JEE Main Online Exam 2025

Questions & Solution

02nd April 2025 | Morning

MATHEMATICS

Section-A: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct..

Q.1 Let $a \in R$ and A be a matrix of order 3×3 such that $\det(A) = -4$ and $A + I = \begin{bmatrix} 1 & a & 1 \\ 2 & 1 & 0 \\ a & 1 & 2 \end{bmatrix}$, where I is the identity matrix of order 3×3. If $\det((a + 1)\,\text{adj}((a-1)A))$ is $2^m 3^n$, $m, n \in \{0, 1, 2, \ldots, 20\}$, then $m + n$ is equal to :

(1) 16 (2) 17 (3) 14 (4) 15

Ans. **[1]**

Sol. $A + I = \begin{bmatrix} 1 & a & 1 \\ 2 & 1 & 0 \\ a & 1 & 2 \end{bmatrix}$

$A = \begin{bmatrix} 0 & a & 1 \\ 2 & 0 & 0 \\ a & 1 & 1 \end{bmatrix}$

$|A| = -4$

$-2a + 2 = -4$

$\Rightarrow \boxed{a = 3}$

$|4\,\text{adj}(2A)| = 4^3|\text{adj}(2A)|$

$= 4^3|2A|^2$

$= 4^3 \times 2^6|A|^2$

$= 4^3 \times 2^6 \times (-4)^2$

$= 2^{16} \times 3^0$

$= m + n = 16$

Q.2 Let $P_n = \alpha^n + \beta^n$, $n \in N$. If $P_{10} = 123$, $P_9 = 76$, $P_8 = 47$ and $P_1 = 1$, then the quadratic equation having roots $\frac{1}{\alpha}$ and $\frac{1}{\beta}$ is :

(1) $x^2 + x - 1 = 0$ (2) $x^2 - x + 1 = 0$

(3) $x^2 + x + 1 = 0$ (4) $x^2 - x - 1 = 0$

Ans. **[1]**

Sol. $\because P_{10} = P_9 + P_8$

$\Rightarrow P_{10} - P_9 - P_8 = 0$

By Newton's method

Therefore, the equation is

$x^2 - x - 1 = 0$

as $P_1 = 1$

$\alpha + \beta = 1$

and $\alpha\beta = -1$

$\therefore$ Quadratic equation whose roots are $\frac{1}{\alpha}$ and $\frac{1}{\beta}$ is

$x^2 - \left(\frac{1}{\alpha} + \frac{1}{\beta}\right)x + \frac{1}{\alpha\beta} = 0$

$x^2 - \left(\frac{\alpha+\beta}{\alpha\beta}\right)x + \frac{1}{\alpha\beta} = 0$

$x^2 - (-1)x - 1 = 0$

$x^2 + x - 1 = 0$

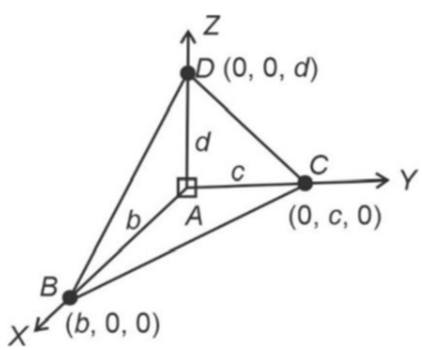
Q.3 Let ABCD be a tetrahedron such that the edges AB, AC and AD are mutually perpendicular. Let the areas of the triangles ABC, ACD and ADB be 5, 6 and 7 square units respectively. Then the area (in square units) of the ΔBCD is equal to :

(1) $\sqrt{110}$ (2) $7\sqrt{3}$

(3) $\sqrt{340}$ (4) 12

Ans. **[1]**

Sol.



ar(ΔABC) = 5

$\frac{1}{2} \times bc = 5$

$\Rightarrow$ bc = 10

ar(ΔACD) = 6

$\frac{1}{2} \times cd = 6$

$\Rightarrow$ cd = 12

ar(ΔABD) = 7

bd = 14

area (ΔBCD) = $\frac{1}{2} \left|\overrightarrow{BC} \times \overrightarrow{BD}\right|$

$\overrightarrow{BC} = \langle -b, c, 0 \rangle$

$\overrightarrow{BD} = \langle -b, 0, d \rangle$

$\left|\overrightarrow{BC} \times \overrightarrow{BD}\right| = \sqrt{c^2d^2 + b^2d^2 + b^2c^2}$

$= \sqrt{12^2 + 14^2 + 10^2}$

$= \sqrt{440}$

ar(ΔBCD) = $\sqrt{110}$

Q.4 If S and S′ are the foci of the ellipse $\frac{x^2}{18} + \frac{y^2}{9} = 1$ and P be a point on the ellipse, then min.(SP·S′P) + max.(SP·S′P) is equal to :

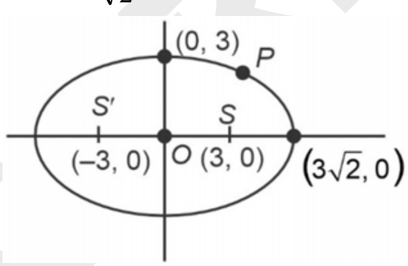
(1) 27 (2) $3(1 + \sqrt{2})$

(3) $3(6 + \sqrt{2})$ (4) 9

Ans. **[1]**

Sol. $a = 3\sqrt{2}$, b = 3

$\Rightarrow e = \frac{1}{\sqrt{2}}$



$PS + PS' = 2a = 6\sqrt{2}$

$\frac{PS + PS'}{2} \geq \sqrt{PS \cdot PS'}$

$\Rightarrow$ (PS × PS′) max = 18

Minima happens when P lies on major axis

$\Rightarrow P = (3\sqrt{2}, 0)$

$PS = (3\sqrt{2} - 3).PS' = (3\sqrt{2} + 3)$

(PS · PS′) min = 9

(PS · PS′)min + (PS · PS′)max = 27

Option (1)

Q.5 Let f : R → R be a twice differentiable function such that (sinx cosy)(f(2x + 2y) – f(2x – 2y)) = (cosx siny)(f(2x + 2y) + f(2x – 2y)), for all x, y ∈ R.

If $f'(0) = \frac{1}{2}$, then the value of $24 f''\left(\frac{5\pi}{3}\right)$ is :

(1) 3 (2) 2 (3) –3 (4) –2

Ans. **[3]**

Sol. sin(x – y) f (2x + 2y) = f (2x – 2y) sin (x + y)

$\frac{f(2x+2y)}{\sin(x+y)} = \frac{f(2x-2y)}{\sin(x-y)} = k(\text{say})$

f(2x + 2y) = k sin (x + y)

f(2x) = ksinx $\quad (\because y = 0)$

$f(x) = k \sin \frac{x}{2}$

$f'(x) = \frac{k}{2} \cos \frac{x}{2}$

$f'(0) = \frac{1}{2} \Rightarrow k = 1$

$f(x) = \sin \frac{x}{2} \Rightarrow f'(x) = \frac{1}{2} \cos \frac{x}{2}$

$f''(x) = -\frac{1}{4} \sin \frac{x}{2}$

$24 f''\left(\frac{5\pi}{3}\right) = -3$

Q.6 The term independent of x in the expansion of $\left(\frac{(x+1)}{(x^{2/3}+1-x^{1/3})} - \frac{(x-1)}{(x-x^{1/2})}\right)^{10}$, x > 1, is :

(1) 120 (2) 240 (3) 210 (4) 150

Ans. **[3]**

Sol. $(x + 1) = [(x^{1/3}) + 1]\,[x^{2/3} - x^{1/3} + 1]$

$(x + 1) = (\sqrt{x} - 1)(\sqrt{x} + 1)$

Now,

$\left(\frac{x+1}{x^{2/3} - x^{1/3} + 1}\right) = (x^{1/3} + 1)$

$\frac{x-1}{x - x^{1/2}} = \frac{(\sqrt{x}-1)(\sqrt{x}+1)}{(\sqrt{x})^2 - \sqrt{x}}$

$$\left[(x^{1/3}+1)\left(1+\frac{1}{\sqrt{x}}\right)\right]^{10} = \left[x^{1/3}-\frac{1}{x^{1/2}}\right]^{10}$$

$$T_{r+1} = {}^{10}C_r\left[-\frac{1}{x^{1/2}}\right]^r . (x^{1/3})^{10-r}$$

$$\Rightarrow x^{\left(\frac{10-r}{3}-\frac{r}{2}\right)} . {}^{10}C_r(-1)^r$$

The term independent of x when exponent of x is 0.

$\Rightarrow r = 4$

So, term $\rightarrow {}^{10}C_4\,(-1)^4x^0 = 210$

Q.7 Let A be the set of all function f : Z → Z and R be a relation on A such that R = {(f, g} : f(0) = g(1) and f(1) = g(0). Then R is :

(1) Symmetric and transitive but not reflective
(2) Reflexive but neither symmetric nor transitive
(3) Transitive but neither reflexive nor symmetric
(4) Symmetric but neither reflective nor transitive

Ans. **[4]**

Sol. For R to be reflexive, (f, f) must be in R.
The means f(0) = f(1) and f(1) = f(0) must be true for all f.
But f(0) ≠ f(1) always
Therefore, R is not reflexive
If (f, g) ∈ R, then f(0) = g(1) and f(1) = g(0)
∵ f(0) = g(1) ⇒ g(1) = f(0)
and f(1) = g(0) ⇒ g(0) = f(1)
R is symmetric
If (f, g) ∈ R and (g, h) ∈ R, then f(0) = g(1),
f(1) = g(0), g(0) = n(1) & g(1) = h(0)
Since, f(0) = g(1) and g(1) = h(0), then f(0) is not necessarily equal to h(0).
Therefore, R is not transitive.
∴ The relation R is symmetric but not reflexive or transitive.

Q.8 Let a_1, a_2, a_3, be in an A.P. such that $\sum_{k=1}^{12} a_{2k-1} = -\frac{72}{5}a_1, a_1 \neq 0$. If $\sum_{k=1}^{n} a_k = 0$, then n is :

(1) 17 (2) 18 (3) 11 (4) 10

Ans. **[3]**

Sol. $\sum_{k=1}^{12} a_{2k-1} = -\frac{72}{5}a_1$

$a_1 + a_3 + \ldots + a_{23} = -\frac{72}{5}a_1$

$a + a + 2d + \ldots + a + 22d = -\frac{72}{5}a$

$12a + 2d\,(1 + 2 + \ldots 11) = -\frac{72}{5}a$

$\Rightarrow 12a + 2d\left(\frac{11\times 12}{2}\right) = \frac{72}{5}a$

$\Rightarrow 132d = -\frac{132}{5}a$

$\Rightarrow a = -5d$ ...(i)

Also $\sum_{k=1}^{n} a_k = 0$

$\Rightarrow S_n = 0$

$\Rightarrow \frac{n}{2}[2a + (n-1)d] = 0$

$\Rightarrow 2a = -(n-1)d$ ...(ii)

From equation (i) and (ii)

$(n-1)d = 10d$

$\therefore n = 11$

Q.9 If the system of linear equations

$3x + y + \beta z = 3$
$2x + \alpha y - z = -3$
$x + 2y + z = 4$

has infinitely many solutions, then the value of $22\beta - 9\alpha$ is:

(1) 43 (2) 49 (3) 37 (4) 31

Ans. **[4]**

Sol. $3x + y + \beta z = 3$
$2x + \alpha y - z = -3$
$x + 2y + z = 4$
has infinite solution

$\Rightarrow \Delta = 0, \Delta_1 = \Delta_2 = \Delta_3$

$$\Delta = 0 \Rightarrow \begin{vmatrix} 3 & 1 & \beta \\ 2 & \alpha & -1 \\ 1 & 2 & 1 \end{vmatrix} = 0$$

$$\Delta_2 = 0 \Rightarrow \begin{vmatrix} 3 & 3 & \beta \\ 2 & -3 & -1 \\ 1 & 4 & 1 \end{vmatrix} = 0$$

$\Rightarrow\ 3(-3+4) - 3(2+1) + \beta(8+3) = 0$

$\Rightarrow 3 - 9 + 11\beta = 0$

$\Rightarrow \beta = \dfrac{6}{11}$

$\Delta = 0 \Rightarrow \begin{vmatrix} 3 & 1 & 3 \\ 2 & \alpha & -3 \\ 1 & 2 & 4 \end{vmatrix} = 0$

$\Rightarrow\ 3(4\alpha + 6) - 1(8+3) + 3(4-\alpha) = 0$

$12\alpha + 18 - 11 + 12 - 3\alpha = 0$

$9\alpha = -19$

$\alpha = \dfrac{-19}{9}$

$\therefore\ \boxed{22\beta - 9\alpha = 31}$

Q.10 The number of sequences of ten terms, whose terms are either 0 or 1 or 2, that contain exactly five 1s and exactly three 2s, is equal to :

(1) 1820 (2) 2520
(3) 360 (4) 45

Ans. [2]

Sol. Exactly five 1s and exactly three 2s

$\Rightarrow$ 2 zeros must be used

$\Rightarrow$ 10 places to arrange 5 ones, 3 twos and 2 zeros

$\Rightarrow \dfrac{10!}{2!3!5!} = 2520$

Q.11 Let $A = \begin{bmatrix} \alpha & -1 \\ 6 & \beta \end{bmatrix}$, $\alpha > 0$, such that det(A) = 0 and $\alpha + \beta = 1$. If I denotes 2×2 identity matrix, then the matrix, $(I + A)^8$ is :

(1) $\begin{bmatrix} 766 & -255 \\ 1530 & -509 \end{bmatrix}$ (2) $\begin{bmatrix} 257 & -64 \\ 514 & -127 \end{bmatrix}$

(3) $\begin{bmatrix} 4 & -1 \\ 6 & -1 \end{bmatrix}$ (4) $\begin{bmatrix} 1025 & -511 \\ 2024 & -1024 \end{bmatrix}$

Ans. [1]

Sol. Let $|A| = 0 \Rightarrow \alpha\beta - (-6) = 0 \Rightarrow \alpha\beta = -6$

and $\alpha + \beta = 1 \Rightarrow \alpha\beta$ are roots of the equation

$x^2 - x - 6 = 0 \Rightarrow x = 3, -2$. Since $\alpha > 0$

$\Rightarrow \alpha = 3, \beta = -2$

$\Rightarrow\ A = \begin{bmatrix} 3 & -1 \\ 6 & -2 \end{bmatrix} \Rightarrow\ I + A = \begin{bmatrix} 4 & -1 \\ 6 & -1 \end{bmatrix}$

$(I+A)^2 = \begin{bmatrix} 10 & -3 \\ 18 & -5 \end{bmatrix} \Rightarrow (I+A)^4 = \begin{bmatrix} 46 & -15 \\ 90 & -29 \end{bmatrix}$

$\Rightarrow (I+A)^8 = \begin{bmatrix} 766 & -255 \\ 1530 & -509 \end{bmatrix}$

Q.12 The largest $n \in N$ such that 3^n divides 50! is :

(1) 22 (2) 21 (3) 20 (4) 23

Ans. [1]

Sol. $V_3 = (50!) = \left[\dfrac{50}{3}\right] + \left[\dfrac{50}{9}\right] + \left[\dfrac{50}{27}\right] + \left[\dfrac{50}{81}\right] + \ldots + \left[\dfrac{50}{3^n}\right]$

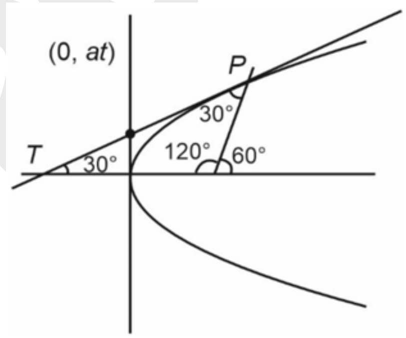
$= 16 + 5 + 1 + 0 + 0 + 0\ldots = 22$

Q.13 Let the focal chord PQ of the parabola $y^2 = 4x$ make an angle of 60° with the positive x-axis, where P lies in the first quadrant. If the circle, whose one diameter is PS, S being the focus of the parabola, touches the y-axis at the point $(0, \alpha)$, then $5\alpha^2$ is equal to :

(1) 15 (2) 25 (3) 30 (4) 20

Ans. [1]

Sol.



PT : $ty = x + at^2$

PS = PT

$M_t = \dfrac{1}{t} = \tan 30^\circ = \dfrac{1}{\sqrt{3}}$

$t = \sqrt{3}$

$\alpha = at = \sqrt{3}$ (a = 1)

$\therefore\ 5\alpha^2 = 15$

Q.14 Let the vertices Q and R of the triangle PQR lie on the line $\dfrac{x+3}{5} = \dfrac{y-1}{2} = \dfrac{z+4}{3}$, QR = 5 and the coordinates of the point P be (0, 2, 3). If the area of the triangle PQR is $\dfrac{m}{n}$ then :

(1) $2m - 5\sqrt{21}\,n = 0$

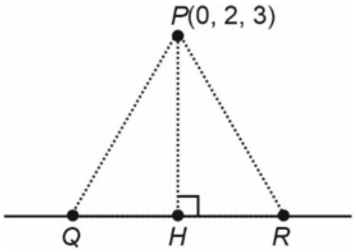
(2) $m - 5\sqrt{21}\,n = 0$

(3) $5m - 21\sqrt{2}\,n = 0$

(4) $5m - 2\sqrt{21}\,n = 0$

Ans. [1]

Sol.



H : $(5\lambda - 3, 2\lambda + 1, 3\lambda - 4)$

< DR of PH>

$< 5\lambda - 3, 2\lambda - 1, 3\lambda - 7 >$

$\overrightarrow{PH} \cdot \overrightarrow{QR} = 0$

$(5\lambda - 3)5 + (2\lambda - 1)2 + (3\lambda - 7)3 = 0$

$\Rightarrow 25\lambda - 15 + 4\lambda - 2 + 9\lambda - 21 = 0$

$\Rightarrow 38\lambda = 38$

$\Rightarrow \lambda = 1$

$\therefore$ H (2, 3, –1)

$PH = \sqrt{4+1+16} = \sqrt{21}$

$\therefore$ Area $= \frac{1}{2} \times PH \times QR$

$= \frac{1}{2} \times \sqrt{21} \times 5 = \frac{5\sqrt{21}}{2} = \frac{m}{n}$

$2m - 5\sqrt{21}\,n = 0$

Q.15 If the function $f(x) = 2x^3 - 9ax^2 + 12a^2x + 1$, where $a > 0$, attains its local maximum and local minimum values at p and q, respectively, such that $p^2 = q$, then f(3) is equal to :

(1) 10 (2) 37 (3) 23 (4) 55

Ans. [2]

Sol. $f(x) = 2x^3 - 9ax^2 + 12a^2x + 1,\ a > 0$

$f'(x) = 6x^2 - 18ax + 12a^2 = 0$

$= 6(x^2 - 3ax + 2a^2)$

$= 6\,(x - a)\,(x - 2a) = 0$

$x = a, 2a$

$\therefore$ x = a is point of maxima

x = 2a is point of minima

$\therefore$ p = a, q = 2a

$p^2 = q$ (Given)

$a^2 = 2a$

$\Rightarrow a = 2$

$f(x) = 2x^3 - 18x^2 + 48x + 1$

$f(3) = 37$

Q.16 If $\theta \in [-2\pi, 2\pi]$, then the number of solutions of $2\sqrt{2}\cos^2\theta + (2 - \sqrt{6})\cos\theta - \sqrt{3} = 0$, is equal to :

(1) 10 (2) 6 (3) 8 (4) 12

Ans. [3]

Sol. $2\sqrt{2}\cos^2\theta + (2 - \sqrt{6})\cos\theta - \sqrt{3} = 0$

$2\sqrt{2}\cos^2\theta + 2\cos\theta - \sqrt{6}\cos\theta - \sqrt{3} = 0$

$(2\cos\theta - \sqrt{3})(\sqrt{2}\cos\theta + 1) = 0$

$\Rightarrow \cos\theta = \frac{\sqrt{3}}{2}$ or $\cos\theta = \frac{-1}{\sqrt{2}}$

$\theta = \left\{\frac{-11\pi}{6}, \frac{-5\pi}{4}, \frac{-3\pi}{4}, \frac{-\pi}{6}, \frac{\pi}{6}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{11\pi}{6}\right\}$

$\Rightarrow$ 8 (solution)

Q.17 For $\alpha, \beta, \gamma \in R$, if $\lim_{x\to 0} \frac{x^2 \sin\alpha x + (\gamma - 1)e^{x^2}}{\sin 2x - \beta x} = 3$, then $\beta + \gamma - \alpha$ is equal to :

(1) 4 (2) 6 (3) 7 (4) –1

Ans. [3]

Sol. At $x \to 0$

$\sin 2x - \beta x \to 0$

$\Rightarrow \frac{0}{0}$ form

$\Rightarrow (\gamma - 1)e^0 + 0\sin(\alpha x) \to 0$

$\Rightarrow (\gamma - 1) = 0$

$\Rightarrow \gamma = 1$

$\Rightarrow \lim_{x\to 0} \frac{x^2 \sin(\alpha x)}{(\sin 2x - \beta x)} = 3$

$\Rightarrow \lim_{x\to 0} \frac{x^2\left[\alpha x - \frac{(\alpha x)^3}{3!} + \frac{(\alpha x)^5}{5!} - \ldots\right]}{\left[(2x) - \frac{(2x)^3}{3!} + \frac{(2x)^5}{5!}\right] - \beta x}$

$\Rightarrow \lim_{x\to 0} \frac{\alpha x^3 - \frac{\alpha^3 x^5}{3!} + \frac{\alpha^5 x^7}{5!} - \ldots}{x(2 - \beta) - \frac{8x^3}{6} + \frac{2^5.x^5}{5!} - \ldots} = 3$

$\Rightarrow 2 - \beta = 0$ and $\frac{\alpha}{\frac{-8}{6}} = 3$

$\Rightarrow \beta = 2$

$\alpha = 3\left(\frac{8}{6}\right) = -4$

$\Rightarrow \gamma = 1, \beta = 2, \alpha = -4$

$\Rightarrow \beta + \gamma - \alpha = 7$

Q.18 Let one focus of the hyperbola H : $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ be at ($\sqrt{10}$, 0) and the corresponding directrix be x = $\frac{9}{\sqrt{10}}$. If e and l respectively are the eccentricity and the length of the latus rectum of H, then $9(e^2 + l)$ is equal to :

(1) 15 (2) 14 (3) 12 (4) 16

Ans. **[4]**

Sol. $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

Directrix : $x = \frac{9}{\sqrt{10}} = \frac{a}{e}$ …(i)

Focus : $(\sqrt{10}, 0) \equiv (ae, 0)$

$ae = \sqrt{10}$ …(ii)

(i) × (ii)

$\Rightarrow a^2 = 9 \Rightarrow a = 3$

Substitute in (ii)

$e = \frac{\sqrt{10}}{3}$

Now $e^2 = 1 + \frac{b^2}{a^2}$

$\frac{10}{9} = 1 + \frac{b^2}{a}$

$\Rightarrow \boxed{b = 1}$

$l = \frac{2b^2}{a} = \frac{2\times 1}{3} = \frac{2}{3}$

$a(e^2 + l) = 9\left[\frac{10}{9} + \frac{2}{3}\right] = 10 + 6 = 16$

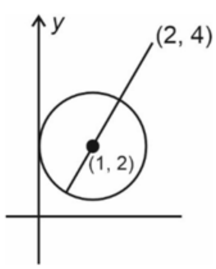
Q.19 Let z be a complex number such that |z| = 1. If $\frac{2+k^2z}{k+\bar{z}} = kz$, k ∈ R, then the maximum distance of $k + ik^2$ from the circle |z – (1 + 2i)| = 1 is :

(1) $\sqrt{5} + 1$ (2) 3

(3) $\sqrt{3} + 1$ (4) 2

Ans. **[1]**

Sol.



$\frac{2+k^2z}{k+\bar{z}} = kz$

$\Rightarrow 2 + k^2z = k^2z + kz\bar{z}$

$\Rightarrow 2 = k|z|^2 \quad (z\bar{z} = |z|^2, |z| = 1)$

$\Rightarrow \boxed{2 = k}$

$\therefore k + k^2 i = 2 + 4i$

$\therefore$ The maximum distance is

$= \sqrt{(4-2)+(2-1)^2} + \text{radius}$

$= \sqrt{(2)^2 + (1)^2} + 1$

$= \sqrt{5} + 1$

Q.20 If $\vec{a}$ is a non-zero vector such that its projections on the vectors $2\hat{i} - \hat{j} + 2\hat{k}$, $\hat{i} + 2\hat{j} - 2\hat{k}$ and $\hat{k}$ are equal, then a unit vector along $\vec{a}$ is :

(1) $\frac{1}{\sqrt{155}}(7\hat{i} + 9\hat{j} + 5\hat{k})$

(2) $\frac{1}{\sqrt{155}}(7\hat{i} + 9\hat{j} - 5\hat{k})$

(3) $\frac{1}{\sqrt{155}}(-7\hat{i} + 9\hat{j} - 5\hat{k})$

(4) $\frac{1}{\sqrt{155}}(-7\hat{i} + 9\hat{j} + 5\hat{k})$

Ans. **[1]**

Sol. Projection of $\vec{a}$ on $\vec{v}$

$= \frac{\vec{a}\cdot\vec{v}}{|\vec{v}|}$

$\Rightarrow \frac{\vec{a}\cdot(2\hat{i} - \hat{j} + 2\hat{k})}{3} = \frac{\vec{a}\cdot\hat{k}}{1} = \frac{\vec{a}\cdot(\hat{i} + 2\hat{j} - 2\hat{k})}{3}$

$\Rightarrow \vec{a}\cdot(2\hat{i} - \hat{j} - \hat{k}) = 0$ and $\vec{a}\cdot(\hat{i} + 2\hat{j} - 5\hat{k}) = 0$

$\Rightarrow \vec{a} \perp (2\hat{i} - \hat{j} - \hat{k})$ and $(\hat{i} + 2\hat{j} - 5\hat{k})$

$\Rightarrow \vec{a} \parallel (2\hat{i} - \hat{j} - \hat{k}) \times (\hat{i} + 2\hat{j} - 5\hat{k})$

$\Rightarrow \vec{a} = \pm k\begin{vmatrix} \hat{i} & -\hat{j} & \hat{k} \\ 2 & -1 & -1 \\ 1 & 2 & -5 \end{vmatrix} = \pm k(7\hat{i} + 9\hat{j} - 5\hat{k})$

$\Rightarrow$ Unit vector will be $\frac{1}{\sqrt{155}}(7\hat{i} + 9\hat{j} + 5\hat{k})$

Section-B: Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

Q.21 Lef $f : R \rightarrow R$ be a thrice differentiable odd function satisfying $f'(x) \geq 0$, $f''(x) = f(x)$, $f(0) = 0$, $f'(0) = 3$. Then $9f(\log_e 3)$ is equal to ________.

Ans. **[36]**

Sol. $f'(x) \geq 0,\ f''(x) = f(x)$

Second order differential equation

$f(x) = Ae^{x} + Be^{-x}$

$f(0) = 0 \Rightarrow A = -B$

$\Rightarrow f(x) = A(e^{x} - e^{-x})$

$f'(x) = Ae^{x} + Ae^{-x} = A(e^{x} + e^{-x})$

$f'(0) = 3 = A(e^{0} + e^{-0}) = 2A \Rightarrow A = \frac{3}{2}$

$f(x) = \frac{3}{2}(e^{x} - e^{-x})$

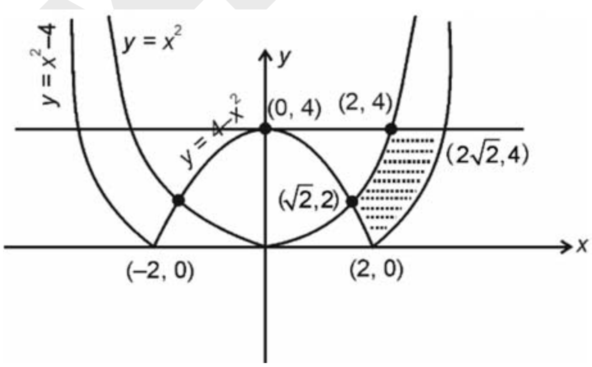
$\text{If}(\ln 3) = \frac{27}{2}(e^{\ln 3} - e^{-\ln 3}) = \frac{27}{2}\left(3 - \frac{1}{3}\right) = \frac{27}{2} \cdot \frac{8}{3}$

$= 36$

Q.22 If the area of the region $\{(x, y) : |4 - x^2| \leq y \leq x^2,\ y \leq 4,\ x \geq 0\}$ is $\left(\frac{80\sqrt{2}}{\alpha} - \beta\right)$, $\alpha, \beta \in N$, then $\alpha + \beta$ is equal to ______.

Ans. **[22]**

Sol. $\text{Area} = \int_{\sqrt{2}}^{2} (x^2 - (4 - x^2))\,dx + (2\sqrt{2} - 2) \times 4 - \int_{2}^{2\sqrt{2}} (x^2 - 4)\,dx$



$= \left[\frac{2x^3}{3} - 4x\right]_{\sqrt{2}}^{2} + 8\sqrt{2} - 8 - \left[\frac{x^3}{3} - 4x\right]_{2}^{2\sqrt{2}}$

$= \frac{40\sqrt{2}}{3} - 16$

$\Rightarrow \alpha = 6, \beta = 16 \Rightarrow \alpha + \beta = 22$

Q.23 Let $[\cdot]$ denote the greatest integer function. If $\int_{0}^{e^3}\left[\frac{1}{e^{x-1}}\right]dx = \alpha - \log_e 2$, then α^3 is equal to …..

Ans. **[8]**

Sol. $I = \int_{0}^{e^3}\left[\frac{1}{e^{x-1}}\right]dx = \int_{0}^{1-\ln 2} 2dx + \int_{1-\ln 2}^{1} dx + \int_{1}^{e^3} 0dx$

$= 2(1 - \ln 2) + (1 - (1 - \ln 2) + 0 = 2 - \ln 2$

$\Rightarrow \alpha = 2 \Rightarrow \alpha^3 = 8$

Q.24 Three distinct numbers are selected randomly from the set $\{1, 2, 3, \ldots, 40\}$. If the probability, that the selected numbers are in an increasing G.P., is $\frac{m}{n}$, $\gcd(m, n) = 1$, then $m + n$ is equal to ______.

Ans. **[4949]**

Sol.

Common ratio	Last triplet	Total
r = 2	10, 20, 40	10
r = 3	4, 12, 36	4
r = 4	2, 8, 32	2
r = 5	1, 5, 25	1
r = 6	1, 6, 36	1
	Total = 18	

Total choices $= {}^{40}C_3 = 9880$

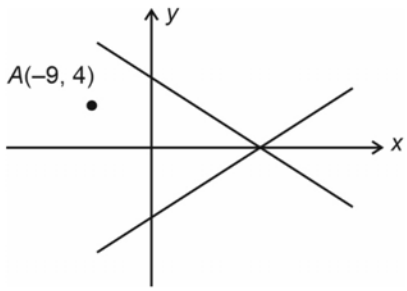
Required probability $= \frac{18}{9880} = \frac{9}{4940} = \frac{m}{n}$

$m + n = 4949$

Q.25 The absolute difference between the squares of the radii of the two circles passing through the point $(-9, 4)$ and touching the lines $x + y = 3$ and $x - y = 3$, is equal to ______.

Ans. **[768]**

Sol.



$\because$ x + y = 3 and x – y = 3 are tangents

$\therefore$ Both circle centre will lie on x-axis

$\therefore$ $(x - a)^2 + y^2 = r^2$

Hence centre is C(α, 0)

$r = \sqrt{(\alpha+9)^2+16}$ …(1)

Also $\left|\frac{\alpha-3}{\sqrt{2}}\right| = r$ …(2)

$\sqrt{(\alpha+9)^2+16} = \left|\frac{\alpha-3}{\sqrt{2}}\right|$

$\Rightarrow$ a = –5 or – 37

$r = \left|\frac{-5-3}{\sqrt{2}}\right|$ or $\left|\frac{-37-3}{\sqrt{2}}\right|$

$= 4\sqrt{2}$ or $20\sqrt{2}$

$|r_1^2 - r_2^2| = |32 - 800| = 768$

PHYSICS

Section-A: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct..

Q.26 The battery of a mobile phone is rated as 4.2 V, 5800 mAh. How much energy is stored in it when fully charged?

(1) 43.8 kJ (2) 48.7 kJ
(3) 87.7 kJ (4) 24.4 kJ

Ans. [3]

Sol. Energy stored in battery $= \frac{(\text{mAh}) \times V \times 3600}{1000}$

$= \frac{58000 \times 4.2 \times 3600}{1000} = 87.7$ kJ

Q.27 A river is flowing from west to east direction with speed of 9 km h^{-1}. If a boat capable of moving at a maximum speed of 27 km h^{-1} in still water, crosses the river in half a minute, while moving with maximum speed at an angle of 150° to direction of river flow, then the width of the river is :

(1) 112.5 m (2) 75 m
(3) 300 m (4) $112.5 \times \sqrt{3}$ m

Ans. [1]

Sol. $t = \frac{w}{v_b \cos(60)}$

$w = t v_b \cos(60)$

$= 30 \times 27 \times \frac{5}{18} \times \frac{1}{2}$

$= 112.5$ m

Q.28 Moment of inertia of a rod of mass 'M' and length 'L' about an axis passing through its center and normal to its length is 'α'. Now the rod is cut into two equal parts and these parts are joined symmetrically to form a cross shape. Moment of inertia of cross about an axis passing through its center and normal to plane containing cross is :

(1) α/4 (2) α/2 (3) α/8 (4) α

Ans. [1]

Sol. $\alpha = \frac{ML^2}{12}$

$I = \frac{\left(\frac{M}{2}\right)\left(\frac{L}{2}\right)^2}{12} \times 2$

$I = \frac{\alpha}{4}$

Q.29 Considering Bohr's atomic model for hydrogen atom :

(A) The energy of H atom in ground state is same as energy of He^+ ion in its first excited state.

(B) The energy of H atom in ground state is same as that for Li^{++} ion in its second excited state.

(C) The energy of H atom in its ground state is same as that of He^+ ion for its ground state.

(D) The energy of He^+ ion in its first excited state is same as that for Li^{++} ion in its ground state.

Choose the correct answer from the options given below:

(1) (A), (D) only (2) (A), (B) only
(3) (A), (C) only (4) (B), (D) only

Ans. [2]

Sol. $E = -13.6 \frac{z^2}{n^2}$ eV

H_{He^+} 1^{st} excited state = –13.6 eV

$H_{Li^{2+}}$ 2^{nd} excited state = –13.6 eV

(A), (B) are correct.

Q.30 The equation for real gas is given by $\left(P + \frac{a}{V^2}\right)(V - b) = RT$, where P, V, T and R the pressure, volume, temperature and gas constant, respectively. The dimension of ab^{-2} is equivalent to that of :

(1) Strain (2) Compressibility

(3) Planck's constant (4) Energy density

Ans. [4]

Sol. $P \equiv \frac{a}{V^2}$

$a \equiv PV^2$

$b \equiv V$

$\frac{a}{b^2} \equiv P$

Dimensions of P is same as energy density.

Q.31 The relationship between the magnetic susceptibility (χ) and the magnetic permeability (μ) is given by :

(μ_0 is the permeability of free space and μ_r is relative permeability)

(1) $\chi = \mu_r + 1$ (2) $\chi = \frac{\mu}{\mu_0} - 1$

(3) $\chi = \frac{\mu}{\mu_0} + 1$ (4) $\chi = 1 - \frac{\mu}{\mu_0}$

Ans. [2]

Sol. $\mu = \mu_r \mu_0$

$\mu_r = (\chi + 1)$

$\mu = (\chi + 1)\mu_0$

$\chi = \left(\frac{\mu}{\mu_0} - 1\right)$

Q.32 In an adiabatic process, which of the following statements is true?

(1) The molar heat capacity is zero

(2) The molar heat capacity is infinite

(3) The internal energy of the gas decreases as the temperature increases

(4) Work done by the gas equals the increase in internal energy

Ans. [1]

Sol. In adiabatic process

$Q = 0 = nC\Delta T$

$\therefore C = 0$

Q.33 Match List-I with List-II.

LIST-I		LIST-II	
(A)	Coefficient of viscosity	(I)	$[ML^0T^{-3}]$
(B)	Intensity of wave	(II)	$[ML^{-2}T^{-2}]$
(C)	Pressure gradient	(III)	$[M^{-1}LT^2]$
(D)	Compressibility	(IV)	$[ML^{-1}T^{-1}]$

Choose the correct answer from the options given below.

(1) (A)-(IV), (B)-(I), (C)-(II), (D)-(III)

(2) (A)-(II), (B)-(III), (C)-(IV), (D)-(I)

(3) (A)-(IV), (B)-(II), (C)-(I), (D)-(III)

(4) (A)-(I), (B)-(IV), (C)-(III), (D)-(II)

Ans. [1]

Sol. $\eta \equiv \frac{F}{rv} \equiv [ML^{-1}T^{-1}]$ A → IV

$I \equiv \frac{P}{A} \equiv [MT^{-3}]$ B → I

$\frac{dP}{dx} \equiv [ML^{-2}T^{-2}]$ C → II

$K \equiv \frac{1}{P} \equiv [M^{-1}LT^2]$ D → III

Q.34 A light wave is propagating with plane wave fronts of the type x + y + z = constant. The angle made by the direction of wave propagation with the x-axis is :

(1) $\cos^{-1}\left(\frac{2}{3}\right)$ (2) $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$

(3) $\cos^{-1}\left(\frac{1}{3}\right)$ (4) $\cos^{-1}\left(\sqrt{\frac{2}{3}}\right)$

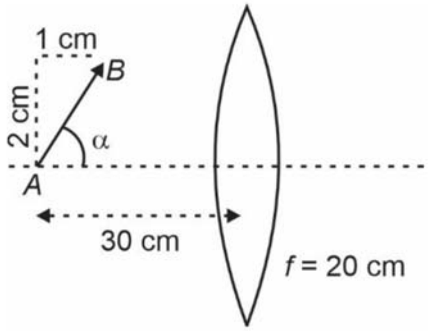
Ans. [2]

Sol. Direction of propagation $A = \vec{A} = \hat{i} + \hat{j} + \hat{k}$

Angle with x-axis $= \cos^{-1}\left(\frac{A_x}{\sqrt{A_x^2 + A_y^2 + A_z^2}}\right)$

$= \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$

Q.35 A slanted object AB is placed on one side of convex lens as shown in the diagram. The image is formed on the opposite side. Angle made by the image with principal axis is :



(1) –45° (2) –α

(3) +45° (4) $-\frac{\alpha}{2}$

Ans. [1]

Sol. Image of A

$$\frac{1}{v_A} - \frac{1}{-30} = \frac{1}{20}$$

$V_A = 60 \Rightarrow (60, 0)$

$$m = \frac{v}{u} = \frac{60}{-30} = -2$$

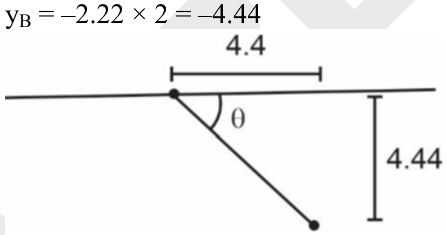
Image of B

$$\frac{1}{v_B} - \frac{1}{-29} = \frac{1}{20}$$

$$v_B = \frac{20 \times 29}{9} \Rightarrow 64.4$$

$$m = \frac{64.4}{-29} = -2.22$$

$y_B = -2.22 \times 2 = -4.44$



$\theta \approx -45°$

Q.36 Let B_1 be the magnitude of magnetic field at centre of a circular coil or radius R carrying current I. Let B_2 be the magnitude of magnetic field at an axial distance 'x' from the center. For $\frac{x}{R} = \frac{3}{4}$, $\frac{B_2}{B_1}$ is :

(1) 16 : 25 (2) 25 : 16

(3) 64 : 125 (4) 4 : 5

Ans. [3]

Sol. $B_1 = \frac{\mu_0 i}{2R}$

$$x = \frac{3R}{4}$$

$$B_2 = \frac{\mu_0 i R^2}{2(R^2 + x^2)^{3/2}} = \frac{\mu_0 i R^2}{2\left(\frac{5R}{4}\right)^3}$$

$$= \frac{64}{125}\left(\frac{\mu_0 i}{2R}\right)$$

$$\frac{B_2}{B_1} = \frac{64}{125}$$

Q.37 A particle is subjected to two simple harmonic motions as:

$x_1 = \sqrt{7}\sin 5t$ cm and

$x_2 = 2\sqrt{7}\sin\left(5t + \frac{\pi}{3}\right)$ cm

where x is displacement and t is time in seconds.

The maximum acceleration of the particle is $x \times 10^{-2}$ ms^{-2}. The value of x is :

(1) $5\sqrt{7}$ (2) $25\sqrt{7}$

(3) 125 (4) 175

Ans. [4]

Sol. $\omega = 5$

$$A = \sqrt{(\sqrt{7})^2 + (2\sqrt{7})^2 + 2 \times 2 \times 7\cos\left(\frac{\pi}{3}\right)}$$

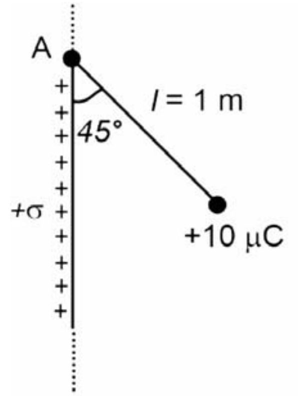
$= 7$

$a_{max} = \omega^2 A = 25 \times 7$

$= 175$

Q.38 A small bob of mass 100 mg and charge + 10 μC is connected to an insulating string of length 1 m. It is brought near to an infinitely long non-conducting sheet of charge density 'σ' as shown in figure. If string subtends and angle of 45° with the sheet at equilibrium the charge density of sheet will be.

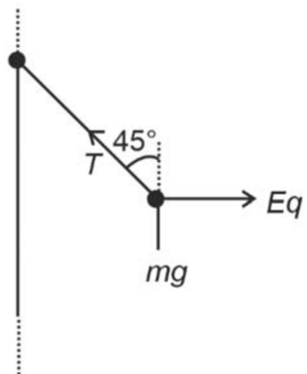
{Given $\varepsilon_0 = 8.85 \times 10^{-12}$ $\frac{F}{m}$ and acceleration due to gravity, $g = 10\ \frac{m}{s^2}$ }



(1) 17.7 nC/m^2 (2) 885 nC/m^2
(3) 0.885 nC/m^2 (4) 1.77 nC/m^2

Ans. **[4]**

Sol.



$T \cos 45^o = Eq$
$T \sin 45^o = mg$
$Eq = mg$

$$E = \frac{mg}{q} = \frac{\sigma}{2\varepsilon_0}$$

$$\sigma = \frac{2\varepsilon_0 mg}{q}$$

$$= 1.77\ nC/m^2$$

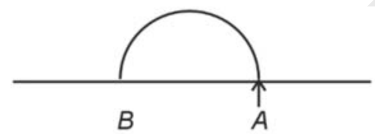
Q.39 A monochromatic light is incident on a metallic plate having work function ϕ. An electron, emitted normally to the plate from a point A with maximum kinetic energy, enters a constant magnetic field, perpendicular to the initial velocity of electron. The electron passes through a curve and hits back the plate at a point B. The distance between A and B is:
(Given: The magnitude of charge of an electron is e and mass is m, h is Planck's constant and c is velocity of light. Take the magnetic field exists throughout the path of electron)

(1) $\sqrt{m\left(\frac{hc}{\lambda}-\phi\right)}/eB$ (2) $\sqrt{8m\left(\frac{hc}{\lambda}-\phi\right)}/eB$

(3) $2\sqrt{m\left(\frac{hc}{\lambda}-\phi\right)}/eB$ (4) $\sqrt{2m\left(\frac{hc}{\lambda}-\phi\right)}/eB$

Ans. **[2]**

Sol.



AB = 2R

$$R = \frac{vm}{Bq} = \frac{\sqrt{2mK}}{Be}$$

$$K = \left(\frac{hc}{\lambda}-\phi\right)$$

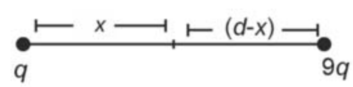
$$2R = \frac{2\sqrt{2m\left(\frac{hc}{\lambda}-\phi\right)}}{Be}$$

Q.40 A point charge + q is placed at the origin. A second point charge +9q is placed at (d, 0, 0) in Cartesian coordinate system. The point in between them where the electric field vanishes is:

(1) (d/4, 0, 0) (2) (d/3, 0, 0)
(3) (3d/4,0,0) (4) (4d/3, 0,0)

Ans. **[1]**

Sol.



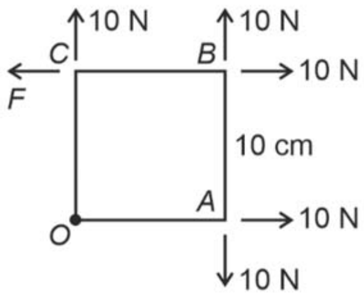
$$\frac{9}{x^2} = \frac{2q}{(d-x)^2}$$

$d - x = 3x$

$$x = \frac{d}{4}$$

$\therefore$ E = 0 at (d/4, 0, 0)

Q.41 A square Lamina OABC of length 10 cm is pivoted at 'O'. Forces act at Lamina as shown in figure. If Lamina remains stationary, then the magnitude of F is :



(1) 0 (Zero) (2) 10 N

(3) $10\sqrt{2}$ N (4) 20N

Ans. [2]

Sol. For stationary, net torque about O = 0

$-Fa + 10a - 10a + 10a = 0$

F = 10 N

Q.42 A zener diode with 5 V zener voltage is used to regulate an unregulated dc voltage input of 25 V. For a 400 Ω resistor connected in series, the zener current is found to be 4 times load current. The load current (I_L) and load resistance (R_L) are :

(1) I_L = 20 mA; R_L = 250 Ω

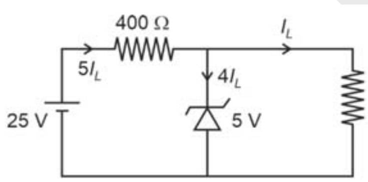
(2) I_L = 0.02 mA; R_L = 250 Ω

(3) I_L = 10 mA; R_L = 500 Ω

(4) I_L = 10 A; R_L = 0.5 Ω

Ans. [3]

Sol.



$$5I_L = \frac{(25-5)}{400} = \frac{1}{20} = 50 \text{ mA}$$

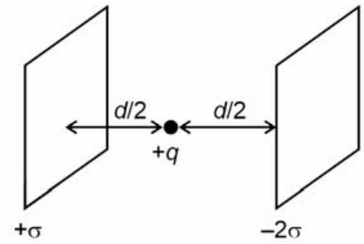
I_L = 10 mA

$5 = I_L R_L$

$$R_L = \frac{5}{I_L} = \frac{5}{10\times10^{-3}} = 500\ \Omega$$

Q.43 Consider two infinitely large plane parallel conducting plates as shown below. Two plates are uniformly charged with a surface charge density +σ and –2σ.

The force experienced by a point charge +q placed at the mid point between two plates will be :



(1) $\frac{3\sigma q}{4\varepsilon_0}$ (2) $\frac{\sigma q}{2\varepsilon_0}$

(3) $\frac{3\sigma q}{2\varepsilon_0}$ (4) $\frac{\sigma q}{4\varepsilon_0}$

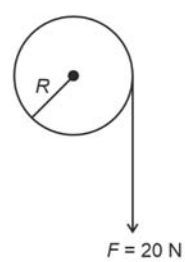
Ans. [3]

Sol. $$E = \frac{2\sigma q}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0} = \frac{3\sigma}{2\varepsilon_0}$$

$$F = Eq = \frac{3\sigma q}{2\varepsilon_0}$$

Q.44 A cord of negligible mass is around the rim of a wheel supported by spokes with negligible mass. The mass of wheel is 10 kg and radius is 10 cm and it can freely rotate without any friction. Initially the wheel is at rest.

If a steady pull of 20 N is applied on the cord, the angular velocity of the wheel, after the cord is unwound by 1 m, would be :



(1) 10 rad/s (2) 20 rad/s

(3) 30 rad/s (4) 0 rad/s

Ans. [2]

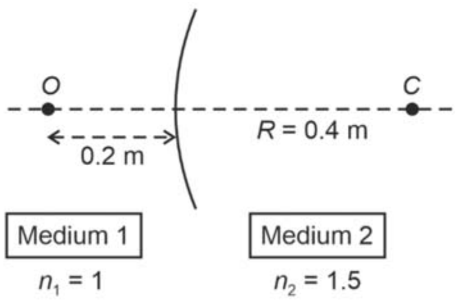
Sol. W = F.S = 20 × 1 = 20 J

$$= 20 \text{ J} = \frac{1}{2}(MR^2)\omega^2$$

$$20 = \frac{1}{2} \times 10 \times \frac{1}{100} \times \omega^2$$

ω = 20 rad/s

Q.45



A spherical surface separates two media of refractive indices 1 and 1.5 as shown in figure. Distance of the image of an object 'O', is
(C is the center of curvature of the spherical surface and R is the radius of curvature)
(1) 0.4 m left to the spherical surface
(2) 0.24 m left to the spherical surface
(3) 0.24 m right to the spherical surface
(4) 0.4 m right to the spherical surface

Ans. **[1]**

Sol. For spherical surfaces

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

$$\frac{1.5}{v} - \frac{1}{-0.2} = \frac{0.5}{0.4}$$

$v = -0.4$ m

Section-B: Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

Q.46 A person travelling on a straight line moves with a uniform velocity v_1 for a distance x and with a uniform velocity v_2 for the next $\frac{3}{2}x$ distance. The average velocity in this motion is $\frac{50}{7}$ m/s. If v_1 is 5 m/s then v_2 = ______ m/s.

Ans. **[10]**

Sol.
$$v_{avg} = \frac{x + \frac{3x}{2}}{\frac{x}{5} + \frac{3x}{2v_2}} = \frac{50}{7}$$

$v_2 = 10$ m/s

Q.47 γ_A is the specific heat ratio of monoatomic gas A having 3 translational degrees of freedom. γ_B is the specific heat ratio of polyatomic gas B having 3 translational, 3 rotational degrees of freedom and 1 vibrational mode. If $\frac{\gamma_A}{\gamma_B} = \left(1 + \frac{1}{n}\right)$, then the value of n is ________.

Ans. **[3]**

Sol. $\gamma = 1 + \frac{2}{f}$

$f_A = 3, \Rightarrow \gamma_A = \frac{5}{3}$

$f_B = 3 + 3 + 2 = 8, \quad \gamma_B = 1 + \frac{2}{8} = \frac{5}{4}$

$\frac{\gamma_A}{\gamma_B} = \frac{4}{3}$

$\therefore \quad n = 3$

Q.48 A vessel with square cross-section and height of 6 m is vertically partitioned. A small window of 100 cm^2 with hinged door is fitted at a depth of 3 m in the partition wall. One part of the vessel is filled completely with water and the other side is filled with the liquid having density 1.5×10^3 kg/m^3. What force one needs to apply on the hinged door so that it does not get opened?
(Acceleration due to gravity = 10 m/s^2)

Ans. **[150]**

Sol. $F = (P_2 - P_1)A$
$= gh(\rho_2 - \rho_1)A$
$= 10 \times 3 \times 500 \times 100 \times 10^{-4}$
$= 150$ N

Q.49 If the measured angular separation between the second minimum to the left to the central maximum and the third minimum to the right of the central maximum is 30° in a single slit diffraction pattern recorded using 628 nm light, then the width of the slit is _________ μm.

Ans. **[6]**

Sol. 2^{nd} minima $\Rightarrow \theta_1 = \left(\frac{2\lambda}{a}\right)$

3^{rd} minima $\Rightarrow \theta_2 = \frac{3\lambda}{a}$

$\theta_1 + \theta_2 = \frac{\pi}{6}$

$\frac{5\lambda}{a} = \frac{\pi}{6}$

$a = \frac{30\lambda}{\pi} = \frac{30 \times 0.628}{3.14} = 6\ \mu m$

Q.50 A steel wire of length 2 m and Young's modulus 2.0×10^{11} Nm^{-2} is stretched by a force. If Poisson ratio and transverse strain for the wire are 0.2 and 10^{-3} respectively, then the elastic potential energy density of the wire is ________ $\times 10^5$ (in SI units).

Ans. **[25]**

Sol. $U = \frac{1}{2} \times y \times (\text{longitudinal strain})^2$

$$\alpha = \left|\frac{\text{transverse strain}}{\text{longitudinal strain}}\right|$$

$$\text{Longitudinal strain} = \frac{10^{-3}}{0.2} = 5\times 10^{-3}$$

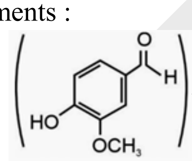
$$U = \frac{1}{2} \times 2 \times 10^{11} \times 25 \times 10^{-6}$$

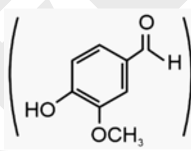
$$= 25 \times 10^5 \text{ J/m}^3$$

CHEMISTRY

Section-A: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct..

Q.51 Given below are two statements :

Statement (I) : Vanillin () will react with NaOH and also with Tollen's reagent.

Statement (II) : Vanillin () will undergo self aldol condensation very easily.

In the light of the above statements, choose the most appropriate answer from the options given below :

(1) Both **Statement I** and **Statement II** are correct

(2) Both Statement **I** and **Statement II** are incorrect

(3) **Statement I** is correct but **Statement II** is incorrect

(4) **Statement I** is incorrect but **Statement II** are correct

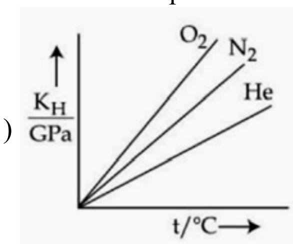
Ans. **[3]**

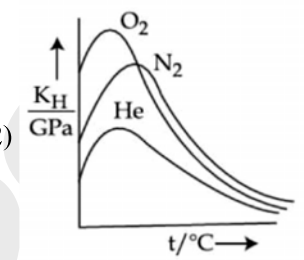
Sol. Vanillin will react with NaOH to give Cannizzaro reaction.

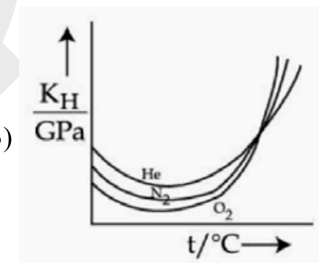
Vanillin also gives Tollen's reagent test.

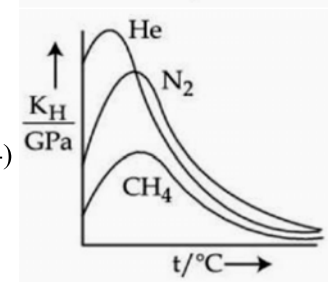
Vanillin does not undergo self aldol as it does not have α H-atom.

Q.52 Which of the following graph correctly represents the plots of K_H at 1 bar for gases in water versus temperature?

(1) 

(2) 

(3) 

(4) 

Ans. **[4]**

Sol. At low temperatures value of K_H first increases with temperature, passes through maximum and then decreases.

Q.53 A solution is made by mixing one mole of volatile liquid A with 3 moles of volatile liquid B. The vapour pressure of pure A is 200 mm Hg and that of the solution is 500 mm Hg. The vapour pressure of pure B and the least volatile component of the solution, respectively, are

(1) 600 mm Hg, B
(2) 600 mm Hg, A
(3) 1400 mm Hg, A
(4) 1400 mm Hg, B

Ans. [2]

Sol. $p_{total} = p_A^o x_A + p_B^o x_B$

$500 = 200\left(\frac{1}{4}\right) + p_B^o\left(\frac{3}{4}\right)$

$2000 = 200 + 3p_B^o$

$p_B^o = 600$

$p_A^o = 200$

A is least volatile

Q.54 The correct order of basic nature in aqueous solution for the bases NH_3, $H_2N - NH_2$, $CH_3CH_2NH_2$, $(CH_3CH_2)_2NH$ and $(CH_3CH_2)_3N$ is :

(1) $NH_3 < H_2N - NH_2 < (CH_3CH_2)_3N < CH_3CH_2NH_2 < (CH_3CH_2)_2NH$

(2) $NH_3 < H_2N - NH_2 < CH_3CH_2NH_2 < (CH_3CH_2)_2NH < (CH_3CH_2)_3N$

(3) $NH_2 - NH_2 < NH_3 < CH_3CH_2NH_2 < (CH_3CH_2)_3N < (CH_3CH_2)_2NH$

(4) $H_2N - NH_2 < NH_3 < (CH_3CH_2)_3N < CH_3CH_2NH_2 < (CH_3CH_2)_2NH$

Ans. [3]

Sol. Order of basic strength of ethyl amines in water is

2° > 3° > 1°. Also NH_3 is more basic than $H_2N — NH_2$

Q.55 Given below are two statements :

Statement (I): The metallic radius of Al is less than that of Ga.

Statement (II) : The ionic radius of Al^{3+} is less than that of Ga^{3+}

In the light of the above statements, choose the **most appropriate answer** from the options given below :

(1) **Statement-I** is incorrect but **Statement-II** is correct

(2) Both **Statement-I** and **Statement-II** are incorrect

(3) Both **Statement-I** and **Statement-II** are correct

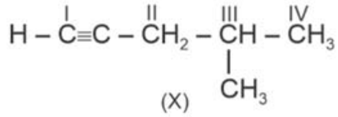
(4) **Statement-I** is correct but **Statement-II** is incorrect

Ans. [1]

Sol. Metallic radius of Al is, greater than that of Ga due to ineffective shielding of d-electrons.

But ionic radius of Al^{3+} is less than that of Ga^{3+} due to the absence of valence electrons.

Q.56 Consider the following compound (X)



(X)

The most stable and least stable carbon radicals, respectively, produced by homolytic cleavage of corresponding C–H bond are :

(1) II, IV (2) III, II (3) II, I (4) I, IV

Ans. [3]

Sol. Most stable : II (due to resonance)

Least stable : I (due to electron deficient sp hybrid C-atom)

Q.57 Among SO_2, NF_3, NH_3, XeF_2, ClF_3 and SF_4, the hybridization of the molecule with non-zero dipole moment and highest number of lone-pairs of electrons on the centre atom is :

(1) sp^3 (2) dsp^2 (3) sp^3d^2 (4) sp^3d

Ans. [4]

Sol. ClF_3 has highest no. of lone pairs on central atom which is sp^3d hybrid and has non-zero dipole moment

Q.58 Given below are two statements :

Statement (I) : In octahedral complexes, when $\Delta_0 < P$ high spin complex are formed.

Statement (II) : In tetrahedral complexes because of $\Delta_t < P$, low spin complexes are rarely formed.

In the light of the above statements, choose the **most appropriate answer** from the options given below :

(1) Both **Statement-I** and **Statement-II** are correct

(2) **Statement-I** is correct but **Statement-II** is incorrect

(3) Both **Statement-I** and **Statement-II** are incorrect

(4) **Statement-I** is incorrect but **Statement-II** is correct

Ans. [1]

Sol. When $\Delta_0 < P \Rightarrow$ No pairing

$\Rightarrow$ High spin complex

When $\Delta_0 > P \Rightarrow$ Pairing

$\Rightarrow$ Low spin complex

Statement-I is correct

In tetrahedral complexes, $\Delta_t < P$

$\Rightarrow$ High spin complexes are generally observed

$\Rightarrow$ Low spin complexes are rarely observed

$\Rightarrow$ Statement-II is correct

Q.59 A molecule with the formula AX_4Y has all its elements from p-block. Element A is rarest, monoatomic, non-radioactive from its group and has the lowest ionization enthalpy value among A, X and Y. Elements X and Y have first and second highest electronegativity values respectively among all the known elements. The shape of the molecule is :

(1) Trigonal bipyramidal

(2) Square pyramidal

(3) Octahedral

(4) Pentagonal planar

Ans. **[2]**

Sol. A is xenon

X is fluorine

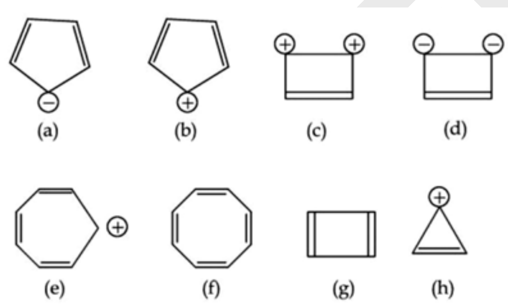
Y is oxygen

$XeOF_4$

Shape : Square pyramidal

Hybridisation : sp^3d^2

Q.60



Designate whether each of the following compounds is aromatic or not aromatic.

(1) a, b, c, d aromatic and e, f, g, h not aromatic

(2) e, g aromatic and a, b, c, d, f, h not aromatic

(3) a c, d, e, h aromatic and b, f, g not aromatic

(4) b, e, f, g aromatic and a, c, d, h not aromatic

Ans. **[3]**

Sol. a, c, d, e, f $\Rightarrow$ aromatic

b, f, g - Not aromatic

Q.61 An optically active alkyl halide C_4H_9Br [A] reacts with hot KOH dissolved in ethanol and forms alkene [B] as major product which reacts with bromine to give dibromide [C]. The compound [C] is converted into a gas [D] upon reacting with alcoholic $NaNH_2$.

During hydration 18 gram of water is added to 1 mole of gas [D] on warming with mercuric sulphate and dilute acid at 333 K to form compound [E]. The IUPAC name of compound [E] is :

(1) Butan-1-al (2) But-2-yne

(3) Butan-2-one (4) Butan-2-ol

Ans. **[3]**

Sol. A : $CH_3-CH_2-\underset{\underset{Br}{|}}{CH}-CH_3$

B : $CH_3-CH=CH-CH_3$

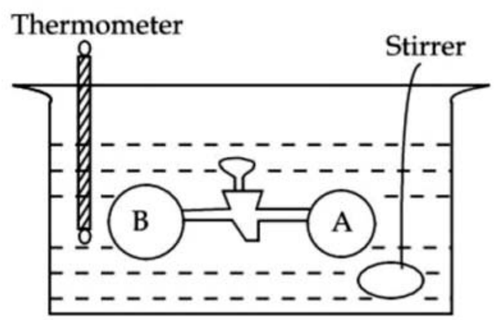
C : $CH_3-\underset{\underset{Br}{|}}{CH}-\overset{\overset{Br}{|}}{CH}-CH_3$

D : $CH_3-C\equiv C-CH_3$

E : $CH_3-\overset{\overset{O}{||}}{C}-CH_2-CH_3$

E is Butan-2-one

Q.62



Two vessels A and B are connected via stopcock.

The vessel A is filled with a gas at a certain pressure. The entire assembly is immersed in water and is allowed to come to thermal equilibrium with water. After opening the stopcock the gas from vessel A expands into vessel B and no change in temperature is observed in the thermometer. Which of the following statement is **true**?

(1) $dq \neq 0$

(2) $dw \neq 0$

(3) $dU \neq 0$

(4) The pressure in the vessel B before opening the stopcock is zero

Ans. **[4]**

Sol. The pressure in the vessel B before opening the stopcock is zero because dT = 0. Therefore,

dq = 0

dw = 0

dU = 0

Q.63 If equal volumes of AB_2 and XY (both are salts) aqueous solutions are mixed, which of the following combination will give a precipitate of AY_2 at 300 K?

(Given K_{sp} (at 300 K) for AY_2- = 5.2×10^{-7})

(1) 2.0×10^{-2} M AB_2, 2.0×10^{-2} M XY

(2) 2.0×10^{-4} M AB_2, 0.8×10^{-3} M XY

(3) 3.6×10^{-3} M AB_2, 5.0×10^{-4} M XY

(4) 1.5×10^{-4} M AB_2, 1.5×10^{-3} M XY

Ans. **[1]**

Sol. For precipitation

$[A^{2+}]\,[Y^-]^2 \geq 5.2 \times 10^{-7}$

In option (1)

$[A^{2+}] = 10^{-2}$ M

$[Y^-] = 10^{-2}$ M

$[A^{2+}]\,[Y^-]^2 = 10^{-6}$ which is greater than 5.2×10^{-7}

Q.64 On complete combustion 1.0g of an organic compound (X) gave 1.46 g of CO_2 and 0.567 g of H_2O. The empirical formula mass of compound (X) is ______g.

(Given molar mass in g mol^{-1} C : 12, H : 1, O : 16)

(1) 30 (2) 60 (3) 15 (4) 45

Ans. **[1]**

Sol. % by mass of C = 39.8

% by mass of H = 6.3

% by mass of O = 53.9

Emperical formula = CH_2O

Emperical formula mass of (X) = 30 g

Q.65 The property/properties that show irregularity in first four elements of group-17 is/are

(A) Covalent radius

(B) Electron affinity

(C) Ionic radius

(D) First ionization energy

Choose the **correct** answer from the options given below:

(1) A and C only (2) B only

(3) B and D only (4) A, B C and D

Ans. **[2]**

Sol. Exception in electron affinity only

Electron affinity order :

Cl > F > Br > I

Q.66 Choose the correct tests with respective observations.

(A) $CuSO_4$ (acidified with acetic acid) + $K_4[Fe(CN)_6] \rightarrow$ Chocolate brown precipitate.

(B) $FeCl_3$ + $K_4[Fe(CN)_6] \rightarrow$ Prussian blue precipitate.

(C) $ZnCl_2$ + $K_4[Fe(CN)_6]$, neutralised with $NH_4OH \rightarrow$ White or bluish white precipitate.

(D) $MgCl_2 + K_4[Fe(CN)_6] \rightarrow$ Blue precipitate.

(E) $BaCl_2$ + $K_4[Fe(CN)_6]$, neutralised with $NaOH \rightarrow$ White precipitate.

Choose the **correct** answer from the options given below :

(1) A, D and E only (2) B, D and E only

(3) C, D and E only (4) A, B and C only

Ans. **[4]**

Sol. $Cu^{2+} + K_4[Fe(CN)_6] \rightarrow \underset{\text{Chocolate Brown ppt}}{Cu_2[Fe(CN)_6]}\downarrow$

$Fe^{3+} + K_4[Fe(CN)_6] \rightarrow \underset{\text{Prussian blue ppt}}{Fe_4[Fe(CN)_6]_3}$

$Zn^{2+} + k_4\,[Fe(CN)_6] \rightarrow \underset{\text{White ppt}}{K_2Zn_3[Fe(CN)_6]_2}$

A, B, C are correct

Q.67 According to Bohr's model of hydrogen atom, which of the following statement is **incorrect**?

(1) Radius of 4^{th} orbit is four times larger than that of 2^{nd} orbit

(2) Radius of 6^{th} orbit is three times larger than that of 4^{th} orbit

(3) Radius of 8^{th} orbit is four times larger than that of 4^{th} orbit

(4) Radius of 3^{rd} orbit is nine times larger than that of 1^{st} orbit

Ans. **[2]**

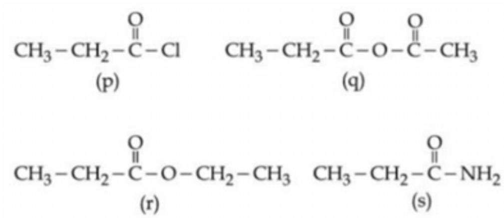
Sol. $r \propto n^2$

$r_6 = \frac{36}{16} r_4$

$\boxed{r_6 = \frac{9}{4} r_4}$

Option (2) is incorrect

Q.68 Consider the following molecules:



The correct order of rate of hydrolysis is :

(1) r > q > p > s (2) p > q > r > s

(3) p > r > q > s (4) q > p > r > s

Ans. **[2]**

Sol. Order of Hydrolysis

Acid chloride > Anhydride > Ester > Amide

p > q > r > s

Option (2) is correct

Q.69 $CaCO_3(s) + 2HCl(aq) \rightarrow CaCl_2(aq) + CO_2(g) + H_2O(l)$

Consider the above reaction, what mass of $CaCl_2$ will be formed if 250 mL of 0.76 M HCl reacts with 1000 g of $CaCO_3$?

(Given: Molar mass of Ca, C, O, H and Cl are 40, 12, 16, 1 and 35.5 g mol^{-1}, respectively)

(1) 3.908 g (2) 2.636 g

(3) 10.545 g (4) 5.272 g

Ans. **[3]**

Sol. $\underset{10\text{ mole}}{CaCO_3} + \underset{LR}{2HCl} \rightarrow CaCl_2 + CO_2 + H_2O$

Moles of $CaCl_2 = \dfrac{0.250 \times 0.76}{2}$

Mass of $CaCl_2 = \dfrac{0.250 \times 0.76}{2} \times 111 = 10.545$ g

Q.70 Identify the correct statement among the following:

(1) All naturally occurring amino acids except glycine contain one chiral centre.

(2) Glutamic acid is the only amino acid that contains a –COOH group at the side chain.

(3) Amino acid, cysteine can easily undergo dimerisation due to the presence of free SH group.

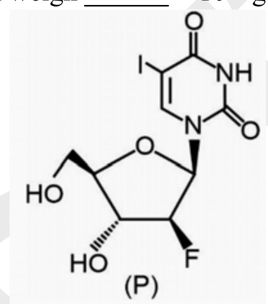
(4) All naturally occurring amino acids are optically active.

Ans. **[3]**

Sol. Amino acid, cysteine can easily undergo dimerisation due to free SH group.

Section-B: Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

Q.71 0.1 mol of the following given antiviral compound (P) will weigh ______ × 10^{-1} g



(Given : molar mass in g mol^{-1} H : 1, C : 12, N : 14, O : 16, F : 19, I : 127)

Ans. **[372]**

Sol. Formula of compound (P)

$= C_9O_5FIN_2H_{10}$

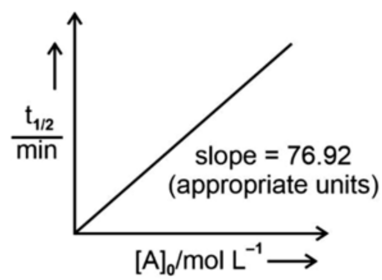
Molar mass of (P)

$= 372 \dfrac{gm}{mole}$

0.1 mole mass = 37.2 g

$= 372 \times 10^{-1}$ g

Q.72 For the reaction A → products.



The concentration of A at 10 minutes is ___ × 10^{-3} mol L^{-1} (nearest integer).

The reaction was started with 2.5 mol L^{-1} of A.

Ans. **[2435]**

Sol. $t_{1/2} \propto [A_0] \Rightarrow$ zero order reaction

$\dfrac{1}{2k} = 76.92$

$k = \dfrac{1}{2 \times 76.92} = 6.5 \times 10^{-3}$

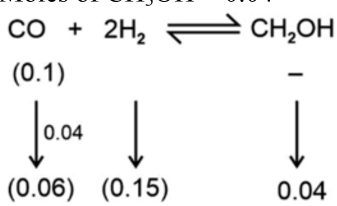
$C_t = C_0 - kt$
$C_t = (2.5) - (6.5 \times 10^{-3})(10)$
$= 2.5 - 6.5 \times 10^{-2}$
$= 2.435$
$= 2435 \times 10^{-3}$
Nearest integer = 2435

Q.73 Consider the following equilibrium,
$CO(g) + 2H_2(g) \rightleftharpoons CH_3OH(g)$
0.1 mol of CO along with a catalyst is present in a 2 dm^3 flask maintained at 500 K. Hydrogen is introduced into the flask until the pressure is 5 bar and 0.04 mol of CH_3OH is formed. The k_p is ___ $\times 10^{-3}$ (nearest integer).
Given : R = 0.08 dm^3 bar K^{-1} mol^{-1}
Assume only methanol is formed as the product and the system follows ideal gas behaviour.

Ans. **[74]**

Sol. n_{total} at equilibrium : $\dfrac{5 \times 2}{(0.08) \times 500}$

$= 0.250$
Moles of $CH_3OH = 0.04$



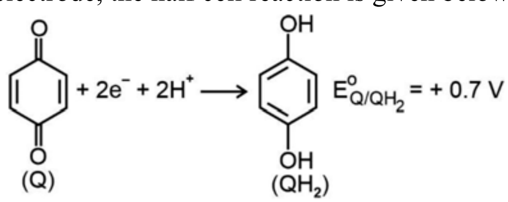
$P_{CH_2OH} = \dfrac{(0.04) \times (0.08) \times 500}{2} = 0.8$ bar

$P_{H_2} = 3$ bar

$P_{CO} = 1.2$ bar

$K_P = \dfrac{0.8}{(1.2)(3)^2} = 0.07407 = 74.07 \times 10^{-3}$

Q.74 Consider the following electrochemical cell at standard condition.
$Au(s)|QH_2,Q|NH_4X(0.01\ M)||Ag^+(1\ M)|Ag(s)$
$E_{cell} = +0.4$ V
The couple QH_2/Q represents quinhydrone electrode, the half cell reaction is given below :



$\left[\text{Given} : E^o_{Ag^+/Ag} = +0.8\text{V and } \dfrac{2.303RT}{F} = 0.06\text{V}\right]$

The pK_b value of the ammonium halide salt (NH_4X) used here is ______. (nearest integer)

Ans. **[6]**

Sol. $E^0_{cell} = 0.8 - 0.7 = 0.1$V
$E_{cell} = 0.4$ V

$0.4 = 0.1 - \dfrac{0.06}{1}\log[H^+]$

$0.3 = 0.06$ pH
pH = 5

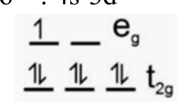
$5 = 7 - \left(\dfrac{pK_b + \log C}{2}\right)$

$pK_b + \log 0.01 = 4$
$pK_b - 2 = 4$
$pK_b = 6$

Q.75 A transition metal (M) among Mn, Cr, Co and Fe has the highest standard electrode potential (M^{3+}/M^{2+}). If M forms a metal complex of the type $[M(CN)_6]^{4-}$. The number of electrons present in the e_g orbital of the complex is ______.

Ans. **[1]**

Sol. $E^0_{M^{3+}/M^{2+}}$ is highest for Co
$[Co(CN)_6]^{4-}$
Co^{2+} : $4s^0 3d^7$



No. of electrons in e_g orbital = 1