CAREER POINT

JEE Advanced Exam 2023
(Paper & Solution)

Date : 04 / 06 / 2023

PAPER-1

MATHEMATICS

SECTION – 1 (Maximum Marks: 12)

- This section contains **THREE (03)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is(are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:

  Full Marks : +4 **ONLY** if (all) the correct option(s) is(are) chosen;
  Partial Marks : +3 If all the four options are correct but **ONLY** three options are chosen;
  Partial Marks : +2 If three or more options are correct but **ONLY** two options are chosen, both of which are correct;
  Partial Marks : +1 If two or more options are correct but **ONLY** one option is chosen and it is a correct option;
  Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
  Negative Marks : –2 In all other cases.
- For example, in a question, if (A), (B) and (D) are the ONLY three options corresponding to correct answers, then
  choosing ONLY (A), (B) and (D) will get +4 marks;
  choosing ONLY (A) and (B) will get +2 marks;
  choosing ONLY (A) and (D) will get +2 marks;
  choosing ONLY (B) and (D) will get +2 marks;
  choosing ONLY (A) will get +1 mark;
  choosing ONLY (B) will get +1 mark;
  choosing ONLY (D) will get +1 mark;
  choosing no option (i.e. the question is unanswered) will get 0 marks; and
  choosing any other combination of options will get –2 marks.

Q.1 Let $S = (0,1)\cup(1,2)\cup(3,4)$ and $T = \{0,1, 2,3\}$. Then which of the following statements is(are) true?

(A) There are infinitely many functions from S to T

(B) There are infinitely many strictly increasing functions from S to T

(C) The number of continuous functions from S to T is at most 120

(D) Every continuous function from S to T is differentiable

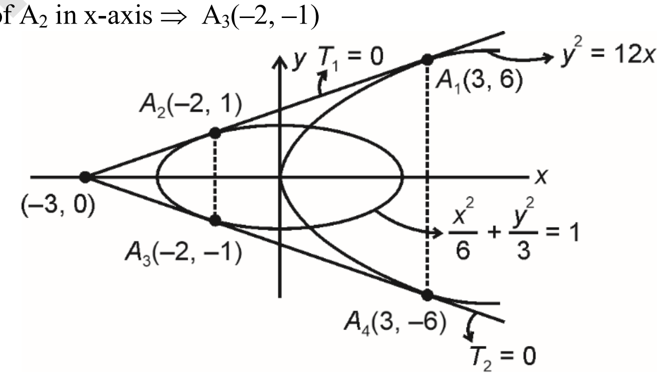
Ans. **[A,C,D]**

Sol. $S = (0, 1) \cup (1, 2) \cup (3, 4)$ and $T = \{0, 1, 2, 3\}$.
Let domain and co-domain of a function $y = f(x)$ are S and T respectively.
(A) There are infinitely many elements in domain and four elements in co-domain.
$\Rightarrow$ There are infinitely many functions from S to T.
$\Rightarrow$ Option (A) is correct
(B) If number of elements in domain is greater than number of elements in co-domain, then number of strictly increasing function is zero.
$\Rightarrow$ Option (B) is incorrect
(C) Maximum number of continuous functions $= 4 \times 4 \times 4 = 64$
(Every subset (0, 1), (1, 2), (3, 4) has four choices)
$\because 64 < 120 \Rightarrow$ option (C) is correct.
(D) For every point at which $f(x)$ is continuous, $f'(x) = 0$
$\Rightarrow$ Every continuous function from S to T is differentiable.
Option (D) is correct.

Q.2 Let T_1 and T_2 be two distinct common tangents to the ellipse $E : \frac{x^2}{6} + \frac{y^2}{3} = 1$ and the parabola $P : y^2 = 12x$. Suppose that the tangent T_1 touches P and E at the points A_1 and A_2, respectively and the tangent T_2 touches P and E at the points A_4 and A_3, respectively. Then which of the following statements is(are) true?
(A) The area of the quadrilateral $A_1A_2A_3A_4$ is 35 square units
(B) The area of the quadrilateral $A_1A_2A_3A_4$ is 36 square units
(C) The tangents T_1 and T_2 meet the x-axis at the point (–3,0)
(D) The tangents T_1 and T_2 meet the x-axis at the point (–6,0)

Ans. **[A,C]**

Sol. $E : \frac{x^2}{6} + \frac{y^2}{3} = 1$, Tangent : $y = m_1x \pm \sqrt{6m_1^2 + 3}$
$P : y^2 = 12x$, Tangent: $y = m_2x + \frac{3}{m_2}$
For common tangent
$m = m_1 = m_2$, $\sqrt{6m_1^2 + 3} = \frac{3}{m_2}$
$\Rightarrow m = \pm 1$
$\Rightarrow$ equation of common tangents $y = x + 3$ and $y = -x - 3$ point of contact for parabola is $\left(\frac{a}{m^2}, \frac{2a}{m}\right)$
$\Rightarrow A_1 \equiv (3, 6)$, $A_4\ (3 - 6)$
Let $A_2(x_1, y_1) \Rightarrow$ tangent to E is $\frac{xx_1}{6} + \frac{yy_1}{3} = 1$
A_3 is mirror image of A_2 in x-axis $\Rightarrow A_3(-2, -1)$



Intersection point of $T_1 = 0$ and $T_2 = 0$ is $(-3, 0)$
Area of quadrilateral $A_1A_2A_3A_4 + \frac{1}{2}\ (12 + 2) \times 5 = 35$square units

Q.3 Let $f : [0,1] \rightarrow [0,1]$ be the function defined by $f(x) = \frac{x^3}{3} - x^2 + \frac{5}{9}x + \frac{17}{36}$. Consider the square region $S = [0,1] \times [0,1]$. Let $G = \{(x, y) \in S : y > f(x)\}$ be called the green region and $R = \{(x, y) \in S : y < f(x)\}$ be called the red region. Let $L_h = \{(x, h) \in S : x \in [0,1]\}$ be the horizontal line drawn at a height $h \in [0, 1]$. Then which of the following statements is(are) true?

(A) There exists an $h \in \left[\frac{1}{4}, \frac{2}{3}\right]$ such that the area of the green region above the line L_h equals the area of the green region below the line L_h

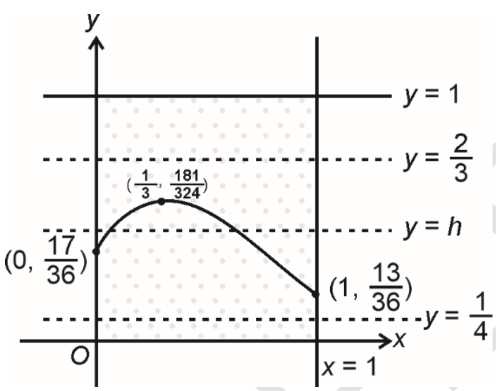
(B) There exists an $h \in \left[\frac{1}{4}, \frac{2}{3}\right]$ such that the area of the red region above the line L_h equals the area of the red region below the line L_h

(C) There exists an $h \in \left[\frac{1}{4}, \frac{2}{3}\right]$ such that the area of the green region above the line L_h equals the area of the red region below the line L_h

(D) There exists an $h \in \left[\frac{1}{4}, \frac{2}{3}\right]$ such that the area of the red region above the line L_h equals the area of the green region below the line L_h

Ans. **[B,C,D]**

Sol.



$f(x) = \frac{x^3}{3}\ x^2 + \frac{5}{9}x + \frac{17}{36}$, $f'(x) = x^2 - 2x + \frac{5}{9}$

For maxima/minima, $f'(x) = 0 \Rightarrow x = \frac{1}{3}$

$A_R = \int_0^1 f(x)dx = \frac{1}{2} \Rightarrow A_G = \frac{1}{2}$

(A) $1 - h = h - \frac{1}{2} \Rightarrow h = \frac{3}{4}, \frac{3}{4} > \frac{2}{3}$ option (A) is incorrect

(B) $h = \frac{1}{2} - h \Rightarrow h = \frac{1}{4}$ $\Rightarrow$ option (B) is correct.

(C) $\int_0^1 f(x)dx = \frac{1}{2}, \int_0^1 \frac{1}{2}dx = \frac{1}{2} \Rightarrow \int_0^1 \left(f(x) - \frac{1}{2}\right)dx = 0$

$\Rightarrow h = \frac{1}{2}$ $\Rightarrow$ option (C) is correct.

(D) $\because$ Option (C) is correct $\Rightarrow$ option (D) is also correct.

SECTION – 2 [Maximum Mark : 12]

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme :

Full Marks	: **+3**	If **ONLY** the correct option is chosen.
Zero Marks	: **0**	If none of the option is chosen (i.e. the question is unanswered).
Negative Marks	: **–1**	In all other cases.

Q.4 Let $f : (0,1) \to \mathbf{R}$ be the function defined as $f(x) = \sqrt{n}$ if $x \in \left[\frac{1}{n+1}, \frac{1}{n}\right)$ where $n \in \mathbf{N}$. Let $g : (0,1) \to \mathbf{R}$ be a function such that $\int_{x^2}^{x} \sqrt{\frac{1-t}{t}}\, dt < g(x) < 2\sqrt{x}$ for all $x \in (0,1)$. Then $\lim_{x\to 0} f(x)g(x)$

(A) does NOT exist (B) is equal to 1 (C) is equal to 2 (D) is equal to 3

Ans. **[C]**

Sol. We need to solve 1 sided limit here to get some answer, otherwise $\lim_{x\to 0^-}$ doesn't exist here (not in domain)

$f(x) = \sqrt{\left(\frac{1}{x}\right) - 1}$ where $(\cdot)$ = least integer function

$$\lim_{x\to 0^+} \int_{x^2}^{x} \sqrt{\frac{1-t}{t}} dt \cdot \sqrt{\left(\frac{1}{x}\right) - 1} \le \lim_{x\to 0^+} f(x)\cdot g(x) \le \lim_{x\to 0^+} \sqrt{\left(\frac{1}{x}\right) - 1} \times 2\sqrt{x}$$

Now $\lim_{x\to 0^+} \sqrt{\left(\frac{1}{x}\right) - 1} \times 2\sqrt{x} = \lim_{x\to 0^+} 2\sqrt{x} \sqrt{\left[\frac{1}{x}\right]} \left(\frac{1}{x} \notin Z\right)$

$$= \lim_{x\to 0^+} 2\sqrt{x\left(\frac{1}{x} - \left\{\frac{1}{x}\right\}\right)} = 2$$

$$= \lim_{x\to 0^+} 2\sqrt{x\left(\frac{1}{x}\right)} = 2 \; ; \left(\frac{1}{x} \notin Z\right)$$

$$\lim_{x\to 0^+} \int_{x^2}^{x} \sqrt{\frac{1-t}{t}} dt \cdot \sqrt{\frac{1}{x} - \left\{\frac{1}{x}\right\}} = \frac{\int_{x^2}^{x} \sqrt{\frac{1-t}{t}} dt \cdot \sqrt{1 - x\left\{\frac{1}{x}\right\}}}{\sqrt{x}}$$

$$\lim_{x\to 0^+} \frac{\int_{x^2}^{x} \sqrt{\frac{1-t}{t}} dt}{\sqrt{x}} = \lim_{x\to 0^+} \frac{\sqrt{\frac{1-x}{x}} - 2x\sqrt{\frac{1-x^2}{x^2}}}{\frac{1}{2\sqrt{x}}}$$

$$\lim_{x\to 0^+} 2\sqrt{1-x} - 4\sqrt{x}\cdot\sqrt{1-x^2} = 2$$

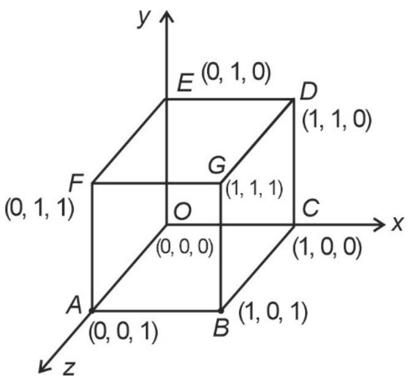
Similarly for $\frac{1}{x} \in Z$ is equal to 2.

Q.5 Let Q be the cube with the set of vertices $\{(x_1, x_2, x_3) \in \mathbf{R}^3 : x_1, x_2, x_3 \in \{0, 1\}\}$. Let F be the set of all twelve lines containing the diagonals of the six faces of the cube Q. Let S be the set of all four lines containing the main diagonals of the cube Q; for instance, the line passing through the vertices (0,0,0) and (1,1,1) is in S. For lines ℓ_1 and ℓ_2, let $d(\ell_1, \ell_2)$ denote the shortest distance between them. Then the maximum value of $d(\ell_1, \ell_2)$, as ℓ_1 varies over F and ℓ_2 varies over S, is

(A) $\frac{1}{\sqrt{6}}$ (B) $\frac{1}{\sqrt{8}}$ (C) $\frac{1}{\sqrt{3}}$ (D) $\frac{1}{\sqrt{12}}$

Ans. **[A]**

Sol.



Equation of OD line is

$\vec{r} = \vec{0} + \lambda(\hat{i} + \hat{j})$

Equation of diagonal BE is

$\vec{r}_1 = \hat{j} + \alpha(\hat{i} - \hat{j} + \hat{k})$

$\text{S.D} = \left|\frac{\hat{j}\cdot(\hat{i} - \hat{j} - 2\hat{k})}{\sqrt{6}}\right| = \frac{1}{\sqrt{6}}$

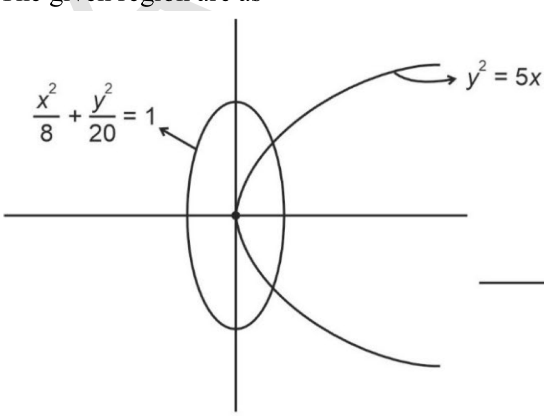
In other case S.D is zero.

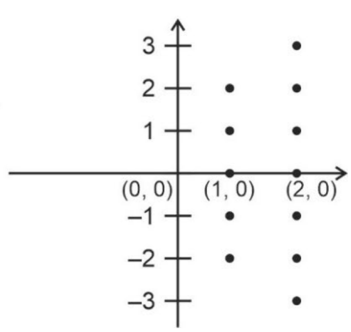
Q.6 Let $X = \left\{(x, y) \in \mathbb{Z} \times \mathbb{Z} : \frac{x^2}{8} + \frac{y^2}{20} < 1 \text{ and } y^2 < 5x\right\}$. Three distinct points P, Q and R are randomly chosen from X. Then the probability that P, Q and R form a triangle whose area is a positive integer, is

(A) $\frac{71}{220}$ (B) $\frac{73}{220}$ (C) $\frac{79}{220}$ (D) $\frac{83}{220}$

Ans. **[B]**

Sol. The given region are as





The points inside region are {(2, 1), (2, –1), (2, 2), (2, –2), (2, 3), (2, –3), (2, 0), (1, 1), (1, –1), (1, 2), (1, –2), (1, 0)}.

Total number of ways to select three points = ${}^{12}C_3 = 220$

Required number of triangle = $4 \times {}^7C_1 + 9 \times {}^5C_1 = 73$

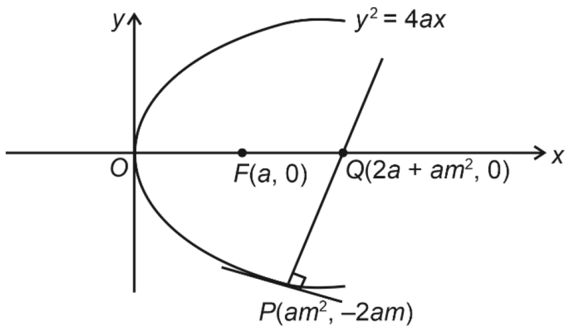
Points are taken such a way that distance between two points are multiple of 2.

Q.7 Let P be a point on the parabola $y^2 = 4ax$, where a > 0. The normal to the parabola at P meets the x-axis at a point Q. The area of the triangle PFQ, where F is the focus of the parabola, is 120. If the slope m of the normal and a are both positive integers, then the pair (a, m) is

(A) (2, 3) (B) (1, 3) (C) (2, 4) (D) (3, 4)

Ans. **[A]**

Sol.



Equation of normal at $P(am^2, -2am)$ is $y = mx - 2am - am^3$

$\Rightarrow$ Area of $\Delta PFQ = \frac{1}{2}(a + am^2) \times 2am = 120$

$am^2(1 + m^2) = 120$

Pair (a, m) ≡ (2, 3) satisfies above equation

SECTION – 3 [Maximum Mark : 24]

- This section contains **SIX (06)** questions.
- The answer to each question is a **NON-NEGATIVE INTEGER**.
- For each question, enter the correct integer corresponding to the answer using the mouse and the onscreen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme :

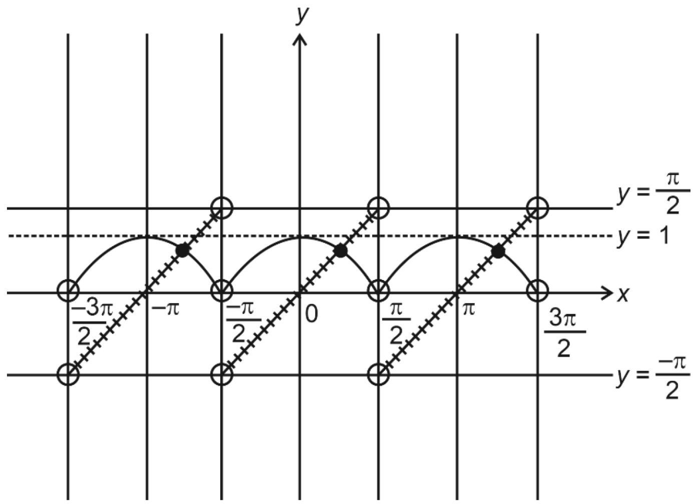
Full Marks	: **+4**	If **ONLY** the correct integer is entered;
Negative Marks	: 0	In all other cases.

Q.8 Let $\tan^{-1}(x) \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, for $x \in R$. Then the number of real solutions of the equation $\sqrt{1+\cos(2x)} = \sqrt{2}\tan^{-1}(\tan x)$ in the set $\left(-\frac{3\pi}{2}, -\frac{\pi}{2}\right) \cup \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$ is equal to

Ans. [3]

Sol. $\sqrt{1+\cos 2x} = \sqrt{2}\tan^{-1}(\tan x)$

$\Rightarrow |\cos x| = \tan^{-1}(\tan x)$



Number of solutions = Number of intersection points = 3

Q.9 Let $n \geq 2$ be a natural number and $f : [0, 1] \rightarrow R$ be the function defined by

$$f(x) = \begin{cases} n(1-2nx) & \text{if } 0 \leq x \leq \frac{1}{2n} \\ 2n(2nx-1) & \text{if } \frac{1}{2n} \leq x \leq \frac{3}{4n} \\ 4n(1-nx) & \text{if } \frac{3}{4n} \leq x \leq \frac{1}{n} \\ \frac{n}{n-1}(nx-1) & \text{if } \frac{1}{n} \leq x \leq 1 \end{cases}$$

If n is such that the area of the region bounded by the curves x = 0, x = 1, y = 0 and y = f(x) is 4, then the maximum value of the function f is

Ans. [8]

Sol.
$$f(x) = \begin{cases} n(1-2nx), & 0 \leq x \leq \frac{1}{2n} \\ 2n(2nx-1), & \frac{1}{2n} \leq x \leq \frac{3}{4n} \\ 4n(1-nx), & \frac{3}{4n} \leq x \leq \frac{1}{n} \\ \frac{n}{n-1}(nx-1), & \frac{1}{n} \leq x \leq 1 \end{cases}$$

$x \in [0, 1]$

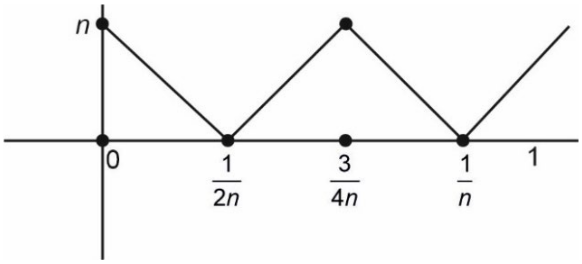
$f(x)$ is decreasing in $\left[0, \frac{1}{2n}\right]$

increasing in $\left[\frac{1}{2n}, \frac{3}{4n}\right]$

decreasing in $\left[\frac{3}{4n}, \frac{1}{n}\right]$

increasing in $\left[\frac{1}{n}, 1\right]$

Graph



$f(x) \in [0, n]$

Area = 4 $\Rightarrow$ n = 8 = and $f(x)$max = n = 8

Q.10 Let $\overbrace{75...57}^{r}$ denote the (r + 2) digit number where the first and the last digits are 7 and the remaining r digits are 5. Consider the sum S = 77 + 757 + 7557 + $\overbrace{75...57}^{98}$. If S = $\frac{\overbrace{75...57}^{99} + m}{n}$, where m and n are natural numbers less than 3000, then the value of m + n is

Ans. **[1219]**

Sol. $S = 77 + 757 + 7557 + \ldots + \overbrace{75...57}^{98}$

$= 7(10 + 10^2 + \ldots + 10^{99}) + 50(1 + 11 + \ldots + \overbrace{111...1}^{98}) + 7 \times 99$

$= 70\left(\frac{10^{99}-1}{9}\right) + \frac{50}{9}[(10-1) + (10^2-1) + \ldots + (10^{98}-1)] + 7 \times 99$

$= 77\left(\frac{10^{99}-1}{9}\right) + \frac{50}{9}\left[10\left(\frac{10^{98}-1}{9}\right) - 98\right] + 7 \times 99$

$= \frac{7 \times 10^{100}}{9} - \frac{70}{9} + \frac{50}{9}\left[\frac{10^{99}-1-9}{9} - 98\right] + 7 \times 99$

$= \frac{7 \times 10^{100}}{9} - \frac{70}{9} + \frac{50}{9}[\overbrace{111...1}^{99} - 99] + 7 \times 99$

$= \frac{7 \times 10^{100} - 70 + \overbrace{555...50}^{99}}{9} - 550 + 693$

$= \frac{\overbrace{7555...5}^{99} - 70 + 143 \times 9}{9} = \frac{\overbrace{755...57}^{99} + 1210}{9}$

m + n = 1219

Q.11 Let $A = \left(\frac{1967 + 1686 i \sin\theta}{7 - 3i\cos\theta} : \theta \in R\right)$. If A contains exactly one positive integer n, then the value of n is

Ans. **[281]**

Sol. $z = \frac{1967 + 1686 i \sin\theta}{7 - 3i\cos\theta}$ is a positive integer.

$z = \frac{(1967 + 1686 i \sin\theta)(7 + 3i\cos\theta)}{(7 - 3i\cos\theta)(7 + 3i\cos\theta)}$

$1967 = 281 \times 7$; $1686 = 281 \times 6$

$z = \frac{1967 \times 7 - 1686 \times 3\sin\theta\cos\theta + i(1686 \times 7\sin\theta + 1967 \times 3\cos\theta)}{49 + 9\cos^2\theta}$

$(281 \times 6) \times 7\sin\theta + (281 \times 7) \times 3\cos\theta = 0$

$\tan\theta = -\frac{1}{2}$

$\Rightarrow \cos^2\theta = \frac{4}{5}$; $\sin\theta\cos\theta = -\frac{2}{5}$

$$z = \frac{(281 \times 7 \times 7) - (281 \times 6) \times 3\left(-\frac{2}{5}\right)}{49 + 9 \times \frac{4}{5}} = \frac{281\left(49 + \frac{36}{5}\right)}{\left(49 + \frac{36}{5}\right)} = 281$$

Q.12 Let P be the plane $\sqrt{3}\,x + 2y + 3z = 16$ and let

$S = \left\{\alpha\hat{i} + \beta\hat{j} + \gamma\hat{k} : \alpha^2 + \beta^2 + \gamma^2 = 1 \text{ and the distance of } (\alpha, \beta, \gamma) \text{ from the plane P is } \frac{7}{2}\right\}$. Let $\vec{u}$, $\vec{v}$ and $\vec{w}$ be three distinct vectors in S such that $|\vec{u} - \vec{v}| = |\vec{v} - \vec{w}| = |\vec{w} - \vec{u}|$. Let V be the volume of the parallelepiped determined by vectors $\vec{u}$, $\vec{v}$ and $\vec{w}$. Then the value of $\frac{80}{\sqrt{3}}$ V is

Ans. **[45]**

Sol. P : $\sqrt{3}\,x + 2y + 3z = 16$

S = $\alpha\hat{i} + \beta\hat{j} + \gamma\hat{k} : \alpha^2 + \beta^2 + \gamma^2 = 1$

$d(\alpha, \beta, \gamma)$ from P = $\frac{7}{2}$

$|\vec{u} - \vec{v}| = |\vec{v} - \vec{w}| = |\vec{w} - \vec{u}|$

V : volume of parallelepiped by vectors $\vec{u}$, $\vec{v}$, $\vec{w}$

$\frac{80}{\sqrt{3}}$ V = ?

$d(\alpha, \beta, \gamma)$ from $\beta = \frac{7}{2}$ (Given)

$\Rightarrow \frac{|\sqrt{3}\alpha + 2\beta + 3\gamma - 16|}{\sqrt{3 + 4 + 9}} = \frac{7}{2}$

$= \frac{|\sqrt{3}\alpha + 2\beta + 3\gamma - 16|}{4} = \frac{7}{2}$

$|\sqrt{3}\alpha + 2\beta + 3\gamma - 16| = 14$…(i)

$\alpha^2 + \beta^2 + \gamma^2 = 1$ …(ii)

Volume of parallelepiped by vectors $\vec{u}$, $\vec{v}$, $\vec{w}$

$V = [\vec{u}\ \vec{v}\ \vec{w}]$

$= \vec{u}\cdot(v \times w)$ …(iii)

$|\vec{u}| = |\vec{v}| = |\vec{w}| = 1$ (Given) …(iv)

$|\vec{u} - \vec{v}| = |\vec{v} - \vec{w}| = |\vec{w} - \vec{u}|$(Given)

$\Rightarrow |\vec{u} - \vec{v}|^2 = |\vec{v} - \vec{w}|^2 = |\vec{w} - \vec{u}|^2$

$\Rightarrow u^2 + v^2 - 2\,\vec{u}\cdot\vec{v} = v^2 + w^2 - 2\,\vec{v}\cdot\vec{w} = w^2 + u^2 - 2\,\vec{w}\cdot\vec{u}$

(A) (B) (C)

(A) and (B)

$\Rightarrow u^2 + v^2 - 2\,\vec{u}\cdot\vec{v} = v^2 + w^2 - 2\,\vec{v}\cdot\vec{w}$

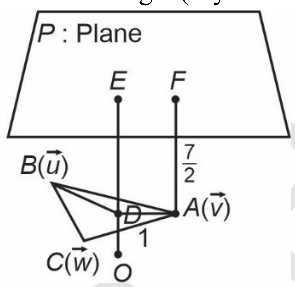
$\Rightarrow u^2 - w^2 = 2\,\vec{u}\cdot\vec{v} - 2\,\vec{v}\cdot\vec{w}$ $[\because |\vec{u}| = |\vec{w}| = 1$ (Given)|

$\Rightarrow \vec{u}\cdot\vec{v} = \vec{v}\cdot\vec{w}$

Hence, by using (B) and (C) also, we will get

$\vec{u}\cdot\vec{v} = \vec{v}\cdot\vec{w} = \vec{w}\cdot\vec{u} = m$ (say)

$\Rightarrow$ $\vec{u}$, $\vec{v}$, $\vec{w}$ are the vectors of an equilateral triangle (say ΔABC)



$d(O, P) = \dfrac{16}{\sqrt{3+4+9}} = \dfrac{16}{4} = 4$ units

$\overrightarrow{OA} = \vec{u}, \overrightarrow{OB} = \vec{v}, \overrightarrow{OC} = \vec{w}$

$|\overrightarrow{OA}| = |\overrightarrow{OB}| = |\overrightarrow{OC}| = 1$ (Given)

In an equilateral triangle, circumcentre, orthrocentre and centroid coincide.

Let D be the circumcentre of ΔABC, then

$\angle ADB = 120°$

$\text{Given} = \dfrac{DA^2 + DB^2 - AB^2}{2(DA)\cdot(DB)}$ …(vi)

OE = OD + DE

= OD + AF

$\Rightarrow 4 = OD + \dfrac{7}{2}$

$\Rightarrow OD = 4 - \dfrac{7}{2} = \dfrac{1}{2}$

$\Rightarrow DA = \sqrt{OA^2 - OD^2}$

$= \sqrt{1 - \dfrac{1}{4}}$

$DA = \dfrac{\sqrt{3}}{2}$

$\Rightarrow DA = DB = \dfrac{\sqrt{3}}{2}$ …(vii)

From (vi) and (vii),

$$-\frac{1}{2} = \frac{\frac{3}{4} + \frac{3}{4} - AB^2}{2\frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2}} =$$

$$-\frac{1}{2} = \frac{\frac{3}{2} - AB^2}{\frac{3}{2}}$$

$$\Rightarrow -\frac{1}{2} \times \frac{3}{2} = \frac{3}{2} - AB^2$$

$$\Rightarrow AB^2 = \frac{3}{2} + \frac{3}{4}$$

$$\Rightarrow AB^2 = \frac{9}{4}$$

$$\Rightarrow AB - \frac{3}{2} = |\vec{u} - \vec{v}|$$

$$\Rightarrow AB^2 = \frac{9}{4} = u^2 + v^2 - 2\,\vec{u} - \vec{v}$$

$$\Rightarrow \frac{9}{4} = 1 + 1 - 2m$$

$$\Rightarrow 2m = 2 - \frac{9}{4} = -\frac{1}{4}$$

$$\Rightarrow m = -\frac{1}{8} \qquad \ldots(viii)$$

Volume of parallelepiped,

$V = [\vec{u}\ \vec{v}\ \vec{w}]$

$$|\vec{u}\ \vec{v}\ \vec{w}|^2 = \begin{vmatrix} 1 & \vec{u}\cdot\vec{v} & \vec{u}\cdot\vec{w} \\ \vec{u}\cdot\vec{v} & 1 & \vec{v}\cdot\vec{w} \\ \vec{w}\cdot\vec{u} & \vec{w}\cdot\vec{v} & 1 \end{vmatrix} = \begin{vmatrix} 1 & m & m \\ m & 1 & m \\ m & m & 1 \end{vmatrix}$$

$= 1(1 - m^2) - m(m - m^2) + m(m^2 - m)$

$= 1 - m^2 - m^2 + m^3 + m^3 - m^2$

$= 1 - 3m^2 + 2m^3$

$|\vec{u}\ \vec{v}\ \vec{w}|^2 = 2m^3 - 3m^2 + 1$

$= (m - 1)\,[2m^2 - m - 1]$

$= (m - 1)\,[2m^2 - 2m + m - 1]$

$= (m - 1)\,(m - 1)\,(2m + 1)$

$= (m - 1)^2\,(2m + 1)$

$$\Rightarrow |\,[\vec{u}\ \vec{v}\ \vec{w}]\,| = (m - 1)\sqrt{(2m+1)} = v$$

$$\left|\left(-\frac{1}{8} - 1\right)\sqrt{2 \times -\frac{1}{8} + 1}\right|$$

$$V = \frac{9}{8} \times \frac{\sqrt{3}}{2}$$

$$\frac{80}{\sqrt{3}}v = \frac{80}{\sqrt{3}} \times \frac{9}{8} \times \frac{\sqrt{3}}{2} = 45$$

Q.13 Let a and b be two nonzero real numbers. If the coefficient of x^5 in the expansion of $\left(ax^2 + \frac{70}{27bx}\right)^4$ is equal to the coefficient of x^{-5} in the expansion of $\left(ax - \frac{1}{bx^2}\right)^7$, then the value of 2b is

Ans. [3]

Sol. $T_{r+1} = {}^4C_r (ax^2)^{4-r}\left(\frac{70}{27bx}\right)^r$

For coefficient of x^5, $8 - 2r - r = 5 \Rightarrow r = 1$

$\Rightarrow$ Coefficient of $x^5 = {}^4C_1 a^3\left(\frac{70}{27b}\right)$

$t_{r+1} = {}^7C_r (ax)^{7-r}\left(-\frac{1}{bx^2}\right)^r$

For coefficient of x^{-5}, $7 - r - 2r = -5 \Rightarrow r = 4$

$\Rightarrow$ coefficient of $x^{-5} = {}^7C_4 a^3 \frac{1}{b^4}$

$\Rightarrow {}^4C_1 a^3\left(\frac{70}{27b}\right) = {}^7C_4 a^3 \frac{1}{b^4} \Rightarrow 2b = 3$

SECTION 4 (Maximum Marks: 12)

- This section contains **FOUR (04)** Matching List Sets.
- Each set has **ONE** Multiple Choice Question.
- Each set has **TWO** lists: **List-I** and **List-II**.
- **List-I** has **Four** entries (P), (Q), (R) and (S) and **List-II** has **Five** entries (1), (2), (3), (4) and (5).
- **FOUR** options are given in each Multiple Choice Question based on **List-I** and **List-II** and **ONLY ONE** of these four options satisfies the condition asked in the Multiple Choice Question.
- Answer to each question will be evaluated according to the following marking scheme:
  Full Marks : +3 **ONLY** if the option corresponding to the correct combination is chosen;
  Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
  Negative Marks : –1 In all other cases.

Q.14 Let α, β and γ be real numbers. Consider the following system of linear equations

$x + 2y + z = 7$

$x + \alpha z = 11$

$2x - 3y + \beta z = \gamma$

Match each entry in List-I to the correct entries in List-II.

List-I	List-II
(P) If $\beta = \frac{1}{2}(7\alpha - 3)$ and $\gamma = 28$, then the system has	(1) a unique solution
(Q) If $\beta = \frac{1}{2}(7\alpha - 3)$ and $\gamma \neq 28$, then the system has	(2) no solution
(R) If $\beta \neq \frac{1}{2}(7\alpha - 3)$ where $\alpha = 1$ and $\gamma \neq 28$, then the system has	(3) infinitely many solutions
(S) If $\beta \neq \frac{1}{2}(7\alpha - 3)$ where $\alpha = 1$ and $\gamma = 28$, then the system has	(4) $x = 11$, $y = -2$ and $z = 0$ as a solution
	(5) $x = -15$, $y = 4$ and $z = 0$ as a solution

The correct option is:

(A) (P) → (3); (Q) → (2); (R) → (1); (S) → (4)
(B) (P) → (3); (Q) → (2); (R) → (5); (S) → (4)
(C) (P) → (2); (Q) → (1); (R) → (4); (S) → (5)
(D) (P) → (2); (Q) → (1); (R) → (1); (S) → (3)

Ans. **[A]**

Sol. $x + 2y + z = 7$

$x + \alpha z = 11$

$2x - 3y + \beta z = \gamma$

$$\Delta = \begin{vmatrix} 1 & 2 & 1 \\ 1 & 0 & \alpha \\ 2 & -3 & \beta \end{vmatrix} = 0$$

$3\alpha - 2(\beta - 2\alpha) - 3 = 0$

$7\alpha - 2\beta = 3$

$\Rightarrow \beta = \frac{1}{2}(7\alpha - 3)$

$$\Delta_1 = \begin{vmatrix} 7 & 2 & 1 \\ 11 & 0 & \alpha \\ \gamma & -3 & \beta \end{vmatrix}, \Delta_2 = \begin{vmatrix} 1 & 7 & 1 \\ 1 & 11 & \alpha \\ 2 & \gamma & \beta \end{vmatrix}, \Delta_3 = \begin{vmatrix} 1 & 2 & 7 \\ 1 & 0 & 11 \\ 2 & -3 & \gamma \end{vmatrix}$$

$\Delta_3 = 0$

$\Rightarrow 33 - 2(\gamma - 22) + 7(-3) = 0$

$\gamma = 28$

$\Delta_1 = 21\alpha - 2(11\beta - \alpha\gamma) - 33$

$= 21\alpha - 22\beta + 2\alpha\gamma - 33$

$\Delta_2 = 11\beta - \alpha\gamma - 7(\beta - 2\alpha) + \gamma - 22$

$= 14\alpha + 4\beta + \gamma - \alpha\gamma - 22$

(P) If $\beta = \frac{1}{2}(7\alpha - 3)$ and $\gamma = 28$

$\Delta = 0, \Delta_1 = 0, \Delta_2 = 0, \Delta_3 = 0$

Infinitely many solutions

$x = 11, y = -2$ and $z = 0$ will satisfy all the three given equations, so it is a solution.

(Q) If $\beta = \frac{1}{2}(7\alpha - 3)$ and $\gamma \neq 28$ then

$\Delta = 0$, but $\Delta_3 \neq 0$ so no solution

(R) If $\beta \neq \frac{1}{2}(7\alpha - 3)$, $\alpha = 1$ and $\gamma \neq 28$

$\Delta \neq 0, \Delta_3 \neq 0$ so a unique solution

(S) If $\beta \neq \frac{1}{2}(7\alpha - 3)$, $\alpha = 1$, $\gamma = 28$

$\Delta \neq 0, \Delta_3 = 0, \Delta_1 \neq 0, \Delta_2 \neq 0$, so a unique solution

$x = 11, y = -2$ and $z = 0$ will satisfy all the three equations

Option A is correct.

Q.15 Consider the given data with frequency distribution

x_i 3 8 11 10 5 4

fi 5 2 3 2 4 4

Match each entry in List-I to the correct entries in List-II.

List-I	List-II
(P) The mean of the above data is	(1) 2.5
(Q) The median of the above data is	(2) 5
(R) The mean deviation about the mean of the above data is	(3) 6
(S) The mean deviation about the median of the above data is	(4) 2.7

The correct option is:

(A) (P) → (3); (Q) → (2); (R) → (4); (S) → (5)
(B) (P) → (3); (Q) → (2); (R) → (1); (S) → (5)
(C) (P) → (2); (Q) → (3); (R) → (4); (S) → (1)
(D) (P) → (3); (Q) → (3); (R) → (5); (S) → (5)

Ans. **[A]**

Sol. x … 3 4 5 8 10 11 (ascending order)

f … 5 4 4 2 2 3

$$\text{Mean} = \frac{3\times5+8\times2+11\times3+10\times2+5\times4+4\times4}{5+2+3+2+4+4}$$

$$= \frac{15+16+33+20+20+16}{20} = \frac{120}{20} = 6$$

$$\text{Median} = \frac{1}{2}(10^{th}\ 11^{th}\ \text{observation})$$

$$= \frac{1}{2}(5+5) = 5$$

Mean deviation about mean

$$= \frac{3\times5+2\times4+1\times4+2\times2+4\times2+5\times3}{20} = \frac{54}{20} = 2.7$$

Mean deviation about median

$$= \frac{2\times5+1\times4+0+3\times2+5\times2+6\times3}{20} = \frac{4.8}{20} = 2.4$$

P → 3; Q → 2; R → 4; S → 5

∵ Option A is correct.

Q.16 Let ℓ_1 and ℓ_2 be the lines $\vec{r}_1 = \lambda(\hat{i}+\hat{j}+\hat{k})$ and $\vec{r}_2 = (\hat{j}-\hat{k})+\mu(\hat{i}+\hat{k})$, respectively. Let X be the set of all the planes H that contain the line ℓ_1. For a plane H, let d(H) denote the smallest possible distance between the points of ℓ_2 and H. Let H_0 be a plane in X for which $d(H_0)$ is the maximum value of d(H) as H varies over all planes in X .

Match each entry in List-I to the correct entries in List-II.

List-I	List-II
(P) The value of $d(H_0)$ is	(1) $\sqrt{3}$
(Q) The distance of the point (0, 1, 2) from H_0 is	(2) $\frac{1}{\sqrt{3}}$
(R) The distance of origin from H_0 is	(3) 0
(S) The distance of origin from the point of intersection of planes y = z, x = 1 and H_0 is	(4) $\sqrt{2}$
	(5) $\frac{1}{\sqrt{2}}$

The correct option is:

(A) (P) → (2); (Q) → (4); (R) → (5); (S) → (1)
(B) (P) → (5); (Q) → (4); (R) → (3); (S) → (1)
(C) (P) → (2); (Q) → (1); (R) → (3); (S) → (2)
(D) (P) → (5); (Q) → (1); (R) → (4); (S) → (2)

Ans. **[B]**

Sol. H_0 will be the plane containing the line ℓ_1 and parallel to ℓ_2.

$\therefore$ Normal vector of plane parallel ℓ_1 and ℓ_2 is $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{vmatrix} = \hat{j}(1) - \hat{j}(1-1) + \hat{k}(-1)$

$\therefore H_0 : x - z = c \,|_{(0,\,0,\,0)}$

$\Rightarrow C = 0$

$\therefore H_0 : x - z = 0$

(P) $d(H_0) = 1$ distance of point (0, 1, –1) from H.

$d = \left|\frac{0-(-1)}{\sqrt{2}}\right| = \frac{1}{\sqrt{2}} \quad \therefore P \to 5$

(Q) $d = \left|\frac{0-2}{\sqrt{2}}\right| = \sqrt{2} \quad \therefore Q \to 4$

(R) $d = \left|\frac{0}{\sqrt{2}}\right| = 0 \quad \therefore R \to 3$

(S) Point of intersection will be (1, 1, 1) $\therefore S \to 1$

$d = \sqrt{1+1+} = \sqrt{3}$

$\therefore$ Option (B) is correct.

Q.17 Let z be a complex number satisfying $|z|^3 + 2z^2 + 4\bar{z} - 8 = 0$, where $\bar{z}$ denotes the complex conjugate of z. Let the imaginary part of z be non-zero.

Match each entry in List-I to the correct entries in List-II.

List-I	**List-II**
(P) $|z|^2$ is equal to	(1) 12
(Q) $|z - \bar{z}|^2$ is equal to	(2) 4
(R) $|z|^2 + |z + \bar{z}|^2$ is equal to	(3) 8
(S) $|z + 1|^2$ is equal to	(4) 10
	(5) 7

The correct option is:

(A) (P) → (1); (Q) → (3); (R) → (5); (S) → (4)
(B) (P) → (2); (Q) → (1); (R) → (3); (S) → (5)
(C) (P) → (2); (Q) → (4); (R) → (5); (S) → (1)
(D) (P) → (2); (Q) → (3); (R) → (5); (S) → (4)

Ans. **[B]**

$|z|^3 + 2z^2 + 4\bar{z} - 8 = 0$

Sol. $\underline{|z|^3 + 2\bar{z}^2 + 4z - 8 = 0}$

$2(z^2 - \bar{z}^2) + 4(\bar{z} - z) = 0$

$(z - \bar{z})[2(z + \bar{z}) - 4] = 0$

$\because z = \bar{z}$ (not possible) or $4x = 4 \Rightarrow x = 1$.

$z = 1 + \lambda i \Rightarrow |z| = \sqrt{1+\lambda^2} \Rightarrow \bar{z} = 1 - \lambda i$

$(1 + \lambda^2)^{3/2} + 2(1 - \lambda^2 + 2\lambda i) + 4(1 - \lambda i) - 8 = 0$

$\Rightarrow (1 + \lambda^2)^{3/2} + 2(1 - \lambda^2) = 4$

$(1 + \lambda^2)^{3/2} = 2(1 + \lambda^2)$

$(1 + \lambda^2)[\sqrt{1+\lambda^2} - 2] = 0$

$\Rightarrow \lambda^2 = 3$

Now

(P) $|z|^2 = 1 + \lambda^2 = 1 + 3 = 4$

(Q) $|z - \bar{z}|^2 = |1 + \lambda i - (1 - \lambda i)|^2 = |2\lambda i|^2 = 4\lambda^2 = 12$

(R) $|z|^2 + |z + \bar{z}|^2 = 4 + |(1 + \lambda i) + (1 - \lambda i)|^2 = 4 + 4 = 8$

(S) $|z + 1|^2 = |1 + \lambda i + 1|^2 = 4 + \lambda^2 = 4 + 3 = 7$

$\therefore$ P $\to$ (2), Q $\to$ (1), R $\to$ (3), S $\to$ (5)

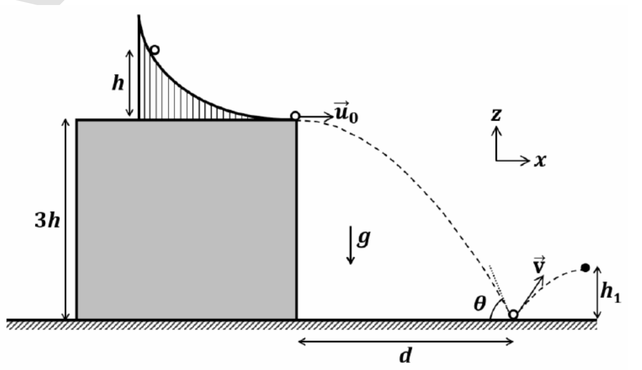
PHYSICS

SECTION – 1 (Maximum Marks: 12)

- This section contains **THREE (03)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is(are) correct answer(s).
- For each question, **choose the option(s)** corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:

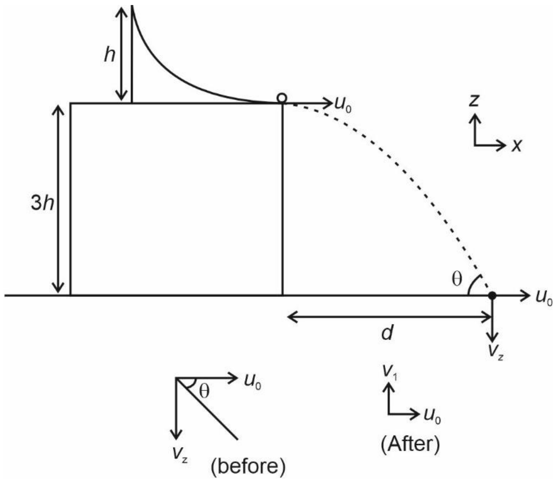
  Full Marks : +4 **ONLY** if (all) the correct option(s) is(are) chosen;
  Partial Marks : +3 If all the four options are correct but **ONLY** three options are chosen;
  Partial Marks : +2 If three or more options are correct but **ONLY** two options are chosen, both of which are correct;
  Partial Marks : +1 If two or more options are correct but **ONLY** one option is chosen and it is a correct option;
  Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
  Negative Marks : –2 In all other cases.
- For example, in a question, if (A), (B) and (D) are the ONLY three options corresponding to correct answers, then
  choosing ONLY (A), (B) and (D) will get +4 marks;
  choosing ONLY (A) and (B) will get +2 marks;
  choosing ONLY (A) and (D) will get +2 marks;
  choosing ONLY (B) and (D) will get +2 marks;
  choosing ONLY (A) will get +1 mark;
  choosing ONLY (B) will get +1 mark;
  choosing ONLY (D) will get +1 mark;
  choosing no option (i.e. the question is unanswered) will get 0 marks; and
  choosing any other combination of options will get –2 marks.

Q.1 A slide with a frictionless curved surface, which becomes horizontal at its lower end, is fixed on the terrace of a building of height 3h from the ground, as shown in the figure. A spherical ball of mass m is released on the slide from rest at a height h from the top of the terrace. The ball leaves the slide with a velocity $\vec{u}_0 = u_0\hat{x}$ and falls on the ground at a distance d from the building making an angle θ with the horizontal. It bounces off with a velocity $\vec{v}$ and reaches a maximum height h_1. The acceleration due to gravity is g and the coefficient of restitution of the ground is $1/\sqrt{3}$. Which of the following statement(s) is(are) correct?



(A) $\vec{u}_0 = \sqrt{2gh}\,\hat{x}$ (B) $\vec{v} = \sqrt{2gh}\,(\hat{x} - \hat{z})$ (C) θ = 60º (D) $d/h_1 = 2\sqrt{3}$

Ans. **[A,C,D]**
Sol.



$u_0 = \sqrt{2gh}$

$v_z = \sqrt{2g(3h)}$

$\tan\theta = \frac{v_z}{u} = \sqrt{3}$

$\boxed{\theta = 60^\circ}$

$d = u_0 T = u_0\sqrt{2\left(\frac{3h}{g}\right)} = \sqrt{(2gh)}\sqrt{(2)\left(\frac{3h}{g}\right)}$

Velocity after collision, only velocity along z-direction change

$v_1 = ev_z = \sqrt{2gh}$

$\vec{v} = v_1\hat{k} + u_0\hat{i} = \sqrt{2gh}\,[\hat{i} + \hat{k}]$

$h_1 = \frac{v_1^2}{2g} = h$

Finally, $u_0 = \sqrt{2gh}$, $\theta = 60^\circ$, $\frac{d}{h} = 2\sqrt{3}$

Q.2 A plane polarized blue light ray is incident on a prism such that there is no reflection from the surface of the prism. The angle of deviation of the emergent ray is $\delta = 60°$ (see Figure-1). The angle of minimum deviation for red light from the same prism is $\delta_{min} = 30°$ (see Figure-2). The refractive index of the prism material for blue light is $\sqrt{3}$. Which of the following statement(s) is(are) correct?

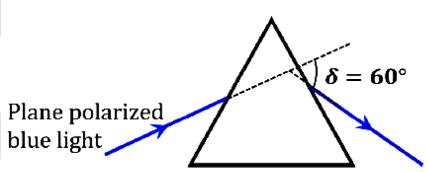


Figure-1

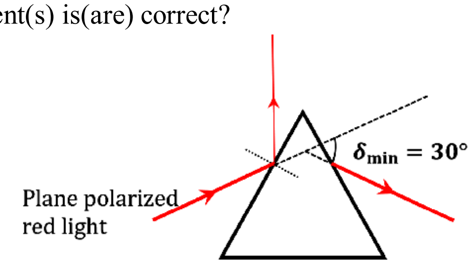


Figure-2

(A) The blue light is polarized in the plane of incidence.

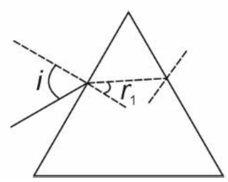
(B) The angle of the prism is 45°.

(C) The refractive index of the material of the prism for red light is $\sqrt{2}$.

(D) The angle of refraction for blue light in air at the exit plane of the prism is 60°.

Ans. **[A,C,D]**

Sol. For no reflection



$\tan i = \sqrt{3}$

$i = 60$

$\frac{\sin i}{\sin r_1} = \sqrt{3}$, $r_1 = 30^o$

$\delta = i + e - r_1 - r_2 = 60^o$

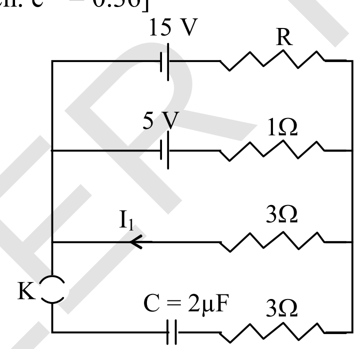
$\boxed{e = 60^o}$ $r_2 = 30^o$

$\angle A = 60^o$

For red light,

$$\mu = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)} = \sqrt{2}$$

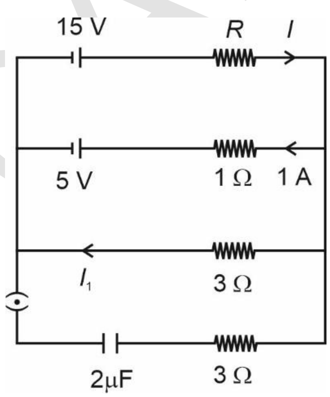
Q.3 In a circuit shown in the figure, the capacitor C is initially uncharged and the key K is open. In this condition, a current of 1 A flows through the 1 Ω resistor. The key is closed at time $t = t_0$. Which of the following statement(s) is(are) correct? [Given: $e^{-1} = 0.36$]

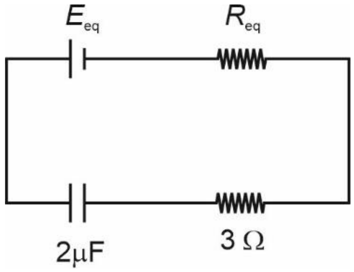


(A) The value of the resistance R is 3 Ω.
(B) For $t < t_0$, the value of current I_1 is 2 A.
(C) At $t = t_0 + 7.2$ μs, the current in the capacitor is 0.6 A.
(D) For $t \to \infty$, the charge on the capacitor is 12 μC.

Ans. **[A,B,C,D]**

Sol.





15 – IR = 6

$I_1[3] = 6$

$\boxed{I_1 = 2A}$

$I = I_1 + 1 = 3$

15 – 3R = 6

$\Rightarrow \boxed{R = 3\Omega}$

Eq. circuit is

$$\frac{1}{R_{eq}} = \frac{1}{3} + \frac{1}{3} + 1$$

$$R_{eq} = \frac{3}{5}\Omega$$

$$E_{eq} = 5 + 5 + 0 = 10$$

$$E_{eq} = 10 \times \frac{3}{5} = 6 \text{ V}$$

Current in circuit is $\dfrac{6}{\left(\frac{3}{5}+3\right)} e^{-t/CR}$

$$= \frac{6.5}{18} e^{-\frac{7.2\times10^{-6}}{2\times10^{-6}\times3.6}}$$

$$= \frac{30}{18} \times e^{-1} = \frac{30}{18} \times 0.36 = \boxed{0.6A}$$

At steady state, voltage across capacitor = 6V

Q = 6 × 2 = 12μ C

SECTION – 2 [Maximum Mark : 12]

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme :

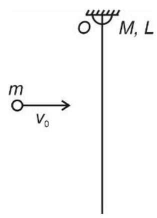
Full Marks	: **+3**	If **ONLY** the correct option is chosen.
Zero Marks	: **0**	If none of the option is chosen (i.e. the question is unanswered).
Negative Marks	: **–1**	In all other cases.

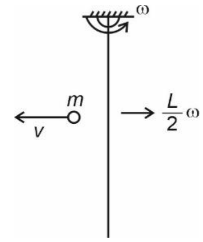
Q.4 A bar of mass M = 1.00 kg and length L = 0.20 m is lying on a horizontal frictionless surface. One end of the bar is pivoted at a point about which it is free to rotate. A small mass m = 0.10 kg is moving on the same horizontal surface with 5.00 m s^{-1} speed on a path perpendicular to the bar. It hits the bar at a distance L/2 from the pivoted end and returns back on the same path with speed v. After this elastic collision, the bar rotates with an angular velocity ω. Which of the following statement is correct?

(A) $\omega = 6.98$ rad s^{-1} and $v = 4.30$ m s^{-1}
(B) $\omega = 3.75$ rad s^{-1} and $v = 4.30$ m s^{-1}
(C) $\omega = 3.75$ rad s^{-1} and $v = 10.0$ m s^{-1}
(D) $\omega = 6.80$ rad s^{-1} and $v = 4.10$ m s^{-1}

Ans. **[A]**

Sol. C.O.A.M. about point O





$$mv_0\frac{L}{2} = \frac{ML^2}{3}\omega - \frac{mvL}{2} \quad \ldots \text{(i)}$$

e = 1

$$\Rightarrow v_0 = v + \frac{L\omega}{2} \quad \ldots \text{(ii)}$$

Solve equation (i) and (ii)

m = 0.1 kg, M = 1 kg, L = 0.20 m

v_0 = 5 m/s

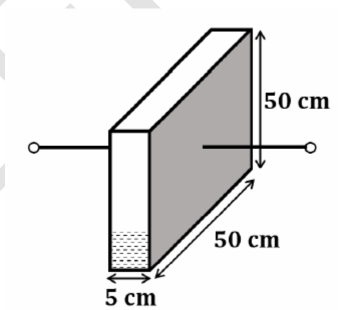
Solve (i) and (ii)

We get, ω = 6.98 rad/s

and v = 4.3 m/s

Q.5 A container has a base of 50 cm × 5 cm and height 50 cm, as shown in the figure. It has two parallel electrically conducting walls each of area 50 cm × 50 cm. The remaining walls of the container are thin and non-conducting. The container is being filled with a liquid of dielectric constant 3 at a uniform rate of 250 $cm^3\ s^{-1}$. What is the value of the capacitance of the container after 10 seconds?

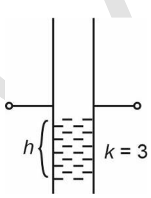
[Given: Permittivity of free space $\in_0 = 9 \times 10^{-12}\ C^2N^{-1}m^{-2}$, the effects of the non-conducting walls on the capacitance are negligible]



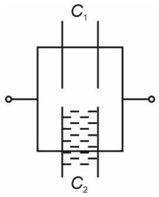
(A) 27 pF (B) 63 pF (C) 81 pF (D) 135 pF

Ans. **[B]**

Sol.



$$h = \frac{250 \times 10}{50 \times 5} = 10 \text{ cm}$$



$C_1 = \dfrac{(0.40 \times 0.50) \times 9 \times 10^{-12}}{5 \times 10^{-2}}$

$= 0.36 \times 10^{-10}$ F

$C_2 = \dfrac{3 \times 0.10 \times 0.5 \times 9 \times 10^{-12}}{5 \times 10^{-2}}$

$C_2 = 0.27 \times 10^{-10}$ F

$C = C_1 + C_2$

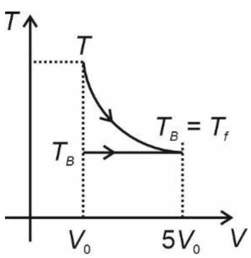
$= 63$ pF

Q.6 One mole of an ideal gas expands adiabatically from an initial state (T_A, V_0) to final state (T_f, $5V_0$). Another mole of the same gas expands isothermally from a different initial state (T_B, V_0) to the same final state (T_f, $5V_0$). The ratio of the specific heats at constant pressure and constant volume of this ideal gas is γ. What is the ratio T_A/T_B?

(A) $5^{\gamma-1}$ (B) $5^{1-\gamma}$ (C) 5^{γ} (D) $5^{1+\gamma}$

Ans. **[A]**

Sol.



For Adiabatic process

$TV^{\gamma-1} = C$

$\Rightarrow T_A V_0^{\gamma-1} = T_f(5V_0)^{\gamma-1}$ ...(i)

For Isothermal process

$T_B = T_f$ ...(ii)

Equation (i) ÷ equation (ii)

$\Rightarrow \dfrac{T_A}{T_B} = 5^{\gamma-1}$

Q.7 Two satellites P and Q are moving in different circular orbits around the Earth (radius R). The heights of P and Q from the Earth surface are h_P and h_Q, respectively, where $h_P = R/3$. The accelerations of P and Q due to Earth's gravity are g_P and g_Q, respectively. If $g_P/g_Q = 36/25$, what is the value of h_Q?

(A) 3R/5 (B) R/6 (C) 6R/5 (D) 5R/6

Ans. **[A]**

Sol. Given $h_P = \dfrac{R}{3}$

$h_Q = ?$

gravitational acceleration at height

$g_{ht} = \dfrac{GM}{(R+h)^2}$

$\dfrac{g_P}{g_Q} = \dfrac{36}{25} = \dfrac{\dfrac{GM}{(R+h_P)^2}}{\dfrac{GM}{(R+h_Q)^2}}$

Put $h_P = \dfrac{R}{3}$ solving

$h_Q = \dfrac{3R}{5}$

SECTION – 3 [Maximum Mark : 24]

- This section contains **SIX (06)** questions.
- The answer to each question is a **NON-NEGATIVE INTEGER**.
- For each question, enter the correct integer corresponding to the answer using the mouse and the onscreen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme :

Full Marks	: **+4**	If **ONLY** the correct integer is entered;
Negative Marks	: 0	In all other cases.

Q.8 A Hydrogen-like atom has atomic number Z. Photons emitted in the electronic transitions from level n = 4 to level n = 3 in these atoms are used to perform photoelectric effect experiment on a target metal. The maximum kinetic energy of the photoelectrons generated is 1. 95 eV. If the photoelectric threshold wavelength for the target metal is 310 nm, the value of Z is _______.
[Given: hc = 1240 eV-nm and Rhc = 13.6 eV, where R is the Rydberg constant, h is the Planck's constant and c is the speed of light in vacuum]

Ans. **[3]**

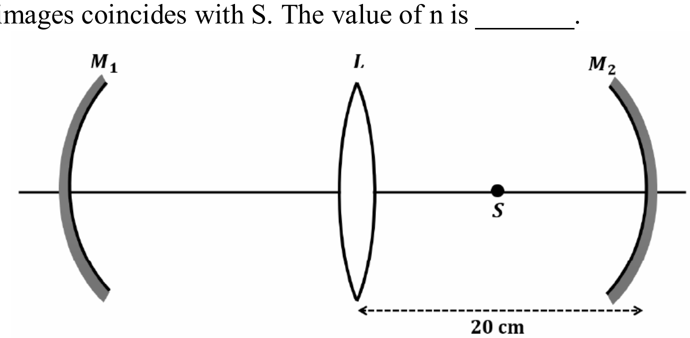
Sol. $\Delta E_{4 \text{ to } 3} = 1.95 \text{ eV} + \dfrac{1240}{310} \text{ eV}$

$13.6\, Z^2 \left(\dfrac{1}{3^2} - \dfrac{1}{4^2}\right) = 5.95 \Rightarrow 13.6\, Z^2 \dfrac{7}{9 \times 16} = 5.95$

$Z^2 = \dfrac{5.95 \times 9 \times 16}{13.6}$

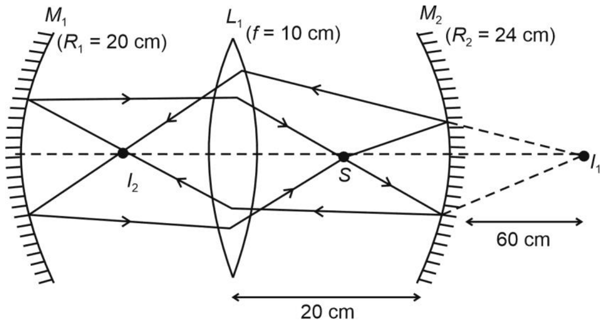
Solving Z = 3

Q.9 An optical arrangement consists of two concave mirrors M_1 and M_2, and a convex lens L with a common principal axis, as shown in the figure. The focal length of L is 10 cm. The radii of curvature of M_1 and M_2 are 20 cm and 24 cm, respectively. The distance between L and M_2 is 20 cm. A point object S is placed at the mid-point between L and M_2 on the axis. When the distance between L and M_1 is n/7 cm, one of the images coincides with S. The value of n is _______.



Ans. **[80 or 150 or 220]**

Sol.



For reflection from M_2

$$\frac{1}{v}+\frac{1}{(-10)}=\frac{1}{(-12)}$$

$$\frac{1}{v}=\frac{1}{10}-\frac{1}{12}$$

v = +60 cm (for I_1)

For refraction from L

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$$

$$\frac{1}{v}-\frac{1}{(-80)}=\frac{1}{10}$$

$$v=+\frac{80}{7}\text{ (For } I_2)$$

This image should be at focus of M_1

$$\therefore\ \frac{20}{2}+\frac{80}{7}=\frac{n}{7}$$

$$\boxed{n=150}$$

Also,

If I_2 is formed at pole of M_1

then $\frac{n}{7}=\frac{80}{7}$

$$\boxed{n=80}$$

And further if I_2 is formed at centre of curvature of M_1 then

$$\frac{n}{7}=\frac{80}{7}+20$$

$$\therefore\ \boxed{n=220}$$

Q.10 In an experiment for determination of the focal length of a thin convex lens, the distance of the object from the lens is 10 ± 0.1 cm and the distance of its real image from the lens is 20 ± 0.2 cm. The error in the determination of focal length of the lens is n %. The value of n is _______.

Ans. **[1]**

Sol. Object distance = 10 ± 0.1 cm

Image distance = 20 ± 0.2 cm

Applying lens formula

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f} \quad \dots \text{(i)}$$

$\Rightarrow \frac{1}{20} - \frac{1}{(-10)} = \frac{1}{f}$

$\Rightarrow f = \frac{20}{3}$ cm

Differentiate equation (i)

$-\frac{1}{v^2}dv + \frac{1}{u^2}du = \frac{-1}{f^2}df$

For calculating error

$\frac{1}{f^2}df = \frac{1}{v^2}dv + \frac{1}{u^2}du$

$\left(\frac{df}{f}\right)\times 100 = \left(\frac{0.2}{20^2} + \frac{0.1}{10^2}\right)\frac{20}{3}\times 100$

$= \left(\frac{0.2}{4} + \frac{0.1}{1}\right)\frac{20}{3} = 1$

$\therefore \frac{df}{f} \times 100 = 1\%$

Q.11 A closed container contains a homogeneous mixture of two moles of an ideal monatomic gas ($\gamma = 5/3$) and one mole of an ideal diatomic gas ($\gamma = 7/5$). Here, γ is the ratio of the specific heats at constant pressure and constant volume of an ideal gas. The gas mixture does a work of 66 Joule when heated at constant pressure. The change in its internal energy is ________ Joule.

Ans. **[121]**

Sol. $\Delta u = n_1C_1\Delta T + n_2C_2\Delta T$

$= (n_1C_1 + n_2C_2)\Delta T$ … (i)

Work done = $P\Delta v$

$= (n_1 + n_2)R\Delta T$ … (ii)

Divide (i) by (ii)

$\frac{\Delta u}{W} = \frac{(n_1C_1 + n_2C_2)\Delta T}{(n_1 + n_2)R\Delta T}$

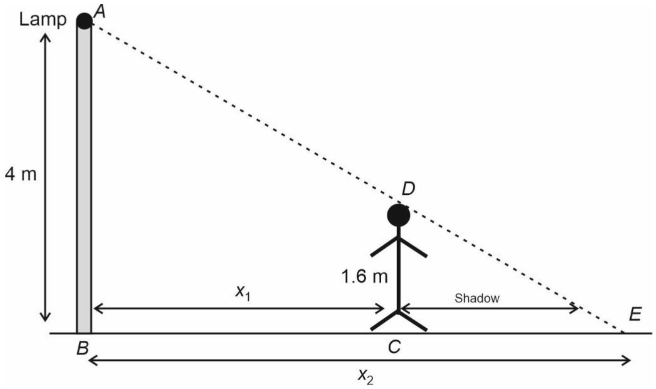
$\Delta u = \frac{W}{R}\left(\frac{n_1C_1 + n_2C_2}{n_1 + n_2}\right)$

$= \frac{66}{R}\frac{\left[\frac{3R}{2}\times 2 + \frac{5R}{2}\times 1\right]}{2+1} = 121$ J

Q.12 A person of height 1.6 m is walking away from a lamp post of height 4 m along a straight path on the flat ground. The lamp post and the person are always perpendicular to the ground. If the speed of the person is 60 cm s^{-1}, the speed of the tip of the person's shadow on the ground with respect to the person is _______ cm s^{-1}.

Ans. **[40]**

Sol. Given that $\frac{dx_1}{dt}$ = speed of person = 60 cm/s



Also $\frac{dx_2}{dt}$ = speed of tip of person's shadow

Applying similar triangle rule in ΔABE & ΔDCE

$$\frac{4}{x_2} = \frac{1.6}{x_2 - x_1}$$

$4x_2 – 4x_1 = 1.6x_2$

$2.4x_2 = 4x_1$

Differentiate both sides w.r.t. t

$$2.4 \frac{dx_2}{dt} = 4\frac{dx_1}{dt}$$

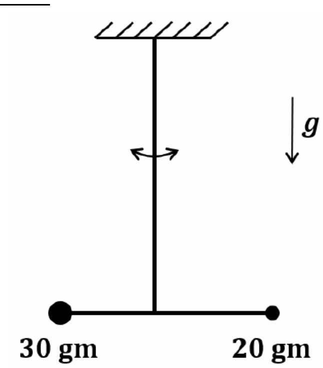
$$\frac{dx_2}{dt} = \frac{4}{2.4}(60) = 100 \text{ cm/s}$$

$\vec{v}_{SP} = \vec{v}_{SG} - \vec{v}_{PG}$

$v_{SP} = 100 \text{ cm s}^{-1} – 60 \text{ cm s}^{-1}$

$= 40 \text{ cm s}^{-1}$

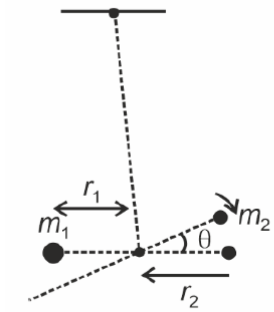
Q.13 Two point-like objects of masses 20 gm and 30 gm are fixed at the two ends of a rigid massless rod of length 10 cm. This system is suspended vertically from a rigid ceiling using a thin wire attached to its center of mass, as shown in the figure. The resulting torsional pendulum undergoes small oscillations. The torsional constant of the wire is 1.2×10^{-8} N m rad^{-1}. The angular frequency of the oscillations in $n \times 10^{-3}$ rad s^{-1}. The value of n is _____.



Ans. **[10]**

Sol. $m_1 = 30$ gm

$m_2 = 20$ gm



Moment of inertia about the axis of rotation is

$I = m_1r_1^2 + m_2r_2^2$

Clearly $r_1 = 4$ cm

And $r_2 = 6$ cm

$\therefore\ I = (30 \times 10^{-3} \times 16 \times 10^{-4}) + (20 \times 10^{-3} \times 36 \times 10^{-4})$

$\Rightarrow I = 1200 \times 10^{-7}$ kg m^2

If the system is rotated by small angle 'θ', the restoring torque is $\tau_{(R)} = -k\theta$

And $\frac{d^2\theta}{dt^2} = \frac{-k}{I}.\theta = -\omega^2\theta = \frac{-1.2\times10^{-8}}{1200\times10^{-7}}.\theta$

$\therefore\ \omega^2 = 10^{-4}$

So, $\omega = \frac{1}{100}$ rad/s $\Rightarrow \omega = 10 \times 10^{-3}$ rad/s

SECTION – 4 [Maximum Mark : 24]

- This section contains **FOUR (04)** Matching List Sets.
- Each set has **ONE** Multiple Choice Question.
- Each set has **TWO** lists: List-I and List-II.
- **List-I** has Four entries (P), (Q), (R) and (S) and **List-II** has **Five** entries (1), (2), (3), (4) and (5).
- **FOUR** options are given in each Multiple Choice Question based on **List-I** and **List-II** and **ONLY ONE** of these four options satisfies the condition asked in the Multiple Choice Question.
- Answer to each question will be evaluated according to the following marking scheme :

Full Marks	: **+3**	**ONLY** if the option corresponding to the correct combination is chosen;
Zero Marks	: 0	If none of the options is chosen (i.e. the question is unanswered);
Negative Marks	: –1	In all other cases.

Q.14 List-I shows different radioactive decay processes and List-II provides possible emitted particles. Match each entry in List-I with an appropriate entry from List-II, and choose the correct option.

	List-I		List-II
(P)	${}^{238}_{92}U \to {}^{234}_{91}Pa$	(1)	one α particle and one β^+ particle
(Q)	${}^{214}_{82}Pb \to {}^{210}_{82}Pb$	(2)	three β^- particles and one α particle
(R)	${}^{210}_{81}Tl \to {}^{206}_{82}Pb$	(3)	two β− particles and one α particle
(S)	${}^{228}_{91}Pa \to {}^{224}_{88}Ra$	(4)	one α particle and one β^- particle
		(5)	one α particle and two β^+ particles

(A) P → 4, Q → 3, R → 2, S → 1
(B) P → 4, Q → 1, R → 2, S → 5
(C) P → 5, Q → 3, R → 1, S → 4
(D) P → 5, Q → 1, R → 3, S → 2

Ans. **[A]**

Sol. Option (A) is correct answer.

• In α decay mass number decreases by 4 unit and atomic number decreases by 2 unit.
• In β⁻ decay mass number does not change but atomic number increases by 1 unit.
• In β⁺ decay mass number does not change but atomic number decreases by 1 unit.

Q.15 Match the temperature of a black body given in List-I with an appropriate statement in List-II, and choose the correct option.

[Given: Wien's constant as 2.9×10^{-3} m-K and $\frac{hc}{e} = 1.24 \times 10^{-6}$ V-m]

	List-I		List-II
(P)	2000 K	(1)	The radiation at peak wavelength can lead to emission of photoelectrons from a metal of work function 4 eV.
(Q)	3000 K	(2)	The radiation at peak wavelength is visible to human eye.
(R)	5000 K	(3)	The radiation at peak emission wavelength will result in the widest central maximum of a single slit diffraction.
(S)	10000 K	(4)	The power emitted per unit area is 1/16 of that emitted by a blackbody at temperature 6000 K.
		(5)	The radiation at peak emission wavelength can be used to image human bones.

(A) $P \to 3, Q \to 5, R \to 2, S \to 3$
(B) $P \to 3, Q \to 2, R \to 4, S \to 1$
(C) $P \to 3, Q \to 4, R \to 2, S \to 1$
(D) $P \to 1, Q \to 2, R \to 5, S \to 3$

Ans. **[C]**

Sol. (P) 2000 K $\quad \lambda_m T = b$

$$\lambda_m = \frac{b}{T} = \frac{2.9\times10^{-3}}{2000} = 1.45 \times 10^{-6} \text{ m} = 1450 \text{ nm}$$

(Q) 3000 K $\quad \lambda_m T = b \Rightarrow \lambda_m = \frac{2.9\times10^{-3}}{3000} = 966.66$ nm

(R) 5000 K $\quad \lambda_m T = b \Rightarrow \lambda_m = \frac{2.9\times10^{-3}}{5000} = 580$ nm

(S) 10000 K $\quad \lambda_m T = b \Rightarrow \lambda_m = \frac{2.9\times10^{-3}}{10000} = 290$ nm

List-II

(1) $\lambda_{th} = \frac{hc}{\phi} = \frac{1.24\times10^{-6}}{4} = 0.31 \times 10^{-6}$ m = 310 nm

As, $\lambda \le \lambda_{th}$ $\quad \boxed{S \to 1}$

(2) $400 < \lambda < 700$ nm $\quad \boxed{R \to 2}$

(3) Central maxima is widest for maximum wavelength

$\Rightarrow \boxed{P \to 3}$

(4) $\sigma A(T_1)^4 = \frac{1}{16}\sigma A T_2^4 \Rightarrow T_1 = \frac{1}{2}T_2 = 3000$ K $\quad \boxed{Q \to 4}$

(5) For imaging bones X-rays are used (1–10 nm)

None of the option in List-II

Q.16 A series LCR circuit is connected to a 45 sin(ωt) Volt source. The resonant angular frequency of the circuit is 10^5 rad s^{-1} and current amplitude at resonance is I_0. When the angular frequency of the source is $\omega = 8 \times 10^4$ rad s^{-1}, the current amplitude in the circuit is 0.05 I_0. If L = 50 mH, match each entry in List-I with an appropriate value from List-II and choose the correct option.

	List-I		List-II
(P)	I_0 in mA	(1)	44.4
(Q)	The quality factor of the circuit	(2)	18
(R)	The bandwidth of the circuit in rad s^{-1}	(3)	400
(S)	The peak power dissipated at resonance in Watt	(4)	2250
		(5)	500

(A) P → 2, Q → 3, R → 5, S → 1
(B) P → 3, Q → 1, R → 4, S → 2
(C) P → 4, Q → 5, R → 3, S → 1
(D) P → 4, Q → 2, R → 1, S → 5

Ans. **[B]**

Sol. As per the given information :

$$\frac{1}{\sqrt{LC}} = 10^5 \qquad \dots (i)$$

$$I_0 = \frac{45}{R} \qquad \dots (ii)$$

$$0.05I_0 = \frac{45}{\sqrt{R^2 + \left(0.8X_{L_0} - \frac{5}{4}X_{C_0}\right)^2}}$$

Where $X_{L_0} = X_{C0}$ are at resonant frequencies

On solving, $R = \frac{450\Omega}{4} \Rightarrow I_0 = 400$ mA

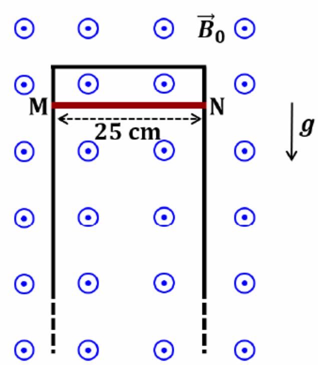
Quality factor $Q = \frac{1}{R}\sqrt{\frac{L}{C}} = 44.44$

$Q = \frac{\omega_0}{\Delta\omega} \Rightarrow \Delta\omega = 2250$ rad/s

Peak power $= 45 \times \frac{400}{10000}$ W = 18

⇒ Correct match is option (B)

Q.17 A thin conducting rod MN of mass 20 gm, length 25 cm and resistance 10 Ω is held on frictionless, long, perfectly conducting vertical rails as shown in the figure. There is a uniform magnetic field $B_0 = 4$ T directed perpendicular to the plane of the rod-rail arrangement. The rod is released from rest at time t = 0 and it moves down along the rails. Assume air drag is negligible. Match each quantity in List-I with an appropriate value from List-II, and choose the correct option.
[Given: The acceleration due to gravity $g = 10$ m s^{-2} and $e^{-1} = 0.4$]



	List-I		List-II
(P)	At t = 0.2 s, the magnitude of the induced emf in Volt	(1)	0.07
(Q)	At t = 0.2 s, the magnitude of the magnetic force in Newton	(2)	0.14
(R)	At t = 0.2 s, the power dissipated as heat in Watt	(3)	1.20
(S)	The magnitude of terminal velocity of the rod in m s^{-1}	(4)	0.12
		(5)	2.00

(A) $P \to 5, Q \to 2, R \to 3, S \to 1$

(B) $P \to 3, Q \to 1, R \to 4, S \to 5$

(C) $P \to 4, Q \to 3, R \to 1, S \to 2$

(D) $P \to 3, Q \to 4, R \to 2, S \to 5$

Ans. **[D]**

Sol. Induced emf $\varepsilon = B\ell v$

$\Rightarrow$ Induced current $i = \frac{\varepsilon}{R} = \frac{B\ell V}{R}$

$\Rightarrow mg - i\ell B = ma$ [Applying 2nd law]

$$\Rightarrow mg - \frac{B^2\ell^2 v}{R} = m\frac{dv}{dt}$$

$$\Rightarrow \frac{dv}{mg - \frac{B^2\ell^2 v}{R}} = \frac{dt}{m} \quad \Rightarrow \frac{\ln\left[mg - \frac{B^2\ell^2 v}{R}\right]_0^v}{\frac{-B^2\ell^2}{R}} = \frac{t}{m}$$

$$\Rightarrow \frac{mg - \frac{B^2\ell^2 v}{R}}{mg} = e^{\frac{-B^2\ell^2}{mR}t}$$

$\Rightarrow v = 2[1 - e^{-5t}]$

$\Rightarrow$ At t = 0.2 s, $v = 2\left[1 - \frac{1}{e}\right]$

$\Rightarrow \varepsilon = B\ell \times 2\left[1 - \frac{1}{e}\right] = 1.2$ volts

and magnetic force = $i\ell B$ = 0.12 N

and power dissipated = 0.144 W

also, Terminal velocity = 2 m/s

$\Rightarrow$ Correct match is (D)

CHEMISTRY

SECTION – 1 (Maximum marks : 12)

- This section contains **THREE (03)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is(are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +4 **ONLY** if (all) the correct option(s) is(are) chosen;
Partial Marks : +3 If all the four options are correct but **ONLY** three options are chosen;
Partial Marks : +2 If three or more options are correct but **ONLY** two options are chosen, both of which are correct;
Partial Marks : +1 If two or more options are correct but **ONLY** one option is chosen and it is a correct option;
Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
Negative Marks : −2 In all other cases.

- For example, in a question, if (A), (B) and (D) are the ONLY three options corresponding to correct answers, then
choosing ONLY (A), (B) and (D) will get +4 marks;
choosing ONLY (A) and (B) will get +2 marks;
choosing ONLY (A) and (D) will get +2 marks;
choosing ONLY (B) and (D) will get +2 marks;
choosing ONLY (A) will get +1 mark;
choosing ONLY (B) will get +1 mark;
choosing ONLY (D) will get +1 mark;
choosing no option (i.e. the question is unanswered) will get 0 marks; and
choosing any other combination of options will get −2 marks.

Q.1 The correct statement(s) related to processes involved in the extraction of metals is(are)
(A) Roasting of Malachite produces Cuprite.
(B) Calcination of Calamine produces Zincite.
(C) Copper pyrites is heated with silica in a reverberatory furnace to remove iron.
(D) Impure silver is treated with aqueous KCN in the presence of oxygen followed by reduction with zinc metal.

Ans. **[B, C, D]**

Sol. $Cu(OH)_2.CuCO_3 \rightarrow CuO + H_2O + CO_2$

$$\underset{\text{Calamine}}{ZnCO_3} \xrightarrow{\Delta} \underset{\text{Zincite}}{ZnO} + CO_2$$

$$CuFeS_2 + O_2 \longrightarrow Cu_2S + FeS + SO_2$$

$$FeS + O_2 \longrightarrow FeO + SO_2$$

$$FeO + SiO_2 \longrightarrow \underset{\text{Slage}}{FeSiO_3}$$

$$4Ag + 8CN^- + 2H_2O + O_2 \longrightarrow 4[Ag(CN)_2]^- + 4OH^-$$

$$2[Ag(CN)_2]^- + Zn(s) \longrightarrow [Zn(CN)_4]^{2-} + 2Ag$$

Q.2 In the following reactions, **P**, **Q**, **R**, and **S** are the major products.

$$CH_3CH_2CH(CH_3)CH_2CN \xrightarrow[\text{(ii) PhMgBr, then } H_2O]{\text{(i) PhMgBr, then } H_3O^{\oplus}} \mathbf{P}$$

$$\text{Ph-H} + CH_3\overset{O}{\overset{||}{C}}Cl \xrightarrow[\text{(ii) PhMgBr, then } H_2O]{\text{(i) Anhyd. } AlCl_3} \mathbf{Q}$$

$$CH_3CH_2\overset{O}{\overset{||}{C}}Cl \xrightarrow[\text{(ii) PhMgBr, then } H_2O]{\text{(i) } 1/2\ (PhCl_2)_2Cd} \mathbf{R}$$

$$PhCH_2CHO \xrightarrow[\text{(iii) HCN; (iv) } H_2SO_4, \Delta]{\text{(i) PhMgBr, then } H_2O;\ \text{(ii) } CrO_3,\ \text{dil. } H_2SO_4} \mathbf{S}$$

The correct statement(s) about **P**, **Q**, **R**, and **S** is(are)

(A) Both **P** and **Q** have asymmetric carbon(s).

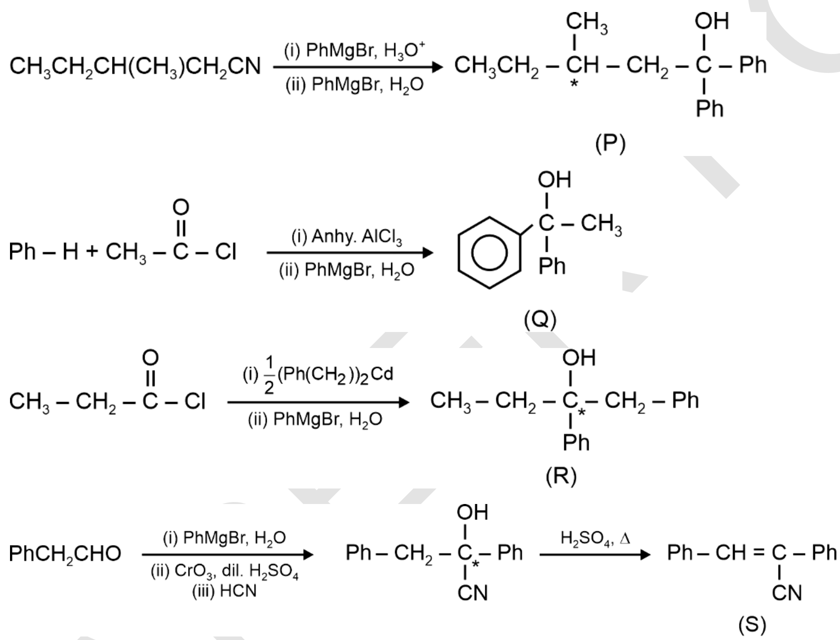
(B) Both **Q** and **R** have asymmetric carbon(s).

(C) Both **P** and **R** have asymmetric carbon(s).

(D) **P** has asymmetric carbon(s), **S** does **not** have any asymmetric carbon.

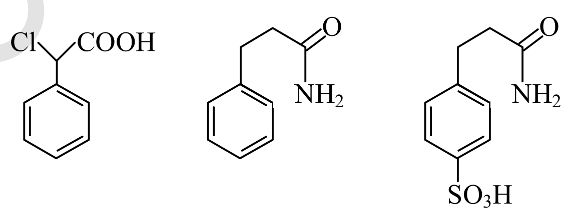
Ans. **[C, D]**

Sol.

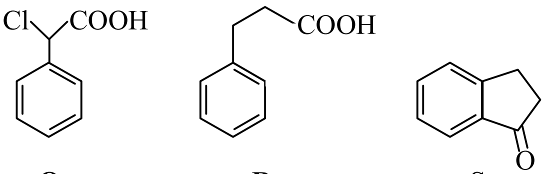


Q.3 Consider the following reaction scheme and choose the correct option(s) for the major products **Q**, **R** and **S**.

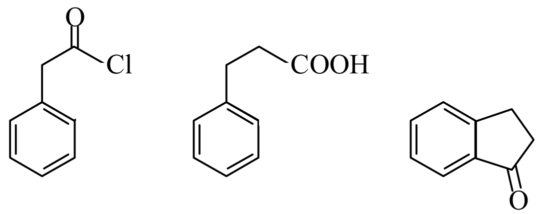
$$\text{Styrene} \xrightarrow[\text{(ii) NaOH, } H_2O_2,\ H_2O]{\text{(i) } B_2H_6} \mathbf{P} \xrightarrow[\text{(ii) } Cl_2,\ \text{Red phosphate; (iii) } H_2O]{\text{(i) } CrO_3,\ H_2SO_4} \mathbf{Q} \xrightarrow[\text{(ii) NaCN; (iii) } H_3O^+, \Delta]{\text{(i) } SOCl_2} \mathbf{R} \xrightarrow{\text{conc. } H_2SO_4} \mathbf{S}$$

(A) 

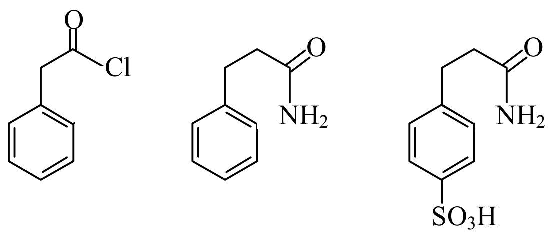
Q **R** **S**

(B) 

Q **R** **S**

(C) 

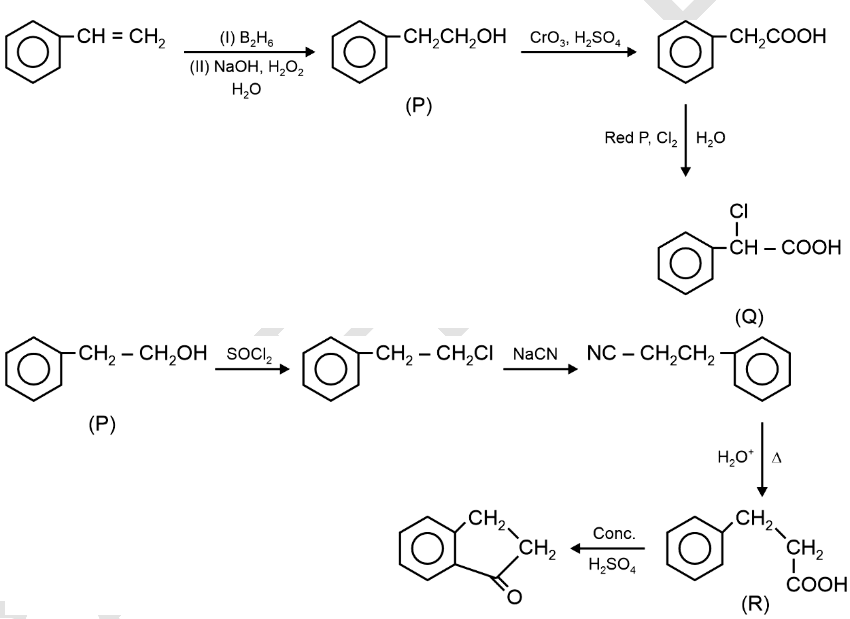
Q **R** **S**

(D) 

Q **R** **S**

Ans. **[B]**

Sol.



SECTION – 2 (Maximum Marks : 12)

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:

  Full Marks : +3 If **ONLY** the correct option is chosen;
  Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
  Negative Marks : −1 In all other cases.

Q.4 In the scheme given below, **X** and **Y**, respectively, are

$$\text{Metal halide} \xrightarrow{\text{aq. NaOH}} \text{White precipitate } (\mathbf{P}) + \text{Filtrate } (\mathbf{Q})$$

$$\mathbf{P} \xrightarrow[\text{Heat}]{\text{aq. } H_2SO_4,\ PbO_2\text{(excess)}} \mathbf{X} \text{ (a colour species in solution)}$$

$$\mathbf{Q} \xrightarrow[\text{Warm}]{MnO(OH)_2,\ \text{Cons. } H_2SO_4} \mathbf{Y} \text{ (gives blue-coloration with KI-starch paper}$$

(A) CrO_4^{2-} and Br_2 (B) MnO_4^{2-} and Cl_2 (C) MnO_4^- and Cl_2 (D) $MnSO_4$ and HOCl

Ans. **[C]**

Sol.

$$MnCl_2 \xrightarrow{\text{aq NaOH}} \underset{(P)}{Mn(OH)_2} + \underset{(Q)}{NaCl}$$

$$\underset{P}{Mn(OH)_2} \xrightarrow[H_2SO_4,\ \Delta]{PbO_2} \underset{(X)}{MnO_4^-}$$

$$NaCl \xrightarrow[\text{Conc. } H_2SO_4,\ \Delta]{MnO(OH)_2} \underset{(Y)}{Cl_2} \xrightarrow[\text{starch}]{\text{KI}} \text{blue}$$

Q.5 Plotting $1/\Lambda_m$ against $c\Lambda_m$ for aqueous solutions of a monobasic weak acid (HX) resulted in a straight line with y-axis intercept of P and slope of S. The ratio P/S is

[Λ_m = molar conductivity

Λ_m^o = limiting molar conductivity

c = molar concentration

K_a = dissociation constant of HX]

(A) $K_a\Lambda_m^o$ (B) $K_a\Lambda_m^o/2$ (C) $2K_a\Lambda_m^o$ (D) $1/(K_a\Lambda_m^o)$

Ans. **[A]**

Sol. $$\alpha = \frac{\Lambda_m}{\Lambda_m^o}$$

$$K_a = \frac{c\alpha^2}{1-\alpha}$$

$$K_a = \frac{c(\Lambda_m/\Lambda_m^o)^2}{1-(\Lambda_m/\Lambda_m^o)} = \frac{c\Lambda_m^2}{\Lambda_m^o(\Lambda_m^o - \Lambda_m)}$$

$$K_a\Lambda_m^{o2} - K_a\Lambda_m^o\Lambda_m = c\Lambda_m^2$$

$$\frac{K_a\Lambda_m^{o2}}{\Lambda_m} - K_a\Lambda_m^o = c\Lambda_m$$

$$\frac{K_a\Lambda_m^{o2}}{\Lambda_m} = c\Lambda_m + K_a\Lambda_m^o$$

$$\frac{1}{\Lambda_m} = \left(\frac{c\Lambda_m}{K_a\Lambda_m^{o2}}\right) + \frac{1}{\Lambda_m^o}$$

$$P = \frac{1}{\Lambda_m^o}\ ;\ S = \frac{1}{K_a\Lambda_m^{o2}}$$

$$\frac{P}{S} = \left(\frac{\frac{1}{\Lambda_m^o}}{\frac{1}{K_a\Lambda_m^{o2}}}\right) = K_a\Lambda_m^o$$

Q.6 On decreasing the pH from 7 to 2, the solubility of a sparingly soluble salt (MX) of a weak acid (HX) increased from 10^{-4} mol L^{-1} to 10^{-3} mol L^{-1}. The pK_a of HX is

(A) 3 (B) 4 (C) 5 (D) 2

Ans. **[B]**

Sol.

$$MX \rightleftharpoons M^{\oplus} + X^{\ominus}$$

$$X^{\ominus} + H^{\oplus} \rightleftharpoons HX$$

$$S = \sqrt{k_{sp}\left(1+\frac{H^{\oplus}}{k_a}\right)}$$

$$10^{-4} = \sqrt{k_{sp}\left(1+\frac{10^{-7}}{k_a}\right)} \qquad ....(1)$$

$$10^{-3} = \sqrt{k_{sp}\left(1+\frac{10^{-2}}{k_a}\right)} \qquad ....(2)$$

Equation (1)/(2) gives

$$10^{-2} = \frac{\left(1+\frac{10^{-7}}{k_a}\right)}{\left(1+\frac{10^{-2}}{k_a}\right)}$$

$$10^{-2} + \frac{10^{-4}}{k_a} = 1 + \frac{10^{-7}}{k_a}$$

$$\frac{10^{-4}-10^{-7}}{k_a} = 0.99$$

$$\frac{10^{-4}}{k_a} = 0.99$$

$$k_a = \frac{10^{-4}}{0.99} = \frac{1}{99} \times 10^{-2}$$

$pK_a = 2 + \log 99 = 4$

Q.7 In the given reaction scheme, **P** is a phenyl alkyl ether, **Q** is an aromatic compound; **R** and **S** are the major products.

$$\mathbf{P} \xrightarrow{HI} \mathbf{Q} \xrightarrow[\text{(iii) } H_3O^+]{\text{(i) NaOH, (ii) } CO_2} \mathbf{R} \xrightarrow[\text{(ii) } H_3O^+]{\text{(i) } (CH_3CO)_2O} \mathbf{S}$$

The correct statement about **S** is

(A) It primarily inhibits noradrenaline degrading enzymes.

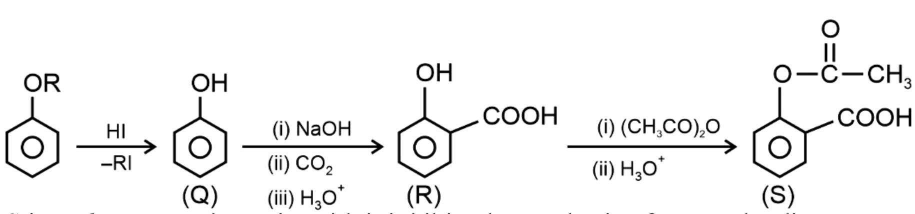
(B) It inhibits the synthesis of prostaglandin.

(C) It is a narcotic drug.

(D) It is *ortho*-acetylbenzoic acid.

Ans. **[B]**

Sol.



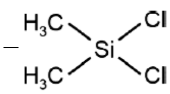
S is *ortho*-acetoxybenzoic acid, it inhibits the synthesis of prostaglandin.

SECTION – 3 (Maximum Marks : 24)

- This section contains **SIX (06)** questions.
- The answer to each question is a **NON-NEGATIVE INTEGER**.
- For each question, enter the correct integer corresponding to the answer using the mouse and the onscreen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme:
  Full Marks : +4 If **ONLY** the correct integer is entered;
  Zero Marks : 0 In all other cases.

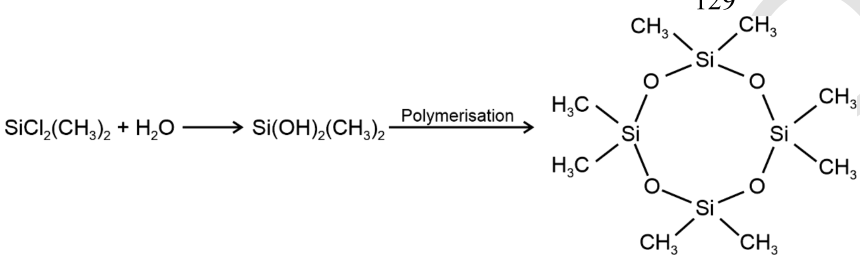
Q.8 The stoichiometric reaction of 516 g of dimethyldichlorosilane with water results in a tetrameric cyclic product **X** in 75% yield. The weight (in g) of **X** obtained is___.
[**Use**, molar mass (g mol^{-1}): H = 1, C = 12, O = 16, Si = 28, Cl = 35.5]

Ans. **[222]**

Sol. Dimethyldichlorosilane – 

mol. mass – 129 g/mol

Number of moles of dimethyldichlorosilane initially taken = $\frac{516}{129}$ = 4 moles



X

Applying POAC on Si atom

Moles of tetrameric cyclic product formed = $\frac{4}{4} \times \frac{75}{100}$ = 0.75 moles

Molar mass of product formed = 296 g/moles
The mass of product formed = 296 × 0.75 = 222 g

Q.9 A gas has a compressibility factor of 0.5 and a molar volume of 0.4 dm^3 mol^{-1} at a temperature of 800 K and pressure **x** atm. If it shows ideal gas behaviour at the same temperature and pressure, the molar volume will be **y** dm^3 mol^{-1}. The value of **x/y** is ___.
[**Use**: Gas constant, R = 8 × 10^{-2} L atm K^{-1} mol^{-1}]

Ans. **[100]**

Sol. Compressibility factor (Z) = $\frac{V_{real}}{V_{ideal}} = 0.5$

$V_{real} = 0.4\ dm^3\ mol^{-1} = 0.4$ L/mol

$\therefore V_{ideal} = \frac{0.4}{0.5} = 0.8$ L/mol

$\therefore$ $\boxed{y = 0.8 \text{ L/mol}}$

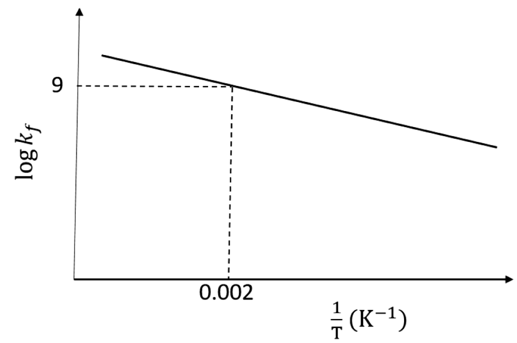
Using ideal gas equation : PV = nRT

$$P = \frac{1 \times 8 \times 10^{-2} \times 800}{0.8}$$

$\boxed{x = 80 \text{ atm}}$

$$\therefore \frac{x}{y} = \frac{80}{0.8} = 100$$

Q.10 The plot of log k_f versus $\frac{1}{T}$ for a reversible reaction A (g) $\rightleftharpoons$ P (g) is shown.



Pre-exponential factors for the forward and backward reactions are 10^{15} s^{-1} and 10^{11} s^{-1}, respectively. If the value of log K for the reaction at 500 K is 6, the value of | log k_b | at 250 K is ___.
[K = equilibrium constant of the reaction
k_f= rate constant of forward reaction
k_b = rate constant of backward reaction]

Ans. **[5]**

Sol. From the question

$A_f = 10^{15}$, Ab = 10^{11}, f = Forward reaction
log K at 500 K = 6 b = Backward reaction
log k_f at 500 K = 9 (from graph)
log k_b at 500 K :

$$\log K = \log\left(\frac{k_f}{k_b}\right) \text{ since} \Rightarrow K = \frac{k_f}{k_b}$$

6 = log k_f – log k_b
6 = 9 – log k_b
log k_b = 3 at 500 K

$$\log\frac{k_2}{k_1} = \frac{-E_a}{R}\left(\frac{1}{T_2} - \frac{1}{T_1}\right)$$

$$k_b = A_b\, e^{\frac{-E_{ab}}{RT}}$$

$$\ln k_b = \ln\left(A_b e^{\frac{-E_{ab}}{RT}}\right)$$

$$\ln k_b = \ln A_b - \frac{-E_{ab}}{RT}$$

$$2.303\log k_b = 2.303\log A_b - \frac{E_{ab}}{500R}$$

$$\frac{E_a}{500R} = 2.303(\log A_b - \log k_b)$$

$$\frac{E_a}{500R} = 2.303(\log 10^{11} - 3)$$

$$\frac{E_a}{500R} = 2.303(11 - 3) = 2.303 \times 8$$

Ea = 2.303 × 8 × 500R

$$\ln\left(\frac{k_2}{k_1}\right) = \frac{-E_a}{R}\left(\frac{1}{T_2} - \frac{1}{T_1}\right)$$

$$\ln\left(\frac{k_{250K}}{k_{500K}}\right) = \frac{-E_a}{R}\left(\frac{1}{250} - \frac{1}{500}\right)$$

$$\ln\left(\frac{k_{250K}}{k_{500K}}\right) = \frac{-2.303\times 8\times 500R}{R}\left(\frac{1}{500}\right)$$

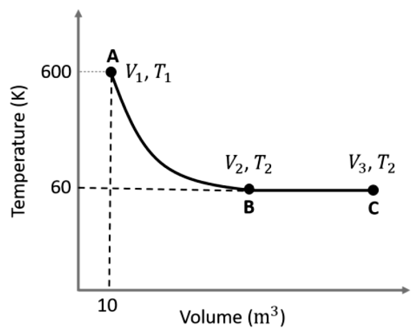
$2.303(\log k_{250\,K} - \log k_{500\,K}) = -2.303 \times 8$

$\log k_{250\,K} - 3 = -8$

$\log k_{250\,K} = -5$

$|\log k_{250\,K}| = 5$

Q.11 One mole of an ideal monoatomic gas undergoes two reversible processes ($A \rightarrow B$ and $B \rightarrow C$) as shown in the given figure:



$A \rightarrow B$ is an adiabatic process. If the total heat absorbed in the entire process ($A \rightarrow B$ and $B \rightarrow C$) is RT_2 ln10, the value of 2 log V_3 is ___.

[**Use**, molar heat capacity of the gas at constant pressure, $C_{p,m} = \frac{5}{2}R$]

Ans. **[7]**

Sol. $q_{A \rightarrow C} = RT_2 \ln 10$

$q_{A \rightarrow B} = 0$ ($\because$ adiabatic)

$q_{A \rightarrow C} = q_{A \rightarrow B} + q_{B \rightarrow C}$

$q_{A \rightarrow C} = q_{B \rightarrow C}$

$$q_{A \rightarrow C} = nRT_2\ln\left(\frac{V_3}{V_2}\right) \qquad ....(1)$$

For $B \rightarrow C$

$\Delta E = q + w$

$\Delta E = 0$ (since isothermic)

$q = -w$

$$= -\left(-nRT_2\ln\frac{V_3}{V_2}\right)$$

$$= nRT_2\ln\left(\frac{V_3}{V_2}\right)$$

$$q_{B\rightarrow C} = nRT_2\ln\left(\frac{V_3}{V_2}\right)$$

$$q_{B\rightarrow C} = RT_2\ln\left(\frac{V_3}{V_2}\right) \qquad \text{[Since n = 1]}$$

From A → B

$T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$

$600V_1^{\gamma-1} = 60V_2^{\gamma-1}$

$10 \times 10^{\frac{5}{3}-1} = V_2^{\gamma-1}$

$10^{5/3} = V_2^{\frac{5}{3}-1}$

$10^{5/3} = V_2^{2/3}$

$V_2 = 10^{\frac{5}{3}\times\frac{3}{2}} = 10^{\frac{5}{2}}$

$V_2 = 10^{\frac{5}{2}}$ ....(2)

From equation (1)

$q_{A\to C} = nRT_2 \text{In}\left(\frac{V_3}{V_2}\right)$

Given, $q_{A\to C} = RT_2 \ln 10$

$RT_2 \ln 10 = RT_2 \text{In}\left(\frac{V_3}{V_2}\right)$

$\ln 10 = \text{In}\left(\frac{V_3}{V_2}\right)$

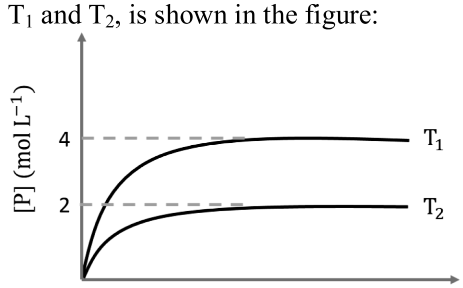
$\ln 10 = \text{In}\left(\frac{V_3}{10^{\frac{5}{2}}}\right)$

$10 = \frac{V_3}{10^{\frac{5}{2}}}$

$V_3 = 10^{1+\frac{5}{2}} = 10^{\frac{7}{2}}$

$2\log V_3 = 2\log 10^{7/2} = 7$

Q.12 In a one-litre flask, 6 moles of A undergoes the reaction A (g) $\rightleftharpoons$ P (g). The progress of product formation at two temperatures (in Kelvin), T_1 and T_2, is shown in the figure:



If $T_1 = 2T_2$ and $(\Delta G_2^{\Theta} - \Delta G_1^{\Theta}) = RT_2 \ln x$, then the value of x is ____.

[ΔG_1^{Θ} and ΔG_2^{Θ} are standard Gibb's free energy change for the reaction at temperatures T_1 and T_2, respectively.]

Ans. [8]

Sol.

$$A(g) \rightleftharpoons P(g)$$

	A(g)	P(g)
Initially	6	0
At equilibrium	6–a	a

at T_1 a = 4 **at T_2** a = 2

$\therefore (K_{eq})_1 = \frac{4}{2} = 2$ $\therefore (K_{eq})_2 = \frac{2}{4} = \frac{1}{2}$

$\Delta G_1^{\ominus} = -RT_1 \ln(K_{eq})_1$

$\Delta G_1^{\ominus} = -2RT_2 \ln(K_{eq})_1$ [Given: $T_1 = 2T_2$]

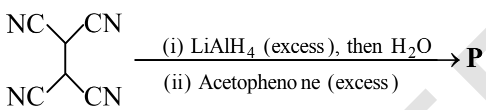
$\Delta G_2^{\ominus} = -RT_2 \ln(K_{eq})_2$

$$\therefore \Delta G_2^{\ominus} - \Delta G_1^{\ominus} = RT_2 \ln\left(\frac{K_{eq_1}}{K_{eq_2}}\right)^2$$

$$= RT_2 \ln \frac{2^2}{\frac{1}{2}} = RT_2 \ln 8$$

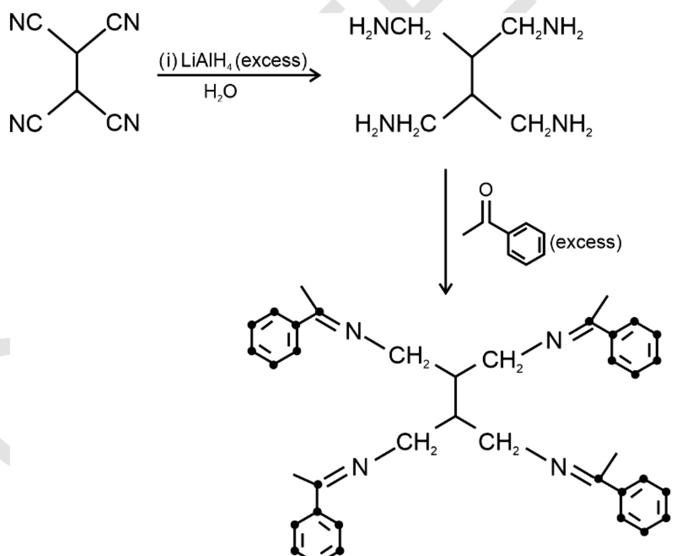
$\therefore \Delta G^{\ominus} = RT \ln \times$ has x = 8

Q.13 The total number of sp^2 hybridised carbon atoms in the major product **P** (a non-heterocyclic compound) of the following reaction is ___.



Ans. **[28]**

Sol.



All marked C–atoms are sp^2 hybridised.

SECTION – 4 (Maximum marks : 12)

- This section contains **FOUR (04)** Matching List Sets.
- Each set has **ONE** Multiple Choice Question.
- Each set has **TWO** lists: **List-I** and **List-II**.
- **List-I** has **Four** entries (P), (Q), (R) and (S) and **List-II** has **Five** entries (1), (2), (3), (4) and (5).
- **FOUR** options are given in each Multiple Choice Question based on **List-I** and **List-II** and **ONLY ONE** of these four options satisfies the condition asked in the Multiple Choice Question.
- Answer to each question will be evaluated according to the following marking scheme:
  Full Marks : +3 **ONLY** if the option corresponding to the correct combination is chosen;
  Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
  Negative Marks : −1 In all other cases.

Q.14 Match the reactions (in the given stoichiometry of the reactants) in List-I with one of their products given in List-II and choose the correct option.

	List-I		List-II
(P)	$P_2O_3 + 3H_2O \rightarrow$	(1)	$P(O)(OCH_3)Cl_2$
(Q)	$P_4 + 3NaOH + 3H_2O \rightarrow$	(2)	H_3PO_3
(R)	$PCl_5 + CH_3COOH \rightarrow$	(3)	PH_3
(S)	$H_3PO_2 + 2H_2O + 4AgNO_3 \rightarrow$	(4)	$POCl_3$
		(5)	H_3PO_4

(A) P → 2; Q → 3; R → 1; S → 5
(B) P → 3; Q → 5; R → 4; S → 2
(C) P → 5; Q → 2; R → 1; S → 3
(D) P → 2; Q → 3; R → 4; S → 5

Ans. **[D]**

Sol. $P_2O_3 + H_2O \rightarrow H_3PO_3$

$P_4 + NaOH + H_2O \rightarrow PH_3 + NaH_2PO_2$

$PCl_5 + CH_3COOH \rightarrow CH_3C(=O){-}Cl + POCl_3 + HCl$

$H_3PO_2 + H_2O + AgNO_3 \rightarrow H_3PO_4 + HNO_3 + Ag\downarrow$

Hence :-
P → 2
Q → 3
R → 4
S → 5

Q.15 Match the electronic configurations in List-I with appropriate metal complex ions in List-II and choose the correct option.
[Atomic Number: Fe = 26, Mn = 25, Co = 27]

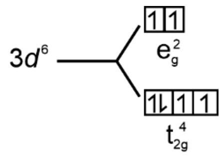
	List-I		List-II
(P)	$t_{2g}^6 e_g^0$	(1)	$[Fe(H_2O)_6]^{2+}$
(Q)	$t_{2g}^3 e_g^2$	(2)	$[Mn(H_2O)_6]^{2+}$
(R)	$e^2 t_2^3$	(3)	$[Co(NH_3)_6]^{3+}$
(S)	$t_{2g}^4 e_g^2$	(4)	$[FeCl_4]^-$
		(5)	$[CoCl_4]^{2-}$

(A) P → 1; Q → 4; R → 2; S → 3
(B) P → 1; Q → 2; R → 4; S → 5
(C) P → 3; Q → 2; R → 5; S → 1
(D) P → 3; Q → 2; R → 4; S → 1

Ans. **[D]**

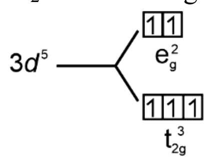
Sol. (1) $[Fe(H_2O)_6]^{2+} \rightarrow 3d^6$

$H_2O \rightarrow$ weak ligand



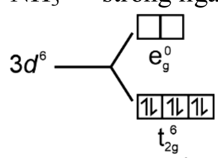
(2) $[Mn(H_2O)_6]^{2+} \rightarrow 3d^5$

$H_2O \rightarrow$ weak ligand



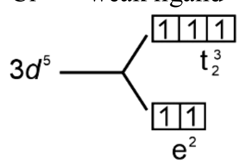
(3) $[Co(NH_3)_6]^{3+} \rightarrow 3d^6$

$NH_3 \rightarrow$ strong ligand



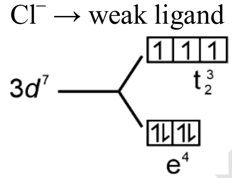
(4) $[FeCl_4]^{-} \rightarrow Fe^{+3} \rightarrow 3d^5$

$Cl^{-} \rightarrow$ weak ligand



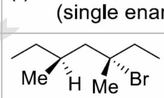
(5) $[CoCl_4]^{2-} \rightarrow Co^{+2} \rightarrow 3d^7$

$Cl^{-} \rightarrow$ weak ligand



$\therefore$ P → 3, Q → 2, R → 4, S → 1

Q.16 Match the reactions in List-I with the features of their products in List-II and choose the correct option.

	List-I		List-II
(P)	(-)-1-Bromo-2-ethylpentane (single enantiomer) $\xrightarrow[S_N2 \text{ reaction}]{\text{aq. NaOH}}$	(1)	Inversion of configuration
(Q)	(-)-2-Bromopentane (single enantiomer) $\xrightarrow[S_N2 \text{ reaction}]{\text{aq. NaOH}}$	(2)	Retention of configuration
(R)	(-)-3-Bromo-3-methylhexane (single enantiomer) $\xrightarrow[S_N1 \text{ reaction}]{\text{aq. NaOH}}$	(3)	Mixture of enantiomers
(S)	 (single enantiomer) $\xrightarrow[S_N1 \text{ reaction}]{\text{aq. NaOH}}$	(4)	Mixture of structural isomers
		(5)	Mixture of diastereomers

(A) P → 1; Q → 2; R → 5; S → 3
(B) P → 2; Q → 1; R → 3; S → 5
(C) P → 1; Q → 2; R → 5; S → 4
(D) P → 2; Q → 4; R → 3; S → 5

Ans. **[B]**

Sol. (P) Configuration at chiral carbon is same.

P → 2 [reaction does not occur at chiral carbon]

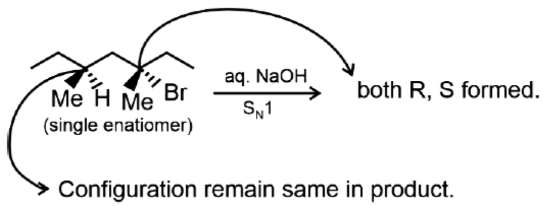
(Q) Configuration at chiral carbon changes.

Q → 1

(R) S_N1 → Mixture of enantiomers formed.

R → 3

(S)



∴ So mixture of diastereomers are formed.

S → 5

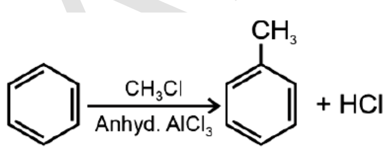
Q.17 The major products obtained from the reactions in List-II are the reactants for the named reactions mentioned in List-I. Match List-I with List-II and choose the correct option.

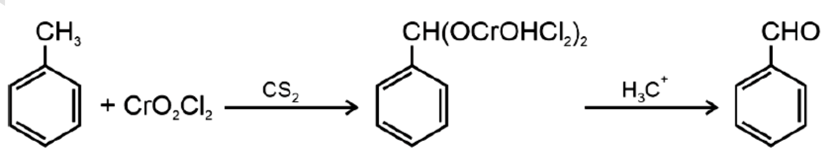
	List-I		**List-II**
(P)	Etard reaction	(1)	Acetophenone $\xrightarrow{\text{Zn-Hg, HCl}}$
(Q)	Gattermann reaction	(2)	Toluene $\xrightarrow[\text{(ii) SOCl}_2]{\text{(i) KMnO}_4\text{, KOH, }\Delta}$
(R)	Gattermann-Koch reaction	(3)	Benzene $\xrightarrow[\text{anhyd. AlCl}_3]{\text{CH}_3\text{Cl}}$
(S)	Rosenmund reduction	(4)	Aniline $\xrightarrow[\text{273-278 K}]{\text{NaNO}_2\text{/HCl}}$
		(5)	Phenol $\xrightarrow{\text{Zn, }\Delta}$

(A) P → 2; Q → 4; R → 1; S → 3
(B) P → 1; Q → 3; R → 5; S → 2
(C) P → 3; Q → 2; R → 1; S → 4
(D) P → 3; Q → 4; R → 5; S → 2

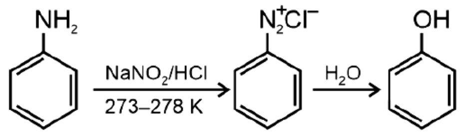
Ans. **[D]**

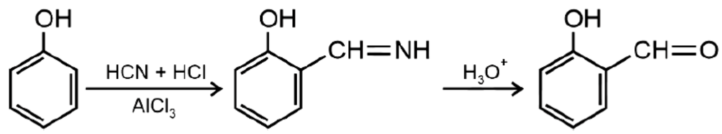
Sol. **P → 3**



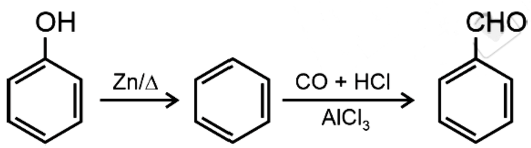


Q → 4

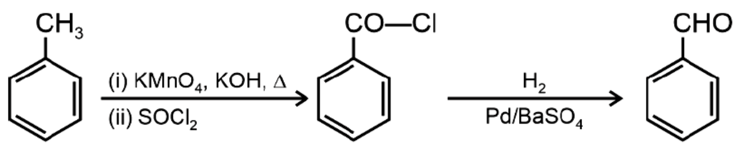

NH2
NaNO2/HCl
273–278 K
N2+Cl−
H2O
OH


OH
HCN + HCl
AlCl3
CH=NH
H3O+
CH=O

R → 5


OH
Zn/Δ
CO + HCl
AlCl3
CHO

S → 2


CH3
(i) KMnO4, KOH, Δ
(ii) SOCl2
CO—Cl
H2
Pd/BaSO4
CHO