JEE Main Online Exam 2025

Questions & Solution

07nd April 2025 | Evening

MATHEMATICS

Section-A: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct..

Q.1 The number of solutions of the equation $\cos 2\theta \cos\frac{\theta}{2} + \cos\frac{5\theta}{2} = 2\cos^3\frac{5\theta}{2}$ in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ is :

(1) 9 (2) 7 (3) 6 (4) 5

Ans. **[2]**

Sol. $2(\cos 2\theta) \times \left(\cos\frac{\theta}{2}\right) + \cos\frac{5\theta}{2} = 4\cos^3\left(\frac{5\theta}{2}\right)$

$\Rightarrow \cos\left(\frac{5\theta}{2}\right) + \cos\frac{3\theta}{2} + 2\cos\left(\frac{5\theta}{2}\right)$

$= \left(\cos\frac{15\theta}{2} + 3\cos\frac{5\theta}{2}\right)$

$\Rightarrow \cos\left(\frac{3\theta}{2}\right) + \cos\left(\frac{15\theta}{2}\right)$

$\Rightarrow \cos\left(\frac{3\theta}{2}\right) - \cos\frac{15\theta}{2} = 0$

$\Rightarrow 2\sin\left(\frac{9\theta}{2}\right)\sin\left(\frac{6\theta}{2}\right) = 0, 3\theta = 2n\pi$

$\therefore \frac{9\theta}{2} = \eta\pi \rightarrow \theta = \frac{2n\pi}{9}$

$\Rightarrow \theta = \frac{2n\pi}{9}$

$\therefore \theta = -\frac{4\pi}{9}, -\frac{3\pi}{9}, -\frac{2\pi}{9}, 0, \frac{2\pi}{9}, \frac{3\pi}{9}, \frac{4\pi}{9}$

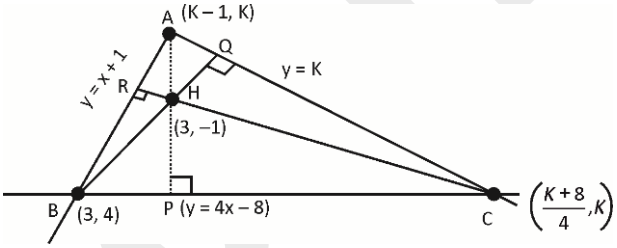
Q.2 If the orthocenter of the triangle formed by the lines $y = x + 1$, $y = 4x - 8$ and $y = mx + c$ is at $(3, -1)$, then $m - c$ is:

(1) –2 (2) 0 (3) 4 (4) 2

Ans. **[2]**

Sol. $y = x + 1$

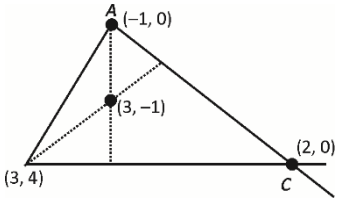
$y = 4x - 8$



$CR \perp y = x + 1$

$\Rightarrow CR : x + y - 2 = 0$

$\frac{K+8}{4} + K - 2 = 0 \Rightarrow K = 0$



Line AC : $y = 0$

Q.3 If the equation of the line passing through the point $\left(0, -\frac{1}{2}, 0\right)$ and perpendicular to the lines $\vec{r} = \lambda(\hat{i} + a\hat{j} + b\hat{k})$ and $\vec{r} = (\hat{i} - \hat{j} - 6\hat{k}) + \mu(-b\hat{i} + a\hat{j} + 5\hat{k})$ is $\frac{x-1}{-2} = \frac{y+4}{d} = \frac{z-c}{-4}$, then $a + b + c + d$ is equal to :

(1) 14 (2) 10 (3) 12 (4) 13

Ans. **[1]**

Sol. Direction ratio of the given line are – 2, d and – 4

$\Rightarrow -2 + ad - 4b = 0$ …(i)

and $2b + ad - 20 = 0$ …(ii)

Subtracting equation (i) and (ii)

$6b - 18 = 0$

$b = 3$

also, line passes through the point $\left(0, -\frac{1}{2}, 0\right)$

$\Rightarrow \frac{0-1}{-2} = \frac{-\frac{1}{2}+4}{d} = \frac{0-c}{-4}$

$\Rightarrow d = 7, c = 2$

From equation (i)

$-2 + 7a - 12 = 0$

$\Rightarrow a = 2$

$a + b + c + d = 14$

Q.4 Let $\vec{a}$ and $\vec{b}$ be the vectors of the same magnitude such that $\frac{|\vec{a}+\vec{b}|+|\vec{a}-\vec{b}|}{|\vec{a}+\vec{b}|-|\vec{a}-\vec{b}|} = \sqrt{2}+1$. Then $\frac{|\vec{a}+\vec{b}|^2}{|\vec{a}|^2}$ is

(1) $4+2\sqrt{2}$ (2) $2+4\sqrt{2}$

(3) $2+\sqrt{2}$ (4) $1+\sqrt{2}$

Ans. **[3]**

Sol. $\frac{|\vec{a}+\vec{b}|}{|\vec{a}-\vec{b}|} = \frac{\sqrt{2}+2}{\sqrt{2}}$ Let $|\vec{a}| = |\vec{b}| = k$, $\cos\theta = \vec{a} \wedge \vec{b}$

$\left(\sqrt{2}|\vec{a}+\vec{b}|\right)^2 = (\sqrt{2}+2)^2 \ |\vec{a}-\vec{b}|^2$

$2(k^2 + k^2 + 2k^2\cos\theta) = (6+4\sqrt{2})$

$(k^2 + k^2 - \cos\theta)$

$(1+\cos\theta) = (3+2\sqrt{2})(1-\cos\theta)$

$\frac{1+\cos\theta}{1-\cos\theta} = 3+2\sqrt{2}$

$\frac{1}{\cos\theta} = \frac{4+2\sqrt{2}}{2+2\sqrt{2}}$

$\cos\theta = \frac{1+\sqrt{2}}{2+\sqrt{2}}$

Now $\frac{|\vec{a}+\vec{b}|^2}{|\vec{a}|^2} = \frac{k^2+k^2+2\cos\theta k^2}{k^2}$

$= 2 + 2\cos\theta = 2 + \frac{2+2\sqrt{2}}{2+\sqrt{2}}$

$= \frac{6+4\sqrt{2}}{2+\sqrt{2}}$

$= 2 + \sqrt{2}$

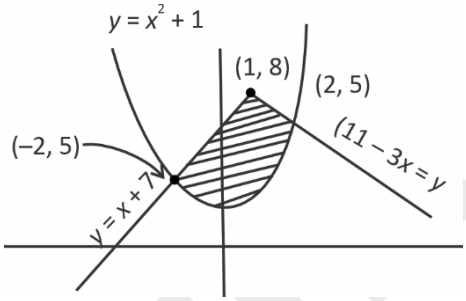
Q.5 If the area of the region $\{(x, y) : 1 + x^2 \le y \le \min\{x + 7, 11 - 3x\}\}$ is A, then 3A is equal to

(1) 46 (2) 49

(3) 50 (4) 47

Ans. **[3]**

Sol.



$A = \int_{-2}^{1}((x+7)-(x^2+1))dx \int_{1}^{2}((1-3x)-(x^2+1))dx$

$= \frac{50}{3}$

$\therefore \ 3A = 50$

Q.6 Let a random variable X take values 0, 1, 2, 3 with $P(X = 0) = P(X = 1) = p$, $P(X = 2) = P(X = 3)$ and $E(X^2) = 2E(X)$. Then the value of $8p - 1$ is :

(1) 0 (2) 3 (3) 2 (4) 1

Ans. **[3]**

Sol. $P(X = 0) = P(X = 1) = p$ and $P(X = 2) = P(X = 3) = q$

$2p + 2q = 1$

$\Rightarrow p + q = \frac{1}{2}$ ...(1)

$E(X^2) = 2E(x)$

$P(0^2) + p(1)^2 + q(2)^2 + q(3)^2$

$= 2(p(0) + p(1) + q(2) + q(3))$

$\Rightarrow \boxed{p = 3q}$ ...(2)

For (1) and (2)

$q = \frac{1}{8}$ and $p = \frac{3}{8}$

$8p - 1 = 2$

Q.7 Let p be the number of all triangles that can be formed by joining the vertices of a regular polygon P of n sides and q be the number of all quadrilaterals that can be formed by joining the vertices of P. If $p + q = 126$, then the eccentricity of the ellipse $\frac{x^2}{16} + \frac{y^2}{n} = 1$ is :

(1) $\frac{\sqrt{7}}{4}$ (2) $\frac{1}{\sqrt{2}}$ (3) $\frac{1}{2}$ (4) $\frac{3}{4}$

Ans. **[2]**

Sol. $P = {}^nC_3 = \frac{n(n-1)(n-2)}{6}$

$q = {}^nC_4 = \frac{n(n-1)(n-2)(n-3)}{24}$

$p + q = 126$

$\therefore (n^2 - n)(n^2 - n - 2) = 3024$

Let $m = n^2 - n$

$m(m-2) = 3024$

$m^2 - 2m - 3024 = 0$

$m = 56$

or $n^2 - n = 56$

$n = 8$

$\therefore$ Eqn. of ellipse, $\frac{x^2}{6} + \frac{y^2}{8} = 1$

$e = \sqrt{1 - \frac{b^2}{a^2}} = \frac{1}{\sqrt{2}}$

Q.8 If the range of the function $f(x) = \frac{5-x}{x^2-3x+2}$, $x \neq 1, 2$, is $(-\infty, \alpha] \cup [\beta, \infty)$, then $\alpha^2 + \beta^2$ is equal to :

(1) 194 (2) 192 (3) 188 (4) 190

Ans. **[1]**

Sol. $y = \frac{5-x}{x^3-3x+2}$

$x^2y - 3xy + 2y = 5 - x$

$x^2y + x(1 - 3y) + 2y - 5 = 0$

For x to be real

$D > 0$

$(1-3y)^2 - 4y(2y-5) > 0$

$1 + 9y^2 - 6y - 8y^2 + 20y > 0$

$y^2 + 14y + 1 > 0$

$y \in (-\infty, \alpha) \cup (\beta, \infty)$

$\alpha + \beta = -14$

$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$= 194$

Q.9 If the locus of $Z \in C$, such that $\text{Re}\left(\frac{z-1}{2z+i}\right) + \text{Re}\left(\frac{\bar{z}-1}{2\bar{z}-i}\right) = 2$, is a circle of radius r and center (a, b), then $\frac{15ab}{r^2}$ is equal to :

(1) 12 (2) 24 (3) 18 (4) 16

Ans. **[3]**

Sol. $\text{Re}(z) + \text{Re}(\bar{z}) = 2$

$\Rightarrow \text{Re}(z) = 1$

$\Rightarrow \text{Re}\left(\frac{x+iy-1}{2x+i(2y+1)}\right) = 1$

$\Rightarrow \text{Re}\left(\frac{(x-1)+iy}{2x+i(2y+1)}\right) = 1$

$\Rightarrow \text{Re}\frac{((x-1)+iy)(2x-i(2y+1))}{((2x+i(2y+1))(2x-i(2y+1))} = 1$

$\Rightarrow \frac{2x^2 - 2x + 2y^2 + y}{4x^2 + (2y+1)^2} = 1$

$\Rightarrow x^2 + y^2 + x + \frac{3}{2}y + \frac{1}{2} = 0$

Centre $(a, b) = \left(-\frac{1}{2}, \frac{-3}{4}\right)$

Radius $= \frac{\sqrt{5}}{4}$

$\frac{15ab}{r^2} = 15 \cdot \frac{3}{8} \times \frac{16}{5} = 18$

Q.10 Let a_n be the n^{th} term of an A.P. If $S_n = a_1 + a_2 + a_3 + \ldots + a_n = 700$, $a_6 = 7$ and $S_7 = 7$, then a_n is equal to :

(1) 65 (2) 64

(3) 56 (4) 70

Ans. **[2]**

Sol. $a_6 = 7 = a + 5d = 7$ …(i)

$S_7 = \frac{7}{2}(2a + 6d) = 7(a + 3d) = 7$

$\Rightarrow a + 3d = 1$ …(ii)

Using (i) & (ii)

$\Rightarrow 2d = 6 \Rightarrow d = 3, a_1 = -8$

$S_n = \frac{n}{2}(2a + (n-1)d) = 700$

$= \frac{n}{2}(-16 + (n-1)3) = 700$

$= n(3n - 19) = 1400, n > 0 \Rightarrow n = 25$

$\Rightarrow a_{25} = (-8) + 24 \cdot (3) = 72 - 8 = 64$

Q.11 A bag contains 19 unbiased coins and one coin with head on both sides. One coin drawn at random is tossed and head turns up. If the probability that the drawn coin was unbiased, is $\frac{m}{n}$, gcd (m, n) = 1, then $n^2 - m^2$ is equal to :

(1) 64 (2) 72 (3) 80 (4) 60

Ans. **[3]**

Sol. Let U : let unbiased coin is drawn
H : head turns up

$$\Rightarrow P\left(\frac{U}{H}\right) = \frac{P(U)\cdot P\left(\frac{H}{U}\right)}{P(U)P\left(\frac{H}{U}\right)+P(\overline{U})\cdot P\left(\frac{H}{\overline{U}}\right)}$$

$$= \frac{\frac{19}{20}\cdot\frac{1}{2}}{\frac{19}{20}\cdot\frac{1}{2}+\frac{1}{20}\cdot\left(\frac{2}{2}\right)} = \left(\frac{19}{21}\right) = \frac{m}{n}$$

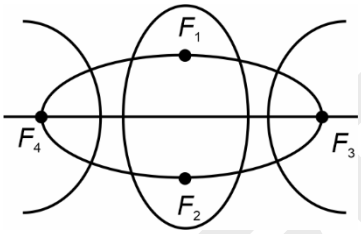
$n^2 - m^2 = 21^2 - 19^2 = (21 + 19)(21 - 19) = 80$

Q.12 Let e_1 and e_2 be the eccentricities of the ellipse $\frac{x^2}{b^2} + \frac{y^2}{25} = 1$ and the hyperbola $\frac{x^2}{16} - \frac{y^2}{b^2} = 1$, respectively. If $b < 5$ and $e_1e_2 = 1$, then the eccentricity of the ellipse having its axes along the coordinate axes and passing through all four foci (two of the ellipse and two of the hyperbola) is:

(1) $\frac{3}{5}$ (2) $\frac{\sqrt{7}}{4}$ (3) $\frac{\sqrt{3}}{2}$ (4) $\frac{4}{5}$

Ans. **[1]**

Sol.



$$e_1^2 : 1 - \frac{b^2}{25} \Rightarrow e_1^2e_2^2 = 1 = \left(1-\frac{b^2}{25}\right)\left(1+\frac{b^2}{16}\right) = 1$$

$$e_2^2 : \left(1+\frac{b^2}{16}\right)$$

$$\frac{-b^4}{400} + b^2\left[\frac{-1}{25}+\frac{1}{16}\right] = 0$$

$$\Rightarrow \frac{b^2}{400} = \frac{9}{400} \Rightarrow b^2 = 9 \Rightarrow e_1 = \frac{4}{5}, e_2 = \frac{5}{4}$$

$$F_1F_2 = 2be = (2)(5)\left(\frac{4}{5}\right) = 8$$

$$F_3F_4 = 2ae = (2)(4)\cdot\left(\frac{5}{4}\right) = 10$$

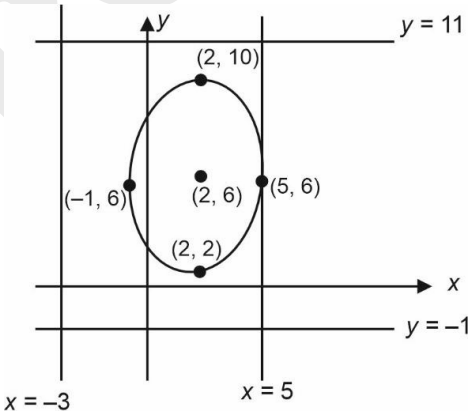
$$\Rightarrow \text{eccentricity} = \sqrt{1-\left(\frac{8}{10}\right)^2} = \frac{6}{10} = \frac{3}{5}$$

Q.13 Let $A = \{(\alpha, \beta) \in R \times R : |\alpha - 1| \le 4 \text{ and } |\beta - 5| \le 6\}$ and $B = \{(\alpha, \beta) \in R \times R : 16(\alpha - 2)^2 + 9(\beta - 6)^2 \le 144\}$
Then
(1) $B \subset A$
(2) $A \cup B = \{(x, y) : -4 \le x \le 4, -1 \le y \le 11\}$
(3) Neither $A \subset B$ nor $B \subset A$
(4) $A \subset B$

Ans. **[1]**

Sol. Set A : For α
$-4 \le \alpha - 1 \le 4$
$\Rightarrow -3 \le \alpha \le 5$
For β
$-6 \le \beta - 5 \le 6$
$\Rightarrow -1 \le \beta \le 11$
Set B :

$$\frac{(\alpha-2)^2}{9} + \frac{(\beta-6)^2}{16} \le 1$$



clearly $B \subset A$

Q.14 If the sum of the second, fourth and sixth terms of a G.P. of positive terms is 21 and the sum of its eighth, tenth and twelfth terms is 15309, then the sum of its first nine terms is:

(1) 755 (2) 750 (3) 760 (4) 757

Ans. **[4]**

Sol. $ar + ar^3 + ar^5 = 21$ …(i)
$ar^7 + ar^9 + ar^{11} = 15309$ …(ii)
(ii)/(i)

$$\frac{r^7}{r} = 729 \Rightarrow r = 3 \qquad (r > 0)$$

Using (i)
$a(3 + 27 + 243) = 21$

$\Rightarrow a = \frac{21}{273} = \frac{1}{13}$

$S_9 = \frac{\frac{1}{13}(3^9 - 1)}{3-1}$

$= \frac{1}{26}(3^9 - 1)$

$= 3^6 + 3^3 + 1$

$= 757$

Q.15 Let the length of a latus rectum of an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ be 10. If its eccentricity is the minimum value of the function $f(t) = t^2 + t + \frac{11}{12}$, $t \in R$, then $a^2 + b^2$ is equal to.

(1) 125 (2) 120 (3) 126 (4) 115

Ans. **[3]**

Sol. $\frac{2b^2}{a} = 10 \Rightarrow b^2 = 5a$

$e = f(t)\big|_{min} = t^2 + t + \frac{11}{12}\Big|_{t=\frac{1}{4}} = \frac{1}{4} - \frac{1}{2} + \frac{11}{12}$

$= \frac{3-6+11}{12} = \frac{8}{12} = \frac{2}{3}$

$\Rightarrow \sqrt{1 - \frac{b^2}{a^2}} = \frac{2}{3}$

$\Rightarrow 1 - \frac{b^2}{a^2} = \frac{4}{9} \Rightarrow \frac{b^2}{a^2} = \frac{5}{9} \Rightarrow b^2 = \frac{5a^2}{9}$

$\Rightarrow 5a = \frac{5a^2}{9} \Rightarrow a = 9$; So, $b^2 = 45$

Hence, $a^2 + b^2 = 81 + 45 = 126$

Q.16 Let $y = y(x)$ be the solution of the differential equation $(x^2 + 1)y' - 2xy = (x^4 + 2x^2 + 1)\cos x$, $y(0) = 1$. Then $\int_{-3}^{3} y(x)dx$ is :

(1) 24 (2) 18
(3) 30 (4) 36

Ans. **[1]**

Sol. $(1 + x^2)\frac{dy}{dx} - 2xy = (x^4 + 2x^2 + 1)\cos x$

$\frac{dy}{dx} - \left(\frac{2x}{1+x^2}\right)y = \frac{(x^2+1)\cos x}{(x^2+1)}$

$IF = e^{-\int \frac{2x}{1+x^2}dx} = \frac{1}{1+x^2}$

$\frac{y}{1+x^2} = \int \cos x dx$

$\frac{y}{1+x^2} = \int \sin x + c$

$\because y(0) = 1$

$\Rightarrow 1 = c$

$\therefore y = (1 + \sin x)(1 + x^2)$

$\int_{-3}^{3} y(x)dx \int_{-3}^{3} (1 + \sin x)(1 + x^2)dx$

$= \int_{-3}^{3} 2(1 + x^2)dx$

$= 2x + \frac{2x^3}{3}\Big]_0^3$

$= 6 + 18 = 24$

Q.17 Consider the lines $L_1 : x - 1 = y - 2 = z$ and $L_2 : x - 2 = y = z - 1$. Let the feet of the perpendiculars from the point P(5, 1, –3) on the lines L_1 and L_2 be Q and R respectively. If the area of the triangle PQR is A, then $4A^2$ is equal to

(1) 147 (2) 143 (3) 139 (4) 151

Ans. **[1]**

Sol. P(5, 1, –3)

$L_1 : x - 1 = y - 2 = z = \lambda$

$L_2 : x - 2 = y = z - 1 = \mu$

Any point of L_1 is Q $(\lambda + 1, \lambda + 2, \lambda)$

Any point of L_2 is R$(\mu + 2, \mu, \mu + 1)$

Now $PQ < \lambda - 4, \lambda + 1, \lambda + 3 > \cdot < 1, 1, 1 > = 0$

$\lambda - 4 + \lambda + 1 + \lambda + 3 = 0$

$3\lambda = 0$

$\Rightarrow \lambda = 0$

$\therefore$ Q(1, 2, 0)

Also,

$PR < \mu - 3, \mu - 1, \mu + 4 > \cdot < 1, 1, 1 > = 0$

$\mu - 3 + \mu - 1 + \mu + 4 = 0$

$\Rightarrow \mu = 0$

R (2, 0, 1)

$\text{Area} = \frac{1}{2}\left|\overrightarrow{PQ} \times \overrightarrow{PR}\right| = \left|\frac{1}{2}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -1 & -3 \\ 3 & 1 & -4 \end{vmatrix}\right|$

$= \frac{1}{2}\left|7\hat{i} + 7\hat{j} + 7\hat{k}\right| = \frac{1}{2} \times 7 \times \sqrt{3} = \frac{7\sqrt{3}}{2}$

$\because A = \frac{7\sqrt{3}}{2}$

$\Rightarrow 4A^2 = \left(7\sqrt{3}\right)^2$

$= 49 \times 3$

$= 147$

Q.18 The number of real roots of the equation $x|x-2| + 3|x-3| + 1 = 0$ is

(1) 3 (2) 4 (3) 2 (4) 1

Ans. **[4]**

Sol. $x|x-2| + 3|x-3| + 1 = 0$

Case I : $x < 2$

$-x(x-2) - 3(x-3) + 1 = 0$

$x^2 + 2x - 3x + 9 + 1 = 0$

$x^2 + x - 10 = 0$

$x = \frac{-1-\sqrt{41}}{2}$ or $\frac{-1+\sqrt{41}}{2}$ (rejected)

1 sol^n.

Case II : $2 \le x < 3$

$x(x-2) - 3(x-3) + 1 = 0$

$x^2 - 5x + 8 = 0$

No solution

Case III : $x \ge 3$

$x(x-2) + 3(x-3) + 1 = 0$

$x^2 + x - 8 = 0$

$x = \frac{-1 \pm \sqrt{33}}{2}$ (no sol^n)

$\therefore x = \frac{-1-\sqrt{41}}{2}$ is the only solution

Q.19 Let the system of equations

$x + 5y - z = 1$

$4x + 3y - 3z = 7$

$24x + y + \lambda z = m$

$\lambda, \mu \in \mathbf{R}$, have infinitely many solutions. Then the number of the solutions of this system, if x, y, z are integers and satisfy $7 \le x + y + z \le 77$, is

(1) 6 (2) 3 (3) 5 (4) 4

Ans. **[2]**

Sol. $\Delta = \begin{vmatrix} 1 & 5 & -1 \\ 4 & 3 & -3 \\ 24 & 1 & \lambda \end{vmatrix}$

$= (3\lambda + 3) - 5(4\lambda + 72) - (4 - 72)$

$= -17\lambda - 289 = 0$

$\Rightarrow \lambda = -17$

$\Delta_1 = \begin{vmatrix} 1 & 5 & -1 \\ 7 & 3 & -3 \\ \mu & 1 & -17 \end{vmatrix}$

$= (-51 + 3) - 5(-119 + 3\mu) - (7 - 3\mu)$

$= 540 - 12\mu$

$\mu = \frac{540}{12} = 45$

$\therefore x + 5y - z = 1$

$4x + 3y - 3z = 7$

$24x + y - 17z = 45$

$z = x + 5y - 1$

$\therefore 4x + 3y - 3x - 15y + 3 = 7$

$\Rightarrow x - 12y = 4$

$\Rightarrow x = 4 + 12y$

$\therefore z = 4 + 12y + 5y - 1$

$= 3 + 17y$

$\therefore (x, y, z) \equiv (4 + 12k, k, 3 + 17k)$ ($\because$ Assume $y = k$)

$7 \le 7 + 30k \le 77$

$0 \le 30k < 70$

$0 \le k \le 2{\cdot}3$

$\Rightarrow k = 0, 1, 2$ Three solutions possible

Q.20 Let $f : \mathbf{R} \to \mathbf{R}$ be a polynomial function of degree four having extreme values at $x = 4$ and $x = 5$. If $\lim\limits_{x \to 0} \frac{f(x)}{x^2} = 5$, then f(2) is equal to

(1) 14 (2) 10 (3) 12 (4) 8

Ans. **[2]**

Sol. $f(x) = ax^4 + bx^3 + cx^2 + dx + e$

$f'(4) = 0, \ f'(5) = 0$

Also $\lim\limits_{x \to 0} \frac{f(x)}{x^2} = 5$

$\Rightarrow \lim\limits_{x \to 0} \left(\frac{ax^4 + bx^3 + cx^2 + dx + e}{x^2} \right) = 5$

$\Rightarrow d = e = 0$

and $c = 5$

$\therefore f(x) = ax^4 + bx^3 + 5x^2$

$f'(x) = 4ax^3 + 3bx^2 + 10x$

$f'(4) = 256a + 48b + 40 = 0$ ...(i)

$f'(5) = 500a + 75b + 50 = 0$ ...(ii)

Solving equation (i) and (ii)

We get $a = \frac{1}{8}, b = \frac{-3}{2}$

$\therefore f(x) = \frac{1}{8}x^4 - \frac{3}{2}x^3 + 5x^2$

$f(2) = \frac{16}{8} - \frac{3}{2}(2)^3 + 5(2)^2$

$f(2) = 2 - 12 + 20$

$f(2) = 10$

Section-B: Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

Q.21 For $t > -1$, let α_t and β_t be the roots of the equation

$$\left((t+2)^{\frac{1}{7}}-1\right)x^2+\left((t+2)^{\frac{1}{6}}-1\right)x+\left((t+2)^{\frac{1}{21}}-1\right)=0.$$

If $\lim\limits_{t\to -1^+}\alpha_t = a$ and $\lim\limits_{t\to -1^+}\beta_t = b$, then $72(a+b)^2$ is equal to _____.

Ans. **[98]**

Sol. $\alpha+\beta=\dfrac{(t+2)^{1/6}-1}{(t+2)^{1/7}-1}$

$$\alpha\beta=\frac{(t+2)^{1/21}-1}{(t+2)^{1/7}-1}$$

$$\lim_{t\to -1}(\alpha+\beta)=\frac{-\frac{1}{6}}{\frac{1}{7}}=\frac{-7}{6}=a+b$$

$$\lim_{t\to -1}(\alpha\beta)=\frac{\frac{1}{21}}{\frac{1}{7}}=\frac{7}{21}=\frac{1}{3}=ab$$

$$\Rightarrow (a+b)^2=\frac{49}{36}$$

$$\Rightarrow 72(a+b)^2=98$$

Q.22 If $\int\left(\frac{1}{x}+\frac{1}{x^3}\right)\left(\sqrt[23]{3x^{-24}+x^{-26}}\right)dx$

$$=-\frac{\alpha}{3(\alpha+1)}\left(3x^{\beta}+x^{\gamma}\right)^{\frac{\alpha+1}{\alpha}}+C,\ x>0,\ (\alpha,\beta,\gamma\in Z),$$

where C is the constant of integration, then $\alpha+\beta+\gamma$ is equal to _____.

Ans. **[19]**

Sol. $I=\int\left(\frac{1}{x}+\frac{1}{x^3}\right)\int\left(\frac{3}{x^{24}}+\frac{1}{x^{26}}\right)^{\frac{1}{23}}dx$

$$=\int\left(\frac{1}{x^2}+\frac{1}{x^4}\right)\left(\frac{3}{x}+\frac{1}{x^3}\right)^{\frac{1}{23}}dx$$

Put $\frac{3}{x}+\frac{1}{x^3}=t\Rightarrow\left(-\frac{3}{x^2}-\frac{3}{x^4}\right)dx=dt$

$$\Rightarrow I=-\frac{1}{3}\int t^{\frac{1}{23}}dt=-\frac{1}{3}\frac{t^{\frac{1}{23}+1}}{\frac{1}{23}+1}+C$$

$$=-\frac{1}{3}\times\frac{23}{24}(3x^{-1}+x^{-3})^{\frac{24}{23}}+C$$

$$\Rightarrow \alpha=23,\ \beta=-1,\ \gamma=-3$$

$$\Rightarrow \alpha+\beta+\gamma=19$$

Q.23 The sum of the series

$2\times1\times{}^{20}C_4-3\times2\times{}^{20}C_5+4\times3\times{}^{20}C_6-5\times4\times{}^{20}C_7+.....+18\times17\times{}^{20}C_{20}$, is equal to _____.

Ans. **[34]**

Sol. $(1+x)^{20}={}^{20}C_0+{}^{20}C_1x+{}^{20}C_2x^2+{}^{20}C_2x^3+{}^{20}C_4x^4+{}^{20}C_5x^5+.....+{}^{20}C_{20}x^{20}$

$$\frac{(1+x)^{20}}{x^2}=\frac{1}{x^2}+\frac{20}{x}+{}^{20}C_2+{}^{20}C_3x+{}^{20}C_4x^2+{}^{20}C_5x^3+.....+{}^{20}C_{20}x^{18}$$

d.w.r. to x

$$\frac{x^2\cdot20(1+x)^{19}+(1+x)^{20}\cdot2x}{x^4}=\frac{-2}{x^3}-\frac{20}{x^2}+{}^{20}C_3+2\cdot{}^{20}C_4x+3\cdot{}^{20}C_5x^2+.....+18\ {}^{20}C_{20}x^{17}$$

$$\frac{2x(1+x)^{19}(11x+1)}{x^4}=\frac{-2}{x^3}-\frac{20}{x^2}+{}^{20}C_3+2\cdot{}^{20}C_4x+3\cdot{}^{20}C_5x^2+....+18\ {}^{20}C_{20}x^{17}$$

Differentiate w.r. to x

$$\frac{2\left[x^3(1+x)^{19}\times11+x^3(11x+1)\cdot19(1+x)^{18}-(1+x)^{19}(11x+1)\cdot3x^2\right]}{x^6}$$

$$=\frac{6}{x^4}+\frac{40}{x^3}+2\cdot1\cdot{}^{20}C_4+3\cdot2\ {}^{20}C_5x+....+18\cdot17\ {}^{20}C_{20}x^{16}$$

Put $x=-1$ in above equation

$\Rightarrow 0=6-40+R\Rightarrow R=34$

Q.24 Let the lengths of the transverse and conjugate axes of a hyperbola in standard form 2a and 2b, respectively, and one focus and the corresponding directrix of this hyperbola be (–5, 0) and $5x+9=0$, respectively. If the product of the focal distances of a point $(\alpha,2\sqrt{5})$ on the hyperbola is p, then 4p is equal to ____.

Ans. **[189]**

Sol. Equation of hyperbola is $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

Directrix : $x = \frac{-9}{5}$ and corresponding foci $(-5, 0)$

$-\frac{a}{e} = -\frac{9}{5}$ and $-ae = -5$

$\Rightarrow \frac{9e^2}{5} = 5 \Rightarrow e = \sqrt{\frac{25}{9}} = \frac{5}{3} \Rightarrow a = 3$

$\therefore b^2 = a^2(e^2 - 1) = 9\left(\frac{25}{9} - 1\right) = 16$

Hyperbola $\frac{x^2}{9} - \frac{y^2}{16} = 1$

$(\alpha, 2\sqrt{5})$ lie on it

$\Rightarrow \frac{\alpha^2}{9} - \frac{20}{16} = 1 \Rightarrow \alpha^2 = \frac{36}{16} \times 9 = \frac{81}{4}$

Product for distance of (x_1, y_1) from the two foci

$= (ex_1 + a)|ex_1 - a|$

$= e^2x_1^2 - a^2$

For $(\alpha, 2\sqrt{5}) \Rightarrow P = \frac{25}{9} \cdot \frac{81}{4} - 9 = \frac{189}{4}$

$4P = 189$

Q.25 If the function $f(x) = \frac{\tan(\tan x) - \sin(\sin x)}{\tan x - \sin x}$ is continuous at $x = 0$, then $f(0)$ is equal to _____.

Ans. **[2]**

Sol. $\lim\limits_{x\to 0} f(x) = \lim\limits_{x\to 0} \frac{\tan(\tan x) - \sin(\sin x)}{\tan x - \sin x}$

$= \lim\limits_{x\to 0} \frac{\tan(\tan x) - \tan x + \sin x - \sin(\sin x) + (\tan x - \sin x)}{\tan x - \sin x}$

$= 1 + \lim\limits_{x\to 0} \left(\left(\frac{\tan(\tan x) - \tan x}{\tan^3 x}\right)\frac{\tan^3 x}{x^3} + \left(\frac{\sin x - \sin(\sin x)}{\sin^3 x}\right)\frac{\sin^3 x}{x^3}\right) \cdot \frac{x^3}{\tan x - \sin x}$

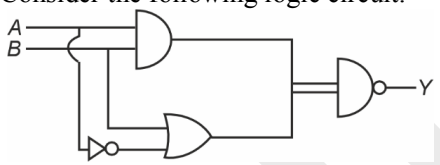
$= 1 + \left(\frac{1}{6} + \frac{1}{3}\right) \times \lim\limits_{x\to 0} \left(\frac{x^2}{1 - \cos x}\right)\cos x \cdot \left(\frac{x}{\sin x}\right)$

$= 1 + \frac{1}{2} \cdot 2 = 2$

PHYSICS

Section-A: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct..

Q.26 Consider the following logic circuit.



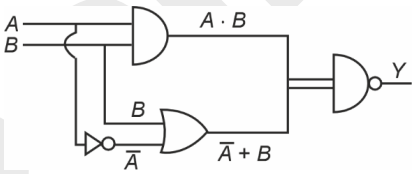
The output is Y = 0 when :

(1) A = 0 and B = 1 (2) A = 0 and B = 0

(3) A = 1 and B = 1 (4) A = 1 and B = 0

Ans. **[3]**

Sol.



$Y = \overline{A \cdot B}$

Q.27 The unit of $\sqrt{\frac{2I}{\epsilon_0 c}}$ is :

(I = intensity of an electromagnetic wave, c : speed of light)

(1) NC (2) Vm (3) NC^{-1} (4) Nm

Ans. **[3]**

Sol. For $\sqrt{\frac{2I}{\epsilon \cdot c}}$

Unit of $I \Rightarrow \frac{J}{S \cdot m^2}$

Unit of $C \Rightarrow m/s$

Unit of $\epsilon_0 \Rightarrow C^2 m^{-2} N^{-1}$,

Putting $= \frac{N}{C}$

Q.28 A dipole with two electric charges of 2 μC magnitude each, with separation distance 0.5 μm, is placed between the plates of a capacitor such that its axis is parallel to an electric field established between the plates when a potential difference of 5 V is applied. Separation between the plates is 0.5 mm. If the dipole is rotated by 30° from the axis, it tends to realign in the direction due to a torque.

The value of torque is :

(1) 5×10^{-3} Nm

(2) 2.5×10^{-12} Nm

(3) 5×10^{-9} Nm

(4) 2.5×10^{-9} Nm

Ans. **[3]**

Sol. $E = \frac{5}{0.5\times10^{-3}} = 10^4$ N/C

$P = 10^{-12}$ C · m

$\tau = PE \sin\theta$

$= 5 \times 10^{-9}$ N.m

Q.29 Given below are two statements : one is labelled as **Assertion (A)** and the other is labelled as **Reason (R).**

Assertion (A) : Magnetic monopoles do not exist.

Reason (R) : Magnetic field lines are continuous and form closed loops.

In the light of the above statements, choose the most appropriate answer from the options given below :

(1) (A) is not correct but (R) is correct

(2) (A) is correct but (R) is not correct

(3) Both (A) and (R) are correct but (R) is not the correct explanation of (A)

(4) Both (A) and (R) are correct and (R) is the correct explanation of (A)

Ans. **[4]**

Sol. The magnet does not exist in form of monopole and magnetic lines never intersect, continuous and form a closed loop. Line initiates from north pole and end to south outside while inside vice versa.

Q.30 Match List-I with List-II.

List -I		List-II	
a.	Mass density	(i)	$[ML^2T^{-3}]$
b.	impulse	(ii)	$[MLT^{-1}]$
c.	power	(iii)	$[ML^2T^0]$
d.	Moment of inertia	(iv)	$[ML^{-3}T^0]$

Choose the correct answer from the options given below :

(1) a(iv), b(ii), c(i), d(iii)

(2) a(i), b(iii), c(iv), d(ii)

(3) a(ii), b(iii), c(iv), d(i)

(4) a(iv), b(ii), c(iii), d(i)

Ans. **[1]**

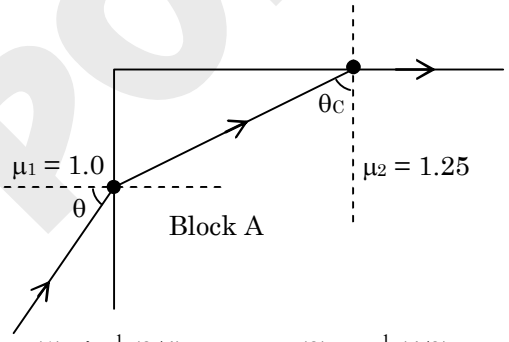
Sol. $\rho = \frac{kg}{m^3} \Rightarrow [ML^{-3}T^0]$ ...(iv)

$I = F \cdot \Delta T \Rightarrow [MLT^{-1}]$ …(ii)

$P = \frac{w}{t} \Rightarrow [ML^2T^{-3}]$ …(i)

$I = \frac{MR^2}{n} \Rightarrow [ML^2T^0]$ …(iii)

Q.31 A transparent block A having refractive index $\mu = 1.25$ is surrounded by another medium of refractive index $\mu = 1.0$ as shown in figure. A light ray is incident on the flat face of the block with incident angle θ as shown in figure. What is the maximum value of θ for which light suffers total internal reflection at the top surface of the block?



(1) $\sin^{-1}(3/4)$ (2) $\tan^{-1}(4/3)$

(3) $\cos^{-1}(3/4)$ (4) $\tan^{-1}(3/4)$

Ans. **[1]**

Sol. At 1st refractions

$1 \times \sin\theta = \frac{5}{4}\sin\alpha$

$\sin\alpha = \frac{4}{5}\sin\theta$ …(i)

At point of TIR

$\frac{5}{4}\sin(\theta_c) = 1$

$\sin(90-\alpha) = \frac{4}{5}$

$\cos\alpha = \frac{4}{5}$

$\sin\alpha = \frac{3}{5}$ …(ii)

From (i) and (ii)

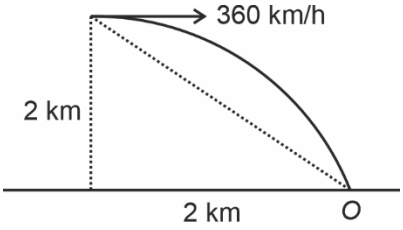
$\theta = \sin^{-1}\left(\frac{3}{4}\right)$

Q.32 A helicopter flying horizontally with a speed of 360 km/h at an altitude of 2 km, drops an object at an instant. The object hits the ground at a point O, 20 s after it is dropped. Displacement of 'O' from the position of helicopter where the object was released is :
(use acceleration due to gravity g = 10 m/s^2 and neglect air resistance)

(1) $2\sqrt{2}$ km (2) 4 km
(3) $2\sqrt{5}$ km (4) 7.2 km

Ans. **[1]**

Sol.



R = 100 × 20
R = 2 km
S = $\sqrt{2^2+2^2}$
S = $2\sqrt{2}$ km
Displacement = $2\sqrt{2}$ km

Q.33 Given below are two statements : one is labelled as
Assertion (A) and the other is labelled as Reason (R).
Assertion (A) : The outer body of an air craft is made of metal which protects persons sitting inside from lightning-strikes.
Reason (R) : The electric field inside the cavity enclosed by a conductor is zero.
In the light of the above statements, choose the most appropriate answer from the options given below :

(1) Both (A) and (R) are correct but (R) is not the correct explanation of (A)
(2) (A) is correct but (R) is not correct
(3) Both (A) and (R) are correct and (R) is the correct explanation of (A)
(4) (A) is not correct but (R) is correct

Ans. **[3]**

Sol. The outer body of air craft made with metal to provide electrostatic shield, such that electric field inside craft remain zero for any charge on outer surface.

Q.34 The dimension of $\sqrt{\frac{\mu_0}{\varepsilon_0}}$ is equal to that of :
(μ_0 = Vacuum permeability and ε_0 = Vacuum permittivity)

(1) inductance (2) Resistance
(3) Capacitance (4) Voltage

Ans. **[2]**

Sol. $[\mu_0] = [MLT^{-2}A^{-2}]$
$[\varepsilon_0] = [M^{-1}L^{-3}T^4A^2]$
$\left[\sqrt{\frac{\mu_0}{\varepsilon_0}}\right] = [ML^2T^{-3}A^{-2}] \simeq [R]$

Q.35 The helium and argon are put in the flask at the same room temperature (300 K). The ratio of average kinetic energies (per molecule) of helium and argon is (Give : Molar mass of helium = 4 g/mol, Molar mass of argon = 40 g/mol)

(1) 1 : 10 (2) 10 : 1
(3) 1 : $\sqrt{10}$ (4) 1 : 1

Ans. **[4]**

Sol. Average K.E. = $\frac{3}{2}$ RT, is not depends on molar/molecular mass.
So, $\frac{(K.E.)_{He}}{(K.E.)_{Ar}} = 1$

Q.36 A capillary tube of radius 0.1 mm is partly dipped in water (surface tension 70 dyn/cm and glass water contact angle ≈ 0°) with 30° inclined with the vertical.
The length of water risen in the capillary is _____ cm.
(Take g = 9.8 m/s^2

(1) $\frac{82}{5}$ (2) $\frac{68}{5}$
(3) $\frac{71}{5}$ (4) $\frac{57}{2}$

Ans. **[1]**

Sol. $h = \frac{2T\cos\theta}{\rho g r}$
$= \frac{100}{7}$
$h' = \frac{h}{\cos\theta} = \frac{100}{7\cos 30°}$
$h' = \frac{200}{7\sqrt{3}} \approx 16.49$

Q.37 Given below are two statements : one is labelled as
Assertion (A) and the other is labelled as Reason (R).
Assertion (A) : Refractive index of glass is higher than that of air.
Reason (R) : Optical density of a medium is directly proportionate to its mass density which results in a proportionate refractive index.
In the light of the above statements, choose the most appropriate answer from the options given below
(1) (A) is correct but (R) is not correct
(2) (A) is not correct but (R) is correct
(3) Both (A) and (R) are correct but (R) is not the correct explanation of (A)
(4) Both (A) and (R) are correct and (R) is the correct explanation of (A)

Ans. **[1]**

Sol. Speed of light $v = \frac{c}{\mu}$
for air $v \approx c$
for glass $v < c$
so $\mu_g > \mu_a$
and optical density (μ) does not depends on density of medium
so A is correct but R is incorrect

Q.38 Given below are two statements : one is labelled as
Assertion (A) and the other is labelled as Reason (R).
Assertion (A) : The radius vector from the Sun to a planet sweeps out equal areas in equal intervals of time and thus areal velocity of planet is constant.
Reason (R) : For a central force field the angular momentum is a constant.
In the light of the above statements, choose the most appropriate answer from the options given below.
(1) Both (A) and (R) are correct but (R) is not the correct explanation of (A)
(2) (A) is correct but (R) is not correct
(3) Both (A) and (R) are correct and (R) is the correct explanation of (A)
(4) (A) is not correct but (R) is correct

Ans. **[3]**

Sol. From Kepler's 2nd law
$\frac{dA}{dt} = \frac{L}{2m} = \text{constant}$
and angular momentum for central force is constant.
so, both (A) and (R) are correct and (R) explain (A)

Q.39 Match List-I with List-II.

List-I		List-II	
(A)	Isothermal	(I)	ΔW (work done) = 0
(B)	Adiabatic	(II)	ΔQ (supplied heat) = 0
(C)	Isobaric	(III)	ΔU (change in internal energy) $\neq 0$
(D)	Isochoric	(IV)	$\Delta U = 0$

Choose the correct answer from the options given below.
(1) (A)-(II), (B)-(IV), (C)-(I), (D)-(III)
(2) (A)-(IV), (B)-(II), (C)-(III), (D)-(I)
(3) (A)-(IV), (B)-(I), (C)-(III), (D)-(II)
(4) (A)-(III), (B)-(II), (C)-(I), (D)-(IV)

Ans. **[2]**

Sol. For isothermal,
$T = \text{constant}, \Delta T = 0, \Delta U = nC_v\Delta T = 0$ … (IV)
For Adiabatic,
$\Delta Q = 0, \Delta W = -\Delta U$ … (II)
For Isobaric,
$p = \text{constant}, \Delta U = nC_V \Delta T \neq 0,$
$\Delta Q \neq 0, \Delta W \neq 0$ … (III)
For Isochoric,
$v = \text{constant}, \Delta W = 0, \Delta Q = \Delta U \neq 0$ …(I)

Q.40 A photoemissive substance is illuminated with a radiation of wavelength λ_i so that it releases electrons with de-Broglie wavelength λ_e. The longest wavelength of radiation that can emit photoelectron is λ_0. Expression for de-Broglie wavelength is given by:
(m: mass of the electron, h: Planck's constant and c: speed of light)

(1) $\lambda_e = \sqrt{\frac{h\lambda_i}{2mc}}$

(2) $\lambda_e = \sqrt{\frac{h}{2mc\left(\frac{1}{\lambda_i - \lambda_0}\right)}}$

(3) $\lambda_e = \sqrt{\frac{h\lambda_0}{2mc}}$

(4) $\lambda e = \frac{h}{\sqrt{2mc\left(\frac{1}{\lambda_i} - \frac{1}{\lambda_0}\right)}}$

Ans. **[2]**

Sol. $\frac{hc}{\lambda_i} = \phi + KE$

$\phi = \frac{hc}{\lambda_0}$, $KE = \left(\frac{h}{\lambda_e}\right)^2 \cdot \frac{1}{2m}$

$\frac{hc}{\lambda_i} = \frac{hc}{\lambda_0} + \left(\frac{h}{\lambda_e}\right)^2 \cdot \frac{1}{2m}$

$\left(\frac{\lambda_e}{h}\right)^2 = \frac{1}{2mhc\left(\frac{1}{\lambda_i} - \frac{1}{\lambda_0}\right)}$

$\lambda_e = \sqrt{\frac{h}{2mc\left(\frac{1}{\lambda_i} - \frac{1}{\lambda_0}\right)}}$

Q.41 An object with mass 500 g moves along x-axis with speed $v = 4\sqrt{x}$ m/s. The force acting on the object is :

(1) 5 N (2) 4 N

(3) 6 N (4) 8 N

Ans. **[2]**

Sol. $\frac{dx}{dt} = 4\sqrt{x}$

$x = 4t^2$

$v = \frac{dx}{dt} = 8t$

$a = 8\ m/s^2$

$F = ma = 4N$

Q.42 Given below are two statements : one is labelled as Assertion (A) and the other is labelled as Reason (R).

Assertion (A) : The density of the copper $\left({}^{64}_{29}Cu\right)$ nucleus is greater than that of the carbon $\left({}^{12}_{6}C\right)$ nucleus.

Reason (R) : The nucleus of mass number A has a radius proportional to $A^{1/3}$.

In the light of the above statements, choose the most appropriate answer from the options given below.

(1) Both (A) and (R) are correct and (R) is the correct explanation of (A)

(2) (A) is not correct but (R) is correct

(3) Both (A) and (R) are correct and (R) is not the correct explanation of (A)

(4) (A) is correct but (R) is not correct

Ans. **[2]**

Sol. Density of nucleus independent on mass number (A) and radius is proportional to $A^{1/3}$ so Assertion (A) is incorrect but Reason (R) is correct.

Q.43 Which one of the following forces cannot be expressed in terms of potential energy ?

(1) Gravitational force

(2) Restoring force

(3) Frictional force

(4) Coulomb's force

Ans. **[3]**

Sol. Potential energy can be expressed for conservative force, so it can not be expressed for friction because it is non-conservative force

Q.44 A mirror is used to produce an image with magnification of $\frac{1}{4}$ If the distance between object and its image is 40 cm, then the focal length of the mirror is _____.

(1) 10.7 cm

(2) 15 cm

(3) 12.7 cm

(4) 10 cm

Ans. **[1]**

Sol. here $\left|\frac{v}{u}\right| = \frac{1}{4}$

$u = 4v$

and $u + v = 40$

so $u = 32$, $v = 8$

$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$

$F = 10.66\ cm \approx 10.7\ cm$

Q.45 The equation of a wave travelling on a string is $y = \sin[20\pi x + 10\pi t]$, where x and t are distance and time in SI units. The minimum distance between two points having the same oscillating speed is :

(1) 10 cm (2) 2.5 cm

(3) 20 cm (4) 5.0 cm

Ans. **[4]**

Sol. $k = 20\pi = \frac{2\pi}{\lambda}$

$\lambda = 10\ cm$

Minimum distance $= \frac{\lambda}{2} = 5\ cm$

Section-B: Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

Q.46 The electric field in a region is given by $\vec{E} = (2\hat{i} + 4\hat{j} + 6\hat{k}) \times 10^3$ N/C. The flux of the field through a rectangular surface parallel to x-z plane is 6.0 Nm^2C^{-1}. The area of the surface is _____cm^2.

Ans. **[15]**

Sol. $\vec{E} = (2\hat{i} + 4\hat{j} + 6\hat{k}) \times 10^3$ N/C

$\phi = \vec{E} \cdot \vec{S}$

$= \left[(2\hat{i} + 4\hat{j} + 6\hat{k}) - (a\hat{j})\right] \times 10^3$

$6 = A4 \times 10^3$

$A = \dfrac{6}{4 \times 10^3}$ m^2

$= 15$ cm^2

Q.47 An inductor of reactance 100 Ω, a capacitor of reactance 50 Ω, and a resistor of resistance 50 Ω are connected in series with an AC source of 10 V, 50 Hz. Average power dissipated by the circuit is _____ W.

Ans. **[1]**

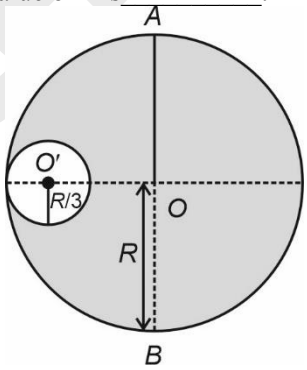
Sol. $P = \dfrac{I_0 V_0}{2} \cos\phi$

$\cos\phi = \dfrac{R}{Z}$, $Z = \sqrt{R^2 + (X_L - X_C)^2} = 50\sqrt{2}$ Ω

$\cos\phi = \dfrac{1}{2}$

$P = 1$ W

Q.48 M and R be the mass and radius of a disc. A small disc of radius $\dfrac{R}{3}$ is removed from the bigger disc as shown in figure. The moment of inertia of remaining part of bigger disc about an axis AB passing through the centre O and perpendicular to the plane of disc is $\dfrac{4}{x}MR^2$. The value of x is_____.



Ans. **[9]**

Sol. $m_1 = M$

$I_0 = \dfrac{MR^2}{2}$

$I_{0'} = \dfrac{\frac{M}{9}\left(\frac{R}{3}\right)^2}{2} = \dfrac{MR^2}{81 \times 2}$

$(I_{net})_O = \dfrac{MR^2}{2} - \left\{\dfrac{MR^2}{81 \times 2} + \dfrac{M}{9} \times \left(\dfrac{2R}{3}\right)^2\right\}$

$= \dfrac{4}{9}MR^2$

Q.49 Two cylindrical rods A and B made of different materials, are joined in a straight line. The ratios of lengths, radii and thermal conductivites of these rods are: $\dfrac{L_A}{L_B} = \dfrac{1}{2}$, $\dfrac{r_A}{r_B} = 2$ and $\dfrac{K_A}{K_B} = \dfrac{1}{2}$. The free ends of rods A and B are maintained at 400 K, 200 K respectively. The temperature of rods interface is _____K, when equilibrium is established.

Ans. **[360]**

Sol. $\dfrac{K_A \pi r_A^2 (400 - T)}{L_A} = \dfrac{K_B \pi r_B^2 (T - 200)}{L_B}$

$\dfrac{K_A}{K_B}\left(\dfrac{r_A}{r_B}\right)^2 (400 - T) = \dfrac{L_A}{L_B}(T - 200)$

$T = 360$ K

Q.50 A parallel plate capacitor has charge 5×10^{-6} C. A dielectric slab is inserted between the plates and almost fills the space between the plates. If the induced charge on one face of the slab is 4×10^{-6} C then the dielectric constant of the slab is _____.

Ans. **[5]**

Sol. $E_{net} = E_0 - E_{in}$

$E_{in} = E_0\left(1 - \dfrac{1}{k}\right)$

$Q_{in} = Q_0 = \left(1 - \dfrac{1}{k}\right)$

$4 \times 10^{-6} = 5 \times 10^{-6}\left(1 - \dfrac{1}{k}\right)$

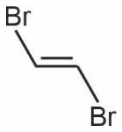
$k = 5$

CHEMISTRY

Section-A: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct..

Q.51 Given below are two statements :

Statement I :  is more polar than  .

Statement II : Boiling point of  is lower than  but it is more polar than 

In the light of the above statements, choose the **most appropriate answer** from the options given below.

(1) **Statement I** is correct but **Statement II** is incorrect

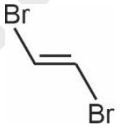
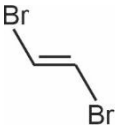
(2) **Statement I** is incorrect but **Statement II** is correct

(3) Both **Statement I** and **Statement II** are correct

(4) Both **Statement I** and **Statement II** are incorrect

Ans. **[1]**

Sol.  is more polar than  because C – Cl bond is more polar than C – Br bond.

Boiling point of  is less than that of  because of non-polar nature and hence lower intermolecular forces.  is non-polar.

Q.52 Given below are two statements :

1 M aqueous solutions of each of $Cu(NO_3)_2$, $AgNO_3$, $Hg_2(NO_3)_2$; $Mg(NO_3)_2$ are electrolysed using inert electrodes.

Given : $E^{o}_{Ag^+/Ag} = 0.80$ V, $E^{o}_{Hg_2^{2+}/Hg} = 0.79$ V,

$E^{0}_{Cu^{2+}/Cu} = 0.24$ V and $E^{0}_{Mg^{2+}/Mg} = -2.37$ V

Statement I : With increasing voltage, the sequence of deposition of metals on the cathode will be Ag, Hg and Cu.

Statement II : Magnesium will not be deposited at cathode instead oxygen gas will be evolved at the cathode.

In the light of the above statements, choose the **most appropriate answer** from the options given below.

(1) **Statement I** is incorrect but **Statement II** is correct

(2) Both **Statement I** and **Statement II** are incorrect

(3) **Statement I** is correct but **Statement II** is incorrect

(4) Both **Statement I** and **Statement II** are correct

Ans. **[3]**

Sol. Due to higher SRP, Ag will deposit first followed by Hg and Cu. Magnesium will not deposit, rather H_2O will get reduced due to higher SRP and H_2 gas will be produced

$2H_2O + 2e^- \rightleftharpoons H_2 + 2OH^-$ $E° = -0.827$ V

Q.53 Mixture of 1 g each of chlorobenzene, aniline and benzoic acid is dissolved in 50 mL ethyl acetate and placed in a separating funnel. 5 M NaOH (30 mL) was added in the same funnel. The funnel was shaken vigorously and then kept aside. The ethyl acetate layer in the funnel contains

(1) Benzoic acid

(2) Benzoic acid and aniline

(3) Chlorobenzene and aniline

(4) Benzoic acid and chlorobenzene

Ans. **[3]**

Sol. NaOH and benzoic acid will react to form salt. Aniline being a base and chlorobenzene will remain in ethyl acetate layer.

Q.54 Match List-I with List-II.

List-I		List-II	
(A)	Solution of chloroform and acetone	(i)	Minimum boiling azeotrope
(B)	Solution of ethanol and water	(ii)	Dimerizes
(C)	Solution of benzene and toluene	(iii)	Maximum boiling azeotrope
(D)	Solution of acetic acid in benzene	(iv)	$\Delta V_{mix} = 0$

Choose the **correct** answer from the options given below.

(1) (A)-(III), (B)-(IV), (C)-(I), (D)-(II)

(2) (A)-(II), (B)-(I), (C)-(IV), (D)-(III)

(3) (A)-(II), (B)-(IV), (C)-(I), (D)-(III)

(4) (A)-(III), (B)-(I), (C)-(IV), (D)-(II)

Ans. **[4]**

Sol. Acetone + chloroform shows negative deviation and form maximum boiling point azeotrope.

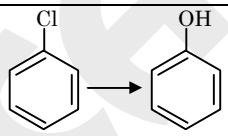
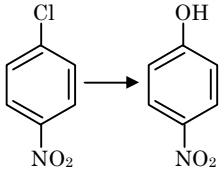
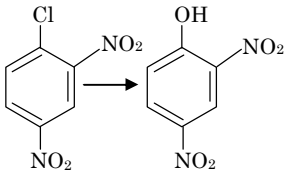
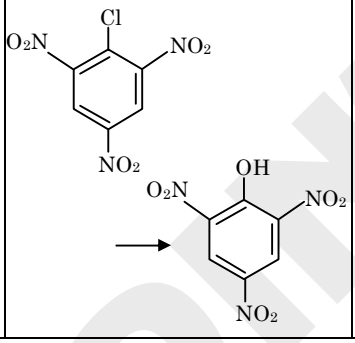
Ethanol and water shows positive deviation and form minimum boiling point azeotrope.

Benzene + toluene $\Rightarrow$ Ideal solution

Hence, $\Delta V_{mix} = 0$

Acetic acid will dimerise in benzene.

Q.55 Match List-I with List-II.

List-I (Conversion)		List-II (Reagents, Conditions used)	
(A)		(I)	Warm, H_2O
(B)		(II)	(a) NaOH, 368 K ; (b) H_3O^+
(C)		(III)	(a) NaOH, 443 K ; (b) H_3O^+
(D)		(IV)	(a) NaOH, 623 K, 300 atm ; (b) H_3O^+

Choose the **correct** answer from the options given below.

(1) (A)-(II), (B)-(III), (C)-(I), (D)-(IV)

(2) (A)-(IV), (B)-(III), (C)-(II), (D)-(I)

(3) (A)-(IV), (B)-(III), (C)-(I), (D)-(II)

(4) (A)-(III), (B)-(IV), (C)-(II), (D)-(I)

Ans. **[2]**

Sol. Presence of EWG at ortho and para-position of chlorobenzene will increase rate of aromatic nucleophilic substitution due to stable intermediate.

A-IV, B-III, C-II, D-I

Q.56 $A(g) \rightarrow B(g) + C(g)$ is a first order reaction.

Time	t	∞
P_{system}	p_t	p_∞

The reaction was started with reactant A only. Which of the following expression is correct for rate constant k ?

(1) $k = \frac{1}{t}\ln\frac{p_\infty}{p_t}$

(2) $k = \frac{1}{t}\ln\frac{p_\infty}{(p_\infty - p_t)}$

(3) $k = \frac{1}{t}\ln\frac{2(p_\infty - p_t)}{p_t}$

(4) $k = \frac{1}{t}\ln\frac{p_\infty}{2(p_\infty - p_t)}$

Ans. **[4]**

Sol.

	A(g)	⟶ B(g)	+ C(g)
t = 0	p_0	0	0
t = t	$p_0 - p$	p	p
t = ∞	0	p_0	p_0

At $t = t$ $\quad p_t = p_0 + p \Rightarrow p = p_t - p_0$

$t = \infty \quad 2p_0 = p_\infty$

For first order reaction

$$k = \frac{1}{t}\ln\frac{(p_0)A}{(p_t)A} \qquad p_0 - p = \frac{p_\infty}{2} - pt + \frac{p_\infty}{2}$$

$$= \frac{1}{t}\ln\frac{p_0}{p_0 - p} \qquad = p_\infty - pt$$

$$= \frac{1}{t}\ln\frac{p_\infty}{2(p_\infty - p_t)}$$

Q.57 "P" is an optically active compound with molecular formula $C_6H_{12}O$. When 'P' is treated with 2, 4–dinitrophenlhydrazine, it gives a positive test. However, in presence of Tollens reagent, "P" gives a negative test.

Predict the structure of "P"

(1) $CH_3 - \overset{\overset{O}{\|}}{C} - \underset{\underset{CH_2 - CH_3}{|}}{CH} - CH_3$

(2) $H - \overset{\overset{O}{\|}}{C} - CH_2 - \underset{\underset{CH_2 - CH_3}{|}}{CH} - CH_3$

(3) $CH_3 - \overset{\overset{O}{\|}}{C} - CH_2 - \underset{\underset{CH_3}{|}}{CH} - CH_3$

(4) $CH_3 - \overset{\overset{O}{\|}}{C} - CH_2 - CH_2 - CH_2 - CH_3$

Ans. **[1]**

Sol. P gives negative test with Tollen's reagent so it is not an aldehyde.

Also P is optically active so (1) is correct.

Q.58 The number of unpaired electrons responsible for the paramagnetic nature of the following complex species are respectively:

$[Fe(CN)_6]^{3-}$, $[FeF_6]^{3-}$, $[CoF_6]^{3-}$, $[Mn(CN)_6]^{3-}$

(1) 1, 5, 4, 2

(2) 1, 4, 4, 2

(3) 1, 1, 4, 2

(4) 1, 5, 5, 2

Ans. **[1]**

Sol. $[Fe(CN)_6]^{3-} \Rightarrow Fe^{3+}, d^5, t_{2g}^5 e_g^0$

$\Rightarrow$ 1 unpaired electron

$[FeF_6]^{3-} \Rightarrow Fe^{3+}, d^5, t_{2g}^3 e_g^2$

$\Rightarrow$ 5 unpaired electron

$[CoF_6]^{3-} \Rightarrow Co^{3+}, d^6, t_{2g}^4 e_g^2$

$\Rightarrow$ 4 unpaired electron

$[Mn(CN)_6]^{3-}$ $Mn^{3+}, d^4, t_{2g}^4 e_g^0$

$\Rightarrow$ 2 unpaired electrons

Q.59 The correct statements from the following are:

(A) Tl^{3+} is a powerful oxidising agent

(B) Al^{3+} does not get reduced easily

(C) Both Al^{3+} and Tl^{3+} are very stable in solution

(D) Tl^{+} is more stable than Tl^{3+}

(E) Al^{3+} and Tl^{+} are highly stable

Choose the **correct** answer from the options given below:

(1) (A), (B), (C), (D) and (E)

(2) (A), (C) and (D) only

(3) (A), (B), (D) and (E) only

(4) (B), (D) and (E) only

Ans. **[3]**

Sol. Tl^{3+} is powerful oxidising agent as Tl^{+} is more stable

Tl^{3+} is not stable in solution due to inert pair effect.

Q.60 Liquid A and B form an ideal solution. The vapour pressures of pure liquids A and B are 350 and 750 mm Hg respectively at the same temperature. If x_A and x_B are the mole fraction of A and B in solution while y_A and y_B are the mole fraction of A and B in vapour phase then,

(1) $\frac{x_A}{x_B} = \frac{y_A}{y_B}$

(2) $\frac{x_A}{x_B} < \frac{y_A}{y_B}$

(3) $\frac{x_A}{x_B} > \frac{y_A}{y_B}$

(4) $(x_A - y_A) < (x_B - y_B)$

Ans. **[3]**

Sol. $P_A^0 = 350$ mm Hg $\quad P_B^0 = 750$ mm Hg

$$y_A = \frac{350x_A}{P_T} \qquad y_B = \frac{350x_B}{P_T}$$

$$\frac{y_A}{y_B} = \frac{350x_A}{750x_B}$$

$$\Rightarrow \frac{y_A}{y_B} < \frac{x_A}{x_B}$$

Q.61 The correct statement amongst the following is

(1) $\Delta_f H^0_{298}$ is zero for O(g)

(2) $\Delta_f H^0_{500}$ is zero for O_2(g)

(3) The term 'standard state' implies that the temperature is 0°C

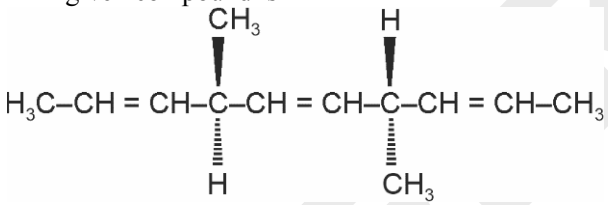
(4) The standard state of a pure gas is the pure gas at a pressure of 1 bar and temperature 273 K

Ans. **[2]**

Sol. $\Delta_f H^0_{298}$ for O_2 is zero and not for O

The term 'Standard state' does not necessarily means 0°C temperature

Q.62 The number of optically active products obtained from the complete ozonolysis of the given compound is

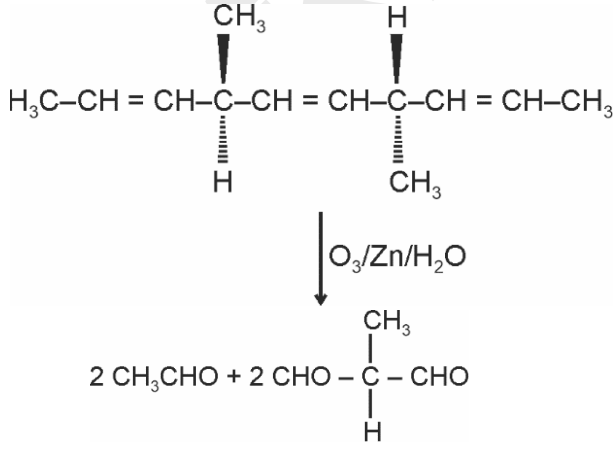


(1) 0 (2) 4

(3) 1 (4) 2

Ans. **[1]**

Sol.



None is optically active.

Q.63 In SO_2, NO_2^- and N_3^- the hybridization at the central atom are respectively

(1) sp^2, sp^2 and sp

(2) sp^2, sp^2 and sp^2

(3) sp, sp^2 and sp

(4) sp^2, sp and sp

Ans. **[1]**

Sol. $SO_2 \Rightarrow 2\sigma$ bond + 1 l.p. $\Rightarrow sp^2$ hybridisation

$NO_2^- \Rightarrow 2\sigma$ bond + 1 l.p. $\Rightarrow sp^2$ hybridisation

$N_3^- \Rightarrow 2\sigma$ bond $\Rightarrow$ sp hybidisation

Q.64 Given below are two statements

Statement I: On hydrolysis, oligo peptides give rise to fewer number of α-amino acids while proteins give rise to a large number of β-amino acids.

Statement II: Natural proteins are denatured by acids which convert the water soluble form of fibrous proteins to their water insoluble form.

In the light of the above statements, choose the **most appropriate answer** from the options given below

(1) **Statement I** is incorrect but **statement II** is correct

(2) Both **statement I** and **statement II** are incorrect

(3) **Statement I** is correct but **statement II** is incorrect

(4) Both **statement I** and **statement II** are correct

Ans. **[2]**

Sol. Protein give rise to alpha amino acid, incorrect statement.

Natural protein are denatured by acid. Due to this, globules unfold and helix get uncoiled incorrect.

Q.65 'X' is the number of acidic oxides among VO_2, V_2O_3, CrO_3, V_2O_5 and Mn_2O_7. The primary valency of cobalt in $[CO(H_2NCH_2CH_2NH_2)_3]_2(SO_4)_3$ is Y. The value of X + Y is _____.

(1) 4 (2) 3 (3) 2 (4) 5

Ans. **[4]**

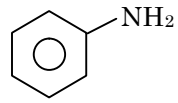
Sol. Among VO_2 , V_2O_3 V_2O_5 CrO_3 and Mn_2O_7, only CrO_3 & Mn_2O_7 is acidic X = 2
Primary valency of cobalt in $[Co(H_2NCH_2CH_2NH_2)_3]_2$ (SO_4) is 3 as cobalt have + 3 charge
Y = 3
X + Y = 5

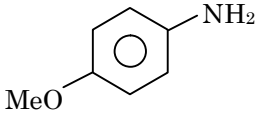
Q.66 Choose the incorrect trend in the atomic radii (r) of the elements.
(1) $r_{Br} < r_K$ (2) $r_{Rb} < r_{Cs}$
(3) $r_{Mg} < r_{Al}$ (4) $r_{At} < r_{Cs}$

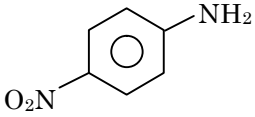
Ans. **[3]**

Sol. $r_{Mg} > r_{Al}$ due to lower effective nuclear change.

Q.67 The descending order of basicity of following amines is :

(A) 

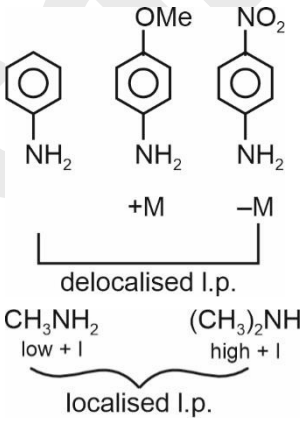
(B) 

(C) 

(D) CH_3NH_2
(E) $(CH_3)_2$ NH
Choose the **correct** answer from the options given below:
(1) B > E > D > A > C
(2) E > D > B > A > C
(3) E > D > A > B > C
(4) E > A > D > C > B

Ans. **[2]**

Sol.



E > D > B > A > C

Q.68 The extra stability of half-filled subshell is due to :
(A) Symmetrical distribution of electrons
(B) Smaller coulombic repulsion energy
(C) The presence of electrons with the same spin in non-degenerate orbitals
(D) Larger exchange energy
(E) Relatively smaller shielding of electrons by one another
Identify the **correct** statements:
(1) (B), (D) and (E) only
(2) (A), (B), (D) and (E) only
(3) (A), (B) and (D) only
(4) (B), (C) and (D) only

Ans. **[2]**

Sol. Extra stability of half filled subshell is due to symmetrical distribution of electrons, smaller coulombic repulsion, larger exchange energy and smaller shielding of electrons.

Q.69 The hydration energies of K^+ and Cl^- are –x and –y kJ/mol respectively. If lattice energy of KCl is –z kJ/mol, then the heat of solution of KCl is : –
(1) x + y + z (2) – z – (x + y)
(3) + x – y – z (4) z – (x + y)

Ans. **[4]**

Sol. $\Delta H_{solution} = (-\Delta H_{L.E.}) + \Delta H_{hydration}$
$= -(-z) + (-x - y)$
$= z - (x + y)$

Q.70 Match List-I with List-II.

	List-I Complex		**List-II Primary valency and Secondary valency**	
(A)	$[Co(en)_2Cl_2]Cl$	(I)	3	6
(B)	$[Pt(NH_3)_2Cl(NO_2)]$	(II)	3	4
(C)	$Hg[CO(SCN)_4]$	(III)	2	6
(D)	$[Mg(EDTA)]^{2-}$	(IV)	2	4

Choose the **correct** answer from the options given below :
(1) (A)-(I), (B)-(IV), (C)-(II), (D)-(III)
(2) (A)-(III), (B)-(I), (C)-(II), (D)-(IV)
(3) (A)-(I), (B)-(III), (C)-(II), (D)-(IV)
(4) (A)-(II), (B)-(III), (C)-(IV), (D)-(I)

Ans. **[1]**

Sol.

		P.V	S.V
(A)	$[Co(en)_2Cl_2]Cl$	3	6
(B)	$[Pt(NH_3)_2Cl(NO_2)]$	2	4
(C)	$Hg[CO(SCN)_4]$	3	4
(D)	$[Mg(EDTA)]^{2-}$	2	6

(A)-(I), (B)-(IV), (C)-(II), (D)-(III)

Section-B: Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

Q.71 Butane reacts with oxygen to produce carbon dioxide and water following the equation given below.

$$C_4H_{10(g)} + \frac{13}{2}O_2(g) \rightarrow 4CO_2(g) + 5H_2O(l)$$

If 174.0 kg of butane is mixed with 320.0 kg of O_2, the volume of water formed in liters is ____.

(Nearest integer)

[Given : (a) Molar mass of C, H, O are 12, 1, 16g mol^{-1} respectively, (b) Density of water = 1 g mL^{-1}]

Ans. **[138]**

Sol. $C_4H_{10}(g) + \frac{13}{2}O_2 \longrightarrow 4CO_2(g) + 5H_2O(l)$

Moles of $C_4H_{10} = \frac{174kg}{58g} = 3 \times 10^3$ moles

Moles of $O_2 = \frac{320kg}{32g}$

$= 10 \times 10^3$ moles

1 mole $C_4H_{10} \Rightarrow \frac{13}{2}$ moles O_2

3×10^3 moles $C_4H_{10} \Rightarrow \frac{13}{2} \times 3 \times 10^3$ mole O_2

O_2 is limiting reagent

Moles of H_2O formed $= \frac{10}{13} \times 5 \times 2 \times 10^3$

Volume of H_2O formed $= \frac{10}{13} \times 5 \times 2 \times 18$ L

$= 138.46$ L

Q.72 One litre buffer solution was prepared by adding 0.10 mol each of NH_3 and NH_4Cl in deionised water.

The change in pH on addition of 0.05 mol of HCl to the above solution is _____ $\times 10^{-2}$. (Nearest integer)

Given : pK_b of $NH_3 = 4.745$ and $\log_{10}3 = 0.477$

Ans. **[48]**

Sol. Initially

$$pOH = pK_b + \log\frac{[NH_4Cl]}{[NH_3]} = pK_b + \log\frac{0.1}{0.1}$$

$pOH = pK_b$

$pH = 14 - pOH \Rightarrow pH = 9.255$

When 0.05 mol HCl is added

$NH_3 + H^+ \rightleftharpoons NH_4^+$

0.1 0.05

0.05 0 0.15

$$pOH = pK_b + \log\frac{0.15}{0.05} = 5.222$$

$pH = 8.778$

Change in pH = 0.477 or 47.7×10^{-2}

Q.73 The number of paramagnetic metal complex species among $[Co(NH_3)_6]^{3+}$, $[Co(C_2O_4)_3]^{3-}$, $[MnCl_6]^{3-}$, $[Mn(CN_6)]^{3-}$, $[CoF_6)^{3-}$, $[Fe(CN)_6]^{3-}$ and $[FeF_6]^{3-}$ with same number of unpaired electrons is_____.

Ans. **[2]**

Sol. $[Co(NH_3)_6]^{3+} \Rightarrow Co^{3+}, d^6, t_{2g}^6e_g^0$

$\Rightarrow$ diamagnetic

$[Co(C_2O_4)_3]^{3-} \Rightarrow Co^{3+}, d^6, t_{2g}^6e_g^0$

$\Rightarrow$ diamagnetic

$[MnCl_6]^{3-} \Rightarrow Mn^{3+}, d^4, t_{2g}^3e_g^1$

$\Rightarrow$ 4 unpaired electron, paramagnetic

$[CoF_6]^{3-} \Rightarrow Co^{3+}, d^6, t_{2g}^4e_g^2$

$\Rightarrow$ 4 unpaired electron, paramagnetic

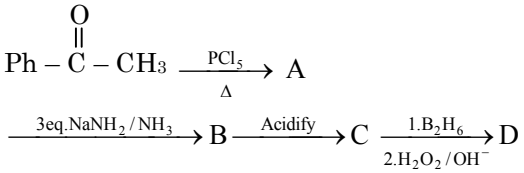
$[Fe(CN)_6]^{3-} \Rightarrow Fe^{3+}, d^5, t_{2g}^5e_g^0$

$\Rightarrow$ 1 unpaired electron, paramagnetic

$[FeF_6]^{3-} \Rightarrow Fe^{3+}, d^5, t_{2g}^3e_g^2$

$\Rightarrow$ 5 unpaired electron

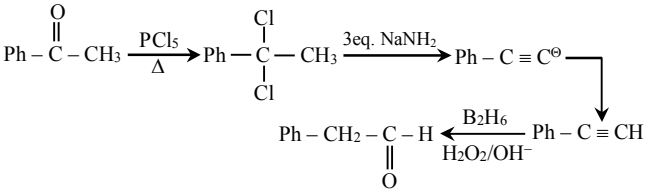
Q.74 Identify the structure of the final product (D) in the following sequence of the reactions :



Total number of sp^2 hybridised carbon atom in product D is _____.

Ans. **[7]**

Sol.



Number of sp^2 hybridised carbon = 7

Q.75 In Dumas' method 292 mg of an organic compound released 50 mL of nitrogen gas (N_2) at 300 K temperature and 715 mm Hg pressure. The percentage composition of 'N' in the organic compound is _____% (Nearest integer)
(Aqueous tension at 300 K = 15 mm Hg)

Ans. **[18]**

Sol. V_{N_2} at STP $= \dfrac{273\times(715-15)\times 50}{300\times 760}$

$= 41.9$ mL

$M_{N_2} = \dfrac{41.9}{22400} \times 28 = 0.052$ g

$\%\ N = \dfrac{0.052g}{0.292} \times 100 = 17.94\ \%$