



JEE Main Online Exam 2025

Questions & Solution

02nd April 2025 | Evening

MATHEMATICS

Section-A: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct..

Q.1 If $\lim_{x \rightarrow 0} \frac{\cos(2x) + a \cos(4x) - b}{x^4}$ is finite, then $(a + b)$ is equal to :

(1) $\frac{1}{2}$ (2) $\frac{3}{4}$ (3) -1 (4) 0

Ans. [1]

Sol. $\lim_{x \rightarrow 0} \frac{\cos(2x) + a \cos(4x) - b}{x^4}$
 $\lim_{x \rightarrow 0} \frac{\left(1 - \frac{4x^2}{2!} + \frac{(2x)^4}{4!} - \dots\right) + a\left(1 - \frac{(4x)^2}{2!} + \frac{(4x)^4}{4!} - \dots\right) - b}{x^4}$

For limit to be finite

$$1 + a - b = 0$$

$$\text{And } -2 - 8a = 0$$

$$\Rightarrow 8a = -2$$

$$\Rightarrow a = \frac{-1}{4}$$

$$\Rightarrow b = 1 + a$$

$$= 1 - \frac{1}{4}$$

$$b = +\frac{3}{4}$$

$$a + b = \frac{-1}{4} + \frac{3}{4} = \frac{1}{2}$$

Option (1) is correct

Q.2 If $\sum_{r=0}^{10} \left(\frac{10^{r+1} - 1}{10^r} \right) \cdot {}^{11}C_{r+1} = \frac{\alpha^{11} - 11^{11}}{10^{10}}$, then α is equal to :

(1) 20 (2) 24 (3) 15 (4) 11

Ans. [1]

Sol. $\sum_{r=0}^{10} \left(\frac{10^{r+1} - 1}{10^r} \right) \cdot {}^{11}C_{r+1}$

$$\begin{aligned}
 &= (10 - 1) {}^{11}C_1 + \left(\frac{10^2 - 1}{10} \right) {}^{11}C_2 \\
 &\quad + \dots + \left(\frac{10^{11} - 1}{10^{10}} \right) {}^{11}C_{11} \\
 &= 10({}^{11}C_1 + {}^{11}C_2 + \dots + {}^{11}C_{11}) \\
 &\quad - \left({}^{11}C_1 + \frac{{}^{11}C_2}{10} + \dots + \frac{{}^{11}C_{11}}{10^{10}} \right) \\
 &= 10(2^{11} - 1) - 10 \left(\frac{{}^{11}C_1}{10} + \frac{{}^{11}C_2}{10^2} + \dots + \frac{{}^{11}C_{11}}{10^{11}} \right) \\
 &= 10(2^{11} - 1) - 10 \left(\left(1 + \frac{1}{10} \right)^{11} - 1 \right) \\
 &= 10(2^{11} - 1) - 10 \left(\frac{11^{11} - 10^{11}}{10^{11}} \right) \\
 &= 10(2^{11} - 1) - \left(\frac{11^{11} - 10^{11}}{10^{10}} \right) \\
 &= \frac{10^{11}(2^{11} - 1) - 11^{11} + 10^{11}}{10^{10}} \\
 &= \frac{2^{11}(10^{11}) - 11^{11}}{10^{10}} = \frac{20^{11} - 11^{11}}{10^{10}} \\
 &= \alpha = 20
 \end{aligned}$$

Q.3

Let (a, b) be the point of intersection of the curve $x^2 = 2y$ and the straight line $y - 2x - 6 = 0$ in the second quadrant. Then the integral $I = \int_a^b \frac{9x^2}{1+5^x} dx$ is equal to :

(1) 21 (2) 27 (3) 24 (4) 18

Ans. [3]

Sol. $x^2 = 2y$ and $y - 2x - 6 = 0$

$$\frac{x^2}{2} - 2x - 6 = 0$$

$$x^2 - 4x - 12 = 0$$

$$x^2 - 6x + 2x - 12 = 0$$



$$x(x-6) + 2(x-6) = 0$$

$$(x-6)(x+2) = 0$$

Point of intersection are (6, 18) and (-2, 2)

(-2, 2) is in second quadrant

$$a = -2, b = 2$$

$$I = \int_{-2}^2 \frac{9x^2}{1+5^x} dx \quad \dots(i)$$

$$I = \int_{-2}^2 \frac{9x^2}{1+5^{-x}} dx \quad \dots(ii)$$

Adding (i) and (ii)

$$2I = \int_{-2}^2 9x^2 dx$$

$$I = 9 \int_0^2 x^2 dx$$

$$I = 9 \left(\frac{x^3}{3} \right)_0^2 \Rightarrow I = 24$$

Q.4 If $\theta \in \left[-\frac{7\pi}{6}, \frac{4\pi}{3} \right]$, then the number of solutions of $\sqrt{3} \operatorname{cosec}^2 \theta - 2(\sqrt{3}-1) \operatorname{cosec} \theta - 4 = 0$, is equal to :

(1) 8 (2) 10 (3) 7 (4) 6

Ans. [4]

$$\begin{aligned} \operatorname{cosec} \theta &= \frac{2\sqrt{3} - 2 \pm \sqrt{(2(\sqrt{3}-1))^2 + 4\sqrt{3}(4)}}{2\sqrt{3}} \\ &= 2\sqrt{3} - 2 \pm \sqrt{4(3+1-2\sqrt{3})+16\sqrt{3}} \\ &= 2\sqrt{3} - 2 \pm 2(\sqrt{3}+1) \\ &= \frac{2\sqrt{3} - 2 \pm 2(\sqrt{3}+1)}{2\sqrt{3}} \\ \Rightarrow \operatorname{cosec} \theta &= \frac{-2}{\sqrt{3}}, 2 \\ \theta &= \frac{-7\pi}{6}, \frac{-2\pi}{3}, \frac{-\pi}{3}, \frac{\pi}{6}, \frac{5\pi}{6}, \frac{4\pi}{3} \end{aligned}$$

Q.5 If the mean and the variance of 6, 4, a, 8, b, 12, 10, 13 are 9 and 9.25 respectively, then a + b + ab is equal to :

(1) 103 (2) 100 (3) 106 (4) 105

Ans. [1]

$$\text{Sol. Mean} = \frac{6+4+a+8+b+12+10+13}{8} = 9$$

$$53 + a + b + = 72$$

$$\Rightarrow a + b = 19 \text{ and}$$

And

$$\frac{36+16+a^2+64+b^2+144+100+169}{8} - 9^2$$

$$= 9.25$$

$$\Rightarrow \frac{529+a^2+b^2}{8} = 90.25$$

$$\Rightarrow a^2 + b^2 = 193$$

$$\therefore a = 7, b = 12$$

$$\text{Hence, } a + b + ab = 19 + 84 = 103$$

Q.6

If the domain of the function

$$f(x) = \frac{1}{\sqrt{10+3x-x^2}} + \frac{1}{\sqrt{x+|x|}}$$

(1 + a)² + b² is equal to :

(1) 26 (2) 30 (3) 29 (4) 25

Ans. [1]

$$\text{Sol. } x + |x| = \begin{cases} 2x, x \geq 0 \\ 0, x < 0 \end{cases}$$

$$\Rightarrow \frac{1}{\sqrt{x+|x|}}, \text{ domain is } x > 0, \text{ as } 2x \neq 0$$

Similarly,

$$\frac{1}{\sqrt{10+3x-x^2}} \text{ is defined when } 10+3x-x^2 > 0$$

$$\Rightarrow x^2 - 3x - 10 < 0$$

$$(x-5)(x+2) < 0$$

$$\Rightarrow x \in (-2, 5)$$

$$\Rightarrow \text{Domain will be } (0, \infty) \cap (-2, 5) = (0, 5)$$

$$\Rightarrow (1+a)^2 + b^2 = 1 + 25 = 26$$

Q.7

Given three identical bags each containing 10 balls, whose colours are as follows :

| | Red | Blue | Green |
|----------------|-----|------|-------|
| Bag I | 3 | 2 | 5 |
| Bag II | 4 | 3 | 3 |
| Bag III | 5 | 1 | 4 |

A person chooses a bag at random and takes out a ball. If the ball is Red, the probability that it is from bag I is p and if the ball is Green, the probability that it is from bag III is q, then the

value of $\left(\frac{1}{p} + \frac{1}{q} \right)$ is :

(1) 8 (2) 9 (3) 7 (4) 6

Ans. [3]

Sol. $p(B_1/R) = \frac{p(B_1) \cdot p(R/B_1)}{p(R)}$

$$= \frac{\frac{1}{3} \times \frac{3}{10}}{\frac{1}{3} \times \frac{3}{10} + \frac{1}{3} \times \frac{4}{10} + \frac{1}{3} \times \frac{5}{10}}$$

$$= \frac{\frac{1}{4}}{\frac{1}{4}} = p$$

$$p(B_3/G) = \frac{p(B_3) \cdot p(G/B_3)}{p(G)}$$

$$= \frac{\frac{1}{3} \times \frac{4}{10}}{\frac{1}{3} \times \frac{5}{10} + \frac{1}{3} \times \frac{3}{10} + \frac{1}{3} \times \frac{4}{10}}$$

$$= \frac{\frac{1}{3}}{\frac{1}{3}} = q$$

$$\therefore \left(\frac{1}{p} + \frac{1}{q} \right) = 7$$

Q.8 The line L_1 is parallel to the vector $\vec{a} = -3\hat{i} + 2\hat{j} + 4\hat{k}$ and passes through the point $(7, 6, 2)$ and the line L_2 is parallel to the vector $\vec{b} = 2\hat{i} + \hat{j} + 3\hat{k}$ and passes through the point $(5, 3, 4)$. The shortest distance between the lines L_1 and L_2 is :

(1) $\frac{21}{\sqrt{38}}$ (2) $\frac{23}{\sqrt{38}}$ (3) $\frac{21}{\sqrt{57}}$ (4) $\frac{23}{\sqrt{57}}$

Ans. [2]

Sol. Eqn. of $L_1 : 7\hat{i} + 6\hat{j} + 2\hat{k} + \lambda(-3\hat{i} + 2\hat{j} + 4\hat{k})$

Eqn. of $L_2 : 5\hat{i} + 3\hat{j} + 4\hat{k} + \lambda(2\hat{i} + \hat{j} + 3\hat{k})$

Shortest distance between L_1 and L_2

$$= \left| \frac{(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)}{|\vec{b}_1 \times \vec{b}_2|} \right|$$

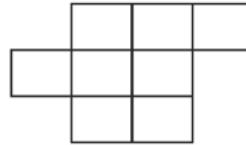
$$= \left| \frac{(-2\hat{i} - 3\hat{j} + 2\hat{k}) \cdot (2\hat{i} + 17\hat{j} - 7\hat{k})}{\sqrt{2^2 + (-17)^2 + (-7)^2}} \right|$$

$$= \left| \frac{-4 - 51 - 14}{\sqrt{4 + 289 + 49}} \right|$$

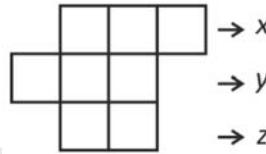
$$= \left| \frac{-69}{\sqrt{342}} \right| = \frac{23}{\sqrt{38}}$$

Q.9

The number of ways, in which the letters A, B, C, D, E can be placed in the 8 boxes of the figure below so that no row remains empty and at most one letter can be placed in a box, is :



(1) 960 (2) 5760 (3) 840 (4) 5880

Ans. [2]


Let x, y, z be the number of boxes which are filled

$$\Rightarrow 1 \leq x \leq 3, 1 \leq y \leq 3, 1 \leq z \leq 2$$

| x | y | z | Number of ways |
|---|---|---|--|
| 3 | 1 | 1 | ${}^3C_3 \cdot {}^3C_1 \cdot {}^2C_1 = 6$ |
| 2 | 2 | 1 | ${}^3C_2 \cdot {}^3C_2 \cdot {}^2C_1 = 18$ |
| 1 | 3 | 1 | ${}^3C_1 \cdot {}^3C_3 \cdot {}^2C_1 = 6$ |
| 2 | 1 | 2 | ${}^3C_2 \cdot {}^3C_1 \cdot {}^2C_2 = 9$ |
| 3 | 2 | 2 | ${}^3C_1 \cdot {}^3C_2 \cdot {}^2C_2 = 9$ |

Total ways = (48) to fill boxes

Now to arrange a, b, c, d and e

Number of ways will be $48 \cdot 5! = 5760$

Q.10

Let $f : [1, \infty) \rightarrow [2, \infty)$ be a differentiable function. If $10 \int_1^x f(t) dt = 5xf(x) - x^5 - 9$ for all

$x \geq 1$, then the value of $f(3)$ is :

(1) 26 (2) 32 (3) 18 (4) 22

Ans. [2]

Sol. $10 \int_1^x f(t) dt = 5xf(x) - x^5 - 9$

differentiate w.r.t. x both sides

$$10f(x) = 5f(x) + 5xf'(x) - 5x^4$$

$$\Rightarrow 5f(x) = 5xf'(x) - 5x^4$$

$$\text{Let } y = f(x) \Rightarrow \frac{dy}{dx} = f'(x)$$

$$5y = 5x \frac{dy}{dx} - 5x^4 \Rightarrow \frac{dy}{dx} - \frac{y}{x} = x^3$$

I.F. of this linear differential equation

$$\Rightarrow \text{I.F.} = e^{\int \frac{-1}{x} dx} = e^{-\ln x} = e^{\ln\left(\frac{1}{x}\right)} = \frac{1}{x}$$

$$\Rightarrow 1 \left(\frac{1}{x} \right) = \int x^3 - \frac{1}{x} dx = \frac{x^3}{3} + C$$

$$\Rightarrow y = \frac{x^4}{3} + Cx, \text{ at } x = 1$$

$$10 \int_1^1 f(t) dt = 5f(1) - 1 - 9 \Rightarrow f(1) = 2$$

$$\Rightarrow f(1) = \frac{1}{3} + C = 2 \Rightarrow C = \frac{5}{3}$$

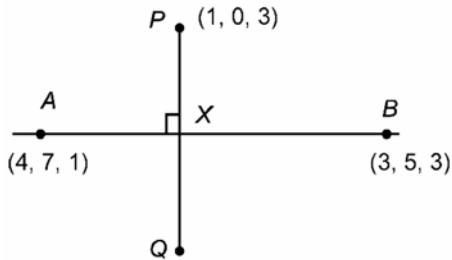
$$\Rightarrow f(3) = \frac{3^4}{3} + \frac{5}{3} \cdot 3 = 3^3 + 5 = 32$$

Q.11 If the image of the point $P(1, 0, 3)$ in the line joining the points $A(4, 7, 1)$ and $B(3, 5, 3)$ is $Q(\alpha, \beta, \gamma)$, then $\alpha + \beta + \gamma$ is equal to :

(1) $\frac{46}{3}$ (2) $\frac{47}{3}$ (3) 18 (4) 13

Ans. [1]

Sol.



Let X be mid point of P and Q , which would be also feet of perpendicular.

Let X divides A and B in $\lambda : 1$, $\lambda \neq -1$

$$X = \left(\frac{3\lambda + 4}{\lambda + 1}, \frac{5\lambda + 7}{\lambda + 1}, \frac{3\lambda + 1}{\lambda + 1} \right)$$

Now $PX \perp AB \Rightarrow \vec{PX} \cdot \vec{AB} = 0$

$$\left(\frac{3\lambda + 4}{\lambda + 1} - 1 \right) \cdot (4 - 3) + \left(\frac{5\lambda + 7}{\lambda + 1} - 0 \right) (7 - 5) + \left(\frac{3\lambda + 1}{\lambda + 1} - 3 \right) \cdot (1 - 3) = 0$$

$$\frac{2\lambda + 3}{\lambda + 1} + \frac{10\lambda + 14}{\lambda + 1} + \frac{4}{\lambda + 1} = 0$$

$$\Rightarrow \frac{12\lambda + 21}{\lambda + 1} = 0 \Rightarrow \lambda = \frac{-7}{4}$$

$$X = \left(\frac{5}{3}, \frac{7}{3}, \frac{17}{3} \right)$$

X is mid point of PQ

$$Q \equiv \left(2 \cdot \frac{5}{3} - 1, 2 \cdot \frac{7}{3} - 0, 2 \cdot \frac{17}{3} - 3 \right) \equiv (\alpha, \beta, \gamma)$$

$$\Rightarrow \alpha + \beta + \gamma = \frac{2(5 + 7 + 17)}{3} - 4 = \frac{58}{3} - 4 = \frac{46}{3}$$

Q.12

Let $A = \{1, 2, 3, \dots, 100\}$ and R be a relation on A such that $R = \{(a, b) : a = 2b + 1\}$. Let $(a_1, a_2), (a_2, a_3), (a_3, a_4), \dots, (a_k, a_k + 1)$ be a sequence of k elements of R such that the second entry of an ordered pair is equal to the first entry of the next ordered pair. Then the largest integer k , for which such a sequence exists, is equal to:

(1) 6 (2) 8 (3) 5 (4) 7

Ans.

[3]

The sequence with k terms be

$$(a_1, a_2), (a_2, a_3), \dots, (a_k, a_k + 1)$$

Where $a_i = 2a_{i+1} + 1$, using A on A relation $a_1 = 2a_2 + 1$, a_1 will be odd.

$$a_2 = 2a_3 + 1 \Rightarrow a_1 = 2(2a_3 + 1) + 1 = 4a_3 + 3$$

$$a_3 = 2a_4 + 1 \quad a_1 = 4(2a_4 + 1) + 3 = 8a_4 + 7$$

$$\vdots \quad = 2^3 \cdot a_4 + 7$$

$$a_k = 2a_{k+1} + 1 \quad a_1 = 2^k \cdot a_{k+1} + (2^k - 1) \in A$$

$$\frac{a_1 + 1 - 2^k}{2^k} = a_{k+1} \Rightarrow 2^k |(a_1 + 1), \text{ we need to}$$

find highest k . $a_1 + 1 \in \{2, \dots, 101\}$

k maximum when $k = 6$, as at $k = 7$, $2^k = 128$

$128 | e_i \forall e_i \in A \Rightarrow a_1 = 95$ and $k = 6$

$(95, 47), (47, 23), (23, 11), (11, 5), (5, 2)$ will be sequence.

Q.13

Let the point P of the focal chord PQ of the parabola $y^2 = 16x$ be $(1, -4)$. If the focus of the parabola divides the chord PQ in the ratio $m : n$, $\text{gcd}(m, n) = 1$, then $m^2 + n^2$ is equal to :

(1) 17 (2) 26 (3) 10 (4) 37

Ans.

Sol.

$$P(at^2, 2at) \equiv P(4t^2, 8t) \equiv (1, -4)$$

$$8t = -4 \Rightarrow t = -\frac{1}{2}$$

$$\therefore Q \left(\frac{a}{t^2}, \frac{-2a}{t} \right) \quad (t_1 \cdot t_2 = -1)$$

$S(4, 0)$ is the focus

$$PS = a + at^2$$

$$QS = a + \frac{a}{t^2} = \frac{at^2 + a}{t^2}$$

$$\frac{PS}{QS} = t^2 = \frac{1}{4} = \frac{m}{n}$$

$$\therefore m^2 + n^2 = 17$$

Q.14 If the length of the minor axis of an ellipse is equal to one fourth of the distance between the foci, then the eccentricity of the ellipse is:

(1) $\frac{\sqrt{5}}{7}$ (2) $\frac{4}{\sqrt{17}}$ (3) $\frac{3}{\sqrt{19}}$ (4) $\frac{\sqrt{3}}{16}$

Ans. [2]

Sol. $2b = \frac{1}{4}(2ae)$

$$\Rightarrow b = \frac{ae}{4}$$

$$\Rightarrow \frac{b^2}{a^2} = \frac{e^2}{16}$$

$$\Rightarrow \frac{17e^2}{16} = 1 \Rightarrow e = \frac{4}{\sqrt{17}}$$

Q.15 Let $\vec{a} = 2\hat{i} - 3\hat{j} + \hat{k}$, $\vec{b} = 3\hat{i} + 2\hat{j} + 5\hat{k}$ and a vector \vec{c} be such that $(\vec{a} - \vec{c}) \times \vec{b} = -18\hat{i} - 3\hat{j} + 12\hat{k}$ and $\vec{a} \cdot \vec{c} = 3$. If $\vec{b} \times \vec{c} = \vec{d}$, then

$|\vec{a} \cdot \vec{d}|$ is equal to :

(1) 9 (2) 12 (3) 18 (4) 15

Ans. [4]

Sol. $(\vec{a} - \vec{c}) \times \vec{b} = -18\hat{i} - 3\hat{j} + 12\hat{k}$

$$\vec{a} \times \vec{b} - \vec{c} \times \vec{b} = -18\hat{i} - 3\hat{j} + 12\hat{k}$$

$$\Rightarrow \vec{a} \times \vec{b} + \vec{d} = -18\hat{i} - 3\hat{j} + 12\hat{k}$$

Take dot with \vec{a}

$$\vec{a} \cdot \vec{d} = -36 + 9 + 12 = -15$$

$$\therefore |\vec{a} \cdot \vec{d}| = 15$$

Q.16 $4 \int_0^1 \left(\frac{1}{\sqrt{3+x^2}} + \frac{1}{\sqrt{1+x^2}} \right) dx - 3 \log_e (\sqrt{3})$ is equal to :

(1) $2 + \sqrt{2} - \log_e (1 + \sqrt{2})$
 (2) $2 - \sqrt{2} - \log_e (1 + \sqrt{2})$
 (3) $2 + \sqrt{2} + \log_e (1 + \sqrt{2})$
 (4) $2 - \sqrt{2} + \log_e (1 + \sqrt{2})$

Ans. [3]

Sol. $I = 4 \int_0^1 \frac{1}{\sqrt{3+x^2}} dx$
 $= 2 \int_0^1 \sqrt{3+x^2} - \sqrt{1+x^2} dx$
 $= 2 \left[\int_0^1 \sqrt{3+x^2} dx - \int_0^1 \sqrt{1+x^2} dx \right]$
 $= 2 \left[\left(\frac{1}{2}x\sqrt{x^2+3} + \frac{3}{2} \ln \left| \sqrt{x^2+3} + x \right| \right) - \left(\frac{1}{2}x\sqrt{1+x^2} + \frac{1}{2} \ln \left| \sqrt{1+x^2} + x \right| \right) \right]_0^1$
 $= 2 \left[\left(1 + \frac{3}{2} \ln 3 - \frac{3}{2} \ln \sqrt{3} \right) - \left(\frac{\sqrt{2}}{2} + \frac{1}{2} \ln(\sqrt{2} + 1) \right) \right]$
 $= 2 \left(1 + \frac{3}{4} \ln 3 - \frac{1}{\sqrt{2}} - \frac{1}{2} \ln(\sqrt{2} + 1) \right)$
 $= 3 \ln \sqrt{3} + 2 - \sqrt{2} - \ln(\sqrt{2} + 1)$
 $I = 3 \ln \sqrt{3} = 2 + \sqrt{2} + \ln(\sqrt{2} + 1)$

Q.17

The number of terms of an A.P. is even, the sum of all the odd terms is 24, the sum of all the even terms is 30 and the last term exceeds the first by $\frac{21}{2}$. Then the number of terms

which are integers in the A.P. is:

(1) 10 (2) 6 (3) 4 (4) 8

Ans. [4]

Sol. Let the number of terms be $2n$

$$\begin{aligned} T_1 + T_3 + T_5 + \dots + T_{2n-1} &= 24 \\ T_2 + T_4 + T_6 + \dots + T_{2n} &= 30 \\ (T_2 - T_1) + (T_4 - T_3) + \dots + (T_{2n} - T_{2n-1}) &= 6 \\ nd &= 6 \end{aligned}$$

$$(a + (2n+1)d) - a = \frac{21}{2}$$

$$\Rightarrow 2nd - d = \frac{21}{2}$$

$$\Rightarrow 12 - \frac{21}{2} = d$$

$$\Rightarrow d = \frac{3}{2}$$

$$\therefore n = 4$$

$$\therefore \text{Total terms} = 8$$

Q.18 If the system of equations

$$2x + \lambda y + 3z = 5$$

$$3x + 2y - z = 7$$

$$4x + 5y + \mu z = 9$$

has infinitely many solutions, then $(\lambda^2 + \mu^2)$ is equal to :

(1) 30 (2) 18 (3) 26 (4) 22

Ans. [3]

Sol. $2x + \lambda y + 3z = 5$

$$3x + 2y - z = 7$$

$$4x + 5y + \mu z = 9$$

For infinite solutions $\Rightarrow \Delta = 0 = \Delta_1 = \Delta_2 = \Delta_3$

$$\Delta = \begin{vmatrix} 2 & \lambda & 3 \\ 3 & 2 & -1 \\ 4 & 5 & \mu \end{vmatrix} = 0$$

$$\Rightarrow -4\lambda - 3\lambda\mu + 4\mu + 31 = 0$$

$$\Delta_1 = \begin{vmatrix} 5 & \lambda & 3 \\ 7 & 2 & -1 \\ 9 & 5 & \mu \end{vmatrix} = 0 \Rightarrow -9\lambda - 7\lambda\mu + 10\mu + 76 = 0$$

$$\Delta_2 = \begin{vmatrix} 2 & 3 & 5 \\ 3 & -1 & 7 \\ 4 & \mu & 9 \end{vmatrix} = 0 \Rightarrow \mu + 5 = 0 \Rightarrow \mu = -5$$

$$\Delta_3 = \begin{vmatrix} 2 & \lambda & 5 \\ 3 & 2 & 7 \\ 4 & 5 & 9 \end{vmatrix} = 0 \Rightarrow \lambda + 1 = 0 \Rightarrow \lambda = -1$$

\therefore For infinite solution $\mu = -5$ and $\lambda = -1$

Now $\mu^2 + \lambda^2 = 25 + 1 = 26$

Q.19 Let A be a 3×3 real matrix such that $A^2(A - 2I) - 4(A - I) = 0$, where I and O are the identity and null matrices, respectively. If $A^5 = \alpha A^2 + \beta A + \gamma I$, where α , β and γ are real constants, then $\alpha + \beta + \gamma$ is equal to :

(1) 4 (2) 20 (3) 12 (4) 76

Ans. [3]

Sol. $A^2(A - 2I) - 4(A - I) = 0$

$$A^3 - 2A^2 - 4A + 4I = 0$$

Multiply by A

$$A^4 = 2A^3 + 4A^2 - 4A$$

$$A^4 = 2(2A^2 + 4A - 4I) + 4A^2 - 4A$$

$$A^4 = 8A^2 + 4A - 8I$$

Multiply again by A

$$\Rightarrow A^5 = 8A^3 + 4A^2 - 8A$$

$$\Rightarrow A^5 = 8(2A^2 + 4A - 4I) + 4A^2 - 8A$$

$$\Rightarrow A^5 = 20A^2 + 24A - 32I$$

Comparing with $A^5 = \alpha A^2 + \beta A + \gamma I$

$$\alpha = 20, \beta = 24, \gamma = -32$$

$$\therefore \alpha + \beta + \gamma = 20 + 24 - 32$$

$$= 44 - 32$$

$$= 12$$

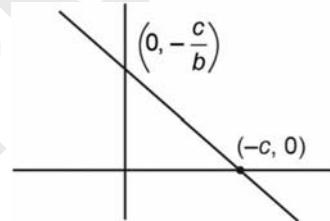
Q.20

Let the area of the triangle formed by a straight line $L : x + by + c = 0$ with co-ordinate axes be 48 square units. If the perpendicular drawn from the origin to the line L makes an angle of 45° with the positive x-axis, then the value of $b^2 + c^2$ is:

(1) 97 (2) 90 (3) 93 (4) 83

Ans. [1]

Sol.



$$L : x + by + c = 0$$

$$\therefore \frac{1}{2} \left| (-c) \cdot \left(-\frac{c}{b} \right) \right| = 48$$

$$\therefore \left| \frac{c^2}{b} \right| = 96 \quad \dots (i)$$

$$\text{Slope of line } L = -\frac{1}{b}$$

\therefore Slope of line perpendicular to L is b .

$$\therefore b = 1$$

$$\therefore c^2 = 96$$

$$\therefore b^2 + c^2 = 97$$

Section-B: Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

Q.21 If the sum of the first 10 terms of the series

$$\frac{4.1}{1+4.1^4} + \frac{4.2}{1+4.2^4} + \frac{4.3}{1+4.3^4} + \dots \text{ is } \frac{m}{n}, \text{ where } \text{gcd}(m, n), \text{ then } m + n \text{ is equal to } \underline{\hspace{2cm}}$$

Ans. [441]

$$\text{Sol. } \frac{4.1}{1+4.1^4} + \frac{4.2}{1+4.2^4} + \frac{4.3}{1+4.3^4} + \dots$$

$$T_r = \frac{4r}{1+4r^4} = \frac{4r}{4r^4 + 4r^2 + 1 - 4r^2}$$

$$= \frac{4r}{(2r^2 + 1)^2 - (2r)^2}$$

$$T_r = \frac{4r}{(2r^2 - 2r + 1)(2r^2 + 2r + 1)}$$

$$T_r = \frac{(2r^2 + 2r + 1) - (2r^2 - 2r + 1)}{(2r^2 - 2r + 1)(2r^2 + 2r + 1)}$$

$$\sum_{r=1}^{10} T_r = \left(\frac{1}{O^2 + 1^2} - \frac{1}{1^2 + 2^2} + \frac{1}{1^2 + 2^2} - \frac{1}{2^2 + 3^2} + \dots \right)$$

$$= \frac{1}{9^2 + 10^2} - \frac{1}{10^2 + 11^2}$$

$$= 1 - \frac{1}{221}$$

$$= \frac{220}{221}$$

$$\therefore m + n = 220 + 221$$

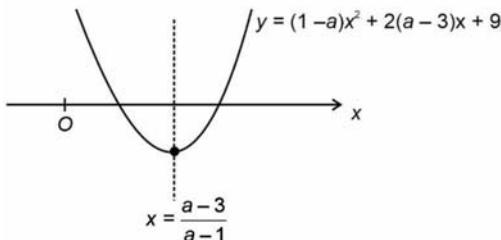
$$= 441$$

Q.22 If the set of all $a \in \mathbb{R} - \{1\}$, for which the roots of the equation $(1-a)x^2 + 2(a-3)x + 9 = 0$ are positive is $(-\infty, -\alpha] \cup [\beta, \gamma)$, then $2\alpha + \beta + \gamma$ is equal to _____.
Ans. [7]

Sol. $f(x) = (1-a)x^2 + 2(a-3)x + 9$, $f(0) = 9 > 0$

$$D \geq 0 \Rightarrow 4(a-3)^2 \geq 4(1-a) \cdot 9$$

$$\Rightarrow [a \in (-\infty, -3] \cup [0, \infty)] \quad \dots (i)$$



$$x_1 + x_2 = \frac{-2(a-3)}{1-a}, x_1 x_2 = \frac{9}{1-a}$$

$$x_1 + x_2 > 0 \Rightarrow \frac{a-3}{a-1} > 0$$

$$\Rightarrow [a \in (-\infty, 1) \cup (3, \infty)] \quad \dots (ii)$$

$$x_1 x_2 > 0 \Rightarrow 1-a > 0$$

$$\Rightarrow [a \in (-\infty, 1)] \quad \dots (iii)$$

\Rightarrow Interaction of (i), (ii) and (iii)

$$a \in (-\infty, -3) \cup [0, 1)$$

$$\Rightarrow \alpha = 3, \beta = 0, \gamma = 1 \Rightarrow 2\alpha + \beta + \gamma = 7$$

Q.23 If $y = \cos\left(\frac{\pi}{3} + \cos^{-1}\frac{x}{2}\right)$, then $(x-y)^2 + 3y^2$ is equal to

Ans. [3]

$$\text{Sol. } y = \cos\left(\frac{\pi}{3} + \cos^{-1}\frac{x}{2}\right)$$

$$= \cos\left(\frac{\pi}{3}\right) \cos\left(\cos^{-1}\left(\frac{x}{2}\right)\right) - \sin\left(\frac{\pi}{3}\right) \sin\left(\cos^{-1}\left(\frac{x}{2}\right)\right)$$

$$= \frac{1}{2} \cdot \frac{x}{2} - \frac{\sqrt{3}}{2} \cdot \sqrt{1 - \frac{x^2}{4}}$$

$$\Rightarrow 4y = x - \sqrt{3} \sqrt{4-x^2}$$

$$\Rightarrow (4y-x)^2 = 3(4-x^2)$$

$$\Rightarrow 16y^2 + x^2 - 8xy = 12 - 3x^2$$

$$x^2 + 4y^2 - 2xy = 3$$

$$(x-y)^2 + 3y^2 = 3$$

Q.24

Let $y = y(x)$ be the solution of the differential equation

$$\frac{dy}{dx} + 2y \sec^2 x = 2\sec^2 x + 3\tan x \cdot \sec^2 x$$

$$\text{such that } y(0) = \frac{5}{4}.$$

Then $12 \left(y\left(\frac{\pi}{4}\right) - e^{-2} \right)$ is equal to

Ans. [21]

$$\text{Sol. } \frac{dy}{dx} + 2y \sec^2 x = 2\sec^2 x + 3\tan x \sec^2 x$$

$$\text{I.F.} = e^{\int 2\sec^2 x dx}$$

$$\text{I.F.} = e^{2\tan x}$$

$$y \cdot e^{2\tan x} = \int e^{2\tan x} (2 + 3\tan x) \sec^2 x dx$$

$$\text{Put } \tan x = u$$

$$\sec^2 x dx = du$$

$$y \cdot e^{2u} = \int e^{2u} (2 + 3u) du$$

$$y \cdot e^{2u} \Rightarrow \frac{2e^{2u}}{2} + 3 \int e^{2u} \cdot u du$$

$$y \cdot e^{2u} = e^{2u} + 3 \left[\frac{ue^{2u}}{2} - \int \frac{e^{2u}}{2} \right]$$

$$ye^{2u} = e^{2u} + 3 \left[\frac{ue^{2u}}{2} - \frac{e^{2u}}{4} \right] + C$$

$$ye^{2\tan x} = e^{2\tan x} + 3 \left[\frac{\tan x e^{2\tan x}}{2} - \frac{e^{2\tan x}}{4} \right] + C$$

$$F(0) = \frac{5}{4}$$

$$\frac{5}{4} = 1 - \frac{3}{4} + C$$

$$\frac{5}{4} - \frac{1}{4} = C$$

$$1 = C$$

$$y = 1 + 3 \left(\frac{\tan x}{2} - \frac{1}{4} \right) + 1 \cdot e^{-2\tan x}$$

$$y\left(\frac{\pi}{4}\right) = 1 + 3 \left(\frac{1}{2} - \frac{1}{4} \right) + \frac{1}{e^2}$$

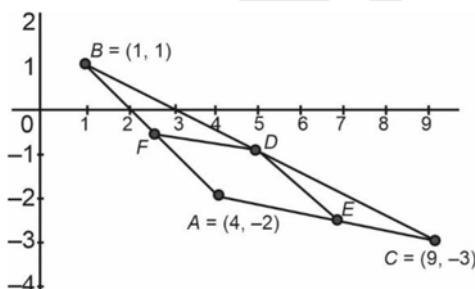
$$y\left(\frac{\pi}{4}\right) = \frac{7}{4} + \frac{1}{e^2}$$

$$12 \left(y\left(\frac{\pi}{4}\right) - \frac{1}{e^2} \right) = 12 \left(\frac{7}{4} + \frac{1}{e^2} - \frac{1}{e^2} \right) = 21$$

Q.25 Let A(4, -2), B(1, 1) and C(9, -3) be the vertices of a triangle ABC. Then the maximum area of the parallelogram AFDE, formed with vertices D, E and F on the sides BC, CA and AB of the triangle ABC respectively, is _____.

Ans. [3]

Sol. The maximum area of such a parallelogram AFDE, with one vertex fixed at A and the other three points lying on the sides of triangle ABC, is half the area of triangle ABC.



Using the determinant formula for area of triangle with vertices A(x₁, y₁), B(x₂, y₂), C(x₃, y₃) :

Area ΔABC

$$= \frac{1}{2} |x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$$

Substitute the coordinates :

$$= \frac{1}{2} |4(1 - (-3)) + 1((-3) - (-2)) + 9((-2) - 1)|$$

$$= \frac{1}{2} |4(4) + 1(-1) + 9(-3)|$$

$$= \frac{1}{2} |16 - 1 - 27| = \frac{1}{2} |-12| = \frac{12}{2} = 6$$

Maximum area of parallelogram AFDE

$$= \frac{1}{2} \times \text{area of triangle} = \frac{1}{2} \times 6 = 3$$

PHYSICS

Section-A: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct..

Q.26 Given below are two statements : one is labelled as

Assertion (A) and the other is labelled as **Reason (R)**.

Assertion (A) : Net dipole moment of a polar linear isotropic dielectric substance is not zero even in the absence of an external electric field.

Reason (R) : In absence of an external electric field, the different permanent dipoles of a polar dielectric substance are oriented in random directions.

In the light of the above statements, choose the most appropriate answer from the options given below.

- (1) Both (A) and (R) are correct but (R) is not the correct explanation of (A)
- (2) (A) is correct but (R) is not correct
- (3) (A) is not correct but (R) is correct
- (4) Both (A) and (R) are correct and (R) is the correct explanation of (A)

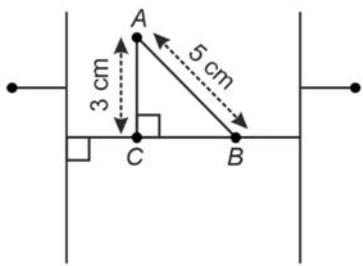
Ans. [3]

Sol. Net dipole moment of polar dielectric substance is zero in absence of electric field as dipoles are randomly oriented.

(A) is not correct but (R) is correct.

Q.27

Two large plane parallel conducting plates are kept 10 cm apart as shown in figure. The potential difference between them is V. The potential difference between the points A and B (shown in the figure) is



(1) $\frac{3}{V}$ (2) 1V (3) $\frac{2}{5}V$ (4) $\frac{1}{4}V$

Ans.

[3]

Sol. Field lines are perpendicular to the plates

$$V_{AB} = V_{CB} = \left(\frac{4}{10} \right) V = \frac{2V}{5}$$

Q.28 Energy released when two deuterons (${}_1H^2$) fuse to form a helium nucleus (${}_2He^4$) is (Given: Binding energy per nucleon of ${}_1H^2$ = 1.1 MeV and binding energy per nucleon of ${}_2He^4$ = 7.0 MeV)

(1) 23.6 MeV (2) 5.9 MeV
(3) 8.1 MeV (4) 26.8 MeV

Ans.

[1]

Sol. $2 {}_1H^2 \rightarrow {}_2He^4$
 $Q = (4 \times 7 - 4 \times 1.1) \text{ MeV}$
 $= 23.6 \text{ MeV}$

Q.29 The moment of inertia of a circular ring of mass M and diameter r about a tangential axis lying in the plane of the ring is :

(1) $\frac{3}{8}Mr^2$ (2) $2Mr^2$ (3) VMr^2 (4) $\frac{1}{2}Mr^2$

Ans.

[1]

Sol.



$$I_1 = \frac{MR^2}{2}$$

$$I' = I_1 + MR^2 = \frac{3}{2}MR^2$$

$$r = 2R$$

$$\therefore r = \frac{3}{8}Mr^2$$

Q.30

Consider a circular loop that is uniformly charged and has a radius $a\sqrt{2}$. Find the position along the positive z-axis of the cartesian coordinate system where the electric field is maximum if the ring was assumed to be placed in xy plane at the origin :

(1) a (2) $a/2$ (3) $\frac{a}{\sqrt{2}}$ (4) 0

Ans.

[1]

Sol.

$$E = \frac{KQx}{(R^2 + x^2)^{3/2}}$$

$$\text{For } E_{\max}, \frac{dE}{dx} = 0$$

$$x = \frac{R}{\sqrt{2}}$$

$$R = a\sqrt{2}$$

$$\therefore x = a$$

Q.31

Two water drops each of radius 'r' coalesce to form a bigger drop. If 'T' is the surface tension, the surface energy released in this process is :

(1) $4\pi r^2 T [\sqrt{2} - 1]$ (2) $4\pi r^2 T \left[2 - 2^{\frac{2}{3}} \right]$
(3) $4\pi r^2 T \left[2 - 2^{\frac{1}{3}} \right]$ (4) $4\pi r^2 T [1 + \sqrt{2}]$

Ans.

[2]

Sol.

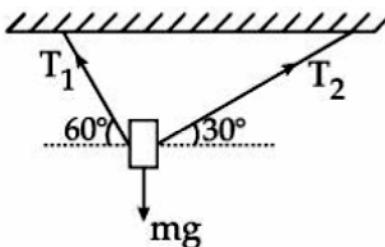
$$2 \times \frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3$$

$$R = (2)^{1/3} r$$

$$\begin{aligned} \Delta Q &= E_i - E_f \\ &= 2 \times 4\pi r^2 T - 4\pi(2)^{2/3} r^2 T \\ &= 4\pi R^2 T \left[2 - (2)^{2/3} \right] \end{aligned}$$

Q.32

A body of mass 1 kg is suspended with the help of two strings making angles as shown in figure.



Magnitudes of tensions T_1 and T_2 , respectively, are (in N) (Take acceleration due to gravity 10 m/s^2)

(1) $5\sqrt{3}, 5$ (2) $5\sqrt{3}, 5\sqrt{3}$
 (3) $5, 5\sqrt{3}$ (4) $5, 5$

Ans. [1]

Sol. $T_1 \sin 60 + T_2 \sin 30 = mg \quad \dots(\text{i})$

$T_1 \cos 60 = T_2 \cos 30 \quad \dots(\text{ii})$

$T_1 = \sqrt{3} T_2$

$$\sqrt{3} T_2 \times \frac{\sqrt{3}}{2} + \frac{T_2}{2} = mg$$

$$T_2 = \frac{mg}{2} = 5\text{N}$$

$$T_1 = \frac{\sqrt{3}mg}{2} = 5\sqrt{3} \text{ N}$$

Q.33 If μ_0 and ϵ_0 are the permeability and permittivity of free space, respectively, then the

dimension of $\left(\frac{1}{\mu_0 \epsilon_0}\right)$ is :

(1) T^2/L^2 (2) L^2/T^2 (3) T^2/L (4) L/T^2

Ans. [2]

Sol. $c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$

$$\mu_0 \epsilon_0 = \frac{1}{c^2} = [L^{-2} T^2]$$

$$\therefore \left[\frac{1}{\mu_0 \epsilon_0} \right] = \frac{L^2}{T^2}$$

Q.34 Match List - I with List - II.

| List - I | | List - II | |
|----------|--------------------------------|-----------|--|
| (A) | Heat capacity of body | (I) | $J \text{ kg}^{-1}$ |
| (B) | Specific heat capacity of body | (II) | $J \text{ K}^{-1}$ |
| (C) | Latent heat | (III) | $J \text{ kg}^{-1} \text{ K}^{-1}$ |
| (D) | Thermal conductivity | (IV) | $J \text{ m}^{-1} \text{ K}^{-1} \text{ s}^{-1}$ |

Choose the correct answer from the options given below.

(1) (A)-(II), (B)-(III), (C)-(I), (D)-(IV)
 (2) (A)-(IV), (B)-(III), (C)-(II), (D)-(I)
 (3) (A)-(III), (B)-(I), (C)-(II), (D)-(IV)
 (4) (A)-(III), (B)-(IV), (C)-(I), (D)-(II)

Ans.

[1]

Heat capacity = J/K , A \rightarrow II

Specific heat capacity = $J/\text{kg K}$, B \rightarrow III

Latent heat = J/kg , C \rightarrow I

Thermal conductivity

$$k \equiv \frac{Q}{A} \frac{\Delta x}{\Delta T} \equiv J/\text{mKs}, \quad D \rightarrow IV$$

Q.35

A bi-convex lens has radius of curvature of both the surfaces same as $1/6$ cm. If this lens is required to be replaced by another convex lens having different radii of curvatures on both sides ($R_1 \neq R_2$), without any change in lens power then possible combination of R_1 and R_2 is :

(1) $\frac{1}{5}$ cm and $\frac{1}{7}$ cm (2) $\frac{1}{3}$ cm and $\frac{1}{7}$ cm
 (3) $\frac{1}{6}$ cm and $\frac{1}{9}$ cm (4) $\frac{1}{3}$ cm and $\frac{1}{3}$ cm

Ans.

[1]

Sol.

$$\frac{1}{f} = (\mu - 1) \left(\frac{2}{R} \right) = (\mu - 1) 12$$

$$\frac{1}{f'} = (\mu - 1) \left(\frac{1}{R_1} + \frac{1}{R_2} \right)$$

$$f = f'$$

$$12 = \frac{1}{R_1} + \frac{1}{R_2}$$

Q.36

Given a charge q , current I and permeability of vacuum μ_0 . Which of the following quantity has the dimension of momentum?

(1) $q\mu_0/I$ (2) qI/μ_0 (3) $q\mu_0 I$ (4) $q^2\mu_0 I$

Ans.

[3]

Momentum = mv

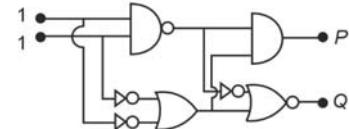
$$F \equiv Bqv$$

$$\frac{mv}{t} = \frac{\mu_0 I v q}{r}$$

$$mv \equiv \mu_0 I q$$

Q.37

In the digital circuit shown in the figure, for the given inputs the P and Q values are



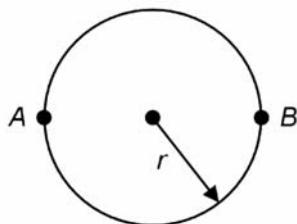
(1) P = 1, Q = 0 (2) P = 1, Q = 1
 (3) P = 0, Q = 0 (4) P = 0, Q = 1

Ans. [3]

Sol. $P = \overline{AB}(\overline{A} + \overline{B}) \Rightarrow 0$

$Q = \overline{(\overline{A} + \overline{B}) + AB} = 0$

Q.38 A sportsman runs around a circular track of radius r such that he traverses the path ABAB. The distance travelled and displacement, respectively, are :



(1) $3\pi r, 2r$ (2) $3\pi r, \pi r$
 (3) $2r, 3\pi r$ (4) $\pi r, 3r$

Ans. [1]

Sol. Distance $= 3 \times \pi r = 3\pi r$
 Displacement $= 2r$

Q.39 An electron with mass 'm' with an initial velocity ($t = 0$) $\vec{v} = v_0 \hat{i}$ ($v_0 > 0$) enters a magnetic field $\vec{B} = B_0 \hat{j}$. If the initial de-Broglie wavelength at $t = 0$ is λ_0 then its value after time 't' would be :

(1) $\frac{\lambda_0}{\sqrt{1 - \frac{e^2 B_0^2 t^2}{m^2}}}$ (2) $\frac{\lambda_0}{\sqrt{1 + \frac{e^2 B_0^2 t^2}{m^2}}}$
 (3) $\lambda_0 \sqrt{1 + \frac{e^2 B_0^2 t^2}{m^2}}$ (4) λ_0

Ans. [4]
Sol. The speed of the electron will not change in presence of magnetic field.
 $\therefore \lambda = \lambda_0$

Q.40 A solenoid having area A and length ℓ is filled with a material having relative permeability 2. The magnetic energy stored in the solenoid is:

(1) $B^2 A \ell$ (2) $\frac{B^2 A \ell}{\mu_0}$
 (3) $\frac{B^2 A \ell}{2\mu_0}$ (4) $\frac{B^2 A \ell}{4\mu_0}$

Ans. [4]

Sol. $E = \left(\frac{B^2}{2\mu} \right) \times \text{Volume}$
 $\mu = 2\mu_0 = \frac{B^2 A \ell}{4\mu_0}$

Q.41

Two identical objects are placed in front of convex mirror and concave mirror having same radii of curvature of 12 cm, at same distance of 18 cm from the respective mirrors. The ratio of sizes of the images formed by convex mirror and by concave mirror is:

(1) 2 (2) 3 (3) $\frac{1}{3}$ (4) $\frac{1}{2}$

Ans. [4]

Sol. $m \left(\frac{f}{f-u} \right)$
 $h_i = mh_0$
 $\frac{h_{i_1}}{h_{i_2}} = \left(\frac{f_1}{f_1-u} \right) \left(\frac{f_2-u}{f_2} \right)$
 $f_1 = 6, f_2 = -6$
 $u = -18$
 $= \frac{6}{24} \times \frac{(12)}{(-6)}$
 $\left| \frac{h_{i_1}}{h_{i_2}} \right| = \left| \frac{-1}{2} \right| = \frac{1}{2}$

Q.42

A sinusoidal wave of wavelength 7.5 cm travels a distance of 1.2 cm along the x-direction in 0.3 sec. The crest P is at $x = 0$ at $t = 0$ sec and maximum displacement of the wave is 2 cm. Which equation correctly represents this wave?

(1) $y = 2\cos(3.35x - 0.83t)$ cm
 (2) $y = 2\cos(0.83x - 3.35t)$ cm
 (3) $y = 2\sin(0.83x - 3.5t)$ cm
 (4) $y = 2\cos(0.13x - 0.5t)$ cm

Ans. [2]

Sol. $\lambda = 7.5$ cm

$\lambda = \frac{2\pi}{K}$

$K = \frac{2\pi}{7.5} = 0.83 \text{ cm}^{-1}$

$v = \frac{1.2}{0.3} = \frac{\omega}{K}$

$\omega = 4 \times 0.83 = 3.35$

At $t = 0, x = 0$, there is a crest

$\therefore y = 2 \cos(0.83x - 3.35t)$ cm

Q.43 In a moving coil galvanometer, two moving coils M_1 and M_2 have the following particulars :
 $R_1 = 5\Omega$, $N_1 = 15$, $A_1 = 3.6 \times 10^{-3} \text{ m}^2$, $B_1 = 0.25 \text{ T}$
 $R_2 = 7\Omega$, $N_2 = 21$, $A_2 = 1.8 \times 10^{-3} \text{ m}^2$, $B_2 = 0.50 \text{ T}$
Assuming that torsional constant of the springs are same for both coils, what will be the ratio of voltage sensitivity of M_1 and M_2 ?

(1) 1 : 2 (2) 1 : 3 (3) 1 : 4 (4) 1 : 1

Ans. [4]

Sol. $K\theta = NI AB$

$$\frac{\theta}{I} = \frac{NAB}{K}$$

$$\frac{\theta}{V} = \frac{\theta}{RI} = \frac{NAB}{KR}$$

Ratio of voltage sensitivity

$$= \left(\frac{N_1 A_1 B_1}{R_1} \right) \left(\frac{R_2}{N_2 A_2 B_2} \right)$$

$$= \frac{5}{7} \times \frac{2}{1} \times \frac{1}{2} \times \frac{7}{5} = 1$$

Q.44 Identify the characteristics of an adiabatic process in a monoatomic gas.

(A) Internal energy is constant.
(B) Work done in the process is equal to the change in internal energy.
(C) The product of temperature and volume is a constant.
(D) The product of pressure and volume is a constant.
(E) The work done to change the temperature from T_1 to T_2 is proportional to $(T_2 - T_1)$.

Choose the correct answer from the options given below :

(1) (A), (C), (E) only (2) (B), (E) only
(3) (A), (C), (D) only (4) (B), (D) only

Ans. [2]

Sol. For adiabatic process

$$Q = 0$$

$$\therefore \Delta U = -W$$

B, E are correct

Q.45 Assuming the validity of Bohr's atomic model for hydrogen like ions the radius of Li^{++} ion in its ground state is given by $\frac{1}{X} a_0$, where

$$X = \text{_____}$$

(Where a_0 is the first Bohr's radius.)

(1) 2 (2) 9 (3) 3 (4) 1

Ans. [3]

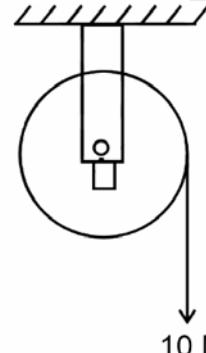
$$\text{Sol. } r = \frac{a_0 n^2}{z}$$

$$\therefore r_{\text{Li}} (n=1) = \frac{a_0 (1)}{3}$$

$$\therefore X = 3$$

Section-B: Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

Q.46



A wheel of radius 0.2 m rotates freely about its center when a string that is wrapped over its rim is pulled by force of 10 N as shown in figure. The established torque produces an angular acceleration of 2 rad/s^2 . Moment of inertia of the wheel is _____ kgm^2 . (Acceleration due to gravity = 10 m/s^2)

Ans.

$$\tau = FR = I\alpha$$

$$I = \frac{FR}{\alpha} = \frac{10 \times 0.2}{2} = 1$$

Q.47

The length of a light string is 1.4 m when the tension on it is 5 N. If the tension increases to 7 N, the length of the string is 1.56 m. The original length of the string is _____ m.

Ans.

Sol.

$$\ell = \ell_0 + \Delta\ell = \ell_0 + \frac{T\ell_0}{AY}$$

$$\ell = \ell_0(1 + KT)$$

$$1.4 = \ell_0(1 + 5K) \quad \dots(i)$$

$$1.56 = \ell_0(1 + 7K) \quad \dots(ii)$$

On solving

$$\ell_0 = 1$$

Q.48 The internal energy of air in $4 \text{ m} \times 4 \text{ m} \times 3 \text{ m}$ sized room at 1 atmospheric pressure will be _____ $\times 10^6 \text{ J}$.
(Consider air as diatomic molecule)

Ans. [12]

Sol. $U = nC_v T$

$$= n \times \frac{5R}{2} T$$

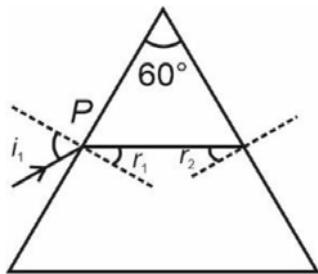
$$= \frac{5}{2} (nRT) = \frac{5}{2} PV$$

$$U = \frac{5}{2} \times 10^5 \times 4 \times 4 \times 3 \\ = 12 \times 10^6 \text{ J}$$

Q.49 A ray of light suffers minimum deviation when incident on a prism having angle of the prism equal to 60° . The refractive index of the prism material is $\sqrt{2}$. The angle of incidence (in degrees) is _____.

Ans. [45]

Sol.



$$\text{For minimum deviation } r_1 = r_2 = \frac{A}{2} = 30^\circ$$

At P

$$1 \times \sin i = \mu \times \sin 30$$

$$\sin i = \sqrt{2} \times \frac{1}{2} = \frac{1}{\sqrt{2}}$$

$$i = 45^\circ$$

Q.50 A satellite of mass 1000 kg is launched to revolve around the earth in an orbit at a height of 270 km from the earth's surface. Kinetic energy of the satellite in this orbit is _____ $\times 10^{10} \text{ J}$.

(Mass of earth = $6 \times 10^{24} \text{ kg}$, Radius of earth = $6.4 \times 10^6 \text{ m}$, Gravitational constant = $6.67 \times 10^{-11} \text{ Nm}^2 \text{ kg}^{-2}$)

Ans. [3]

$$\begin{aligned} \text{Sol. } K &= -\frac{U}{2} = \frac{GMm}{2r} \\ &= \frac{6.67 \times 10^{-11} \times 10^{24} \times 10^3 \times 6}{2 \times 6670 \times 10^3} \\ &= 3 \times 10^{10} \text{ J} \end{aligned}$$

CHEMISTRY

Section-A: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct..

Q.51 Arrange the following in order of magnitude of work done by the system/on the system at constant temperature.

- (a) $|W_{\text{reversible}}|$ for expansion in infinite stages
- (b) $|W_{\text{irreversible}}|$ for expansion in single state
- (c) $|W_{\text{reversible}}|$ for compression in infinite stages
- (d) $|W_{\text{irreversible}}|$ for compression in single stage

Choose the **correct** answer from the options given below.

- (1) $c = a > d > b$
- (2) $d > c = a > b$
- (3) $a > c > b > d$
- (4) $a > b > c > d$

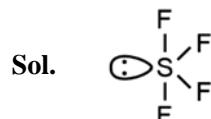
Ans. [2]

Sol. $|W_{\text{reversible expansion}}| > |W_{\text{irreversible expansion}}|$
 $|W_{\text{reversible expansion}}| = |W_{\text{reversible compression}}|$
 $|W_{\text{irreversible compression}}| > |W_{\text{reversible compression}}|$

Q.52 Which among the following molecules is (a) involved in sp^3d hybridization, (b) has different bond lengths and (c) has lone pair of electrons on the central atom?

- (1) XeF_4
- (2) XeF_2
- (3) PF_5
- (4) SF_4

Ans. [4]



$$(S - F)_a > (S - F)_e$$

SF_4 has 4 bond pairs of electrons and 1 lone pair of electrons.

Since, steric number is 5,

Hybridisation of SF_4 = sp^3d

Q.53 Match List-I with List-II.

| List-I (Purification technique) | | List-II (Mixture of organic compounds) |
|---|----------------------------|--|
| (A) Distillation (simple) | (I) Diesel + Petrol | |
| (B) Fractional distillation | (II) Aniline + Water | |
| (C) Distillation under reduced pressure | (III) Chloroform + Aniline | |
| (D) Steam distillation | (IV) Glycerol + Spent-lye | |

Choose the **correct** answer from the options given below.

- (1) (A)-(II), (B)-(IV), (C)-(I), (D)-(III)
- (2) (A)-(III), (B)-(I), (C)-(IV), (D)-(II)
- (3) (A)-(II), (B)-(III), (C)-(IV), (D)-(I)
- (4) (A)-(III), (B)-(IV), (C)-(II), (D)-(I)

Ans. [2]

Sol. Boiling point of aniline is 547 K and B.P. of CHCl_3 is 334 K. So, they are separated by simple distillation.

Q.54 Formation of $\text{Na}_4[\text{Fe}(\text{CN})_5\text{NO}]$, a purple coloured complex formed by addition of sodium nitroprusside in sodium carbonate extract of salt indicates the presence of

- (1) Sulphate ion (2) Sulphide ion
- (3) Sulphite ion (4) Sodium ion

Ans. [2]

Sol. Test for sulphide ion

Q.55 Electronic configuration of four elements A, B, C and D are given below :

- (A) $1s^2 2s^2 2p^3$ (B) $1s^2 2s^2 2p^4$
- (C) $1s^2 2s^2 2p^5$ (D) $1s^2 2s^2 2p^2$

Which of the following is the correct order of increasing electronegativity (Pauling's scale)?

- (1) A < C < B < D (2) A < B < C < D
- (3) D < A < B < C (4) A < D < B < C

Ans. [3]

Sol. (A) $1s^2 2s^2 2p^3 \Rightarrow \text{N}$
(B) $1s^2 2s^2 2p^4 \Rightarrow \text{O}$
(C) $1s^2 2s^2 2p^5 \Rightarrow \text{F}$
(D) $1s^2 2s^2 2p^2 \Rightarrow \text{C}$

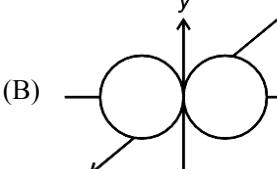
Order of electro negativity

F > O > N > C

C > B > A > D

Q.56 Which of the following statements are **true**?

- (A) The subsidiary quantum number ℓ describes the shape of the orbital occupied by the electron.

(B) 

surface diagram of the $2px$ orbital.

- (C) The + and - signs in the wave function of the $2px$ orbital refer to charge.
- (D) The wave function of $2px$ orbital is zero everywhere in the xy plane.

- (1) (B) and (D) only

- (2) (C) and (D) only

- (3) (A) and (B) only

- (4) (A), (B) and (C) only

Ans. [3]

Sol. Subsidiary quantum number or azimuthal quantum number (ℓ) describes shape of orbital. The given surface diagram is of the $2px$ orbital.

Q.57 A tetrapeptide, "x" on complete hydrolysis produce glycine (Gly), alanine (Ala), valine (Val), leucine (Leu) in equimolar proportion each. The number of tetrapeptides (sequences) possible involving each of these amino acids is:

- (1) 32 (2) 24 (3) 8 (4) 16

Ans. [2]

Sol. Let A = Gly

B = Ala

C = Val

D = Leu

Total 24 tetrapeptides are formed. The 24 tetrapeptides formed including all the four amino acids are

ABCD BACD CABD DABC
ABDC BADC CADB DACB
ACBD BDAC CBAD DBAC
ACDB BDCA CBDA DBCA
ADBC BCAD CDAB DCAB
ADCB BCDA CDBA DCBA

Total = 24

Q.58 Given below are two statements:

Statement (I): Neopentane forms only one monosubstituted derivative.

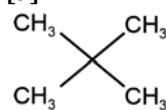
Statement (II): Melting point of neopentane is higher than n-pentane.

In the light of the above statements, choose the **most appropriate answer** from the options given below:

- (1) **Statement I** is incorrect but **Statement II** is correct
- (2) Both **Statement I** and **Statement II** are incorrect
- (3) Both **Statement I** and **Statement II** are correct
- (4) **Statement I** is correct but **Statement II** is incorrect

Ans.

[3]



Sol.

neopentane has only one type of hydrogen so form only one monosubstituted derivative.

Q.59

The nature of oxide (TeO_2) and hydride (TeH_2) formed by Te, respectively are:

- (1) Oxidising and basic
- (2) Reducing and acidic
- (3) Oxidising and acidic
- (4) Reducing and basic

Ans.

[3]

Sol.

TeO_2 is oxidising in nature TeH_2 is reducing in nature and acidic in nature.

Q.60

The type of hybridization and the magnetic property of $[\text{MnCl}_6]^{3-}$ are,

- (1) sp^3d^2 , paramagnetic with two unpaired electrons
- (2) d^2sp^3 , paramagnetic with two unpaired electrons
- (3) sp^3d^2 , paramagnetic with four unpaired electrons
- (4) d^2sp^3 , paramagnetic with four unpaired electrons

Ans.

[3]

Sol.

Cl^- with Mn^{3+} is a weak field ligand

$$\text{Mn}^{3+} \Rightarrow 3\text{d}^4 \Rightarrow \text{t}_{2g}^3 \text{e}_g^1$$

Hybridisation = sp^3d^2 with 4 unpaired electron and paramagnetic.

Q.61

The d-orbital electronic configuration of the complex among $[\text{Co}(\text{en})_3]^{3+}$, $[\text{CoF}_6]^{3-}$, $[\text{Mn}(\text{H}_2\text{O})_6]^{2+}$ and $[\text{Zn}(\text{H}_2\text{O})]^6$ that has the highest CFSE is

- (1) $\text{t}_{2g}^3 \text{e}_g^2$
- (2) $\text{t}_{2g}^6 \text{e}_g^4$
- (3) $\text{t}_{2g}^6 \text{e}_g^0$
- (4) $\text{t}_{2g}^4 \text{e}_g^2$

Ans.

[3]

Sol. $[\text{Co}(\text{en})_3]^{3+}$ has highest CFSE
 Co^{3+} with SFL (en)

$$= \text{t}_{2g}^6 \text{e}_g^0$$

$$\text{CFSE} = (6 \times (-0.4) + 0 \times 0.6)\Delta_0 \\ = -2.4 \Delta_0$$

Q.62

In Dumas' method for estimation of nitrogen, 0.5 gram of an organic compound gave 60 mL of nitrogen collected at 300 K temperature and 715 mm Hg pressure. The percentage composition of nitrogen in the compound (Aqueous tension at 300 K = 15 mm Hg) is %.

- (1) 12.57
- (2) 20.87
- (3) 1.257
- (4) 18.67

Ans.

[1]

Sol.

$$\text{P}_{\text{N}_2} = \text{P}_{\text{moist gas}} - \text{aqueous tension} \\ = (715 - 15) \text{ mm Hg} \\ = 700 \text{ mm Hg}$$

$$V_{\text{N}_2} = 60 \text{ mL}$$

$$= 0.060 \text{ L}$$

$$T = 300 \text{ K}$$

$$n_{\text{N}_2} = \frac{PV}{RT} = \frac{700 \times 0.06}{760 \times 0.0821 \times 300} \\ = 0.022 \text{ mol}$$

$$\text{Mass of N}_2 = 0.0224 \times 28 = 0.0628$$

$$\% \text{ N}_2 = \frac{0.0628}{0.5} \times 100 = 12.58\%$$

Q.63

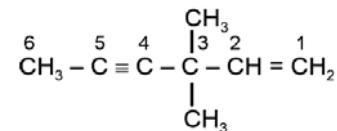
In 3,3-dimethylhex-1-en-4-yne, there are _____ sp^3 , _____ sp^2 and _____ sp hybridised carbon atoms respectively.

- (1) 2, 4, 2
- (2) 2, 2, 4
- (3) 3, 3, 2
- (4) 4, 2, 2

Ans.

[4]

Sol.



$$\text{sp}^3 \text{C} = 4$$

$$\text{sp}^2 \text{C} = 2$$

$$\text{sp C} = 2$$

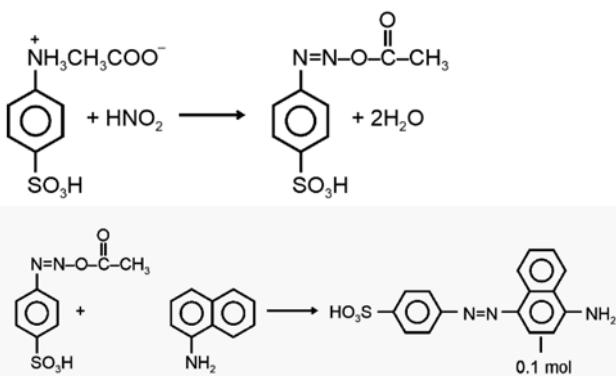
Q.64

When a concentrated solution of sulphanilic acid and 1-naphthylamine is treated with nitrous acid (273 K) and acidified with acetic acid, the mass (g) of 0.1 mole of product formed is (Given molar mass in g mol⁻¹ H : 1, C : 12, N : 14, O : 16, S : 32)

- (1) 343
- (2) 33
- (3) 66
- (4) 330

Ans.

[2]

Sol.


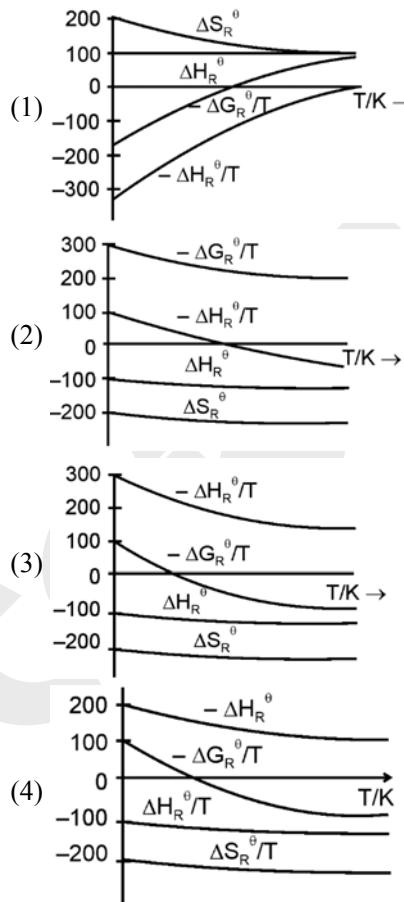
Molar mass of product formed

$$= (16 \times 12) + (14 \times 3) + (16 \times 3) + 32 + 13 \\ = 327 \text{ g}$$

Mass of 0.1 mol product

$$= 32.7 \text{ g} \approx 33 \text{ g}$$

Q.65 Which of the following graphs correctly represents the variation of thermodynamic properties of Haber's process?


Ans. [3]

Sol. $\text{N}_2(\text{g}) + 3\text{H}_2(\text{g}) \rightleftharpoons 2\text{NH}_3(\text{g})$
 $\Delta S_r = -\text{ve}$
 $\Delta H_f = -\text{ve}$
 $\frac{-\Delta H_r^0}{T} \Rightarrow \text{first decreases then constant}$
 $\frac{-\Delta G_r^0}{T} \Rightarrow \text{First decreases then constant}$

Q.66 Match List-I with List-II.

| List-I (Reaction) | List-II (Name of Reaction) |
|--|---|
| (A) $2 \text{C}_6\text{H}_5\text{X} + 2\text{Na} \xrightarrow[\text{Ether}]{\text{Dry}} \text{C}_6\text{H}_5\text{C}_6\text{H}_5 + 2\text{NaX}$ | (I) <i>Lucas reaction</i> |
| (B) $\text{ArN}_2^+ \text{X}^- \xrightarrow[\text{HCl}]{\text{Cu}} \text{ArCl} + \text{N}_2 \uparrow + \text{CuX}$ | (II) <i>Finkelstein reaction</i> |
| (C) $\text{C}_2\text{H}_5\text{Br} + \text{NaI} \xrightarrow[\text{Acetone}]{\text{Dry}} \text{C}_2\text{H}_5\text{I} + \text{NaBr}$ | (III) <i>Fitting reaction</i> |
| (D) $\text{CH}_3\text{C}(\text{OH}(\text{CH}_3)\text{CH}_3 \xrightarrow[\text{ZnCl}_2]{\text{HCl}} \text{CH}_3\text{C}(\text{Cl})(\text{CH}_3)\text{CH}_3$ | (IV) <i>Gatterman reaction</i> |

 Choose the **correct** answer from the options given below:

(1) (A)-(III), (B)-(IV), (C)-(II), (D)-(I)
 (2) (A)-(IV), (B)-(III), (C)-(I), (D)-(II)
 (3) (A)-(IV), (B)-(I), (C)-(II), (D)-(III)
 (4) (A)-(III), (B)-(II), (C)-(IV), (D)-(I)

Ans. [1]

Sol.

| | |
|--|-----------------------------------|
| (A) $2 \text{C}_6\text{H}_5\text{X} + 2\text{Na} \xrightarrow[\text{Ether}]{\text{Dry}} \text{C}_6\text{H}_5\text{C}_6\text{H}_5 + 2\text{NaX}$ | (I) <i>Fitting Reaction</i> |
| (B) $\text{ArN}_2^+ \text{X}^- \xrightarrow[\text{HCl}]{\text{Cu}} \text{ArCl} + \text{N}_2 \uparrow + \text{CuX}$ | (II) <i>Gatterman reaction</i> |
| (C) $\text{C}_2\text{H}_5\text{Br} + \text{NaI} \xrightarrow[\text{Acetone}]{\text{Dry}} \text{C}_2\text{H}_5\text{I} + \text{NaBr}$ | (III) <i>Finkelstein reaction</i> |
| (D) $\text{CH}_3\text{C}(\text{OH}(\text{CH}_3)\text{CH}_3 \xrightarrow[\text{ZnCl}_2]{\text{HCl}} \text{CH}_3\text{C}(\text{Cl})(\text{CH}_3)\text{CH}_3$ | (IV) <i>Lucas reaction</i> |

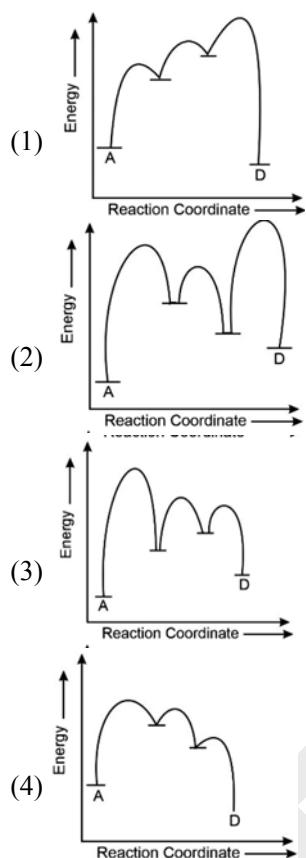
Q.67 Reactant A converts to product D through the given mechanism (with the net evolution of heat) :

A \rightarrow B Slow; $\Delta H = +ve$

B \rightarrow C Fast; $\Delta H = -ve$

C \rightarrow D Fast; $\Delta H = -ve$

Which of the following represents the above reaction mechanism?

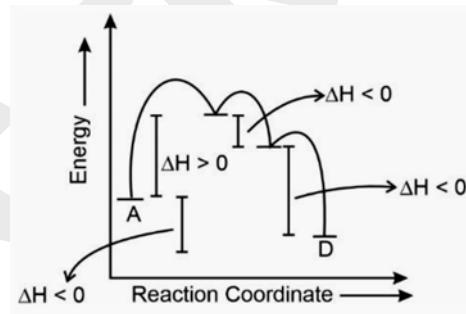


Ans.

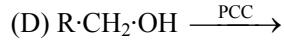
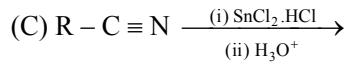
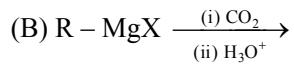
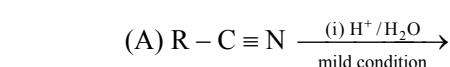
[4]

Sol. As given net evolution of heat takes place.

A \rightarrow D $\Delta H < 0$



Q.68 Consider the following reactions. Form these reactions which reaction will give carboxylic acid as a major product?



Choose the **correct** answer from the options given below:

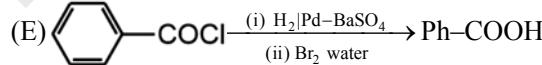
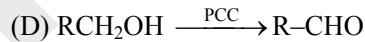
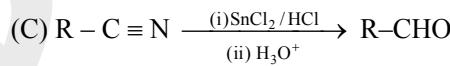
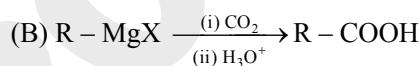
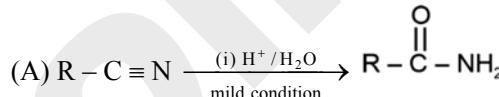
(1) B and E only (2) B, C and E only

(3) A and D only (4) A, B and E only

Ans.

[1]

Sol.



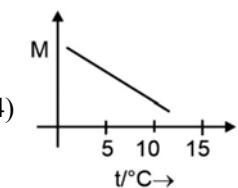
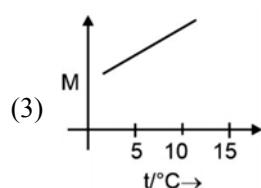
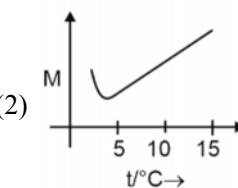
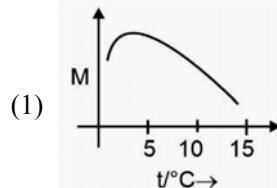
Br_2 water oxidises aldehyde to carboxylic acid.

Q.69

'x' g of NaCl is added to water in a beaker with a lid.

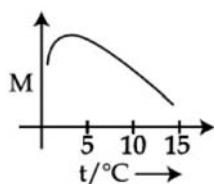
The temperature of the system is raised from 1°C to 25°C. Which out of the following plots, is best suited for the change in the molarity (M) of the solution with respect to temperature?

[Consider the solubility of NaCl remains unchanged over the temperature range]



Ans.

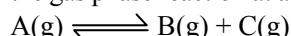
[1]

Sol.


Volume is minimum at 4°C, so molarity will be maximum at 4°C.

Q.70

Consider the following chemical equilibrium of the gas phase reaction at a constant temperature:



If p being the total pressure, K_p is the pressure equilibrium constant and α is the degree of dissociation, then which of the following is true at equilibrium?

- (1) If K_p value is extremely high compared to p , α becomes much less than unity
- (2) When p increases α increases
- (3) If p value is extremely high compared to K_p , $\alpha \approx 1$
- (4) When p increases α decreases

Ans.
[4]

Sol. According to Le-Chatelier's principle as pressure increases in above reaction the reaction moves in backward direction hence α decreases.

Section-B: Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

Q.71

When 1 g each of compounds AB and AB_2 are dissolved in 15 g of water separately, they increased the boiling point of water by 2.7 K and 1.5 K respectively. The atomic mass of A (in amu) is _____ $\times 10^{-1}$ (Nearest integer) (Given: Molal boiling point elevation constant is 0.5 K kg mol $^{-1}$)

Ans.
[25]
Sol.

For AB

$$\Delta T_b = 2.7$$

$$K_b = 0.5 \text{ K kg mol}^{-1}$$

$$m = \frac{\Delta T_b}{K_b} = 5.4 \text{ m}$$

For AB_2

$$\Delta T_b = 1.5 \text{ K}$$

$$m = \frac{1.5}{0.5} = 5.4 \text{ m}$$

$$\text{Moles of AB} = 0.081 \text{ mol}$$

$$\text{Moles of } AB_2 = 0.045 \text{ mol}$$

$$\text{Molar mass of AB} = 12.35 \text{ g/mol}$$

$$\text{Molar mass of } AB_2 = 22.22 \text{ g/mol}$$

$$\text{Let atomic mass of A} = a, \text{B} = b$$

$$a + b = 12.35$$

$$a + 2b = 22.22$$

$$\text{On solving } a = 2.48$$

$$= 24.8 \times 10^{-1}$$

Q.72

0.2% (w/v) solution of NaOH is measured to have resistivity 870.0 m Ω m. The molar conductivity of the solution will be _____ $\times 10^2$ mS dm 2 mol $^{-1}$. (Nearest integer)

Ans.

[23] We have 0.2 g of NaOH in 100 mL solution

$$M = \frac{0.2}{40 \times 100} \times 1000 = 0.05 \text{ M}$$

$$K = \frac{1}{0.870} = 1.15 \text{ Sm}^{-1}$$

$$= 1.15 \times 10^{-2} \text{ S cm}^{-1}$$

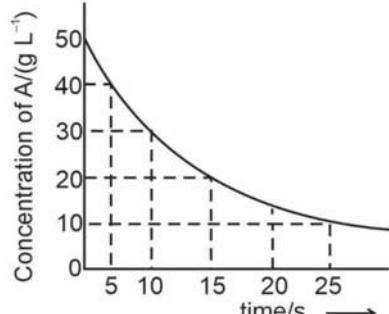
$$\begin{aligned} \hat{\kappa}_m &= \frac{\kappa \times 1000}{m} = \frac{1.15 \times 10^{-2} \times 1000}{0.05} \\ &= 230 \text{ S cm}^2 \text{ mol}^{-1} \\ &= 23 \times 10^4 \text{ m S cm}^2 \text{ mol}^{-1} \\ &= 23 \times 10^2 \text{ m S dm}^2 \text{ mol}^{-1} \end{aligned}$$

Q.73

For the reaction $A \rightarrow B$ the following graph was obtained. The time required (in seconds) for the concentration of A to reduce to 2.5 g L $^{-1}$ (if the initial concentration of A was 50 g L $^{-1}$) is _____.

(Nearest integer)

Given: $\log 2 = 0.3010$


Ans. [47]

Sol. The graph given is for 1st order reaction

$$k = \frac{2.303}{t} \log \frac{[A_0]}{[A_t]}$$

$$k = \frac{2.303}{25} \log \frac{[50]}{[10]}$$

$$k = \frac{2.303}{25} \log 5$$

$$= \frac{2.303 \times 0.7}{25} \text{ s}^{-1}$$

$$= \frac{2.303 \times 0.7}{25} = \frac{2.303}{t} \log \frac{50}{2.5}$$

$$\frac{0.699}{25} = \frac{1}{t} \log 20$$

$$t = \frac{1.301 \times 25}{0.699} = 46.53 \text{ s} \approx 47 \text{ s}$$

Q.74 The spin-only magnetic moment value of Mn⁺ ion formed among Ni, Zn, Mn and Cu that has the least enthalpy of atomisation is _____. (in nearest integer) Here n is equal to the number of diamagnetic complexes among K₂[NiCl₄], [Zn(H₂O)₆]Cl₂, K₃[Mn(CN)₆] and [Cu(PPh₃)₃I]

Ans. [0]

Sol. K₂[NiCl₄] \Rightarrow Ni²⁺ \Rightarrow 3d⁸ \Rightarrow sp³

\Rightarrow Paramagnetic

[Zn(H₂O)₆]Cl₂ \Rightarrow Zn²⁺ \Rightarrow 3d¹⁰ \Rightarrow diamagnetic

K₃[Mn(CN)₆] \Rightarrow Mn³⁺ \Rightarrow 3d⁴ \Rightarrow paramagnetic

[Cu(PPh₃)₃I] \Rightarrow Cu⁺¹ \Rightarrow 3d¹⁰ \Rightarrow diamagnetic

n = 2

Zn has least enthalpy of atomisation

Zn²⁺ \Rightarrow $\mu = 0$

Q.75



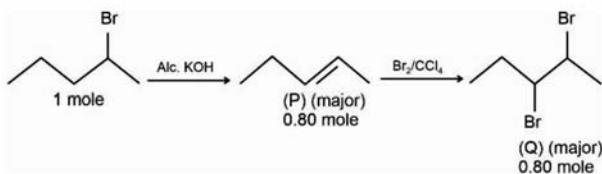
Consider the above sequence of reactions. 151 g of 2-bromopentane is made to react. Yield of major product P is 80% whereas Q is 100%.

Mass of product Q obtained is _____ g.

(Given molar mass in g mol⁻¹ H:1, C:12, O:16, Br:80)

Ans. [184]

Sol.



Molecular mass of Q = 230 g mol⁻¹

Mass of Q = 0.8 \times 230 = 184 g