CAREER POINT

JEE Main Online Exam 2025

Questions & Solution

04th April 2025 | Morning

MATHEMATICS

Section-A: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct..

Choose the correct answer:

Q.1 $1 + 3 + 5^2 + 7 + 9^2 + , \ldots$ upto 40 terms is equal to

(1) 33980 (2) 41880

(3) 40870 (4) 43890

Ans. [2]

Sol. $1 + 3 + 5^2 + 7 + 9^2 + \ldots$ upto 40 terms

$(1^2 + 5^2 + 9^2 + \ldots) + (3 + 7 + 11 + \ldots)$

$= \left(\sum_{k=1}^{20}(4k-3)^2\right) + \frac{20}{2}[6 + (20-1)4]$

$= 16\sum_{k=1}^{20}k^2 - 24\sum_{k=1}^{20}k + 9 \times 20 + 10[82]$

$= 16\left(\frac{20\times21\times41}{6}\right) - 24\left(\frac{20\times21}{2}\right) + 1000$

$= 45920 - 5040 + 1000$

$= 41880$

Q.2 Considering the principal values of the inverse trigonometric functions, $\sin^{-1}\left(\frac{\sqrt{3}}{2}x + \frac{1}{2}\sqrt{1-x^2}\right)$, $-\frac{1}{2} < x < \frac{1}{\sqrt{2}}$, is equal to

(1) $\frac{\pi}{4} + \sin^{-1}x$ (2) $\frac{\pi}{6} + \sin^{-1}x$

(3) $\frac{5\pi}{6} - \sin^{-1}x$ (4) $\frac{-5\pi}{6} - \sin^{-1}x$

Ans. [2]

Sol. $\sin^{-1}\left(\frac{\sqrt{3}}{2}x + \frac{1}{2}\sqrt{1-x^2}\right), -\frac{1}{2} < x < \frac{1}{\sqrt{2}}$

Let $x = \cos\theta$, $\theta \in \left(\frac{\pi}{4}, \frac{2\pi}{3}\right)$

$\Rightarrow \sqrt{1-x^2} = \sin\theta$ as $\sin\theta > 0$

$\sin^{-1}\left(\frac{\sqrt{3}}{2}\cos\theta + \frac{1}{2}\sin\theta\right)$

$= \sin^{-1}\left(\sin\left(\frac{\pi}{3}+\theta\right)\right)$ $\quad \frac{\pi}{3}+\theta \in \left(\frac{7\pi}{2}, \pi\right)$

$= \sin^{-1}\left(\sin\left(\pi - \left(\frac{\pi}{3}+\theta\right)\right)\right)$

$= \sin^{-1}\left(\sin\left(\frac{2\pi}{3}-\theta\right)\right)$

$= \frac{2\pi}{3} - \theta$

$= \frac{2\pi}{3} - \cos^{-1}x$

$= \frac{2\pi}{3} - \left(\frac{\pi}{2} - \sin^{-1}x\right)$

$= \frac{\pi}{6} - \sin^{-1}x$

Q.3 Let A = {1, 6, 11, 16, …} and B {9, 16, 23, 30, …} be the sets consisting of the first 2025 terms of two arithmetic progressions. Then n $(A \cup B)$ is

(1) 4003 (2) 3814

(3) 4027 (4) 3761

Ans. [4]

Sol. 1st A.P. : 1, 6, 11… $\Rightarrow T_n = Sn - 4$

2nd A.P.: 9, 16, 23… $\Rightarrow T_m = 2 + 7m$

Let's find when they are equal for the first time:

$5n - 4 = 2 + 7m$

$\Rightarrow 5n - 7m = 6$

$\Rightarrow n = 4, m = 2$

$\Rightarrow$ 16 is the first term, common difference will be

LCM (d_1, d_2) = LCM (5, 7) = 35

$\Rightarrow$ Common terms will be 16, 51, 86 …

The last term of 1st A.P.

$= T_{2025} = 5 \times 2025 - 4 = 10121$

$\Rightarrow$ Common term must be less than that

$\Rightarrow 35n - 19$

$\Rightarrow 35n - 19 \le 10121 \Rightarrow 35n \le 10140$

$\Rightarrow n \le 289.7$

$\Rightarrow \boxed{n = 289}$

$\Rightarrow$ in $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$= 2025 + 2025 - 289$

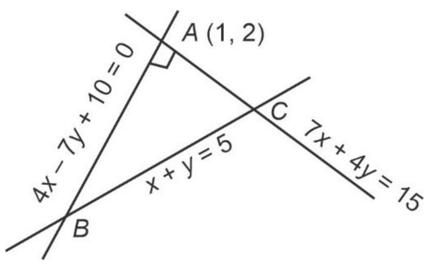
$= 3761$

Q.4 Let the three sides of a triangle are on the lines $4x - 7y + 10 = 0$, $x + y = 5$ and $7x + 4y = 15$. Then the distance of its orthocentre from the orthocenter of the triangle formed by the lines $x = 0$, $y = 0$ and $x + y = 1$ is

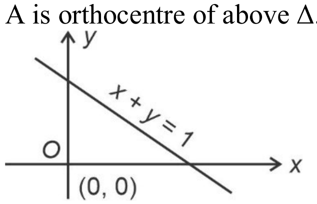
(1) $\sqrt{20}$ (2) $\sqrt{5}$ (3) 5 (4) 20

Ans. [2]

Sol.



A is orthocentre of above Δ.



O is orthocentre of above Δ.

$OA = \sqrt{5}$

Q.5 Consider two vectors $\vec{u} = 3\hat{i} - \hat{j}$ and $\vec{v} = 2\hat{i} + \hat{j} - \lambda\hat{k}$, $\lambda > 0$. The angle between them is given by $\cos^{-1}\left(\frac{\sqrt{5}}{2\sqrt{7}}\right)$. Let $\vec{v} = \vec{v}_1 + \vec{v}_2$, where $\vec{v}_1$ is parallel to $\vec{u}$ and $\vec{v}_2$ is perpendicular to $\vec{u}$. Then the value $|\vec{v}_1|^2 + |\vec{v}_2|^2$ is equal to

(1) $\frac{23}{2}$ (2) $\frac{25}{2}$ (3) 14 (4) 10

Ans. [3]

Sol. $\vec{u}.\vec{v} = |u|.|v|.\cos\theta$

$\Rightarrow 6 - 1 = \sqrt{10}\,.\,\sqrt{5+\lambda^2}\,.\,\frac{\sqrt{5}}{2\sqrt{7}}$

$\Rightarrow 1 = \sqrt{2}\,.\sqrt{5+\lambda^2}\,.\,\frac{1}{2\sqrt{7}}$

$\Rightarrow 14 = 5 + \lambda^2$

$\Rightarrow \lambda^2 = 9$

$\Rightarrow \lambda = 3$

$v_1 = k\ \vec{u}$

$\vec{v} = \vec{v}_1 + \vec{v}_2$

$\Rightarrow \vec{v} = k\vec{u} + \vec{v}_2$

$\vec{v}.\vec{u} = k.|\vec{u}|^2$

$\Rightarrow 5 = k.10 \Rightarrow k = \frac{1}{2}$

$\therefore \vec{v}_1 = \frac{\vec{u}}{2} = \frac{3\hat{i}}{2} - \frac{\hat{j}}{2}$

$|\vec{v}_1|^2 = \frac{10}{4}$

$\vec{v}_2 = \vec{v} - \vec{v}_1$

$= \frac{1}{2}\hat{i} + \frac{3\hat{j}}{2} - 3\hat{k}$

$|\vec{v}_2|^2 = \frac{10}{4} + 9$

$|\vec{v}_1|^2 + |\vec{v}_2|^2 = \frac{10}{4} + \frac{10}{4} + 9 = 14$

Q.6 Let $f, g : (1, \infty) \to \mathbb{R}$ be defined as $f(x) = \frac{2x+3}{5x+2}$ and $g(x) = \frac{2-3x}{1-x}$. If the range of the function $fog : [2, 4] \to \mathbb{R}$ is $[\alpha, \beta]$, then $\frac{1}{\beta - \alpha}$ is equal to

(1) 68 (2) 29 (3) 2 (4) 56

Ans. [4]

Sol. $g(2) = 4$, $g(4) = \frac{10}{3}$

f is decreasing in $\left(\frac{10}{3}, 4\right)$

$\therefore \alpha = f(4) = \frac{1}{2}$

$\beta = f\left(\frac{10}{3}\right) = \frac{29}{56}$

$\frac{1}{\beta - \alpha} = \frac{1}{\frac{29}{56} - \frac{1}{2}} = 56$

Q.7 Let $f : [0, \infty) \to \mathbb{R}$ be a differentiable function such that $f(x) = 1 - 2x + \int_0^x e^{x-t} f(t)dt$ for all $x \in [0, \infty)$. Then the area of the region bounded by $y = f(x)$ and the coordinate axes is

(1) $\sqrt{2}$ (2) 2 (3) $\frac{1}{2}$ (4) $\sqrt{5}$

Ans. [3]

Sol. $\because$ $f(x) = 1 - 2x + \int_0^x e^{x-t} f(t)dt$

or $f(x) = 1 - 2x + e^x \int_0^x e^{-t} f(t)dt$

on differentiating both sides w.r.t. x we get

$f'(x) = -2 + e^x \int_0^x e^{-t} f(t)dt + e^x.e^{-x} f(x)$

$f'(x) = -2 + f(x) + 2x - 1 + f(x)$ {from eq. (1)}

$\therefore$ $f'(x) - 2f(x) = 2x - 3$

I.F. = $e^{\int -2dx} = e^{-2x}$

$\therefore e^{-2x} . f(x) = \int e^{-2x} (2x - 3)dx$

$e^{-2x} . f(x) = (2x - 3) . \frac{e^{-2x}}{-2} - \int 2.\frac{e^{-2x}}{-2} dx$

$x^{-2x} .f(x) = \frac{(2x-3)e^{-2x}}{-2} + \frac{e^{-2x}}{-2} + c$

$f(x) = -x + 1 + c'e^{2x}$

$\because$ $f(x) = 1$ from eq. (1)

$\therefore 1 = 0 + 1 + c' \Rightarrow c' = 0$

$\therefore f(x) = -x + 1 \Rightarrow$ Area $= \frac{1}{2}$

Q.8 In the expansion of $\left(\sqrt[3]{2} + \frac{1}{\sqrt[3]{3}}\right)^n$, $n \in \mathbb{N}$, if the ratio of 15^{th} term from the beginning to the 15^{th} term from the end is $\frac{1}{6}$, then the value of ${}^nC_3$ is

(1) 4960 (2) 4060 (3) 2300 (4) 1040

Ans. [3]

Sol. In the expansion of $(a + b)^n$

15th term from beginning : $T_{15} = {}^nC_{14} a^{n-14} b^{14}$

15th term from end : $T'_{15} = {}^nC_{14} b^{n-14} a^{14}$

$\therefore \frac{T_{15}}{T'_{15}} = \frac{1}{6}$

$\left(\frac{a}{b}\right)^{n-28} = \frac{1}{6}$

$\left(6^{\frac{1}{3}}\right)^{n-28} = 6^{-1} \Rightarrow \frac{n-28}{3} = -1$

$n = 25$

$\therefore {}^{25}C_3 = 2300$

Q.9 The probability, of forming a 12 persons committee from 4 engineers, 2 doctors and 10 professors containing at least 3 engineers and at least 1 doctor, is

(1) $\frac{103}{182}$ (2) $\frac{19}{26}$ (3) $\frac{17}{26}$ (4) $\frac{129}{182}$

Ans. [4]

Sol.

3E,	1D,	8P
3E,	2D,	7P
4E,	1D,	7P
4E,	2D,	6P

$P = \frac{{}^4C_3.{}^2C_1.{}^{10}C_8 + {}^4C_3.{}^2C_2.{}^{10}C_7 + {}^4C_4.{}^2C_1.{}^{10}C_7 + {}^4C_4.{}^2C_2.{}^{10}C_6}{{}^{10}C_{12}}$

$= \frac{129}{182}$

Q.10 If $10 \sin^4\theta + 15 \cos^4\theta = 6$, then the value of $\frac{27 \operatorname{cosec}^6 \theta + 8 \sec^6 \theta}{16 \sec^8 \theta}$ is

(1) $\frac{2}{5}$ (2) $\frac{1}{5}$ (3) $\frac{3}{5}$ (4) $\frac{3}{4}$

Ans. [1]

Sol. $10 \sin^4\theta + 15 \cos^4\theta = 6$

$\Rightarrow 10 \sin^4\theta + 10 \cos^4\theta + 5 \cos^4\theta = 6$

$\Rightarrow 10 [(\sin^2\theta + \cos^2\theta)^2 - 2 \sin^2\theta \cos^2\theta] + 5 \cos^4\theta = 6$

$\Rightarrow 10 - 20(1 - \cos^2\theta) \cos^2\theta + 5 \cos^4\theta = 6$

Let $\cos^2\theta = x$

$10 - 20 (x - x^2) + 5x^2 = 6$

$\Rightarrow 25x^2 - 20x + 4 = 0$

$(5x - 2)^2 = 0 \Rightarrow x = \frac{2}{5}$

$\Rightarrow \cos^2 \theta = \frac{2}{5} \Rightarrow \sin^2 \theta = \frac{3}{5}$,

$\sec^2\theta = \frac{5}{2}$, $\operatorname{cosec}^2\theta = \frac{5}{2}$

$\frac{27 \operatorname{cosec}^6\theta + 8 \sec^6 \theta}{16 \sec^8 \theta} = \frac{27\left(\frac{5}{3}\right)^3 + 8\left(\frac{5}{2}\right)^3}{16\left(\frac{5}{2}\right)^4}$

$= \frac{5^3 + 5^3}{5^4} = \frac{2.5^3}{5^4} = \frac{2}{5}$

Q.11 Consider the equation $x^2 + 4x - n = 0$ where $n \in [20, 100]$ is a natural number. Then the number of all distinct values of n, for which the given equation has integral roots, is equal to

(1) 8
(2) 6
(3) 7
(4) 5

Ans. [2]

Sol. $x^2 + 4x - n = 0$ has integer roots

$\Rightarrow x = \frac{-4 \pm \sqrt{16 + 4n}}{2} = -2 \pm \sqrt{4 + n}$

For x to be integer 4 + n must be perfect squares

$n \in [20, 100]$

$n + 4 \in [24, 104] = S$

$\{25, 36, \ldots 10^2\} \in S \Rightarrow 5^2, 6^2, \ldots 10^2 \Rightarrow 6$ values of n

Q.12 A box contains 10 pens of which 3 are defective. A sample of 2 pens is drawn at random and let X denote the number of defective pens. Then the variance of X is

(1) $\frac{11}{15}$ (2) $\frac{2}{15}$ (3) $\frac{28}{75}$ (4) $\frac{3}{5}$

Ans. [3]

Sol.

X	P(X)	XP(X)	$(X_i - \mu)^2$	$P_iX(X_i - \mu)^2$
X = 0	$\frac{^7C_2}{^{10}C_2}$	0	$\left(0-\frac{3}{5}\right)^2$	$\frac{7}{15}\left(\frac{9}{25}\right)$
X = 1	$\frac{^7C_1\,^3C_1}{^{10}C_2}$	$\frac{7}{15}$	$\left(1-\frac{3}{5}\right)^2$	$\frac{7}{15}\left(\frac{4}{25}\right)$
X = 2	$\frac{^7C_0\,^3C_2}{^{10}C_2}$	$\frac{2}{15}$	$\left(2-\frac{3}{5}\right)^2$	$\frac{2}{15}\left(\frac{49}{25}\right)$

$$\mu = \sum X_i P(X_i) = 0 + \frac{7}{15} + \frac{2}{15} = \frac{3}{5}$$

$$\text{Variance (X)} = \sum p_i (X_i - \mu)^2$$

$$= \frac{7}{15}\left(\frac{9}{25}\right) + \frac{7}{15}\left(\frac{4}{25}\right) + \frac{2}{15}\left(\frac{49}{25}\right) = \frac{28}{75}$$

Q.13 The value of

$$\int_{-1}^{1} \frac{(1+\sqrt{|x|-x})e^x + (\sqrt{|x|-x})e^{-x}}{e^x + e^{-x}} dx \text{ is equal to}$$

(1) $2 + \frac{2\sqrt{2}}{3}$ (2) $1 + \frac{2\sqrt{2}}{3}$

(3) $1 - \frac{2\sqrt{2}}{3}$ (4) $3 - \frac{2\sqrt{2}}{3}$

Ans. [2]

Sol.
$$I = \int_{-1}^{1} \frac{(1+\sqrt{|x|-x})e^x + (\sqrt{|x|-x})e^{-x}}{x^e + e^{-x}} dx$$

$$= \int_0^1 \left(\frac{(1+\sqrt{|x|-x})e^x + (\sqrt{|x|-x})e^{-x}}{e^x + e^{-x}} + \left(\frac{(1+\sqrt{|x|-x})e^{-x} + (\sqrt{|x|-x})e^{-x}}{e^{-x} + e^x} \right) dx \right.$$

$$= \int_0^1 \frac{(1+\sqrt{|x|-x} + \sqrt{|x|+x}(e^x + e^{-x})}{e^x + e^{-x}} dx$$

$$= \int_0^1 (1+\sqrt{|x|-x} + \sqrt{|x|+x}\, dx$$

$$= \int_0^1 (1+\sqrt{2x})dx$$

$$= x\Big|_0^1 + \frac{\sqrt{2}x^{\frac{3}{2}}}{\frac{3}{2}}\Bigg|_0^1$$

$$= 1 + \frac{2\sqrt{2}}{3}$$

Q.14 If $\lim_{x \to 1^+} \frac{(x-1)(6+\lambda\cos(x-1)) + \mu\sin(1-x)}{(x-1)^3} = -1$,

where $\lambda, \mu \in \mathbb{R}$, then $\lambda + \mu$ is equal to

(1) 20 (2) 18 (3) 19 (4) 17

Ans. [2]

Sol. $\lim_{x \to 1^+} \frac{(x-1)(6+\lambda\cos(x-1)) + \mu\sin(1-x)}{(x-1)^3} = -1$,

Let $x - 1 = t$

$$\lim_{t \to 0^+} \frac{6t + \lambda t\cos t - \mu\sin t}{t^3} = -1$$

$$= \lim_{t \to 0^+} \frac{6t + \lambda t\left(1 - \frac{t^2}{2!} + \frac{t^4}{4!} + \ldots\ldots\right) - \mu\left(t - \frac{t^3}{3!} + \ldots\ldots\right)}{t^3} = -1$$

$$= \lim_{t \to 0^+} \frac{t(6+\lambda-\mu) + t^3\left(-\frac{\lambda}{2} + \frac{\mu}{6}\right) + \ldots\ldots}{t^3} = -1$$

$\therefore \quad \lambda - \mu + 6 = 0$ …..(i)

$\frac{\mu}{6} - \frac{\lambda}{2} = -1$ …..(ii)

Solving (i) and (ii)

$\lambda = 6, \mu = 12$

$\lambda + \mu = 18$

Q.15 For an integer $n \geq 2$, if the arithmetic mean of all coefficients in the binomial expansion of $(x + y)^{2n-3}$ is 16, then the distance of the point $P(2n - 1, n^2 - 4n)$ from the line $x + y = 8$ is

(1) $2\sqrt{2}$ (2) $\sqrt{2}$ (3) $5\sqrt{2}$ (4) $3\sqrt{2}$

Ans. [4]

Sol. Mean =

$$\frac{{}^{2n-3}C_0 + {}^{2n-3}C_1 + {}^{2n-3}C_2 + ... {}^{2n-3}C_{2n-3}}{2n-2} = 16$$

$= 2^{2n-3} = 16(2n-2)$

$= 2^{2n-3} = 2^5(n-1)$

$\Rightarrow n = 5$

$\therefore P(9, 5)$

$d = \left|\frac{9+5-8}{\sqrt{2}}\right| = \frac{6}{\sqrt{2}} = 3\sqrt{2}$

Q.16 Find the length of latus rectum of an ellipse, whose foci are (2, 5) and (2, –3) and the eccentricity of the ellipse is $\frac{4}{5}$

(1) $\frac{32}{3}$ (2) $\frac{32}{5}$ (3) $\frac{18}{5}$ (4) $\frac{16}{5}$

Ans. **[3]**

Sol. F_1 : (2, 5) and F_2 : (2, –3), notice major axis along y-axis

$\Rightarrow F_1F_2 = 8 = 2be$

$\Rightarrow b = \frac{8}{2e} = \frac{4}{4/5} = 5$

$\Rightarrow e^2 = 1 - \frac{a^2}{b^2} = 1 - \frac{a^2}{25} = \frac{16}{25}$

$\Rightarrow a^2 = 9$

$\Rightarrow a = 3$

The length of latus rectum :

$\frac{2a^2}{b} = \frac{2(9)}{5} = \frac{18}{5}$

Q.17 Let the shortest distance between the lines $\frac{x-3}{3} = \frac{y-\alpha}{-1} = \frac{z-3}{1}$ and $\frac{x+3}{-3} = \frac{y+7}{2} = \frac{z-\beta}{4}$ be $3\sqrt{30}$. Then the positive value of $5\alpha + \beta$ is

(1) 42 (2) 40 (3) 46 (4) 48

Ans. **[3]**

Sol. $L_1 : \frac{x-3}{3} = \frac{y-\alpha}{-1} = \frac{z-3}{1}$

$L_2 : \frac{x+3}{-3} = \frac{y+7}{2} = \frac{z-\beta}{4}$

$a_1 : (3, \alpha, 3)$

$a_2 = (-3, -7, \beta)$

$\vec{b}_1 = 3\hat{i} - \hat{j} + \hat{k}$

$\vec{b}_2 = -3\hat{i} - 2\hat{j} + 4\hat{k}$

$d = \frac{|(\vec{a}_2 - \vec{a}_1).(\vec{b}_1 \times \vec{b}_2)|}{|\vec{b}_1 \times \vec{b}_2|} = 3\sqrt{30}$

$$= \frac{\begin{vmatrix} 6 & \alpha+7 & 3-\beta \\ 3 & -1 & 1 \\ -3 & 2 & 4 \end{vmatrix}}{\sqrt{6^2+15^2+3^2}} = 3\sqrt{30}$$

$\Rightarrow |-15\alpha - 3\beta - 132| = 270$

$|5\alpha + \beta + 44| = 90$

$\Rightarrow 5\alpha + \beta = 90 - 44 = 46$

Q.18 Let A and B be two distinct points on the line L : $\frac{x-6}{3} = \frac{y-7}{2} = \frac{z-7}{-2}$. Both A and B are at a distance $2\sqrt{17}$ from the foot of perpendicular drawn from the point (1, 2, 3) on the line L. If O is then $\overrightarrow{OA}.\overrightarrow{OB}$ is equal to –

(1) 62 (2) 47 (3) 21 (4) 49

Ans. **[2]**

Sol. L : $\frac{x-6}{3} = \frac{y-7}{2} = \frac{z-7}{-2}$

Point $A(3\lambda + 6, 2\lambda + 7, 7 - 2\lambda)$

$B(3\mu + 6, 2\mu + 7, 7 - 2\mu)$

Let $P(3k + 6, 2k + 7, 7 - 2k)$ be foot of perpendicular from $P'(1, 2, 3)$

$\therefore PP' < 3, 2, -2 > = 0$

$3(3k + 5) + (2k + 5)2 + (4 - 2k)(-2) = 0$

$9k + 15 + 4k + 10 - 8 + 4k = 0$

$17k + 17 = 0$

$\Rightarrow k = -1$ $\therefore P(3, 5, 9)$

$|\overrightarrow{AP}| = 2\sqrt{17}$

$\Rightarrow (3\lambda + 3)^2 + (2\lambda + 2)^2 + (-2 - 2\lambda)^2 = 17 \times 4$

$= 17(\lambda + 1)^2 = 17 \times 4$

$\Rightarrow \lambda + 1 = \pm 2$

$\Rightarrow \lambda = 1$ or $\lambda = -3$

$\therefore A(9, 9, 5), B(-3, 1, 13)$

$\overrightarrow{OA}.\overrightarrow{OB} = (9\hat{i} + 9\hat{j} + 5\hat{k}).(-3\hat{i} + \hat{j} + 13\hat{k})$

$= -27 + 9 + 65 = 47$

Q.19 Consider the sets $A = \{(x, y) \in \mathbb{R} \times \mathbb{R} : x^2 + y^2 = 25\}$, $B = \{(x, y) \in \mathbb{R} \times \mathbb{R} : x^2 + 9y^2 = 144\}$, $C = \{(x, y) \in \mathbb{Z} \times \mathbb{Z} : x^2 + y^2 \le 4\}$ and $D = A \cap B$. The total number of one-one functions from the set D to the set C is :

(1) 18290 (2) 15120 (3) 17160 (4) 19320

Ans. **[3]**

Sol. $A = \{(x, y) \in R \times R: x^2 + y^2 = 25\}$, $B = \{(x, y) \in \mathbb{R} \times \mathbb{R} : x^2 + 9y^2 = 144\}$

$x^2 + 9y^2 - (x^2 + y^2) = 144 - 25$

Plug in $y^2 = \frac{119}{8}$ into either equation to find x.

$$x^2 = 25 - \frac{119}{8}$$

$$x^2 = \frac{200-119}{8}$$

$$x^2 = \frac{81}{8}$$

$$x = \pm\sqrt{\frac{81}{8}},\ y = \pm\sqrt{\frac{119}{8}}$$

Now, $C = \{(x, y) \in \mathbb{Z} \times \mathbb{Z} : x^2 + y^2 \le 4\}$

Valid points are (–2, 0), (–1, –1), (–1, 0), (–1, 1), (0, –2), (0, –1), (0, 0), (0, 1), (0, 2), (1, –1), (1, 0), (1, 1)

∴ Total valid points in C = 13

⇒ There are 4 distinct real points in set D

∴ The number of one-one functions from D to C

$$\Rightarrow {}^{13}P_4 \Rightarrow \frac{13!}{(13-4)!} = \frac{13!}{9!} = 17160$$

Q.20 Let $f : \mathbb{R} \to \mathbb{R}$ be a continuous function satisfying f(0) = 1 and f(2x) – f(x) = x for all $x \in \mathbb{R}$.

If $\lim_{n\to\infty}\left\{f(x) - f\left(\frac{x}{2^n}\right)\right\} = G(x)$, then $\sum_{r=1}^{10} G(r^2)$ is equal to

(1) 385 (2) 420 (3) 215 (4) 540

Ans. [1]

Sol. f(0) = 1, f(2x) – f(x) = x

Replace $x \to \frac{x}{2}$

$$f(x) - f\left(\frac{x}{2}\right) = \frac{x}{2} \quad .....(i)$$

Again, Replace $x \to \frac{x}{2}$

$$f\left(\frac{x}{2}\right) - f\left(\frac{x}{2^2}\right) = \frac{x}{2^2} \quad ...(2)$$

⋮ ⋮ ⋮

$$f\left(\frac{x}{2^{n-1}}\right) - f\left(\frac{x}{2^n}\right) = \frac{x}{2^n} \quad ....(n)$$

Adding (1) + (2) + (3) + ….+(n)

We get $f(x) - f\left(\frac{x}{2^n}\right) = \frac{x}{2} + \frac{x}{2^2} + ... + \frac{x}{2^n}$

$$\lim_{n\to\infty}\left(f(x) - f\left(\frac{x}{2^n}\right)\right) = \lim_{n\to\infty}\left(\frac{x}{2} + \frac{x}{2^n} + ... + \frac{x}{2^n}\right)$$

$$f(x) - f(0) = \frac{\frac{x}{2}}{\frac{1}{2}}$$

⇒ G(x) = x

⇒ $G(r^2) = r^2$

$$\Rightarrow \sum_{r=1}^{10} G(r^2) = \sum_{r=1}^{10} G(r^2)$$

$$= \frac{(10)(11)(21)}{6} = (55)7$$

⇒ 385

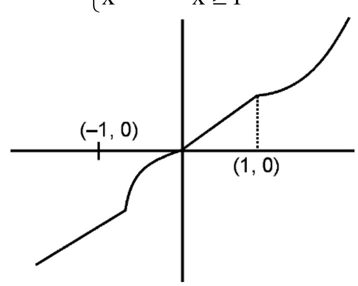
Section-B: Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

Q.21 Let m and n be the number of points at which the function $f(x) = \max\{x, x^3, x^5, \ldots x^{21}\}$, $x \in \mathbb{R}$, is not differentiable and not continuous, respectively. Then m + n is equal to ________

Ans. [3]

Sol. for $x \ge 1$, $x^{21} \ge x^{19} \ge \ldots \ge x$.

$$f(x) = \begin{cases} x & x < -1 \\ x^{21} & -1 \le x \le 0 \\ x & 0 < x < 1 \\ x^{21} & x \ge 1 \end{cases}$$



Clearly, f(x) is continuous everywhere.

⇒ n = 0

$$f'(x) = \begin{cases} 1\ ; & x < -1 \\ 21x^{20}\ ; & -1 \le x \le 0 \\ 1\ ; & 0 < x < 1 \\ 21.x^{20}\ ; & x \ge 1 \end{cases}$$

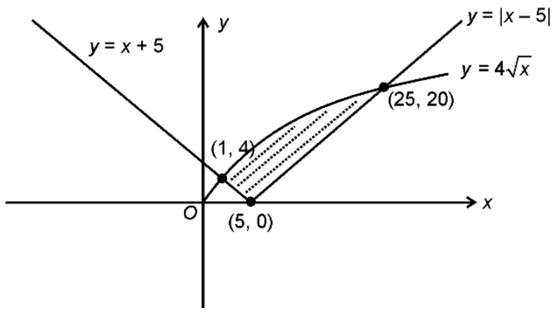
⇒ m = 3

⇒ m + n = 3

Q.22 If the area of the region $\{(x, y) : |x - 5| \le y \le 4\sqrt{x}\}$ is A, then 3A is equal to ________

Ans. **[368]**

Sol.



Area = $\int_1^{25} 4\sqrt{x}dx - \frac{1}{2} \times (5-1) \cdot 4 - \frac{1}{2} \times (25 - 5) \times 20$

$= 4\left[\frac{2}{3}x^{\frac{2}{3}}\right]_1^{25} - 8 - 200 = \frac{368}{3}$

$\Rightarrow 3A = 368$

Q.23 Let A = $\{z \in C : |z - 2 - i| = 3\}$, B = $\{z \in C : \text{Re}(z - iz) = 2\}$ and S = A∩B. Then $\sum_{z \in S} |z|^2$ is equal to ________.

Ans. **[22]**

Sol. Let $z = x + iy$

$|z - 2 - i| = 3 \Rightarrow (x-2)^2 + (y-1)^2 = 3^2$

$\text{Re}(z - iz) = \text{Re}(x + iy - ix + y)$

$= x + y \Rightarrow x + y = 2$

$\Rightarrow A = \{(x, y) : (x-2)^2 + (y-1)^2 = 3^2, x, y \in R\}$,

$B = \{(x, y) : x + y = 2\}$

$\Rightarrow x - 2 = -y \Rightarrow y^2 + (y-1)^2 = 3^2$

$\Rightarrow 2y^2 - 2y - 8 = 0 \Rightarrow y^2 - y - 4 = 0$

$y_1 + y_2 = 1, y_1y_2 = -4$

$\Rightarrow y_1^2 + y_2^2 = (y_1 + y_2)^2 - 2y_1y_2 = 9$

$\Rightarrow x_1 + x_2 = 4(y_1 + y_2) = 3$,

$x_1x_2 = (2 - y_1)(2 - y_2)$

$= 4 - 2(y_1 + y_2) + y_1y_2 = -2$

$\Rightarrow x_1^2 + x_2^2 = (x_1 + x_2)^2 - 2x_1x_2 = 13$

$\because S = \{(x_1, y_1), (x_2, y_2)\}$

$\Rightarrow \sum_{z \in S} |z|^2 = (x_1^2 + y_1^2) + (x_2^2 + y_2^2) = 22$

Q.24 Let C be the circle $x^2 + (y-1)^2 = 2$, E_1 and E_2 be two ellipses whose centres lie at the origin and major axes lie on x-axis and y-axis respectively. Let the straight-line $x + y = 3$ touch the curves C, E_1 and E_2 at $P(x_1, y_1)$, $Q(x_2, y_2)$ and $R(x_3, y_3)$ respectively. Given that P is the midpoint of the line segment QR and PQ = $\frac{2\sqrt{2}}{3}$, the value of $9(x_1y_1 + x_2y_2 + x_3y_3)$ is equal to ________

Ans. **[46]**

Sol. Solving the line $x + y = 3$, and the circle $x^2 + (y-1)^2 = 2$

Substitute $y = 3 - x$:

$x^2 + (3 - x - 1)^2 = 2$

$\Rightarrow x^2 - 2x + 1 = 0$

$\Rightarrow x = 1 \Rightarrow y = 2$

So, $P = (x_1, y_1) = (1, 2) \Rightarrow x_1y_1 = 1 \cdot 2 = 2$

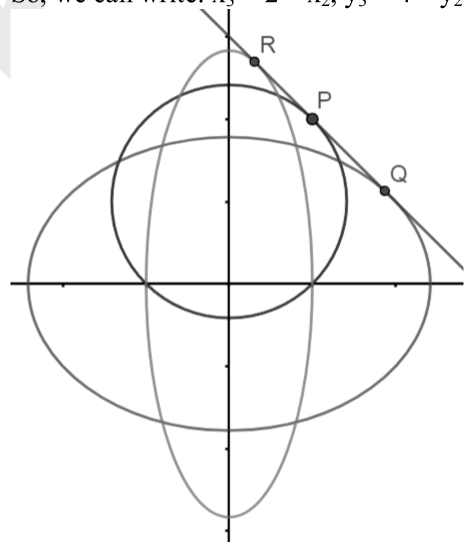
Use midpoint condition

Let $Q = (x_2, y_2)$, $R = (x_3, y_3)$.

Since P is the midpoint of QR:

$x_2 + x_3 = 2x_1 = 2$, $y_2 + y_3 = 2y_1 = 4$

So, we can write: $x_3 = 2 - x_2$, $y_3 = 4 - y_2$



Given,

$PQ = \frac{2\sqrt{2}}{3} \Rightarrow PQ^2 = (x_2 - 1)^2 + (y_2 - 2)^2 = \frac{8}{9}$

Let's denote : $x_2 = a$, $y_2 = b$, $x_3 = 2 - a$, $y_3 = 4 - b$

$(a-1)^2 + (b-2)^2 = \frac{8}{9}$

$\Rightarrow a^2 - 2a + 1 + b^2 - 4b + 4 = \frac{8}{9}$

$\Rightarrow a^2 + b^2 - 2a - 4b + 5 = \frac{8}{9}$

$\Rightarrow 9a^2 + 9b^2 - 18a - 36b + 37 = 0$

Hence, $a = \frac{5}{3}, b = \frac{4}{3}$

$x_1y_1 + x_2y_2 + x_3y_3 = 2 + ab + (2 - a)(4 - b)$

$9(x_1y_1 + x_2y_2 + x_3y_3) = 9(10 + 2ab - 2b - 4a)$

$= 90 + 18ab - 18b - 36a = 46$

Q.25 Let $A = \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix}$. If for some $\theta \in (0, \pi)$, $A^2 = A^T$, then the sum of the diagonal elements of the matrix $(A + I)^3 + (A - I)^3 - 6A$ is equal to _______.

Ans. **[6]**

Sol. Note that A is orthogonal:

$AA^T = A^TA = I$ and $A^T = A^{-1}$

Given $A^2 = A^T$, then:

$A^3 = I$

$Tr(A + I)^3 + (A - I)^3 - 6A = Tr(2A^3 + 6A - 6A)$

$= Tr(2A^3) = Tr(2I)$

(Using $(A + I)^3 + (A - I)^3 = 2A^3 + 6A$ and $2A^3 = 2I) = 6$

PHYSICS

Section-A: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct..

Choose the correct answer :

Q.26 When an object is placed 40 cm away from a spherical mirror an image of magnification $\frac{1}{2}$ is produced. To obtain an image with magnification of $\frac{1}{3}$, the object is to be moved

(1) 20 cm towards the mirror
(2) 20 cm away from the mirror
(3) 80 cm away from the mirror
(4) 40 cm away from the mirror

Ans. **[4]**

Sol. $m = \frac{f}{f - u}$

$\frac{1}{2} = \frac{f}{f - (-40)}$

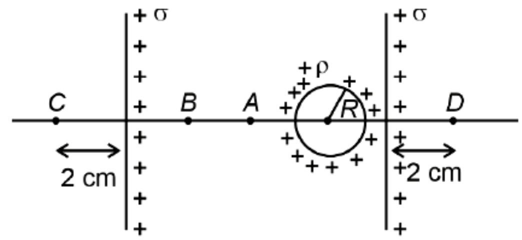
$f = 40$

$\frac{40}{40 - (+u)} = \frac{1}{3}$

$u = -80$

Move 40 cm away from mirror.

Q.27 Two infinite identical charged sheets and a charged spherical body of charge density 'ρ' are arranged as shown in figure. Then the correct relation between the electrical fields at A, B, C and D points is



(1) $\vec{E}_C \neq \vec{E}_D; \vec{E}_A > \vec{E}_B$
(2) $|\vec{E}_A| = |\vec{E}_B|; \vec{E}_C > \vec{E}_D$
(3) $\vec{E}_A = \vec{E}_B; \vec{E}_C = \vec{E}_D$
(4) $\vec{E}_A > \vec{E}_B; \vec{E}_C = \vec{E}_D$

Ans. **[1]**

Sol. At point A and B sum of field due to plates is zero. A is closer to the body so $\vec{E}_A > \vec{E}_B$. D and C are not symmetric w.r.t. body so $E_C \neq E_D$

Q.28 Considering the Bohr model of hydrogen like atoms, the ratio of the radius of 5th orbit of the electron in Li^{2+} and He^{+} is

(1) $\frac{4}{9}$ (2) $\frac{2}{3}$ (3) $\frac{3}{2}$ (4) $\frac{9}{4}$

Ans. **[2]**

Sol. $r \propto \frac{n^2}{Z}$

$\frac{r_{Li^{+2}}}{r_{He^{+2}}} = \frac{2}{3}$

Q.29 The Boolean expression $Y = A\bar{B}C + \bar{A}\,\bar{C}$ can be realised with which of the following gate configurations.

A. One 3-input AND gate, 3 NOT gates and one 2-input OR gate, One 2-input AND gate,
B. One 3-input AND gate, 1 NOT gate, One 2-input NOR gate and one 2-input OR gate
C. 3-input OR gate, 3 NOT gates and one 2-input AND gate

Choose the **correct** answer from the given below.

(1) A, C only (2) A, B, C only
(3) A, B only (4) B, C only

Ans. **[3]**

Sol. $Y = A\bar{B}C + \bar{A}\,\bar{C}$

A. $Y = A\bar{B}C + \bar{A}\,\bar{C} = Y = A\bar{B}C + A + B$

B. $A\bar{B}C + A + B$

Q.30 Consider the sound wave travelling in ideal gases of He, CH_4 and CO_2. All the gases have the same ratio $\frac{P}{\rho}$, where P is the pressure and ρ is the density. The ratio of the speed of sound through the gases $v_{He} : v_{CH_4} : v_{CO_2}$ is given by

(1) $\sqrt{\frac{5}{3}} : \sqrt{\frac{4}{3}} : \sqrt{\frac{4}{3}}$ (2) $\sqrt{\frac{5}{3}} : \sqrt{\frac{4}{3}} : \sqrt{\frac{7}{5}}$

(3) $\sqrt{\frac{4}{3}} : \sqrt{\frac{5}{3}} : \sqrt{\frac{7}{5}}$ (4) $\sqrt{\frac{7}{5}} : \sqrt{\frac{5}{3}} : \sqrt{\frac{4}{3}}$

Ans. [2]

Sol. $v = \sqrt{\frac{\gamma P}{\rho}}$

$$v_{He} : v_{CH_4} : v_{CO_2} = \sqrt{\frac{5}{3}} : \sqrt{\frac{4}{3}} : \sqrt{\frac{7}{5}}$$

Q.31 The mean free path and the average speed of oxygen molecules at 300 K and 1 atm are 3×10^{-7} m and 600 m/s, respectively. Find the frequency of its collisions.

(1) 9×10^5 /s (2) 5×10^8 /s

(3) 2×10^{10} /s (4) 2×10^9 /s

Ans. [4]

Sol. Frequency of collision $= \frac{\text{Average speed}}{\text{Mean free path}}$

$$= \frac{600}{3 \times 10^{-7}} = 2 \times 10^9 \text{ /s}$$

Q.32 Given below are two statements: one is labelled as **Assertion A** and the other is labelled as **Reason R**.

Assertion A : In photoelectric effect, on increasing the intensity of incident light the stopping potential increases.

Reason R: Increase in intensity of light increases the rate of photoelectrons emitted, provided the frequency of incident light is greater than threshold frequency.

In the light of the above statements, choose the **correct** answer from the options given below

(1) Both **A** and **R** are true but **R** is **NOT** the correct explanation of **A**

(2) **A** is true but **R** is false

(3) Both **A** and **R** are true and **R** is the correct explanation of **A**

(4) **A** is false but **R** is true

Ans. [4]

Sol. Intensity ∝ number of photons.

Stopping potential depends on frequency.

Q.33 In a Young's double slit experiment, the slits are separated by 0.2 mm. If the slits separation is increased to 0.4 mm, the percentage change of the fringe width is

(1) 25%

(2) 50%

(3) 0%

(4) 100%

Ans. [2]

Sol. $\beta_1 = \frac{\lambda D}{d}$

$\beta_2 = \frac{\lambda D}{2d} = \frac{\beta_1}{2}$

% change = 50%

Q.34 A small mirror of mass m is suspended by a massless thread of length ℓ. Then the small angle through which the thread will be deflected when a short pulse of laser of energy E falls normal on the mirror

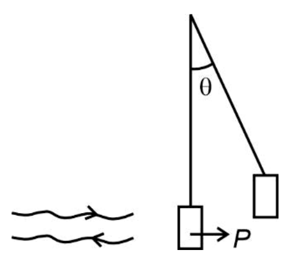
(c = speed of light in vacuum and g = acceleration due to gravity)

(1) $\theta = \frac{2E}{mc\sqrt{g\ell}}$ (2) $\theta = \frac{E}{2mc\sqrt{g\ell}}$

(3) $\theta = \frac{3E}{4mc\sqrt{g\ell}}$ (4) $\theta = \frac{E}{mc\sqrt{g\ell}}$

Ans. [1]

Sol.



Momentum of photon $= \frac{h}{\lambda} = \frac{E}{c}$

$P - \frac{E}{c} = \frac{E}{c}$

$KE_i = PE_f$; $p = \frac{2E}{c}$

$$\frac{\left(\frac{2E}{c}\right)^2}{2m} = mg\ell(1 - \cos\theta)$$

$$\frac{4E^2}{2c^2m} = mg\ell\frac{\theta^2}{2} \Rightarrow \theta = \frac{2E}{mc\sqrt{g\ell}}$$

Q.35 Which of the following are correct expression for torque acting on a body?

A. $\vec{\tau} = \vec{r} \times \vec{L}$

B. $\vec{\tau} = \frac{d}{dt}(\vec{r} \times \vec{p})$

C. $\vec{\tau} = \vec{r} \times \frac{d\vec{p}}{dt}$

D. $\vec{\tau} = I\vec{r}$

E. $\vec{\tau} = \vec{r} \times \vec{F}$

($\vec{r}$ = position vector; $\vec{p}$ = linear momentum; $\vec{L}$ = angular momentum; $\vec{\alpha}$ = angular acceleration; I = moment of inertia; $\vec{F}$ = force; t = time)

Choose the correct answer from the options given below:

(1) B, D and E Only
(2) A, B, D and E Only
(3) B, C, D and E Only
(4) C and D Only

Ans. [3]

Sol.

$\vec{\tau} = \vec{r} \times \vec{L}$	Wrong
$\vec{\tau} = \frac{d}{dt}(\vec{r} \times \vec{p})$	Correct
$\vec{\tau} = \vec{r} \times \frac{d\vec{p}}{dt} = \vec{r} \times \vec{F}$	Correct
$\vec{\tau} = I\vec{\alpha}$	Correct
$\vec{\tau} = \vec{r} \times \vec{F}$	Correct

Q.36 If $\vec{L}$ and $\vec{P}$ represent the angular momentum and linear momentum respectively of a particle of mass 'm' having position vector as $\vec{r} = a(\hat{i}\cos\omega t + \hat{j}\sin\omega t)$. The direction of force is

(1) Opposite to the direction of $\vec{L}$
(2) Opposite to the direction of $\vec{r}$
(3) Opposite to the direction of $\vec{P}$
(4) Opposite to the direction of $\vec{L} \times \vec{P}$

Ans. [2]

Sol. $\vec{r} = a\cos\omega i\hat{i} + a\sin\omega t\hat{j}$

$\vec{v} = \frac{d\vec{r}}{dt} = -a\,\omega\sin\omega t\hat{i} + a\omega\cos\omega t\hat{j}$

$\vec{a} = \frac{d\vec{v}}{dt} = -a\omega^2\cos\omega t\hat{i} - a\omega^2\sin\omega t\hat{j}$

$\vec{F} = m\vec{a}$

$\vec{a}$ is antiparallel to $\vec{r}$

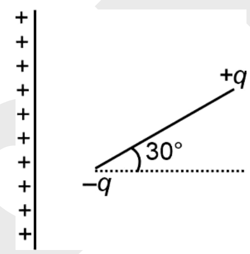
Q.37 Two small spherical balls of mass 10 g each with charges –2 μC and 2 μC, are attached to two ends of very light rigid rod of length 20 cm. The arrangement is now placed near an infinite nonconducting charge sheet with uniform charge density of 100 μC/m^2 such that length of rod makes an angle of 30° with electric field generated by charge sheet. Net torque acting on the rod is:

(Take $\varepsilon_0 = 8.85 \times 10^{-12}$ C^2/Nm^2)

(1) 1.12 Nm (2) 2.24 Nm
(3) 112 Nm (4) 11.2 Nm

Ans. [1]

Sol.



τ = PE sinθ

= q dE sinθ

$= 2 \times 10^{-6} \times 0.2 \times \frac{100\times10^{-6}}{2\times8.85\times10^{-12}} \times \frac{1}{2}$

τ = 1.12 Nm

Q.38 Two simple pendulums having lengths ℓ_1 and ℓ_2 with negligible string mass undergo angular displacements θ_1 and θ_2, from their mean positions, respectively. If the angular accelerations of both pendulums are same, then which expression is correct?

(1) $\theta_1\ell_1^2 = \theta_2\ell_2^2$
(2) $\theta_1\ell_2^2 = \theta_2\ell_1^2$
(3) $\theta_1\ell_1 = \theta_2\ell_2$
(4) $\theta_1\ell_2 = \theta_2\ell_1$

Ans. [4]

Sol. $\theta = \theta_0\sin(\omega t + \phi)$

$\frac{d^2\theta}{dt^2} = -\theta_0\omega^2\sin(\omega t + \varphi)$

$= -\theta_0.\frac{g}{\ell}\sin(\omega t + \varphi)$

$\frac{\theta_1}{\ell_1} = \frac{\theta_2}{\ell_2}$

$\theta_1\ell_2 = \theta_2\ell_1$

Q.39 Current passing through a wire as function of time is given as I(t) = 0.02 t + 0.01 A. The charge that will flow through the wire from t = 1 s to t = 2 s is

(1) 0.07 C
(2) 0.06 C
(3) 0.02 C
(4) 0.04 C

Ans. **[4]**

Sol. $i(t) = 0.02t + 0.01$

$q = \int i\,dt$

$= \int (0.02)t\,dt + 0.01\int dt$

$= \frac{0.02}{2} \times (2^2 - 1^2) + 0.01 \times 1$

$= 0.04$ C

Q.40 Given below are two statements: one is labelled as **Assertion A** and the other is labelled as **Reason R**

Assertion A : The kinetic energy needed to project a body of mass m from earth surface to infinity is $\frac{1}{2}mgR$, where R is the radius of earth.

Reason R: The maximum potential energy of a body is zero when it is projected to infinity from earth surface.

In the light of the above statements, choose the **correct** answer from the options given below.

(1) **A** is true but **R** is false
(2) **A** is false but **R** is true
(3) Both **A** and **R** are true but **R** is **NOT** the correct explanation of **A**
(4) Both **A** and **R** are true but **R** is the correct explanation of **A**

Ans. **[2]**

Sol. $\frac{1}{2}mv^2 - \frac{GMm}{R} = 0$

$KE = \frac{GM}{R^2} \cdot mR$

$= mgR$

A is false

PE at infinity is zero

R is true.

Q.41 A body of mass m is suspended by two strings making angles θ_1 and θ_2 with the horizontal ceiling with tensions T_1 and T_2 simultaneously. T_1 and T_2 are related by $T_1 = \sqrt{3}\,T_2$, the angles θ_1 and θ_2 are

(1) $\theta_1 = 30^\circ$ $\theta_2 = 60^\circ$ with $T_2 = \frac{3mg}{4}$

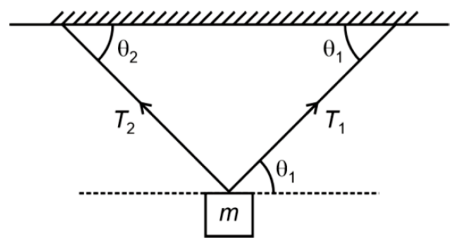
(2) $\theta_1 = 45^\circ$ $\theta_2 = 45^\circ$ with $T_2 = \frac{3mg}{4}$

(3) $\theta_1 = 60^\circ$ $\theta_2 = 30^\circ$ with $T_2 = \frac{mg}{2}$

(4) $\theta_1 = 30^\circ$ $\theta_2 = 60^\circ$ with $T_2 = \frac{4mg}{5}$

Ans. **[3]**

Sol.



$T_1 \sin\theta_1 + T_2 \sin\theta_2 = mg$

$T_1 \cos\theta_1 = T_2 \cos\theta_2$

$\sqrt{3}\,T_2 \cos\theta_1 = T_2 \cos\theta_2$

$\sqrt{3}\cos\theta_1 = \cos\theta_2$

$\theta_1 = 60^\circ, \theta_2 = 30^\circ$

$\sqrt{3}\,T_2 \cdot \frac{\sqrt{3}}{2} + T_2 \cdot \frac{1}{2} = mg$

$T_2 = \frac{mg}{2}$

Q.42 An alternating current is represented by the equation, $i = 100\sqrt{2}\sin(100\pi t)$ ampere. The RMS value of current and the frequency of the given alternating current are

(1) $50\sqrt{2}$ A, 50 Hz
(2) $100\sqrt{2}$ A, 100 Hz
(3) 100 A, 50 Hz
(4) $\frac{100}{\sqrt{2}}$ A, 100 Hz

Ans. **[3]**

Sol. $I = 100\sqrt{2}\,\sin(100\pi t)$

$i_{rms} = \frac{100\sqrt{2}}{\sqrt{2}} = 100$ A

$2\pi f = 100\pi$

$f = 50$ Hz

Q.43 Two liquids A and B have θ_A and θ_B as contact angles in a capillary tube. If $K = \frac{\cos\theta_A}{\cos\theta_B}$, K then identify the correct statement:

(1) K is negative, then liquid A has concave meniscus and liquid B has convex meniscus.
(2) K is zero, then liquid A has convex meniscus and liquid B has concave meniscus.
(3) K is negative, then liquid A and B have convex meniscus.
(4) K is negative, then liquid A and liquid B have concave meniscus.

Ans. **[1]**

Sol. For concave meniscus $\theta_A < 90°$

For convex meniscus $\theta_B > 90°$

$$\frac{\cos\theta_A}{\cos\theta_B} < 0$$

Q.44 In an electromagnetic system, the quantity representing the ratio of electric flux and magnetic flux has dimension of $M^P L^Q T^R A^S$, where value of 'Q' and 'R' is

(1) (–2, 1) (2) (1, –1)
(3) (–2, 2) (4) (3, –5)

Ans. **[2]**

Sol. $\frac{\varphi_E}{\varphi_M} = \frac{E_A}{B_A} = C$

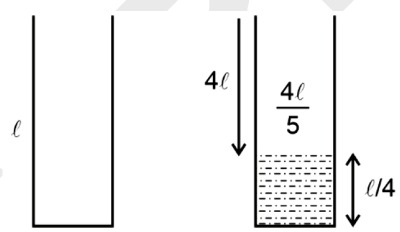
$= LT^{-1}$

Q.45 In an experiment with a closed organ pipe, it is filled with water by $\left(\frac{1}{5}\right)$th of its volume. The frequency of the fundamental note will change by

(1) –25% (2) –20%
(3) 25% (4) 20%

Ans. **[3]**

Sol



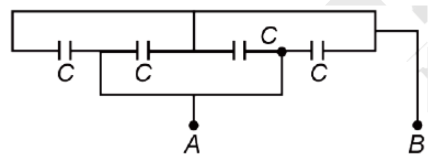
$f_1 = \frac{v}{2\ell}$ $f_2 = \frac{v}{2\left(\frac{4\ell}{5}\right)}$

$= \frac{5}{4} f_1 = 1.25 f_1$

$100 \times \frac{\Delta f}{f} = 25\%$

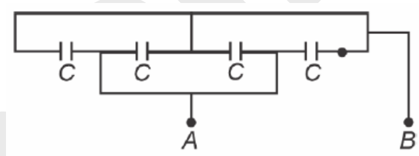
Section-B: Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

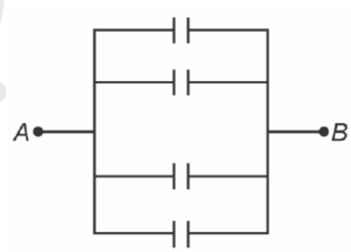
Q.46 Four capacitors each of capacitance 16 μF are connected as shown in the figure. The capacitance between points A and B is : __________ (in μF).



Ans. **[64]**

Sol.

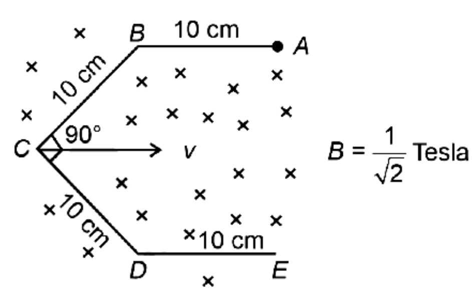




$C_{eq} = 4C = 4 \times 16$

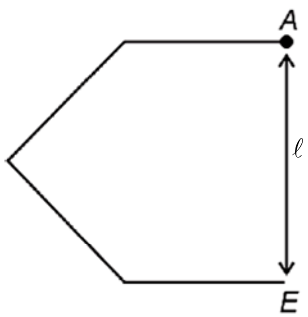
$= 64\ \mu F$

Q.47 Conductor wire ABCDE with each arm 10 cm in length is placed in magnetic field of $\frac{1}{\sqrt{2}}$ Tesla, perpendicular to its plane. When conductor is pulled towards right with constant velocity of 10 cm/s, induced emf between points A and E is ______ mV.



Ans. **[10]**

Sol.



$\ell = 10\sqrt{2}$ cm

$= \frac{\sqrt{2}}{10}$ m

EMF = Bvℓ

$= \frac{1}{\sqrt{2}} \times (0.1)\left(\frac{\sqrt{2}}{10}\right)$

= 0.01

= 10 mV

Q.48 Distance between object and its image (magnified by $-\frac{1}{3}$) is 30 cm. The focal length of the mirror used is $\left(\frac{x}{4}\right)$ cm, where magnitude of value of x is ______

Ans. **[45]**

Sol. $m = \frac{f}{f-u} = -\frac{1}{3}$

$4f = u$

$\frac{1}{v} + \frac{1}{4f} = \frac{1}{f}$

$v = \frac{4f}{3}$

$(u - v) = \frac{8f}{3} = 30$

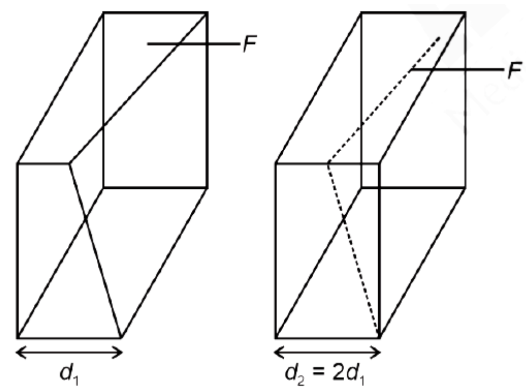
$f = \frac{90}{8} = \frac{45}{4}$

x = 45

Q.49 Two slabs with square cross section of different materials (1, 2) with equal sides (ℓ) and thickness d_1 and d_2 such that $d_2 = 2d_1$ and $\ell > d_2$. Considering lower edges of these slabs are fixed to the floor, we apply equal shearing force on the narrow faces. The angle of deformation is $\theta_2 = 2\theta_1$. If the shear moduli of material 1 is 4×10^9 N/m^2, then shear moduli of material 2 is $x \times 10^9$ N/m^2, where value of x is ______.

Ans. **[1]**

Sol.



$\frac{F}{A_1} = 4 \times 10^9\,\theta_1$ $\qquad$ $\frac{F}{A_2} = x \times 10^9 \times (2\theta_1)$

$\frac{F}{\ell d_1} = 4 \times 10^9\,\theta_1$ $\qquad$ $\frac{F}{2\ell d_1} = x \times 10^9 \times 2\theta_1$

x = 1

Q.50 A circular ring and a solid sphere having same radius roll down on an inclined plane from rest without slipping. The ratio of their velocities when reached at the bottom of the plane is $\sqrt{\frac{x}{5}}$ where x = ______

Answer (None option matches)

Sol. For ring

$mgh = \frac{1}{2}mV^2 + \frac{1}{2}(mR^2)\left(\frac{V}{R}\right)^2$

$mgh = mV^2$

$\sqrt{gh} = V_R$

For solid sphere

$mgh = \frac{1}{2}mV^2 + \frac{1}{2}\left(\frac{2}{5}mR^2\right)\left(\frac{V}{R}\right)^2$

$mgh = \frac{7}{10}mV^2$

$V = \sqrt{\frac{10}{7}gh}$

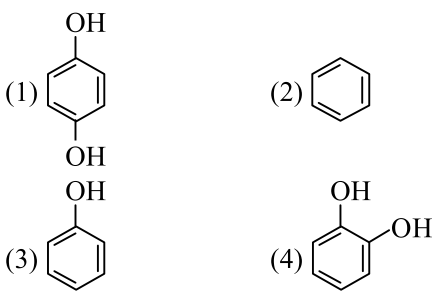
$\frac{V_R}{V_S} = \sqrt{\frac{7}{10}}$

Answer not matched.

CHEMISTRY

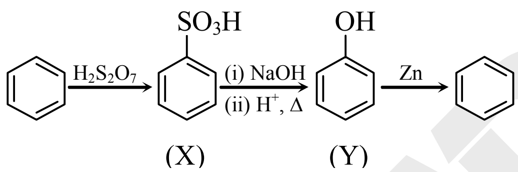
Section-A: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct..

Q.51 Benzene is treated with oleum to produce compound (X) which when further heated with molten sodium hydroxide followed by acidification produces compound (Y). The compound (Y) is treated with zinc metal to produce compound (Z). Identify the structure of compound (Z) from the following option.



Ans. [2]

Sol.



Q.52 On charging the lead storage battery, the oxidation state of lead changes from x_1 to y_1 at the anode and from x_2 to y_2 at the cathode. The values of x_1, y_1, x_2, y_2 are respectively

(1) +4, +2, 0, +2 (2) +2, 0, +2, +4

(3) 0, +2, +4, +2 (4) +2, 0, 0, +4

Ans. [2]

Sol. On charging, lead storage battery

$\overset{+2}{Pb}SO_4 \rightarrow \overset{0}{Pb}$ at anode

$\overset{+2}{Pb}SO_4 \rightarrow \overset{+4}{Pb}O_2$ at cathode

Q.53 An organic compound (X) with molecular formula C_3H_6O is not readily oxidised. On reduction it gives C_3H_8O (Y) which reacts with HBr to give a bromide (Z) which is converted to Grignard reagent. This Grignard reagent on reaction with (X) followed by hydrolysis give 2, 3-dimethylbutan-2-ol.

Compounds (X), (Y) and (Z) respectively are

(1) CH_3COCH_3, $CH_3CH(OH)CH_3$, $CH_3CH(Br)CH_3$

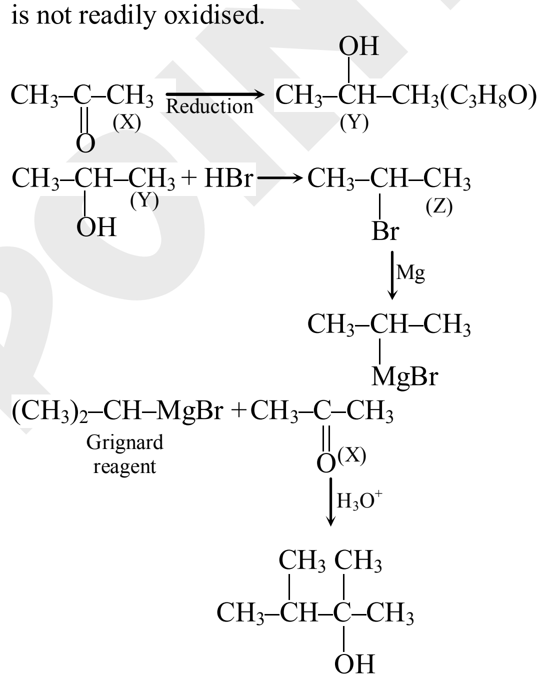
(2) CH_3COCH_3, $CH_3CH_2CH_2OH$, $CH_3CH(Br)CH_3$

(3) CH_3CH_2CHO, $CH_3CH_2CH_2OH$, $CH_3CH_2CH_2Br$

(4) CH_3CH_2CHO, $CH_3CH = CH_2$, $CH_3CH(Br)CH_3$

Ans. [1]

Sol. $X \Rightarrow CH_3-\underset{\underset{O}{||}}{C}-CH_3$ as the organic compound is not readily oxidised.



(2, 3-dimethylbutan-2-ol)

Q.54 One mole of an ideal gas expands isothermally and reversibly from 10 dm^3 to 20 dm^3 at 300 K. ΔU, q and work done in the process respectively are

Given : R = 8.3 J K^{-1} mol^{-1}

In 10 = 2.3

log 2 = 0.30

log 3 = 0.48

(1) 0, 21.84 kJ, 21.84 kJ

(2) 0, 21.84 kJ, –1.726 J

(3) 0, –17.18 kJ, 1.718 J

(4) 0, 1.718 kJ, –1.718 kJ

Ans. [4]

Sol. $\Delta U = 0$, for isothermal process

$\therefore q = -w$

$$w = -nRT \ln\left(\frac{V_2}{V_1}\right)$$

$$w = -1 \times 8.3 \times 300 \times \ln\left(\frac{20}{10}\right)$$

$$w = -8.3 \times 300 \times 2.3 \times \log 2$$

$w = -1.718$ kJ $w = -1.718$ kJ

$\therefore q = +1.718$ kJ

Q.55 Given below are two statements.

Statement-I : The dipole moment of

$\overset{4}{C}H_3-\overset{3}{C}H=\overset{2}{C}H-\overset{1}{C}H=O$ is greater than

$\overset{4}{C}H_3-\overset{3}{C}H_2-\overset{2}{C}H_2-\overset{1}{C}H=O$

Statement-II : $C_1 - C_2$ bond length of

$\underset{4}{C}H_3-\underset{3}{C}H=\underset{2}{C}H-\underset{1}{C}H=O$ is greater than

$C_1 - C_2$ bond length of

$\underset{4}{C}H_3-\underset{3}{C}H_2-\underset{2}{C}H_2-\underset{1}{C}H=O$

In the light of the above statements, choose the **correct** answer from the options given below :

(1) Both **Statement-I** and **Statement-II** are false

(2) **Statement-I** is true but **Statement-II** is false

(3) Both **Statement-I** and **Statement-II** are true

(4) **Statement-I** is false but **Statement-II** is true

Ans. **[2]**

Sol. Dipole moment of Butanal < Crotonaldehyde. Due to conjugation in crotonaldehyde, electron flow is in the same direction as of inductive effect and hence, it has higher dipole moment than butanal.

$\therefore$ Statement-I is correct

Statement-II is incorrect $\rightarrow$ since, $C_1–C_2$ bond in crotonaldehyde have partial double bond character, hence it is smaller than that of Butanal, in which $C_1–C_2$ bond is single bond.

Q.56 Given below are the pairs of group 13 elements showing their relation in terms of atomic radius.

(B < Al), (Al < Ga), (Ga < ln) and (ln < Tl)

Identify the elements present in the incorrect pair and in that pair find out the element (X) that has higher ionic radius (M^3) than the other one. The atomic number of the element (X) is

(1) 31 (2) 81 (3) 13 (4) 49

Ans. **[1]**

Sol. The correct order of atomic radius is

B < Ga < Al < ln ≤ Tl

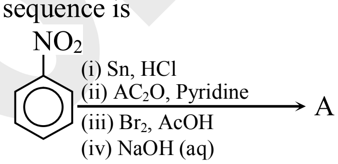
Incorrect pair given is $\rightarrow$ (Al < Ga)

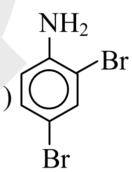
$Al^{3+} < Ga^{3+}$, ionic radius

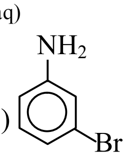
X = Ga

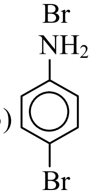
Atomic no. $\Rightarrow$ 31

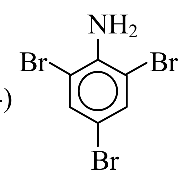
Q.57 The major product (A) formed in the following reaction sequence is



(1) 

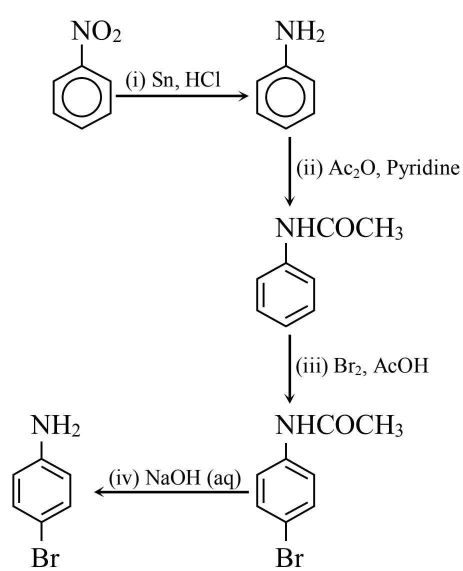
(2) 

(3) 

(4) 

Ans. **[3]**

Sol.



Q.58 Pair of transition metal ions having the same number of unpaired electrons is

(1) V^{2+}, Co^{2+} (2) Ti^{2+}, Co^{2+}
(3) Fe^{3+}, Cr^{2+} (4) Ti^{3+}, Mn^{2+}

Ans. [1]

Sol.

$V^{2+} \Rightarrow 3d^3 4s^0 \rightarrow$ [↿][↿][↿][ ][ ] $\rightarrow$ 3 unpaired e^-

$Co^{2+} \Rightarrow 3d^7 4s^0 \Rightarrow$ [⇅][⇅][↿][↿][↿] $\rightarrow$ 3 unpaired e^-

$Ti^{2+} \Rightarrow 3d^2 4s^0 \Rightarrow$ [↿][↿][ ][ ][ ] $\rightarrow$ 2 unpaired e^-

$Fe^{3+} \Rightarrow 3d^5 4s^0 \Rightarrow$ [↿][↿][↿][↿][↿] $\rightarrow$ 5 unpaired e^-

$Cr^{2+} \Rightarrow 3d^4 4s^0 \rightarrow$ [↿][↿][↿][↿][ ] $\rightarrow$ 4 unpaired e^-

$Ti^{3+} \Rightarrow 3d^1 4s^0 \Rightarrow$ [↿][ ][ ][ ][ ] $\rightarrow$ 1 unpaired e^-

$Mn^{2+} \Rightarrow 3d^5 4s^0 \Rightarrow$ [↿][↿][↿][↿][↿] $\rightarrow$ 5 unpaired e^-

Q.59 Given below are two statements :

Statement I: Nitrogen forms oxides with +1 to +5 oxidation states due to the formation of pπ - pπ bond with oxygen.

Statement II: Nitrogen does not form halides with +5 oxidation state due to the absence of d-orbital in it.

In the light of given statements, choose the **correct** answer from the options given below.

(1) Both Statement **I** and Statement **II** are False
(2) Statement **I** is true but Statement **II** is false
(3) Both Statement **I** and Statement **II** are true
(4) Statement **I** is false but Statement **II** are true

Ans. [3]

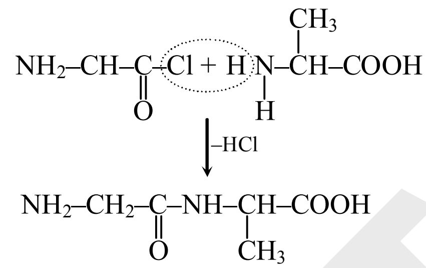
Sol. Nitrogen is a 2nd period element, it do not contain d–orbitals hence cannot form NX_5 type halides. But it can form oxides with +5 O.S.

Q.60 Identify the pair of reactants that upon reaction, with elimination of HCl will give rise to the dipeptide Gly-Ala.

(1) $NH_2–CH_2–COOH$ and $NH_2–CH(CH_3)–COCl$
(2) $NH_2–CH_2–COOH$ and $NH_2–CH(CH_3)–COOH$
(3) $NH_2–CH_2–COCl$ and $\overset{\oplus}{N}H_2–CH(CH_3)–COCl$
(4) $NH_2–CH_2–COCl$ and $NH_2–CH(CH_3)–COOH$

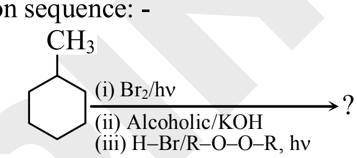
Ans. [4]

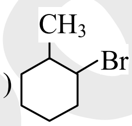
Sol.

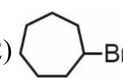


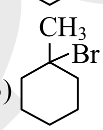
Gly-Ala ⇒ dipeptide formed.

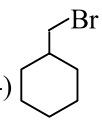
Q.61 Predict the major product of the following reaction sequence: -



(1) 

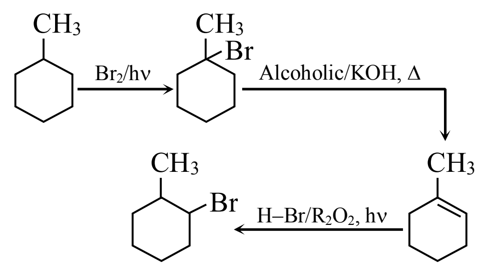
(2) 

(3) 

(4) 

Ans. [1]

Sol.



Q.62 Rate law for a reaction between A and B is given by $r = k\,[A]^n\,[B]^m$

If concentration of A is doubled and concentration of B is halved from their initial value, the ratio of new rate of reaction to the initial rate of reaction $\left(\frac{r_2}{r_1}\right)$ is

(1) $2^{(n-m)}$ (2) $(m + n)$
(3) $\frac{1}{2^{m+n}}$ (4) $(n – m)$

Ans. [1]

Sol. $r_1 = k[A]^n\,[B]^m$

$r_2 = k\,(2[A])^n \left(\frac{1}{2}[B]\right)^m$

$$r_2 = k[A]^n[B]^m \cdot 2^n \cdot \left(\frac{1}{2}\right)^m$$

$$\frac{r_2}{r_1} = 2^n \cdot \left(\frac{1}{2}\right)^m = 2^{(n-m)}$$

Q.63 Which of the following molecule(s) show/s paramagnetic behavior?

A. O_2 B. N_2 C. F_2 D. S_2
E. Cl_2

Choose the correct answer from the options given below.

(1) A & D only (2) A & C only
(3) A & E only (4) B only

Ans. **[1]**

Sol. O_2 and S_2 are paramagnetic N_2, F_2 and Cl_2 are diamagnetic.

Q.64 Number of stereoisomers possible for the complexes, $[CrCl_3(py)_3]$ and $[CrCl_2(ox)_2]^{3-}$ are respectively

(py = pyridine, ox = oxalate)

(1) 2 & 2
(2) 1 & 2
(3) 2 & 3
(4) 3 & 3

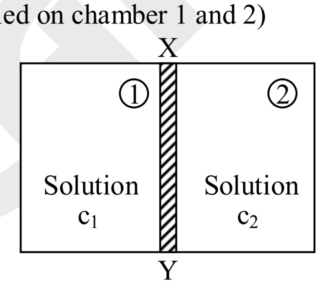
Ans. **[3]**

Sol. $[CrCl_3(py)_3]$ $\Rightarrow$ facial and meridional, 2 isomers (stereoisomers) possible.

$[CrCl_2(ox)_2]$ $\Rightarrow$ 3 isomers (2 geometrical $\rightarrow$ cis/trans).

The cis isomer also has an optical isomer so, total 3 isomers possible.

Q.65 XY is the membrane/partition between two chambers 1 and 2 containing sugar solutions of concentration c_1 and c_2 ($c_1 > c_2$) mol L^{-1}. For the reverse osmosis to take place identify the correct condition. (Here p_1 and p_2 are pressures applied on chamber 1 and 2)



A. Membrane/Partition : Cellophane, $p_1 > \pi$
B. Membrane/Partition : Porous, $p_2 > \pi$
C. Membrane/Partition : Parchment paper, $p_1 > \pi$
D. Memberane/Partition : Cellophane, $p_2 > \pi$

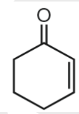
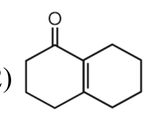
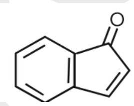
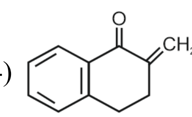
Choose the **correct** answer from the option given below.

(1) A and D only (2) A and C only
(3) C only (4) B and D only

Ans. **[2]**

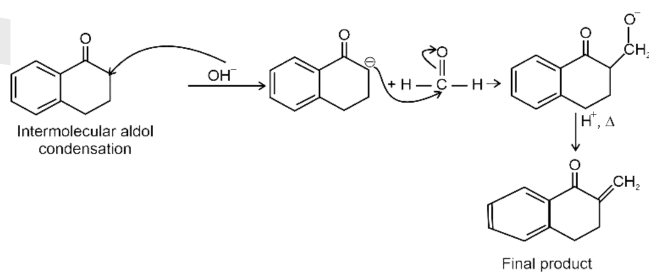
Sol. Membrane should be cellophane or parchment paper i.e., semipermeable membrane. And for reverse osmosis, the pressure p_1 > osmotic pressure (π).

Q.66 Aldol condensation is a popular and classical method to prepare α, β-unsaturated carbonyl compounds. This reaction can be both intermolecular and intramolecular. Predict which one of the following is not a product of intra–molecular aldol condensation?

(1) 
(2) 
(3) 
(4) 

Ans. **[4]**

Sol.



Q.67 Which one of the following complexes will have $\Delta_O = 0$ and μ= 5.96 B.M ?

(1) $[Fe(CN)_6]^{4-}$ (2) $[Mn(SCN)_6]^{4-}$
(3) $[FeF_6]^{4-}$ (4) $[Co(NH_3)_6]^{3+}$

Ans. **[2]**

Sol. Complex having d^5 electronic configuration, with weak field ligand,

has CFSE = 0.

$[Mn(SCN)_6]^{4-} \Rightarrow Mn^{2+} \Rightarrow t_{2g}^3e_g^2$

CFSE = $[3 \times (-0.4) + 2 \times (0.6)]\ \Delta_0$

$Mn^{2+} = \boxed{\uparrow}\boxed{\uparrow}\boxed{\uparrow}\boxed{\uparrow}\boxed{\uparrow} \Rightarrow d^5$

n = 5

$\mu = \sqrt{5(5+2)} = \sqrt{5 \times 7} = 5.96$ BM

Q.68 Let us consider a reversible reaction at temperature, T.
In this reaction, both ΔH and ΔS were observed to have positive values. If the equilibrium temperature is Te, then the reaction becomes spontaneous at:
(1) Te = 5T
(2) Te > T
(3) T > Te
(4) T = Te

Ans. **[3]**

Sol. $\Delta G = \Delta H - T\Delta S$
If ΔH and ΔS are +ve
ΔG = –ve at high temperature.

Q.69 For $A_2 + B_2 \rightleftharpoons 2AB$
Ea for forward and backward reaction are 180 and 200 kJ mol^{-1} respectively. If catalyst lowers E_a for both reaction by 100 kJ mol^{-1}.
Which of the following statement is correct ?
(1) The enthalpy change for the catalysed reaction is different from that of uncatalysed reaction
(2) Catalyst does not alter the Gibbs energy change of a reaction
(3) Catalyst can cause non-spontaneous reactions to occur
(4) The enthalpy change for the reaction is +20 kJ mol^{-1}

Ans. **[2]**

Sol. Catalyst cannot cause non-spontaneous reaction to occur, it only reduces activation energy of the reaction.

Q.70 Which one of the following about an electron occupying the 1s orbital in a hydrogen atom is incorrect ?
(Bohr's radius is represented by a_0)
(1) The probability density of finding the electron is maximum at the nucleus
(2) The 1s orbital is spherically symmetrical
(3) The total energy of the electron is maximum when it is at a distance a_0 from the nucleus
(4) The electron can be found at a distance $2a_0$ from the nucleus

Ans. **[3]**

Sol. More far the electron is from nucleus, more is its energy. Total energy of on electron is given by $-\frac{13.6z^2}{n^2}$. So as n $\uparrow \Rightarrow$ energy $\uparrow$

Section-B: Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

Q.71 $KMnO_4$ acts as an oxidising agent in acidic medium. "X" is the difference between the oxidation states of Mn in reactant and product, "Y" is the number of 'd' electrons present in the brown red precipitate formed at the end of the acetate ion test with neutral ferric chloride. The value of X + Y is ____

Ans. **[10]**

Sol. $K\overset{+7}{Mn}O_4 \rightarrow Mn^{2+}$ (when act as oxidising agent)
difference in O.S. = 5, $\therefore$ X = 5
$CH_3COO^- + FeCl_3 \rightarrow \underset{\text{(Brown ppt.)}}{(CH_3COO)_3Fe}$
$Fe^{3+} \Rightarrow d^5 \Rightarrow$ 5 d-electrons present, $\therefore$ Y = 5
$\therefore$ X + Y = 10

Q.72 The pH of a 0.01 M weak acid HX ($K_a = 4 \times 10^{-10}$) is found to be 5. Now the acid solution is diluted with excess of water so that the pH of the solution changes to 6. The new concentration of the diluted weak acid is given as x × 10^{-4} M. The value of x is _____ (nearest integer)

Ans. **[25]**

Sol. After dilution
pH = 6 $\qquad [H^+] = 10^{-6}$
$[H^+] = 10^{-6} = \sqrt{K_aC}$
$10^{-6} = \sqrt{4\times10^{-10}\times C}$
$\frac{10^{-12}}{4\times10^{-10}} = C$
$0.25 \times 10^{-2} = C$
$25 \times 10^{-4} = C \qquad \boxed{\therefore x = 25}$

Q.73 Fortification of food with iron is done using $FeSO_4.7H_2O$. The mass in grams of the $FeSO_4.7H_2O$ required to achieve 12 ppm of iron in 150 kg of wheat is _____ (Nearest integer)
[Given : Molar mass of Fe, S and O respectively are 56, 32 and 16 g mol^{-1}]

Ans. **[9]**

Sol. $\text{ppm of Fe} = \frac{\text{mass of Fe}}{\text{mass of wheat}} \times 10^6$
$12 = \frac{w}{150\times10^3} \times 10^6$

$1.8 \text{ gm} = w_{Fe}$

$\therefore$ mass of $FeSO_4.7H_2O$ required $= \dfrac{278 \times 1.8}{56}$

$= 8.93 \text{ gm} \approx 9$ grams

Q.74 The total number of hydrogen bonds of a DNA double Helix strand whose one strand has the following sequence of bases is _____
5'-G-G-C-A-A-A-T-C-G-G-C-T-A-3'

Ans. **[33]**

Sol.

5′-G-G-C-A-A-A-T-C-G-G-C-T-A-3′
||| ||| ||| || || || || ||| ||| ||| ||| || ||
3′-C-C-G-T-T-T-A-G-C-C-G-A-T-5′

There are 2 H-bonds, between A and T and there are 3 H-bonds, between G and C base pairs.

Q.75 In Dumas' method for estimation of nitrogen 1 g of an organic compound gave 150 mL of nitrogen collected at 300 K temperature and 900 mm Hg pressure. The percentage composition of nitrogen in the compound is ______% (nearest integer)
(Aqueous tension at 300 K = 15 mm Hg)

Ans. **[20]**

Sol. Pressure = 885 nm

V_{N_2} at STP $= \dfrac{273 \times 885 \times 150}{300 \times 760} = 158.95$ mL

Mass of $N_2 = \dfrac{158.95}{224.00} \times 28 = 0.2$ g

Mass of organic compound = 1 g

$\therefore$ % of N $= \dfrac{0.2}{1} \times 100 = 20\%$