y - x = 2$ and $x + y = 2$, respectively, and the points B and C lie on x-axis. If P is the orthocentre of the triangle ABC, then the area of the triangle PBC is equal to

(1) 4 (2) 6 (3) 8 (4) 10

Ans. [2]

Sol. Equation of line AB is $3y - x = 2$

And AC is $x + y = 2$

In line AB,

When $y = 0$, $x = -2$

$\therefore$ B(–2, 0)

In line AC,

When $y = 0$, $x = 2$

$\therefore$ C(2x, 0)

Equation of altitude of BC,

$Y = x + 2$

Similarly, equation of altitude of AB,

$y = -3x + 6$

$\therefore$ On solving, orthocentre P(1, 3)

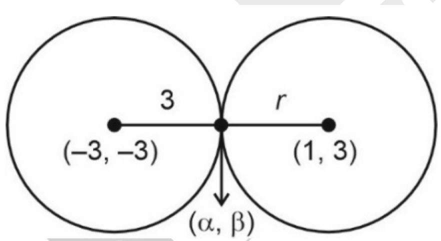
$\therefore$ ar(ΔPBC) = 6

Q.8 Let C_1 be the circle in the third quadrant of radius 3, that touches both coordinate axes. Let C_2 be the circle with centre (1, 3) that touches C_1 externally at the point (α, β). If $(\beta - \alpha)^2 = \frac{m}{n}$, gcd(m, n)=1, then $m + n$ is equal to

(1) 22 (2) 31 (3) 13 (4) 9

Ans. [1]

Sol.



$\sqrt{16+36} = r + 3$

$\Rightarrow r = \sqrt{52} - 3$

$\Rightarrow \frac{3-3r}{-r+3} = \alpha,\ \beta = \frac{9-3r}{3+r}$

$\Rightarrow \frac{(9-3r-3+3r)^2}{(r+3)^2}$

$\Rightarrow \frac{36}{12} = \frac{9}{13} = \frac{m}{n} \Rightarrow m + n = 22$

Q.9 Let the angle θ, $0 < \theta < \frac{\pi}{2}$ between two unit vectors $\hat{a}$ and $\hat{b}$ be $\sin^{-1}\left(\frac{\sqrt{65}}{9}\right)$. If the vector $\vec{c} = 3\hat{a} + 6\hat{b} + 9(\hat{a} \times \hat{b})$, then the value of $9(\vec{c}.\,\hat{a}) - 3(\vec{c}.\,\hat{b})$ is

(1) 24 (2) 27 (3) 31 (4) 29

Ans. [4]

Sol. $\vec{c}.\hat{a} = 3 + 6\hat{a}.\,\hat{b}$

$\vec{c}.\hat{b} = 3\hat{a}.\hat{b} + 6$

So, $9\ (\vec{c}.\,\hat{a}) - 3(\vec{c}.\,\hat{b}) = 27 - 18 + (54 - 9)\ \hat{a}\,.\,\hat{b}$

$= 9 + 45\ \hat{a}\,.\,\hat{b}$

$= 9 + 45 \times \frac{\sqrt{81-65}}{9}$

$= 9 + 5\sqrt{16} = 29$

Q.10 The integral $\int_0^{\pi} \frac{(x+3)\sin x}{1+3\cos^2 x}dx$ is equal to

(1) $\frac{\pi}{3\sqrt{3}}(\pi+6)$ (2) $\frac{\pi}{2\sqrt{3}}(\pi+4)$

(3) $\frac{\pi}{\sqrt{3}}(\pi+1)$ (4) $\frac{\pi}{\sqrt{3}}(\pi+2)$

Ans. [1]

Sol. $I = \int_0^{\pi} \frac{(x+3)\sin x}{1+3\cos^2 x}dx$ ....(i)

$= \int_0^{\pi} \frac{(\pi - x + 3)\sin(\pi - x)}{1+3\cos^2(\pi - x)}dx$

$= \int_0^{\pi} \frac{(x+3)\sin x - x\sin x}{1+3\cos^2 x}dx$ ....(ii)

Add (i) & (ii)

$2I = \int_0^{\pi} \frac{(\pi+6)\sin x}{1+3\cos^2 x}dx$

Let $\cos x = t \Rightarrow -\sin x\,dx = dt$

$(\pi + 6)\int_1^{-1} \frac{-dt}{(1+3t^2)} = \left(\frac{\pi+6}{3}\right)\int_{-1}^{1} \frac{dt}{t^2 + \left(\frac{1}{\sqrt{3}}\right)^2}$

$$\Rightarrow 2I = \left(\frac{\pi+6}{3}\right).\frac{1}{\left(\frac{1}{\sqrt{3}}\right)}.\tan^{-1}\frac{t}{\frac{1}{\sqrt{3}}}\Bigg|_{-1}^{1}$$

$$\Rightarrow 2I \left(\frac{\pi+6}{\sqrt{3}}\right)\left[\tan^{-1}\left(\sqrt{3}\right) - \tan^{-1}\left(-\sqrt{3}\right)\right]$$

$$= \left(\frac{\pi+6}{\sqrt{3}}\right)\left(\frac{\pi}{3} - \left(-\frac{\pi}{3}\right)\right)$$

$$= \left(\frac{\pi+6}{\sqrt{3}}\right).\frac{2\pi}{3}$$

$$\Rightarrow I = \frac{\pi(\pi+6)}{3\sqrt{3}}$$

Q.11 From a group of 7 batsmen and 6 bowlers, 10 players are to be chosen for a team, which should include atleast 4 batsmen and atleast 4 bowlers.

One batsmen and one bowler who are captain and vice-captain respectively of the team should be included. Then the total number of ways such a selection can be made, is

(1) 135 (2) 165 (3) 145 (4) 155

Ans. **[4]**

Sol. 1 Captain, 1 vice-captain are already present

$\Rightarrow$ We need to select 8 players such that atleast 3 batsman and bowler must be there

Batsman	Bowler	Number of ways
3	5	${}^6C_3.{}^5C_5 = 20$
4	4	${}^6C_4.{}^5C_4 = 75$
5	3	${}^6C_5.{}^5C_3 = 60$

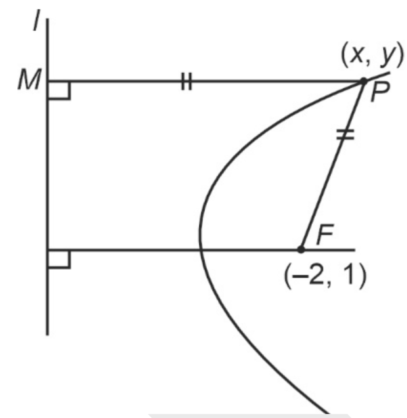
Total = 155 ways

Q.12 Let P be the parabola, whose focus is (–2, 1) and directrix is 2x + y + 2 = 0. Then the sum of the ordinates of the points on P, whose abscissa is –2, is

(1) $\frac{1}{4}$ (2) $\frac{3}{2}$ (3) $\frac{3}{4}$ (4) $\frac{5}{2}$

Ans. **[2]**

Sol.



PM = PF

$$\Rightarrow \frac{|2x+y+2|}{\sqrt{5}} = \sqrt{(x+2)^2 + (y-1)^2}$$

Now abscissa of P is – 2 $\Rightarrow$ x = –2

$$\left|\frac{y-2}{\sqrt{5}}\right| = \sqrt{0+(y-1)^2} \Rightarrow \frac{|y-2|}{\sqrt{5}} = |y-1|$$

$\Rightarrow (y-2)^2 = 5(y-1)^2$

$\Rightarrow 4y^2 - 6y + 1 = 0 \Rightarrow$ Sum of ordinates

$$= -\left(\frac{-6}{4}\right) = \frac{3}{2}$$

Q.13 Among the statements

(S1): The set $\{z \in C - \{-i\} : |z| = 1$ and $\frac{z-i}{z+i}$ is purely real$\}$ contains exactly two elements, and

(S2): The set $\{z \in C - \{-1\} : |z| = 1$ and $\frac{z-1}{z+1}$ is purely imaginary$\}$ contains infinitely many elements.

(1) Both are correct

(2) Only (S1) is correct

(3) Only (S2) is correct

(4) Both are incorrect

Ans. **[3]**

Sol. $\frac{z-i}{z+i} = \frac{\bar{z}+i}{\bar{z}-i}$

$= z\bar{z} - i\bar{z} - iz - 1 = z\bar{z} + zi + i\bar{z} - 1$

$= z + \bar{z} = 0$

$= 2x = 0$

$= x = 0$ (y-axis)

$|z| = 1$

$\therefore$ z = i (z $\neq$ – i is given)

Statement 1 is incorrect

$$\frac{z-i}{z+i} = \frac{\bar{z}-i}{\bar{z}+i} = 0$$

$= z\bar{z} - \bar{z} + z - 1 + z\bar{z} - z + \bar{z} - 1 = 0$

$= z\bar{z} = 1$

$= |z| = 1$

Statement 2 is correct

Q.14 Let x_1, x_2, x_3, x_4 be in a geometric progression. If 2, 7, 9, 5 are subtracted respectively from x_1, x_2, x_3, x_4, then the resulting numbers are in an arithmetic progression. Then the value of $\frac{1}{24}(x_1 x_2 x_3 x_4)$ is :

(1) 72 (2) 18 (3) 216 (4) 36

Ans. [3]

Sol. $x_1 = a;\ x_2 = ar;\ x_3 = ar^2;\ x_4 = ar^3$

$a - 2,\ ar - 7,\ ar^2 - 9,\ ar^3 - 5 \to$ A.P.

$a_2 - a_1 = a_3 - a_2$

$(ar - 7) - (a - 2) = (ar^2 - 9) - (ar - 7)$

$= a(r - 1) - 5 = ar(r - 1) - 2$

$a(r - 1)(r - 1) = -3$ …(i)

$a_2 - a_1 = a_4 - a_3$

$(ar - 7) - (a - 2) = (ar^3 - 5) - (ar^2 - 9)$

$= a(r - 1) - 5 = ar^2(r - 1) + 4$

$= a(r - 1)(r^2 - 1) = -9$ …(ii)

(ii)/(i)

$\Rightarrow r + 1 = 3$

$\Rightarrow r = 2$

Using (i)

$a(1)(1) = -3$

$a = -3$

$x_1 = -3,\ x_2 = -6,\ x_3 = -12,\ x_4 = -24$

$\frac{1}{24}(x_1 . x_2 . x_3 . x_4) = 216$

Q.15 If the shortest distance between the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x}{1} = \frac{y}{\alpha} = \frac{z-5}{1}$ is $\frac{5}{\sqrt{6}}$, then the sum of all possible values of α is

(1) –3 (2) $\frac{3}{2}$ (3) 3 (4) $-\frac{3}{2}$

Ans. [1]

Sol. $$d = \left|\frac{(\vec{a}-\vec{b}).\vec{p}_1 \times \vec{p}_2}{|\vec{p}_1 \times \vec{p}_2|}\right| = \frac{5}{\sqrt{6}}$$

$\vec{a} - \vec{b} = \hat{i} + 2\hat{j} - 2\hat{k}$

$$\vec{p}_1 \times \vec{p}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 1 & \alpha & 1 \end{vmatrix}$$

$= \hat{i}(3 - 4\alpha) - \hat{j}(-2) + \hat{k}(2a - 3)$

$(\vec{a} - \vec{b}).|\vec{p}_1 \times \vec{p}_2| = 3 - 4\alpha + 4 - 4\alpha + 6$

$= 13 - 8\alpha$

$$\left|\frac{13-8\alpha}{\sqrt{(3-4\alpha)^2 + 4 + (2\alpha-3)^2}}\right| = \frac{5}{\sqrt{6}}$$

$$\left|\frac{13-8\alpha}{\sqrt{20\alpha^2 - 36\alpha + 22}}\right| = \frac{5}{\sqrt{6}}$$

$= 6(13 - 8\alpha)^2 = 25(20\alpha^2 - 36\alpha + 22)$

$= 116\alpha^2 + 348\alpha - 464 = 0$

Sum of roots $= -3$

Q.16 $$\lim_{x\to 0^+} \frac{\tan\left(5(x)^{\frac{1}{3}}\right)\log_e(1+3x^2)}{\left(\tan^{-1} 3\sqrt{x}\right)^2\left(e^{5x^{\frac{4}{3}}} - 1\right)}$$ is equal to

(1) $\frac{1}{15}$ (2) $\frac{5}{3}$ (3) $\frac{1}{3}$ (4)1

Ans. [3]

Sol. $$\lim_{x\to 0^+} \frac{\tan\left(5(x)^{\frac{1}{3}}\right)\ln(1+3x^3)}{\left(\tan^{-1}(3\sqrt{x})\right)^2\left(e^{5x^{\frac{4}{3}}} - 1\right)}\ \text{ss}$$

$$\lim_{x\to 0^+} \frac{\frac{\tan\left(5(x)^{\frac{1}{3}}\right)}{5(x)^{\frac{1}{3}}} \frac{\ln(1+3x^2)}{3x^2} \times 5(x)^{\frac{1}{3}}(3x^2)}{\frac{\left(\tan^{-1}(3\sqrt{x})\right)^2}{(3\sqrt{x})^2} \frac{\left(e^{5x^{\frac{4}{3}}} - 1\right)}{5x^{\frac{4}{3}}} \times 9x \times 5x^{\frac{4}{3}}}$$

$$\lim_{x\to 0^+} \frac{\frac{\tan\left(5(x)^{\frac{1}{3}}\right)}{5(x)^{\frac{1}{3}}} \frac{\ln(1+3x^3)}{3x^2} \times 15x^{\frac{7}{3}}}{\frac{\left(\tan^{-1}(3\sqrt{x})\right)^2}{(3\sqrt{x})^2} \frac{\left(e^{5(x)^{\frac{4}{3}}} - 1\right)}{5x^{\frac{1}{3}}} \times 45x^{\frac{7}{3}}} = \frac{1}{3}$$

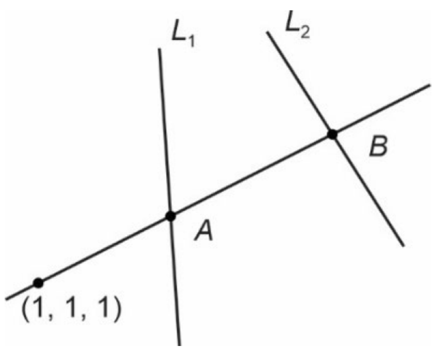
Q.17 Let the line L pass through (1, 1, 1) and intersect the lines $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-1}{4}$ and $\frac{x-3}{1}=\frac{y-4}{2}=\frac{z}{1}$. Then, which of the following points lies on the line L ?

(1) (10, – 29, – 50) (2) (7, 15, 13)

(3) (5, 4, 3) (4) (4, 22, 7)

Ans. [2]

Sol.



$L : \frac{x-1}{a}=\frac{y-1}{b}=\frac{z-1}{c}$

L_1: $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-1}{4} = \lambda$ (say)

Any point on L_1 be A $(2\lambda + 1, 3\lambda + 1, 4\lambda + 1)$

$L_2 : \frac{x-3}{1}=\frac{y-4}{2}=\frac{z}{1} = \mu$ (say)

Any point on L_2 be B $(\mu + 3, 2\mu – 4, \mu)$

D of L be: $< 2\lambda, 3\lambda – 2, 4\lambda >$ or $< \mu + 2, 2\mu + 3, \mu – 1 >$

Now $\frac{2\lambda}{\mu+2}=\frac{3\lambda-7}{2\mu+3}=\frac{4\lambda}{\mu-1}$

$\Rightarrow \lambda = \frac{-6}{5}$ $\mu = – 5$

$\therefore < a,b,c > \equiv < – 3, – 7, – 6 >$ or $< 3, 7, 6 >$

$\therefore L : \frac{x-1}{3}=\frac{y-1}{7}=\frac{z-1}{6}$

(7, 15, 13) lies on the line.

Q.18 The remainder when $\left((64)^{(64)}\right)^{(64)}$ is divided by 7 is equal to

(1) 3 (2) 1 (3) 6 (4) 4

Ans. [2]

Sol. $64^{64} \Rightarrow (63 + 1)^{64} = 63\lambda + 1$

$64^{64^{64}} \Rightarrow (63+1)^{64^{64}}$

$63\lambda_1 + 1$

Required remainder when divided 7 is 1.

Q.19 Let the system of equations:

$2x + 3y + 5z = 9,$

$7x + 3y – 2z = 8,$

$12x + 3y – (4 + \lambda)z = 16 – \mu.$

have infinitely many solutions. Then the radius of the circle centred at (λ, μ) and touching the line $4x = 3y$ is

(1) $\frac{7}{5}$ (2) 7 (3) $\frac{17}{5}$ (4) $\frac{21}{5}$

Ans. [1]

Sol. $\Delta = \begin{vmatrix} 2 & 3 & 5 \\ 7 & 3 & -2 \\ 12 & 3 & -(4+\lambda) \end{vmatrix}$

$= 2(–12 – 3\lambda + 6)– 3(–28 – 7\lambda + 24) + 5(21–36)$

$= –12 – 6\lambda +12 + 21\lambda – 75$

$= 15\lambda – 75$

$\Rightarrow 15\lambda – 75 = 0$

$\Rightarrow \lambda = 5$

$\Delta_1 = \begin{vmatrix} 9 & 3 & 5 \\ 8 & 3 & -2 \\ 16-\mu & 3 & -9 \end{vmatrix}$

$= 9(–27 + 6) – 3(–72 + 32 – 2\mu) + 5(24 – 48 + 3\mu)$

$= – 189 + 120 + 6\mu – 120 + 15\mu$

$= 21\mu – 189 = 0$

$\Rightarrow \mu = 9$

$\therefore r = \left|\frac{4(5)-3(9)}{\sqrt{(4)^2+(3)^2}}\right|$

$r = \frac{7}{5}$

Q.20 If for $\theta \in \left[-\frac{\pi}{3},0\right]$, the points (x, y) = $\left(3\tan\left(\theta+\frac{\pi}{3}\right),2\tan\left(\theta+\frac{\pi}{6}\right)\right)$ lie on $xy + \alpha x + \beta y + \gamma = 0$, then $\alpha^2 + \beta^2 + \gamma^2$ is equal to

(1) 80 (2) 96 (3) 72 (4) 75

Ans. [4]

Sol. Let $\phi = \theta + \frac{\pi}{3} \Rightarrow \theta = \phi - \frac{\pi}{6}$

$x = 3\tan\left(\theta+\frac{\pi}{3}\right) = 3\tan\left(\theta+\frac{\pi}{6}\right)$

$y = 2\tan\phi$

$$\tan\left(\phi + \frac{\pi}{6}\right) = \frac{\tan\phi + \frac{1}{\sqrt{3}}}{1 - \tan\phi . \frac{1}{\sqrt{3}}}$$

$$\frac{x}{3} = \frac{\frac{y}{2} + \frac{1}{\sqrt{3}}}{1 - \frac{y}{2}.\frac{1}{\sqrt{3}}}$$

$$\Rightarrow x = \frac{3(y\sqrt{3}+2)}{2\sqrt{3}-y}$$

$xy + \alpha x + \beta y + \gamma = 0$

$$3\left(\frac{y\sqrt{3}+2}{2\sqrt{3}-y}\right) + \alpha\left(3\frac{(y\sqrt{3}+2)}{(2\sqrt{3}-y)}\right) + \beta y + \gamma = 0$$

$$= (3\sqrt{3}-\beta)y^2 + (6 + 3\sqrt{3}\alpha + 2\sqrt{3}\beta - y)y + (6\alpha + 2\sqrt{3}y) = 0$$

For this identity to hold for all θ, coefficients must be 0

$\therefore \beta = 3\sqrt{3}$

$\gamma = -\alpha\sqrt{3}$

$6 + 3\sqrt{3}\alpha + (2\sqrt{3})(3\sqrt{3}) + \alpha\sqrt{3} = 0$

$\Rightarrow \alpha = -2\sqrt{3}$

$\Rightarrow \beta = 6$

$\alpha^2 + \beta^2 + \gamma^2 = 75$

Section-B: Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

Q.21 The number of singular matrices of order 2, whose elements are from the set {2, 3, 6, 9}, is ________.

Ans. **[36.00]**

Sol. Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

for A to be singular matrix

ad = bc

Case 1: exactly 1 number is used $\Rightarrow {}^4C_1$ ways

Case 2: exactly 2 numbers is used $\Rightarrow {}^4C_2$ ways

Case 3: exactly 3 numbers used $\Rightarrow$ none will be singular.

Case 4: exactly 4 numbers is used

$\Rightarrow ab = cd \Rightarrow 2 \times 9 = 3 \times 6$

$\Rightarrow {}^4C_1 \times 2! = 8$ matrix.

$\therefore$ Total ways $\Rightarrow 4 + 6 \times 4 + 8 = 36$ matrices.

Q.22 For $n \geq 2$, let S_n denote the set of all subsets of {1, 2, ….., n} with **no** two consecutive numbers. For example {1, 3, 5} $\in S_6$ but {1, 2, 4} $\notin S_6$. Then n (S_5) is equal to ________.

Ans. **[13]**

Sol. A = {1, 2, 3, 4, 5}

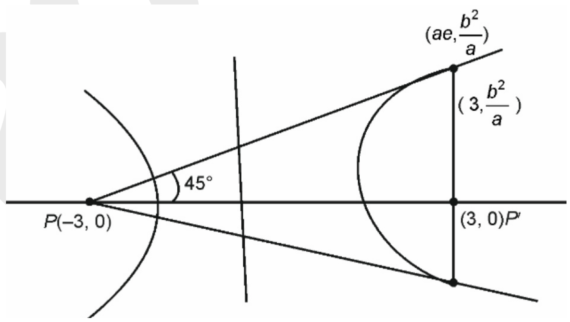
S_5 = {{}, {1}, {2}, {3}, {4}, {5}, {1, 3}, {1, 4}, {1, 5},
{2, 4}, {2, 5}, {3, 5}, {1, 3, 5}}

$^n(S_5) = 13$

Q.23 Consider the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ having one of its focus at P (–3, 0) . If the latus rectum through its other focus subtends a right angle at P and $a^2b^2 = \alpha\sqrt{2} - \beta$, α, β $\in$ N, then α + β is ________ .

Ans. **[1944]**

Sol.



$ae = 3$, $\tan 45° = ae = 3\ \frac{\frac{b^2}{a}}{6} \Rightarrow \frac{b^2}{a} = 6$ ....(i)

$$a\sqrt{\left(1 + \frac{b^2}{a^2}\right)} = 3$$

$a^2 + b^2 = 9$ …(ii)

$\Rightarrow a^2 - 6a + 9 = 0 \Rightarrow a = 3(\sqrt{2} - 1)$

$\Rightarrow a^2b^2 = 9(3 - 2(\sqrt{2})) . 6.3\ (\sqrt{2} - 1)$

$= 162\ (5\sqrt{2} - 7)$

$\Rightarrow \alpha = 162 \times 5,\ \ \beta = 162 \times 7$

$\Rightarrow \alpha + \beta = 162 \times 12 = 1944$

Q.24 The number of relations on the set A = {1,2,3}, containing at most 6 elements including (1, 2), which are reflexive and transitive but not symmetric, is _______.

Ans. **[5]**

Sol. Since relation needs to be reflexive the ordered pairs (1, 1), (2, 2), (3, 3) need to be there and (1, 2) is also to be included.

Let's call $R_0 = \{(1, 1), (2, 2), (3, 3), (1, 2)\}$ the base relation.

$\because$ A × A contain 3 × 3 = 9 ordered pairs, remaining 5 ordered are

(2, 1), (1, 3), (3, 1), (2, 3), (3, 2)

We have to add at most two ordered pairs to R_0 such that resulting relation is reflexive, transitive but not symmetric.

Following are the only possibilities.

$R = R_0 \cup \{(1, 3)\}$

OR $R_0 \cup \{(3, 2)\}$

OR $R_0 \cup \{(1, 3),(3, 1)\}$

OR $R_0 \cup \{(1, 3),(3, 2)\}$

OR $R_0 \cup \{(3, 1),(3, 2)\}$

Q.25 The number of points of discontinuity of the function f(x) = $\left[\frac{x^2}{2}\right] - [\sqrt{x}]$, x ∈ [0, 4] where [.] denotes the greatest integer function is ________.

Ans. **[8]**

Sol. Probable values of x where $\left[\frac{x^2}{2}\right]$ may be discontinuous on x ∈ [0, 4] are

= 1, 2, 3, 4, 5, 6, 7, 8

$x = \sqrt{2}, 2, \sqrt{6}, 2\sqrt{2}, \sqrt{10}, 2\sqrt{3}, \sqrt{14}, 4$

And for $[\sqrt{x}]$ corresponding values are x = 1, 2

On checking for continuity at these points we get the f(x) is discontinuous at

$x = 1, \sqrt{2}, 2, \sqrt{6}, 2\sqrt{2}, \sqrt{10}, 2\sqrt{3}, \sqrt{14}$

Hence, f(x) is discontinuous for 8 values of x ∈ [0, 4]

PHYSICS

Section-A: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct..

Q.26 For a hydrogen atom, the ratio of the largest wavelength of Lyman series to that of the Balmer series is

(1) 5 : 27 (2) 5 : 36 (3) 3 : 4 (4) 27 : 5

Ans. **[1]**

Sol. $\frac{\lambda_C}{\lambda_L} = E_0\left[1 - \frac{1}{4}\right] = \frac{E_0 3}{4}$ ......(1)

$\frac{\lambda_C}{\lambda_B} = E_0\left[\frac{1}{4} - \frac{1}{9}\right] = \frac{E_0 5}{4 \times 9}$ ......(2)

So, $\frac{\lambda_C \lambda_L}{\lambda_B \times \lambda_C} = \frac{5E_0 \times 4}{4 \times 9 \times 3E_0} = \frac{5}{27}$

Q.27 An object of mass 1000 g experiences a time dependent force $\vec{F} = (2t\hat{i} + 3t^2\hat{j})$ N. The power generated by the force at time t is:

(1) $(2t^2 + 3t^3)$ W (2) $(3t^3 + 5t^5)$ W

(3) $(2t^2 + 18t^3)$ W (4) $(2t^3 + 3t^5)$ W

Ans. **[4]**

Sol. m = 1000 gram

$\vec{F} = (2t\hat{i} + 3t^2\hat{j})$

So, $\frac{dv}{dt} = \frac{F}{m} = (2t\hat{i} + 3t^2\hat{j})$

So $v = \int_0^v dv = \int_0^t (2t\hat{i} + 3t^2\hat{j})$

So $v = ((t\hat{i} + t^3\hat{j}))$

So power $= \vec{F}.\vec{v} = 2t^2 + 3t^5$

Q.28 Two wires A and B are made of same material having ratio of lengths $\frac{L_A}{L_B} = \frac{1}{3}$ and their diameters ratio $\frac{d_A}{d_B} = 2$. If both the wires are stretched using same force, what would be the ratio of their respective elongations?

(1) 1 : 3 (2) 1 : 6 (3) 1 : 12 (4) 3 : 4

Ans. **[3]**

Sol. Strain $= \frac{\Delta L}{L} = \frac{\text{Stress}}{y} = \frac{F}{Ay}$

So, $\frac{\Delta L(A)}{\Delta L(B)} = \frac{L_A A_{(B)}}{A_{(A)} L_B} = \frac{1}{3} \times \frac{1}{4} = \frac{1}{12}$

Q.29 Two plane polarized light waves combine at a certain point whose electric field components are

$E_1 = E_0 \text{ Sin } \omega t$

$E_2 = E_0 \text{ Sin } \omega t \left(\omega t + \frac{\pi}{3}\right)$

Find the amplitude of the resultant wave.

(1) 1.7 E_0 (2) 0.9 E_0

(3) E_0 (4) 3.4 E_0

Ans. [1]

Sol. $E_1 = E_0 \sin(\omega t)$

$E_2 = E_0 \text{ Sin } \omega t \left(\omega t + \frac{\pi}{3}\right)$

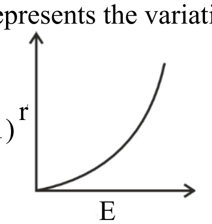
$$E_R^2 = E_1^2 + E_2^2 + 2E_1E_2 \cos(\phi)$$

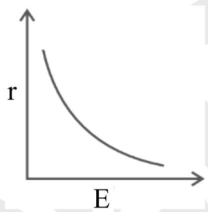
$$E_R^2 = E_0^2 + E_0^2 + 2E_0^2 \cos\phi = 2E_0^2 \ (1 + \cos\phi)$$

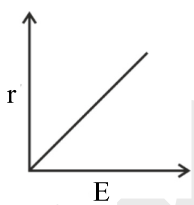
$$E_R^2 = 2E_0^2 \times 2\cos^2\left(\frac{\pi}{6}\right) = 4E_0^2 . \frac{3}{4}$$

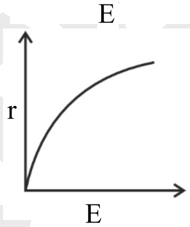
Hence, $E_R = E_0\sqrt{3} \approx 1.7 E_0$

Q.30 A particle of charge q, mass m and kinetic energy E enters in magnetic field perpendicular to its velocity and undergoes a circular arc of radius (r). Which of the following curves represents the variation of r with E?

(1) 

(2) 

(3) 

(4) 

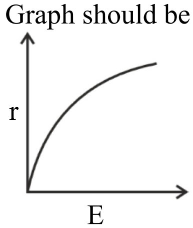
Ans. [4

Sol. $\frac{1}{2} mv^2 = E \Rightarrow (mv)^2 = 2mE$

Also, $r = \frac{mv}{qB} \Rightarrow r = \frac{\sqrt{2mE}}{qB}$

So, $r \propto (E)^{\frac{1}{2}}$

Graph should be like



Q.31 An ac current is represented as

$i = 5\sqrt{2} + 10 \cos\left(650\pi t + \frac{\pi}{6}\right)$ Amp

The r.m.s value of the current is

(1) 10 Amp (3) 50 Amp

(3) 100 Amp (4) $5\sqrt{2}$ Amp

Ans. [1]

Sol. $i = 5\sqrt{2} + 10 \cos\left(650\pi t + \frac{\pi}{6}\right)$

$$i_{RMS}^2 = I_{1(RMS)}^2 + I_{2(RMS)}^2$$

$$\Rightarrow i_{(RMS)}^2 = 50 + \frac{100}{2} = 100$$

$\Rightarrow i_{RMS} = 10$ Amp

Q.32 A cubic block of mass m is sliding down on an inclined plane at 60° with an acceleration of $\frac{g}{2}$, the value of coefficient of kinetic friction is

(1) $\frac{\sqrt{2}}{3}$ (2) $1 - \frac{\sqrt{3}}{2}$ (3) $\frac{\sqrt{3}}{2}$ (4) $\sqrt{3} - 1$

Ans. [4]

Sol. $a = \frac{mg\sin\theta - \mu mg\cos\theta}{m} = \frac{g\sqrt{3}}{2} - \frac{\mu g}{2}$

$$\Rightarrow \frac{g}{2} = \frac{g}{2}\left(\sqrt{3} - \mu\right) \Rightarrow \sqrt{3} - \mu = 1$$

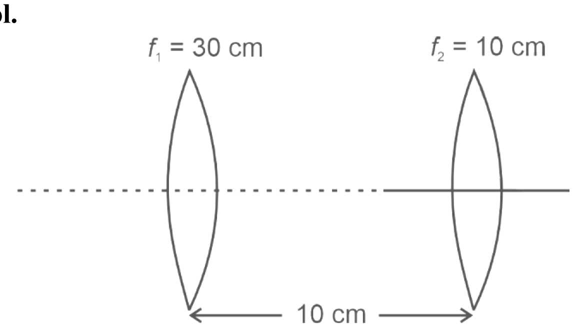
$$\Rightarrow \mu = \left(\sqrt{3} - 1\right) \approx 1.73 - 1 = 0.73$$

Q.33 Two thin convex lenses of focal lengths 30 cm and 10 cm are placed coaxially, 10 cm apart. The power of this combination is:

(1) 10 D (2) 1 D (3) 5 D (4) 20 D

Ans. [1]

Sol.



For parallel range

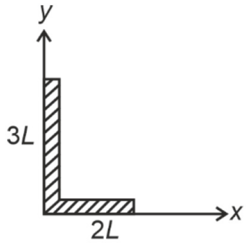
$$\frac{1}{F}=\frac{1}{F_1}+\frac{1}{F_2}-\frac{d}{F_1F_2}$$

$$\Rightarrow \frac{1}{F}=\frac{1}{30}+\frac{1}{10}-\frac{10}{10\times 30}=\frac{1}{10}$$

$$\Rightarrow F=\left(\frac{1}{10}\right)\text{ m}$$

So, power $p=\frac{1}{F}=10$ D

Q.34 A rod of length 5 L is bent right angle keeping one side length as 2 L.



The position of the centre of mass of the system: (Consider L = 10 cm)

(1) $3\hat{i}+7\hat{j}$ (2) $4\hat{i}+9\hat{j}$ (3) $2\hat{i}+3\hat{j}$ (4) $5\hat{i}+8\hat{j}$

Ans. [2]

Sol. $m_1=(2m);\ x_1=L,\ y_1=0$

$m_2=(3m);\ x_2=0,\ y_2=\frac{3L}{2}$

So, $x_{cm}=\frac{2m(L)+0}{5m}=\frac{2L}{5}=4\hat{i}$ (cm)

$$y_{cm}=\frac{(0)+\frac{9mL}{2}}{5m}=\frac{9L}{10}=9\hat{j}\text{ (cm)}$$

So $\vec{r}_{cm}=(4\hat{i}+9\hat{j})$ cm

Q.35 In a hydrogen like ion, the energy difference between the 2nd excitation energy state and ground is 108.8 eV. The atomic number of the ion is:

(1) 2 (2) 1 (3) 3 (4) 4

Ans. [3]

Sol. $\Delta E=E_0(Z)^2\left[1-\frac{1}{9}\right]=108.8$ eV

$$\Rightarrow 13.6(Z)^2\times\frac{8}{9}=108.8$$

$$\Rightarrow Z^2=\frac{108.8\times 9}{8\times 13.6}=9$$

$$\Rightarrow Z=3$$

Q.36 If ϵ_0 denotes the permittivity of free space and ϕ_E is the flux of the electric field through the area bounded by the closed surface, then dimensions of $\left(\epsilon_0\frac{d\phi_E}{dt}\right)$ are that of:

(1) electric current (2) electric charge

(3) electric field (4) electric potential

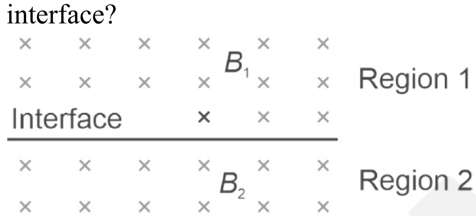
Ans. [1]

Sol. Dimension of $\epsilon_0\left(\frac{d\phi_E}{dt}\right)$ will be

Same as electric current

So $\left[\epsilon_0\frac{d\phi_E}{dt}\right]=A$

Q.37 Uniform magnetic fields of different strengths (B_1 and B_2), both normal to the plane of the paper exist as shown in the figure. A charged particle of mass m and charge q, at the interface at an instant, moves into the region 2 with velocity v and returns to the interface. It continues to move into region 1 and finally reaches the interface. What is the displacement of the particle during this movement along the interface?



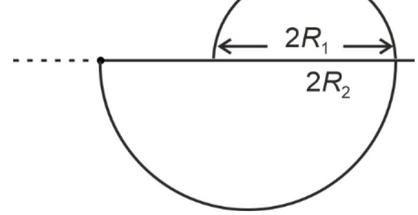
(1) $\frac{mv}{qB_1}\left(1-\frac{B_1}{B_2}\right)$ (2) $\frac{mv}{qB_1}\left(1-\frac{B_2}{B_1}\right)$

(3) $\frac{mv}{qB_1}\left(1-\frac{B_2}{B_1}\right)\times 2$ (4) $\frac{mv}{qB_1}\left(1-\frac{B_1}{B_2}\right)\times 2$

Ans. [4]

Sol. $R_1=\frac{mv}{qB_1}$

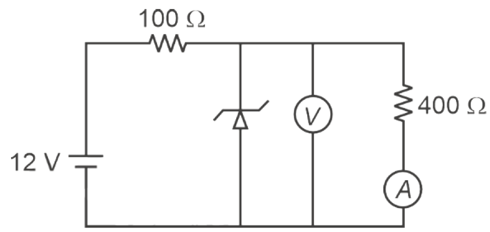
$R_2=\frac{mv}{qB_2}$



$$|\Delta r| = |2R_2 - 2R_1| = \frac{2mv}{q}\left[\frac{1}{B_2} - \frac{1}{B_1}\right]$$

$$\Rightarrow \Delta r = \left|\frac{2mv}{qB_1}\left[\frac{B_1}{B_2} - 1\right]\right| = \left|\frac{2mv}{qB_1}\left(1 - \frac{B_1}{B_2}\right)\right|$$

Q.38 In the following circuit, the reading of the ammeter will be (Take Zener breakdown voltage = 4 V)



(1) 10 mA (2) 24 mA
(3) 60 mA (4) 80 mA

Ans. [1]

Sol. Voltage across 400 Ω resistor is 4 volts

So $I = \frac{4}{400} = 10$ mA

Q.39 Two projectiles are fired from ground with same initial speeds from same point at angles (45° + α) and (45° – α) with horizontal direction. The ratio of their times of flights is

(1) $\frac{1+\sin 2\alpha}{1-\sin 2\alpha}$ (2) $\frac{1+\tan\alpha}{1-\tan\alpha}$

(3) $\frac{1-\tan\alpha}{1+\tan\alpha}$ (4) 1

Ans. [2]

Sol. Time of flight for 1st projectile

$$T_1 = \frac{24\sin(45+\alpha)}{g}$$

And $T_2 = \frac{24\sin(45-\alpha)}{g}$

$$\text{So, } \frac{T_1}{T_2} = \frac{\sin(45+\alpha)}{\sin(45-\alpha)} = \frac{\frac{\cos\alpha}{\sqrt{2}} + \frac{\sin\alpha}{\sqrt{2}}}{\frac{\cos\alpha}{\sqrt{2}} - \frac{\sin\alpha}{\sqrt{2}}}$$

$$\Rightarrow \frac{T_1}{T_2} = \frac{1+\tan\alpha}{1-\tan\alpha}$$

Q.40 A lens having refractive index 1.6 has focal length of 12 cm, when it is in air. Find the focal length of the lens when it is placed in water. (Take refractive index of water as 1.28)

(1) 555 mm (2) 655 mm
(3) 355 mm (4) 288 mm

Ans. [4]

Sol. $\frac{\mu_1}{f} - \frac{\mu_1}{\alpha} = (\mu - \mu_1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$

For air $\mu_1 = 1$ and $\mu = 1.6$

So $\frac{1}{f} = 0.6\left[\frac{1}{R_1} - \frac{1}{R_2}\right] = \frac{1}{12}$ cm

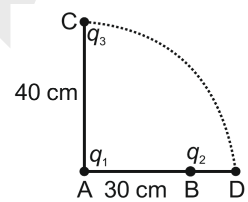
For water $\mu_1 = 1.28$ and $\mu = 1.6$

So,

$$\frac{\mu_1}{f_w} = 0.32\left[\frac{1}{R_1} - \frac{1}{R_2}\right] \Rightarrow \frac{1}{f_w} = \frac{0.32}{1.28} \times \frac{1}{0.6\times 12}$$

$$\Rightarrow f_w = \frac{1.28\times 0.6\times 12}{0.32} = 28.8 \text{ cm} \approx 288 \text{ mm}$$

Q.41 Two charges q_1 and q_2 are separated by a distance of 30 cm. A third charge q_3 initially at 'C' as shown in the figure, is moved along the circular path of radius 40 cm from C to D. If the difference in potential energy due to movement of q_3 from C to D is given by $\frac{q_3 K}{4\pi\varepsilon_0}$ the value of K is



(1) $8q_1$ (2) $8q_2$ (3) $6q_2$ (4) $6q_1$

Ans. [2]

Sol. $u(C) = \left(\frac{kq_1}{40} + \frac{kq_2}{50}\right)q_3$

$u(D) = \left(\frac{kq_1}{40} + \frac{kq_2}{10}\right)q_3$

So, $\Delta u = |u(D) - u(C)| = kq_2\left[\frac{1}{10} - \frac{1}{50}\right]q_3$

$\Rightarrow \Delta u = \frac{kq_2 4}{50}q_3 = \frac{4q_2q_3}{4\pi\varepsilon_0 50}$

$\Rightarrow \left(\frac{4q_2q_3 \times 2}{4\pi\varepsilon_0}\right)$ SI unit $\Leftrightarrow \frac{q_3k}{4\pi\varepsilon_0}$

$\Rightarrow k = 8q_2$

Q.42 Two harmonic waves moving in the same direction superimpose to form a wave x = a cos (1.5t) cos (50.5t) where t is in seconds. Find the period with which they beat. (close to nearest integer)

(1) 1 s (2) 6 s (3) 4 s (4) 2 s

Ans. [4]

Sol. x = a cos (1.5t) cos (50.5t)

Clearly,

$x = \frac{a}{2}\cos(50.5t + 1.5t) + \frac{a}{2}\cos(50.5t - 1.5t)$

$\Rightarrow x = \frac{a}{2}\cos(52t) + \frac{a}{2}\cos(49t)$

$f_1 = \frac{52}{2\pi}$ and $f_2 = \frac{49}{2\pi}$

$\Rightarrow \Delta f = \frac{3}{2\pi}$

$\Delta T = \frac{1}{\Delta f} = \frac{2\pi}{3} = 2.09$

$\Rightarrow \Delta T \approx 2$ s

Q.43 Match the **LIST-I** with **LIST-II**

LIST-I		LIST-II	
A	Triatomic rigid gas	I	$\frac{C_p}{C_v} = \frac{5}{3}$
B	Diatomic non-rigid gas	II	$\frac{C_p}{C_v} = \frac{7}{5}$
C	Monoatomic gas	III	$\frac{C_p}{C_v} = \frac{4}{3}$
D	Diatomic rigid gas	IV	$\frac{C_p}{C_v} = \frac{9}{7}$

Choose the correct answer from the options given below:

(1) A-II, B-IV, C-I, D-III

(2) A-III, B-II, C-IV, D-I

(3) A-III, B-IV, C-I, D-II

(4) A-IV, B-II, C-III, D-I

Ans. [3]

Sol. For triatomic rigid gas $\frac{C_p}{C_v} = \frac{4}{3}$

For diatomic non rigid gas $\frac{C_p}{C_v} = \frac{9}{7}$

For monoatomic gas $\frac{C_p}{C_v} = \frac{5}{3}$

For diatomic rigid gas $\frac{C_p}{C_v} = \frac{7}{5}$

Q.44 The percentage increase in magnetic field (B) when space within a current carrying solenoid is filled with magnesium (magnetic susceptibility $\chi_{Mg} = 1.2 \times 10^{-5}$) is

(1) $\frac{6}{5} \times 10^{-3}\%$ (2) $\frac{5}{3} \times 10^{-5}\%$

(3) $\frac{5}{6} \times 10^{-5}\%$ (4) $\frac{5}{6} \times 10^{-4}\%$

Ans. [1]

Sol. $\frac{\Delta B}{B} = \frac{(1 + x_B)B - B}{B} = x_B$

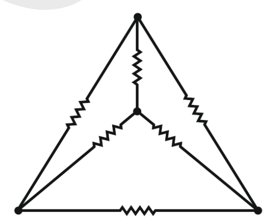
So percentage increase = $(x_B \times 100)$

$= 1.2 \times 10^{-5} \times 100$

$= \frac{6}{5} \times 10^{-3}$

Q.45 A wire of resistance R is bent into a triangular pyramid as shown in figure with each segment having same length. The resistance between points A and B is R/n.

The value of n is



(1) 10 (2) 12 (3) 16 (4) 14\

Ans. [2]

Sol. Clearly, $R_1 = \frac{R}{6}$

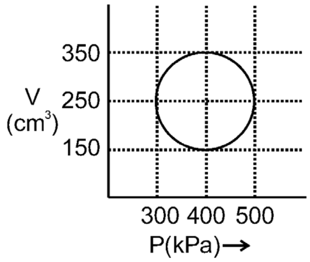
So $R_{AB} = R_1 \| 2R_1 \| 2R_1$

$\Rightarrow \frac{1}{R_{AB}} = \frac{1}{R_1} + \frac{1}{2R_1} + \frac{1}{2R_1} = \frac{4}{2R_1} = \frac{2}{R_1}$

$\Rightarrow R_{AB} = \frac{R_1}{2} = \frac{R}{12}$

Section-B: Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

Q.46 An ideal gas has undergone through the cyclic process as shown in the figure. Work done by the gas in the entire cycle is ______ $\times 10^{-1}$ J. (Take $\pi = 3.14$)



Ans. [314]

Sol. $\Delta W = \pi\left(\frac{\Delta v}{2}\right)\left(\frac{\Delta p}{2}\right)$

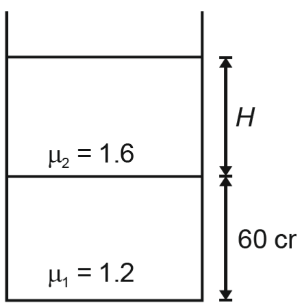
$\Delta W = \pi \times 100 \times 10^{-6} \times 100 \times 10^{3}$

$= 3.14 \times 10 = 31.4\text{ J} = 314 \times 10^{-1}\text{ J}$

Q.47 A container contains a liquid with refractive index of 1.2 up to a height of 60 cm and another liquid having refractive index 1.6 is added to height H above first liquid. If viewed from above, the apparent shift in the position of bottom of container is 40 cm. The value of H is cm (Consider liquids are immiscible).

Ans. [80]

Sol.



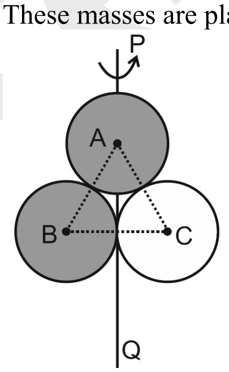
$\Delta t = 60\left(1-\frac{1}{1.2}\right)+H\left(1-\frac{1}{1.6}\right)$

$\Rightarrow 40 = \left(\frac{1}{6}\right)60 + H\left(\frac{3}{8}\right)$

$\Rightarrow 30 = H \times \frac{3}{8}$

$\Rightarrow H = 80$ cm

Q.48 A, B and C are disc, solid sphere and spherical shell respectively with same radii and masses. These masses are placed as shown in figure.



The moment of inertia of the given system about PQ axis is $\frac{x}{15}$ I, where I is the moment of inertia of the disc about its diameter. The value of x is _______.

Ans. [199]

Sol. (disk) $I_A = \frac{mR^2}{4}$; $\quad I = \frac{mR^2}{4}$

(solid sphere) $I_B = \frac{7}{5}mR^2$

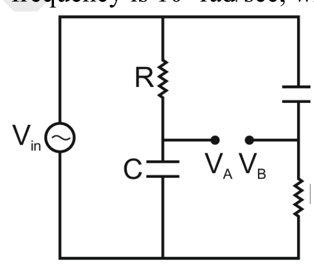
(Spherical shell) $I_C = \frac{5}{3}mR^2$

$I_{PQ} = mR^2\left[\frac{1}{4}+\frac{7}{5}+\frac{5}{3}\right] = mR^2\left(\frac{199}{4}\right)\times\frac{1}{15}$

So, $\frac{x}{15}\times\frac{mR^2}{4} = \frac{mR^2\times 199}{4\times 15}$

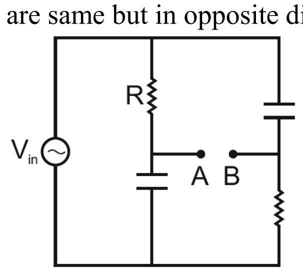
$\Rightarrow x = 199$

Q.49 For ac circuit shown in figure, R = 100 kΩ and C = 100 pF and the phase difference between V_{in} and $(V_B - V_A)$ is 90°. The input signal frequency is 10^x rad/sec, where 'x' is ________



Ans. [5]

Sol. Since both branch are identical. So phase difference between V_A and V_{in} and V_B and V_{in} are same but in opposite direction.



So, phase difference between V_{in} and V_A must be 45° as V_{in} and $|V_A - V_D|$ has difference of 90°. So, clearly $|R| = (x_C)$

$\Rightarrow 100 \times 10^3 = \frac{10^{12}}{w\times 100}$

$\Rightarrow W = 10^5$ rad/s

Q.50 A wire of length 10 cm and diameter 0.5 mm is used in a bulb. The temperature of the wire is 1727°C and power radiated by the wire is 94.2 W. Its emissivity is $\frac{x}{8}$ where x = __________.

(Given $\sigma = 6.0 \times 10^{-8}$ Wm^{-2} K^{-4}, $\pi = 3.14$ and assume that the emissivity of wire material is same at all wavelength.)

Ans. **[5]**

Sol. $A = \pi D \times \ell = \pi \times 5 \times 10^{-4} \times 10 \times 10^{-2}$ m^2

$\Rightarrow P = \sigma e A T^4$

$\Rightarrow 94.2 = \frac{\sigma x}{8}(5\pi \times 10^{-5}) \times (2000)^4$

$\Rightarrow x = \frac{94.2 \times 8}{5 \times 3.14 \times 10^{-5} \times (2000)^4 \times 6.0 \times 10^{-8}}$

$\Rightarrow x = 5$

CHEMISTRY

Section-A: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct..

Q.51 Reaction $A(g) \rightarrow 2B(g) + C(g)$ is a first order reaction. It was started with pure A

t/min	Pressure of system at time t/mm Hg
10	160
∞	240

Which of the following option is **incorrect**?

(1) Partial pressure of A after 10 minute is 40 mm Hg

(2) Initial pressure of A is 80 mm Hg

(3) The reaction never goes to completion

(4) Rate constant of the reaction is 1.693 min^{-1}

Ans. **[4]**

Sol.

	A	→ 2B	+	C
t – 0		a_0		
t – t		$a_0 - x$	2x	x
t – ∞		0	$2a_0$	a_0

Now, $a_0 + 2x = 160$

$3a_0 = 240$

$a_0 = 80$

$x = 40$

So, now

$A \rightarrow 2B + C$

t = 0 80

t = t 80 – 40 = 40

$k = \frac{2.303}{10} \times \log \frac{80}{40} = 0.0693$ min^{-1}

Q.52 When a salt is treated with sodium hydroxide solution it gives gas X. On passing gas X through reagent Y a brown coloured precipitate is formed. X and Y respectively, are

(1) $X = NH_3$ and $Y = K_2HgI_4 + KOH$

(2) $X = NH_3$ and $Y = HgO$

(3) $X = HCl$ and $Y = NH_4Cl$

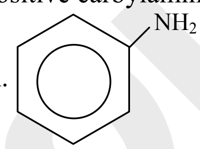
(4) $X = NH_4Cl$ and $Y = KOH$

Ans. **[1]**

Sol. $NH_4^+ - \text{Salt} \xrightarrow{NaOH} \underset{(X)}{NH_3} \xrightarrow{K_2HgI_4(Y)}$

Iodide of Millon's base (brown ppt)

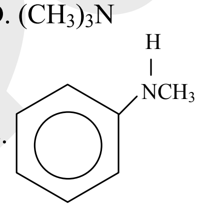
Q.53 Which of the following amine(s) show(s) positive carbylamine test?

A. 

B. $(CH_3)_2NH$

C. CH_3NH_2

D. $(CH_3)_3N$

E. 

Choose the **correct** answer from the options given below.

(1) C only (2) A and E only

(3) B, C and D only (4) A and C only

Ans. **[4]**

Sol. Only 1° amines (aliphatic or aromatic) respond to carbylamine test.

$RNH_2 \xrightarrow[CHCl_3 + KOH]{} RNC$

Q.54 Given below are two statements

Statement I: D-(+)-glucose + D-(+) fructose $\xrightarrow{-H_2O}$ Sucrose $\xrightarrow{\text{hydrolysis}}$ D-(+) glucose + D-(+) fructose

Statement II: Invert sugar is formed during sucrose hydrolysis.

In the light of the above statements, choose the **correct** answer from the options given below.

(1) Both Statement I and Statement II are true

(2) Both Statement I and Statement II are false

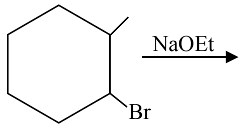
(3) Statement I is true but Statement II is false

(4) Statement I is false but Statement II is true

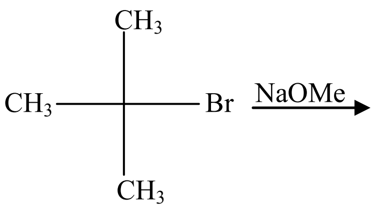
Ans. **[4]**

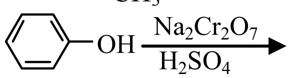
Sol. Invert sugar is a 1 : 1 molar mixture of D-(+)-glucose and D-(–)-fructose.

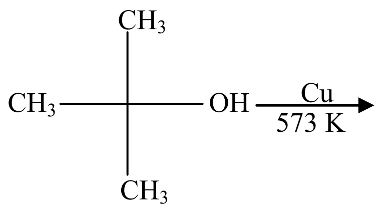
Q.55 The reactions which cannot be applied to prepare an alkene by elimination, are

A. 

B. $CH_3-CH_2-\underset{\underset{Br}{|}}{CH}-CH_3 \xrightarrow{KOH(Aq.)}$

C. 

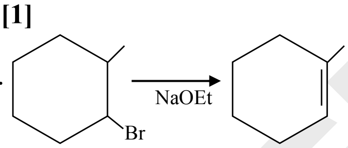
D. 

E. 

Choose the **correct** answer from the options given below :

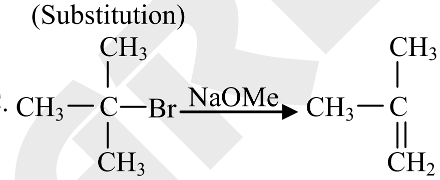
(1) B & D only (2) B, C & D only
(3) A, C & D only (4) B & E only

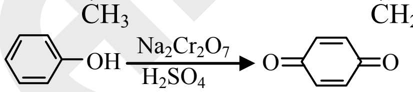
Ans. [1]

Sol. A. 

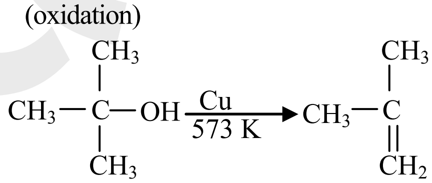
B. $CH_3-CH_2-\underset{\underset{Br}{|}}{CH}-CH_3 \xrightarrow[KOH]{aq.} CH_3-CH_2-\underset{\underset{OH}{|}}{CH}-CH_3$

(Substitution)

C. 

D. 

(oxidation)

E. 

Q.56 The number of valence electrons present in the metal among Cr, Co, Fe and Ni which has the lowest enthalpy of atomisation is

(1) 8 (2) 10 (3) 6 (4) 9

Ans. [3]

Sol. Enthalpy of atomisation **(kJ/mol)**

Cr → 397
Co → 425
Fe → 416
Ni → 430

Cr has the lowest value of enthalpy of atomisation, its electronic configuration is $Ar^{18}.\ 3d^5 4s^1$

Q.57 Match the **List-I** with **List-II**

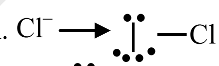
List-I Molecule/Ion		**List-II Bond pair : lone pair (on the central atom)**	
A.	ICl_2^-	I	4 : 2
B.	H_2O	II.	4 : 1
C.	SO_2	III.	2 : 3
D.	XeF_4	IV.	2 : 2

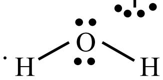
Choose the **correct** answer from the options given below :

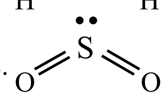
(1) A-III, B-IV, C-II, D-I
(2) A-II, B-I, C-IV, D-III
(3) A-III, B-IV, C-I, D-II
(4) A-IV, B-III, C-II, D-I

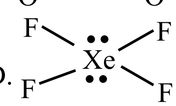
Ans. [1]

Sol. **Bp : lp**

A.  2 : 3

B.  2 : 2

C.  4 : 1

D.  4 : 2

Q.58 Given below are two statements :

Statement-I : Mohr's salt is composed of only three types of ions-ferrous, ammonium and sulfate.

Statement-II: If the molar conductance at infinite dilution for ferrous, ammonium and sulfate ions are x_1, x_2 and x_3 S cm^2 mol^{-1}, respectively then the molar conductance for Mohr's salt solution at infinite dilution would be given by $x_1 + x_2 + 2x_3$

In the light of the given statements, choose the **correct** answer from the options given below:

(1) Both **Statements-I** and **Statement-II** are false
(2) Both **Statements-I** and **Statement-II** are true
(3) **Statement-I** is false but **Statement-II** is true
(4) **Statement-I** is true but **Statement-II** are false

Ans. [4]

Sol. Mohr's salt is $FeSO_4(NH_4)_2SO_4 \cdot 6H_2O$

$(\lambda^{\circ}_M)_{\text{Mohr salt}} = x_1 + 2x_2 + 2x_3$

Q.59 At the sea level, the dry air mass percentage composition is given as nitrogen gas: 70.0, oxygen gas: 27.0 and argon gas: 3.0. If total pressure is 1.15 atm, then calculate the ratio of following respectively:

(i) partial pressure of nitrogen gas to partial pressure of oxygen gas

(ii) partial pressure of oxygen gas to partial pressure of argon gas

(Given: Molar mass of N, O and Ar are 14, 16 and 40 g mol^{-1} respectively.)

(1) 2.59, 11.85 (2) 5.46, 17.8

(3) 2.96, 11.2 (4) 4.26, 19.3

Ans. **[3]**

Sol.

$$P_{O_2} = \frac{\frac{27}{32}}{\frac{27}{32}+\frac{70}{28}+\frac{3}{40}} \times 1.15 \text{ atm}$$

$$P_{N_2} = \frac{\frac{70}{28}}{\frac{27}{32}+\frac{70}{28}+\frac{3}{40}} \times 1.15 \text{ atm}$$

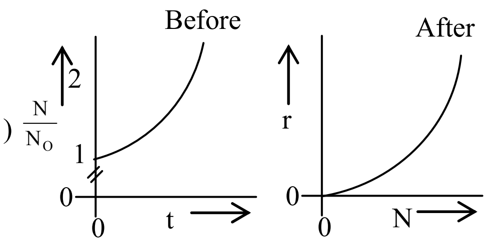
$$P_{Ar} = \frac{\frac{3}{40}}{\frac{27}{32}+\frac{70}{28}+\frac{3}{40}} \times 1.15 \text{ atm}$$

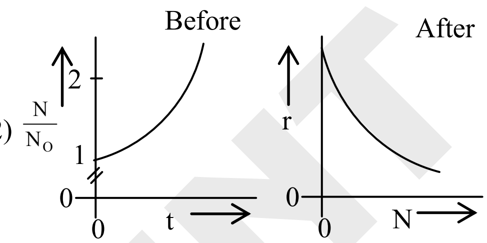
$$\frac{P_{N_2}}{P_{O_2}} = 2.95$$

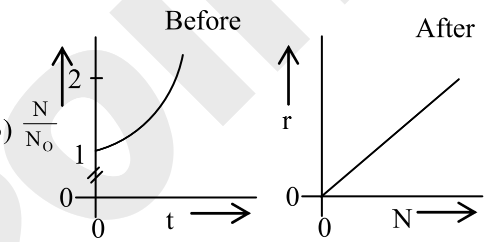
$$\frac{P_{O_2}}{P_{Ar}} = 11.18$$

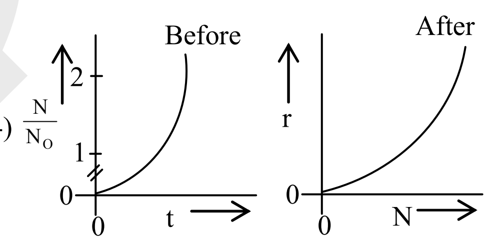
Q.60 A person's wound was exposed to some bacteria and then bacterial growth started to happen at the same place. The wound was later treated with some antibacterial medicine and the rate of bacterial decay(r) was found to be proportional with the square of the existing number of bacteria at any instance. Which of the following set of graphs correctly represents the 'before' and 'after' situation of the application of the medicine?

[Given: N = No. of bacteria, t = time, bacterial growth follows 1st order kinetics.]

(1)


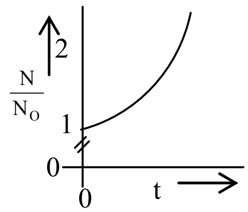
(2)


(3)


(4)


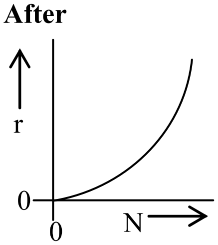
Ans. **[1]**

Sol. **Before**



Bacteria growing in the initial phase.

After



More is the no. of bacteria, more is the rate of bacterial decay.

$r \propto (N)^2$.

Q.61 The group 14 elements A and B have the first ionisation enthalpy values of 708 and 715 kJ mol^{-1} respectively. The above values are lowest among their group members. The nature of their ions A^{2+} and B^{4+} respectively is
(1) both reducing
(2) oxidising and reducing
(3) both oxidising
(4) reducing and oxidising

Ans. [4]

Sol. $\Rightarrow A^{2+}$ has the tendency to lose e^- – so reducing nature
$\Rightarrow B^{4+}$ has the tendency to gain e^- – so oxidising nature

Q.62 The first transition series metal 'M' has the highest enthalpy of atomisation in its series. One of its aquated ion (M^{n+}) exists in green colour. The nature of the oxide formed by the above M^{n+} ion is :
(1) amphoteric (2) neutral
(3) basic (4) acidic

Ans. [3]

Sol. V has the highest enthalpy of atomisation (515 kJ/mol).
Its oxide will be basic in nature.

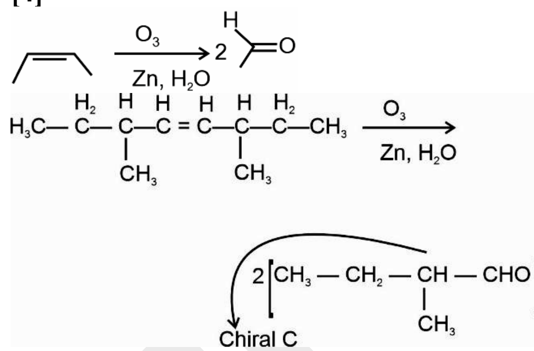
Q.63 Total enthalpy change for freezing of 1 mol of water at 10°C to ice at –10°C is_______
(Given : $\Delta_{fus} H = x$ kJ/mol
$C_p(H_2O(\ell)) = y \text{ J mol}^{-1} \text{ K}^{-1}$
$C_p(H_2O(s)) = z\text{J mol}^{-1} \text{ K}^{-1}$
(1) x – 10y – 10z (2) 10(100x + y + z)
(3) –x – 10y – 10z (4) –10(100 x + y + z)

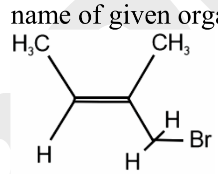
Ans. [4]

Sol. $\underset{10°C}{H_2O(\ell)} \xrightarrow{1} \underset{0°C}{H_2O(\ell)} \underset{2}{\rightleftharpoons} \underset{0°C}{H_2O(s)} \xrightarrow{3} \underset{-10°C}{H_2O(s)}$
(1) $\rightarrow -y \times 10$
(2) $\rightarrow -x \times 1000$
(3) $\rightarrow -z \times 10$
$\Delta H_{net} = -10(y + 100x + z)$

Q.64 Given below are two statements:
Statement I : Ozonolysis followed by treatment with Zn, H_2O of cis-2 butene gives ethanal.
Statement II : The product obtained by ozonolysis followed by treatment with Zn, H_2O of 3, 6- dimethyloct-4-ene has no chiral carbon atom.
In the light of the above statements, choose the correct answer from the options given below
(1) Statement I is false but Statement II is true
(2) Both Statement I and Statement II are False
(3) Both Statement I and Statement II are true
(4) Statement I is true but Statement II is false

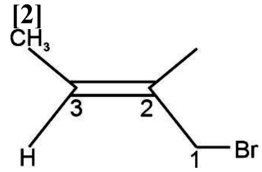
Ans. [4]

Sol.


Q.65 Which of the following is the correct IUPAC name of given organic compound (X)?


(X)
(1) 2-Bromo-2-methylbut-2-ene
(2) 1-Bromo-2-methylbut-2-ene
(3) 4-Bromo-3-methylbut-2-ene
(4) 3-Bromo-3-methylprop-2-ene

Ans. [2]

Sol.


1-Bromo-2-methyl but-2-ene

Q.66 Given below ae two statements
Statement I : Dimethyl ether is completely soluble in water. However, diethyl ether is soluble in water to a very small extent.
Statement II : Sodium metal can be used to dry diethyl ether and not ethyl alcohol.
In the light of given statements, choose the **correct** answer from the options given below.
(1) Statement I is true but Statement II is false
(2) Both Statement I and Statement II are true
(3) Statement I is false but Statement II is true
(4) Both Statement I and Statement II are False

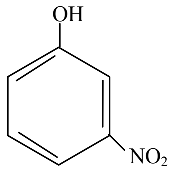
Ans. [2]

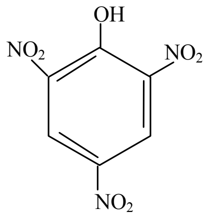
Sol. Diethyl ether is less soluble then dimethyl ether- as hydrophobic portion grows

$$C_2H_5OH + Na \longrightarrow C_2H_5ONa + \frac{1}{2}H_2$$

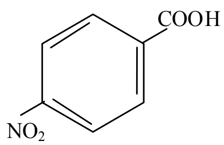
Na metal reacts with alcohol.

Q.67 Which of the following compounds is least likely to give effervescence of CO_2 in presence of aq. $NaHCO_3$?

(1) 

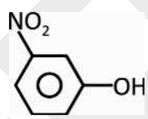
(2) 

(3) $Ph\text{–}\overset{\oplus}{N}H_3\ \overset{\ominus}{Cl}$

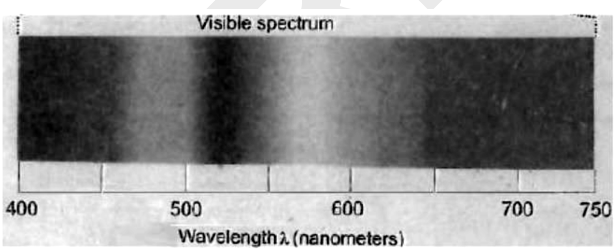
(4) 

Ans. [1]

Sol. $RCOOH + NaHCO_3 \rightarrow RCOO^-Na^+ + H_2CO_3$
Stronger acid
$H_2CO_3 \downarrow H_2O + CO_2$

Strong acids give CO_2 (g)  is weakest acid among given compounds.

Q.68



Which of the following statements are correct, if the threshold frequency of caesium is 5.16×10^{14} Hz?

A. When Cs is placed inside a vacuum chamber with an ammeter connected to it and yellow light is focused on Cs, the ammeter shows the presence of current.

B. When the brightness of the yellow light is dimmed, the value of the current in the ammeter is reduced.

C. When a red light is used instead of the yellow light, the current produced is higher with respect to the yellow light.

D. When a blue light is used, the ammeter shows the formation of current.

E. When a white light is used. the ammeter shows formation of current.

Choose the correct answer from the options given below:

(1) A, C, D and E Only (2) A, B, D and E Only

(3) A, D and E Only (4) B, C and D Only

Ans. [2]

Sol. Where Cs is irradiated it shows photo electric effect.

Red light (λ = 620 – 760 nm) has lesser λ than yellow (570 – 590 nm)

Q.69 An octahedral complex having molecular composition $Co.5NH_3.Cl.SO_4$ has two isomers A and B. The solution of A gives a white precipitate with $AgNO_3$ solution and the solution of B gives white precipitate with $BaCl_2$ solution. The type of isomerism exhibited by the complex is,

(1) Ionisation isomerism

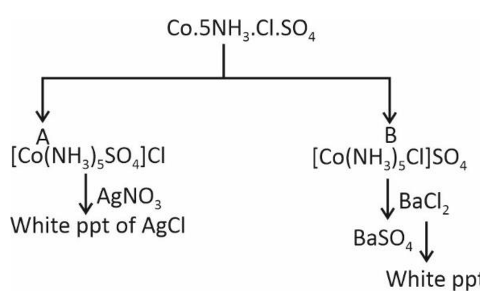
(2) Co-ordinate isomerism

(3) Geometrical isomerism

(4) Linkage isomerism

Ans. [1]

Sol.



Q.70 An aqueous solution of HCl with pH 1.0 is diluted by adding equal volume of water (ignoring dissociation of water). The pH of HCl solution would

(Given log 2 = 0.30)

(1) Increase to 2

(2) Remain same

(3) Reduce to 0.5

(4) Increase to 1.3

Ans. **[4]**

Sol. $pH = 1 \Rightarrow H^+ = 10^{-1}$ M

Now concentration $= \frac{0.1}{2} = 0.05$

$pH = -\log 0.05 = 1.3$

Section-B: Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

Q.71 The number of paramagnetic complex among $[FeF_6]^{3-}$, $[Fe(CN)_6]^{3-}$, $[Mn(CN)_6]^{3-}$, $[Co(C_2O_4)_3]^{3-}$, $[MnCl_6]^{3-}$ and $[CoF_6]^{3-}$, which involved d^2sp^3 hybridization is _______

Ans. **[2]**

Sol. $[FeF_6]^{3-}$ has Fe^{3+} | ↑ | ↑ | ↑ | ↑ | ↑ | sp^3d^2

n = 5

$\mu = \sqrt{35}$ B.M paramagnetic

$[Fe(CN)_6]^{3-}$ has SFL, d^2sp^3. n = 1

$[Mn(CN)_6]^{3-}$ d^4 : $t_{2g}^4 e_g^0$: $d^2\,sp^3$ paramagnetic

$[Co(C_2O_4)_3]^{3-}$ has SFL, d^2sp^3. diamagnetic

$[MnCl_6]^{3-}$ has WFL, sp^3d^2 paramagnetic

$[CoF_6]^{3-}$ has WFL, sp^3d^2 paramagnetic

Q.72 The percentage dissociation of a salt (MX_3) solution at given temperature(van't Hoff factor i = 2) is ______% (Nearest integer)

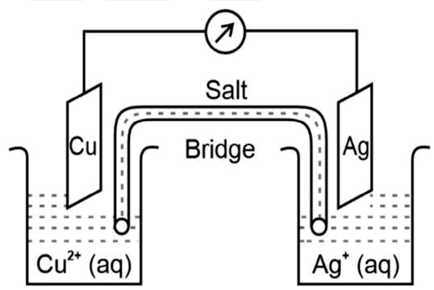
Ans. **[33]**

Sol.

MX_3	→	M^{3+}	+	$3X^-$
1				
$1-\alpha$		α		3α

$1 + 3\alpha = 2$ $\qquad \alpha = 33.33\%$

Q.73 1 Faraday electricity was passed through Cu^{2+} (1.5 M, 1 L)/Cu and 1 Faraday was passed through Ag+ (0.2 M, 1 L)/Ag electrolytic cells. After this the two cells were connected as shown below to make an electrochemical cell. The emf of the cell thus formed at 298 K is _______ mV (nearest integer)



Given : $E^o_{Cu^{2+}/Cu} = 0.34$ V

$E^o_{Ag^+/Ag} = 0.8$V

$\frac{2.303RT}{F} = 0.06$ V

Ans. **[400]**

Sol. $Cu|Cu^{2+}||Ag^+/Ag$

$E_{cell} = E^o - \frac{0.0591}{2}\log\frac{[Cu^{2+}]}{[Ag^+]^2}$

1 F deposits 1 equivalent of $Cu^{2+} = 0.5$ mol

Initial moles of $Cu^{2+} = 1 \times 1.5 = 1.5$

Final moles = 1.5 – 0.5 = 1

$[Cu^{2+}]_{final} = \frac{1}{1} = 1M$ $[Ag^+]_{final} = 0.1$ M

Putting values

$E_{cell} = [0.80-0.34] - \frac{0.06}{2}\log\frac{[Cu^{2+}]}{[Ag^+]^2}$

= 400 mV

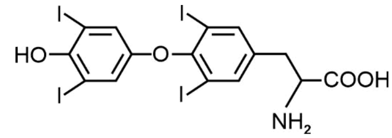
Q.74 An organic compound weighing 500 mg, produced 220 mg of CO_2, on complete combustion. The percentage composition of carbon in the compound is ______%. (nearest integer)

(Given molar mass in g mol^{-1} of C : 12, O : 16)

Ans. **[12]**

Sol. $\% \text{ of C} = \dfrac{\frac{12}{44}\times 220}{500} \times 100 = 12$

Q.75 Thyroxine, the hormone has given below structure



The percentage of iodine in thyroxine is ____%.
(nearest integer)
(Given molar mass in g mol^{-1} C : 12, H : 1, O : 16, N : 14, I : 127)

Ans. **[65]**

Sol. $\% \text{ of I} = \dfrac{\text{mass of iodine}}{\text{mass of thyroxine}} \times 100$

$= \dfrac{4 \times 127}{777} \times 100 = 65\%$