CAREER POINT

JEE Main Online Exam 2025

Questions & Solution

03rd April 2025 | Evening

MATHEMATICS

Section-A: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

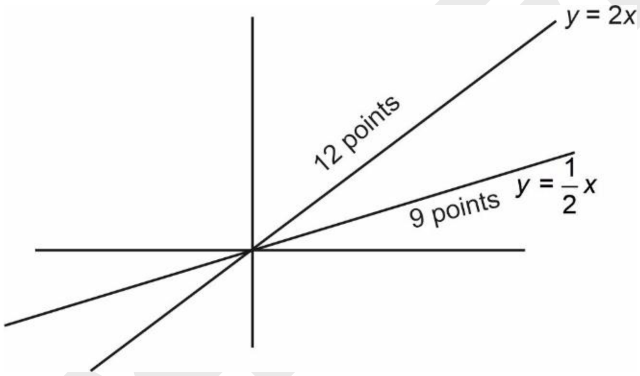
Q.1 Line L_1 of slope 2 and line L_2 of slope $\frac{1}{2}$ intersect at the origin O. In the first quadrant, $P_1, P_2 \ldots P_{12}$ are 12 points on line L_1 and $Q_1, Q_2 \ldots, Q_9$ are 9 points on line L_2. Then the total number of triangles, that can be formed having vertices at three of the 22 points O, $P_1, P_2 \ldots, P_{12}, Q_1 Q_2, \ldots Q_9$, is:

(1) 1080 (2) 1188

(3) 1134 (4) 1026

Ans. [3]

Sol. Total triangles



(2 points as $y = \frac{x}{2}$, 1 point on $y = \frac{x}{2}$)

+2(points and $y = \frac{x}{2}$, 1 point on y = 2x)

+ (1 point on y = 2x, 1) point on $y = \frac{x}{2}$ and origin)

$= {}^9C_2, {}^{12}C_1 + {}^9C_1\ {}^{12}C_2 + {}^9C_1 . {}^{12}C_1 . {}^1C_1$

= 1134

Q.2 Let the Mean and Variance of five observations $x_1 = 1, x_2 = 3, x_3 = a, x_4 = 7$ and $x_5 = b$, $a > b$, be 5 and 10 respectively. Then the Variance of the observations $n + x_n$, $n = 1.2, \ldots, 5$ is

(1) 16.4 (2) 16 (3) 17.4 (4) 17

Ans. [2]

Sol. $5 = \frac{1+3+a+7+b}{5}$

$\Rightarrow a + b = 14$

$\frac{1+9+a^2+49-b^2}{5} - (5)^2 = 10$

$a^2 + b^2 = 116$

$\Rightarrow a = 10, b = 4$

New digits : 2, 5, 13, 11, 9

$\text{Var} = \frac{4+26+169+121+81}{5} - 64$

Var = 16

Q.3 Let the equation $x(x + 2)(12 - k) = 2$ have equal roots. Then the distance of the point $\left(k, \frac{k}{2}\right)$ from the line $3x + 4y + 5 = 0$ is

(1) $5\sqrt{3}$ (2) 15 (3) $15\sqrt{5}$ (4) 12

Ans. [2]

Sol. $x^2 + 2x - \frac{2}{12-k} = 0$

D = 0

$4 - 4\left(\frac{2}{12-k}\right) = 0$

$1 + \frac{2}{12-k} = 0$

$\Rightarrow k = 14$

Point (14, 7)

$\text{Distance} = \left|\frac{3(14)+4(7)+5}{5}\right|$

d = 15

Option (2) is correct

Q.4 The integral $\int_0^{\pi} \frac{8x\,dx}{4\cos^2 x + \sin^2 x}$ is equal to

(1) π^2 (2) $\frac{3\pi^2}{2}$ (3) $4\pi^2$ (4) $2\pi^2$

Ans. [4]

Sol. $I = \int_0^{\pi} \frac{8x}{4\cos^2 x + \sin^2 x}dx$ … (1)

$I = \int_0^{\pi} \frac{8(\pi - x)}{4\cos^2(\pi - x) + \sin^2(\pi - x)}dx$

$I = \int_0^{\pi} \frac{8(\pi - x)}{4\cos^2 x + \sin^2 x}dx$ … (2)

Adding (1) and (2)

$2I = 8\pi \int_0^{\pi} \frac{1}{4\cos^2 x + \sin^2 x}dx$

$I = 4\pi \times 2\int_0^{\pi} \frac{\sec^2 x}{4\tan^2 x}dx$

Put tanx = t

$\sec^2 x\,dx = dt$

$I = 8\pi \int_0^{\infty} \frac{dt}{4+t^2}$

$I = 8\pi \frac{1}{2}\left(\tan^{-1}\frac{t}{2}\right)_0^{\infty}$

$I = 4\pi\left(\frac{\pi}{2}\right)$

$\boxed{I = 2\pi^2}$

Q.5 If the domain of the function $f(x) = \log_7(1 - \log_4(x^2 - 9x + 18))$ is $(\alpha, \beta) \cup (\gamma, \delta)$, then $\alpha + \beta + \gamma + \delta$ is equal to

(1) 17 (2) 18 (3) 15 (4) 16

Ans. [2]

Sol. $1 - \log_4(x^2 - 9x + 18) > 0$

$\log_4(x^2 - 9x + 18) < 1$

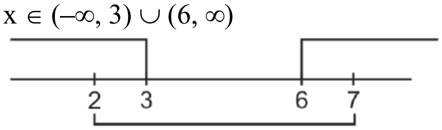
$x^2 - 9x + 18 < 4$

$x^2 - 9x + 14 < 0$

$x \in (2, 7)$

$x^2 - 9x + 18 > 0$

$x \in (-\infty, 3) \cup (6, \infty)$



$x \in (2, 3) \cup (6, 7)$

$\alpha + \beta + \gamma + \delta = 18$

Q.6 Let $f : R \to R$ be a function defined by $f(x) = ||x + 2| - 2|x||$. If m is the number of points of local minima and n is the number of points of local maxima of f, then m + n is

(1) 3 (2) 2 (3) 4 (4) 5

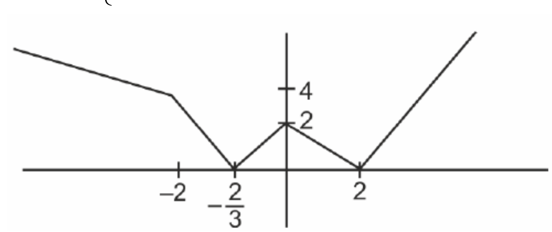
Ans. [1]

Sol. $f(x) = \begin{cases} |-x-2+2x| & x \le -2 \\ |x+2+2x| & -2 \le x \le 0 \\ |x+2-2x| & x \ge 0 \end{cases}$

$f(x) = \begin{cases} |x-2| & x \le -2 \\ |3x+2| & -2 < x \le 0 \\ |2-x| & x > 0 \end{cases}$

$f(x) = \begin{cases} 2-x & x \le -2 \\ -3x-2 & -2 < x \le -\frac{2}{3} \\ 3x+2 & -\frac{2}{3} < x \le 0 \end{cases}$

$f(x) = \begin{cases} 2-x & x \le -2 \\ -3x-2 & -2 < x \le -\frac{2}{3} \\ 3x+2 & -\frac{2}{3} < x \le 0 \\ 2-x & 0 < x < 2 \\ x-2 & x \ge 2 \end{cases}$



No. of maxima = 1

No. of minima = 2

m = 2

n = 1

m + n = 3

Q.7 If the probability that the random variable X takes the value x is given by $P(X = x) = k(x + 1).3^{-x}$, x = 0, 1, 2, 3…, where k is a constant, then $P(X \ge 3)$ is equal to

(1) $\frac{4}{9}$ (2) $\frac{7}{27}$ (3) $\frac{1}{9}$ (4) $\frac{8}{27}$

Ans. [3]

Sol. $s = \frac{k}{3^0} + \frac{2k}{3} + \frac{3k}{3^2} + \ldots$

$\frac{s}{3} = \frac{k}{3} + \frac{2k}{3^2} + \ldots$

$$s = \frac{s}{3} = k + \frac{k}{3} + \frac{k}{3^2} + ...$$

$$\frac{2s}{3} = k\left(1 + \frac{1}{3} + \frac{1}{3^2} + ...\right)$$

$$\frac{2s}{3} = k \times \frac{1}{1 - \frac{1}{3}} = \frac{3k}{2}$$

$$s = \frac{9k}{4} = 1 \qquad \text{(Total probability)}$$

$$k = \frac{4}{9}$$

$$P(x \geq 3) = 1 - (P(x = 0) + P(x = 1) + P(x = 2))$$

$$= 1 - \left(k + \frac{2k}{3} + \frac{3k}{3^2}\right)$$

$$= 1 - 2k$$

$$= 1 - 2 \times \frac{4}{9} = \frac{1}{9}$$

Q.8 Each of the angles β and γ that a given line makes with the positive y and z-axes, respectively, is half of the angle that this line makes with the positive x-axes. Then the sum of all possible values of the angle β is

(1) π (2) $\frac{3\pi}{4}$ (3) $\frac{\pi}{2}$ (4) $\frac{3\pi}{2}$

Ans. [2]

Sol. $\beta = \gamma = \frac{\alpha}{2}$

$\cos^2\alpha + \cos^2\beta \pm \cos^2y = 1$

$\cos^2 2\beta + \cos^2\beta + \cos^2\beta = 1$

$(2\cos^2\beta - 1)^2 + (2\cos^2\beta - 1) = 0$

$(2\cos^2\beta - 1)\,[(2\cos^2\beta - 1 + 1] = 0$

$\cos^2\beta = \frac{1}{2}$ or $\cos^2\beta = 0$

$\therefore \beta = \frac{\pi}{4}, \frac{\pi}{2}$

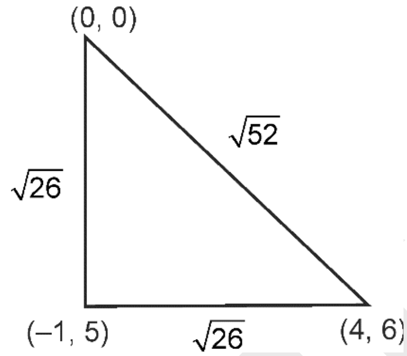
$\therefore$ sum $= \frac{\pi}{4} + \frac{\pi}{2} = \frac{3\pi}{4}$

Q.9 If the four distinct points (4, 6), (–1, 5), (0, 0) and (k, 3k) lie on a circle of radius r, then $10k + r^2$ is equal to

(1) 33 (2) 32 (3) 34 (4) 35

Ans. [4]

Sol.



$2r = \sqrt{52}$

$4r^2 = 52$

$2r^2 = 26$

$r^2 = 13$

Eqn of circle is

$x(x - 4) + y(y - 6) = 0$

$k(k - 4) + 3k(3k - 6) = 0$

$k^2 - 4k + 9k^2 - 18k = 0$

$10k^2 = 22k$

$10k = 22$

$\therefore 10k + r^2 = 35$

Q.10 The sum $1 + \frac{1+3}{2!} + \frac{1+3+5}{3!} + \frac{1+3+5+7}{4!} + ...$ upto ∞ terms, is equal to

(1) 6e (2) 2e (3) 3e (4) 4e

Ans. [2]

Sol. $T_r = \frac{r^2}{r!} = \frac{r}{(r-1)!} = \frac{(r-1)+1}{(r-1)!} = \frac{1}{(r-2)!} + \frac{1}{(r-1)!}$

$$\sum_{r=1}^{\infty} T_r = \sum_{r=1}^{\infty} \frac{1}{(r-2)!} + \frac{1}{(r-1)!} = e + e = 2e$$

Q.11 Let f be a function such that $f(x) + 3f\left(\frac{24}{x}\right) = 4x$, $x \neq 0$. Then f(3) + f(8) is equal to

(1) 10 (2) 12 (3) 13 (4) 11

Ans. [4]

Sol. $f(x) + 3f\left(\frac{24}{x}\right) = 4x, x \neq 0$ …(1)

replace x by $\frac{24}{x}$

$$f\left(\frac{24}{x}\right) + 3f\left(\frac{24}{\frac{24}{x}}\right) = 4\left(\frac{24}{x}\right) = \frac{96}{x} \quad ...(2)$$

$3 \times (2) - (1)$

$\Rightarrow 8f(x) = \frac{96.3}{x} - 4x \Rightarrow f(x) = \frac{36}{x} - \frac{x}{2}$

$f(3) + f(8) = \left(12 - \frac{3}{2}\right) + \left(\frac{36}{8} - 4\right)$

$= 8 + \frac{36}{8} - \frac{12}{8} = 11$

Q.12 The number of solutions of the equation $(4-\sqrt{3})\sin x - 2\sqrt{3}\cos^2 x = \frac{-4}{1+\sqrt{3}}, x \in \left[-2\pi, \frac{5\pi}{2}\right]$ is

(1) 6 (2) 4 (3) 3 (4) 5

Ans. [4]

Sol. $(4-\sqrt{3})\sin x - 2\sqrt{3}\cos^2 x = \frac{-4}{(1+\sqrt{3})}$

Let $\sin x = t \Rightarrow \cos^2 x = 1 - t^2$

$(4-\sqrt{3})t - 2\sqrt{3}(1-t^2) = \frac{-4}{(1+\sqrt{3})}$

$\Rightarrow 2\sqrt{3}t^2 + (4-\sqrt{3})t - 2\sqrt{3} = \frac{-4}{1+\sqrt{3}} = 0$

$2\sqrt{3}t^2 + (4-\sqrt{3})t + \frac{(-2\sqrt{3}-2)}{(1+\sqrt{3})} = 0$

$2\sqrt{3}t^2 + (4-\sqrt{3})t - 2 = 0$

$\Rightarrow t = \frac{(\sqrt{3}-4) \pm (\sqrt{19 - 8\sqrt{3} + 8(2\sqrt{3})}}{4\sqrt{3}}$

$t = \frac{(\sqrt{3}-4) \pm \sqrt{19+8\sqrt{3}}}{4\sqrt{3}} = \frac{(\sqrt{3}-4) \pm (\sqrt{3}+4)}{4\sqrt{3}}$

$= \left(\frac{2\sqrt{3}}{4\sqrt{3}}\right)$ or $\frac{-8}{4\sqrt{3}}$

$\Rightarrow \sin x = \frac{1}{2}$ or $\frac{-2}{\sqrt{3}} < -1 \Rightarrow$ only $\sin x = \frac{1}{2}$

$\Rightarrow$ 5 solutions in $x \in \left[-2\pi, \frac{5\pi}{2}\right]$

Q.13 The shortest distance between the curves $y^2 = 8x$ and $x^2 + y^2 + 12y + 35 = 0$ is :

(1) $3\sqrt{2} - 1$ (2) $2\sqrt{3} - 1$

(3) $2\sqrt{2} - 1$ (4) $\sqrt{2}$

Ans. [3]

Sol. Equation of normal : $y = mx - 2am - am^3$ $(a = 2)$

$y = mx - 4m - 2m^3$

centre of circle: $c(0, -6)$, radius = 1

$-6 = -4m - 2m^3$

$\Rightarrow m = 1$

$P(am^2, -2am)$

$= P(2, -4)$

Shortest distance: $CP - r$

$= \sqrt{4+4} - 1$

$= 2\sqrt{2} - 1$

Q.14 Let $A = \{-2, -1, 0, 1, 2, 3\}$. Let R be a relation on A defined by xRy if and only if $y = \max\{x, 1\}$. Let l be the number of elements in R. Let m and n be the minimum number of elements required to be added in R to make it reflexive and symmetric relations, respectively. Then $l + m + n$ is equal to

(1) 12 (2) 11 (3) 14 (4) 13

Ans. [1]

Sol. $A = \{-2, -1, 0, 1, 2, 3\}$

$xRy \Leftrightarrow y = \max\{x, 1\}$

$\therefore R = \{(-2, 1), (-1, 1), (0, 1), (1, 1), (2, 2), (3, 3)\}$

$l = 6$

Elements to be added to make it reflexive:

$(-2, -2), (-1, -1), (0, 0)$

$\therefore m = 3$

Elements to be added to make it symmetric:

$(1, -2), (1, -1), (1, 0)$

$\therefore n = 3$

$l + m + n = 12$

Q.15 The distance of the point (7, 10, 11) from the line $\frac{x-4}{1} = \frac{y-4}{0} = \frac{z-2}{3}$ along the line $\frac{x-9}{2} = \frac{y-13}{3} = \frac{z-17}{6}$ is

(1) 16 (2) 18 (3) 14 (4) 12

Ans. [3]

Sol. Equation of line passing through P(7, 10, 11) along the line $\frac{x-9}{2} = \frac{y-13}{3} = \frac{z-17}{6}$ is

$\frac{x-7}{2} = \frac{y-10}{3} = \frac{z-11}{6} = \lambda$

Let the point on the line is

$Q(2\lambda + 7, 3\lambda + 10, 6\lambda + 11)$

Q lies on line $\frac{x-4}{2} = \frac{y-4}{3} = \frac{z-2}{3}$

$3\lambda + 10 = 4 \Rightarrow \lambda = -2$

$\therefore Q(3, 4, -1)$

$PQ = \sqrt{16+36+144} = 14$

Q.16 Consider the lines $x(3\lambda + 1) + y(7\lambda + 2) = 17\lambda + 5$, λ being a parameter, all passing through a point P. One of these lines (say L) is farthest from the origin. If the distance of L from the point (3, 6) is d, then the value of d^2 is

(1) 10 (2) 20 (3) 15 (4) 30

Ans. **[2]**

Sol. $x(3\lambda + 1) + y(7\lambda + 2) = 17\lambda + 5$

$(x + 2y - 5) + \lambda(3x + 7y - 17) = 0$

$L_1 + \lambda L_2 = 0$

$\Rightarrow$ P is intersection of L_1 & L_2 i.e. (1, 2)

$y - 2 = m(x - 1)$

$mx - y + 2 - m = 0$

Distance from origin $= \left|\dfrac{2-m}{\sqrt{1+m^2}}\right| = \max$

$2y - 4 = -x + 1$

For $m = -\dfrac{1}{2}$

$\therefore L = y - 2 = \dfrac{-1}{2}(x - 1)$

$L : x + 2y - 5 = 0$

Now, $d = \left|\dfrac{3+12-5}{\sqrt{5}}\right| = \left|\dfrac{10}{\sqrt{5}}\right|$

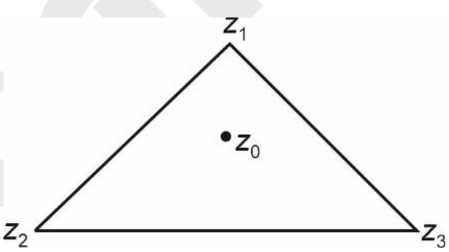
$d^2 = \dfrac{100}{5} = 20$

Q.17 If $z_1, z_2, z_3 \in C$ are the vertices of an equilateral triangle, whose centroid is z_0, then $\sum_{k=1}^{3}(z_k - z_0)^2$ is equal to

(1) 0 (2) 1 (3) i (4) –i

Ans. **[1]**

Sol.



$z_0 = z_1 + z_2 + z_3$

$\sum_{k=1}^{3}(z_k - z_0)^2 = (z_1 - z_0)^2 + (z_2 - z_0)^2 + (z_3 - z_0)^2$

Let z_0 is origin $\Rightarrow z_1, z_2, z_3$ lies on a circle having $|z_0 - z_i| = R$

$\therefore z_1 = Re^{i2\pi/3}\ z_2 = Re^{i4\pi/3}\ z_3 = Re^{i6\pi/3}$

$\Rightarrow z_1^2 + z_2^2 + z_3^2 = R^2\left[e^{i4\pi/3} + e^{i8\pi/3} + e^{i12\pi/3}\right]$

$= 0$

$\therefore \sum_{k=1}^{3}(z_k - z_0)^2 = 0$

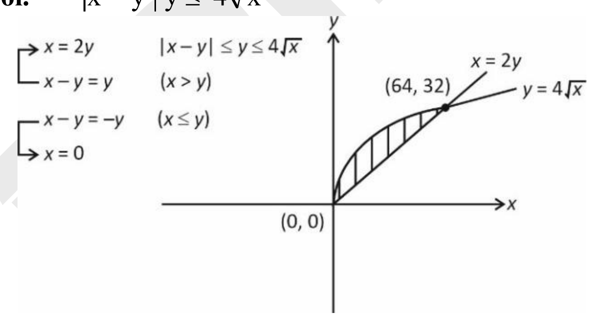
Q.18 The area of the region $\{(x, y) : |x - y| \le y \le 4\sqrt{x}\}$ is

(1) $\dfrac{512}{3}$ (2) $\dfrac{1024}{3}$

(3) $\dfrac{2048}{3}$ (4) 512

Ans. **[2]**

Sol. $|x - y|\ y \le 4\sqrt{x}$



Area $= \int_0^{64}\left(4\sqrt{x} - \dfrac{x}{2}\right)dx$

$= \left.\dfrac{4x^{3/2}}{\frac{3}{2}} - \dfrac{x^2}{4}\right]_0^{64}$

Q.19 Let $y = y(x)$ be the solution of the differential equation $\dfrac{dy}{dx} + 3(\tan^2 x)y + 3y = \sec^2 x$, $y(0) = \dfrac{1}{3} + e^3$. Then $y\left(\dfrac{\pi}{4}\right)$ is equal to

(1) $\dfrac{4}{3} + e^3$ (2) $\dfrac{4}{3}$

(3) $\dfrac{2}{3}$ (4) $\dfrac{2}{3} + e^3$

Ans. **[2]**

Sol. $\dfrac{dy}{dx} + 3(\tan^2 x)y + 3y = \sec^2 x$

$\Rightarrow \dfrac{dy}{dx} + 3\sec^2 xy = \sec^2 x$

$$\text{I.F.} = e^{\int 3\sec^2 x\,dx}$$
$$= e^{3\tan x}$$

$$y.e^{\tan x} = \int e^{3\tan x}.\sec^2 x\,dx + c$$

$$y.e^{\tan x} = \frac{e^{3\tan x}}{3} + c$$

Also $f(0) = \frac{1}{3} + e^3$

$$\Rightarrow \left(\frac{1}{3} + e^3\right) = \frac{1}{3} + c$$

$$\Rightarrow c = e^3$$

$$\therefore\ y.e^{3\tan x} = \frac{e^{3\tan x}}{3} + e^3$$

Put $x = \frac{\pi}{4}$

$$ye^3 = \frac{e^3}{3} + e^3 \Rightarrow \boxed{y = \frac{4}{3}}$$

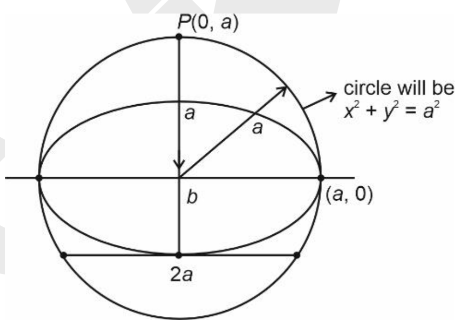
Q.20 Let C be the circle of minimum area enclosing the ellipse E : $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with eccentricity $\frac{1}{2}$ and foci$(\pm 2, 0)$. Let PQR be a variable triangle, whose vertex P is on the circle C and the side QR of length 2a is parallel to the major axis of E and contains the point of intersection of E with the negative y-axis. Then the maximum area of the triangle PQR is:

(1) $8(2+\sqrt{3})$ (2) $6(2+\sqrt{3})$

(3) $6(3+\sqrt{2})$ (4) $8(3+\sqrt{2})$

Ans. [1]

Sol.



$$\text{Area} = \frac{1}{2} \times (a+b).2a = a(a+b)$$

Since, $e = \frac{1}{2}$ $ae = 2 \Rightarrow a = 4,\ b = 2\sqrt{3}$

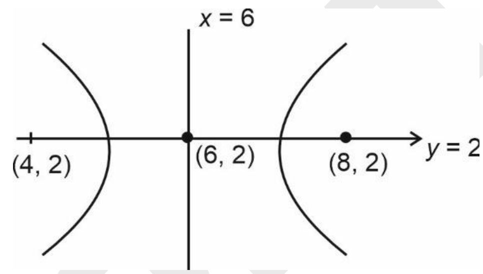
$$\Rightarrow \text{Area} = 4(4 + 2\sqrt{3}) = 8(2+\sqrt{3})$$

Section-B: Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

Q.21 If the equation of the hyperbola with foci (4, 2) and (8, 2) is $3x^2 - y^2 - \alpha x + \beta y + \gamma = 0$, then $\alpha + \beta + \gamma$ is equal to _____.

Ans. [141]

Sol.



Given $\frac{(x-6)^2}{a^2} - \frac{(y-2)^2}{b^2} = 1$

$$\Rightarrow b^2x^2 - a^2y^2 - 12xb^2 + 4ya^2 + 36b^2 - 4a^2 - a^2b^2 = 0$$

Comparing $\frac{b^2}{a^2} = 3$ $\Rightarrow e^2 = 1 + \frac{b^2}{a^2}$

$$\Rightarrow e^2 = 4$$

$$\Rightarrow \boxed{e = 2}$$

Similarly, $2ae = 4$

$$\Rightarrow a = 1 \Rightarrow b = \sqrt{3}$$

$$\frac{(x-6)^2}{1} - \frac{(y-2)^2}{3} = 1$$

$$\Rightarrow 3x^2 - y^2 - 36x + 4y + 108 - 4 - 3 = 0$$

$$\Rightarrow 3x^2 - y^2 - 36x + 4y + 101 = 0$$

$$\Rightarrow \alpha = 36,\ \beta = 4,\ \gamma = 101$$

$$\Rightarrow \alpha + \beta + \gamma = 141$$

Q.22 Let I be the identity matrix of order 3 × 3 and for the matrix $A = \begin{bmatrix} \lambda & 2 & 3 \\ 4 & 5 & 6 \\ 7 & -1 & 2 \end{bmatrix}$, $|A| = -1$. Let B be the inverse of the matrix $\text{adj}(A\ \text{adj}(A^2))$. Then $|\lambda(B + I)|$ is equal to _______ .

Ans. [38]

Sol. $B = [\text{adj}(A\ \text{adj}(A^2))]^{-1}$

$\text{Adj}(A^2) = (\text{adj}A)^2 \Rightarrow A\ \text{adj}(A^2) = A\ \text{adj}(A) \cdot (\text{adj}\ A)$

$= A(|A|A^{-1})^2 = |A|^2(A^{-1}) = A^{-1}$

$$\Rightarrow B = (\text{adj}(A^{-1}))^{-1} = (|(A^{-1})|A)^{-1} = \frac{A^{-1}}{-1} = -A^{-1}$$

$\Rightarrow B = -A^{-1}$

$|A| = -1 = \begin{vmatrix} \lambda & 2 & 3 \\ 4 & 5 & 6 \\ 7 & -1 & 2 \end{vmatrix} = -1 \Rightarrow \lambda = 3$

$|3B + I| = |I - 3A^{-1}| = \dfrac{|A||I - 3A^{-1}|}{|A|} = \dfrac{|A - 3|}{|A|}$

$= \dfrac{|A - 3I|}{-1} = \dfrac{\begin{vmatrix} 0 & 2 & 3 \\ 4 & 2 & 6 \\ 7 & -1 & -1 \end{vmatrix}}{-1} = 38$

$\Rightarrow |3B + I| = 38$

Q.23 If $\lim\limits_{x\to 0}\left(\dfrac{\tan x}{x}\right)^{\frac{1}{x^2}} = p$, then $96\log_e p$ is equal to ______.

Ans. **[32]**

Sol. $\lim\limits_{x\to 0}\left(\dfrac{\tan x}{x}\right)^{\frac{1}{x^2}} = p$

$\therefore$ Form : 1^∞

$\Rightarrow p = e^{\lim\limits_{x\to 0}\left(\frac{\tan x}{x} - 1\right)\frac{1}{x^2}}$

$= e^{\lim\limits_{x\to 0}\left(\frac{\tan x - x}{x^3}\right)}$

$= e^{\lim\limits_{x\to 0}\left(\frac{x + \frac{x^3}{3} + \frac{2}{15}x^5 + ... - x}{x^3}\right)} = e^{\frac{1}{3}}$

$\Rightarrow \log_e p = \dfrac{1}{3}$

$\Rightarrow 96\log_e p = 32$

Q.24 Let $\vec{a} = \hat{i} + 2\hat{j} + \hat{k}$, $\vec{b} = 3\hat{i} - 3\hat{j} + 3\hat{k}$, $\vec{c} = 2\hat{i} - \hat{j} + 2\hat{k}$ and $\vec{d}$ be a vector such that $\vec{b}\times\vec{d} = \vec{c}\times\vec{d}$ and $\vec{a}.\vec{d} = 4$. Then $|\vec{a}\times\vec{d}|^2$ is equal to ______.

Ans. **[128]**

Sol. $\vec{b}\times\vec{d} = \vec{c}\times\vec{d} \Rightarrow (\vec{b} - \vec{c}) \times \vec{d} = 0$

$\Rightarrow (\vec{b} - \vec{c})$ is parallel to $\vec{d}$

$\Rightarrow \vec{d} = \lambda(\hat{i} - 2\hat{j} + \hat{k})$

$\because \vec{a}.\vec{d} = 4$

$\Rightarrow \lambda - 4\lambda + \lambda = 4$

$\Rightarrow \lambda = -2 \Rightarrow \vec{d} = -2\hat{i} + 4\hat{j} - 2\hat{k}$

$\vec{a}\times\vec{d} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 1 \\ -2 & 4 & -2 \end{vmatrix} = -8\hat{i} + 0\hat{j} + 8\hat{k}$

$|\vec{a}\times\vec{d}|^2 = 128$

Q.25 Let $(1 + x + x^2)^{10} = a_0 + a_1x + a_2x^2 + \ldots + a_{20}x^{20}$. If $(a_1 + a_3 + a_5 + \ldots + a_{19}) - 11a_2 = 121k$, then k is equal to ______.

Ans. **[239]**

Sol. Let $f(x) = (1 + x + x^2)^{10} = \sum\limits_{r=0}^{20} a_r x^r$

The sum of odd coefficients: $S_{odd} = a_1 + a_3 + a_5 + \cdots + a_{19}$

Subtracting $11a_2$ from above will give the answer

$S_{odd} = \dfrac{f(1) - f(-1)}{2}$

$f(1) = (1 + 1 + 1)^{10} = 3^{10}$

$f(-1) = (1 - 1 + 1)^{10} = (1)^{10} = 1$

$S_{odd} = \sum\limits_{odd\ r} a_r = \dfrac{3^{10} - 1}{2}$

Now for a_2

$1 + x + x^2 = \dfrac{1 - x^3}{1 - x} \Rightarrow f(x) = \left(\dfrac{1 - x^3}{1 - x}\right) = \dfrac{(1 - x^3)^{10}}{(1 - x)^{10}}$

Now use :

$(1 - x^3)^{10} = \sum\limits_{x=0}^{10} (-1)^k \binom{10}{k} x^{3k}$

$(1 - x)^{-10} = \sum\limits_{r=0}^{\infty} \binom{r+9}{9} x^r$

So

$f(x) = \left(\sum\limits_{k=0}^{10} (-1)^k \binom{10}{k} x^{3k}\right).\left(\sum\limits_{r=0}^{\infty} \binom{r+9}{9} x^r\right)$

Only the term with x^0 from the first sum (i.e., k = 0) can contribute to x^2, since all other $k \geq 1$ gives $x^{3k} \geq x^3$

From $(1 - x^3)^{10}$: the x^0 term is $\binom{10}{0} = 1$

From $(1 - x)^{-10}$: the coefficient of x^2 is

$\binom{2+9}{9} = \binom{11}{9} = 55$

Hence, $a_2 = 1.55 = 55$

Now, $S_{odd} - 11a_2 = \dfrac{3^{10} - 1}{2} - 11.55 = 121\,k$

$3^{10} = 59049$

So :

$S = \dfrac{59049 - 1}{2} - 605 = \dfrac{59048}{2} - 605$

$= 29524 - 605 = 28919$

So :

$121k = 28919 \Rightarrow k = \dfrac{28919}{121} = 239$

PHYSICS

Section-A: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Q.26 A magnetic dipole experiences a torque of $80\sqrt{3}$ N m when placed in uniform magnetic field in such a way that dipole moment makes angle of 60° with magnetic field. The potential energy of the dipole is :

(1) –80 J (2) $-40\sqrt{3}$ J

(3) –60 J (4) 80 J

Ans. [1]

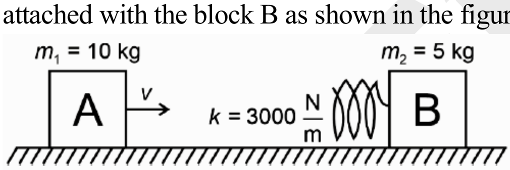
Sol. $\tau = MB\sin\theta$

$U = -MB\cos\theta$

$\frac{U}{\tau} = -\cot\theta$

$U = -80\sqrt{3}\times\frac{1}{\sqrt{3}} = -80$ J

Q.27 Consider two blocks A and B of masses $m_1 = 10$ kg and $m_2 = 5$ kg that are placed on a frictionless table. The block A moves with a constant speed v = 3 m/s towards the block B kept at rest. A spring with spring constant k = 3000 N/m is attached with the block B as shown in the figure.



After the collision, suppose that the blocks A and B, along with the spring in constant compression state, move together, then the compression in the spring is, (Neglect the mass of the spring)

(1) 0.2 m (2) 0.3 m

(3) 0.4 m (4) 0.1 m

Ans. [4]

Sol. COLM

$10\times 3 + 50\times 0 = 15V_C$

$V_C = 2$ m/s

COME

$$\frac{1}{2}kx^2 = \left|\frac{1}{2}(m+m)V_C^2 - \frac{1}{2}mv_0^2\right|$$

$$1500x^2 = \left|\frac{1}{2}\times 15\times 2^2 - \frac{1}{2}\times 10\times 3^2\right|$$

$$1500x^2 = |30-45|$$

$$x^2 = \frac{15}{1500}$$

$$x = \frac{1}{10} = 0.1 \text{ m}$$

Q.28 Given below are two statements : one is labelled as **Assertion A** and the other is labelled as **Reason R**.

Assertion A : The Bohr model is applicable to hydrogen and hydrogen-like atoms only

Reason R : The formulation of Bohr model does not include repulsive force between electrons.

In the light of the above statements, choose the **correct** answer from the options given below :

(1) **A** is true but **R** is false

(2) **A** is false but **R** is true

(3) Both **A** and **R** are true but **R** is **NOT** the correct explanation of **A**

(4) Both **A** and **R** are true and **R** is the correct explanation of **A**

Ans. [4]

Sol. Assertion is correct as a fact, Reason is correct as there are no two electrons for repulsion. Therefore (4) is most appropriate response

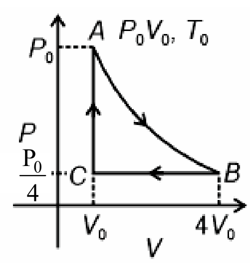
Q.29 An ideal gas exists in a state with pressure P_0, volume V_0. It is isothermally expanded to 4 times of its initial volume (V_0), then isobarically compressed to its original volume. Finally the system is heated isochorically to bring it to its initial state. The amount of heat exchanged in this process is

(1) $P_0V_0(\ln 2 - 0.25)$ (2) $P_0V_0(2\ln 2 - 0.75)$

(3) $P_0V_0(2\ln 2 - 0.25)$ (4) $P_0V_0(\ln 2 - 0.75)$

Ans. [2]

Sol.



$\Delta Q = \Delta U + \Delta W$

For cyclic process $\Delta Q = \Delta W$

$W = W_{AB} + W_{BC} + W_{CA}$

$$= P_0V_0\ln\frac{4V_0}{V_0} + \frac{PV_0}{4}(V_0 - 4V_0) + 0$$

$$= P_0V_0\ln 4 - \frac{3}{4}P_0V_0$$

$$= P_0V_0(2\ln 2 - 0.75)$$

Q.30 A particle is projected with velocity u so that its horizontal range is three times the maximum height attained by it. The horizontal range of the projectile is given as $\frac{nu^2}{25g}$, where value of n is :

(Given, 'g' is the acceleration due to gravity.)

(1) 18 (2) 6 (3) 12 (4) 24

Ans. **[4]**

Sol. R = 3H

$$\frac{u^2 \sin 2\theta}{g} = \frac{3u^2 \sin^2\theta}{2g}$$

$$2 \sin\theta \cos\theta = \frac{3}{2}\sin\theta \sin\theta$$

$$\frac{4}{3} = \tan\theta$$

$$\theta = 53^o$$

$$R = \frac{u^2}{g} \times 2 \times \frac{4}{5} \times \frac{3}{5} = \frac{24}{25}\frac{u^2}{g}$$

Q.31 A monochromatic light of frequency 5×10^{14} Hz travelling through air, is incident on a medium of refractive index '2'. Wavelength of the refracted light will be :

(1) 400 nm (2) 300 nm

(3) 600 nm (4) 500 nm

Ans. **[2]**

Sol. $v = \nu\lambda$

As $\mu = 2$ therefore; speed $c \rightarrow \frac{C}{2}$

$$\lambda_r = \frac{C}{2\nu} = \frac{3\times10^8}{2\times5\times10^{14}} = 3 \times 10^{8-15} \text{ m}$$

$$= 3 \times 10^{-7} \times 10^9 \text{ nm} = 300 \text{ nm}$$

Q.32 A block of mass 1 kg, moving along x with speed $v_i = 10$ m/s enters a rough region ranging from x = 0.1 m to x = 1.9 m. The retarding force acting on the block in this range is $F_r = -kx$ N, with k = 10 N/m. Then the final speed of the block as it crosses rough region is.

(1) 6 m/s (2) 10 m/s

(3) 4 m/s (4) 8 m/s

Ans. **[4]**

Sol. $W = \Delta k$ So $|W_f|$ = loss in KE

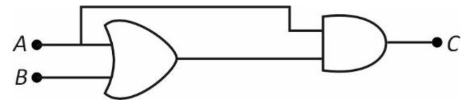
$$\frac{1}{2}k((1.9)^2 - (0.1)^2) = \frac{1}{2} \times 1 \{10^2 - v^2\}$$

$$10 \times 1.8 \times 2 = 100 - v^2$$

$$v^2 = 64$$

$$v = 8 \text{ m/s}$$

Q.33 The truth table corresponding to the circuit given below is :



(1)

A	B	C
0	0	1
0	1	0
1	0	0
1	1	0

(2)

A	B	C
0	0	0
0	1	0
1	0	1
1	1	1

(3)

A	B	C
0	0	1
1	0	0
0	1	0
1	1	0

(4)

A	B	C
0	0	0
1	0	0
0	1	0
1	1	1

Ans. **[2]**

Sol. Loigc : C = (A + B) A = A.A + A.B

= A + A.B

= A(1 + B)

C = A

Q.34 Two monochromatic light beams have intensities in the ratio 1 : 9. An interference pattern is obtained by these beams. The ratio of the intensities of maximum to minimum is

(1) 3 : 1 (2) 9 : 1

(3) 8 : 1 (4) 4 : 1

Ans. **[4]**

Sol.
$$\frac{I_{max}}{I_{min}} = \frac{\left(\sqrt{I_1} + \sqrt{I_2}\right)^2}{\left(\sqrt{I_1} - \sqrt{I_2}\right)^2} = \frac{(1+3)^2}{(1-3)^2} = \frac{16}{4} = 4$$

Q.35 In the resonance experiment, two air columns (closed at one end) of 100 cm and 120 cm long, give 15 beats per second when each one is sounding in the respective fundamental modes. The velocity of sound in the air column is

(1) 370 m/s (2) 340 m/s

(3) 360 m/s (4) 335 m/s

Ans. **[3]**

Sol. $f_0 = \frac{v}{4L}$

$$\Delta f = \frac{v}{4}\left\{\frac{1}{1} - \frac{1}{1.2}\right\}$$

$$\frac{15\times4\times1.2}{0.2} = v$$

$$360 = v$$

Q.36 Width of one of the two slits in a Young's double slit interference experiment is half of the other slit. The ratio of the maximum to the minimum intensity in the interference pattern is

(1) $\left(2\sqrt{2}+1\right):\left(2\sqrt{2}-1\right)$ (2) 3 : 1

(3) 9 : 1 (4) $\left(3+2\sqrt{2}\right):\left(3-2\sqrt{2}\right)$

Ans. **[4]**

Sol. Intensity $\propto$ slit width

$I_1 = I_0$,

$I_2 = 2I_0$;

$$\frac{I_{max}}{I_{min}} = \frac{\left(\sqrt{2I_0}+\sqrt{I_0}\right)^2}{\left(\sqrt{2I_0}-\sqrt{I_0}\right)^2}$$

$$= \frac{(\sqrt{2}+1)^2}{(\sqrt{2}-1)^2}$$

$$= \frac{3+2\sqrt{2}}{3-2\sqrt{2}}$$

Q.37 Given below are two statements: one is labelled as **Assertion A** and the other is labelled as **Reason R**

Assertion A : If Oxygen ion (O^{-2}) and Hydrogen ion (H^+) enter normal to the magnetic field with equal momentum, then the path of O^{-2} ion has a smaller curvature than that of H^+.

Reason R : A proton with same linear momentum as an electron will form a path of smaller radius of curvature on entering a uniform magnetic field perpendicularly.

In the light of the above statements, choose the **correct** answer from the options given below

(1) **A** is true but **R** is false

(2) Both **A** and **R** are true and R is the correct explanation of **A**

(3) Both **A** and **R** are true but **R** is **NOT** the correct explanation of **A**

(4) **A** is false but **R** is true

Ans. **[1]**

Sol. $R = \frac{mv}{qB}$

(A) mv, B are same for O^{2-} & H^+ therefore

$R \propto \frac{1}{|q|}$ therefore (A) is correct

(B) $R = \frac{mv}{qB}$; mv, q, B are same both therefore

(R) is incorrect

Q.38 An electric bulb rated as 100W – 220 V is connected to an ac source of rms voltage 220 V. The peak value of current through the bulb is

(1) 0.64 A (2) 0.32 A

(3) 0.45 A (4) 2.2 A

Ans. **[1]**

Sol. $RB = \frac{V^2}{P} = \frac{220\times 220}{100} = 484$

$$I_{rms} = \frac{V_{rms}}{R} = \frac{220}{484} = \frac{10}{22} = \frac{5}{11}$$

$$I_0 = \frac{5\sqrt{2}}{11} \approx \frac{7.07}{11} \approx 0.64$$

Q.39 Match the **List-I** with **List-II**.

	List-I		List-II
A.	Boltzmann constant	I.	ML^2T^{-1}
B.	Coefficient of viscosity	II.	$MLT^{-3}K^{-1}$
C.	Planck's constant	III.	$ML^2T^{-2}K^{-1}$
D.	Thermal conductivity	IV.	$ML^{-1}T^{-1}$

Choose the **correct** answer from the options given below:

(1) A – III, B – IV, C – I, D – II

(2) A – III, B – II, C – I, D – IV

(3) A – II, B – III, C – IV, D – I

(4) A – III, B – IV, C – II, D – I

Ans. **[1]**

Sol. $kt = \text{Energy} \Rightarrow k = ML^2T^{-2}K^{-1}$

$A \rightarrow III$

$$F = 6\pi\eta rv \Rightarrow \eta = \frac{MLT^{-2}}{LLT^{-1}}$$

$$\eta = ML^{-1}T^{-1}$$

$B \rightarrow IV$

$E = h\nu = h = ML^2T^{-1}$

$C \rightarrow I$

$$H = \frac{kA\Delta T}{l} = k = \frac{ML^2T^{-3}}{LK} = MLT^{-3}K^{-1}$$

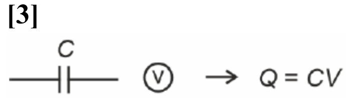
$D \rightarrow II$

Q.40 Using a battery, a 100 pF capacitor is charged to 60 V and then the battery is removed. After that, a second uncharged capacitor is connected to the first capacitor in parallel. If the final voltage across the second capacitor is 20 V, its capacitance is: (in pF)

(1) 100 (2) 400 (3) 200 (4) 600

Ans. **[3]**

Sol.



i.e. Q = 60 × 100 pC

When new capacitance C' is connected

Then Q = (C + C′)V

60 × 100 = (100 + C′) × 20

200 = C′

Q.41 A particle moves along the x-axis and has its displacement x varying with time t according to the equation :

$x = c_0(t^2 - 2) + c(t - 2)^2$

Where c_0 and c are constants of the appropriate dimensions. Then, which of the following statements is correct ?

(1) The acceleration of the particle is $2c_0$
(2) The initial velocity of the particle is 4c
(3) The acceleration of the particle is 2c
(4) The acceleration of the particle is $2(c + c_0)$

Ans. **[4]**

Sol. $x = C_0(t^2 - 2) + C(t^2 + 4 - 4t)$

$V = C_0(2t) + C(2t - 4)$

at t = 0

$v = -4C$

$a = 2C_0 + 2C$

Q.42 A motor operating on 100 V draws a current of 1 A. If the efficiency of the motor is 91.6%, then the loss of power in units of cal/s is

(1) 4 (2) 6.2 (3) 8.4 (4) 2

Ans. **[4]**

Sol. $P_0 = 100 \times 1 = 100$ W

$\text{Loss} = (1 - \eta)p_0 = \frac{8.4}{100} \times 100 = 8.4$ J

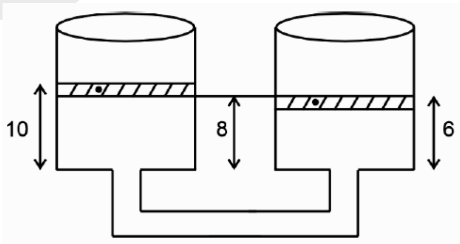
$\text{Loss} = \frac{8.4}{4.2}\text{cal} = 2$

Q.43 Two cylindrical vessels of equal cross sectional area of $2m^2$ contain water upto heights 10m and 6m, respectively. If the vessels are connected at their bottom then the work done by the force of gravity is (Density of water is 10^3 kg/m^3 and g = 10 m/s^2)

(1) 4×10^4 J (2) 8×10^4 J
(3) 6×10^4 J (4) 1×10^5 J

Ans. **[2]**

Sol.



Effectively shaded portion has descended

$|w| = \Delta m g \Delta h$

$= (\rho 2A)g \times 2$

$= 10^3 \times 2 \times 40$

$= 8 \times 10^4$ J

Q.44 A solid steel ball of diameter 3.6 mm acquired terminal velocity 2.45×10^{-2} m/s while falling under gravity through an oil of density 925 kg m^{-3}. Take density of steel as 7825 kg m^{-3} and g as 9.8 m/s^2. The viscosity of the oil in SI unit is

(1) 2.18 (2) 1.68 (3) 2.38 (4) 1.99

Ans. **[4]**

Sol. $V = \frac{2}{9}\frac{r^2 g(\sigma - \rho)}{\eta}$

$\eta = \frac{2}{9} \times \frac{1.8 \times 1.8 \times 10^{-6} \times 10}{2.45 \times 10^{-2}}\{7825 - 925\}$

≈ 2

Q.45 Pressure of an ideal gas, contained in a closed vessel, is increased by 0.4% when heated by 1°C. Its initial temperature must be:

(1) 25°C (2) 250°C
(3) 250 K (4) 2500 K

Ans. **[3]**

Sol. For constant volume

$\frac{P}{T} = \text{constant}$

$\frac{\Delta P}{P} = \frac{\Delta T}{T}$

$\frac{0.4}{100} = \frac{1}{T}$

$T = \frac{100}{0.4} = 250$ K

Section-B: Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

Q.46 Two cells of emfs 1 V and 2 V and internal resistances 2 Ω and 1 Ω, respectively, are connected in series with an external resistance of 6 Ω. The total current in the circuit is I_1. Now the same two cells in parallel configuration are connected to same external resistance. In this case, the total current drawn is I_2. The value of $\left(\frac{I_1}{I_2}\right)$ is $\frac{x}{3}$. The value of x is

Ans. [4]

Sol. $I_1 = \dfrac{\varepsilon_1 + \varepsilon_2}{r_1 + r_2 + R} = \dfrac{1+2}{2+1+6} = \dfrac{3}{9} = \dfrac{1}{3}$

$$I_2 = \dfrac{\dfrac{\dfrac{\varepsilon_1}{r_1} + \dfrac{\varepsilon_2}{r_2}}{\dfrac{1}{r_1} + \dfrac{1}{r_2}}}{\dfrac{r_1 r_2}{r_1 + r_2} + R} = \dfrac{\dfrac{\dfrac{1}{2} + \dfrac{2}{1}}{\dfrac{1}{2} + \dfrac{1}{1}}}{\dfrac{1\times 2}{3} + 6} = \dfrac{5\times 2\times 3}{2\times 3\times 20} = \dfrac{1}{4}$$

$\dfrac{I_1}{I_2} = \dfrac{1}{3} \times \dfrac{4}{1} = \dfrac{4}{3}$

Q.47 An electron in the hydrogen atom initially in the fourth excited state makes a transition to nth energy state by emitting a photon of energy 2.86 eV. The integer value of n will be ___________.

Ans. [2]

Sol. $E_n = 13.6\left(\dfrac{1}{n_1^2} - \dfrac{1}{n_2^2}\right)$

$2.86 = 13.6\left|-\dfrac{1}{25} + \dfrac{1}{n^2}\right|$

$\Rightarrow \dfrac{2.86}{13.6} + \dfrac{1}{25} = \dfrac{1}{n^2}$

$\dfrac{1}{n^2} = 0.25$

$n^2 = 4$

$\Rightarrow \boxed{n = 2}$

Q.48 A physical quantity C is related to four other quantities p, q, r and s as follows $C = \dfrac{pq^2}{r^3\sqrt{s}}$.
The percentage errors in the measurement of p, q, r and s are 1%, 2%, 3% and 2%, respectively. The percentage error in the measurement of C will be ___________ %.

Ans. [15]

Sol. $\dfrac{\Delta C}{C} = \dfrac{\Delta p}{p} + 2\dfrac{\Delta p}{q} + \dfrac{3\Delta r}{r} + \dfrac{1}{2}\dfrac{\Delta s}{s}$

%error $= 1 + 2 \times 2 + 3 \times 3 + \dfrac{1}{2} \times 2$

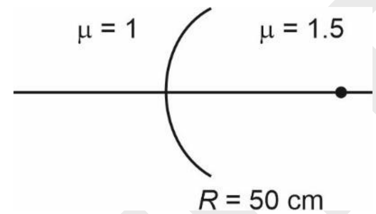
$= 1 + 4 + 9 + 1$

$= 15$

Q.49 Light from a point source in air falls on a spherical glass surface (refractive index, $\mu = 1.5$ and radius of curvature = 50 cm). The image is formed at a distance of 200 cm from the glass surface inside the glass. The magnitude of distance of the light source from the glass surface is ___________ m.

Ans. [4]

Sol.



V = 200 cm

$\dfrac{\mu_2}{v} - \dfrac{\mu_1}{u} = \dfrac{\mu_2 - \mu_1}{R}$

$\dfrac{1.5}{200} - \dfrac{1}{u} = \dfrac{1.5 - 1}{50} = \dfrac{1}{100}$

$\dfrac{1}{u} = \dfrac{15}{2000} - \dfrac{1}{100} = \dfrac{15 - 20}{2000} = \dfrac{-5}{2000}$

$u = -400 \text{ cm} = -4\text{m}$

Q.50 The excess pressure inside a soap bubble A in air is half the excess pressure inside another soap bubble B in air. If the volume of the bubble A is n times the volume of the bubble B, then, the value of n is ___________.

Ans. [8]

Sol. $\dfrac{4s}{r_1} = \dfrac{1}{2}\left(\dfrac{4s}{r_2}\right)$

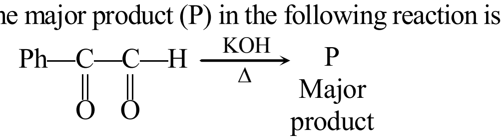
$r_1 = 2r_2$

$\text{Ratio} = \dfrac{\dfrac{4}{3}\pi r_1^3}{\dfrac{4}{3}\pi r_2^3} = 8$

CHEMISTRY

Section-A: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Q.51 The major product (P) in the following reaction is

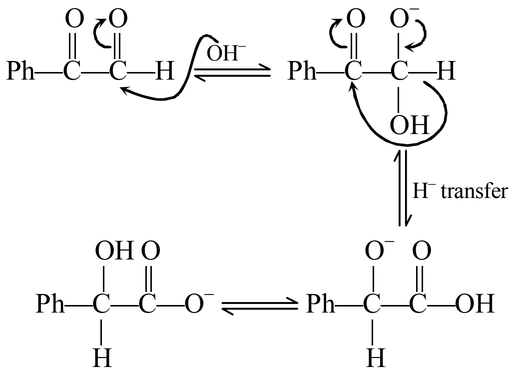


(1) Ph–C(=O)–CH_2OH (2) Ph–CH(OH)–COO^-K^+

(3) Ph–CH(OH)–CH_2OH (4) Ph–C(OH)–COO^-K^+

Ans. **[2]**

Sol.



Q.52 Given below are two statements:

Statement I : Hyperconjugation is not a permanent effect.

Statement II : In general, greater the number of alkyl groups attached to a positively charged C-atom, greater is the hyperconjugation interaction and stabilization of the cation.

In the light of the above statements, choose the **correct** answer from the options given below.

(1) Both **Statement I** and **Statement II** are true
(2) **Statement I** is false but **Statement II** is true
(3) Both **Statement I** and **Statement II** are false
(4) **Statement I** is true but **Statement II** is false

Ans. **[2]**

Sol. Hyperconjugation is a permanent effect.
With increase in number of α-H which increases with number of alkyl groups attached to positively charged C, hyperconjugation increases.

Q.53 Identify the diamagnetic octahedral complex ions from below :

A. $[Mn(CN)_6]^{3-}$ B. $[Co(NH_3)_6]^{3+}$
C. $[Fe(CN)_6]^{4-}$ D. $[Co(H_2O)_3F_3]$

Choose the **correct** answer from the options given below.

(1) B and D only (2) B and C only
(3) A and C only (4) A and D only

Ans. **[2]**

Sol. $[Mn(CN)_6]^{3-}$: Mn^{3+} : $[Ar]4s^0 3d^4$: $t_{2g}^4 e_g^0$
Octahedral, paramagnetic
$[Co(NH_3)_6]^{3+}$: Co^{3+} : $[Ar]4s^0 3d^6$: $t_{2g}^6 e_g^0$
Octahedral, diamagnetic
$[Fe(CN)_6]^{4-}$: Fe^{2+} : $[Ar]4s^0 3d^6$: $t_{2g}^6 e_g^0$
Octahedral, diamagnetic
$[Co(H_2O)_3F_3]$: Co^{3+} : $[Ar]4s^0 3d^6$: $t_{2g}^4 e_g^2$
Octahedral, paramagnetic

Q.54 Given below are two statements:

Statement I : When a system containing ice in equilibrium with water (liquid) is heated, heat is absorbed by the system and there is no change in the temperature of the system until whole ice gets melted.

Statement II : At melting point of ice, there is absorption of heat in order to overcome intermolecular forces of attraction within the molecules of water in ice and kinetic energy of molecules is not increased at melting point.

In the light of the above statements choose the **correct** answer from the options given below.

(1) Both **Statement I** and **Statement II** are true
(2) **Statement I** is true but **Statement II** is false
(3) Both **Statement I** and **Statement II** is false
(4) **Statement I** is false but **Statement II** is true

Ans. **[1]**

Sol. $H_2O(s) \rightleftharpoons H_2O(l)$
In phase transition, heat provided is used to change solid ice to liquid water at constant temperature, hence there is no change in temperature and kinetic energy.

Q.55 Given below are two statements:

Statement I : Wet cotton clothes made of cellulose based carbohydrate takes comparatively longer time to get dried than wet nylon polymer based clothes.

Statement II : Intermolecular hydrogen bonding with water molecule is more in nylon-based clothes than in the case of cotton clothes.

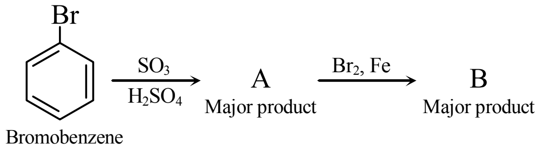
In the light of above statements, choose the **correct** answer from the options given below.

(1) Both **Statement I** and **Statement II** are false
(2) **Statement I** is true but **Statement II** is false
(3) **Statement I** is false but **Statement II** is true
(4) Both **Statement I** and **Statement II** are true

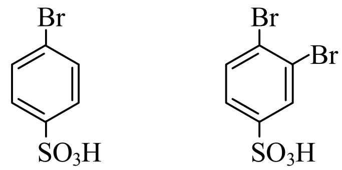
Ans. **[2]**

Sol. Wet cellulose-based cotton clothes take longer time to dry than wet nylon-based clothes due to more number of H-bonds between cellulose and water molecules. So, Statement-I is correct, Statement-II is incorrect.

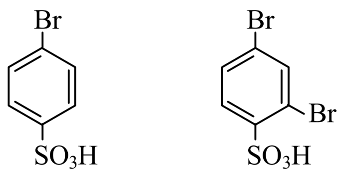
Q.56 In the following series of reactions identify the major products A & B respectively



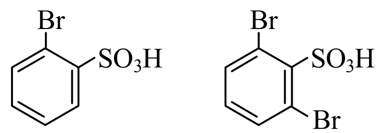
A B

(1) 

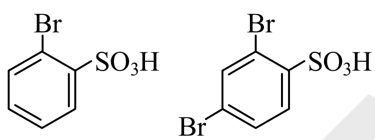
A B

(2) 

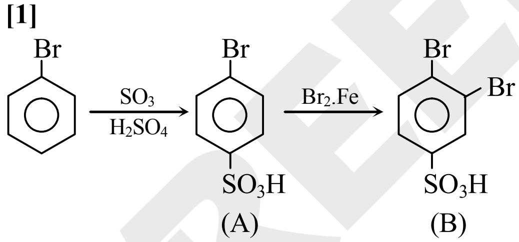
A B

(3) 

A B

(4) 

Ans. [1]

Sol. 

Q.57 Given below are two statements:

Statement I: CrO_3 is a stronger oxidizing agent than MoO_3.

Statement II: Cr(VI) is more stable than Mo(VI)

In the light of the above statements, choose the **correct** answer from the options given below

(1) **Statement I** is true but **Statement II** is false

(2) **Statement I** is false but **Statement II** is true

(3) Both **Statement I** and **Statement II** are false

(4) Both **Statement I** and **Statement II** are true

Ans. [1]

Sol. Oxidising strength : $CrO_3 > MoO_3$

Stability : $Cr^{6+} < Mo^{6+}$

Q.58 Mass of magnesium required to produce 220 mL of hydrogen gas at STP on reaction with excess of dil. HCl is Given: Molar mass of Mg is 24 g mol^{-1}.

(1) 235.7 g (2) 0.24 mg

(3) 2.444 g (4) 236 mg

Ans. [4]

Sol. $Mg + 2HCl \longrightarrow MgCl + \underset{220\text{ mL}}{H_2}$

$$\text{Moles of } H_2 = \frac{220}{22400} \text{ moles}$$

$$\text{Mass of Mg} = \frac{220}{22400} \times 24 = 0.236 \text{ g} = 236 \text{ mg}$$

Q.59 Compounds that should not be used as primary standards in titrimetric analysis are :

A. $Na_2Cr_2O_7$

B. Oxalic acid

C. NaOH

D. $FeSO_4.6H_2O$

E. Sodium tetraborate

Choose the ***most appropriate*** answer from the options given below:

(1) C, D and E only (2) A, C and D only

(3) D and E only (4) B and D only

Ans. [2]

Sol. Compounds like NaOH, $FeSO_4 \cdot 6H_2O$, $Na_2Cr_2O_7$ are not suitable as primary standards in titrimetric analysis because they are hydroscopic, unstable or difficult to obtain in a pure state.

Q.60 Consider the following statements related to temperature dependence of rate constants.

Identify the correct statements.

A. The Arrhenius equation holds true only for an elementary homogenous reaction.

B. The unit of A is same as that of k in Arrhenius equation.

C. At a given temperature, a low activation energy means a fast reaction.

D. A and Ea as used in Arrhenius equation depend on temperature.

E. When Ea >> RT, A and Ea become interdependent.

Choose the correct answer from the options given below.

(1) B and C only (2) B, D and E only

(3) A and B only (4) A, C and D only

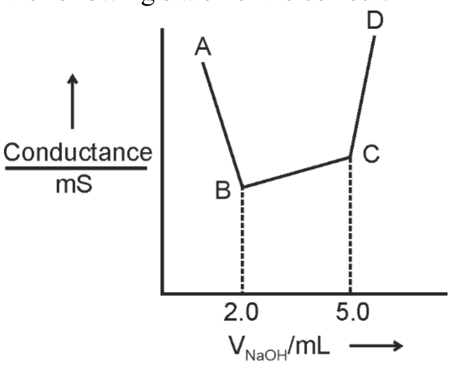
Ans. [1]

Sol. $k = Ae^{-E_a/RT}$

If $E_a \downarrow$: $k \uparrow$

k and A has same units.

Q.61 40 mL of a mixture of CH_3COOH and HCl (aqueous solution) is titrated against 0.1 M NaOH solution conductometrically. Which of the following statement is correct?



(1) The concentration of HCl in the original mixture is 0.005 M

(2) Point 'C' indicates the complete neutralisation of HCl

(3) CH_3COOH is neutralised first followed by neutralisation of HCl

(4) The concentration of CH_3COOH in the original mixture is 0.005 M

Ans. [1]

Sol. $CH_3COOH + NaOH \rightarrow CH_3COONa + H_2O$

$HCl + NaOH \rightarrow NaCl + H_2O$

At first NaOH reacts with HCl, replacing fast moving H^+ ions by slow moving Na^+ ions, hence conductivity decreases.

At point B, eq. of HCl = eq. of NaOH

Hence, $[HCl] = \dfrac{2 \times 0.1}{42} = 0.005M$

Q.62 In Dumas' method for estimation of nitrogen 0.4 g of an organic compound gave 60 mL of nitrogen collected at 300 K temperature and 715 mm Hg pressure. The percentage composition of nitrogen in the compound is

(Given : Aqueous tension at 300 K = 15 mm Hg)

(1) 7.85% (2) 20.95%

(3) 17.46% (4) 15.71%

Ans. [4]

Sol. $P_{N_2} = 715 - 15 = 700$ mm Hg

$$n_{N_2} = \frac{PV}{RT} = \frac{700 \times 60}{760 \times 1000 \times 0.0821 \times 300}$$

$n_{N_2} = 2.24 \times 10^{-3}$

$$\%N_2 = \frac{2.24 \times 10^{-3} \times 28}{0.4} \times 100$$

$= 15.71\%$

Q.63 The standard cell potential $(E^{\ominus}_{cell})$ of a fuel cell based on the oxidation of methanol in air that has been used to power television relay station is measured as 1.21 V. The standard half cell reduction potential for $\left(E^{o}_{O_2/H_2O}\right)$ is 1.229 V.

Choose the correct statement :

(1) The standard half cell reduction potential for the reduction of $CO_2\left(E^{o}_{CO_2/CH_3OH}\right)$ is 19 mV

(2) Reduction of methanol takes place at the cathode

(3) Reactants are fed at one go to each electrode

(4) Oxygen is formed at the anode

Ans. [1]

Sol. Fuel cell reaction

$$CH_3OH(l) + \frac{3}{2}O_2(g) \rightarrow CO_2(g) + 2H_2O(l)$$

Here O_2 reduces to H_2O and CH_3OH oxidises to CO_2.

Hence,

$E^{o}_{cell} = E^{o}_{O_2/H_2O} - E^{o}_{CO_2/CH_3OH}$

$1.21 = 1.229 - E^{o}_{CO_2/CH_3OH}$

$E^{o}_{CO_2/CH_3OH} = 0.019$ V

Q.64 The correct orders among the following are

Atomic radius : B < Al < Ga < In < Tl

Electronegativity : Al < Ga < In < Tl < B

Density : Tl < In < Ga < Al < B

1st Ionisation energy : In < Al < Ga < Tl < B

Choose the correct answer from the options given below :

(1) C and D only (2) A and B only

(3) B and D only (4) A and C only

Ans. [3]

Sol. EN : B > Tl > In > Ga > Al

(2.0) (1.8) (1.7) (1.6) (1.5)

IE_1 : B > Tl > Ga > Al > In

(801) (589) (579) (577) (558) (in kJ/mol)

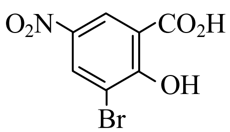
Density : Tl > In > Ga > Al > B

(11.85) (7.31) (5.9) (2.7) (2.35) (in g/ml)

Atomic radius: B < Ga < Al < In < Tl

(85) (135) (143) (167) (170) (in pm)

Q.65 What is the correct IUPAC name of



?

(1) 3-Bromo-4-hydroxy-1-nitrobenzoic acid

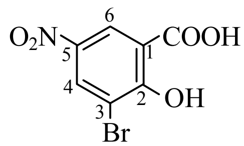
(2) 3-Bromo-2-hydroxy-5-nitrobenzoic acid

(3) 2-Hydroxy-3-bromo-5-nitrobenzoic acid

(4) 5-Nitro-3-bromo-2-hydroxybenzoic acid

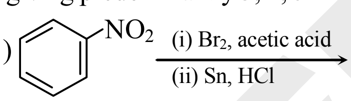
Ans. **[2]**

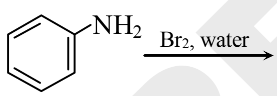
Sol.

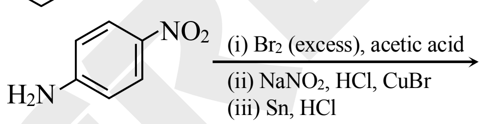


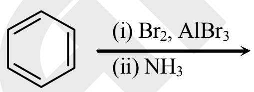
3-Bromo-2-hydroxy-5-nitrobenzoic acid

Q.66 The sequence from the following that would result in giving predominantly 3, 4, 5-Tribromoaniline is

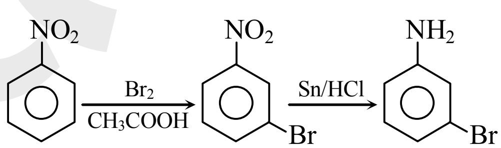
(1) 

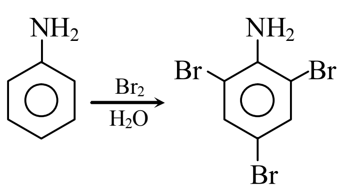
(2) 

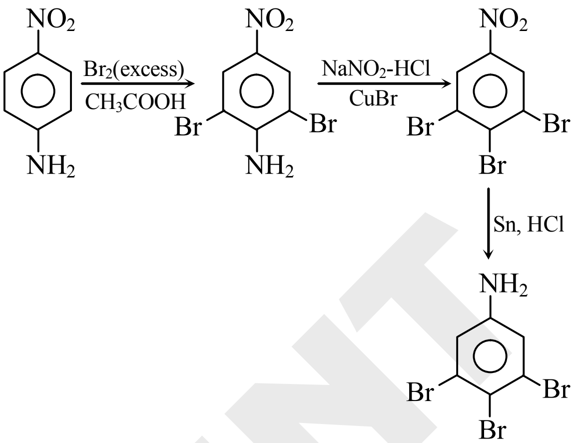
(3) 

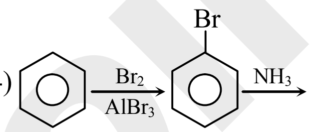
(4) 

Ans. **[3]**

Sol. (1) 

(2) 

(3) 

(4) 

No reaction under normal conditions.

Q.67 Fat soluble vitamins are:

(A) Vitamin B_1 (B) Vitamin C

(C) Vitamin E (D) Vitamin B_{12}

(E) Vitamin K

Choose the correct answer from the options given below:

(1) A and B only (2) C and D only

(3) B and C only (4) C and E only

Ans. **[4]**

Sol. Fat soluble vitamins are : A, D, E and K.

Q.68 Match the List-I with List-II.

	List-I (Family)		List-II (Symbol of Element)
A.	Pnictogen (group 15)	I.	Ts
B.	Chalcogen	II.	Og
C.	Halogen	III.	Lv
D.	Noble gas	IV.	Mc

Choose the **correct** answer from the options given below:

(1) A-IV, B-III, C-I, D-II

(2) A-IV, B-I, C-II, D-III

(3) A-II, B-III, C-IV, D-I

(4) A-III, B-I, C-IV, D-II

Ans. **[1]**

Sol.

A.	Pnictogen (group 15)	IV.	Mc
B.	Chalcogen	III.	Lv
C.	Halogen	I.	Ts
D.	Noble gas	II.	Og

Q.69 10 mL of 2 M NaOH solution is added to 20 mL of 1 M HCl solution kept in a beaker. Now, 10 mL of this mixture is poured into a volumetric flask of 100 mL containing 2 moles of HCl and made the volume upto the mark with distilled water. The solution in this flask.

(1) 10 M HCl solution (2) 20 M HCl solution

(3) Neutral solution (4) 0.2 M NaCl solution

Ans. **[2]**

Sol. 10 mL of 2 M NaOH reacts with 20 mL of 1 M HCl to give completely neutralised 30 mL of NaCl (20 m mole) solution.

10 mL of this solution has $\frac{20}{30}\times 10 = \frac{20}{3}$ m mol of NaCl

In Final 100 mL solution :

$$[HCl] = \frac{2}{100}\times 1000 = 20M$$

$$[NaCl] = \frac{20}{3\times 100} = \frac{1}{15}\ M$$

Q.70 For electrons in '2s' and '2p' orbitals, the orbital angular momentum values, respectively are

(1) 0 and $\sqrt{2}\frac{h}{2\pi}$ (2) 0 and $\sqrt{6}\frac{h}{2\pi}$

(3) $\sqrt{2}\frac{h}{2\pi}$ and 0 (4) $\frac{h}{2\pi}$ and $\sqrt{2}\frac{h}{2\pi}$

Ans. **[1]**

Sol. Orbital angular momentum $= \frac{h}{2\pi}\sqrt{l(l+1)}$

For 2s;

$l = 0$

Orbital angular momentum = 0

For 2p;

$$l = 1 \Rightarrow \frac{h}{2\pi}\sqrt{l(l+1)} = \frac{\sqrt{2}h}{2\pi} = \frac{h}{(\sqrt{2})\pi}$$

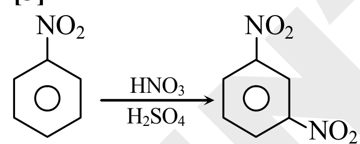
Section-B: Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

Q.71 X g of nitrobenzene on nitration gave 4.2 g of m–dinitrobenzene. X = _________ g. (nearest integer)

[Given: molar mass (in g mol^{-1}) C: 12, H: 1, O: 16, N: 14]

Ans. **[3]**

Sol.



$$\frac{X}{123} = \frac{4.2}{168}$$

X = 3.075 g

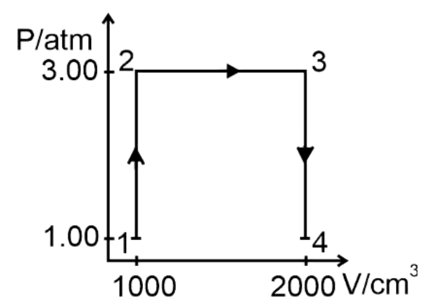
Q.72 A sample of n-octane (1.14 g) was completely burnt in excess of oxygen in a bomb calorimeter, whose heat capacity is 5 kJ K^{-1}. As a result of combustion reaction, the temperature of the calorimeter is increased by 5 K. The magnitude of the heat of combustion of octane at constant volume is _______ kJ mol^{-1} (nearest integer).

Ans. **[2500]**

Sol. $Q = C\Delta T = 5 \times 5 = 25$ kJ

$$Q_v = \frac{25}{1.14} \times 114 = 2500 \text{ kJ mol}^{-1}$$

Q.73



A perfect gas (0.1 mol) having $\overline{C}_v$ = 1.50R (independent of temperature) undergoes the above transformation from point 1 to point 4. If each step is reversible, the total work done (w) while going from point 1 to point 4 is (–) ___________ J (nearest integer)

[Given: R = 0.082 L atm K^{-1} mol^{-1}]

Ans. [304]

Sol. | w | = area under the curve

| w | = 3 × (2000 – 1000)

= 3000 mL atm

= 3 L atm

| w | = 3 × 101.3 J

= 303.9 J

Q.74 Among, Sc, Mn, Co and Cu, identify the element with highest enthalpy of atomisation. The spin only magnetic moment value of that element in its +2 oxidation state is __________ BM (in nearest integer).

Ans. [4]

Sol. Enthalpy of atomisation of : Sc = 326 kJ/mol, Mn = 281 kJ/mol, Co = 425 kJ/mol, Cu = 338 kJ/mol

Co^{2+} : [Ar] $4s_0$ $3d^7$: | $\uparrow\downarrow$ | $\uparrow\downarrow$ | $\uparrow$ | $\uparrow$ | $\uparrow$ |

$\mu = \sqrt{3(3+2)} = \sqrt{15} = 3.87$

Q.75 The total number of structural isomers possible for the substituted benzene derivatives with the molecular formula C_9H_{12} is __________.

Ans. [8]

Sol. Degree of unsaturation = $\dfrac{2\times9+2-12}{2} = 4$