CAREER POINT

JEE Main Online Exam 2025

Questions & Solution

03rd April 2025 | Morning

MATHEMATICS

Section-A: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Q.1 If $\sum_{r=1}^{9}\left(\frac{r+3}{2^r}\right).\,{}^9C_r = \alpha\left(\frac{3}{2}\right)^9 - \beta$, $\alpha, \beta \in N$, then $(\alpha+\beta)^2$ is equal to

(1) 18 (2) 27 (3) 81 (4) 9

Ans. [3]

Sol. $\sum_{r=1}^{9}\left(\frac{r+3}{2^r}\right).\,{}^9C_r = \sum_{r=1}^{9}\frac{r}{2^r}.\frac{9}{r}.\,{}^8C_{r-1} + \sum_{r=1}^{9}3.\,{}^9C_r\left(\frac{1}{2}\right)^r$

$= \frac{9}{2}\sum_{r=1}^{9}{}^8C_{r-1}\left(\frac{1}{2}\right)^{r-1} + 3\sum_{r=1}^{9}{}^9C_r\left(\frac{1}{2}\right)^r$

$= \frac{9}{2}\sum_{r=0}^{8}{}^8C_{r-1}\left(\frac{1}{2}\right)^r + 3\sum_{r=1}^{9}{}^9C_r\left(\frac{1}{2}\right)^r$

$= \frac{9}{2}\left(1+\frac{1}{2}\right)^8 + 3\left[\left(1+\frac{1}{2}\right)^9 - {}^9C_0\left(\frac{1}{2}\right)^0\right]$

$= \frac{9}{2}.\frac{3^8}{2^8} + 3\left[\frac{3^9}{2^9} - 1\right]$

$= \frac{3^{10}}{2^9} + \frac{3^{10}}{2^9} - 3 = 4.\,\frac{3^{10}}{2^{10}} - 3$

$= 4\left(\frac{3}{2}\right)^{10} - 3$

$= 6\left(\frac{3}{2}\right)^9 - 3$

$\alpha = 6, \beta = 3 \Rightarrow (\alpha+\beta)^2 = 81$

Q.2 Let $z \in C$ be such that $\frac{z^2+3i}{z-2+i} = 2+3i$. Then the sum of all possible values of z^2 is

(1) $-19-2i$ (2) $-19+2i$

(3) $19-2i$ (4) $19+2i$

Ans. [1]

Sol. $\frac{z^2+3i}{z-2+i} = 2+3i$

$z^2+3i = (z-2+i)(2+3i)$

$z^2+3i = 2z-4+2i+3iz-6i-3$

$z^2+3i = (2z-7)+i(3z-4)$

$z^2-(2+3i)z+(7+7i) = 0$

This is a quadratic in z.

$z_1+z_2 = 2+3i$

$z_1+z_2 = 7+7i$

$z_1^2+z_2^2 = (z_1+z_2)^2 - 2z_1z_2$

$= (2+3i)^2 - 2(7+7i)$

$= 4-9+12i-14-14i$

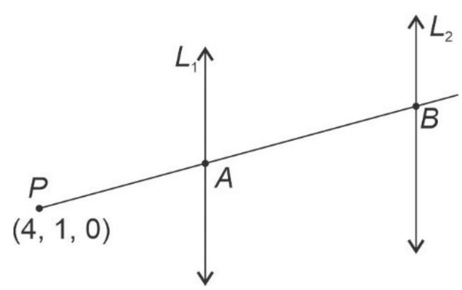
$= -19-2i$

Q.3 Let a line passing through the point (4, 1, 0) intersect the line $L_1: \frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ at the point $A(\alpha, \beta, \gamma)$ and the line $L_2: x-6 = y = -z+4$ at the point B(a, b, c). Then $\begin{vmatrix}1 & 0 & 1\\ \alpha & \beta & \gamma\\ a & b & c\end{vmatrix}$ is equal to

(1) 16 (2) 6 (3) 12 (4) 8

Ans. [4]

Sol.



$A = (\alpha, \beta, 4) = (2\lambda+1, 3\lambda+2, 4\lambda+3)$

and $B = (a, b, c) = (\mu+6, \mu, -\mu+4)$

$\because \frac{\mu+2}{2\lambda-3} = \frac{\mu-1}{3\lambda+1} = \frac{-\mu+4}{4\lambda+3}$

$\therefore \lambda\mu + 8\lambda + 4\mu = 1$ … (1)

and $7\lambda\mu - 16\lambda + 4\mu = 7$ … (2)

from equation (1) and (2)

$\lambda = -1$ and $\mu = 3$

Or $\lambda = -\frac{1}{3}$ and $\mu = 1$

By taking, $\lambda = -1$ and $\mu = 3$, we get

$\therefore$ A(α, β, 4) = (–1, –1, –1)

and B(a, b, c) = (9, 3, 1)

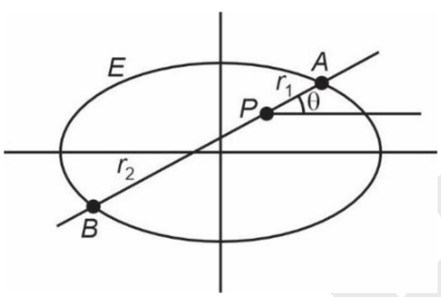
$$\therefore \begin{vmatrix} 1 & 0 & 1 \\ -1 & -1 & -1 \\ 9 & 3 & 1 \end{vmatrix} = 8$$

Q.4 A line passing through the point P($\sqrt{5}$, $\sqrt{5}$) intersects the ellipse $\frac{x^2}{36} + \frac{y^2}{25} = 1$ at A and B such that (PA).(PB) is maximum. Then $5(PA^2 + PB^2)$ is equal to

(1) 377 (2) 218 (3) 338 (4) 290

Ans. [3]

Sol.



$$\frac{x - \sqrt{5}}{\cos\theta} = \frac{y - \sqrt{5}}{\sin\theta} = r$$

$x = r\cos\theta + \sqrt{5}$

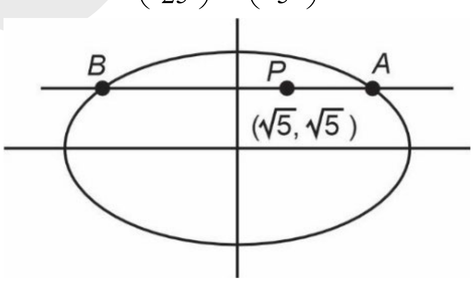
$y = r\sin\theta + \sqrt{5}$

$25x^2 + 36y^2 = 900$, (x, y) lie on E

$$r^2[25\cos^2\theta + 36\sin^2\theta] + r[50\sqrt{5}\cos\theta + 72\sqrt{5}\sin\theta] + 25[5] + 36[5] = 900$$

$$r_1r_2 = \frac{900 - 305}{25 + 11\sin^2\theta},\ (r_1r_2)_{max} = \frac{595}{25} = \frac{119}{5}$$

$$|r_1.r_2|_{max} = \left(\frac{595}{25}\right)^2 = \left(\frac{119}{5}\right) \text{ at } \theta = 0$$



$$\frac{x^2}{36} + \frac{y^2}{25} = 1$$

At $y = \sqrt{5}$

$$\frac{x^2}{36} + \frac{1}{5} = 1$$

$$\Rightarrow \frac{x^2}{36} = \frac{4}{5} \Rightarrow x = \pm\frac{12}{\sqrt{5}}$$

$PA^2 + PB^2 = (PA + PB)^2 - 2PA.PB$

$$= \left(\frac{24}{\sqrt{5}}\right)^2 - 2.\frac{119}{5}$$

$$5(PA^2 + PB^2) = 5\left(\frac{24^2}{5} - \frac{2.119}{5}\right) = 24^2 - 238$$

= 338.

Q.5 Let A = {– 3, –2, –1, 0, 1, 2, 3}. Let R be a relation on A defined by xRy if and only if $0 \le x^2 + 2y \le 4$. Let l be the number of elements in R and m be the minimum number of elements required to be added in R to make it a reflexive relation. Then l + m is equal to

(1) 18 (2) 19 (3) 17 (4) 20

Ans. [1]

Sol. $0 \le x^2 + 2y \le 4$

For y = –3, x = {3, –3}

For y = –2, x = {–2, 2}

For y = –1, x = {–2, 2}

For y = 0, x = {–2, –1, 0, 1, 2}

For y = 1, x = {–1, 0, 1}

For y = 2, x = {0}

R = {(3, –3), (–3, –3), (–2, –2), (2, –2), (–2, –1), (2, –1), (–2, 0), (–1, 0), (0, 0), (1, 0), (2, 0), (–1, 1), (0, 1), (1, 1), (0, 2)}

l = 15

We need to add (–1, –1), (2, 2), (3, 3) to make it reflexive

m = 3 $\Rightarrow$ l + m = 18

Q.6 Let A be a matrix of order 3 × 3 and |A| = 5. If $|2adj(3A\ adj(2A))| = 2^\alpha . 3^\beta . 5^\gamma$, $\alpha, \beta, \gamma \in N$, then $\alpha + \beta + \gamma$ is equal to

(1) 25 (2) 26 (3) 27 (4) 28

Ans. [3]

Sol. $|2adj(3A\ adj(2A))| = 2^3|3A(adj(2A))|^2$

$= 2^3(3^3)^2\ |A|^2\ |adj(2A)^2|$

$= 2^3 \cdot 3^6 \cdot 5^2\ (|(2A)|^2)^2$

$= 2^3 \cdot 3^6 \cdot 5^2\ |2A|^4$

$= 2^3 \cdot 3^6 \cdot 5^2 \cdot (2^3)^4 \cdot |A|^4$

$= 2^{15} \cdot 3^6 \cdot 5^6$

Q.7 Let $f(x) = \begin{cases} (1+ax)^{1/x} & , x<0 \\ 1+b & , x=0 \\ \dfrac{(x+4)^{1/2}-2}{(x+c)^{1/3}-2} & , x>0 \end{cases}$

be continuous at x = 0. Then $e^a bc$ is equal to :

(1) 48 (2) 72 (3) 36 (4) 64

Ans. [1]

Sol. $f(x) = \begin{cases} (1+ax)^{1/x} & , x<0 \\ 1+b & , x=0 \\ \dfrac{(x+4)^{1/2}-2}{(x+c)^{1/3}-2} & , x>0 \end{cases}$

Hence, f(0) = 1 + 6

$$\text{RHL} = \lim_{x\to 0^+} \frac{(x+4)^{1/2}-2}{(x+c)^{1/3}-2} \left[\text{For } \frac{0}{0} \text{ form}, c = 8\right]$$

$$= \lim_{x\to 0} \frac{2\left(1+\frac{x}{4}\right)^{1/2}-2}{\left(1+\frac{x}{8}\right)^{1/3}-2}$$

$$= \lim_{x\to 0} \frac{1+\frac{x}{8}-1}{1+\frac{x}{8}\cdot\frac{1}{3}-1} = \frac{\frac{1}{8}}{\frac{1}{8}\cdot\frac{1}{3}} = 3$$

$\therefore$ 1 + b = 3

$\Rightarrow$ b = 2

$$\text{LHL} = \lim_{x\to 0} (1+ax)^{1/x} = \lim_{x\to 0} e^{\frac{1+ax-1}{x}} = e^a = 3$$

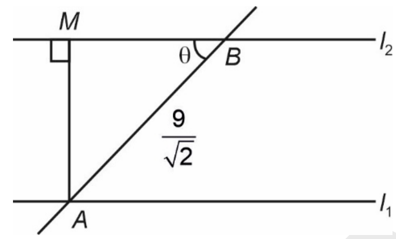
$\therefore e^a \cdot bc = 3.2.8 = 48$

Q.8 A line passes through the origin and makes equal angles with the positive coordinate axes. It intersects the lines $L_1 : 2x + y + 6 = 0$ and $L_2 : 4x + 2y - p = 0$, $p > 0$, at the points A and B, respectively. If $AB = \frac{9}{\sqrt{2}}$ and the foot of the perpendicular from the point A on the line L_2 is M, then $\frac{AM}{BM}$ is equal to

(1) 3 (2) 5 (3) 2 (4) 4

Ans. [1]

Sol.



$$\tan\theta = \left|\frac{-2-1}{1+(-2)(1)}\right| = \frac{3}{1}$$

$$\sin\theta = \frac{3}{\sqrt{10}}$$

$$AM = \frac{3}{\sqrt{10}} \times \frac{9}{\sqrt{2}} = \frac{27}{2\sqrt{5}}$$

Dist. between l_1 & l_2, $AM = \left|\dfrac{\frac{P}{2}+6}{\sqrt{5}}\right| = \dfrac{27}{2\sqrt{5}}$

$$\frac{P}{2} + 6 = \pm\frac{27}{2}$$

$$\frac{P}{2} = \frac{27}{2} - 6 \Rightarrow P = 15, \text{ As } P > 0$$

$$BM = \sqrt{\frac{81}{2} - \left(\frac{27}{2\sqrt{5}}\right)^2}$$

$$= \sqrt{\frac{810-729}{20}} = \sqrt{\frac{81}{20}} = \frac{9}{2\sqrt{5}}$$

$$\text{Now, } \frac{AM}{BM} = \frac{\frac{27}{2\sqrt{5}}}{\frac{9}{2\sqrt{5}}} = 3$$

Q.9 The sum of all rational terms in the expansion of $(2+\sqrt{3})^8$ is

(1) 3763 (2) 18817

(3) 33845 (4) 16923

Ans. [2]

Sol. $S = {}^8C_0(2)^8 + {}^8C_1 2^7(\sqrt{3}) + ... + {}^8C_8(\sqrt{3})^8$

Sum of rational terms

$= {}^8C_0(2)^8 + {}^8C_2 2^6(\sqrt{3})^2 + {}^8C_4(2)^4(\sqrt{3})^4 + {}^8C_6(2)^2(\sqrt{3})^2 + {}^8C_8(\sqrt{3})^8$

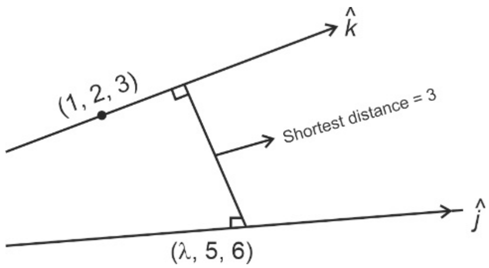
= 18817

Q.10 Line L_1 passes through the point (1, 2, 3) and is parallel to z-axis. Line L_2 passes through the point (λ, 5, 6) and is parallel to y-axis. Let for $\lambda = \lambda_1, \lambda_2, \lambda_2 < \lambda_1$, the shortest distance between the two lines be 3. Then the square of the distance of the point $(\lambda_1, \lambda_2, 7)$ from the line L_1 is

(1) 32 (2) 25 (3) 37 (4) 40

Ans. **[2]**

Sol.

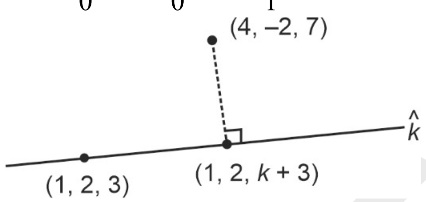


$$3 = \frac{\left|[(\lambda-1)\hat{i}+3\hat{j}+3\hat{k}].(\hat{k}\times\hat{j})\right|}{|\hat{k}\times\hat{j}|} = |\lambda - 1| = 3$$

$\Rightarrow \lambda = 4$ or -2, $\lambda_2 < \lambda_1 \Rightarrow \lambda_2 = -2$

$\lambda_1 = 4$

$$L_1 : \frac{x-1}{0} = \frac{y-2}{0} = \frac{z-3}{1} = k$$



$(3, -4, 4-k) \perp \hat{k}$

$4 - k = 0 \Rightarrow k = 4$

$\Rightarrow$ Distance between (4, –2, 7) & (1, 2 , 7)

$\Rightarrow$ is $\sqrt{3^2+4^2} = 5$

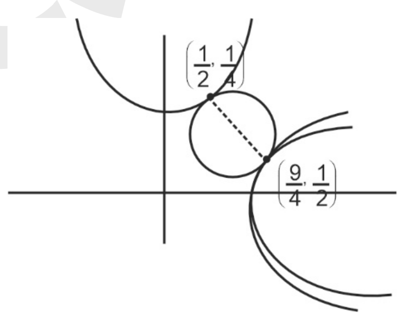
Q.11 The radius of the smallest circle when touches the parabolas $y = x^2 + 2$ and $x = y^2 + 2$ is

(1) $\frac{7\sqrt{2}}{4}$ (2) $\frac{7\sqrt{2}}{16}$

(3) $\frac{7\sqrt{2}}{2}$ (4) $\frac{7\sqrt{2}}{8}$

Ans. **[4]**

Sol.



The circle will have centre on x = y line since parabolas are symmetric about y = x line

The slope of tangent at closest point $y^2 = x - 2$

$$\Rightarrow 2y\left(\frac{dy}{dx}\right) = 1 \Rightarrow y = \frac{1}{2} \Rightarrow \text{point will be}\left(\frac{9}{4}, \frac{1}{2}\right)$$

$$\text{Similarly, on } x^2 = y - 2 \Rightarrow 2x = \frac{dy}{dx} = 1 \Rightarrow \left(\frac{1}{2}, \frac{9}{4}\right)$$

$$2r = \sqrt{\left(\frac{9}{4}-\frac{1}{2}\right)^2 + \left(\frac{1}{2}-\frac{9}{4}\right)^2} = \sqrt{2}\,.\left|\frac{9}{4}-\frac{1}{2}\right|$$

$$= \frac{14}{8}\sqrt{2} = \frac{7\sqrt{2}}{4}$$

$$\Rightarrow r = \frac{7\sqrt{2}}{8}$$

Q.12 The sum 1 + 3 + 11 + 25 + 45 + 71 + ... upto 20 terms, is equal to

(1) 8124 (2) 7240

(3) 7130 (4) 6982

Ans. **[2]**

Sol. $T_r = 3r^2 - 7r + 5$ using second order difference

$$\sum_{r=1}^{20}(3r^2 - 7r + 5) = 3\Sigma r^2 - 7\Sigma r + 5\Sigma(1)$$

$$= \frac{3(n)(n+1)(2n+1)}{6} - \frac{7n(n+1)}{2} - 5n, \ n = 20$$

$= 7240$

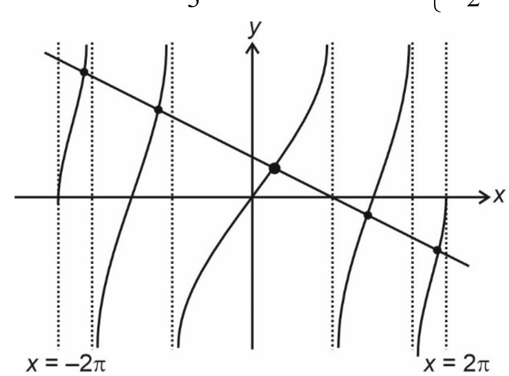
Q.13 The number of solutions of the equation $2x + 3\tan x = \pi$, $x \in [-2\pi, 2\pi] - \left\{\pm\frac{\pi}{2}, \pm\frac{3\pi}{2}\right\}$ is :

(1) 5 (2) 3 (3) 4 (4) 6

Ans. **[1]**

Sol. $2x + 3\tan x = \pi$

$$\Rightarrow \tan x = \frac{\pi - 2x}{3}, \ x \in [-2\pi, \ 2\pi] - \left\{\pm\frac{\pi}{2}, \pm\frac{3\pi}{2}\right\}$$



5 solutions

Q.14 Let the domain of the function $f(x) = \log_2 \log_4 \log_6 (3 + 4x - x^2)$ be (a, b). If $\int_0^{b-a} [x^2]dx = p - \sqrt{q} - \sqrt{r}$, p, q, r ∈ N, gcd(p, q, r) = 1, where [·] is the greatest integer function, then p + q + r is equal to

(1) 8 (2) 10 (3) 11 (4) 9

Ans. **[2]**

Sol. $f(x) = \log_2(\log_4 (\log_6(3 + 4x - x^2)))$

$\log_4(\log_6(3 + 4x - x^2)) > 0$

$\Rightarrow \log_6(3 + 4x - x^2) > 1$

$\Rightarrow 3 + 4x - x^2 > 6$

$\Rightarrow x^2 - 4x + 3 < 0$

$\Rightarrow (x - 1)(x - 3) < 0$

$\Rightarrow x \in (1, 3)$

a = 1, b = 3

$$\int_1^2 [x^2]dx = \int_1^{\sqrt{2}} 1\,dx + \int_{\sqrt{2}}^{\sqrt{3}} 2\,dx + \int_{\sqrt{3}}^{2} 3\,dx = 5 - \sqrt{2} - \sqrt{3}$$

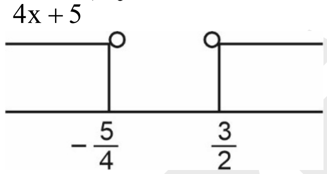
p + q + r = 10

Q.15 If the domain of the function $f(x) = \log_e\left(\frac{2x-3}{5+4x}\right) + \sin^{-1}\left(\frac{4+3x}{2-x}\right)$ is [α, β), then $\alpha^2 + 4\beta$ is equal to

(1) 3 (2) 7 (3) 4 (4) 5

Ans. **[3]**

Sol. $\frac{2x-3}{4x+5} > 0$



$\therefore x \in \left(-\infty, \frac{5}{4}\right) \cup \left(\frac{3}{2}, \infty\right)$ … (i)

$-1 \le \frac{3x+4}{2-x} \le 1$

$\frac{3x+4}{2-x} \le 1$

$\Rightarrow \frac{3x+4}{2-x} - 1 \le 0$

$\Rightarrow \frac{3x+4-2+x}{x-2} \ge 0$

$\Rightarrow \frac{4x+2}{x-2} \ge 0$

$\Rightarrow x \in \left(-\infty, -\frac{1}{2}\right] \cup (2, \infty)$ … (ii)

$\frac{3x+4}{2-x} \ge -1$

$\Rightarrow \frac{3x+4}{2-x} + 1 \ge 0$

$\Rightarrow \frac{3x+4+2-x}{2-x} \ge 0$

$\Rightarrow \frac{2x+6}{x-2} \le 0$

$\therefore x \in [-3, 2)$ … (iii)

Taking intersection of (i), (ii) and (iii)

$x \in \left[-3, -\frac{5}{4}\right)$

$\alpha = -3, \beta = -\frac{5}{4}$

$\alpha^2 + 4\beta = 4$

Q.16 Let $f(x) = \int x^3\sqrt{3-x^2}\,dx$. If $5f(\sqrt{2}) = -4$, then f(1) is equal to

(1) $-\frac{6\sqrt{2}}{5}$ (2) $-\frac{4\sqrt{2}}{5}$

(3) $-\frac{2\sqrt{2}}{5}$ (4) $-\frac{8\sqrt{2}}{5}$

Ans. **[1]**

Sol. $\int x^3\sqrt{3-x^2}\,dx$

$3 - x^2 = t^2$

$-2x\,dx = 2t\,dt$

$= -\int t^3(3-t^2)dt = \int t^4 - 3t^2\,dt$

$= \frac{t^5}{5} - t^3 + c$

$f(x) = \frac{(3-x^2)^{\frac{5}{2}}}{5} - (3-x^2)^{\frac{5}{2}} + c$

$\therefore f(\sqrt{2}) = -\frac{4}{5}$

$\Rightarrow -\frac{4}{5} = \frac{1}{5} - 1 + c \Rightarrow c = 0$

$\therefore f(x) = \frac{(3-x^2)^{\frac{5}{2}}}{5} - (3-x^2)^{\frac{3}{2}}$

Now $f(1) = \frac{2^{\frac{5}{2}}}{5} - 2^{\frac{3}{2}} = 2^{\frac{3}{2}}\left[\frac{2}{5} - 1\right]$

$= 2\sqrt{2}\left(-\frac{3}{5}\right) = \frac{-6\sqrt{2}}{5}$

Q.17 Let $a_1, a_2, a_3, \ldots$ be a G.P of increasing positive numbers. If $a_3a_5 = 729$ and $a_2 + a_4 = \frac{111}{4}$, then $24(a_1 + a_2 + a_3)$ is equal to

(1) 130 (2) 131 (3) 129 (4) 128

Ans. **[3]**

Sol. $a_2 + a_4 = \frac{111}{4}$

$a_3a_5 = 729$

$[ar^2][ar^4] = 729 = (27)^2$

$\Rightarrow ar^3 = 27$ … (i)

$\Rightarrow a_4 = 27$

Now $a_2 = \frac{111}{4} - 27$

$a_2 = \frac{3}{4} = ar$ … (ii)

By (i) & (ii)

$r^2 = \frac{4\times 27}{3} = 4 \times 9 = 36$

$\Rightarrow r = 6$ ($\because$ increasing GP)

$a = \frac{1}{8}$

Now $24[a_1 + a_2 + a_3] = 24\,[a + ar + ar^2]$

$= 24 \times \frac{1}{8}\,[1 + r + r^2] = 3[1 + 6 + 6^2] = 3 \times 43 = 129$

Q.18 Let α and β be the roots of $x^2 + \sqrt{3}\,x - 16 = 0$, and γ and δ be the roots of $x^2 + 3x - 1 = 0$. If $P_n = \alpha^n + \beta^n$ and $Q^n = \gamma^n + \delta^n$, then $\frac{P_{25} + \sqrt{3}P_{24}}{2P_{23}} + \frac{Q_{25} - Q_{23}}{Q_{24}}$ is equal to

(1) 4 (2) 3 (3) 7 (4) 5

Ans. **[4]**

Sol. $x^2 + 3x - 1 = 0 \begin{smallmatrix}\nearrow \gamma \\ \searrow \delta\end{smallmatrix} \Rightarrow x^2 - 1 = -3x$

$\Rightarrow P^n = \gamma^n + \delta^n$

$P_{25} - P_{23} = (\gamma^{25} - \gamma^{23}) + (\delta^{25} - \delta^{23})$

$= \gamma^{23}(\gamma^2 - 1) + \delta^{23}(\delta^2 - 1)$

$= \gamma^{23}(-3\gamma) + \delta^{23}(-3\delta) = -3[\gamma^{24} + \delta^{24}]$

$\Rightarrow \frac{P_{25} - P_{23}}{P_{24}} = (-3)$

Similarly,

$x^2 + \sqrt{3}\,x - 16 = 0 \begin{smallmatrix}\nearrow \alpha \\ \searrow \beta\end{smallmatrix}\; Q_n = \alpha^n + \beta^n$

$\Rightarrow Q_{25} + \sqrt{3}\,Q_{24} = (\alpha^{25} + \sqrt{3}\,\alpha^{24}) + (\beta^{25} + \sqrt{3}\,\beta^{24})$

$= \alpha^{23}(\alpha^2 + \sqrt{3}\,\alpha) + \beta^{23}(\beta^2 + \sqrt{3}\,\beta)$

$= \alpha^{23}(16) + 16\beta^{23}$

$\Rightarrow \frac{Q_{25} + \sqrt{3}Q_{24}}{2.Q_{23}} = \frac{16(\alpha^{23} + \beta^{23})}{2(\alpha^{23} + \beta^{23})} = 8$

$\Rightarrow \frac{Q_{25} + \sqrt{3}Q_{24}}{2Q_{23}} + \frac{(P_{25} - P_{23})}{P_{24}} = 8 + (-3) = 5$

Q.19 Let g be a differentiable function such that $\int_0^x g(t)\,dt = x - \int_0^x tg(t)\,dt$, $x \ge 0$ and let $y = y(x)$ satisfy the differential equation $\frac{dy}{dx} - y\tan x = 2(x + 1)\sec x\, g(x)$, $x \in \left[0, \frac{\pi}{2}\right)$. If $y(0) = 0$, then $y\left(\frac{\pi}{3}\right)$ is equal to

(1) $\frac{2\pi}{3}$ (2) $\frac{4\pi}{3}$ (3) $\frac{2\pi}{3\sqrt{3}}$ (4) $\frac{4\pi}{3\sqrt{3}}$

Ans. **[2]**

Sol. $\int_0^x g(t)\,dt = x - \int_0^x tg(t)\,dt$

Differentiate both sides w.r.t x.

$g(x) = 1 - xg(x)$

$g(x)(1 + x) = 1$

$g(x) = \frac{1}{1+x}$

Now $\frac{dy}{dx} - y\tan x = 2(x + 1)\sec x\, g(x)$

$I.F = e^{-\int \tan x\,dx}$

$I.F = e^{-(-\ln\cos x)}$

$I.F = \cos x$

$y.\cos x = \int 2(x+1)\sec x.\frac{1}{(1+x)}\cos x\,dx$

$y\cos x = 2x + c$ …(i)

$y(0) = 0$

$\Rightarrow \boxed{c = 0}$

$\therefore y\cos x = 2x$

Put $x = \frac{\pi}{3}$

$\Rightarrow y\left(\frac{1}{2}\right) = \frac{2\pi}{3} \Rightarrow \boxed{y = \frac{4\pi}{3}}$

Q.20 If $y(x) = \begin{vmatrix} \sin x & \cos x & \sin x + \cos x + 1 \\ 27 & 28 & 27 \\ 1 & 1 & 1 \end{vmatrix}, x \in R,$

then $\dfrac{d^2y}{dx^2} + y$ is equal to

(1) 28 (2) 1 (3) –1 (4) 27

Ans. [3]

Sol. $f(x) = \begin{vmatrix} \sin x & \cos x & \sin x + \cos x + 1 \\ 27 & 28 & 27 \\ 1 & 1 & 1 \end{vmatrix}$

$f(x) = \sin x(1) - \cos x(0) + (\sin x + \cos x + 1)(-1)$

$f(x) = \sin x - \sin x - \cos x - 1$

$f(x) = -\cos x - 1$

$f'(x) = \sin x$

$f''(x) = \cos x$

$\Rightarrow f''(x) + f(x) = -\cos x - 1 + \cos x = -1$

Section-B: Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

Q.21 Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = 3\hat{i} + 2\hat{j} - \hat{k}$, $\vec{c} = \lambda\hat{j} + \mu\hat{k}$ and $\hat{d}$ be a unit vector such that $\vec{a} \times \hat{d} = \vec{b} \times \hat{d}$ and $\vec{c} \cdot \hat{d} = 1$. If $\vec{c}$ is perpendicular to $\vec{a}$, then $|3\lambda \hat{d} + \mu \vec{c}|^2$ is equal to ________

Ans. [5]

Sol. $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = 3\hat{c} + 2\hat{j} - \hat{k}$, $\vec{c} = \lambda\hat{j} + \mu\hat{k}$

$\vec{a} \times \vec{d} = \vec{b} \times \vec{d}$

$\Rightarrow (\vec{a} - \vec{b}) \times \vec{d} = 0$

$\Rightarrow \vec{d} \parallel (\vec{a} - \vec{b}) = (-2\hat{i} + \hat{j} + 2\hat{k})$

$\Rightarrow \hat{d} = \dfrac{-2}{3}\hat{i} - \dfrac{1}{3}\hat{j} + \dfrac{2}{3}\hat{k}$

Given $\hat{c}.\hat{d} = 1$

$\dfrac{-\lambda}{3} + \dfrac{2\mu}{3} = 1$

$\Rightarrow -\lambda + 2\mu = 3$ … (i)

Also $\vec{a}$ is perpendicular to $\vec{c}$

$\Rightarrow \vec{c} \cdot \vec{a} = 0$

$\Rightarrow \lambda + \mu = 0$

$\Rightarrow \boxed{\lambda = -\mu}$

Put in (i)

$\mu + 2\mu = 3 \Rightarrow 3\mu = 3$

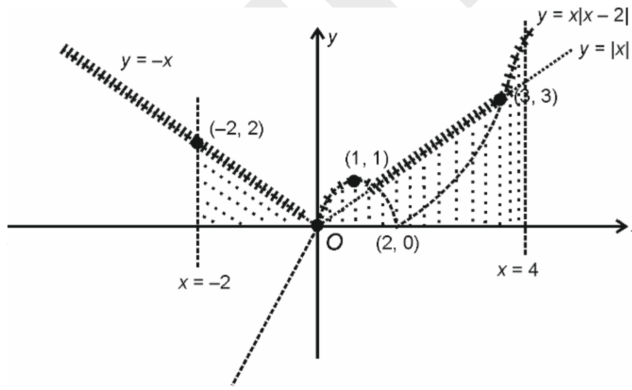
$\Rightarrow \mu = -1 \quad \Rightarrow \lambda = 1$

$\therefore \left|3\lambda\hat{d} + \mu\hat{c}\right|^2 \Rightarrow 9\lambda^2\left|\hat{d}\right|^2 + \mu^2|\vec{c}|^2 + 2.\,3.\,\lambda.\,\mu\, \vec{c} \cdot \vec{d}$

Q.22 The area of the region bounded by the curve y = max{|x|, x | x– 2 |}, the x-axis and the lines x = –2 and x = 4 is equal to ________

Ans. [12]

Sol.



$$\text{Area} = \frac{1}{2} \times 2 \times 2 + \int_0^1 (2x - x^2)\,dx + \int_1^3 x\,dx + \int_3^4 (x^2 - 2x)\,dx$$

$$= 2 + \left[\frac{2x^2}{2} - \frac{x^3}{3}\right]_0^1 + \left[\frac{x^2}{2}\right]_1^3 + \left[\frac{x^3}{3} - \frac{2x^2}{2}\right]_3^4$$

= 12 square units

Q.23 Let the product of the focal distances of the point $(4, 2\sqrt{3})$ on the hyperbola $H : \dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$ be 32. Let the length of the conjugate axis of H be p and the length of its latus rectum be q. Then $p^2 + q^2$ is equal to ______.

Ans. [120]

Sol. $P(4, 2\sqrt{3})$ lies on $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$

$\Rightarrow \dfrac{16}{a^2} - \dfrac{12}{b^2} = 1$ … (i)

$SP = e\left(4 - \dfrac{a}{e}\right)$, $S'P = \left(4 + \dfrac{a}{e}\right)$

$\Rightarrow$ SP · S′P $= 16e^2 - a^2 = 32$

$\because e^2 = 1 + \frac{b^2}{a^2}$

$\Rightarrow 16\left(1 + \frac{b^2}{a^2}\right) - a^2 = 32$ … (ii)

From (i) and (ii), $a^2 = 8$, $b^2 = 12$

$\Rightarrow \because p^2 + q^2 = (2b)^2 + \left(\frac{2b^2}{9}\right)^2 = 4b^2 + \frac{4b^2}{a^2}$

$= 48\left(1 + \frac{12}{8}\right) = 120$

Q.24 All five letter words are made using all the letters A, B, C, D, E and arranged as in an English dictionary with serial numbers. Let the word at serial number n be denoted by W_n. Let the probability $P(W_n)$ of choosing the word W_n satisfy $P(W_n) = 2P(W_n - 1)$, $n > 1$.

If $P(CDBEA) = \frac{2^\alpha}{2^\beta - 1}$, $\alpha, \beta \in N$, then $\alpha + \beta$ is equal to : _______

Ans. **[183]**

Sol. We are given that the probability of choosing W_n is :

$P(W_n) = 2P(W_{n-1})$ for $n > 1$

$\Rightarrow P(W_1) = p$, $P(W_2) = 2p$, $P(W_3) = 4p$, …, $P(W_n) = 2^{n-1}$ p

To find the value of p, we use :

$\sum_{n-1}^{120} P(W_n) = 1$ (total probability)

So, $p(1 + 2 + 2^2 + ... + 2^{119}) = 1$

$p(2^{120} - 1) = 1 \Rightarrow p = \frac{1}{2^{120} - 1}$

Thus, $P(W_n) = \frac{2^{n-1}}{2^{120} - 1}$ … (i)

Since the first letter of CDBEA is C

Words starting with A : 4! = 24

Words starting with B : 4! = 24

CA _ _ _: 3! = 6

CB _ _ _: 3! = 6

CDA _ _: 2! = 2

CDBA _ : 1! = 1

Total before CDBEA = 63

Position of CDBEA = 64^{th}

Putting in (i)

$P(CDBEA) = P(W_{64}) = \frac{2^{63}}{2^{120} - 1}$

Compare with the given form:

$P(CDBEA) = \frac{2^\alpha}{2^\beta - 1}$

So, $\alpha = 63$, $\beta = 120$

$\alpha + \beta = 63 + 120 = 183$

Q.25 If the number of seven-digit numbers, such that the sum of their digits is even, is $m \cdot n \cdot 10^n$, $m, n \in \{1, 2, 3, ..., 9\}$, then m + n is equal to _______.

Ans. **[14]**

Sol. When numbers are uniformly distributed, half of them have even digit sums and half have odd digits sums.

Number of 7-digit numbers with even digit sum

$= \frac{1}{2} . 9 . 10^6 = 9.5.10^5$

Note that $9. 5. 10^5 = m. n. 10^n$

m + n = 9 + 5 = 14

PHYSICS

Section-A: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Q.26 Match the **List-I** with **List-II**.

	List-I		List-II
A.	${}_0^1n + {}_{92}^{235}U \rightarrow {}_{54}^{140}Xe + {}_{38}^{94}Sr + 2{}_0^1n$	I.	Chemical reaction
B.	$2H_2 + O_2 \rightarrow 2H_2O$	II.	Fusion with +ve Q value
C.	${}_1^2H + {}_1^2H \rightarrow {}_2^3He + {}_0^1n$	III.	Fission
D.	${}_1^1H + {}_1^3H \rightarrow {}_1^2H + {}_1^2H$	IV.	Fusion with –ve Q value

Choose the **correct** answer from the options given below:

(1) A-III, B-I, C-IV, D-II

(2) A-II, B-I, C-IV, D-III

(3) A-III, B-I, C-II, D-IV

(4) A-II, B-I, C-III, D-IV

Ans. **[3]**

Sol. (A) is fission reaction

(B) is a chemical reaction

(C) deuteron + deuteron → helium 3 is exothermic process

(D) H + 3He → 2deuteron is endothermic process

Q.27 A particle is released from height S above the surface of the earth. At certain height its kinetic energy is three times its potential energy. The height from the surface of the earth and the speed of the particle at that instant are respectively.

(1) $\frac{S}{2}, \sqrt{\frac{3gS}{2}}$ (2) $\frac{S}{4}, \sqrt{\frac{3gS}{2}}$

(3) $\frac{S}{4}, \frac{3gS}{2}$ (4) $\frac{S}{2}, \frac{3gS}{2}$

Ans. **[2]**

Sol. PE = mgh

KE = mg (S – h)

given mg(S – h) = 3mgh

S = 4h

$h = \frac{S}{4}$

$U = \sqrt{\frac{2g(3S)}{4}} = \sqrt{\frac{3gS}{2}}$

Q.28 Consider following statements for refraction of light through prism, when angle of deviation is minimum.

A. The refracted ray inside prism becomes parallel to the base.

B. Larger angle prisms provide smaller angle of minimum deviation.

C. Angle of incidence and angle of emergence becomes equal.

D. There are always two sets of angle of incidence for which deviation will be same except at minimum deviation setting.

E. Angle of refraction becomes double of prism angle.

Choose the correct answer from the options given below:

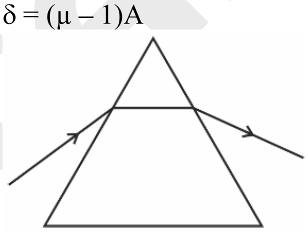
(1) A, B and E only (2) B, D and E only

(3) B, C and D only (4) A, C and D only

Ans. **[4]**

Sol. Because of symmetry where i = r, and $r = \frac{A}{2}$

$\delta = (\mu - 1)A$



(A) is correct

(B) is incorrect

(C) is correct

(D) is correct as $\delta = i + e - A$

(E) is incorrect

Q.29 The radiation pressure exerted by a 450 W light source on a perfectly reflecting surface placed at 2 m away from it, is

(1) 1.5×10^{-8} Pascals (2) 0

(3) 3×10^{-8} Pascals (4) 6×10^{-8} Pascals

Ans. **[4]**

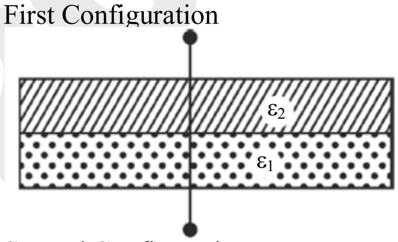
Sol. $P = \frac{2I}{C}$

$I = \frac{P}{A} = \frac{450}{4\pi \times 2^2}$

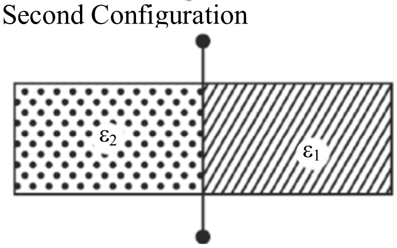
$\text{Pressure} = \frac{2 \times 450}{16\pi \times 3 \times 10^8} \approx 6 \times 10^{-8}$

Q.30 A parallel plate capacitor is filled equally (half) with two dielectrics of dielectric constants ε_1 and ε_2, as shown in figures. The distance between the plates is d and area of each plate is A. If capacitance in first configuration and second configuration are C_1 and C_2 respectively, then $\frac{C_1}{C_2}$ is

First Configuration



Second Configuration



(1) $\frac{4\varepsilon_1\varepsilon_2}{(\varepsilon_1 + \varepsilon_2)^2}$ (2) $\frac{\varepsilon_0(\varepsilon_1 + \varepsilon_2)}{2}$

(3) $\frac{\varepsilon_1\varepsilon_2}{\varepsilon_1 + \varepsilon_2}$ (4) $\frac{\varepsilon_1\varepsilon_2^2}{(\varepsilon_1 + \varepsilon_2)^2}$

Ans. **[1]**

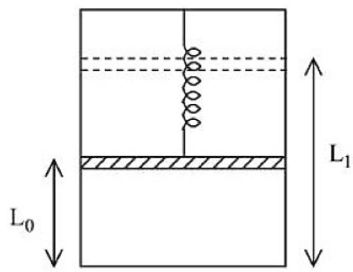
Sol. $C = \frac{k\varepsilon_0 A}{d}$

$C_1 = \frac{\varepsilon_0 A}{\frac{d}{2\varepsilon_1} + \frac{d}{2\varepsilon_2}} = \frac{\varepsilon_0 A}{d}\left\{\frac{1}{\frac{\varepsilon_2 + \varepsilon_1}{2\varepsilon_1\varepsilon_2}}\right\}$

$C_2 = \frac{\varepsilon_0}{d}\left(\varepsilon_1\frac{A}{2} + \varepsilon_2\frac{A}{2}\right) = \frac{\varepsilon_0 A}{d}\left\{\frac{\varepsilon_1}{2} + \frac{\varepsilon_2}{2}\right\}$

$\frac{C_1}{C_2} = \frac{2\varepsilon_1\varepsilon_2}{(\varepsilon_2 + \varepsilon_1)\frac{(\varepsilon_1 + \varepsilon_2)}{2}} = \frac{4\varepsilon_1\varepsilon_2}{(\varepsilon_1 + \varepsilon_2)^2}$

Q.31 A piston of mass M is hung from a massless spring whose restoring force law goes as $F = -kx^3$, where k is the spring constant of appropriate dimension. The piston separates the vertical chamber into two parts, where the bottom part is filled with 'n' moles of an ideal gas. An external work is done on the gas isothermally (at a constant temperature T) with the help of a heating filament (with negligible volume) mounted in lower part of the chamber, so that the piston goes up from a height L_0 to L_1, the total energy delivered by the filament is: (Assume spring to be in its natural length before heating)



(1) $nRT\ln\left(\frac{L_1}{L_0}\right) + Mg(L_1 - L_0) + \frac{k}{4}(L_1^4 - L_0^4)$

(2) $3nRT\ln\left(\frac{L_1}{L_0}\right) + 2Mg(L_1 - L_0) + \frac{k}{3}(L_1^3 - L_0^3)$

(3) $nRT\ln\left(\frac{L_1^2}{L_0^2}\right) + \frac{Mg}{2}(L_1 - L_0) + \frac{k}{4}(L_1^4 - L_0^4)$

(4) $nRT\ln\left(\frac{L_1}{L_0}\right) + Mg(L_1 - L_0) + \frac{3k}{4}(L_1^4 - L_0^4)$

Ans. **[None option matches]**

Sol. From energy conservation

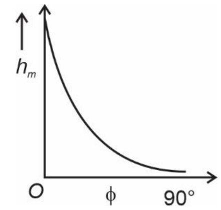
heat given = (ΔU + ΔW) for gas + change in G.P.E. of piston + energy stored in spring

$= 0 + \mu RT\ ln\frac{V_2}{V_1} + Mg(h_2 - h_1) + \frac{1}{4}k(x_2^4 - x_1^4)$

$= nRT\ ln\frac{L_1}{L_0} + Mg(L_1 - L_0) + \frac{k}{4}(L_1 - L_0)^4$

None option matches.

Q.32 The angle of projection of a particle is measured from the vertical axis as ϕ and the maximum height reached by the particle is h_m. Here h_m as function of ϕ can be presented as

(1)


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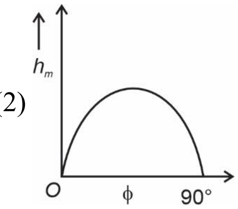
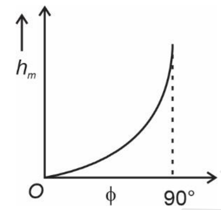
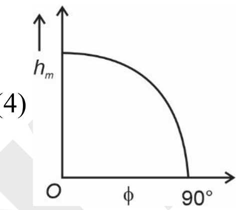


(3)


(4)


Ans. **[4]**

Sol. $h = \frac{u^2 \sin^2\theta}{2g}$ $\theta = 90° - \phi$

$\Rightarrow h = \frac{u^2\cos^2\theta}{2g}$

From ϕ = 0 to ϕ = 90

h decreases with $\cos^2$ function.

Q.33 A person measures mass of 3 different particles as 435.42 g, 226.3 g and 0.125 g. According to the rules for arithmetic operations with significant figures, the addition of the masses of 3 particles will be

(1) 661.8 g (2) 661.84 g
(3) 662 g (4) 661.845 g

Ans. **[1]**

Sol.
435.42
226.3
0.125
―――――
661.85

For addition, minimum decimal number is considered for reporting the measured value

⇒ 661.8

Q.34 During the melting of a slab of ice at 273 K at atmospheric pressure:

(1) Internal energy of ice-water system remains unchanged
(2) Internal energy of the ice-water system decreases
(3) Positive work is done by the ice-water system on the atmosphere
(4) Positive work is done on the ice-water system by the atmosphere

Ans. **[4]**

Sol. Melting of ice requires heat therefore internal energy increases.

But because of decrease in volume the work done on atmosphere is negative or atmosphere does positive work on ice-water system.

Q.35 The work function of a metal is 3 eV. The color of the visible light that is required to cause emission of photoelectrons is

(1) Red (2) Yellow
(3) Green (4) Blue

Ans. **[4]**

Sol. $\lambda = \dfrac{12400}{3}\text{Å} = 4133\ \text{Å}$

and $\lambda_{visible} \in (3800 - 7600)\text{Å}$
V I B G Y O R

Therefore, most appropriate response is blue.

Q.36 Match the **List-I** with **List-II**.

	List-I		List-II
A.	Gravitational constant	I.	$[LT^{-2}]$
B.	Gravitational potential energy	II.	$[L^2T^{-2}]$
C.	Gravitational potential	III.	$[ML^2T^{-2}]$
D.	Acceleration due to gravity	IV.	$[M^{-1}L^3T^{-2}]$

Choose the **correct** answer from the options given below:

(1) A-II, B-IV, C-III, D-I
(2) A-IV, B-III, C-II, D-I
(3) A-I, B-III, C-IV, D-II
(4) A-III, B-II, C-I, D-IV

Ans. **[2]**

Sol. (A) $F = \dfrac{Gm^2}{r^2} \equiv G = \dfrac{Fr^2}{m^2} = \dfrac{MLT^{-2}L^2}{M^2} = M^{-1}L^3T^{-2}$

$A \to IV$

(B) $G.P\,E \equiv \text{Energy} \equiv ML^2T^{-2}$

$B \to III$

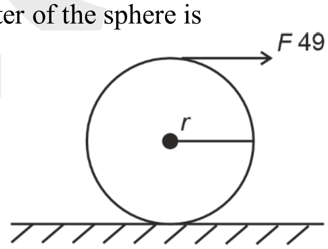
(C) $G.P = \dfrac{mgh}{m} = gh \equiv V^2 \equiv L^2\,T^{-2}$

$C \to II$

(D) $g \equiv LT^{-2}$

$D \to I$

Q.37 A force of 49 N acts tangentially at the highest point of a sphere (solid) of mass 20 kg, kept on a rough horizontal plane. If the sphere rolls without slipping, then the acceleration of the center of the sphere is



(1) 2.5 m/s^2 (2) 0.25 m/s^2
(3) 3.5 m/s^2 (4) 0.35 m/s^2

Ans. **[3]**

Sol. $\tau = I\alpha \Rightarrow 49 \times 2r = \dfrac{7}{5}\ mr^2\alpha$

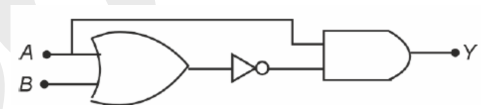
$49 \times 2r = \dfrac{7}{5}\,mr\ a$

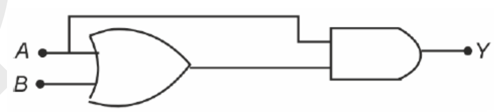
$\dfrac{49 \times 2 \times 5}{7 \times 20} = a$

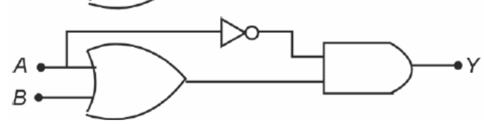
$a = 3.5\ m/s^2$

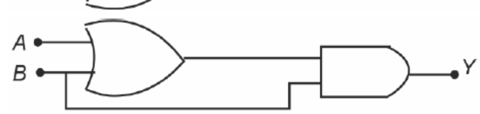
Q.38 Choose the correct logic circuit for the given truth table having inputs A and B.

Inputs		Output
A	B	Y
0	0	0
0	1	0
1	0	1
1	1	1

(1) 

(2) 

(3) 

(4) 

Ans. **[2]**

Sol. Option (1) = $(\overline{A + B})\cdot A$

$= (\bar{A} + \bar{B})\cdot A \equiv 0$

Option (2) = $(A + B)\cdot A$

$A\cdot A + A\cdot B = A$

Option (3) $(A + B)\cdot \bar{A}$

$A\cdot\bar{A} + \bar{A}\cdot B = \bar{A}B$

Option (4)

$(A + B)\cdot(B)$

$A\cdot B + B\cdot B = B$

$\Rightarrow$ option (2) matches the truth table

Q.39 The electrostatic potential on the surface of uniformly charged spherical shell of radius R = 10 cm is 120 V. The potential at the centre of shell, at a distance r = 5 cm from centre, and at a distance r = 15 cm from the centre of the shell respectively, are:

(1) 120V, 120V, 80V (2) 40V, 40V, 80V
(3) 0V, 120V, 40V (4) 0V, 0V, 80V

Ans. [1]

Sol. For $r \le R$ $V = \dfrac{kQ}{R}$

For $r > R$ $V = \dfrac{kQ}{r}$

(i) For r = 0 and r = 5 cm
(< R = 10 cm)
V = 120 V

(ii) For r = 15 cm $V = \dfrac{kQ}{R}\dfrac{R}{r}$

$= 120 \times \dfrac{10}{15}$

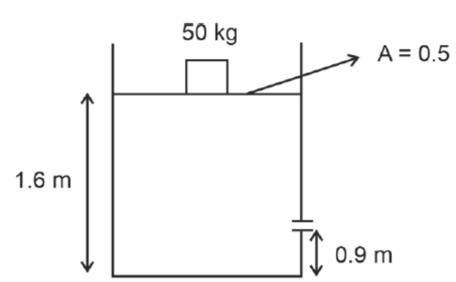
= 80 V

Q.40 Consider a completely full cylindrical water tank of height 1.6 m and of cross-sectional area 0.5 m^2. It has a small hole in its side at a height 90 cm from the bottom. Assume, the cross-sectional area of the hole to be negligibly small as compared to that of the water tank. If a load 50 kg is applied at the top surface of the water in the tank then the velocity of the water coming out at the instant when the hole is opened is: ($g = 10$ m/s^2)

(1) 3 m/s (2) 2 m/s (3) 5 m/s (4) 4 m/s

Ans. [4]

Sol.



$\Delta P = \dfrac{1}{2}\rho V^2$

$\dfrac{mg}{A} + \rho g \Delta h = \dfrac{1}{2}\rho v^2$

$\dfrac{500}{0.5} + 10^3 \times 10 \times 0.7 = \dfrac{1}{2} \times 10^3 v^2$

$1 + 7 = \dfrac{1}{2}v^2$

v = 4 m/s

Q.41 A gas is kept in a container having walls which are thermally non-conducting. Initially the gas has a volume of 800 cm^3 and temperature 27°C. The change in temperature when the gas is adiabatically compressed to 200 cm^3 is : (Take $\gamma = 1.5$; γ is the ratio of specific heats at constant pressure and at constant volume)

(1) 300 K (2) 522 K (3) 327 K (4) 600 K

Ans. [1]

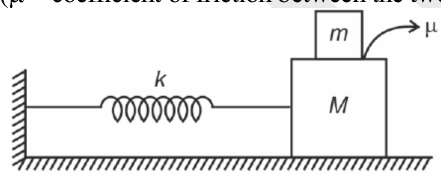
Sol. Being adiabatic process PV^γ = constant, where $\gamma = 1.5$ and also $TV^{\gamma-1}$ = constant

$300\{800\}^{0.5} = T\{200\}^{0.5}$

$T = 300 \times 2 = 600$ K

$\Delta T = 600 - 300 = 300$ K

Q.42 Two blocks of masses m and M, (M > m) are placed on a frictionless table as shown in figure. A massless spring with spring constant k is attached with the lower block. If the system is slightly displaced and released, then (μ = coefficient of friction between the two blocks)



A. The time period of small oscillation of the two blocks is $T = 2\pi\sqrt{\dfrac{(m+M)}{k}}$

B. The acceleration of the blocks is $a = -\dfrac{kx}{M+m}$ (x = displacement of the blocks from the mean position)

C. The magnitude of the frictional force on the upper block is $\dfrac{m\mu|x|}{M+m}$

D. The maximum amplitude of the upper block, if it does not slip, is $\dfrac{\mu(M+m)g}{k}$

E. Maximum frictional force can be $\mu(M+m)g$

Choose the **correct** answer from the options given below :

(1) B, C, D only (2) A, B, C only
(3) A, B, D only (4) C, D, E only

Ans. [3]

Sol. A. Assuming no slipping, $T = 2\pi\sqrt{\dfrac{m_{total}}{k}}$

A is correct.

B. Assuming no slipping, $a = \dfrac{|F|}{m}$

B is correct.

C. $f = (m)(a) = \dfrac{m \times kx}{m+M}$

C is correct.

D. For no slipping $\dfrac{kx_0}{m+M} \le \mu g$

D is correct.

E. $f_{max} = \mu mg$
E is incorrect.

Q.43 The radii of curvature for a thin convex lens are 10 cm and 15 cm respectively. The focal length of the lens is 12 cm. The refractive index of the lens material is

(1) 1.5 (2) 1.2 (3) 1.4 (4) 1.8

Ans. **[1]**

Sol. For biconvex

$R_1 = 10$

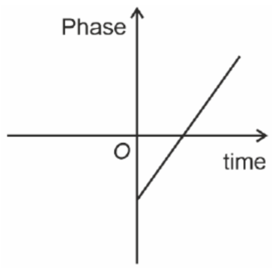
$R_2 = 15$

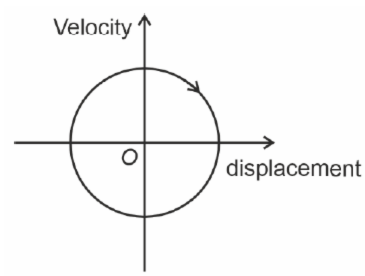
$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

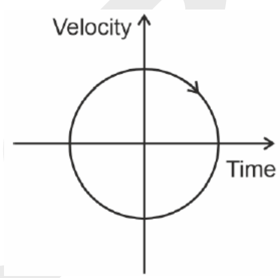
$$\frac{1}{12} = (\mu - 1)\left(\frac{1}{10} + \frac{1}{15}\right) = (\mu - 1)\left(\frac{1}{6}\right)$$

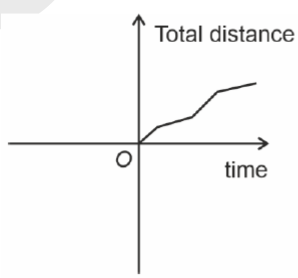
$$\mu - 1 = \frac{1}{2} \Rightarrow \mu = 1.5$$

Q.44 Which of the following curves possibly represent one-dimensional motion of a particle?

A.


B.


C.


D.


Choose the **correct** answer from the options given below :

(1) A and B only (2) A, C and D only

(3) A, B and C only (4) A, B and D only

Ans. **[4]**

Sol. A. Phase increase with time in SHM

$\Rightarrow$ Correct

B. Velocity and displacement are related in elliptical/circular relation

i.e. $\frac{v^2}{v_0^2} + \frac{x^2}{x_0^2} = 1$

$\Rightarrow$ Correct

C. At same time particle can't have two velocities

$\Rightarrow$ Incorrect

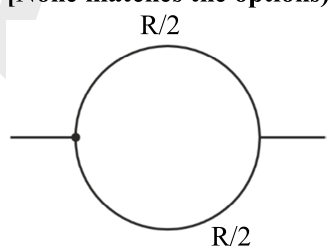
D. Distance always increases

$\Rightarrow$ Correct

Q.45 A wire of length 25 m and cross-sectional area 5 mm^2 having resistivity 2×10^{-6} Ω m is bent into a complete circle. The resistance between diametrically opposite points will be

(1) 100 Ω (2) 50 Ω (3) 12.5 Ω (4) 25 Ω

Ans. **[None matches the options)]**

Sol.


Let R be total resistance across ends of wire, then

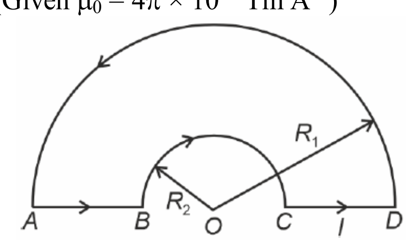
$$R_{eq} = \frac{R}{4} = \frac{\rho\ell}{4A} = \frac{2 \times 10^{-6} \times 25}{4 \times 5 \times 10^{-6}} = 2.5\ \Omega$$

None matches the options.

Section-B: Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

Q.46 A loop ABCDA, carrying current I = 12 A, is placed in a plane, consists of two semi-circular segments of radius $R_1 = 6\pi$ m and $R_2 = 4\pi$ m. The magnitude of the resultant magnetic field at center O is $k \times 10^{-7}$ T. The value of k is ________.

(Given $\mu_0 = 4\pi \times 10^{-7}$ Tm A^{-1})



Ans. [1]

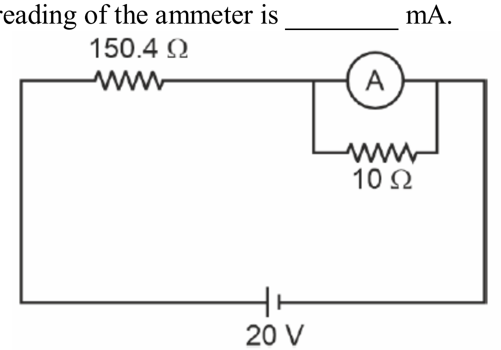
Sol. $B = \frac{\mu_0 I}{2r}\frac{\theta}{2\pi}$ for an arc

For semicircle $B = \frac{\mu_0 I}{4r}$

$$B_{net} = \frac{\mu_0 I}{4\{4\pi\}} - \frac{\mu_0 I}{4(6\pi\}} = \frac{\mu_0 I}{4\pi}\left[\frac{1}{4} - \frac{1}{6}\right]$$

$$= 10^{-7} \times 12 \times \frac{2}{24} = 10^{-7}\ T \Rightarrow k = 1$$

Q.47 In the figure shown below, a resistance of 150.4 Ω is connected in series to an ammeter A of resistance 240 Ω. A shunt resistance of 10 Ω is connected in parallel with the ammeter. The reading of the ammeter is ________ mA.



Ans. [5]

Sol. $$i_0 = \frac{20}{150.4 + \frac{240 \times 10}{250}}$$

$$= \frac{20}{150.4 + 9.6} = \frac{20}{160} = \frac{1}{8} A$$

$$i_A = \frac{1}{8} \times \frac{10}{250} \times 1000\ mA$$

$= 5$ mA

Q.48 Three identical spheres of mass m, are placed at the vertices of an equilateral triangle of length a. When released, they interact only through gravitational force and collide after a time T = 4 seconds. If the sides of the triangle are increased to length 2a and also the masses of the spheres are made 2m, then they will collide after ________ seconds.

Ans. [8]

Sol. As limiting case of elliptical path is straight line therefore proportionality $T^2 \propto \frac{a^3}{M}$ will holds from

$$\left(T^2 = \frac{4\pi^2}{GM}r^3\right)$$

$$\frac{4^2}{T^2} = \frac{a^3}{m}\frac{2m}{(2a)^3} = \frac{2}{8} = \frac{1}{4}$$

$$\frac{4}{T} = \frac{1}{2}$$

T = 8 seconds

Q.49 Two coherent monochromatic light beams of intensities 4I and 9I are superimposed. The difference between the maximum and minimum intensities in the resulting interference pattern is xI. The value of x is ________.

Ans. [24]

Sol. $I_{max} = \left(\sqrt{4I} + \sqrt{9I}\right)^2 = 25I$

$I_{min} = \left(\sqrt{4I} - \sqrt{9I}\right)^2 = I$

$\Delta I = 24I$

Q.50 A 4.0 cm long straight wire carrying a current of 8 A is placed perpendicular to a uniform magnetic field of strength 0.15 T. The magnetic force on the wire is ________ mN.

Ans. [48]

Sol. $F = i\,l\,B$

$= 8 \times 4 \times 10^{-2} \times 0.15$ newton

$= 8 \times 4 \times 10^{-2} \times 0.15 \times 10^3$ mN

$= 32 \times 1.5$ mN

$= 48$ mN

CHEMISTRY

Section-A: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct..

Q.51 2 moles each of ethylene glycol and glucose are dissolved in 500 g of water. The boiling point of the resulting solution is:

(Given : Ebullioscopic constant of water $= 0.52\ K\ kg\ mol^{-1}$)

(1) 377.3 K (2) 375.3 K

(3) 379.2 K (4) 277.3 K

Ans. [1]

Sol. $\Delta T_b = K_b(m)$

$$= (0.52)\left(\frac{4}{0.5}\right)$$

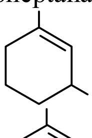
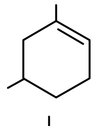
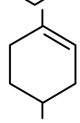
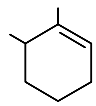
$= (0.52)(8)$

$= 4.16$ K

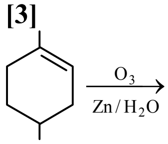
$T_b = 373.15 + 4.16$

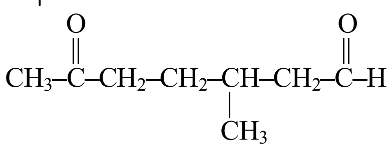
$= 377.3$ K

Q.52 Which compound would give 3-methyl-6-oxoheptanal upon ozonolysis?

(1) 
(2) 
(3) 
(4) 

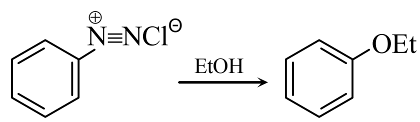
Ans. **[3]**

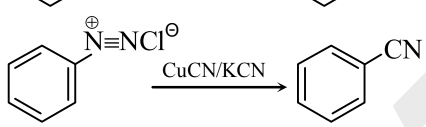
Sol. 

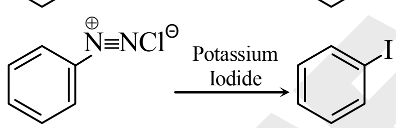


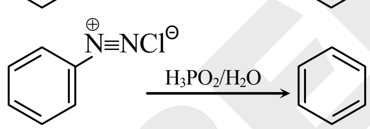
3-methyl-6-oxoheptanal

Q.53 In the following reactions, which one is NOT correct?

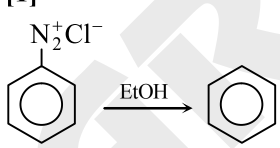
(1) 

(2) 

(3) 

(4) 

Ans. **[1]**

Sol. 

Option (1) is incorrect

Q.54 Among 10^{-9} g (each) of the following elements, which one will have the highest number of atoms? Element: Pb, Po, Pr and Pt

(1) Po (2) Pb (3) Pt (4) Pr

Ans. **[4]**

Sol. Highest atoms in case where atomic mass is lowest

Po = 209

Pb = 207

Pt = 195

Pr = 140

Q.55 Correct order of limiting molar conductivity for cations in water at 298 K is :

(1) $H^+ > Na^+ > K^+ > Ca^{2+} > Mg^{2+}$

(2) $Mg^{2+} > H^+ > Ca^{2+} > K^+ > Na^+$

(3) $H^+ > Ca^{2+} > Mg^{2+} > K^+ > Na^+$

(4) $H^+ > Na^+ > Ca^{2+} > Mg^{2+} > K^+$

Ans. **[3]**

Sol. λ_m^0 order

$H^+ > Ca^{2+} > Mg^{2+} > K^+ > Na^+$

Q.56 Which of the following postulate of Bohr's model of hydrogen atom is not in agreement with quantum mechanical model of an atom?

(1) The electron in a H atom's stationary state moves in a circle around the nucleus

(2) An atom in a stationary state does not emit electromagnetic radiation as long as it stays in the same state.

(3) When an electron makes a transition from a higher energy stationary state to a lower energy stationary state, then it emits a photon of light.

(4) An atom can take only certain distinct energies E_1, E_2, E_3, etc. These allowed states of constant energy are called the stationary states of atom.

Ans. **[1]**

Sol. The electron in a H-atom's stationary state moves in a circle around the nucleus. This is not in agreement with Quantum Mechanical model of an atom.

Q.57 Number of molecules from below which cannot give iodoform reaction is :

Ethanol, Isopropyl alcohol, Bromoacetone, 2-Butanol, 2-Butanone, Butanal, 2-Pentanone, 3- Pentanone, Pentanal and 3-Pentanol.

(1) 2 (2) 5 (3) 3 (4) 4

Ans. **[4]**

Sol. The compounds which doesn't give iodoform test are –

Butanal, 3-pentanone, pentanal, 3-pentanol

Q.58 Given below are two statements :

Statement (I) : A catalyst cannot alter the equilibrium constant (K_C) of the reaction, temperature remaining constant.

Statement (II) : A homogenous catalyst can change the equilibrium composition of a system, temperature remaining constant.

In the light of the above statements, choose the correct answer from the options given below.

(1) Both **Statement-I** and **Statement-II** are false

(2) Both **Statement-I** and **Statement-II** are true

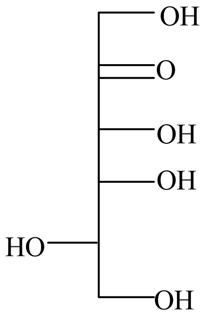
(3) **Statement-I** is true but **Statement-II** is false

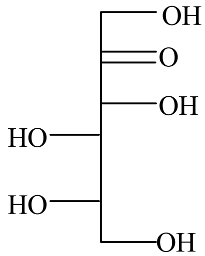
(4) **Statement-I** is false but **Statement-II** is true

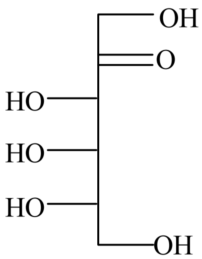
Ans. [3]

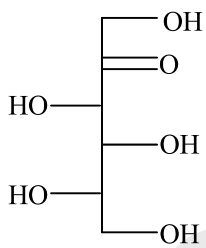
Sol. Catalyst doesn't change K_{eq} catalyst doesn't change composition statement-I is true But statement-II is false.

Q.59 Which of the following is the correct structure of L-Fructose?

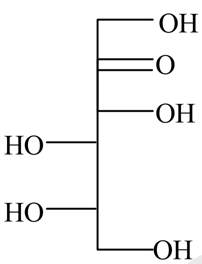
(1)


(2)


(3)


(4)


Ans. [2]

Sol.


Q.60 Match the **List-I** with **List-II**.

	List-I (Molecules/ion)		List-II (Hybridisation of central atmn)
A.	PF_5	I.	dsp^2
B.	SF_6	II.	sp^3d
C.	$Ni(CO)_4$	III.	sp^3d^2
D.	$[PtCl_4]^{2-}$	IV.	sp^3

Choose the **correct** answer from the options given below:

(1) A-III, B-I, C-IV, D-II
(2) A-II, B-III, C-IV, D-I
(3) A-I, B-II, C-III, D-IV
(4) A-IV, B-I, C-II, D-III

Ans. [2]

Sol. (A) $PF_5 - sp^3d$ (II) (B) $SF_6 - sp^3d^2$ (III)
(C) $Ni(CO)_4 - sp^3$ (IV) (D) $[PtCl_4]^{2-} - dsp^2$ (I)

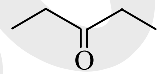
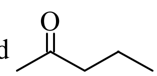
Q.61 In the following system, $PCl_5(g) \rightleftharpoons PCl_3(g) + Cl_2(g)$ at equilibrium, upon addition of xenon gas at constant T & p, the concentration of

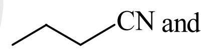
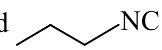
(1) Cl_2 will decrease
(2) PCl_3 will increase
(3) PCl_5 will increase
(4) PCl_5, PCl_3 & Cl_2 remain constant

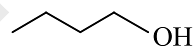
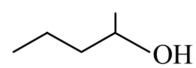
Ans. [2]

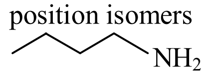
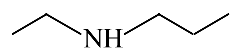
Sol. According to Le-Chatelier's principle
Reaction will shift in forward direction.
Cl_2 will increase
PCl_3 will increase
PCl_5 will decrease

Q.62 Identify the correct statements from the following.

A.  and  are metamers

B.  and  are functional isomers

C.  and  are position isomers

D.  and  are homologous

Choose the **correct** answer from the options given below:

(1) A, B & C Only (2) B & C Only
(3) A & B Only (4) C & D Only

Ans. [3]

Sol. A and B are correct. C ⇒ Homologous

Q.63 In a reaction A + B → C, initial concentrations of A and B are related as $[A]_0 = 8[B]_0$. The half lives of A and B are 10 min and 40 min, respectively, If they start to disappear at the same time, both following first order kinetics, after how much time will the concentration of both the reactants be same?

(1) 40 min (2) 20 min
(3) 80 min (4) 60 min

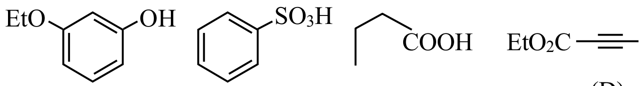
Ans. [1]

Sol. $\underset{8B_0}{A} \xrightarrow{10} 4B_0 \xrightarrow{10} 2B_0 \xrightarrow{10} B_0 \xrightarrow{10} \frac{B_0}{2}$

$\underset{B_0}{B} \xrightarrow{40} \frac{B_0}{2}$

t = 40

Q.64 The least acidic compound, among the following is



(A) (B) (C) (D)

(1) A (2) D (3) C (4)

Ans. [2]

Sol. Terminal alkyne is least Acidic among given compound

Q.65 Which of the following properties will change when system containing solution 1 will become solution 2

10 mol of solute x + 10 L of water	→	1 L of solution 1 + 1 mol of solute x + 1 L of water
(Solution 1)		(Solution 2)

(1) Density (2) Concentration
(3) Gibbs free energy (4) Mol heat capacity

Ans. [3]

Sol. Gibbs free energy will change.

Q.66 The metal ions that have the calculated spin only magnetic moment value of 4.9 B.M. are

A. Cr^{2+} B. Fe^{2+}
C. Fe^{3+} D. Co^{2+}
E. Mn^{3+}

Choose the correct answer from the options given below:

(1) B and E only (2) A, B and E only
(3) A, D and E only (4) A, C and E only

Ans. [2]

Sol. $\mu = 4.9$ BM
n = 4 for Cr^{2+}; Fe^{2+}; Mn^{3+} (where n = unpaired electron)

Q.67 Given below are two statements:

Statement I: The N – N single bond is weaker and longer than that of P – P single bond.
Statement II: Compounds of group 15 elements in +3 oxidation states readily undergo disproportionation reactions

In the light of the above statements, choose the correct answer from the options given below

(1) Statement I is false but Statement II is true
(2) Statement I is true but Statement II is false
(3) Both Statement I and Statement II are true
(4) Both Statement I and Statement II are false

Ans. [4]

Sol. Bond energy : (N – N) < (P – P)
Bond length : (N – N) < (P – P)
Statement-I is false.
Not all elements of Group-15 undergo disproportionation reaction.
Statement-II is false.

Q.68 Which of the following statements are correct?

A. The process of adding an electron to a neutral gaseous atom is always exothermic.
B. The process of removing an electron from an isolated gaseous atom is always endothermic.
C. The 1st ionization energy of boron is less than that of beryllium.
D. The electronegativity of C is 2.5 in CH_4 and CCl_4
E. Li is the most electropositive among elements of group I.

Choose the correct answer from the options given below:

(1) B and D Only
(2) B, C and E Only
(3) A, C and D Only
(4) B and C Only

Ans. [4]

Sol. B and C are correct
IE : B < Be

Q.69 The correct order of the complexes $[Co(NH_3)_5(H_2O)]^{3+}$ (A), $[Co(NH_3)_6]^{3+}$ (B), $[Co(CN)_6]^{3-}$ (C) and $[CoCl(NH_3)_5]^{2+}$ (D) in terms of wavelength of light absorbed is

(1) C > B > A > D (2) D > C > B > A
(3) C > B > D > A (4) D > A > B > C

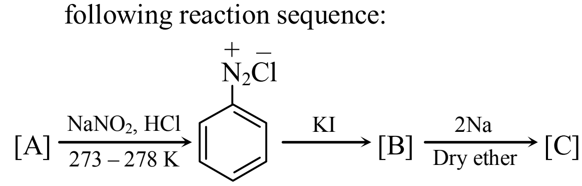
Ans. [4]

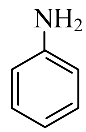
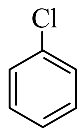
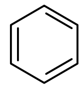
Sol. D > A > B > C

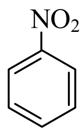
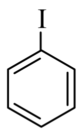
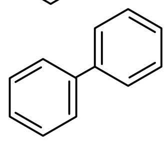
$$\Delta_o = \frac{hc}{\lambda}$$

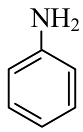
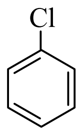
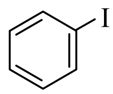
More the value of Δ_o more energy will be obseved Δ_o increases and λ decreases.

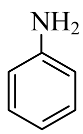
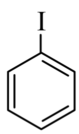
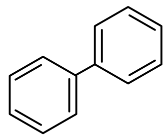
Q.70 Identify [A], [B] and [C], respectively in the following reaction sequence:



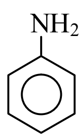
(1) [A]  [B]  [C] 

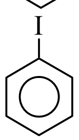
(2) [A]  [B]  [C] 

(3) [A]  [B]  [C] 

(4) [A]  [B]  [C] 

Ans. **[4]**

Sol. A : 

B : 

C : Ph – Ph

Section-B: Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

Q.71 The number of optical isomers exhibited by the iron complex (A) obtained from the following reaction is _____.

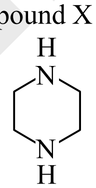
$FeCl_3 + KOH + H_2C_2O_4 \rightarrow A$

Ans. **[2]**

Sol. Complex obtained is $[Fe(C_2O_4)_3]^{3-}$

Total isomers = 2 (d/l form)

Q.72 During estimation of nitrogen by Dumas' method of compound X (0.42 g)



X

_______ mL of N_2 gas will be liberated at STP. (nearest integer)

(Given molar mass in g mol^{-1} : C : 12, H : 1, N : 14)

Ans. **[109]**

Sol. Formula of compound = $C_4H_{10}N_2$

= 86

Moles of compound = 0.00488

Moles of N_2 = 0.00488

Volume of N_2 = 0.1093 L

= 109.3 mL

Nearest integer = 109

Q.73 Given :

$\Delta H^{\ominus}_{sub}$ [C(graphite)] = 710 kJ mol^{-1}

$\Delta_{C-H}H^{\ominus}$ = 414 kJmol^{-1}

$\Delta_{H-H}H^{\ominus}$ = 436 kJmol^{-1}

$\Delta_{C=C}H^{\ominus}$ = 611 kJmol^{-1}

The $\Delta H^{\ominus}_f$ for $CH_2{=}CH_2$ is ____ kJ mol^{-1} (nearest integer value)

Ans. **[25]**

Sol. $\Delta H^{\ominus}_f = 2(710) + 2 \times 436 - 611 - 4(414)$

= 1420 + 872 – 611 – 1656

= 25 kJ $mole^{-1}$

Q.74 0.5 g of an organic compound on combustion gave 1.46 g of CO_2 and 0.9 g of H_2O. The percentage of carbon in the compound is _____. (Nearest integer)

[Given : Molar mass (in g mol^{-1}) C : 12, H : 1, O : 16]

Ans. **[80]**

Sol. Mass of C = $\frac{1.46}{44} \times 12 = 0.398$ g

% by mass of C = $\frac{0.398}{0.5} \times 100$

= 79.63%

Nearest integer = 80

Q.75 Consider the following reactions

$A + \underset{\text{Little amount}}{NaCl} + H_2SO_4 \rightarrow CrO_2Cl_2$ + Side Products

$CrO_2Cl_{2(Vapour)} + NaOH \rightarrow B + NaCl + H_2O$

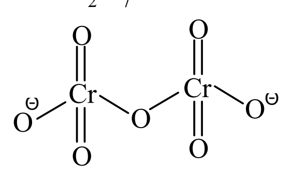
$B + H^+ \rightarrow C + H_2O$

The number of terminal 'O' present in the compound 'C' is _____.

Ans. **[6]**

Sol. B : CrO_4^{2-}

C : $Cr_2O_7^{2-}$



Terminal oxygen atoms = 6