

# JEE Advanced Exam 2019 (Paper & Solution)

Date : 27 / 05 / 2019

## PAPER-2

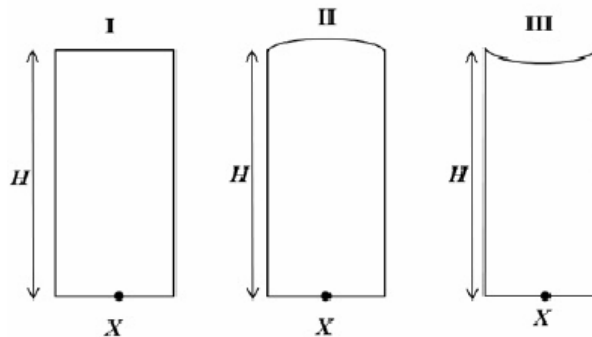
### PART-I (PHYSICS)

#### SECTION – 1 (Maximum Marks : 32)

- This section contains **EIGHT (08)** questions
- Each question has **FOUR** options. **ONE OR MORE THAN ONE** of these four option(s) is (are) correct option(s).
- For each question, choose(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme :

Full Marks	: +4	If only (all) the correct option(s) is (are) chosen.
Partial Marks	: +3	If all the four options are correct but <b>ONLY</b> three options are chosen.
Partial Marks	: +2	If three or more options are correct but <b>ONLY</b> two options are chosen, both of which are correct options.
Partial Marks	: +1	If two or more options are correct but <b>ONLY</b> one option is chosen and it is a correct option.
Zero Marks	: 0	If none of the option is chosen (i.e. the question is unanswered).
Negative Marks	: -1	In all other cases.

- Q.1** Three glass cylinders of equal height  $H = 30$  cm and same refractive index  $n = 1.5$  are placed on a horizontal surface as shown in figure. Cylinder I has a flat top, cylinder II has a convex top and cylinder III has a concave top. The radii of curvature of the two curved tops are same ( $R = 3$  m). If  $H_1$ ,  $H_2$  and  $H_3$  are the apparent depth of a point X on the bottom of the three cylinders, respectively, the correct statement (s) is / are –



- (1)  $0.8 \text{ cm} < (H_2 - H_1) < 0.9 \text{ cm}$                       (2)  $H_3 > H_1$   
(3)  $H_2 > H_3$     (4)  $H_2 > H_1$

**Ans.** [3, 4]

**Sol.** For fig. I

$$H_1 = \frac{H}{1.5} = \frac{30}{1.5} = 20 \text{ cm}$$

$$\frac{1}{-H_2} - \frac{1.5}{-30} = \frac{1-1.5}{-300}$$

**For fig. II**

$$\frac{1}{-H_2} + \frac{1.5}{30} = \frac{1 \setminus 2}{-300} = \frac{1}{600}$$

$$\frac{1}{H_2} = \frac{1.5}{30} - \frac{1}{600} = \frac{1}{20} \left(1 - \frac{1}{30}\right) = \frac{29}{600}$$

$$H_2 = 20.6 \text{ cm.}$$

**For fig. III**

$$\frac{1}{-H_3} - \frac{1.5}{-30} = \frac{1-1.5}{+300}$$

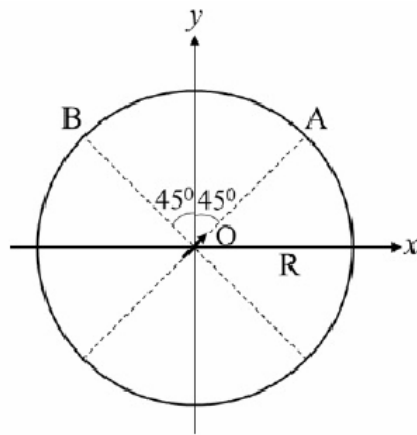
$$+\frac{1}{H_2} = \frac{1}{20} + \frac{1}{600}$$

$$\Rightarrow H_3 = \frac{600}{31} = 19.3$$

$$H_2 - H_1 = 20.6 - 20 = 0.6$$

$$H_2 > H_1, H_3 < H_1, H_2 > H_3$$

- Q.2** An electric dipole with dipole moment  $\frac{P_0}{\sqrt{2}}(\hat{i} + \hat{j})$  is held fixed at the origin O in the presence of a uniform electric field of magnitude  $E_0$ . If the potential is constant on a circle of radius R centered at the origin as shown in figure, then the correct statement (s) is/are : ( $\epsilon_0$  is permittivity of free space  $R \gg$  dipole size)



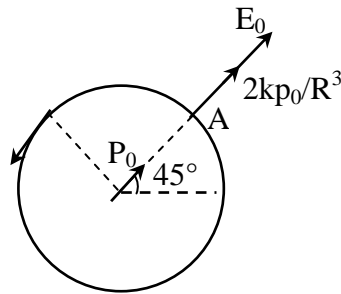
(1) Total electric field at point B is  $\vec{E}_B = 0$

(2) Total electric field at point A is  $\vec{E}_A = \sqrt{2} E_0(\hat{i} + \hat{j})$

(3)  $R = \left(\frac{P_0}{4\pi \epsilon_0 E_0}\right)^{1/3}$

(4) The magnitude of total electric field on any two points of the circle will be same.

**Ans.** [1,3]

**Sol.**

 $E_{\text{Tangential}}$  at all points on circle should be zero to make potential to be constant

$$E_A = \frac{2kp_0}{R^3} + E_0 = 2E_0 + E_0 = 3E_0$$

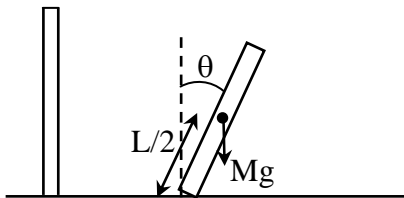
$$E_B = 0 \Rightarrow \frac{kp_0}{R^3} = E_0$$

$$E_0 = \frac{kp_0}{R^3} \Rightarrow R = \left( \frac{kp}{E_0} \right)^{\frac{1}{3}} = \left( \frac{P_0}{4\pi\epsilon_0 E_0} \right)^{\frac{1}{3}}$$

**Q.3** A thin and uniform rod of mass  $M$  and length  $L$  is held vertical on a floor with large friction. The rod is released from rest so that it falls by rotating about its contact-point with the floor without slipping. Which of the following statement (s) is/are correct, when the rod makes an angle  $60^\circ$  with vertical ?  
[ $g$  is the acceleration due to gravity]

- (1) The angular acceleration of the rod will be  $\frac{2g}{L}$
- (2) The radial acceleration of the rod's center of mass will be  $\frac{3g}{4}$
- (3) The angular speed of the rod will be  $\sqrt{\frac{3g}{2L}}$
- (4) The normal reaction force from the floor on the rod will be  $\frac{Mg}{16}$ .

**Ans.** [2,3,4]

**Sol.**


$$\tau = I \alpha \Rightarrow (Mg) \frac{L}{2} \sin \theta = \frac{ML^2}{3} \alpha$$

$$\alpha = \frac{3g}{2L} \sin \theta$$

$$\text{At } \theta = 60^\circ \quad \alpha = \frac{3g}{2L} \cdot \frac{\sqrt{3}}{2} = \frac{3\sqrt{3}g}{4L}$$

$$a_T = \frac{L}{2} \alpha = \frac{L}{2} \frac{3\sqrt{3}g}{4L} = \frac{3\sqrt{3}g}{8}$$

$$Mg \left[ \frac{L}{2} - \frac{L}{4} \right] = \frac{1}{2} \left( \frac{ML^2}{3} \right) \omega^2 \Rightarrow \frac{MgL}{4} = \frac{ML^2}{2 \times 3} \omega^2$$

$$\omega^2 = \frac{3g}{2L} \Rightarrow \omega = \sqrt{3g/2L}$$

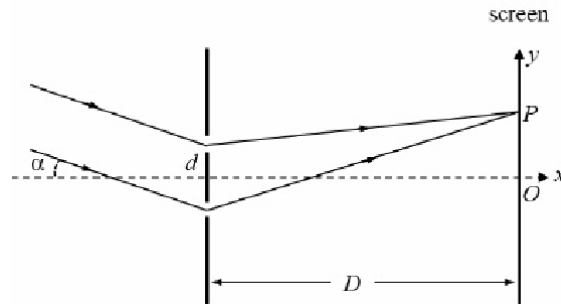
$$\text{Radial acceleration } a_c = \frac{L}{2} \omega^2 = \frac{L}{2} \frac{3g}{2L} = \frac{3g}{4}$$

$$a_{\text{vertical}} = \frac{3g}{4} \cos 60^\circ + \frac{3\sqrt{3}g}{8} \sin 60^\circ = \frac{3g}{8} + \frac{3\sqrt{3}g}{8} \frac{\sqrt{3}}{2}$$

$$= \frac{3g}{8} \left[ 1 + \frac{3}{2} \right] = \frac{15g}{16}$$

$$Mg - N = M \frac{15g}{16} \Rightarrow N = \frac{Mg}{16}$$

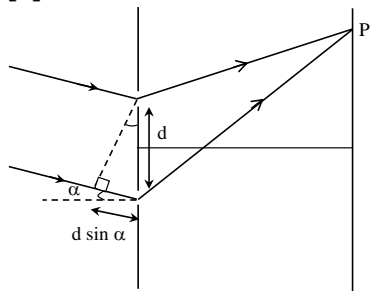
- Q.4** In a Young's double slit experiment, the slit separation  $d$  is 0.3 mm and the screen distance  $D$  is 1m. A parallel beam of light of wavelength 600 nm is incident on the slits at angle  $\alpha$  as shown in figure. On the screen, the point  $O$  is equidistant from the slits and distance  $PO$  is 11.0 mm. which of the following statement(s) is/are correct ?



- (1) For  $\alpha = \frac{0.36}{\pi}$  degree, there will be destructive interference at point O
- (2) For  $\alpha = 0$ , there will be constructive interference at point P
- (3) For  $\alpha = \frac{0.36}{\pi}$  degree, there will be destructive interference at point P.
- (4) Fringe spacing depends on  $\alpha$ .

**Ans.** [3]

**Sol.**



$$\text{Path difference at } p = d \sin \alpha + \frac{dy}{D} \quad \alpha \rightarrow \text{small } \sin \alpha = \alpha$$

$$= d\alpha + \frac{dy}{D}$$

$$= (0.3 \times 10^{-3}) \left[ \alpha + \frac{11 \times 10^{-3}}{1} \right]$$

$$\text{Option (3)} \quad \frac{3 \times 10^{-4} \left[ \frac{0.36 \pi}{\pi \cdot 180} + 11 \times 10^{-3} \right]}{600 \times 10^{-9}} = \frac{10^3}{2} [13 \times 10^{-3}] = \frac{13}{2}$$

$$\frac{\Delta}{\lambda} = \frac{13}{2} = 6.5 \text{ destructive interference.}$$

$$(2) \alpha = 0, \quad \frac{\Delta}{\lambda} = \frac{3 \times 10^{-4} \times 11 \times 10^{-3}}{6 \times 10^{-7}} = \frac{11}{2} = 5.5 (\text{Destructive})$$

$$\frac{\Delta}{\lambda} = \frac{d\alpha}{\lambda} = \frac{3 \times 10^{-4} \times 2 \times 10^{-3}}{6 \times 10^{-7}} = \frac{6 \times 10^{-7}}{6 \times 10^{-7}} = 1 (\text{Constructive})$$

**Q.5** A mixture of ideal gas containing 5 moles of monatomic gas and 1 mole of rigid diatomic gas is initially at pressure  $P_0$ , volume  $V_0$ , and temperature  $T_0$ . If the gas mixture is adiabatically compressed to a volume  $V_0/4$ , then the correct statement(s) is/are, (Given  $2^{1.2} = 2.3$ ;  $2^{3.2} = 9.2$ ; ;  $R$  is gas constant)

- (1) The final pressure of the gas mixture after compression is in between  $9P_0$  and  $10P_0$ .
- (2) The work  $|W|$  done during the process is  $13 RT_0$ .
- (3) Adiabatic constant of the gas mixture is 1.6.
- (4) The average kinetic energy of the gas mixture after compression is in between  $18RT_0$  and  $19RT_0$ .

**Ans.** [1,2,3]

**Sol.** 
$$\gamma_{\text{mix}} = \frac{n_1 C_{p1} + n_2 C_{p2}}{n_1 C_{v1} + n_2 C_{v2}}$$

$$C_{v1} = \frac{3R}{2} \quad C_{p1} = \frac{5R}{2}$$

$$C_{v2} = \frac{5R}{2} \quad C_{p2} = \frac{7R}{2}$$

$$= \frac{5 \left( \frac{5R}{2} \right) + (1) \left( \frac{7R}{2} \right)}{5 \left( \frac{3R}{2} \right) + (1) \left( \frac{5R}{2} \right)} = \frac{32R/2}{20R/2} = \frac{8}{5}$$

$$P_0 V_0^\gamma = P \left( \frac{V_0}{4} \right)^\gamma \quad P = P_0 (4)^\gamma \quad P = P_0 (4)^{8/5} = 9.2 P_0$$

$$W = \frac{P_1 V_1 - P_2 V_2}{\gamma - 1}$$

$$= \frac{P_0 V_0 - (4)^{8/5} P_0 \frac{V_0}{4}}{\frac{8}{5} - 1}$$

$$= P_0 V_0 \left[ \frac{1 - 4^{3/5}}{3/5} \right] = P_0 V_0 \frac{-1.29}{0.6} = -2.15 P_0 V_0$$

$$|W| = 2.15 (6) RT_0 = 13RT_0$$

$$9P_0 < P_f < 10P_0$$

$$|W| = 13 RT_0$$

$$\gamma = \frac{8}{5} = 1.6$$

$$KE = \frac{f}{2} nRT$$

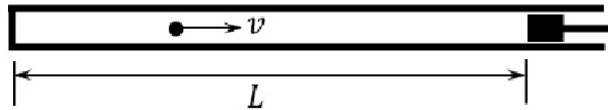
$$\gamma = 1 + \frac{2}{f} \Rightarrow \frac{f}{2} = \frac{1}{\gamma - 1}$$

$$= \frac{f}{2} PV = \frac{1}{3/5} (9.2P_0) \left( \frac{V_0}{4} \right) = \frac{5 \times 9.2 \times 6RT_0}{33 \times 4}$$

$$= 23RT_0$$

options : (1) (2) (3)

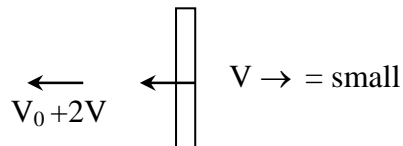
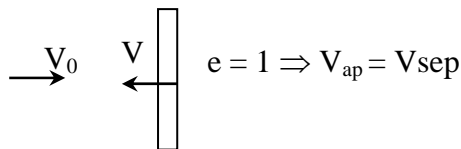
- Q.6** A small particle of mass  $m$  moving inside a heavy, hollow and straight tube along the tube axis undergoes elastic collision at two ends. The tube has no friction and it is closed at one end by a flat surface while the other end is fitted with a heavy movable flat piston as shown in figure. When the distance of the piston from closed end is  $L = L_0$  the particle speed is  $v = v_0$ . The piston is moved inward at a very low speed  $V$  and such that  $V \ll \frac{dL}{L} v_0$ , where  $dL$  is the infinitesimal displacement of the piston. Which of the following statement (s) is/are correct ?



- (1) The particle's kinetic energy increases by a factor of 4 when the piston is moved inward from  $L_0$  to  $\frac{1}{2}L_0$ .
- (2) After each collision with the piston, the particle speed increases by  $2V$ .
- (3) The rate at which the particle strikes the piston is  $v/L$ .
- (4) If the piston moves inward by  $dL$ , the particle speed increases by  $2v \frac{dL}{L}$ .

**Ans.** [1,2]

**Sol.**



$$\text{Frequency of Collision} = \frac{1}{2L/V_0} = \frac{V_0}{2L}$$

$$\text{Speed of particle After 1}^{\text{st}} \text{ collision} = V_0 + 2V$$

$$\text{Speed of particle After 2}^{\text{nd}} \text{ collision} = V_0 + 4V$$

$$\text{Speed of particle After } n^{\text{th}} \text{ collision} = V_0 + 2nV$$

$$\text{time taken by piston to move } L_0/2 = \frac{L_0/2}{V}$$

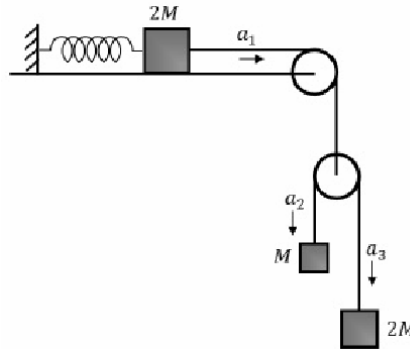
$$\text{no of collision} = \frac{V_0}{2L} \times \frac{L_0}{2V} = \frac{L_0 V_0}{4LV}$$

For  $dL$  displacement  $\Rightarrow t = \frac{dL}{V}$        $N = \frac{V_0}{2L_0} \frac{dL}{V}$

Speed =  $V_0 + 2 \left( \frac{V_0 dL}{2L_0 V} \right) V = V_0 + \frac{V_0 dL}{L}$

Option (1) and (2)

**Q.7** A block of mass  $2M$  is attached to a massless spring with spring-constant  $k$ . This block is connected to two other blocks of masses  $M$  and  $2M$  using two massless pulleys and strings. The accelerations of the block are  $a_1$ ,  $a_2$  and  $a_3$  as shown in the figure. The system is released from rest with the spring in its unstretched state. The maximum extension of the spring is  $x_0$ . Which of the following option (s) is/are correct ? [ $g$  is the acceleration due to gravity. Neglect friction]



(1) When spring achieves an extension of  $\frac{x_0}{2}$  for the first time, the speed of the block connected to the

spring is  $3g \sqrt{\frac{M}{5k}}$ .

(2)  $x_0 = \frac{4Mg}{k}$

(3) At an extension of  $\frac{x_0}{4}$  of the spring, the magnitude of acceleration of the block connected to the

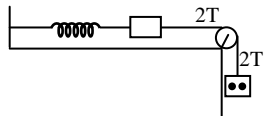
spring is  $\frac{3g}{10}$ .

(4)  $a_2 - a_1 = a_1 - a_3$

**Ans. [4]**

**Sol.**  $a_p = \frac{a_2 + a_3}{2}$  or  $a_2 - a_1 = a_1 - a_3$

Also

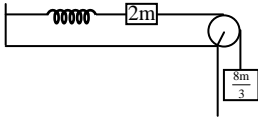


Reduced mass =  $\frac{m \times 2m}{m + 2m} = \frac{2m}{3}$

Tension =  $\frac{2m_1 m_2}{m_1 + m_2} g_{\text{eff}} = \frac{4m}{3} g_{\text{eff}}$

$2T = 2 \times \frac{4m}{3} g_{\text{eff}}$

$$\frac{2T}{g_{\text{eff}}} = \frac{8m}{3}$$



$$\frac{8m}{3} \cdot x_0 g + = \frac{1}{2} k x_0^2$$

$$x_0 = \frac{16mg}{3k}$$

Energy conservation

$$\frac{8m}{3} g \frac{x_0}{2} = \frac{1}{2} k \frac{x_0^2}{4} + \frac{1}{2} (2m)v^2$$

$$\frac{16mg}{3k} \frac{4mg}{3} - \frac{1}{8} k \left( \frac{16mg}{3k} \right)^2 = mv^2 + \frac{4m}{3} v^2$$

$$\frac{32 m^2 g^2}{9 k} = \frac{7}{3} mv^2$$

$$v^2 = \frac{32 mg^2}{21 k}$$

$$v = g \sqrt{\frac{32 mg}{21 k}}$$

**Q.8** A free hydrogen atom after absorbing a photon of wavelength  $\lambda_a$  gets excited from the state  $n = 1$  to the state  $n = 4$ . Immediately after that the electron jumps to  $n = m$  state by emitting a photon of wavelength  $\lambda_e$ . Let the change in momentum of atom due to the adsorption and the emission are  $\Delta p_a$  and  $\Delta p_e$ , respectively. If  $\lambda_a/\lambda_e = \frac{1}{5}$ , which of the option (s) is/are correct ? [Use  $hc = 1242 \text{ eV nm}$ ;  $1\text{nm} = 10^{-9} \text{ m}$ ,  $h$  and  $c$  are Planck's constant and speed of light, respectively]

(1)  $\lambda_e = 418 \text{ nm}$

(2) The ratio of kinetic energy for the electron in the state  $n = m$  to the state  $n = 1$  is  $\frac{1}{4}$ .

(3)  $m = 2$

(4)  $\Delta p_a/\Delta p_e = \frac{1}{2}$

**Ans.** [2,3]

**Sol.**  $\lambda \propto \frac{1}{\Delta E}$

$$\frac{\lambda_a}{\lambda_e} = \frac{\frac{1}{m^2} - \left(\frac{1}{4}\right)^2}{1 - \left(\frac{1}{4}\right)^2} = \frac{1}{5}$$

$$\frac{1}{m^2} - \frac{1}{16} = \frac{1}{5} \times \frac{15}{16} = \frac{3}{16}$$

$$\frac{1}{m^2} = \frac{4}{16} \Rightarrow m = 2$$



$$\frac{hc}{\lambda_e} = 13.6 \left( \frac{1}{4} - \frac{1}{16} \right)$$

$$\frac{1240}{\lambda_e} = 13.6 \left( \frac{12}{16 \times 4} \right)$$

$$\lambda_e = \frac{1240 \times 16 \times 4}{12 \times 13.6} = 487 \text{ nm}$$

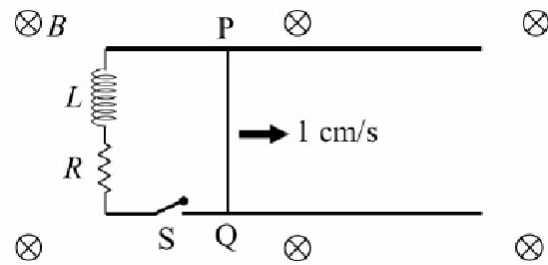
$$KE \propto \frac{1}{n^2} \Rightarrow \frac{KE_{m=2}}{KE_{m=1}} = \frac{1}{4}$$

**SECTION – 2 (Maximum Marks : 18)**

- This section contains **SIX (06)** questions. The answer to each question is a **NUMERICAL VALUE**.
- For each question, enter the correct numerical value using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer. If the numerical value has more than two decimal places, **truncate/round-off** the value to **TWO** decimal places.
- Answer to each question will be evaluated according to the following marking scheme.  
 Full Marks : +3 If **ONLY** the correct numerical value is entered as answer.  
 Zero Marks : 0 In all other cases.

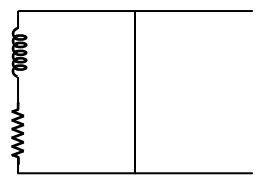
**Q.1** A 10 cm long perfectly conducting wire PQ is moving with a velocity 1 cm/s on a pair of horizontal rails of zero resistance. One side of the rails is connected to an inductor  $L = 1 \text{ mH}$  and a resistance  $R = 1\Omega$  as shown in figure. The horizontal rails, L and R lie in the same plane with a uniform magnetic field  $B = 1\text{T}$  perpendicular to the plan. If the key S is closed at certain instant, the current in the circuit after 1 millisecond is  $x \times 10^{-3} \text{ A}$ , where the value of x is .....

[Assume the velocity of wire PQ remains constant (1 cm/s) after key S is closed. Given :  $e^{-1} = 0.37$ , where e is base of the natural logarithm]



**Ans. [0.63]**

**Sol.**



$$E = v\ell B = (10^{-2} \text{ m/s}) \left( \frac{10}{100} \right) (1) = 10^{-3} \text{ volt}$$

$$i = i_0 (1 - e^{-t/\tau}) \quad \tau = \frac{L}{R} = \frac{10^{-3}}{1}$$

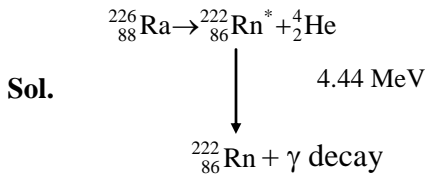
$$= \frac{10^{-3}}{1} \left[ 1 - e^{-10^{-3}/10^{-3}} \right]$$

$$= 10^{-3} \left( 1 - \frac{1}{e} \right) = 0.63 \times 10^{-3}$$

**Q.2** Suppose a  ${}^{226}_{88}\text{Ra}$  nucleus at rest and in ground state undergoes  $\alpha$ -decay to a  ${}^{222}_{86}\text{Rn}$  nucleus in its excited state. The kinetic energy of the emitted  $\alpha$  particle is found to be 4.44 MeV.  ${}^{222}_{86}\text{Rn}$  nucleus then goes to its ground state by  $\gamma$ -decay. The energy of the emitted  $\gamma$  photon is ..... keV.

[Given : atomic mass of  ${}^{226}_{88}\text{Ra} = 226.005$  u, atomic mass of  ${}^{222}_{86}\text{Rn} = 222.000$  u, atomic mass of  $\alpha$  particle = 4.000 u, 1 u = 931 MeV/c<sup>2</sup>, c is speed of the light]

**Ans. [135]**



Mass defect  $\Delta m = (226.005) - [222.000 + 4.000]$   
 $= 226.005 - 226.000 = 0.005$

Q – value =  $0.005 \times 931.5 = 4.6575$  MeV

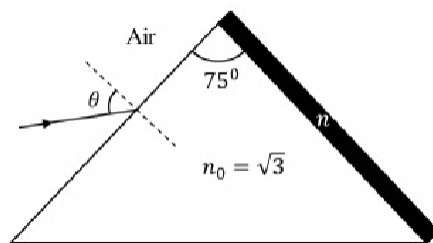
$\vec{P}_{\text{Rn}} = -\vec{P}_{\text{He}} \Rightarrow \text{KE} \propto \frac{1}{m}$

$\frac{(\text{KE})_{\text{Rn}}}{(\text{KE})_{\alpha}} = \frac{m_{\alpha}}{m_{\text{Rn}}} = \frac{4.000}{222.000} \Rightarrow (\text{KE})_{\text{Rn}} = \frac{4}{222} \times 4.44$  MeV

$(\text{KE})_{\text{Rn}} = 0.0799 \approx 0.08$  MeV

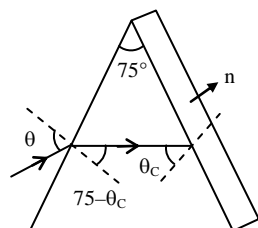
Energy of  $\gamma$  particle =  $4.65 - [4.44 + 0.08]$   
 $= 0.135$  MeV  
 $= 135$  KeV

**Q.3** A monochromatic light is incident from air on a refracting surface of a prism of angle  $75^\circ$  and refractive index  $n_0 = \sqrt{3}$ . The other refracting surface of the prism is coated by a thin film of material of refractive index n as shown in figure. The light suffers total internal reflection at the coated prism surface for an incidence angle of  $\theta \leq 60^\circ$ . The value of  $n^2$  is .....



**Ans. [1.5]**

**Sol.**



$$\sin\theta_c = \frac{n}{\sqrt{3}}$$

$$\sin\theta = \sqrt{3} \sin(75 - \theta_c)$$

$$\theta = 60^\circ, \frac{1}{2} = \sin(75 - \theta_c)$$

$$\sin 30^\circ = \sin(75 - \theta_c)$$

$$75 - \theta_c = 30 \Rightarrow \theta_c = 45^\circ$$

$$\frac{n}{\sqrt{3}} = \frac{1}{\sqrt{2}} \Rightarrow n = \frac{\sqrt{3}}{\sqrt{2}} \Rightarrow n^2 = \frac{3}{2}$$

$$n^2 = 1.50$$

**Q.4** An optical bench has 1.5 m long scale having four equal divisions in each cm. While measuring the focal length of a convex lens, the lens is kept at 75 cm mark of the scale and the object pin is kept at 45 cm mark. The image of the object pin on the other side of the lens overlaps with image pin that is kept at 135 cm mark. In this experiment, the percentage error in the measurement of the focal length of the lens is .....

**Ans.** [1.38 or 1.39]

**Sol.** Least Count =  $\frac{1\text{cm}}{4}$

$$\text{Object distance } u = u_{\text{lens}} - u_{\text{pin}} \Rightarrow du = du_1 + du_2$$

$$= 75 - 45 = 30 \text{ cm} \quad \left| \quad \begin{aligned} du &= \frac{1}{4} + \frac{1}{4} = \frac{1}{2} \\ dv &= \frac{1}{2} \end{aligned} \right.$$

$$v = (135 - 75)\text{cm} = 60 \text{ cm}$$

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{60} - \frac{1}{-30} = \frac{1}{f}$$

$$f = 20 \text{ cm}$$

$$\frac{dv}{v^2} + \frac{du}{u^2} = \frac{df}{f^2} \Rightarrow \frac{1/2}{(60)^2} + \frac{1/2}{(30)^2} = \frac{df}{(20)^2}$$

$$df = \frac{1}{2} \times 400 \left[ \frac{6}{(60)^2} + \frac{1}{(30)^2} \right]$$

$$= \frac{5}{36} \times 2$$

$$\frac{df}{f} \times 100 = \frac{10/36}{20} \times 100 = \frac{50}{36} = 1.388$$

**Q.5** A ball is thrown from ground at an angle  $\theta$  with horizontal and with an initial speed  $u_0$ . For the resulting projectile motion, the magnitude of average velocity of the ball up to the point when it hits the ground for the first time is  $V_1$ . After hitting the ground, the ball rebounds at the same angle  $\theta$  but with a reduced speed of  $u_0/\alpha$ . Its motion continues for a long time as shown in figure. If the magnitude of average velocity of the ball for entire duration of motion is  $0.8 V_1$ , the value of  $\alpha$  is .....



**Ans.** [4]

**Sol.**  $V_{avg} = \frac{\text{Total displacement}}{\text{Time taken}}$

$$2 \left[ \frac{u_0 \cos \theta u_0 \sin \theta}{g} \right] + 2 \left[ \frac{\frac{u_0 \cos \theta}{\alpha} \frac{u_0 \sin \theta}{\alpha}}{g} \right] + \dots$$

$$= \frac{\frac{2u_0 \sin \theta}{g} + \frac{2 \frac{u_0}{\alpha} \sin \theta}{g} + \dots}{\frac{u_0 \cos \theta \left[ 1 + \frac{1}{\alpha^2} + \frac{1}{\alpha^4} + \dots + \frac{1}{\alpha^{2m}} \right]}{1 + \frac{1}{\alpha} + \frac{1}{\alpha^2} + \dots + \frac{1}{\alpha^m}}}$$

$m \rightarrow \infty$

$$= V_1 \left[ \frac{1 + \frac{1}{\alpha^2} + \frac{1}{\alpha^4} + \dots}{1 + \frac{1}{\alpha} + \frac{1}{\alpha^2} + \dots} \right]$$

$$= V_1 \left[ \frac{\frac{1}{1 - \frac{1}{\alpha^2}}}{\frac{1}{1 - \frac{1}{\alpha}}} \right] = v_1 \left[ \frac{\alpha^2}{\alpha^2 - 1} \right] \left[ \frac{\alpha - 1}{\alpha} \right]$$

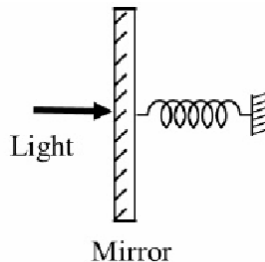
$$V_{avg} = V_1 \left[ \frac{\alpha^2}{\alpha^2 - 1} \frac{\alpha - 1}{\alpha} \right] = \frac{\alpha}{\alpha + 1} V_1$$

$$0.8V_1 = V_1 = \frac{\alpha}{\alpha + 1} \Rightarrow \alpha = 0.8\alpha + 0.8$$

$$0.2\alpha = 0.8$$

$$\alpha = 4$$

**Q.6** A perfectly reflecting mirror of mass  $M$  mounted on a spring constitutes a spring-mass system of angular frequency  $\Omega$  such that  $\frac{4\pi MW}{h} = 10^{24} \text{ m}^{-2}$  with  $h$  as Planck's constant.  $N$  photons of wavelength  $\lambda = 8\pi \times 10^{-6} \text{ m}$  strike the mirror simultaneously at normal incidence such that the mirror gets displaced by  $1\mu\text{m}$ . If the value of  $N$  is  $x \times 10^{12}$ , then the value of  $x$  is ..... [Consider the spring as massless]



**Ans.** [1]**Sol.**  $A = 1 \mu\text{m}$        $N = \text{No. of photons}$ 

$$Ma\omega = Mv_{\text{max}} = 2N \frac{h}{\lambda}$$

$$M(10^{-6})(\Omega) = 2N \frac{h}{8\pi \times 10^{-6}}$$

$$10^{-12} = \frac{Nh}{4\pi M\Omega}$$

$$N = \left( \frac{4\pi M\Omega}{h} \right) 10^{-12}$$

$$N = 10^{24} \times 10^{-12} = 10^{12}$$

### SECTION – 3 (Maximum Marks : 12)

- This section contains **Two (02)** List-Match sets.
- Each List-Match set has **TWO (02)** Multiple Choice Questions.
- Each List-Match set has two lists : **List-I** and **List-II**
- **List-I** has **Four** entries (I), (II), (III) and (IV) and List-II has Six entries (P), (Q), (R), (S), (T) and (U).
- **FOUR** options are given in each Multiple Choice Question based on **List-I** and **List-II** and **ONLY ONE** of these four options satisfies the condition asked in the Multiple Choice Question.
- For each question, marks will be awarded according to the following marking scheme :  
 Full Marks : +3 If **ONLY** the option corresponding to the correct matching is chosen.  
 Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered.)  
 Negative Marks : -1 In all other cases.

**Answer the following by appropriately matching the lists based on the information given in the paragraph.**

**Q.1** A musical instrument is made using four different metal strings, 1, 2, 3 and 4 with mass per unit length  $\mu$ ,  $2\mu$ ,  $3\mu$  and  $4\mu$  respectively. The instrument is played by vibrating the strings by varying the free length in between the range  $L_0$  and  $2L_0$ . It is found that in string-1 ( $\mu$ ) at free length  $L_0$  and tension  $T_0$  the fundamental mode frequency is  $f_0$ .

List-I gives the above four strings while List-II lists the magnitude of some quantity.

#### LIST-I

- (I) String-1 ( $\mu$ )  
 (II) String-2 ( $2\mu$ )  
 (III) String-3 ( $3\mu$ )  
 (IV) String-4 ( $4\mu$ )

#### LIST-II

- (P) 1  
 (Q)  $1/2$   
 (R)  $1/\sqrt{2}$   
 (S)  $1/\sqrt{3}$   
 (T)  $3/16$   
 (U)  $1/16$

If the tension in each string is  $T_0$ , the correct match for the highest fundamental frequency in  $f_0$  units will be,

- (1) I  $\rightarrow$  Q; II  $\rightarrow$  S, III  $\rightarrow$  R ; IV  $\rightarrow$  P  
 (2) I  $\rightarrow$  P; II  $\rightarrow$  R, III  $\rightarrow$  S ; IV  $\rightarrow$  Q  
 (3) I  $\rightarrow$  P; II  $\rightarrow$  Q, III  $\rightarrow$  T ; IV  $\rightarrow$  S  
 (4) I  $\rightarrow$  Q; II  $\rightarrow$  P, III  $\rightarrow$  R ; IV  $\rightarrow$  T

**Ans.** [2]

**Sol.**  $f_0 = \frac{1}{2L_0} \sqrt{\frac{T}{\mu}}$

String (1)  $f_0 = \frac{1}{2L_0} \sqrt{\frac{T_0}{\mu}} \rightarrow P$

String (2)  $f_2 = \frac{1}{2L_0} \sqrt{\frac{T_0}{2\mu}} = \frac{f_0}{\sqrt{2}} \rightarrow R$

String (3)  $f_3 = \frac{1}{2L_0} \sqrt{\frac{T}{3\mu}} = \frac{f_0}{\sqrt{3}} \rightarrow S$

String (4)  $f_4 = \frac{1}{2L_0} \sqrt{\frac{T}{4\mu}} = \frac{f_0}{2} \rightarrow Q$

I  $\rightarrow$  P, II  $\rightarrow$  R, III  $\rightarrow$  S, IV  $\rightarrow$  Q

**Q.2** Answer the following by appropriately matching the lists based on the information given in the paragraph.

A musical instrument is made using four different metal strings, 1, 2, 3 and 4 with mass per unit length  $\mu$ ,  $2\mu$ ,  $3\mu$  and  $4\mu$  respectively. The instrument is played by vibrating the strings by varying the free length in between the range  $L_0$  and  $2L_0$ . It is found that in string-1 ( $\mu$ ) at free length  $L_0$  and tension  $T_0$  the fundamental mode frequency is  $f_0$ .

List-I gives the above four strings while list-II lists the magnitude of some quantity.

**LIST-I**

**LIST-II**

- |                           |                  |
|---------------------------|------------------|
| (I) String-1 ( $\mu$ )    | (P) 1            |
| (II) String-2 ( $2\mu$ )  | (Q) $1/2$        |
| (III) String-3 ( $3\mu$ ) | (R) $1/\sqrt{2}$ |
| (IV) String-4 ( $4\mu$ )  | (S) $1/\sqrt{3}$ |
|                           | (T) $3/16$       |
|                           | (U) $1/16$       |

The length of strings 1, 2, 3 and 4 are kept fixed at  $L_0$ ,  $\frac{3L_0}{2}$ ,  $\frac{5L_0}{4}$  and  $\frac{7L_0}{4}$ , respectively. String 1, 2, 3 and 4 are vibrated at their 1<sup>st</sup>, 3<sup>rd</sup>, 5<sup>th</sup>, and 14<sup>th</sup> harmonics, respectively such that all the strings have same frequency. The correct match for the tension in the four strings in the units of  $T_0$  will be.

- (1) I  $\rightarrow$  P; II  $\rightarrow$  Q, III  $\rightarrow$  T; IV  $\rightarrow$  U  
 (2) I  $\rightarrow$  P; II  $\rightarrow$  Q, III  $\rightarrow$  R; IV  $\rightarrow$  T  
 (3) I  $\rightarrow$  T; II  $\rightarrow$  Q, III  $\rightarrow$  R; IV  $\rightarrow$  U  
 (4) I  $\rightarrow$  P; II  $\rightarrow$  R, III  $\rightarrow$  T; IV  $\rightarrow$  U

**Ans.** [1]

**Sol.**  $f_0 = \frac{1}{2L_0} \sqrt{\frac{T}{\mu}}$

$$f_0 = \frac{3}{2 \left[ \frac{3L_0}{2} \right]} \sqrt{\frac{T_2}{2\mu}} = \frac{1}{\sqrt{2}} \frac{1}{L_0} \sqrt{\frac{T_2}{\mu}} = \frac{1}{2L_0} \sqrt{\frac{T_0}{\mu}}$$

$$T_2 = \frac{T_0}{2}$$

$$f_0 = \frac{5}{2 \left[ \frac{5L_0}{4} \right]} \sqrt{\frac{T_3}{3\mu}} = \frac{2}{\sqrt{3}} \frac{1}{L_0} \sqrt{\frac{T_3}{\mu}} = \frac{1}{2L_0} \sqrt{\frac{T_0}{\mu}}$$

$$T_3 = \frac{3}{16} T_0$$

$$f_0 = \frac{14}{2 \left[ \frac{7L_0}{4} \right]} \sqrt{\frac{T_4}{4\mu}} = \frac{1}{2L_0} \sqrt{\frac{T_0}{\mu}}$$

$$\sqrt{4T_4} = \frac{\sqrt{T_0}}{2} \Rightarrow T_4 = \frac{T_0}{16}$$

I → P, II → Q, III → T, IV → U

**Q.3 Answer the following by appropriately matching the lists based on the information given in the paragraph.**

In a thermodynamic process on an ideal monoatomic gas, the infinitesimal heat absorbed by the gas is given by  $T\Delta X$ , where  $T$  is temperature of the system and  $\Delta X$  is the infinitesimal change in a thermodynamic quantity  $X$  of the system. For a mole of monoatomic ideal gas  $X = \frac{3}{2} R \ln \left( \frac{T}{T_A} \right) + R \ln \left( \frac{V}{V_A} \right)$ . Here,  $R$  is gas constant,  $V$  is volume of gas,  $T_A$  and  $V_A$  are constants.

The List-I below gives some quantities involved in a process and List-II gives some possible values of these quantities.

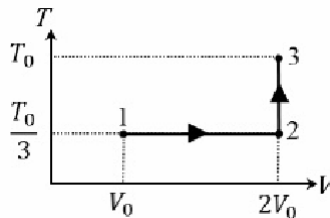
**LIST-I**

- (I) Work done by the system in process 1 → 2 → 3
- (II) Change in internal energy in process 1 → 2 → 3
- (III) Heat absorbed by the system in process 1 → 2 → 3
- (IV) Heat absorbed by the system in process 1 → 2

**LIST-II**

- (P)  $\frac{1}{3} RT_0 \ln 2$
- (Q)  $\frac{1}{3} RT_0$
- (R)  $RT_0$
- (S)  $\frac{4}{3} RT_0$
- (T)  $\frac{1}{3} RT_0 (3 + \ln 2)$
- (U)  $\frac{5}{6} RT_0$

If the process on one mole of monoatomic ideal gas is as shown in the  $TV$ -diagram with  $P_0V_0 = \frac{1}{3} RT_0$ , the correct match is.



- (1) I → P; II → R, III → T; IV → P
- (2) I → P; II → T, III → Q; IV → T
- (3) I → P; II → R, III → T; IV → S
- (4) I → S; II → T, III → Q; IV → U

**Ans.** [1]

**Sol.** (1)  $W_{123} = nR \frac{T_0}{3} \ln 2 + 0 = \frac{RT_0}{3} \ln 2 \quad 1 \rightarrow P$

(2)  $dU_{123} = dU_{12} + dU_{23}$   
 $= 0 + nC_v dT$   
 $= 0 + 1 \times \frac{3R}{2} \left( T_0 - \frac{T_0}{3} \right) = RT_0$

2 → R

(3)  $dQ_{123} = dQ_{12} + dQ_{23}$   
 $= \frac{RT_0}{3} \ln 2 + nC_v dT$   
 $= \frac{RT_0}{3} \ln 2 + RT_0$   
 $= \frac{RT_0}{3} (\ln 2 + 3)$

3 → T

(4)  $dQ_{12} = dW$   
 $= \frac{RT_0}{3} \ln 2$

4 → P

**Q.4** Answer the following by appropriately matching the lists based on the information given in the paragraph.

In a thermodynamic process on an ideal monoatomic gas, the infinitesimal heat absorbed by the gas is given by  $T\Delta X$ , where  $T$  is temperature of the system and  $\Delta X$  is the infinitesimal change in a thermodynamic

quantity  $X$  of the system. For a mole of monoatomic ideal gas  $X = \frac{3}{2}R \ln\left(\frac{T}{T_A}\right) + R \ln\left(\frac{V}{V_A}\right)$ . Here,  $R$  is gas

constant,  $V$  is volume of gas,  $T_A$  and  $V_A$  are constants.

The List-I below gives some quantities involved in a process and List-II gives some possible values of these quantities.

**LIST-I**

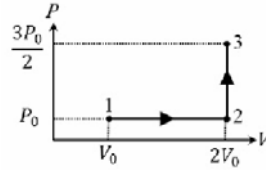
- |   |  |
|---|--|
| <p>(I) Work done by the system in process 1 → 2 → 3</p> <p>(II) Change in internal energy in process 1 → 2 → 3</p> <p>(III) Heat absorbed by the system in process 1 → 2 → 3</p> <p>(IV) Heat absorbed by the system in process 1 → 2</p> | <p>(P) <math>\frac{1}{3}RT_0 \ln 2</math></p> <p>(Q) <math>\frac{1}{3}RT_0</math></p> <p>(R) <math>RT_0</math></p> <p>(S) <math>\frac{4}{3}RT_0</math></p> |
|---|--|

**LIST-II**

- (T)  $\frac{1}{3}RT_0 (3 + \ln 2)$
- (U)  $\frac{5}{6}RT_0$



If the process carried out on one mole of monatomic ideal gas is as shown in the PV-diagram with  $P_0V_0 = \frac{1}{3}RT_0$ , the correct match is.



- (1) I  $\rightarrow$  Q; II  $\rightarrow$  R, III  $\rightarrow$  S ; IV  $\rightarrow$  U  
 (2) I  $\rightarrow$  Q; II  $\rightarrow$  S, III  $\rightarrow$  R ; IV  $\rightarrow$  U  
 (3) I  $\rightarrow$  S; II  $\rightarrow$  R, III  $\rightarrow$  Q ; IV  $\rightarrow$  T  
 (4) I  $\rightarrow$  Q; II  $\rightarrow$  R, III  $\rightarrow$  P ; IV  $\rightarrow$  U

**Ans.** [1]

**Sol.**

$$(1) W_{123} = W_{12} + W_{23}$$

$$= PdV + 0$$

$$= P_0 V_0 = \frac{RT_0}{3}$$

$$(2) dU_{123} = dU_{12} + dU_{23}$$

$$= nC_v (T_3 - T_1)$$

$$= n \frac{3}{2} R(T_3 - T_1)$$

$$= \frac{3}{2} (RT_3 - RT_1)$$

$$= \frac{3}{2} (3P_0V_0 - P_0V_0) = 3 P_0V_0 = RT_0$$

2  $\rightarrow$  R

$$(3) dQ_{123} = dQ_{12} + dQ_{23}$$

$$= nC_p (T_2 - T_1) + nC_v (T_3 - T_2)$$

$$= \frac{5}{2} (RT_2 - RT_1) + \frac{3}{2} (RT_3 - RT_2)$$

$$= \frac{5}{2} (2P_0V_0 - P_0V_0) + \frac{3}{2} (3P_0V_0 - 2P_0V_0)$$

$$= \frac{5}{2} P_0V_0 + \frac{3}{2} P_0V_0 = 4P_0V_0 = \frac{4}{3} P_0V_0$$

3  $\rightarrow$  S

$$(4) dQ_{12} = 5P_0V_0 = \frac{5}{6} RT_0$$

4  $\rightarrow$  U



# JEE Advanced Exam 2019 (Paper & Solution)

Date : 27 / 05 / 2019

## PAPER-2

### PART-I (CHEMISTRY)

#### SECTION – 1 (Maximum Marks : 32)

- This section contains **EIGHT (08)** questions
- Each question has **FOUR** options. **ONE OR MORE THAN ONE** of these four option(s) is (are) correct option(s).
- For each question, choose(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme :
  - Full Marks : +4 If only (all) the correct option(s) is (are) chosen.
  - Partial Marks : +3 If all the four options are correct but **ONLY** three options are chosen.
  - Partial Marks : +2 If three or more options are correct but **ONLY** two options are chosen, both of which are correct options.
  - Partial Marks : +1 If two or more options are correct but **ONLY** one option is chosen and it is a correct option.
  - Zero Marks : 0 If none of the option is chosen (i.e. the question is unanswered).
  - Negative Marks : -1 In all other cases.
- For example, in a question, if (A), (B) and (D) are the **ONLY** three options corresponding to correct answers, then
  - choosing **ONLY** (A), (B) and (D) will get +4 marks;
  - choosing **ONLY** (A) and (B) will get +2 marks;
  - choosing **ONLY** (A) and (D) will get +2 marks;
  - choosing **ONLY** (B) and (D) will get +2 marks;
  - choosing **ONLY** (A) will get +1 marks;
  - choosing **ONLY** (B) will get +1 marks;
  - choosing **ONLY** (D) will get +1 marks;
  - choosing no option (i.e. the question is unanswered) will get 0 marks; and
  - choosing any other combination of options will get -1 mark

**Q.1** The cyanide process of gold extraction involves leaching out gold from its ore with  $\text{CN}^-$  in the presence of Q in water to form R. Subsequently, R is treated with T to obtain Au and Z. Choose the correct option(s)

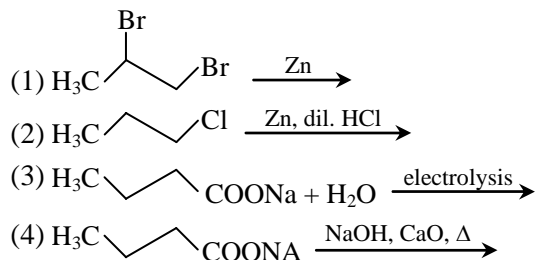
(1) Z is  $[\text{Zn}(\text{CN})_4]^{2-}$       (2) T is Zn      (3) Q is  $\text{O}_2$       (4) R is  $[\text{Au}(\text{CN})_4]^-$

**Ans.** [1,2,3]

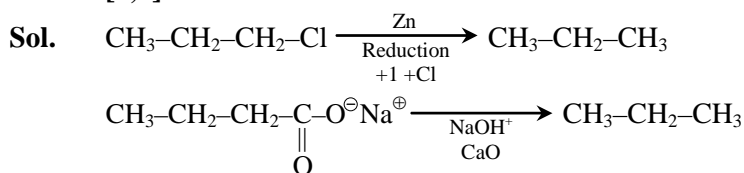
**Sol.** 
$$\text{Au} + \text{CN}^-(\text{aq.}) + \text{O}_2 \xrightarrow[\text{Q}]{\text{H}_2\text{O}} [\text{Au}(\text{CN})_2]^- (\text{aq.}) + \text{OH}^-$$

$$[\text{Au}(\text{CN})_2]^- (\text{R}) (\text{aq.}) + \text{Zn}(\text{s}) \rightarrow [\text{Zn}(\text{CN})_4]^{2-} (\text{Z}) (\text{T}) + 2\text{Au}(\text{s})$$

**Q.2** Which of the following reactions produce(s) propane as a major product ?



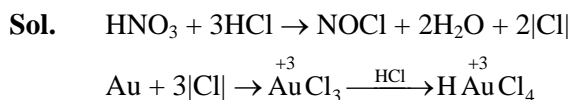
**Ans.** [2,4]



**Q.3** With reference to aqua regia, choose the correct option(s)

- (1) Reaction of gold with aqua regia produces  $\text{NO}_2$  in the absence of air
- (2) Reaction of gold with aqua produces an anion having Au in +3 oxidation state
- (3) The yellow colour of aqua regia is due to the presence of  $\text{NOCl}$  and  $\text{Cl}_2$
- (4) Aqua regia is prepared by mixing conc.  $\text{HCl}$  and conc.  $\text{HNO}_3$  in 3:1 (v/v) ratio

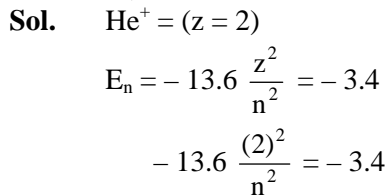
**Ans.** [2,3,4]



**Q.4** The ground state energy of hydrogen atom is  $-13.6$  eV. Consider an electronic state  $\psi$  of  $\text{He}^+$  whose energy, azimuthal quantum number and magnetic quantum number are  $-3.4$  eV, 2 and 0, respectively. Which of the following statement(s) is(are) true for the state  $\psi$  ?

- (1) The nuclear charge experienced by the electron in this state is less than  $2e$ , where  $e$  is the magnitude of the electronic charge
- (2) It has 2 angular nodes
- (3) It has 3 radial nodes
- (4) It is a 4d state

**Ans.** [2,4]



$$n^2 = 4^2$$

$$n = 4$$

$$x = 4, \ell = 2, m = 0 \quad (4dz^2)$$

1. Nuclear charge =  $+2e$

2. Angular Nodes =  $\ell = 2$

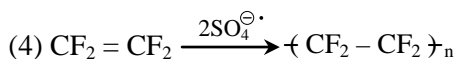
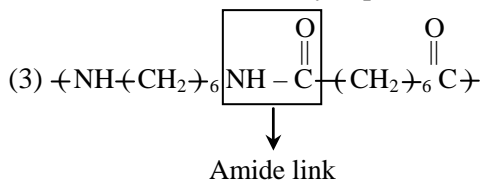
3. Radial Nodes =  $n - \ell - 1 = 4 - 2 - 1 = 1$

4. 4d

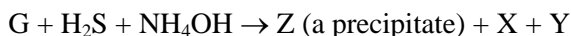
- Q.5** Choose the correct option(s) from the following
- (1) Cellulose has only  $\alpha$ -D-glucose units that are joined by glycosidic linkages
  - (2) Natural rubber is polyisoprene containing trans alkene units
  - (3) Nylon-6 has amide linkages
  - (4) Teflon is prepared by heating tetrafluoroethene in presence of a persulphate catalyst at high

**Ans.** [3,4]

- Sol.** (1) Cellulose has  $\beta$ -D-glucose  
(2) Natural Rubber  $\rightarrow$  Polyisoprene (cis form)



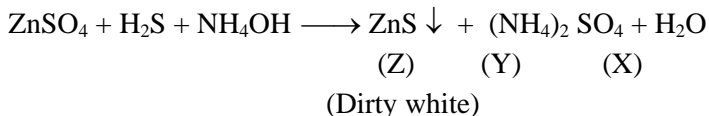
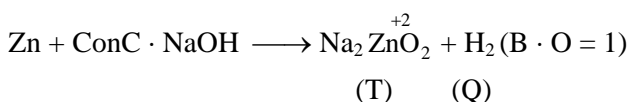
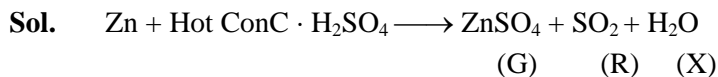
- Q.6** Consider the following reactions (unbalanced)



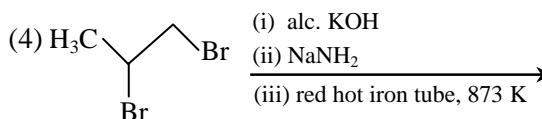
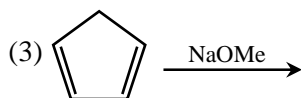
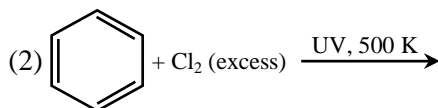
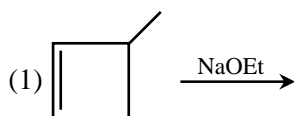
Choose the correct option(s)

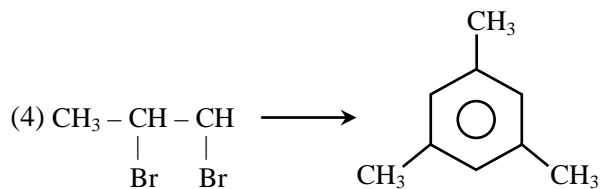
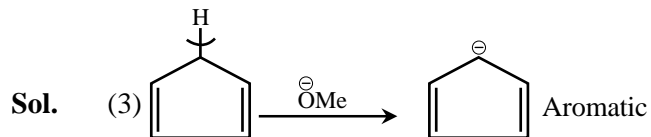
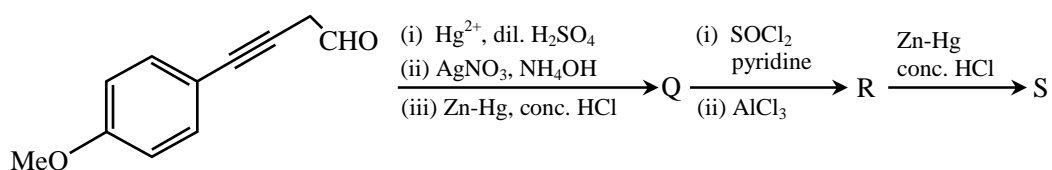
- (1) Bond order of Q is 1 in its ground state
- (2) Z is dirty white is colour
- (3) R is a V-shaped molecule
- (4) The oxidation state of Zn in T is +1

**Ans.** [1,2,3]

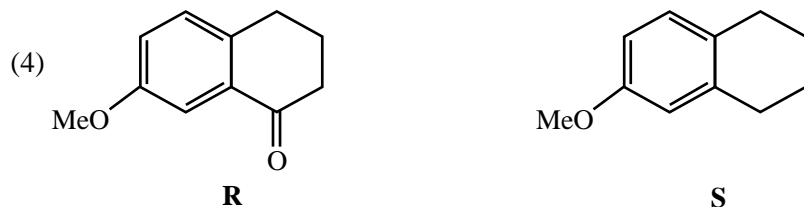
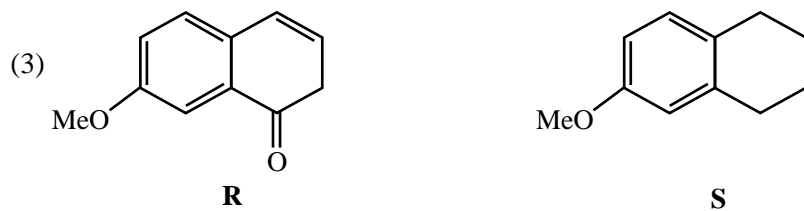
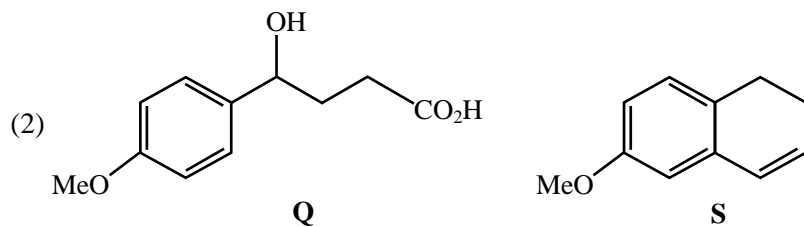
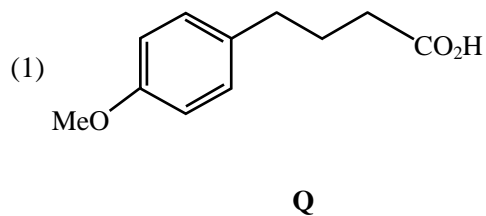


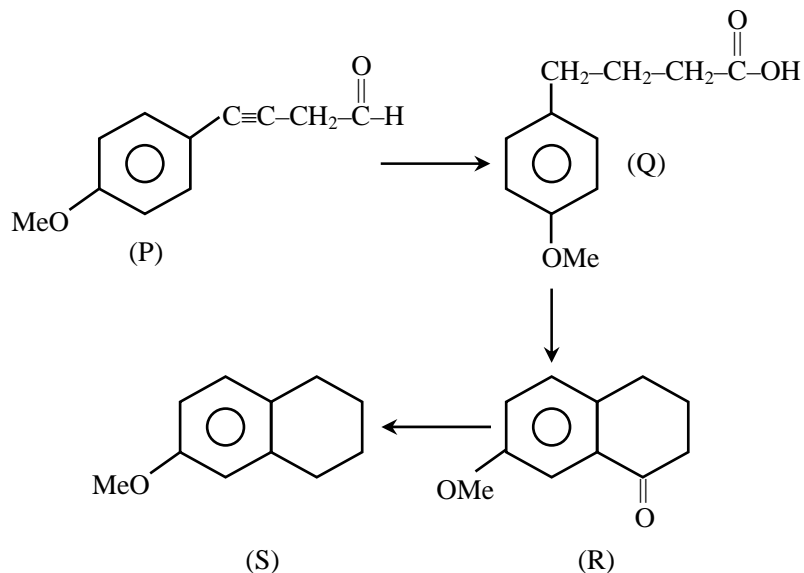
- Q.7** Choose the correct option(s) that give(s) an aromatic compound as the major product



**Ans. [3,4]**

**Q.8** Choose the correct option(s) for the following reaction sequence


Consider Q, R and S as major products



**Ans. [1,4]**
**Sol.**


### SECTION – 2 (Maximum Marks : 18)

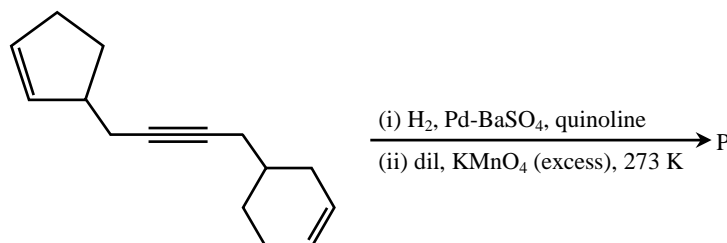
- This section contains **SIX (06)** questions. The answer to each question is a **NUMERICAL VALUE**.
- For each question, enter the correct numerical value using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer. If the numerical value has more than two decimal places, **truncate/round-off** the value to TWO decimal places.
- Answer to each question will be evaluated according to the following marking scheme.  
 Full Marks : +3 If **ONLY** the correct numerical value is entered as answer.  
 Zero Marks : 0 In all other cases.

**Q.1** The amount of water produced (in g) in the oxidation of 1 mole of rhombic sulphur by conc.  $\text{HNO}_3$  to a compound with the highest oxidation state of sulphur is \_\_\_\_\_  
 (Given data : Molar mass of water =  $18 \text{ g mol}^{-1}$ )

**Ans. [288 gm]**

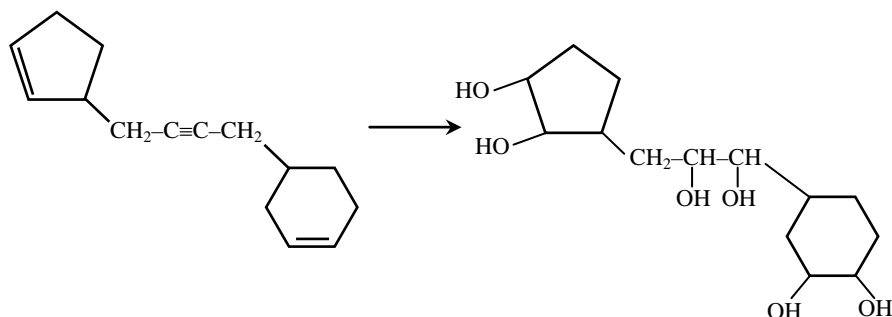
**Sol.**  $\text{S}_8 + 48 \text{HNO}_3 \longrightarrow 8 \text{H}_2\text{SO}_4 + 48 \text{NO}_2 + 16 \text{H}_2\text{O}$   
 Mole of water = 16  
 Amount of water produced =  $16 \times 18 = 288 \text{ gm}$

**Q.2** Total number of hydroxyl groups present in a molecule of the major product P is \_\_\_\_\_



Ans. [6]

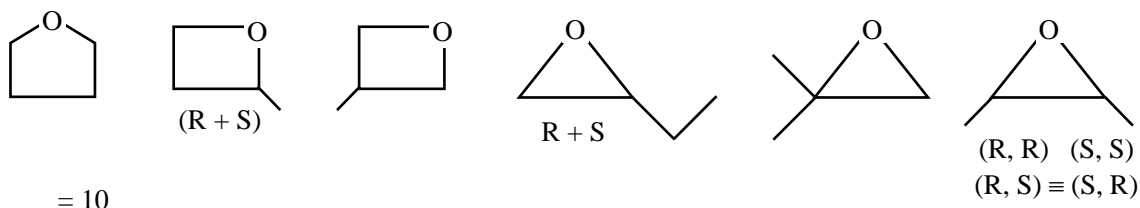
Sol.



**Q.3** Total number of isomers, considering both structural and stereoisomers, of cyclic ethers with the molecular formula  $C_4H_8O$  is \_\_\_\_\_

Ans. [10]

Sol.



**Q.4** The mole fraction of urea in an aqueous urea solution containing 900 g of water is 0.05. If the density of the solution is  $1.2 \text{ g cm}^{-3}$ , the molarity of urea solution is \_\_\_\_\_

(Given data : Molar masses of urea and water are  $60 \text{ g mol}^{-1}$  and  $18 \text{ g mol}^{-1}$ , respectively)

Ans. [2.98]

Sol. mole fraction urea

$$X = \frac{n_A}{n_A + n_B} = 0.05$$

$$\frac{n_A}{n_A + \frac{900}{18}} = 0.05$$

$$\frac{n_A}{n_A + 50} = 0.05$$

$$n_A = 0.05 n_A + 2.5$$

$$0.95 n_A = 2.5$$

$$n_A = \frac{2.5}{0.95} = \frac{250}{95}$$

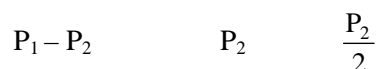
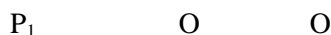
$$\text{In Urea} = \frac{250}{95} \times 60 = 157.89$$

$$M = \frac{\text{Moles (Solute)}}{V_L (\text{Solution})} = \frac{\frac{250}{95}}{\frac{(900 + 157.89)}{1.2 \times 1000}} = \frac{250 \times 1.2 \times 1000}{95 \times 1057.89} = 2.985 = 2.98M$$

**Q.5** The decomposition reaction  $2\text{N}_2\text{O}_5(\text{g}) \xrightarrow{\Delta} 2\text{N}_2\text{O}_4(\text{g}) + \text{O}_2(\text{g})$  is started in a closed cylinder under isothermal isochoric condition at an initial pressure of 1 atm. After  $Y \times 10^3$  s, the pressure inside the cylinder is found to be 1.45 atm. If the rate constant of the reaction is  $5 \times 10^{-4} \text{ s}^{-1}$ , assuming ideal gas behaviour, the value of Y is \_\_\_\_\_

**Ans.** [2.30]

**Sol.**  $2\text{N}_2\text{O}_5(\text{g}) \rightleftharpoons 2\text{N}_2\text{O}_4(\text{g}) + \text{O}_2$



$$P_1 = 1$$

$$P_1 + \frac{P_2}{2} = 1.45$$

$$\frac{P_2}{2} = 1.45 - 1 = 0.45$$

$$P_2 = 0.9$$

$$k = 5 \times 10^{-4} \text{ sec}^{-1}$$

It is I order reaction.

$$k_1 = \frac{2.303}{2 \times t} \log \frac{P_0}{P_t} \qquad P_t = P_1 - P_2 = 1 - 0.45 = 0.55$$

$$5 \times 10^{-4} = \frac{2.303}{2 \times t} \log \frac{1}{0.1}$$

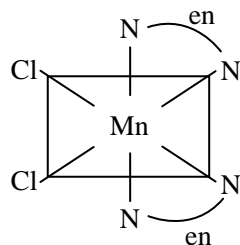
$$t = \frac{2.303}{2 \times 5 \times 10^{-4}} \left[ \log \frac{1.0}{0.1} \right] = \frac{2.303}{10^{-3}} \times 1 = 2.303 \times 10^3 \text{ sec}$$

Hence on comparing  $y = 2.30$  nearly rounding up to two decimal place

**Q.6** Total number of cis N–Mn–Cl bond angles (that is, Mn–N and Mn–Cl bonds in cis positions) present in a molecule of cis-[Mn(en)<sub>2</sub>Cl<sub>2</sub>] complex is \_\_\_\_\_ (en = NH<sub>2</sub>CH<sub>2</sub>CH<sub>2</sub>NH<sub>2</sub>)

**Ans.** [6]

**Sol.** [Mn(en)<sub>2</sub>Cl<sub>2</sub>]





**SECTION – 3 (Maximum Marks : 12)**

- This section contains **Two (02)** List-Match sets.
- Each List-Match set has **TWO (02)** Multiple Choice Questions.
- Each List-Match set has two lists : **List-I** and **List-II**
- **List-I** has **Four** entries (I), (II), (III) and (IV) and List-II has Six entries (P), (Q), (R), (S), (T) and (U).
- **FOUR** options are given in each Multiple Choice Question based on **List-I** and **List-II** and **ONLY ONE** of these four options satisfies the condition asked in the Multiple Choice Question.
- For each question, marks will be awarded according to the following marking scheme :  
Full Marks : +3 If **ONLY** the option corresponding to the correct matching is chosen.  
Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered.)  
Negative Marks : -1 In all other cases.

Answer the following by appropriately matching the lists based on the information given in the paragraph.

- Q.1** Consider the Bohr's model of a one-electron atom where the electron moves around the nucleus. In the following. List-I contains some quantities for the  $n^{\text{th}}$  orbit of the atom and List-II contains options showing how they depend on  $n$ .

**LIST-I**

- (I) Radius of the  $n^{\text{th}}$  orbit  
(II) Angular momentum of the electron in the  $n^{\text{th}}$  orbit  
(III) Kinetic energy of the electron in the  $n^{\text{th}}$  orbit  
(IV) Potential energy of the electron in the  $n^{\text{th}}$  orbit

**LIST-II**

- (P)  $\propto n^{-2}$   
(Q)  $\propto n^{-1}$   
(R)  $\propto n^0$   
(S)  $\propto n^1$   
(T)  $\propto n^2$   
(U)  $\propto n^{1/2}$

(1) (III), (P)

(2) (III), (S)

(3) (IV), (U)

(4) (IV), (Q)

**Ans.** [1]

**Sol.** (I)  $r = 0.529 \frac{n^2}{z}$

$$r \propto n^2$$

I  $\rightarrow$  T

(II) Angular momentum =  $mvr = \left(\frac{z}{n}\right)\left(\frac{n^2}{z}\right) = \frac{nh}{2\pi}$

$$An \propto n^1$$

II  $\rightarrow$  S

(III)  $k \cdot E = -T \cdot E = +13.6 \frac{z^2}{n^2}$

$$k \cdot E \propto n^{-2}$$

(IV)  $P \cdot E = \frac{T \cdot E}{2} = \frac{-13.6 z^2}{2 n^2}$

$$P \cdot E \propto n^{-2}$$

IV  $\rightarrow$  P

**Q.2** Consider the Bohr's model of a one-electron atom where the electron moves around the nucleus. In the following. List-I contains some quantities for the  $n^{\text{th}}$  orbit of the atom and List-II contains options showing how they depend on  $n$ .

LIST-I	LIST-II
(I) Radius of the $n^{\text{th}}$ orbit	(P) $\propto n^{-2}$
(II) Angular momentum of the electron in the $n^{\text{th}}$ orbit	(Q) $\propto n^{-1}$
(III) Kinetic energy of the electron in the $n^{\text{th}}$ orbit	(R) $\propto n^0$
(IV) Potential energy of the electron in the $n^{\text{th}}$ orbit	(S) $\propto n^1$
	(T) $\propto n^2$
	(U) $\propto n^{1/2}$

(1) (I), (P)

(2) (I), (T)

(3) (II), (R)

(4) (II), (Q)

**Ans.** [2]

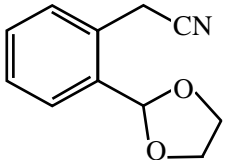
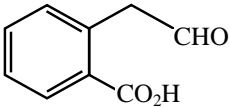
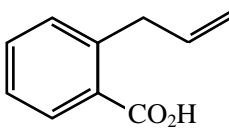
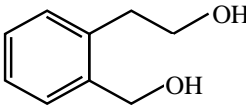
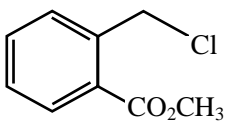
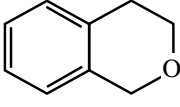
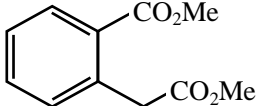
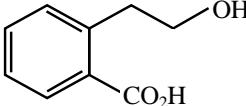
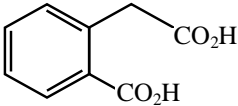
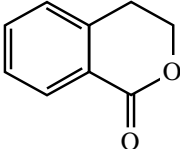
**Sol.**  $r \propto n^2$

$Am \propto n^1$

$K.E \propto n^{-2}$

$P.E \propto n^{-2}$

**Q.3** List-I includes starting materials and reagents of selected chemical reactions. List-II gives structures of compounds that may be formed as intermediate products and/or final products from the reactions of List-I.

LIST-I	LIST-II
<p>(I)  <math>\xrightarrow[\text{(iv) conc. H}_2\text{SO}_4]{\text{(i) DIBAL-H, (ii) dil. HCl, (iii) NaBH}_4}</math></p>	<p>(P) </p>
<p>(II)  <math>\xrightarrow[\text{(iv) conc. H}_2\text{SO}_4]{\text{(i) O}_3, \text{(ii) Zn, H}_2\text{O}, \text{(iii) NaBH}_4}</math></p>	<p>(Q) </p>
<p>(III)  <math>\xrightarrow[\text{(iv) conc. H}_2\text{SO}_4]{\text{(i) KCN, (ii) H}_3\text{O}^+, \Delta, \text{(iii) LiAlH}_4}</math></p>	<p>(R) </p>
<p>(IV)  <math>\xrightarrow[\text{(ii) conc. H}_2\text{SO}_4]{\text{(i) LiAlH}_4}</math></p>	<p>(S) </p> <p>(T) </p> <p>(U) </p>

Which of the following options has correct combination considering List-I and List-II ?

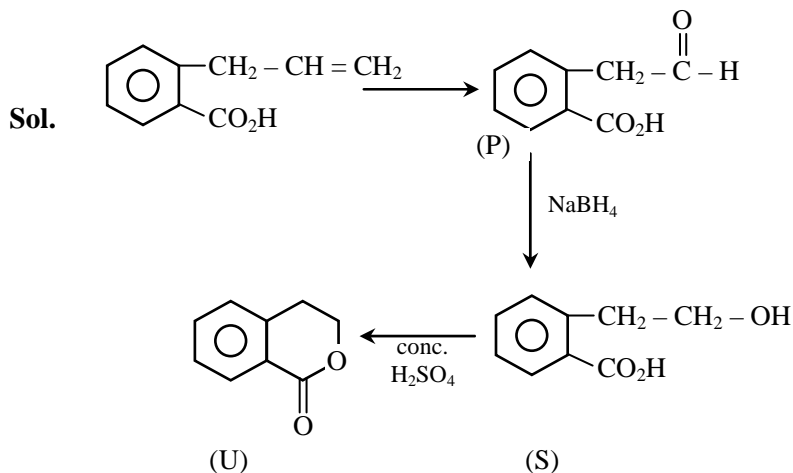
(1) (I), (Q), (T), (U)

(2) (II), (P), (S), (U)

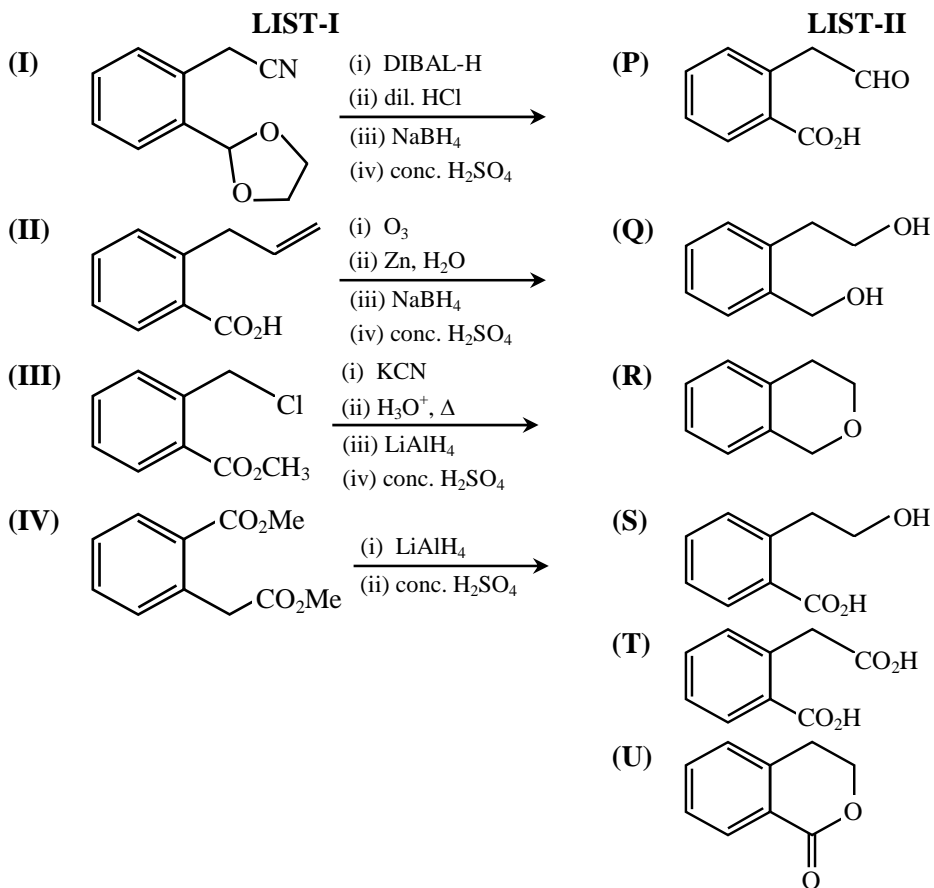
(3) (II), (P), (S), (U)

(4) (I), (S), (Q), (R)

**Ans.** [2]



**Q.4** List-I includes starting materials and reagents of selected chemical reactions. List-II gives structures of compounds that may be formed as intermediate products and/or final products from the reactions of List-I.

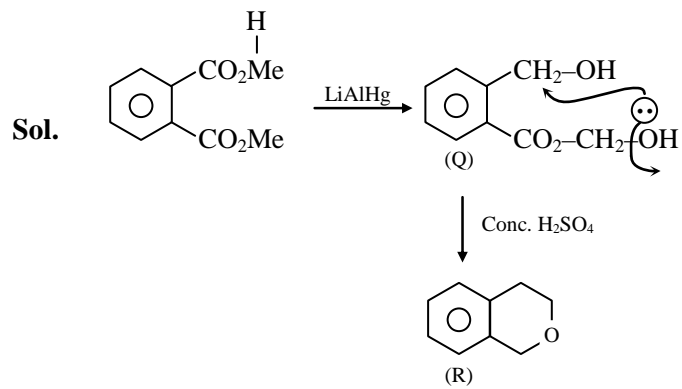


(1) (IV), (Q), (R)

(2) (III), (S), (R)

(3) (IV), (Q), (U)

(4) (III), (T), (U)

**Ans.** [1]



# JEE Advanced Exam 2019 (Paper & Solution)

Date : 27 / 05 / 2019

## PAPER-2

### PART-I (MATHS)

#### SECTION – 1 (Maximum Marks : 32)

- This section contains **EIGHT (08)** questions
- Each question has **FOUR** options. **ONE OR MORE THAN ONE** of these four option(s) is (are) correct Answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme :

Full Marks	: +4	If only (all) the correct option(s) is (are) chosen.
Partial Marks	: +3	If all the four options are correct but <b>ONLY</b> three options are chosen.
Partial Marks	: +2	If three or more options are correct but <b>ONLY</b> two options are chosen, both of which are correct options.
Partial Marks	: +1	If two or more options are correct but <b>ONLY</b> one option is chosen and it is a correct.
Zero Marks	: 0	If none of the option is chosen (i.e. the question is unanswered).
Negative Marks	: -1	In all other cases.
- For example, in a question, if (A), (B) and (D) are the **ONLY** three options corresponding to correct answers, then
  - choosing **ONLY** (A), (B) and (D) will get +4 marks;
  - choosing **ONLY** (A) and (B) will get +2 marks;
  - choosing **ONLY** (A) and (D) will get +2 marks;
  - choosing **ONLY** (B) and (D) will get +2 marks;
  - choosing **ONLY** (A) will get +1 marks;
  - choosing **ONLY** (B) will get +1 marks;
  - choosing **ONLY** (D) will get +1 marks;
  - choosing no option (i.e. the question is unanswered) will get 0 marks; and
  - choosing any other combination of options will get -1 mark

**Q.1** For  $a \in \mathbb{R}$ ,  $|a| > 1$ , let

$$\lim_{n \rightarrow \infty} \left( \frac{1 + \sqrt[3]{2} + \dots + \sqrt[3]{n}}{n^{7/3} \left( \frac{1}{(an+1)^2} + \frac{1}{(an+2)^2} + \dots + \frac{1}{(an+n)^2} \right)} \right) = 54.$$

Then the possible value(s) of  $a$  is/are

- (1) 8                                      (2) -9                                      (3) -6                                      (4) 7

**Ans.** [1,2]

**Sol.**

$$\lim_{n \rightarrow \infty} \left( \frac{\left( \left( \frac{1}{n} \right)^{1/3} + \dots + \left( \frac{n}{n} \right)^{1/3} \right)}{\left( \frac{1}{\left( a + \frac{1}{n} \right)^2} + \frac{1}{\left( a + \frac{2}{n} \right)^2} + \dots + \frac{1}{\left( a + \frac{n}{n} \right)^2} \right)} \right)$$

$$\Rightarrow \frac{\frac{1}{n} \sum_{r=1}^n \left( \frac{r}{n} \right)^{1/3}}{\frac{1}{n} \sum_{r=1}^n \frac{1}{\left( a + \frac{r}{n} \right)^2}} \Rightarrow \frac{\int_0^1 (x)^{1/3} dx}{\int_0^1 \frac{1}{(a+x)^2} dx} \Rightarrow \frac{\left( \frac{x^{4/3}}{4/3} \right)_0^1}{\left( -\frac{1}{(a+x)} \right)_0^1} \Rightarrow \frac{\frac{3}{4}}{\left( \frac{1}{a} - \frac{1}{a+1} \right)} = 54$$

$$\Rightarrow \frac{1}{72} = \frac{1}{a(a+1)}$$

$$a^2 + a = 72$$

$$(a+9)(a-8) = 0$$

$$a = 8 \text{ \& } a = -9$$

**Q.2** Let  $f(x) = \frac{\sin \pi x}{x^2}$ ,  $x > 0$ .

Let  $x_1 < x_2 < x_3 < \dots < x_n < \dots$  be all the points of local maximum of  $f$  and

$y_1 < y_2 < y_3 < \dots < y_n < \dots$  be all the points of local minimum of  $f$ .

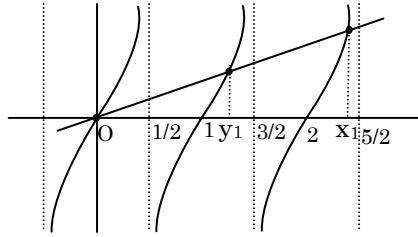
The which of the following options is/are correct?

- (1)  $x_n \in \left( 2n, 2n + \frac{1}{2} \right)$  for every  $n$                                       (2)  $x_1 < y_1$
- (3)  $|x_n - y_n| > 1$  for every  $n$                                       (4)  $x_{n+1} - x_n > 2$  for every  $n$

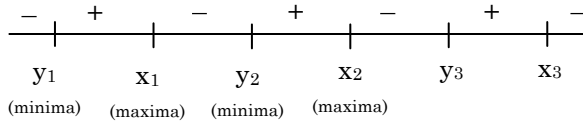
**Ans.** [1,3,4]

**Sol.**  $y = \frac{\sin \pi x}{x^2}$                                        $x > 0$

$$f'(x) = \frac{2x \cos \pi x \left( \frac{\pi x}{2} - \tan \pi x \right)}{x^4}$$



$f'(x) \rightarrow$



**Q.3** Let  $P_1 = I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ ,  $P_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ ,  $P_3 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$P_4 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}, P_5 = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, P_6 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$\text{and } X = \sum_{k=1}^6 P_k \begin{bmatrix} 2 & 1 & 3 \\ 1 & 0 & 2 \\ 3 & 2 & 1 \end{bmatrix} P_k^T$$

where  $P_k^T$  denotes the transpose of the matrix  $P_k$ . Then which of the following options is/are correct?

- (1) X is a symmetric matrix
- (2) If  $X \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \alpha \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ , then  $\alpha = 30$
- (3) The sum of diagonal entries of X is 18
- (4)  $X - 30I$  is an invertible matrix

**Ans.** [1,2,3]

**Sol.**  $\because P_1 = P_1^T = P_1^{-1}$

&  $P_2 = P_2^T = P_2^{-1}$

$\vdots$

$P_6 = P_6^T = P_6^{-1}$

also  $A^T = A$  where  $A = \begin{bmatrix} 2 & 1 & 3 \\ 1 & 0 & 2 \\ 3 & 2 & 1 \end{bmatrix}$

$$\because X = \sum_{k=1}^6 (P_k A P_k^T)$$

$$X = (P_1 A P_1^T + \dots + P_6 A P_6^T)$$

Now  $X^T = P_1 A^T P_1^T + \dots + P_6 A^T P_6^T = X \Rightarrow X$  is symmetric

$$\text{Let } B = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$XB = P_1 A P_1^T B + P_2 A P_2^T B + \dots + P_6 A P_6^T B \quad \because P_1^T B = B$$

$$= P_1 A B + \dots + P_6 A B$$

$$= (P_1 + P_2 + \dots + P_6) A B$$

$$= 30B \Rightarrow \alpha = 30$$

$$\text{trace of } X = \text{tr}(X) = \text{tr}(P_1 A P_1^T) + \text{tr}(P_2 A P_2^T) + \dots = 3 + 3 + \dots 3 \Rightarrow 18$$

$$\because X \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = 30 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$(X - 30I)B = 0$$

$$|(X - 30I)B| = 0$$

$$|X - 30I| |B| = 0$$

$$|X - 30I| = 0, \quad |B| \neq 0$$

**Q.4** Let  $x \in \mathbb{R}$  and let

$$P = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 2 \\ 0 & 0 & 3 \end{bmatrix}, \quad Q = \begin{bmatrix} 2 & x & x \\ 0 & 4 & 0 \\ x & x & 6 \end{bmatrix} \text{ and } R = P Q P^{-1}. \text{ Then which of the following options is/are correct?}$$

$$(1) \text{ For } x = 0, \text{ if } R \begin{bmatrix} 1 \\ a \\ b \end{bmatrix} = 6 \begin{bmatrix} 1 \\ a \\ b \end{bmatrix}, \text{ then } a + b = 5$$

$$(2) \text{ For } x = 1, \text{ there exists a unit vector } \alpha \hat{i} + \beta \hat{j} + \gamma \hat{k} \text{ for which } R \begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

(3) There exists a real number  $x$  such that  $PQ = QP$

$$(4) \det R = \det \begin{bmatrix} 2 & x & x \\ 0 & 4 & 0 \\ x & x & 5 \end{bmatrix} + 8, \text{ for all } x \in \mathbb{R}$$

**Ans.** [1,4]

**Sol.**  $R = P Q P^{-1}$

$$|R| = |P| |Q| |P^{-1}|$$

$$= |Q|$$



$$= 48 - 4x^2$$

$$\text{Now, } R \begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \alpha = \beta = \gamma = 0$$

at  $x = 0$

$$R = PQP^{-1}$$

$$= \begin{bmatrix} 2 & 1 & 2/3 \\ 0 & 4 & 4/3 \\ 0 & 0 & 6 \end{bmatrix}$$

$$\text{Now, } (R - 6I) \begin{bmatrix} 1 \\ a \\ b \end{bmatrix} = \begin{bmatrix} -4 & 1 & 2/3 \\ 0 & -2 & 4/3 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ a \\ b \end{bmatrix}$$

$$-4 + a + \frac{2b}{3} = 0$$

$$-2a + \frac{4b}{3} = 0$$

$$\Rightarrow a = 2, b = 3$$

Now,  $PQ = QP$

$$x + 4 + x = 2 + 2x + 0$$

$$\Rightarrow x \in \phi$$

No value

**Q.5** Let  $f : \mathbb{R} \rightarrow \mathbb{R}$  be a function. We say that  $f$  has

PROPERTY 1 if  $\lim_{h \rightarrow 0} \frac{f(h) - f(0)}{\sqrt{|h|}}$  exists and is finite, and

PROPERTY 2 if  $\lim_{h \rightarrow 0} \frac{f(h) - f(0)}{h^2}$  exists and is finite.

Then which of the following options is/are correct?

(1)  $f(x) = x^{2/3}$  has PROPERTY 1

(2)  $f(x) = x|x|$  has PROPERTY 2

(3)  $f(x) = \sin x$  has PROPERTY 2

(4)  $f(x) = |x|$  has PROPERTY 1

**Ans.** [1,4]

**Sol.**

(1)  $y = x^{2/3}$

$$\lim_{h \rightarrow 0} \frac{h^{2/3} - 0}{\sqrt{|h|}} = 0 \rightarrow \text{finite}$$

(2)  $y = x|x|$

$$\lim_{h \rightarrow 0} \frac{h|h| - 0}{h^2} = \lim_{h \rightarrow 0} \frac{|h|}{h}$$

$$\left. \begin{array}{l} \text{Case -I : } \lim_{h \rightarrow 0^+} \frac{|h|}{h} = 1 \\ \text{Case -II : } \lim_{h \rightarrow 0^-} \frac{|h|}{h} = -1 \end{array} \right\} \rightarrow \text{limit DNE}$$

(3)  $y = \sin x$

$$\lim_{h \rightarrow 0} \frac{\sinh - 0}{h^2} \rightarrow \text{DNE}$$

(4)  $y = |x|$

$$\lim_{h \rightarrow 0} \frac{|h| - 0}{\sqrt{|h|}} = 0 \rightarrow \text{finite}$$

Option (1, 4)

**Q.6** Three lines

$$L_1 : \vec{r} = \lambda \hat{i}, \lambda \in \mathbb{R},$$

$$L_2 : \vec{r} = \hat{k} + \mu \hat{j}, \mu \in \mathbb{R} \text{ and}$$

$$L_3 : \vec{r} = \hat{i} + \hat{j} + \nu \hat{k}, \nu \in \mathbb{R}$$

are given. For which point(s) Q on  $L_2$  can we find a point P on  $L_1$  and a point R on  $L_3$  so that P, Q and R are collinear ?

(1)  $\hat{k} - \frac{1}{2} \hat{j}$

(2)  $\hat{k} + \frac{1}{2} \hat{j}$

(3)  $\hat{k}$

(4)  $\hat{k} + \hat{j}$

**Ans.** [1,2]

**Sol.**  $P(\lambda, 0, 0)$

$$Q(0, \mu, 1)$$

$$R(1, 1, \nu)$$

$$\overrightarrow{PQ} = \overrightarrow{KPR}$$

$$\Rightarrow \frac{\lambda}{\lambda - 1} = \frac{-\mu}{-1} = \frac{-1}{\nu}$$

$$\Rightarrow 1 + \frac{1}{\lambda - 1} = \mu = \frac{1}{\nu}$$

**Case-I**

$$\mu = \frac{1}{\nu}$$

$$\nu = \frac{1}{\mu}$$

$$\mu \neq 0$$

**Case-II**

$$\mu = \frac{\lambda}{\lambda - 1}$$

$$\mu\lambda - \mu = \lambda$$

$$\lambda = \frac{\mu}{\mu - 1}$$

$$\mu \neq 1$$

Option (1) and (2) are correct.

**Q.7** Let  $f : \mathbb{R} \rightarrow \mathbb{R}$  be given by  $f(x) = (x - 1)(x - 2)(x - 5)$ . Define  $F(x) = \int_0^x f(t) dt$ ,  $x > 0$ .

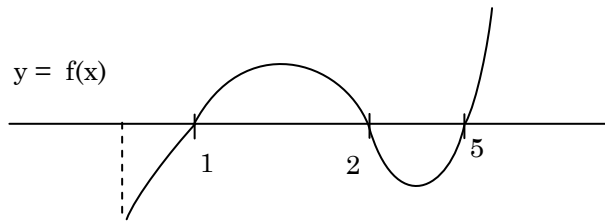
Then which of the following options is/are correct?

- (1)  $F$  has a local minimum at  $x = 1$
- (2)  $F$  has two local maxima and one local minimum in  $(0, \infty)$
- (3)  $F$  has a local maximum at  $x = 2$
- (4)  $F(x) \neq 0$  for all  $x \in (0, 5)$

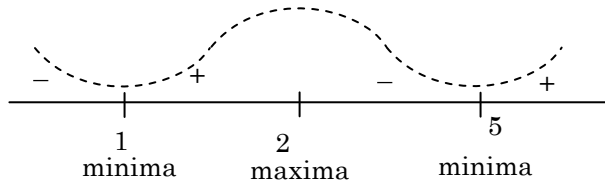
**Ans.** [1,3,4]

**Sol.**  $y = (x - 1)(x - 2)(x - 5)$

$$F(x) = \int_0^x f(t) dt$$



$$F'(x) = f(x) = (x - 1)(x - 2)(x - 5)$$



$$f(x) = x^3 - 8x^2 + 17x - 10$$

$$F(x) = \int_0^x f(x) dx$$

$$F(x) < 0 \quad \forall x \in (0, 5)$$

**Q.8** For non-negative integers  $n$ , let

$$f(n) = \frac{\sum_{k=0}^n \sin\left(\frac{k+1}{n+2}\pi\right) \sin\left(\frac{k+2}{n+2}\pi\right)}{\sum_{k=0}^n \sin^2\left(\frac{k+1}{n+2}\pi\right)}$$

Assuming  $\cos^{-1}x$  takes values in  $[0, \pi]$  which of the following options is/are correct?

$$(1) f(4) = \frac{\sqrt{3}}{2} \qquad (2) \sin(7 \cos^{-1} f(5)) = 0$$

$$(3) \text{ If } \alpha = \tan(\cos^{-1} f(6)), \text{ then } \alpha^2 + 2\alpha - 1 = 0 \qquad (4) \lim_{n \rightarrow \infty} f(n) = \frac{1}{2}$$

**Ans. [1,2,3]**

**Sol.** 
$$f(x) = \frac{\sum_{k=0}^n \left( \cos\left(\frac{\pi}{n+2}\right) - \cos\left(\frac{2k+3}{n+2}\right)\pi \right)}{\sum_{k=0}^n \left( 1 - \cos\left(\frac{2k+2}{n+2}\right)\pi \right)}$$

$$\Rightarrow \frac{(n+1)\cos\left(\frac{\pi}{n+2}\right) - \frac{\cos\left(\frac{n+3}{n+2}\right)\pi \cdot \sin\left(\frac{n+1}{n+2}\right)\pi}{\sin\left(\frac{\pi}{n+2}\right)}}{(n+1) - \frac{\cos\pi \cdot \sin\left(\frac{n+1}{n+2}\right)\pi}{\sin\left(\frac{\pi}{n+2}\right)}}$$
$$= \frac{(n+1)\cos\left(\frac{\pi}{n+2}\right) + \cos\left(\frac{\pi}{n+2}\right)}{(n+1)+1} \Rightarrow \cos\left(\frac{\pi}{n+2}\right)$$

$$f(x) = \cos\left(\frac{\pi}{n+2}\right)$$

$$(1) f(4) = \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$$

$$(2) \sin(7 \cos^{-1} f(5)) = 0$$

$$(3) \alpha = \tan\frac{\pi}{8} \Rightarrow \alpha^2 + 2\alpha - 1 = 0$$

$$(4) \lim_{n \rightarrow \infty} \cos\left(\frac{\pi}{n+2}\right) = 1$$

### SECTION – 2 (Maximum Marks : 18)

- This section contains **SIX (06)** questions. The answer to each question is a **NUMERICAL VALUE**.
- For each question, enter the correct numerical value of the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer. If the numerical value has more than two decimal places, **truncate/round-off** the value to **TWO** decimal places.
- Answer to each question will be evaluated according to the following marking scheme.

Full Marks : +3 If **ONLY** the correct numerical value is entered.

Zero Marks : 0 In all other cases.

**Q.1** Suppose

$$\det \begin{bmatrix} \sum_{k=0}^n k & \sum_{k=0}^n {}^n C_k k^2 \\ \sum_{k=0}^n {}^n C_k k & \sum_{k=0}^n {}^n C_k 3^k \end{bmatrix} = 0$$

holds for some positive integer  $n$ . Then  $\sum_{k=0}^n \frac{{}^n C_k}{k+1}$  equals.....

**Ans. [6.20]**

**Sol.** 
$$\begin{vmatrix} \frac{n(n+1)}{2} & n(n-1)2^{n-2} + n.2^{n-1} \\ n.2^{n-1} & 4^n \end{vmatrix} = 0$$

$$= \frac{n(n+1)}{2} \cdot 4^n - \frac{n^2(n-1)}{8} - \frac{n^2}{4} = 0$$

$$n = 0 \text{ or } 4(n+1) - n(n-1) - 2n = 0$$

$$n = 4$$

$$\therefore \sum_{k=0}^4 \frac{{}^4 C_k}{k+1} = \sum_{k=0}^4 \frac{{}^5 C_{k+1}}{5}$$

$$\Rightarrow \frac{2^5 - 1}{5} = \frac{31}{5} = 6.20$$

**Q.2** Let  $\vec{a} = 2\hat{i} + \hat{j} - \hat{k}$  and  $\vec{b} = \hat{i} + 2\hat{j} + \hat{k}$  be two vectors. Consider a vector  $\vec{c} = \alpha\vec{a} + \beta\vec{b}$ ,  $\alpha, \beta \in \mathbb{R}$ . If the projection of  $\vec{c}$  on the vector  $(\vec{a} + \vec{b})$  is  $3\sqrt{2}$ , then the minimum value of  $(\vec{c} - (\vec{a} \times \vec{b})) \cdot \vec{c}$  equals .....

**Ans. [18]**

**Sol.** 
$$\vec{c} = \alpha(2\hat{i} + \hat{j} - \hat{k}) + \beta(\hat{i} + 2\hat{j} + \hat{k})$$

$$\vec{c} = (2\alpha + \beta)\hat{i} + (\alpha + 2\beta)\hat{j} + (\beta - \alpha)\hat{k}$$

$$\frac{\vec{c} \cdot (\vec{a} + \vec{b})}{|\vec{a} + \vec{b}|} = 3\sqrt{2}$$

$$9(\alpha + \beta) = 18$$

$$\alpha + \beta = 2$$

$$(\vec{c} - \vec{a} \times \vec{b}) \cdot \vec{c} = 6(\alpha^2 - 2\alpha + 4)$$

$$\text{Minimum value} = 18$$

**Q.3** The value of the integral  $\int_0^{\pi/2} \frac{3\sqrt{\cos \theta}}{(\sqrt{\cos \theta} + \sqrt{\sin \theta})^5} d\theta$  equals .....

**Ans.** [0.5]

**Sol.**  $I = \int_0^{\pi/2} \frac{3\sqrt{\cos \theta}}{(\sqrt{\sin \theta} + \sqrt{\cos \theta})^5} d\theta \quad \dots (1)$

$$I = \int_0^{\pi/2} \frac{3\sqrt{\sin \theta}}{(\sqrt{\sin \theta} + \sqrt{\cos \theta})^5} d\theta \quad \dots (2)$$

$$(1) + (2)$$

$$2I = 3 \int_0^{\pi/2} \frac{d\theta}{(\sqrt{\sin \theta} + \sqrt{\cos \theta})^4}$$

$$\frac{2I}{3} = \int_0^{\pi/2} \frac{\sec^2 \theta d\theta}{(\sqrt{\tan \theta} + 1)^4}$$

Put  $\tan \theta = t^2 \Rightarrow \sec^2 \theta d\theta = 2t dt$

$$\frac{2I}{3} = \int_0^{\infty} \frac{2t dt}{(t+1)^4}$$

$$\Rightarrow \frac{I}{3} = \int_0^{\infty} \left[ \frac{1}{(t+1)^3} - \frac{1}{(t+1)^4} \right] dt$$

$$I = \frac{1}{2} = 0.5$$

**Q.4** Five persons A, B, C, D and E are seated in a circular arrangement. If each of them is given a hat of one of the three colours red, blue and green, then the number of ways of distributing the hats such that the persons seated in adjacent seats get different coloured hats is .....

**Ans.** [30]

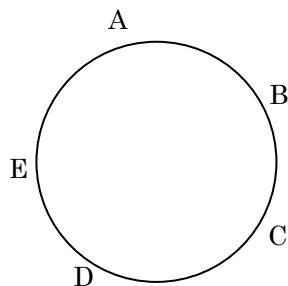
**Sol.** Combinations can only be

(i) RRGGB

(ii) RGGGB

(iii) RRGGB

3 possibility



$$\text{No. of ways} = \underset{\text{No. of possibility}}{3} \times \underset{\text{to distribute non repeating hats}}{5} \times \underset{\text{distributing repeating hats}}{2} = 30$$

**Q.5** The value of  $\sec^{-1}\left(\frac{1}{4}\sum_{k=0}^{10}\sec\left(\frac{7\pi}{12} + \frac{k\pi}{2}\right)\sec\left(\frac{7\pi}{12} + \frac{(k+1)\pi}{2}\right)\right)$  in the interval  $\left[-\frac{\pi}{4}, \frac{3\pi}{4}\right]$  equals ....

**Ans.** [0]

**Sol.**

$$\begin{aligned} & \sec^{-1}\left(\frac{1}{4}\sum_{k=0}^{10}\sec\left(\frac{7\pi}{12} + \frac{k\pi}{2}\right)\sec\left(\frac{7\pi}{12} + \frac{(k+1)\pi}{2}\right)\right) \\ &= \sec^{-1}\left(-\frac{1}{4}\sum_{k=0}^{10}\sec\left(\frac{7\pi}{12} + \frac{k\pi}{2}\right)\operatorname{cosec}\left(\frac{7\pi}{12} + \frac{k\pi}{2}\right)\right) \\ &= \sec^{-1}\left[-\frac{1}{4}\sum_{k=0}^{10}\frac{2}{\sin\left(\frac{7\pi}{6} + k\pi\right)}\right] \\ &= \sec^{-1}\left[-\frac{1}{2}\sum_{k=0}^{10}\frac{1}{(-1)^{k+1}\sin\frac{\pi}{6}}\right] \\ &= \sec^{-1}(1) = 0 \end{aligned}$$

**Q.6** Let  $|X|$  denote the number of elements in a set  $X$ . Let  $S = \{1, 2, 3, 4, 5, 6\}$  be a sample space, where each element is equally likely to occur. If  $A$  and  $B$  are independent events associated with  $S$ , then the number of ordered pairs  $(A, B)$  such that  $1 \leq |B| < |A|$ , equals ....

**Ans.** [422]

**Sol.**  $A$  and  $B$  are independent events

$$P(A)P(B) = P(A \cap B)$$

$$\frac{a}{6} \times \frac{b}{6} = \frac{c}{6} \Rightarrow ab = 6c$$

$$|A| = a, |B| = b, |A \cap B| = c$$

$$(a, b, c) = (3, 2, 1) \quad {}^6C_1 \cdot {}^5C_2 \cdot {}^3C_1 = 180$$

$$= (4, 3, 2) \quad {}^6C_2 \cdot {}^4C_2 \cdot {}^2C_1 = 180$$

$$= (6, 1, 1) \quad {}^6C_1 = 6$$

$$= (6, 2, 2) \quad {}^6C_2 = 15$$

$$= (6, 3, 3) \quad {}^6C_3 = 20$$

$$= (6, 4, 4) \quad {}^6C_4 = 15$$

$$= (6, 5, 5) \quad {}^6C_5 = 6$$

$$\text{Total} = 360 + 62 = 422$$

**SECTION – 3 (Maximum Marks : 12)**

- This section contains **Two (02)** List-Match sets.
- Each List-Match set has **TWO (02)** Multiple Choice Questions.
- Each List-Match set has two lists : **List-I** and **List-II**
- **List-I** has **Four** entries (I), (II), (III) and (IV) and **List-II** has **Six** entries (P), (Q), (R), (S), (T) and (U).
- **FOUR** options are given in each Multiple Choice Question based on **List-I** and **List-II** and **ONLY ONE** of these four options satisfies the condition asked in the Multiple Choice Question.
- Answer to each question will be evaluated according to the following marking scheme :  
Full Marks : +3 If **ONLY** the option corresponding to the correct combination is chosen.  
Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered.)  
Negative Marks : -1 In all other cases.

**Answer the following by appropriately matching the lists based on the information given in the paragraph.**

**Q.1** Let  $f(x) = \sin(\pi \cos x)$  and  $g(x) = \cos(2\pi \sin x)$  be two functions defined for  $x > 0$ . Define the following sets whose elements are written in the increasing order :

$$X = \{x : f(x) = 0\}, Y = \{x : f'(x) = 0\}$$

$$Z = \{x : g(x) = 0\}, W = \{x : g'(x) = 0\}$$

List-I contains the sets X, Y, Z and W. List-II contains some information regarding these sets.

**LIST-I**

(I) X

(II) Y

(III) Z

(IV) W

**LIST-II**

(P)  $\cong \left\{ \frac{\pi}{2}, \frac{3\pi}{2}, 4\pi, 7\pi \right\}$

(Q) an arithmetic progression

(R) NOT an arithmetic progression

(S)  $\cong \left\{ \frac{\pi}{6}, \frac{7\pi}{6}, \frac{13\pi}{6} \right\}$

(T)  $\cong \left\{ \frac{\pi}{3}, \frac{2\pi}{3}, \pi \right\}$

(U)  $\cong \left\{ \frac{\pi}{6}, \frac{3\pi}{4} \right\}$

Which of the following is the only **CORRECT** combination?

(1) (II), (Q), (T)

(2) (I), (P), (R)

(3) (II), (R), (S)

(4) (I), (Q), (U)

**Ans.** [1]

**Sol.**  $X = \{x : f(x) = 0\}$

$$f(x) = 0 \Rightarrow \sin(\pi \cos x) = 0$$

$$\Rightarrow \pi \cos x = n\pi, n \in \mathbb{I}$$

$$\Rightarrow \cos x = n$$

So  $\cos x = -1, 0, 1$

$$\Rightarrow X = n\pi, (2n + 1) \frac{\pi}{2}$$



$$\text{OR } X = \left\{ n \cdot \frac{\pi}{2}, n \in \mathbb{I} \right\}$$

$$Y = \{x : f'(x) = 0\}$$

$$\text{Now, } f'(x) = 0 \Rightarrow \cos(\pi \cos x) (-\pi \sin x) = 0$$

$$\Rightarrow \cos(\pi \cos x) = 0 \text{ or } (-\pi \sin x) = 0$$

$$\Rightarrow \pi \cos x = (2n + 1) \frac{\pi}{2} \text{ or } x = n\pi$$

$$\Rightarrow \cos x = (2n + 1) \frac{1}{2} \text{ or } x = n\pi$$

$$\Rightarrow \cos x = \pm \frac{1}{2} \text{ or } x = n\pi$$

$$Y = \left\{ 2n\pi \pm \frac{\pi}{3}, 2n\pi \pm \frac{2\pi}{3}, n\pi, n \in \mathbb{I} \right\}$$

$$Z = \{x : g(x) = 0\}$$

$$g(x) = 0 \Rightarrow \cos(2\pi \sin x) = 0$$

$$\Rightarrow 2\pi \sin x = (2n + 1) \frac{\pi}{2}, n \in \mathbb{I}$$

$$\Rightarrow \sin x = \frac{2n + 1}{4}$$

$$\Rightarrow \sin x = \pm \frac{1}{4}, \pm \frac{3}{4}$$

$$\Rightarrow Z = \left\{ n\pi \pm \sin^{-1} \frac{1}{4}, n\pi \pm \sin^{-1} \frac{3}{4}, n \in \mathbb{I} \right\}$$

$$W = \{x : g'(x) = 0\}$$

$$g'(x) = 0 \Rightarrow -\sin(2\pi \sin x) (2\pi \cos x) = 0$$

$$\Rightarrow \sin(2\pi \sin x) = 0 \text{ or } 2\pi \cos x = 0$$

$$\Rightarrow 2\pi \sin x = n\pi \text{ or } x = (2n + 1) \frac{\pi}{2}$$

$$\Rightarrow \sin x = \frac{n}{2} \text{ or } x = (2n + 1) \frac{\pi}{2}, n \in \mathbb{I}$$

$$\Rightarrow \sin x = 0, \pm \frac{1}{2}, \pm 1 \text{ or } x = (2n + 1) \frac{\pi}{2}$$

$$W = \left\{ n\pi, (2n + 1) \frac{\pi}{2}, n\pi \pm \frac{\pi}{6}, n \in \mathbb{I} \right\}$$

option (1) (II) (Q) (T)

**Q.2** Let  $f(x) = \sin(\pi \cos x)$  and  $g(x) = \cos(2\pi \sin x)$  be two functions defined for  $x > 0$ . Defined the following set whose elements are written in the increasing order :

$$X = \{x : f(x) = 0\}, \quad Y = \{x : f'(x) = 0\}$$

$$Z = \{x : g(x) = 0\}, \quad W = \{x : g'(x) = 0\}.$$

List –I contains the set X, Y, Z and W. List –II contains some information regarding these sets.

**LIST-I**

(I) X

(II) Y

(III) Z

(IV) W

**LIST-II**

(P)  $\cong \left\{ \frac{\pi}{2}, \frac{3\pi}{2}, 4\pi, 7\pi \right\}$

(Q) an arithmetic progression

(R) NOT an arithmetic progression

(S)  $\cong \left\{ \frac{\pi}{6}, \frac{7\pi}{6}, \frac{13\pi}{6} \right\}$

(T)  $\cong \left\{ \frac{\pi}{3}, \frac{2\pi}{3}, \pi \right\}$

(U)  $\cong \left\{ \frac{\pi}{6}, \frac{3\pi}{4} \right\}$

Which of the following is the only CORRECT combination?

(1) (III), (P), (Q), (U)    (2) (IV), (P), (R), (S)    (3) (III), (R), (U)    (4) (IV), (Q), (T)

**Ans.** [2]

**Sol.** Solution same as above Question (1)

Option (2) (IV) (P) (R) (S)

**Q.3** Let the circles  $C_1 : x^2 + y^2 = 9$  and  $C_2 : (x - 3)^2 + (y - 4)^2 = 16$ , intersect at the points X and Y. Suppose that another circle  $C_3 : (x - h)^2 + (y - k)^2 = r^2$  satisfies the following conditions.

(i) centre of  $C_3$  is collinear with the centres of  $C_1$  and  $C_2$ ,

(ii)  $C_1$  and  $C_2$  both lie inside  $C_3$ , and

(iii)  $C_3$  touches  $C_1$  at M and  $C_2$  at N.

Let the line through X and Y intersect  $C_3$  at Z and W, and let a common tangent of  $C_1$  and  $C_3$  be a tangent to the parabola  $x^2 = 8\alpha y$ .

There are some expressions given in the List-I whose values are given in List-II below :

**LIST-I**

(I)  $2h + k$

(II)  $\frac{\text{Length of } ZW}{\text{Length of } XY}$

(III)  $\frac{\text{Area of triangle } MZN}{\text{Area of triangle } ZMW}$

(IV)  $\alpha$

**LIST-II**

(P) 6

(Q)  $\sqrt{6}$

(R)  $\frac{5}{4}$

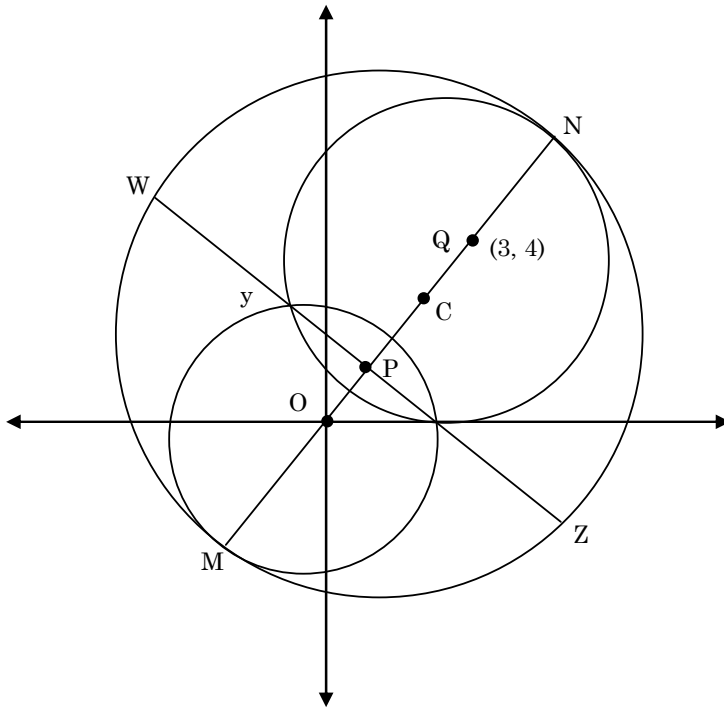
(S)  $\frac{21}{5}$

(T)  $2\sqrt{6}$

(U)  $\frac{10}{3}$

Which of the following is the only INCORRECT combination?

(1) (IV), (U)    (2) (I), (P)    (3) (IV), (S)    (4) (III), (R)

**Ans. [3]**
**Sol.**


$$(I) \text{ Diameter of circle } C_3 \Rightarrow 2r = MN$$

$$= MO + OQ + QN$$

$$= 3 + \sqrt{3^2 + 4^2} + 4$$

$$= 12$$

$$\therefore r = 6$$

Centre C of circle  $C_3$  lies on  $y = \frac{4}{3}x$ .

$$\text{Let } C \left( h, \frac{4}{3}h \right)$$

$$OC = MC - OM$$

$$OC = 6 - 3 = 3$$

$$\therefore \sqrt{h^2 + \frac{16}{9}h^2} = 3$$

$$\Rightarrow h = \frac{9}{5}$$

$$k = \frac{4}{3}h = \frac{12}{5}$$

$$\therefore 2h + k = \frac{18}{5} + \frac{12}{5} = 6$$

(II) Equation of line ZW

$$C_1 - C_2 = 0$$

$$\Rightarrow 3x + 4y = 9$$

Distance of ZW from (0, 0)

$$= \left| \frac{9}{\sqrt{9+16}} \right| = \frac{9}{5}$$

$$\text{Length of XY} = 2\sqrt{9 - \left(\frac{9}{5}\right)^2} = \frac{24}{5}$$

Distance of ZW from C  $\left(\frac{9}{5}, \frac{12}{5}\right)$

$$\left| \frac{3\left(\frac{9}{5}\right) + 4\left(\frac{12}{5}\right) - 9}{\sqrt{9+16}} \right| = \frac{6}{5}$$

$$\text{Length of ZW} = 2\sqrt{36 - \left(\frac{6}{5}\right)^2} = \frac{24\sqrt{6}}{5}$$

$$\therefore \frac{\text{length of ZW}}{\text{length of XY}} = \sqrt{6}$$

$$\text{(III) Area of } \Delta MZN = \frac{1}{2} NM \left( \frac{1}{2} ZW \right) = \frac{72\sqrt{6}}{5}$$

$$\begin{aligned} \text{Area of } \Delta ZMW &= \frac{1}{2} ZW(OM + OP) \\ &= \frac{1}{2} \frac{24\sqrt{6}}{5} \left( 3 + \frac{9}{5} \right) = \frac{288\sqrt{6}}{25} \end{aligned}$$

$$\therefore \frac{\text{Area of } \Delta MZN}{\text{Area of } \Delta ZMW} = \frac{5}{4}$$

(IV) Slope of tangent to  $C_1$  at M =  $-\frac{3}{4}$

$$\therefore \text{Equation of tangent } y = mx - 3\sqrt{1+m^2}$$

$$y = -\frac{3}{4}x - 3\sqrt{1 + \frac{9}{16}}$$

$$\Rightarrow y = -\frac{3x}{4} - \frac{15}{4} \quad \dots(i)$$

Tangent to parabola  $x^2 = 8\alpha y$  is

$$\Rightarrow y = mx - 2\alpha m^2 \quad \dots(\text{ii})$$

Compare (i) & (ii)

$$m = -\frac{3}{4} \text{ and } 2\alpha m^2 = +\frac{15}{4}$$

$$\Rightarrow 2\alpha \left(\frac{9}{16}\right) = \frac{15}{4}$$

$$\Rightarrow \alpha = \frac{10}{3}$$

option (3) (IV) (S)

**Q.4** Let the circles  $C_1 : x^2 + y^2 = 9$  and  $C_2 : (x - 3)^2 + (y - 4)^2 = 16$ , intersect at the points X and Y. Suppose that another circle  $C_3 : (x - h)^2 + (y - k)^2 = r^2$  satisfies the following conditions

(i) centre of  $C_3$  is collinear with the centres of  $C_1$  and  $C_2$ ,

(ii)  $C_1$  and  $C_2$  both lie inside  $C_3$ , and

(iii)  $C_3$  touches  $C_1$  at M and  $C_2$  at N.

Let the line through X and Y intersect  $C_3$  at Z and W, and let a common tangent of  $C_1$  and  $C_3$  be a tangent to the parabola  $x^2 = 8\alpha y$ .

There are some expressions given in the List-I whose values are given in List-II below :

**LIST-I**

(I)  $2h + k$

(II)  $\frac{\text{Length of } ZW}{\text{Length of } XY}$

(III)  $\frac{\text{Area of triangle } MZN}{\text{Area of triangle } ZMW}$

(IV)  $\alpha$

**LIST-II**

(P) 6

(Q)  $\sqrt{6}$

(R)  $\frac{5}{4}$

(S)  $\frac{21}{5}$

(T)  $2\sqrt{6}$

(U)  $\frac{10}{3}$

Which of the following is the only CORRECT combination?

(1) (II), (Q)

(2) (II), (T)

(3) (I), (S)

(4) (I), (U)

**Ans.** [1]

**Sol.** Solution same as above Question (3)  
Option (1); (II) (Q)