CAREER POINT

JEE Advanced Exam 2018

(Paper & Solution)

Date : 20 / 05 / 2018

PAPER-2

PART-I (PHYSICS)

SECTION – 1 (Maximum Marks : 24)

- This section contains **SIX (06)** questions
- Each question has **FOUR** options for correct answer(s). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct option(s).
- For each question, choose the correct option(s) to answer the question.
- Answer to each question will be evaluated according to the following marking scheme :

Full Marks	: **+4**	If only (all) the correct option(s) is (are) chosen.
Partial Marks	: **+3**	If all the four options are correct but ONLY three options are chosen.
Partial Marks	: **+2**	If three or more options are correct but ONLY two options are chosen, both of which are correct options.
Partial Marks	: **+1**	If two or more options are correct but ONLY one option is chosen and it is a correct option.
Zero Marks	: **0**	If none of the option is chosen (i.e. the question is unanswered).
Negative Marks	: **–2**	In all other cases.

- **For Example :** If first, third and fourth are the ONLY three correct options for a question with second option being an incorrect option ; selecting only all the three correct options will result in +4 marks. Selecting only two of the three correct options (e.g. the first and fourth options), without selecting any incorrect option (second option in this case), will result in +2 marks. Selecting only one of the three correct options (either first or third or fourth option), without selecting any incorrect option (second option in this case), will result in +1 marks. Selecting any incorrect option(s) (second option in this case), with or without selection of any correct option (s) will result in –2 marks.

Q.1 A particle of mass m is initially at rest at the origin. It is subjected to a force and starts moving along the x-axis. Its kinetic energy K changes with time as $dK/dt = \gamma t$, where γ is a positive constant of appropriate dimension. Which of the following statements is (are) true?

(A) The force applied on the particle is constant

(B) The speed of the particle is proportional to time

(C) The distance of the particle from the origin increases linearly with time

(D) The force is conservative

Ans. **[A,B,D]**

Sol. $\frac{dK}{dt} = \gamma t$

$K = \frac{\gamma t^2}{2}$

$\frac{1}{2}mv^2 = \frac{\gamma t^2}{2}$

$v = \sqrt{\frac{\gamma}{m}}\, t$

$\therefore \quad a = \frac{dv}{dt} \qquad \Rightarrow \quad a = \sqrt{\frac{\gamma}{m}} \times 1$

$\therefore \quad F = m\frac{\sqrt{\gamma}}{\sqrt{m}} = \sqrt{m\gamma}$

Q.2 Consider a thin square plate floating on a viscous liquid in a large tank. The height h of the liquid in the tank is much less than the width of the tank. The floating plate is pulled horizontally with a constant velocity u_0. Which of the following statements is (are) true?

(A) The resistive force of liquid on the plate is inversely proportional to h

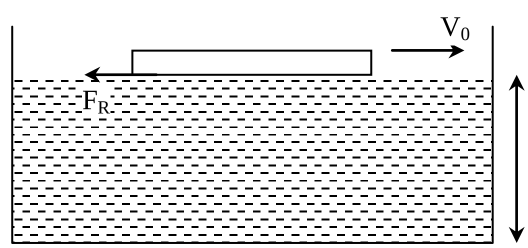
(B) The resistive force of liquid on the plate is independent of the area of the plate

(C) The tangential (shear) stress on the floor of the tank increases with u_0

(D) The tangential (shear) stress on the plate varies linearly with the viscosity η of the liquid

Ans. **[A,C,D]**

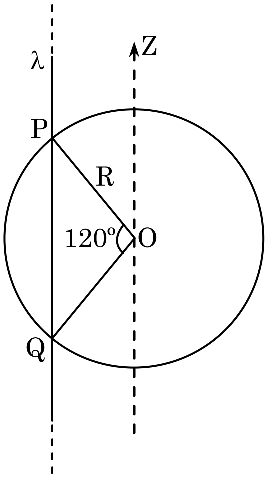
Sol.



Resistance Force = $F_R = \eta A \frac{dV}{dh} = \eta A \frac{V_0}{h}$ Ans. (A)

Tangential stress on floor = $\frac{F_R}{\text{Area}} = \frac{\eta A V_0}{\text{Area}\, h}$ Ans. (C, D)

Q.3 An infinitely long thin non-conducting wire is parallel to the z-axis and carries a uniform line charge density λ. It pierces a thin non-conducting spherical shell of radius R in such a way that the arc PQ subtend an angle 120° at the centre O of the spherical shell, as shown in the figure. The permittivity of free space is ϵ_0. Which of the following statements is (are) true?



(A) The electric flux through the shell is $\sqrt{3}R\lambda/\epsilon_0$

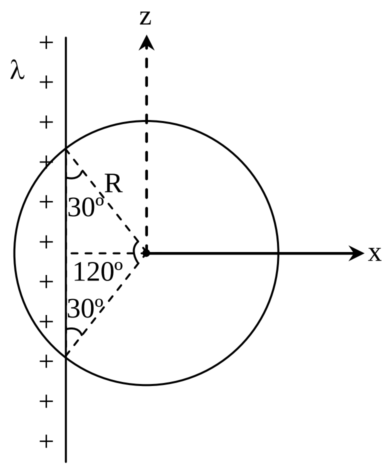
(B) The z-component of the electric filed is zero at all the points on the surface of the shell

(C) The electric flux through the shell is $\sqrt{2}R\lambda/\epsilon_0$

(D) The electric field is normal to the surface of the shell at all points

Ans. **[A,B]**

Sol.



Length of wire inside shell = 2R cos 30 = $\sqrt{3}$ R

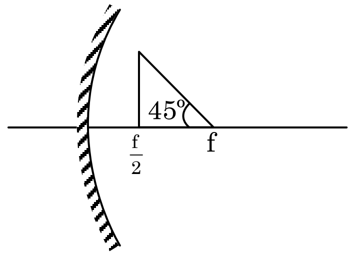
$q_{enclosed} = \sqrt{3}\, R\lambda$

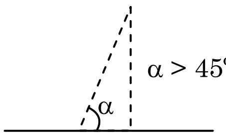
$\phi_{shell} = \dfrac{\sqrt{3}\lambda R}{\varepsilon_0}$ Ans. (A)

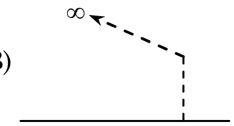
E due to wire at each point is in +ve or –ve x direction.

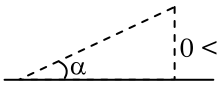
$\therefore$ Z component of E is zero. Ans. (B)

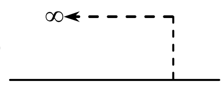
Q.4 A wire is bent in the shape of a right angled triangle and is placed in front of a concave mirror of focal length *f*, as shown in the figure. Which of the figures shown in the four options qualitatively represent(s) the shape of the image of the bent wire? (These figures are not to scale.)



(A) 

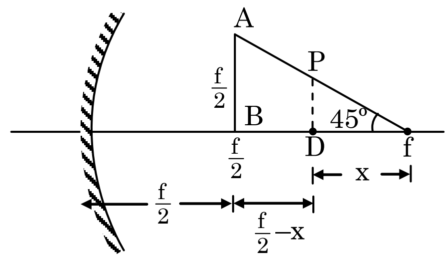
(B) 

(C) 

(D) 

Ans. **[D]**

Sol. Assume rays are paraxial and mirror formula is applicable.



$$\text{m for AB length} = \frac{f}{f-u} = \frac{-f}{-f-\left(\frac{-f}{2}\right)} = \frac{-f}{\frac{-f}{2}} = 2$$

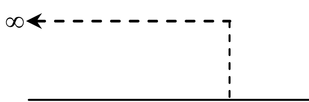
$$\text{size of image of AB} = \frac{f}{2} \times 2 = f$$

Now take any general point P on object are Af.

$$\text{m for PD} = \frac{-f}{-f-[-(f-x)]} = \frac{f}{x}$$

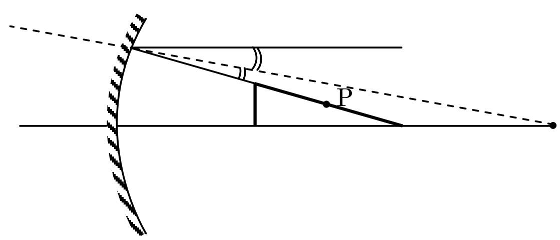
$$\text{Length of image of PD} = \text{PD} \times \frac{f}{x} = x \times \frac{f}{x} = f$$

So image of all point are at same height ∴ answer is



Ans. (D)

If we assume that rays are not paraxial and mirror formula is not valid then (B) can be answer.



Q.5 In a radioactive decay chain, ${}^{232}_{90}\text{Th}$ nucleus decays to ${}^{212}_{82}\text{Pb}$ nucleus. Let N_α and N_β be the number of α and β^- particles, respectively, emitted in this decay process. Which of the following statements in (are) true?

(A) $N_\alpha = 5$ (B) $N_\alpha = 6$ (C) $N_\beta = 2$ (D) $N_\beta = 4$

Ans. **[A,C]**

Sol. $${}^{232}_{90}\text{Th} \xrightarrow{N_\alpha,\, N_\beta} {}^{232-4N_\alpha}_{90-2N_\alpha+N_\beta}\text{Y}$$

$${}^{232-4N_\alpha}_{90+N_\beta-2N_\alpha}\text{Y} = {}^{212}_{82}\text{Pb}$$

$232 - 4N_\alpha = 212$

$N_\alpha = 5$

$90 + N_\beta - 2N_\alpha = 82$

$90 + N_\beta - 2 \times 5 = 82$

$N_\beta = 82 + 10 - 90 = 2$

Q.6 In an experiment to measure the speed of sound by a resonating air column, a tuning fork of frequency 500 *Hz* is used. The length of the air column is varied by changing the level of water in the resonance tube. Two successive resonances are heard at air columns of length 50.7 *cm* and 83.9 *cm*. Which of the following statements is (are) true?

(A) The speed of sound determined from this experiment is 332 ms^{-1}

(B) The end correction in this experiment is 0.9 *cm*

(C) The wavelength of the sound wave is 66.4 *cm*

(D) The resonance at 50.7 *cm* corresponds to the fundamental harmonic

Ans. **[A,C]**

Sol. $\ell_2 - \ell_1 = \frac{\lambda}{2}$

$\lambda = 2(\ell_2 - \ell_1)$

$= 2(33.2) = 66.4$ cm Ans. (C)

$V = f\lambda = 500 \times 66.4 = 332\ ms^{-1}$ Ans. (A)

SECTION – 2 (Maximum Marks : 24)

- This section contains **EIGHT (08)** questions. The answer to each question is a **NUMERICAL VALUE**.
- For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the **second decimal place; e.g.** 6.25, 7.00, –0.33, –.30, 30.27, –127.30) using the mouse and the onscreen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme.

Full Marks : +3 If ONLY the correct numerical value is entered as answer.

Zero Marks : 0 In all other cases.

Q.7 A solid horizontal surface is covered with a thin layer of oil. A rectangular block of mass m = 0.4 kg is at rest on this surface. An impulse of 1.0 *N s* is applied to the block at time t = 0 so that it starts moving along the x-axis with a velocity $v(t) = v_0 e^{-t/\tau}$, where v_0 is a constant and $\tau = 4$ *s*. The displacement of the block, in metres, at $t = \tau$ is ______________. Take $e^{-1} = 0.37$

Ans. **[6.30]**

Sol. J = mV

1 = 0.4 V

$V = \frac{1}{0.4} = 2.5$ m/s

$V = V_0 e^{-\frac{t}{\tau}}$

At t = 0, velocity = V_0 = 2.5

$V = 2.5\ e^{-\frac{t}{4}}$

$$\frac{ds}{dt} = 2.5\ e^{-t/4}$$

$$s = 2.5\int_0^4 e^{-t/4}dt = \frac{2.5(e^{-t/4})_0^4}{-\frac{1}{4}} = -10\ [0.37 - 1]$$

$\Rightarrow$ 10 × 0.63 = 6.3 = 6.30 Ans.

Q.8 A ball is projected from the ground at an angle of 45° with the horizontal surface. It reaches a maximum height of 120 *m* and returns to the ground. Upon hitting the ground for the first time, it loses half of its kinetic energy. Immediately after the bounce, the velocity of the ball makes an angle of 30° with the horizontal surface. The maximum height it reaches after the bounce, in *metres*, is _________________.

Ans. **[30.00]**

Sol. $$\frac{u^2\sin^2\theta}{2g} = 120$$

$$\frac{u^2}{2g}\times\sin^2 45 = 120$$

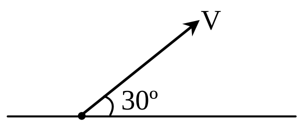
$$\frac{u^2}{20}\times\frac{1}{2} = 120 \qquad \text{....(1)}$$

After hitting the ground new kinetic energy is half of initial kinetic energy.

$$KE_{new} = \frac{1}{2}\ \frac{mu^2}{2}$$

$$\frac{1}{2}\ mv^2 = \frac{1}{4}\ mu^2$$

$$\therefore \qquad v = \frac{u}{\sqrt{2}}$$



$$H_{max} = \frac{v^2\sin^2 30}{2\times 10} = \frac{u^2}{2\times 2\times 10}\times\frac{1}{4}$$

From (1)

$$H_{max} = \frac{120}{4} = 30 \text{ metre} = 30.00$$

Q.9 A particle of mass 10^{-3} *kg* and charge 1.0 *C*, is initially at rest. At time $t = 0$, the particle comes under the influence of an electric field $\vec{E}(t) = E_0 \sin \omega t\,\hat{i}$, where $E_0 = 1.0\ N\ C^{-1}$ and $\omega = 10^3\ rad\ s^{-1}$. Consider the effect of only the electrical force on the particle. Then the maximum speed, in ms^{-1}, attained by the particle at subsequent times is _____________.

Ans. **[2.00]**

Sol. F = qE

$$a = \frac{qE}{m}$$

$a = \dfrac{1 \times 1 \sin \omega t}{10^{-3}}$

$a = 10^3 \sin \omega t$

For maxima of speed

$\dfrac{dv}{dt} = 0$

$a = 0$

$\omega t = 0, \pi, 2\pi$ ....

$\dfrac{dv}{dt} = 10^3 \sin \omega t$

$\int_0^v dv = 10^3 \int_0^t \sin \omega t \, dt$

$v = \dfrac{10^3 [1 - \cos \omega t]_0^t}{\omega}$

$v = \dfrac{10^3}{10^3}(1 - \cos\omega t)$

$v = 1 - \cos \omega t$

$v_{max} = 1 - \cos \pi$

$v_{max} = 1 - (-1) = 2$

$v_{max} = 2.00$ m/s Ans.

Q.10 A moving coil galvanometer has 50 turns and each turn has an area 2×10^{-4} m^2. The magnetic field produced by the magnet inside the galvanometer is 0.02 *T*. The torsional constant of the suspension wire is 10^{-4} *N m rad*$^{-1}$. When a current flows through the galvanometer, a full scale deflection occurs if the coil rotates by 0.2 *rad*. The resistance of the coil of the galvanometer is 50*Ω*. This galvanometer is to be converted into an ammeter capable of measuring current in the range 0 – 1.0 *A*. For this purpose, an shunt resistance is to be added in parallel to the galvanometer. The value of this shunt resistance, in *ohms*, is ____________.

Ans. **[5.55]**

Sol. $NiAB = C\theta$

$50 \times I_g \times 2 \times 10^{-4} \times 0.02 = 10^{-4} \times 0.2$

$1.0 \times 0.02\, I_g = 0.2$

$I_g = \dfrac{0.2}{2} = 0.1$ Amp

$G = 50\ \Omega$

$I = I_s \left[1 + \dfrac{G}{S}\right]$

$1 = 0.1 \left[1 + \dfrac{50}{S}\right]$

$10 = 1 + \dfrac{50}{S}$

$9 = \dfrac{50}{S}$

$S = \dfrac{50}{9} = 5.55\ \Omega$

Q.11 A steel wire of diameter 0.5 *mm* and Young's modulus 2×10^{11} $N\ m^{-2}$ carries a load of mass *M*. The length of the wire with the load is 1.0 *m*. A vernier scale with 10 divisions is attached to the end of this wire. Next to the steel wire is a reference wire to which a main scale, of least count 1.0 *mm*, is attached. The 10 division of the vernier scale correspond to 9 divisions of the main scale. Initially, the zero of vernier scale coincides with the zero of main scale. If the load on the steel wire is increased by1.2 *kg*, the vernier scale division which coincides with a mains scale division is ________. Take g = 10 ms^{-2} and π = 3.2.

Ans. **[3.00]**

Sol. Stress = Y × strain

$$\frac{1.2\times10}{A} = Y \times \frac{\Delta\ell}{\ell}$$

$$\Delta\ell = \frac{\ell\times1.2\times10}{AY}$$

$$\Delta\ell = \frac{1\times1.2\times10\times4}{2\times10^{11}\times3.14\times0.25\times10^{-6}} \text{ metre}$$

$$= \frac{12\times4\times4}{2\times3.14\times10^{5}} \text{ metre}$$

$$= \frac{12\times8}{3.2}\times10^{-5} \text{ metre}$$

$$= \frac{96\times10^{-2}}{3.2} \text{ mm}$$

= 0.3 mm

9 × M.S.D = 10 V.S.D

$$\frac{9\times1}{10} = \text{V.S.D}$$

V.S.D. = 0.9 mm

L.C. = M.S.D – V . S.D

= 1 – 0.9 = 0.1

Third division of vernier coincide

Ans. 3.00

Q.12 One mole of a monatomic ideal gas undergoes an adiabatic expansion in which its volume becomes eight times its initial value. If the initial temperature of the gas is 100 *K* and the universal gas constant $R = 8.0\ J\ mol^{-1}\ K^{-1}$, the decrease in its internal energy, in *Joule*, is ________.

Ans. **[900.00]**

Sol. $TV^{\gamma-1} = C$ where $\gamma = 1 + \frac{2}{f} \Rightarrow \gamma - 1 = \frac{2}{3}$

$100 \times V_1^{2/3} = TV_2^{2/3}$

$$T = 100 \times \left(\frac{V_1}{V_2}\right)^{2/3} = 100 \times \left(\frac{1}{8}\right)^{2/3} = \frac{100}{4} = 25\text{ k}$$

$\Delta U = \frac{3}{2} nR (T_2 - T_1)$

Decrease $= \frac{3}{2} nR (T_1 - T_2)$

$= \frac{3}{2} \times 1 \times 8 (T_1 - T_2)$

$= \frac{3}{2} \times 1 \times 8 (100 - 25)$

$= 3 \times 4 \times 75$

$\Rightarrow$ 900 Joule

Q.13 In a photoelectric experiment a parallel beam of monochromatic light with power of 200 *W* is incident on a perfectly absorbing cathode of work function 6.25 *eV*. The frequency of light is just above the threshold frequency so that the photoelectrons are emitted with negligible kinetic energy. Assume that the photoelectron emission efficiency is 100%. A potential difference of 500 *V* is applied between the cathode and the anode. All the emitted electrons are incident normally on the anode and are absorbed. The anode experiences a force $F = n \times 10^{-4}$ *N* due to the impact of the electrons. The value of n is ____________. Mass of the electron $m_e = 9 \times 10^{-31}$ *kg* and $1.0\ eV = 1.6 \times 10^{-19}$ *J*.

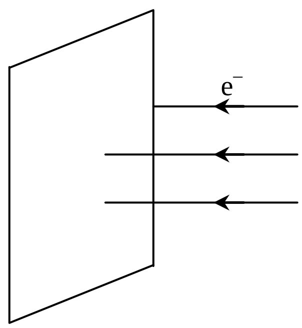
Ans. **[24.00]**

Sol. $P = \dot{N}_P \frac{hc}{\lambda}$

$\dot{N}_P = \frac{P}{\frac{hc}{\lambda}} = \frac{200}{6.25 \times 1.6 \times 10^{-19}}$

$N_e = N_p$ as $\eta = 100\ \%$

Electron reaching at anode have KE = 500 eV as



Anode

Momentum deliver to anode by 1 electron = mV = $\sqrt{2mKE}$

Force $= \frac{dP}{dt}$

$= \dot{N}_e mV$

$= \frac{200}{6.25e} \times \sqrt{2mKE}$

$= \frac{200}{6.25e} \times \sqrt{2 \times 9 \times 10^{-31} \times 500 \times e}$

$= \frac{200}{6.25} \sqrt{\frac{2 \times 9 \times 5 \times 10^{-21} \times 100}{1.6 \times 10^{-19}}}$

$= \frac{200}{6.25} \sqrt{\frac{10 \times 9 \times 10^{-31} \times 10^{21}}{1.6}}$

$= \frac{200}{6.25} \times \frac{3}{4} \times 10 \times 10^{-5}$

$= \frac{2}{625} \times \frac{3}{4} = 0.0024$

$\Rightarrow$ 24×10^{-4} Ans. 24

Q.14 Consider a hydrogen-like ionized atom with atomic number Z with a single electron. In the emission spectrum of this atom, the photon emitted in the $n = 2$ to $n = 1$ transition has energy 74.8 eV higher than the photon emitted in the $n = 3$ to $n = 2$ transition. The ionization energy of the hydrogen atom is 13.6 eV. The value of Z is ____________.

Ans. **[3.00]**

Sol. $E_{n_2} - E_{n_1} = E_{n_3} - E_{n_2} + 74.8$

$13.6\, Z^2 \left[\frac{1}{1^2} - \frac{1}{4}\right] = 13.6 Z^2 \left[\frac{1}{2^2} - \frac{1}{3^2}\right] + 74.8$

$10.2Z^2 - 1.9Z^2 = 74.8$

$8.3\, Z^2 = 74.8$

$Z = 3$

Ans. 3

SECTION – 3 (Maximum Marks : 12)

- This section contains **FOUR (04)** questions.
- Each question has **TWO (02)** matching lists : **LIST-I** and **LIST-II**.
- **FOUR** options are given representing matching of elements from **LIST-I** and **LIST-II**. **ONLY ONE** of these four options corresponds to a correct matching.
- For each question, choose the option corresponding to the correct matching.
- For each question, marks will be awarded according to the following marking scheme :

  Full Marks : +3 If ONLY the option corresponding to the correct matching is chosen.

  Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered.)

  Negative Marks : –1 In all other cases.

Q.15 The electric field E is measured at a point $P(0, 0, d)$ generated due to various charge distributions and the dependence of E on d is found to be different for different charge distribution. List-I contains different relations between E and d. List-II describes different electric charge distributions, along with their locations. Match the functions in List-I with the related charge distributions in List-II

LIST-I		LIST-II	
P.	E is independent of d	**1.**	A point charge Q at the origin
Q.	$E \propto \frac{1}{d}$	**2.**	A small dipole with point charges Q at $(0, 0, l)$ and $-Q$ at $(0, 0, -l)$. Take $2l << d$
R.	$E \propto \frac{1}{d^2}$	**3.**	An infinite line charge coincident with the x-axis, with uniform linear charge density λ
S.	$E \propto \frac{1}{d^3}$	**4.**	Two infinite wires carrying uniform linear charge density parallel to the x-axis. The one along $(y = 0, z = l)$ has a charge density $+\lambda$ and the one along $(y = 0, z = -l)$ has a charge density $-\lambda$. Take $2l << d$
		5.	Infinite plane charge coincident with the xy-plane with uniform surface charge density

(A) P → 5; Q → 3, 4; R → 1; S → 2

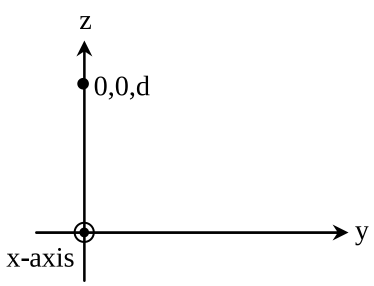
(B) P → 5; Q → 3; R → 1, 4; S → 2

(C) P → 5; Q → 3; R → 1, 2; S → 4

(D) P → 4; Q → 2, 3; R → 1; S → 5

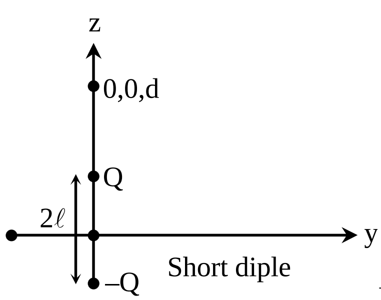
Ans. **[B]**

Sol.



1. Point charge Q at origin $E = \frac{kQ}{d^2}$

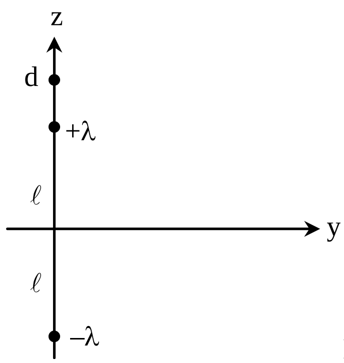
2.



$E = \frac{2kP}{d^3}$

3. E due to infinite wire $\therefore\ E = \dfrac{\lambda}{2\pi\varepsilon_0 d}$

4.



$$E = \frac{\lambda}{2\pi\epsilon_0}\left[\frac{1}{d-\ell} - \frac{1}{d+\ell}\right]$$

$$= \frac{\lambda}{2\pi\epsilon_0}\left[\frac{d+\ell-(d-\ell)}{d^2}\right] \Rightarrow \frac{\lambda 2\ell}{2\pi\epsilon_0 d^2}$$

5. E due to infinite charge sheet = $\dfrac{\sigma}{2\varepsilon_0}$

$P \to 5$; $Q \to 3$; $R \to 1, 4$; $S \to 2$ Ans. (B)

Q.16 A planet of mass M, has two natural satellites with masses m_1 and m_2. The radii of their circular orbits are R_1 and R_2 respectively. Ignore the gravitational force between the satellites. Define v_1, L_1, K_1 and T_1 to be, respectively, the orbital speed, angular momentum, kinetic energy and time period of revolution of satellite 1; and v_2, L_2, K_2 and T_2 to be the corresponding quantities of satellite 2. Given $m_1/m_2 = 2$ and $R_1/R_2 = 1/4$, match the ratios in List-I to the numbers in List-II

LIST-I		LIST-II	
P.	$\dfrac{v_1}{v_2}$	**1.**	$\dfrac{1}{8}$
Q.	$\dfrac{L_1}{L_2}$	**2.**	1
R.	$\dfrac{K_1}{K_2}$	**3.**	2
S.	$\dfrac{T_1}{T_2}$	**4.**	8

(A) $P \to 4$; $Q \to 2$; $R \to 1$; $S \to 3$

(B) $P \to 3$; $Q \to 2$; $R \to 4$; $S \to 1$

(C) $P \to 2$; $Q \to 3$; $R \to 1$; $S \to 4$

(D) $P \to 2$; $Q \to 3$; $R \to 4$; $S \to 1$

Ans. [B]

Sol. $\frac{GMm}{r^2} = \frac{mV^2}{r}$

$V = \sqrt{\frac{GM}{r}}$

$\frac{V_1}{V_2} = \sqrt{\frac{M}{M}} \times \sqrt{\frac{R_2}{R_1}} = \sqrt{\frac{4}{1}} = 2$ $\quad P \to 3$

$L = mvr$

$\frac{L_1}{L_2} = \frac{m_1}{m_2}\ \frac{v_1}{v_2} \times \frac{r_1}{r_2} = 2 \times 2 \times \frac{1}{4} = 1:1$ $\quad Q \to 2$

$k = \frac{1}{2}mV^2$

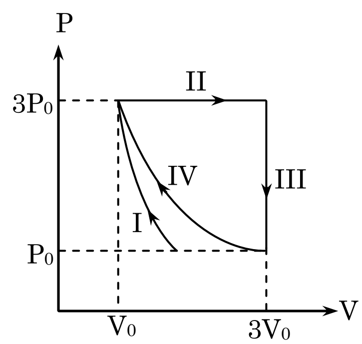
$\frac{k_1}{k_2} = \frac{m_1}{m_2} \times \left(\frac{v_1}{v_2}\right)^2 = 2\,(2)^2 = 8$ $\quad R \to 4$

$T = \frac{2\pi R}{V}$

$\frac{T_1}{T_2} = \frac{R_1}{R_2} \times \frac{V_2}{V_1} = \frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$ $\quad S \to 1$

Ans. (B)

Q.17 One mole of a monatomic ideal gas undergoes four thermodynamic processes as shown schematically in the *PV*-diagram below. Among these four processes, one is isobaric, one is isochoric, one is isothermal and one is adiabatic. Match the processes mentioned in List-I with the corresponding statements in List-II

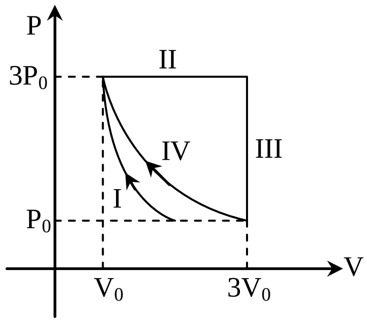


LIST-I		**LIST-II**	
P.	In process I	**1.**	Work done by the gas is zero
Q.	In process II	**2.**	Temperature of the gas remains unchanged
R.	In process III	**3.**	No heat is exchanged between the gas and its surroundings
S.	In process IV	**4.**	Work done by the gas is $6P_0V_0$

(A) $P \to 4;\ Q \to 3;\ R \to 1;\ S \to 2$
(B) $P \to 1;\ Q \to 3;\ R \to 2;\ S \to 4$
(C) $P \to 3;\ Q \to 4;\ R \to 1;\ S \to 2$
(D) $P \to 3;\ Q \to 4;\ R \to 2;\ S \to 1$

Ans. **[C]**

Sol.



I is adiabatic

IV is isothermal

III is isochoric

II is isobaric

P → 3

R → 1

In process-II $W = 3P_0 \times 2V_0 = 6P_0V_0$

Q → 4

S → 2

Ans. (C)

Q.18 In the List-I below, four different paths of a particle are given as functions of time. In these functions, α and β are positive constants of appropriate dimension and $\alpha \neq \beta$. In each case, the force acting on the particle is either zero or conservative. In List-II, five physical quantities of the particle are mentioned: $\vec{p}$ is the linear momentum, $\vec{L}$ is the angular momentum about the origin, *K* is the kinetic energy, *U* is the potential energy and *E* is the total energy. Match each path in List-I with those quantities in List-II, which are **conserved for that path**.

	LIST-I		**LIST-II**
P.	$\vec{r}(t) = \alpha t\hat{i} + \beta t\hat{j}$	**1.**	$\vec{p}$
Q.	$\vec{r}(t) = \alpha \cos \omega t\hat{i} + \beta \sin \omega t\hat{j}$	**2.**	$\vec{L}$
R.	$\vec{r}(t) = \alpha (\cos \omega t\hat{i} + \sin \omega t\hat{j})$	**3.**	*K*
S.	$\vec{r}(t) = \alpha t\hat{i} + \frac{\beta}{2}t^2\hat{j}$	**4.**	*U*
		5.	*E*

(A) P → 1, 2, 3, 4, 5; Q → 2, 5; R → 2, 3, 4, 5; S → 5

(B) P → 1, 2, 3, 4, 5; Q → 3, 5; R → 2, 3, 4, 5; S → 2, 5

(C) P → 2, 3, 4; Q → 5; R → 1, 2, 4; S → 2, 5

(D) P → 1, 2, 3, 5; Q → 2, 5; R → 2, 3, 4, 5; S → 2, 5

Ans. **[A]**

Sol. $\vec{r}(t) = \alpha t\,\hat{i} + \beta t\,\hat{j}$

$\vec{V}(t) = \alpha\hat{i} + \beta\,\hat{j}$

$\vec{V}(t) = \text{constant}$

a = 0

F = 0

P = conserved , K = conserved

L = conserved , U = conserved

E = conserved

P → 1, 2, 3, 4, 5

$\vec{r}(t) = \alpha \cos \omega t\,\hat{i} + \beta \sin \omega t\,\hat{j}$

$\vec{V}(t) = -\alpha\omega \sin \omega t\,\hat{j} + \beta\omega \cos \omega t\,\hat{j}$

$a(t) = -\alpha\omega^2 \cos \omega t\,\hat{i} - \beta\omega^2 \cos \omega t\,\hat{j}$

$L = (\vec{r} \times \vec{v})\,m$

$$\vec{r} \times \vec{v} = \begin{vmatrix} i & j & k \\ \alpha \cos\omega t & \beta \sin \omega t & 0 \\ -\alpha\omega \sin \omega t & \beta\omega \cos \omega t & 0 \end{vmatrix}$$

$\hat{k}\,[\alpha\beta\,\omega \cos^2 \omega t + \alpha\beta\omega \sin^2\omega t] = \alpha\beta\omega\,\hat{k} = \text{const.}$

L = conserved

τ of the force about origin is zero its mean this force is always passed through origin so it is central & conservative

E = conserved.

Q → 2, 5 Ans. (A)

PART-II (CHEMISTRY)

SECTION – 1 (Maximum Marks : 24)

- This section contains **SIX (06)** questions
- Each question has **FOUR** options for correct answer(s). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct option(s).
- For each question, choose the correct option(s) to answer the question.
- Answer to each question will be evaluated according to the following marking scheme :

Full Marks	: **+4**	If only (all) the correct option(s) is (are) chosen.
Partial Marks	: **+3**	If all the four options are correct but ONLY three options are chosen.
Partial Marks	: **+2**	If three or more options are correct but ONLY two options are chosen, both of which are correct options.
Partial Marks	: **+1**	If two or more options are correct but ONLY one option is chosen and it is a correct option.
Zero Marks	: **0**	If none of the option is chosen (i.e. the question is unanswered).
Negative Marks	: **–2**	In all other cases.

- **For Example :** If first, third and fourth are the ONLY three correct options for a question with second option being an incorrect option ; selecting only all the three correct options will result in +4 marks. Selecting only two of the three correct options (e.g. the first and fourth options), without selecting any incorrect option (second option in this case), will result in +2 marks. Selecting only one of the three correct options (either first or third or fourth option), without selecting any incorrect option (second option in this case), will result in +1 marks. Selecting any incorrect option(s) (second option in this case), with or without selection of any correct option (s) will result in –2 marks.

Q.1 The correct option(s) regarding the complex $[Co(en)(NH_3)_3(H_2O)]^{3+}$ (en = $H_2NCH_2CH_2NH_2$) is (are)

(A) It has two geometrical isomers

(B) It will have three geometrical isomers if bidentate 'en' is replaced by two cyanide ligands

(C) It is paramagnetic

(D) It absorbs light at longer wavelength as compared to $[Co(en)(NH_3)_4]^{3+}$

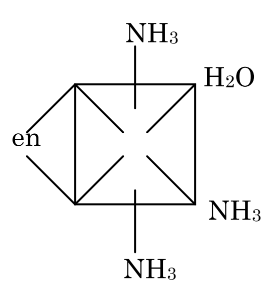
Ans. **[ABD]**

Sol. $[Co(en)(NH_3)_3(H_2O)]^{3+}$

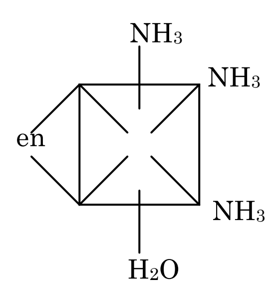
en = $(NH_2 - CH_2 - CH_2 - NH_2)$

(A) it has two Geometric isomer

Like $[M(AA)b_3C]$



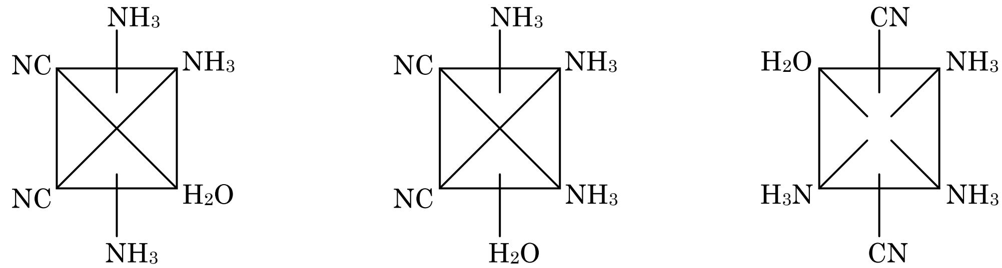
Trans



Cis

(B) If en is replaced by two cynide then G.I. will be 3

Like Ma_2b_3c



(D) $\underset{Value}{CFSE}$ of $[Co(en)\cdot(NH_3)_4]^{3+}$ greater than $[Co(en)(NH_3)_3H_2O]$ So later will absorb light of lower energy and higher wavelength.

Q.2 The correct option(s) to distinguish nitrate salts of Mn^{2+} and Cu^{2+} taken separately is (are)

(A) Mn^{2+} shows the characteristic green colour in the flame test

(B) Only Cu^{2+} shows the formation of precipitate by passing H_2S in acidic medium

(C) Only Mn^{2+} shows the formation of precipitate by passing H_2S in faintly basic medium

(D) Cu^{2+}/Cu has higher reduction potential than Mn^{2+}/Mn (measured under similar conditions)

Ans. **[B&D]**

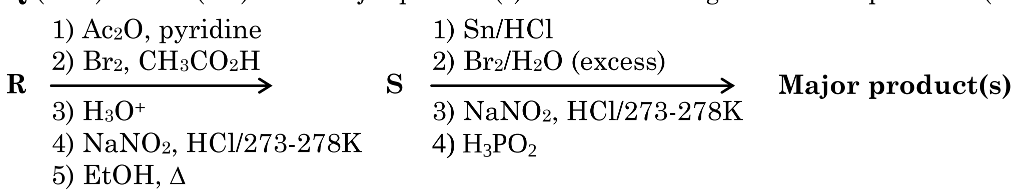
Sol. (B) Cu^{2+} is of II group which has group reagent HCl + H_2S whereas Mn^{2+} is of IVth group which has group reagent $H_2S|OH^{\ominus}$ K_{SP} of CuS is lower than K_{sp} of MnS. That is why in acidic medium only CuS will be precipitated where S^{2-} are less but in basic medium CuS and MnS both will be precipitated (S^{2-} are in higher amount)

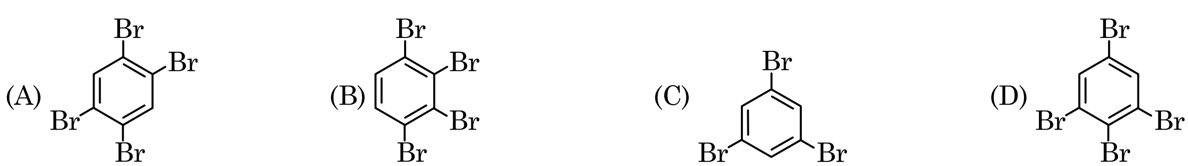
(D) $Cu^{2+}|Cu$ has S.R.P = 0.34 V

$M^{2+}|Mn$ has S.R.P = – 1.18 volt

(due to stable e^- configuration of Mn^{2+})

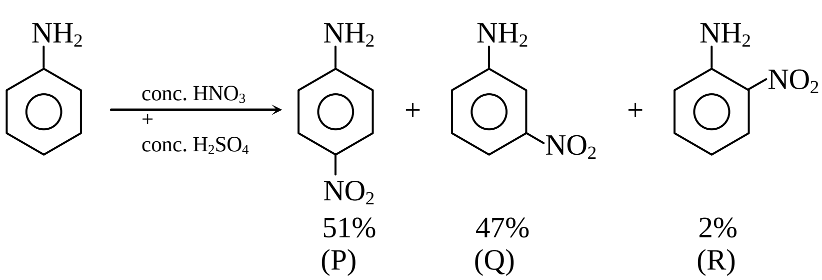
Q.3 Aniline reacts with mixed acid (conc. HNO_3 and conc. H_2SO_4) at 288 K to give P (51%), **Q** (47%) and **R** (2%). The major product(s) of the following reaction sequence is (are)

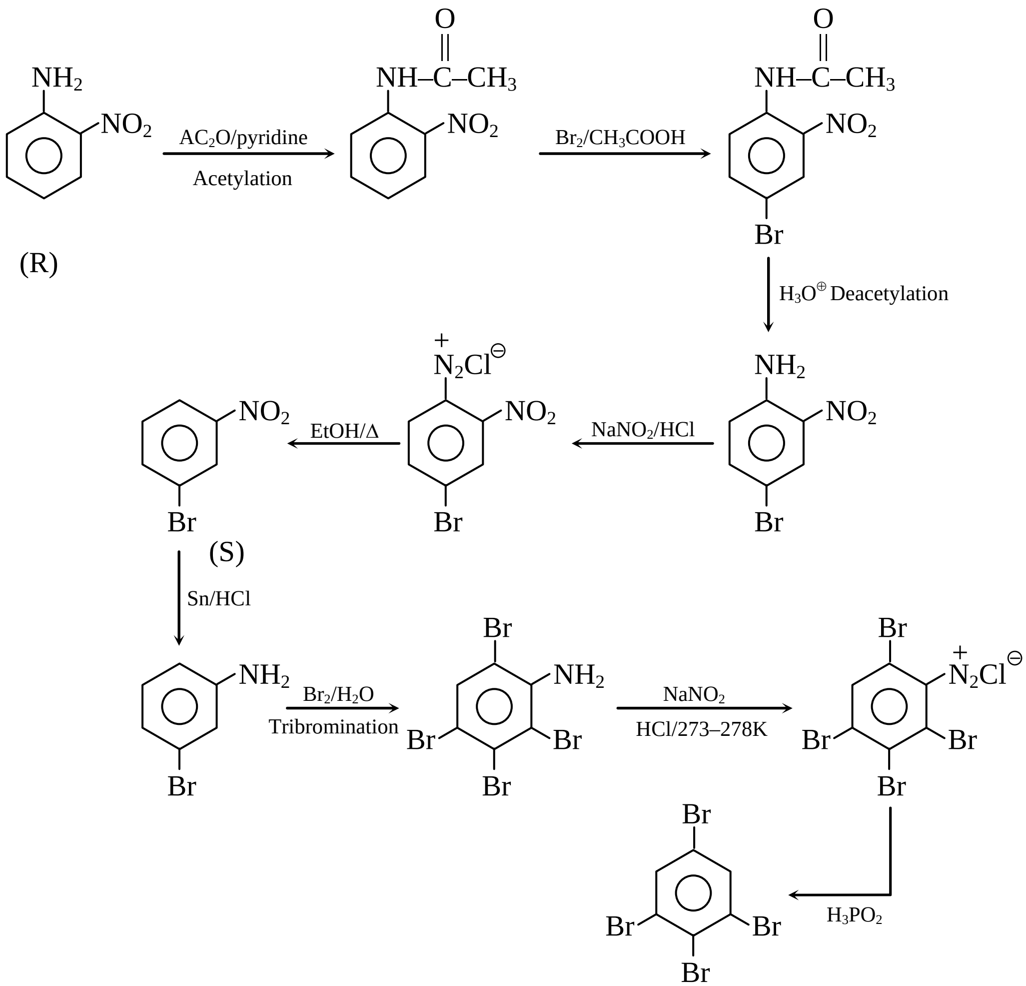




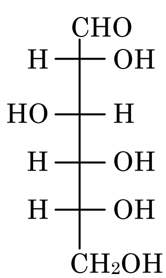
Ans. **[D]**

Sol.



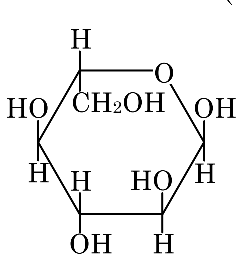


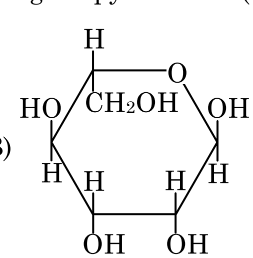
Q.4 The Fischer presentation of D-glucose is given below.

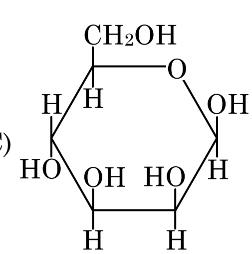


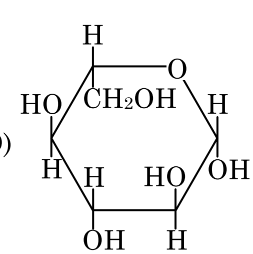
D-glucose

The correct structure(s) of β-L-glucopyranose is (are)

(A) 

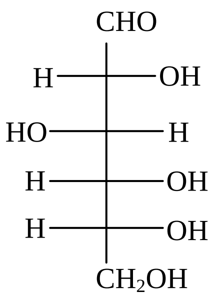
(B) 

(C) 

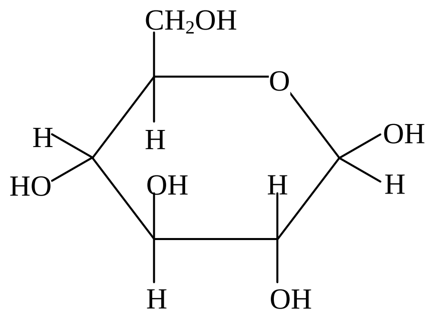
(D) 

Ans. **[D]**

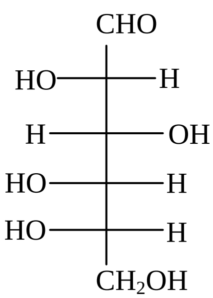
Sol.



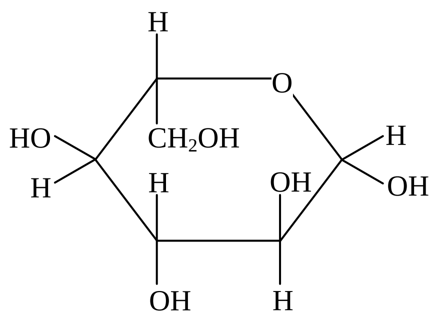
D(+) Glucose



β – D Glucopyranose



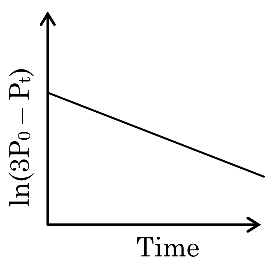
L(–) Glucose

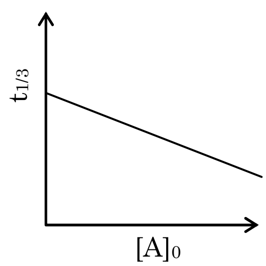


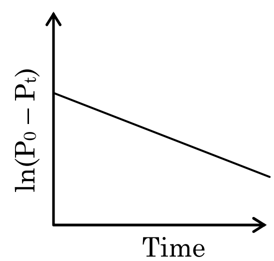
β – L Glucopyranose

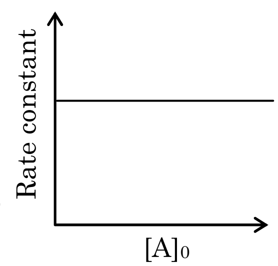
Q.5 For a first order reaction $A(g) \rightarrow 2B(g) + C(g)$ at constant volume and 300 K, the total pressure at the beginning (t = 0) and at time t are P_0 and P_t, respectively. Initially, only A is present with concentration $[A]_0$, and $t_{1/3}$ is the time required for the partial pressure of A to reach $1/3^{rd}$ of it's initial value. The correct option(s) is (are)

(Assume that all these gases behave as ideal gases)

(A)


(B)


(C)


(D)


Ans. **[A,D]**

Sol.

$$A \longrightarrow 2B + C$$

t = 0	P_0	0	0
t = t	$P_0 - P$	2P	P

$\therefore P_t = P_0 + 2P$

$$Kt = \ell n \frac{P_0}{P_0 - P} = \ell n \left[\frac{P_0}{P_0 - \frac{(P_t - P_0)}{2}} \right]$$

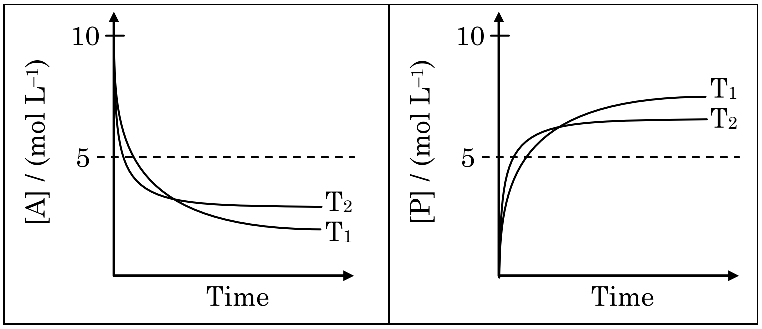
$$\therefore Kt = \ell n \frac{2P_0}{3P_0 - P_t}$$

$\therefore \ell n\, 2P_0 - Kt = \ell n(3P_0 - P_t)$ ...(1)

& $t_{1/3} = \frac{1}{K} \ell n \frac{P_0}{P_0/3} = \frac{1}{K} \ell n\, 3$ = constant

A option is correct using (1) & D is correct because rate constant does not depend on concentration.

Q.6 For a reaction, $A \rightleftharpoons P$, the plots of [A] and [P] with time at temperatures T_1 and T_2 are given below.



If $T_2 > T_1$, the correct statements(s) is (are)

(Assume ΔH^θ and ΔS^θ are independent of temperature and ratio of lnK at T_1 to lnK at T_2 is greater than T_2/T_1. Here H, S, G and K are enthalpy, entropy, Gibbs energy and equilibrium constant, respectively.)

(A) $\Delta H^\theta < 0, \Delta S^\theta < 0$ (B) $\Delta G^\theta < 0, \Delta H^\theta > 0$ (C) $\Delta G^\theta < 0, \Delta S^\theta < 0$ (D) $\Delta G^\theta < 0, \Delta S^\theta > 0$

Ans. **[A,C]**

Sol. $A \rightleftharpoons P$

given that $T_2 > T_1$

Now $\frac{\ell n\, K_1}{\ell n\, K_2} > \frac{T_2}{T_1}$

$\therefore RT_1 \ell n\, K_1 > RT_2 \ell n\, K_2$

$-\Delta G^o_1 > -\Delta G^o_2$

or $(-\Delta H^o + T_1\Delta S^o) > (-\Delta H^o + T_2\Delta S^o)$

$\Rightarrow T_1\Delta S^o > T_2\Delta S^o$

$\therefore \Delta S^o < 0$ $(\because T_2 > T_1)$

SECTION – 2 (Maximum Marks : 24)

- This section contains **EIGHT (08)** questions. The answer to each question is a **NUMERICAL VALUE**.
- For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the **second decimal place; e.g.** 6.25, 7.00, –0.33, –.30, 30.27, –127.30) using the mouse and the onscreen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme.

Full Marks : +3 If ONLY the correct numerical value is entered as answer.

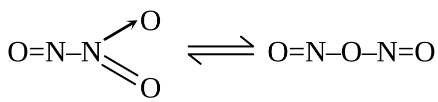
Zero Marks : 0 In all other cases.

Q.7 The total number of compounds having at least one bridging oxo group among the molecules given below is ___.

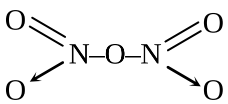
N_2O_3, N_2O_5, P_4O_6, P_4O_7, $H_4P_2O_5$, $H_5P_3O_{10}$, $H_2S_2O_3$, $H_2S_2O_5$

Ans. **[6.00]**

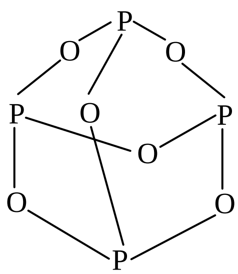
Sol. First six contain bridging oxo group



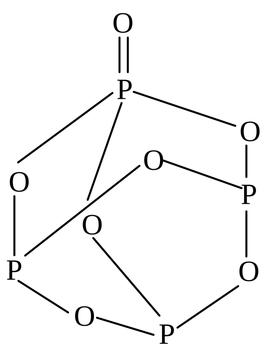
N_2O_3



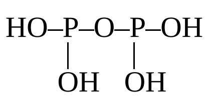
N_2O_5



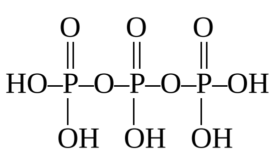
P_4O_6



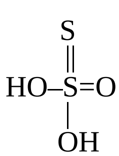
P_4O_7



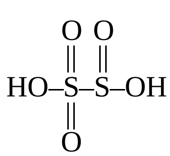
$H_4P_2O_5$



$H_5P_3O_{10}$



$H_2S_2O_3$



$H_2S_2O_5$

Q.8 Galena (an ore) is partially oxidized by passing air through it at high temperature. After some time, the passage of air is stopped, but the heating is continued in a closed furnace such that the contents undergo self-reduction. The weight (in kg) of Pb produced per kg of O_2 consumed is ___.

(Atomic weights in g mol^{-1}: O = 16, S = 32, Pb = 207)

Ans. **[6.47]**

Sol. $2PbS + 3O_2 \longrightarrow 2PbO + SO_2$

$2PbO + PbS \longrightarrow 3Pb + SO_2$

For each mole of O_2 1 mole of Pb is formed

no. of moles of O_2 = no. of moles of Pb = $\frac{1000}{32}$

wt of Pb formed = $\frac{1000}{32} \times 207 = 6468.75$ g

= 6.47 kg

Q.9 To measure the quantity of $MnCl_2$ dissolved in a aqueous solution, it was completely converted to $KMnO_4$ using the reaction,

$MnCl_2 + K_2S_2O_8 + H_2O \rightarrow KMnO_4 + H_2SO_4 + HCl$ (equation not balanced).

Few drops of concentrated HCl were added to this solution and gently warmed. Further, oxalic acid (225 mg) was added in portions till the colour of the permanganate ion disappeared. The quantity of $MnCl_2$ (in mg) present in the initial solution is ___.

(Atomic weights in g mol^{-1} : Mn = 55, Cl = 35.5)

Ans. **[126.00]**

Sol. $MnCl_2 + K_2S_2O_8 + H_2O \longrightarrow KMnO_4 + H_2SO_4 + HCl$

x mole x mole

$C_2O_4^{2-} + MnO_4^- \xrightarrow{H^+} CO_2$

no. of eq^n of $C_2O_4^{2-}$ = no. of eq^n of MnO_4^-

$2 \times \frac{0.225}{90} = x \times 5$

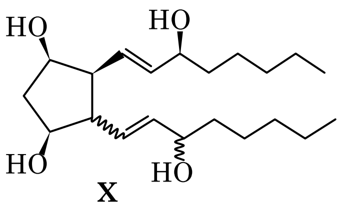
$\therefore$ $x = 1 \times 10^{-3}$

$\therefore$ mass of $MnCl_2 = 1 \times 10^{-3} \times (55 + 2 \times 35.5)$

$= 126 \times 10^{-3}$ g

= 126 mg

Q.10 For the given compound X, the total number of optically active stereoisomers is ___.



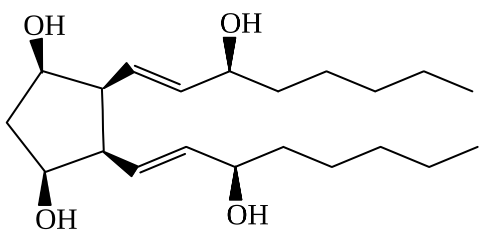
X

This type of bond indicates that the configuration at the specific carbon and the geometry of the double bond is fixed

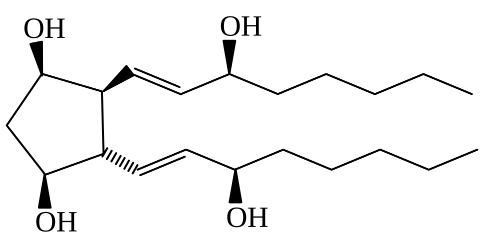
This type of bond indicates that the configuration at the specific carbon and the geometry of the double bond is **NOT** fixed

Ans. **[7.00]**

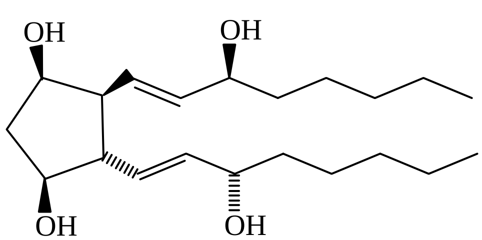
Sol. There are only two chiral centre and 1 D.B where configuration can be changed so total 8 stereoisomer will be formed out of which seven stereoisomer are optically active

(2) 

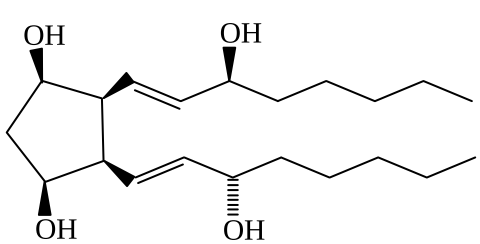
optically inactive if D.B is trans and if D.B. is cis then it is optically active

(2) 

Both optically active it include two stereoisomer one having cis D.B and another having trans

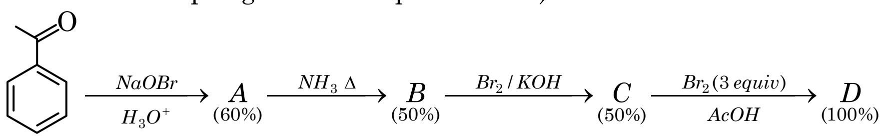
(2) 

Both optically active it include two stereoisomer one having cis D.B and one trans D.B.

(2) 

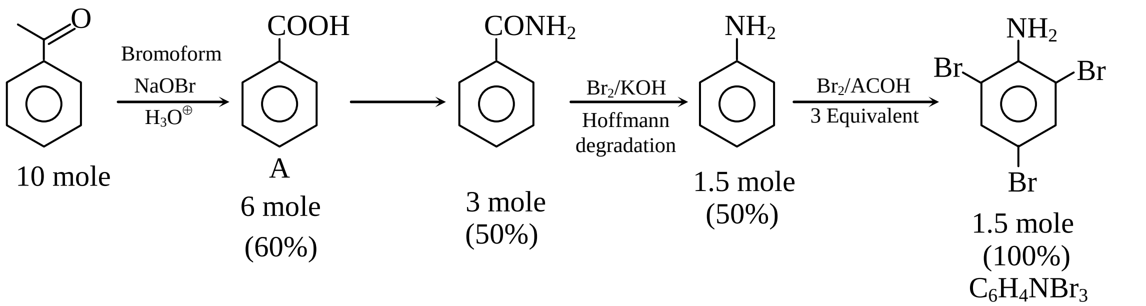
Both optically active it include two stereoisomer one having cis D.B and another trans D.B.

Q.11 In the following reaction sequence, the amount of **D** (in g) formed from 10 moles of acetophenone is ___.
(Atomic weights in g mol^{-1}: H = 1, C = 12, N = 14, O = 16, Br = 80. The yield (%) corresponding to the product in each step is given in the parenthesis)



Ans. **[495.00]**

Sol.



Mass = 1.5 × 330 = 495 g

Q.12 The surface of copper gets tarnished by the formation of copper oxide. N_2 gas was passed to prevent the oxide formation during heating of copper at 1250 K. However, the N_2 gas contains 1 mole % of water vapour as impurity. The water vapour oxidizes copper as per the reaction given below :

$$2Cu(s) + H_2O(g) \rightarrow Cu_2O(s) + H_2(g)$$

P_{H_2} is the minimum partial pressure of H_2 (in bar) needed to prevent the oxidation at 1250 K. The value of ln(P_{H_2}) is ____.

(Given: total pressure = 1 bar, R (universal gas constant) = 8 J K^{-1}mol^{-1}, ln(10) = 2.3. Cu(s) and Cu_2O(s) are mutually immiscible.)
At 1250 K: $2Cu(s) + \frac{1}{2} O_2(g) \rightarrow Cu_2O(s)$; $\Delta G^\theta = -78{,}000$ J mol^{-1}
$H_2(g) + \frac{1}{2} O_2(g) \rightarrow H_2O(g)$; $\Delta G^\theta = -1{,}78{,}000$ J mol^{-1}; G is the Gibbs energy)

Ans. **[–14.60]**

Sol. Given $2Cu + \frac{1}{2}O_2 \longrightarrow Cu_2O_{(s)}$ $\qquad \Delta G^o = -78$ kJ

$H_2 + \frac{1}{2}O_2 \longrightarrow H_2O$ $\qquad \Delta G^o = -178$ kJ

$\therefore\ 2Cu_{(s)} + H_2O_{(g)} \longrightarrow Cu_2O_{(s)} + H_{2(g)}$ $\qquad \Delta G^o = 178 - 78$
$= 100$ kJ

Now $\Delta G = \Delta G^o + RT\ell nQ$

$$0 = 100 + 8 \times 10^{-3} \times 1250\ \ell n\frac{P_{H_2}}{P_{H_2O}} \qquad \left(P_{H_2O} = \frac{1}{100}\right)$$

$\therefore\ \ell n\, P_{H_2} = -14.6$

Q.13 Consider the following reversible reaction,

$$A(g) + B(g) \rightleftharpoons AB(g).$$

The activation energy of the backward reaction exceeds that of the forward reaction by 2RT (in J mol^{-1}). If the pre-exponential factor of the forward reaction is 4 times that of the reverse reaction, the absolute value of ΔG^θ (in J mol^{-1}) for the reaction at 300 K is ___.
(Given; ln(2) = 0.7, RT = 2500 J mol^{-1} at 300 K and G is the Gibbs energy)

Ans. **[8500 J/mol]**

Sol. $E_{ab} - E_{af} = 2RT$

$A_f = 4A_b$

$K_f = A_f e^{-E_{af}/RT}$ ...(1)

$K_b = A_b e^{-E_{ab}/RT}$ ...(2)

(1) ÷ (2)

$$\frac{K_f}{K_b} = K_{eq} = \frac{A_f}{A_b}\ e^{\frac{1}{RT}(E_{ab}-E_{af})}$$

$\therefore K_{eq} = 4e^2$

$\therefore \Delta G^o = -RT\ell nK_{eq}$

$= -2500\ \ln(4e^2)$

$= -\ 8500$ J/mol

$\therefore |\Delta G^o| = 8500$ J/mol

Q.14 Consider an electrochemical cell: A(s) | A^{n+} (aq, 2 M) | | B^{2n+} (aq, 1M) | B(s). The value of ΔH^o for the cell reaction is twice that of ΔG^o at 300 K. If the emf of the cell is zero, the ΔS^θ (in J K^{-1} mol^{-1}) of the cell reaction per mole of B formed at 300 K is ____.
(Given : ln(2) = 0.7, R (universal gas constant) = 8.3 J K^{-1} mol^{-1}. H, S and G are enthalpy, entropy and Gibbs energy, respectively.)

Ans. **[–11.62 J/k-mol]**

Sol. $A \longrightarrow A^{n+} + ne^- \times 2$

$B^{2n+} + 2n\ e^- \longrightarrow B(s)$

$2A_{(s)} + B^{2n+}{}_{(1M)} \longrightarrow 2A^{+n}{}_{(2M)} + B_{(s)}$

$$\therefore \Delta G = \Delta G^o + RT\ \ell n\frac{[A^{+n}]^2}{[B^{+2n}]}$$

$$\therefore \Delta G^o = -RT\ell n\frac{[A^{+n}]^2}{[B^{+2n}]}$$

$= -RT\ell n\ 4$

$\Delta G^o = \Delta H^o - T\Delta S^o$

$\Delta G^o = 2\Delta G^o - T\Delta S^o$ $(\because\ \Delta H^o = 2\Delta G^o)$

$$\therefore \Delta S^o = \frac{\Delta G^o}{T} = \frac{-RT\ell n4}{T}$$

$= -8.3 \times 2 \times 0.7 = -11.62$ J/K-mol

SECTION – 3 (Maximum Marks : 12)

- This section contains **FOUR (04)** questions.
- Each question has **TWO (02)** matching lists : **LIST-I** and **LIST-II**.
- **FOUR** options are given representing matching of elements from **LIST-I** and **LIST-II**. **ONLY ONE** of these four options corresponds to a correct matching.
- For each question, choose the option corresponding to the correct matching.
- For each question, marks will be awarded according to the following marking scheme :
  Full Marks : +3 If ONLY the option corresponding to the correct matching is chosen.
  Zero Marks : 0 In none of the options is chosen (i.e. the question is unanswered.)
  Negative Marks : –1 In all other cases.

Q.15 Match each set of hybrid orbitals from LIST-I with complex(es) given in LIST-II.

LIST-I	**LIST-II**
P. dsp^2	**1.** $[FeF_6]^{4-}$
Q. sp^3	**2.** $[Ti(H_2O)_3Cl_3]$
R. sp^3d^2	**3.** $[Cr(NH_3)_6]^{3+}$
S. d^2sp^2	**4.** $[FeCl_4]^{2-}$
	5. $Ni(CO)_4$
	6. $[Ni(CN)_4]^{2-}$

The correct option is :

(A) **P** → 5; **Q** → 4,6; **R** → 2,3; **S** → 1

(B) **P** → 5,6; **Q** → 4; **R** → 3; **S** → 1,2

(C) **P** → 6; **Q** → 4,5; **R** → 1; **S** → 2,3

(D) **P** → 4,6; **Q** → 5,6; **R** → 1,2; **S** → 3

Ans. **[C]**

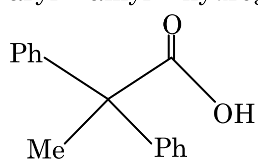
Sol. (P) dsp^2 $\underset{(6)}{[Ni(CN)_4]^{2-}}$

(Q) sp^3 $\underset{(4)}{[FeCl_4]^{2-}}$, $\underset{(5)}{Ni(CO)_4}$

(R) sp^3d^2 $[FeF_6]^{4-}$

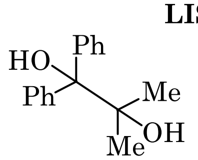
(S) d^2sp^3 $[Ti(H_2O)_3Cl_3]$, $[Cr(NH_3)_6]^{3+}$

Q.16 The desired product **X** can be prepared by reacting the major product of the reactions in LIST-I with one or more appropriate regents in LIST-II.
(given, order of migratory aptitude: aryl > alkyl > hydrogen)

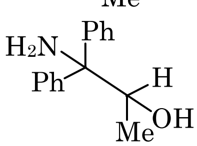


X

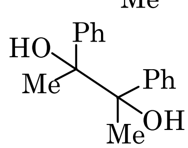
LIST-I / **LIST-II**

P. 

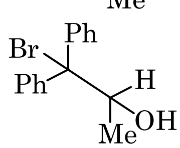
+ H_2SO_4 — **1.** I_2, NaOH

Q. 

+ HNO_2 — **2.** $[Ag(NH_3)_2]OH$

R. 

+ H_2SO_4 — **3.** Fehling solution

S. 

+ $AgNO_3$ — **4.** HCHO, NaOH

5. NaOBr

The correct option is :

(A) **P** → 1; **Q** → 2,3; **R** → 1,4; **S** → 2,4

(B) **P** → 1,5; **Q** → 3,4; **R** → 4,5; **S** → 3

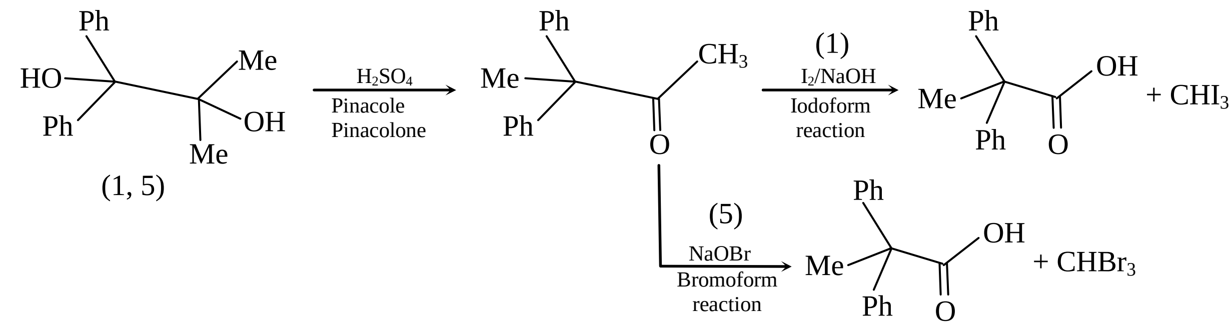
(C) **P** → 1,5; **Q** → 3,4; **R** → 5; **S** → 2,4

(D) **P** → 1,5; **Q** → 2,3; **R** → 1,5; **S** → 2,3

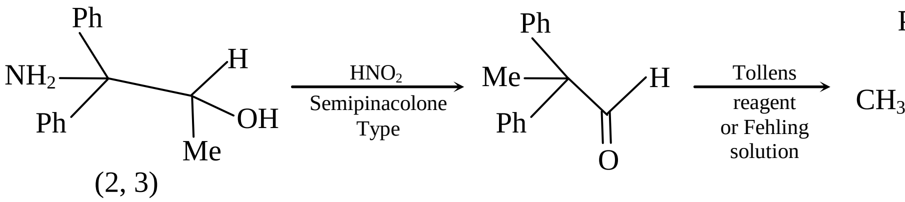
Ans. **[D]**

Sol. This is related with pinacol-pinacolone rearrangement.

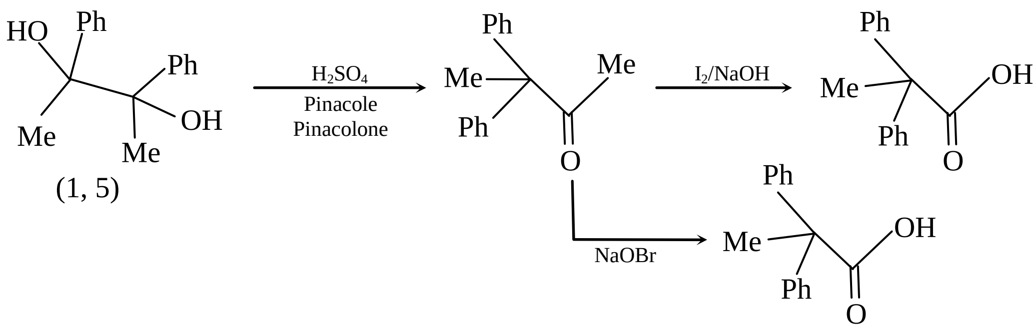
(P)



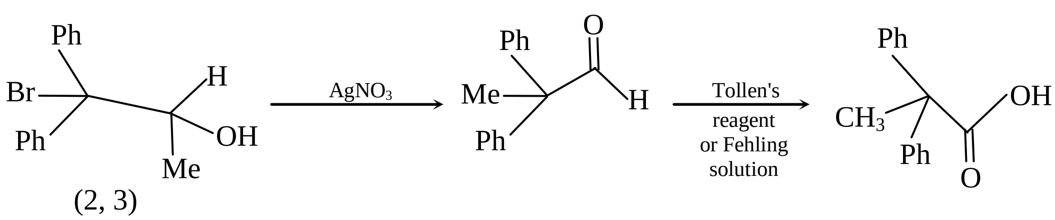
(Q)



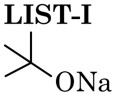
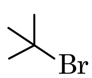
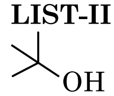
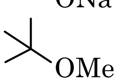
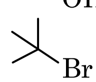
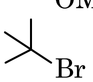
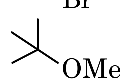
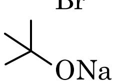
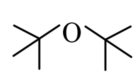
(R)



(S)



Q.17 LIST-I contains reactions and LIST-II contains major products.

LIST-I				LIST-II	
P.		+ 	⟶	**1.**	
Q.		+ HBr	⟶	**2.**	
R.		+ NaOMe	⟶	**3.**	
S.		+ MeBr	⟶	**4.**	
				5.	

Match each reaction in LIST-I with one or more products in LIST-II and choose the correct option.

(A) **P** → 1,5; **Q** → 2; **R** → 3; **S** → 4

(B) **P** → 1,4; **Q** → 2; **R** → 4; **S** → 3

(C) **P** → 1,4; **Q** → 1,2; **R** → 3,4; **S** → 4

(D) **P** → 4,5; **Q** → 4; **R** → 4; **S** → 3,4

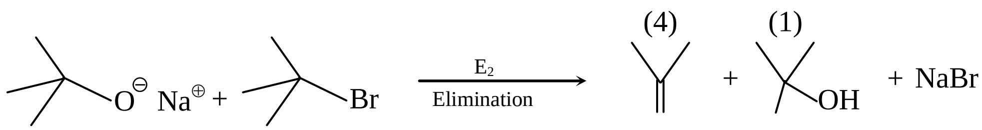
Ans. **[B]**

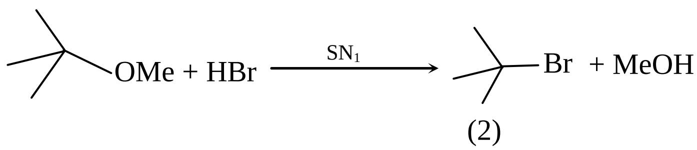
Sol. P → 1, 4

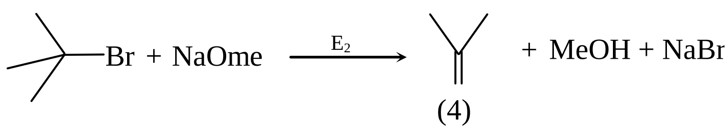
Q → 2

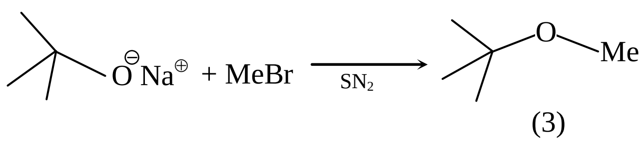
R → 4

S → 3

(P) 

(Q) 

(R) 

(S) 

Q.18 Dilution processes of different aqueous solutions, with water, are given in LIST-I. The effects of dilution of the solutions on $[H^+]$ are given in LIST-II.

(None : Degree of dissociation (α) of weak acid and weak base is << 1; degree of hydrolysis of salt << 1; $[H^+]$ represents the concentration of H^+ ions)

LIST-I	LIST-II
P. (10 mL of 0.1 M NaOH + 20 mL of 0.1 M acetic acid) diluted to 60 mL	**1.** the value of $[H^+]$ does not change on dilution
Q. (20 mL of 0.1 M NaOH + 20 mL of 0.1 M acetic acid) diluted to 80 mL	**2.** the value of $[H^+]$ changes to half of its initial value on dilution
R. (20 mL of 0.1 M HCl + 20 mL of 0.1 M ammonia solution) diluted to 80 mL	**3.** the value of $[H^+]$ changes to two times of its initial value on dilution
S. 10 mL saturated solution of $Ni(OH)_2$ in equilibrium with excess solid $Ni(OH)_2$ is diluted to 20 mL (solid $Ni(OH)_2$ is still present after dilution).	**4.** the value of $[H^+]$ changes to $\frac{1}{\sqrt{2}}$ times of its initial value on dilution
	5. the value of $[H^+]$ changes to $\sqrt{2}$ times of its initial value on dilution

Match each process given in LIST-I with one or more effect(s) in LIST-II. The correct option is

(A) **P** $\rightarrow$ 4; **Q** $\rightarrow$ 2; **R** $\rightarrow$ 3; **S** $\rightarrow$ 1

(B) **P** $\rightarrow$ 4; **Q** $\rightarrow$ 3; **R** $\rightarrow$ 2; **S** $\rightarrow$ 3

(C) **P** $\rightarrow$ 1; **Q** $\rightarrow$ 4; **R** $\rightarrow$ 5; **S** $\rightarrow$ 3

(D) **P** $\rightarrow$ 1; **Q** $\rightarrow$ 5; **R** $\rightarrow$ 4; **S** $\rightarrow$ 1

Ans. **[D]**

Sol. **(**P) $CH_3COOH + NaOH \longrightarrow CH_3COONa + H_2O$

0.1M, 20ml. 0.1M,10ml

since this will make a buffer solution, $[H^+]$ will not change with dilution

$\therefore \boxed{P \rightarrow 1}$

(Q) $CH_3COOH + NaOH \longrightarrow CH_3COONa + H_2O$

2 milli mol 2 milli mol 2 milli mol

0 0 $\frac{2}{40} = 0.05$ M

After dilution$[CH_3COONa] = \frac{2}{80} = 0.025$ M

Now

$CH_3COO^- + H_2O \rightleftharpoons CH_3COOH + OH^-$

c

c – x x x

$$\therefore \frac{K_w}{K_a} = \frac{x^2}{c - x \approx c}$$

$$\therefore x = \sqrt{\frac{K_w c}{K_a}} = [OH^-]$$

$$\therefore [H^+] = \sqrt{\frac{K_w K_a}{c}}$$

$$\therefore \frac{[H^+]_2}{[H^+]_1} = \sqrt{\frac{c_1}{c_2}} = \sqrt{\frac{0.05}{0.025}} = \sqrt{2}$$

$\boxed{Q \rightarrow 5}$

$\therefore$ Option D is correct

PART-III (MATHEMATICS)

SECTION – 1 (Maximum Marks : 24)

- This section contains **SIX (06)** questions
- Each question has **FOUR** options for correct answer(s). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct option(s).
- For each question, choose the correct option(s) to answer the question.
- Answer to each question will be evaluated according to the following marking scheme :

Full Marks	: **+4**	If only (all) the correct option(s) is (are) chosen.
Partial Marks	: **+3**	If all the four options are correct but ONLY three options are chosen.
Partial Marks	: **+2**	If three or more options are correct but ONLY two options are chosen, both of which are correct options.
Partial Marks	: **+1**	If two or more options are correct but ONLY one option is chosen and it is a correct option.
Zero Marks	: **0**	If none of the option is chosen (i.e. the question is unanswered).
Negative Marks	: **–2**	In all other cases.

- **For Example :** If first, third and fourth are the ONLY three correct options for a question with second option being an incorrect option ; selecting only all the three correct options will result in +4 marks. Selecting only two of the three correct options (e.g. the first and fourth options), without selecting any incorrect option (second option in this case), will result in +2 marks. Selecting only one of the three correct options (either first or third or fourth option), without selecting any incorrect option (second option in this case), will result in +1 marks. Selecting any incorrect option(s) (second option in this case), with or without selection of any correct option (s) will result in –2 marks.

Q.1 For any positive integer n, define $f_n : (0, \infty) \to R$ as

$$f_n(x) = \sum_{j=1}^{n} \tan^{-1}\left(\frac{1}{1+(x+j)(x+j-1)}\right) \text{for all } x \in (0, \infty).$$

$\left(\text{Here, the inverse trigonometric function } \tan^{-1} x \text{ assumes values in } \left(-\frac{\pi}{2}, \frac{\pi}{2}\right).\right)$

Then, which of the following statement(s) is (are) TRUE ?

(A) $\sum_{j=1}^{5} \tan^2(f_j(0)) = 55$

(B) $\sum_{j=1}^{10} (1 + f_j'(0)) \sec^2(f_j(0)) = 10$

(C) For any fixed positive integer n, $\lim_{x\to\infty} \tan(f_n(x)) = \frac{1}{n}$

(D) For any fixed positive integer n, $\lim_{x\to\infty} \sec^2(f_n(x)) = 1$

Ans. **[D]**

Sol. $f_n(x) = \sum_{i=1}^{n}(\tan^{-1}(x+j)-\tan^{-1}(x+j-1))$

$= (\tan^{-1}(x+1) - \tan^{-1}x) + (\tan^{-1}(x+2) - \tan^{-1}(x+1))\ .... + (\tan^{-1}(x+n) - \tan^{-1}(x+n-1))$

$f_n(x) = (\tan^{-1}(x+n) - \tan^{-1}x) = \tan^{-1}\dfrac{n}{1+x(x+n)}$

$f_n(0) = \tan^{-1} n$

$f_n'(x) = \dfrac{1}{1+(x+n)^2} - \dfrac{1}{1+x^2} \Rightarrow f_n'(0) = \dfrac{1}{1+n^2} - 1 \Rightarrow (1 + f_n'(0)) = \dfrac{1}{1+n^2}$

$\sum_{j=1}^{5}\tan^2(f_j(0)) = \sum_{j=1}^{5}(\tan(\tan^{-1} j))^2 = \sum_{j=1}^{5} i^2 = 1^2+2^2+.....+5^2 = \dfrac{5.6.(11)}{6} = 55$

$f_j(0) = \tan^{-1} j$

$f_j'(0) = \dfrac{1}{1+j^2} - 1$

$\sum_{j=1}^{10}\left(\dfrac{1}{1+j^2}\right)\left(1+\tan^2(f_j(0))\right) = \sum_{j=1}^{10}\dfrac{1}{(1+j^2)}(1+j^2) = \sum_{j=1}^{10} 1 = 10$

$\lim_{x\to\infty}\tan(f_n(x)) = \lim_{x\to\infty}\dfrac{n}{1+x(x+n)} = 0$

$\lim_{x\to\infty}\sec^2(f_n(x)) = \lim_{x\to\infty} 1+\tan^2(f_n(x)) = 1+0$

Q.2 Let T be the line passing through the points P(–2, 7) and Q(2, –5). Let F_1 but the set of all pairs of circles (S_1, S_2) such that T is tangent to S_1 at P and tangent to S_2 at Q, and also such that S_1 and S_2 touch other other at a point, say, M. Let E_1 be the set representing the locus of M as the pair (S_1, S_2) varies in F_1. Let the set of all straight line segments joining a pair of distinct points of E_1 and passing through the point R (1, 1) be F_2. Let E_2 be the set of the mid-points of the line segments in the set F_2. Then, which of the following statement(s) is (are) TRUE ?

(A) The point (–2, 7) lies in E_1

(B) The point $\left(\dfrac{4}{5},\dfrac{7}{5}\right)$ does **NOT** lie in E_2

(C) The point $\left(\dfrac{1}{2},1\right)$ lies in E_2

(D) The point $\left(0,\dfrac{3}{2}\right)$ does **NOT** line in E_1

Ans. **[B,D]**

Sol. On straight line C_1C_2

$\therefore \angle PMQ = 180 - (\alpha + \beta)$

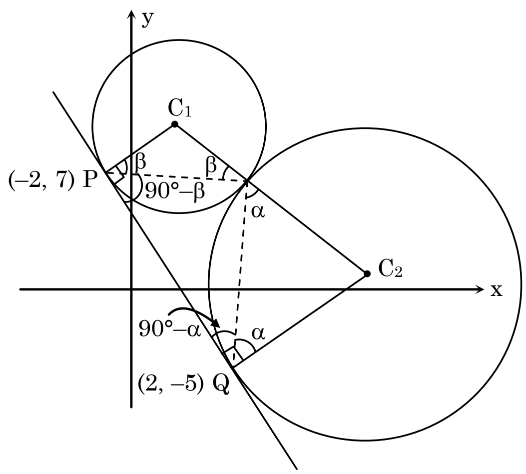
Also in ΔPMQ

$\angle PMQ = \alpha + \beta$

$\therefore \alpha + \beta = 180 - (\alpha + \beta)$

$\alpha + \beta = 90°$

$\Rightarrow \angle PMQ = 90°$



E_1 = Locus of M will be a circle having PQ as diameter

i.e., $(x + 2)(x - 2) + (y - 7)(y + 5) = 0$

$\Rightarrow x^2 + y^2 - 2y - 39 = 0 \quad (x \neq \pm 2)$

$\therefore$ only (D) satisfy E_1

Now, E_2 is set of mid point of line segments in F_2

i.e., locus of mid-point of chord passing through (1, 1) of E_1

$\therefore$ using $T = S_1$, we get

$hx + ky - (y + k) = h^2 + k^2 - 2k$

put (1, 1) we get

$h + k - (1 + k) = h^2 + k^2 - 2k$

$h^2 + k^2 - h - 2k + 1 = 0$

$(h, k) \Rightarrow (x, y)$

$x^2 + y^2 - x - 2y + 1 = 0$

point (4/5, 7/5) does not lie on E_2 because of it is mid-point of chord whose one end point is P(–2, 7).

Q.3 Let S be the set of all column matrices $\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$ such that $b_1, b_2, b_3 \in \mathbb{R}$ and the system of equations (in real variables)

$$-x + 2y + 5z = b_1$$
$$2x - 4y + 3z = b_2$$
$$x - 2y + 2z = b_3$$

has at least one solution. Then, which of the following system(s) (in real variables) has (have) at least one solution for each $\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} \in S$?

(A) $x + 2y + 3z = b_1$, $4y + 5z = b_2$ and $x + 2y + 6z = b_3$

(B) $x + y + 3z = b_1$, $5x + 2y + 6z = b_2$ and $-2x - y - 3z = b_3$

(C) $-x + 2y - 5z = b_1$, $2x - 4y + 10z = b_2$ and $x - 2y + 5z = b_3$

(D) $x + 2y + 5z = b_1$, $2x + 3z = b_2$ and $x + 4y - 5z = b_3$

Ans. **[A, D]**

Sol. $\Delta = \begin{vmatrix} -1 & 2 & 5 \\ 2 & -4 & 3 \\ 1 & -2 & 2 \end{vmatrix} = 0$

∞ solution are possible since no two planer are parallel 1 concident

$P_1 + \lambda P_2 = \mu P_3$.

$(-x + 2y + 5z - b_1) + \lambda(2x - 4y + 3z - b_2) = \mu(x - 2y + 2z - b_3)$

$-1 + 2\lambda = \mu$

$\left.\begin{array}{l} 2 - 4\lambda = -2\mu \\ 5 + 3\lambda = 2\mu \end{array}\right\} \lambda = 7 ; \mu = 13$

$P_1 + 7P_2 = 13P_3$

$b_1 + 7b_2 = 13b_3$

(A) $\Delta = \begin{vmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 1 & 2 & 6 \end{vmatrix} = 12 \neq 0$

Unique solution for any b_1, b_2, b_3

(B) $\Delta = \begin{vmatrix} 1 & 2 & 3 \\ 5 & 2 & 6 \\ -2 & -1 & -3 \end{vmatrix} = 0$

But $b_1 + 7b_2 \neq 13b_3$ in consistent

(C) $\Delta = \begin{vmatrix} -1 & 2 & -5 \\ 2 & -4 & 10 \\ 1 & -2 & 5 \end{vmatrix} = 0$

$b_1 + 7b_2 = 13b_3$

Planes may be parallel. For eg. $b_1 = 0$, $b_2 = 13$, $b_3 = 7$. So every value satisfying $b_1 + 7b_2 = 13b_3$ will not give infinite solutions.

(D) $\Delta = \begin{vmatrix} 1 & 2 & 5 \\ 2 & 0 & 6 \\ 1 & 4 & -5 \end{vmatrix} \neq 0$

Consistent for any b_1, b_2, b_3.

Q.4 Consider two straight lines, each of which is tangent to both the circle $x^2 + y^2 = \frac{1}{2}$ and the parabola $y^2 = 4x$. Let these lines intersect at the point Q. Consider the ellipse whose center is at the origin O(0, 0) and whose semi-major axis is OQ. If the length of the minor axis of this ellipse is $\sqrt{2}$, then which of the following statement(s) is (are) TRUE ?

(A) For the ellipse, the eccentricity is $\frac{1}{\sqrt{2}}$ and the length of the latus rectum is 1

(B) For the ellipse, the eccentricity is $\frac{1}{2}$ and the length of the latus rectum is $\frac{1}{2}$

(C) The area of the region bounded by the ellipse between the lines $x = \frac{1}{\sqrt{2}}$ and $x = 1$ is $\frac{1}{4\sqrt{2}}(\pi - 2)$

(D) The area of the region bounded by the ellipse between the lines $x = \frac{1}{\sqrt{2}}$ and $x = 1$ is $\frac{1}{16}(\pi - 2)$

Ans. **[A,C]**

Sol. $y^2 = 4x$

$y = mx + \frac{1}{m}$ $\quad x^2 + y^2 = \frac{1}{2}$

$$\left|\frac{0-0+\frac{1}{m}}{\sqrt{1+m^2}}\right| = \frac{1}{\sqrt{2}};$$

$2 = m^2 + m^4$

$m^4 + m^2 - 2 = 0$

$(m^2 - 1)(m^2 - 2) = 0$

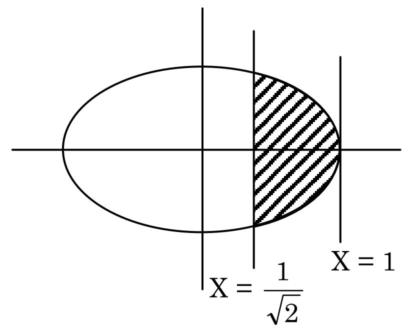
$m^2 = 1 \qquad m^2 = -2$

$m = +1$

$$\left.\begin{matrix} y = x+1 \\ y = -x-1 \end{matrix}\right| Q(-1,0)$$

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ $\quad a = 1$

Ellipse $x^2 + 2y^2 = 1$ $\quad b = \frac{1}{\sqrt{2}}$



$$e = \sqrt{1-\frac{b^2}{a^2}} = \sqrt{1-\frac{1}{2}} = \frac{1}{\sqrt{2}}$$

$$LR = \frac{2b^2}{a} = 1$$

$$\text{Area} = 2 = \int_{\frac{1}{\sqrt{2}}}^{1} \frac{1}{\sqrt{2}}\sqrt{1-x^2}\,dx = \sqrt{2}\left(\frac{x}{2}\sqrt{1-x^2} + \frac{1}{2}\sin^{-1}x\right)_{\frac{1}{\sqrt{2}}}^{1}$$

$$= \sqrt{2}\left(0 + \frac{\pi}{4} - \frac{1}{2\sqrt{2}}\sqrt{1-\frac{1}{2}} - \frac{1}{2}\frac{\pi}{4}\right)$$

$$= \sqrt{2}\left(\frac{\pi}{8} - \frac{1}{4}\right) = \frac{\pi-2}{4\sqrt{2}}$$

Q.5 Let s, t, r be non-zero complex numbers and L be the set of solutions $z = x + iy$ ($x, y \in R$, $i = \sqrt{-1}$) of the equation $sz + t\bar{z} + r = 0$, where $\bar{z} = x - iy$. Then, which of the following statement(s) is (are) TRUE ?

(A) If L has exactly one element, then $|s| \neq |t|$

(B) If $|s| = |t|$, then L has infinitely many elements

(C) The number of elements in $L \cap \{z : |z - 1 + i| = 5\}$ is at most 2

(D) If L has more than one element, then L has infinitely many elements

Ans. **[A,C,D]**

Sol. Let $s = \alpha_1 + i\beta_1$

$t = \alpha_2 + i\beta_2$

$(\alpha_1 + i\beta_1)(x + iy) + (\alpha_2 + i\beta_2)(x - iy) + (\alpha_3 + i\beta_3) = 0$

(A) $\alpha_1 x - \beta_1 y + \alpha_2 x + \beta_2 y + \alpha_3 = 0$ $\quad|\quad$ $\alpha_1 y + \beta_1 x - \alpha_2 y + \beta_2 x + \beta_3 = 0$

$(\alpha_1 + \alpha_2)x + y(\beta_2 - \beta_1) + \alpha_3 = 0$...(i) $\quad|\quad$ $(\beta_1 + \beta_2)x + (\alpha_1 - \alpha_2)y + \beta_3 = 0$...(ii)

for exactly one solution

$$-\frac{\alpha_1 + \alpha_2}{\beta_2 - \beta_1} \neq -\frac{\beta_1 + \beta_2}{\alpha_1 - \alpha_2}$$

$-(\alpha_1 + \alpha_2)(\alpha_1 - \alpha_2) \neq -(\beta_1 + \beta_2)(\beta_2 - \beta_1)$

$\alpha_2^2 - \alpha_1^2 \neq +\beta_1^2 - \beta_2^2$

$\alpha_2^2 + \beta_2^2 \neq \alpha_1^2 + \beta_1^2$

$|s| \neq |t|$

(B) Clearly $|s| = |t|$

slope of line are same

if $\alpha_3 \neq \beta_3$ parallel line

so no solution

(C) Clearly either z is singleton or represent a line (∞ solution) so that these exist at most 2 sol. hence $|z - 1 + i| = 5$ represent circle.

(D) Clearly if Eq. (i) & (ii) has more than one element then these Eq. (i) & (ii) has ∞ no. of solution (Eq. (i) & (ii) are linear equation in x & y.

Q.6 Let $f : (0, \pi) \to R$ be a twice differentiable function such that

$$\lim_{t\to x} \frac{f(x)\sin t - f(t)\sin x}{t - x} = \sin^2 x \text{ for all } x \in (0, \pi).$$

If $f\left(\frac{\pi}{6}\right) = -\frac{\pi}{12}$, then which of the following statement(s) is (are) TRUE ?

(A) $f\left(\frac{\pi}{4}\right) = \frac{\pi}{4\sqrt{2}}$

(B) $f(x) < \frac{x^4}{6} - x^2$ for all $x \in (0, \pi)$

(C) There exists $\alpha \in (0, \pi)$ such that $f'(\alpha) = 0$

(D) $f''\left(\frac{\pi}{2}\right) + f\left(\frac{\pi}{2}\right) = 0$

Ans. **[B,C,D]**

Sol. $f'_1 (0, \pi) \to R$

$$\lim_{t\to x} \frac{f(x)\sin t - f(t)\sin x}{t - x} = \sin^2 x \text{ (using L-H Rule)}$$

$$\lim_{t\to x} \frac{f(x)\cos t - f'(t)\sin x}{1 - 0} = \sin^2 x$$

$f(x)\cos x - f'(x)\sin x = \sin^2 x$

$\frac{dy}{dt} - y \cot x = -\sin x$

$IF = e^{-\int \cot x dx}$

$= e^{-\ln \sin x} = \text{cosec } x$

$y \text{ cosec} = \int -dx \Rightarrow y \text{ cosec } x = -x + c \Rightarrow y = -x \sin x + c \sin x$

$f\left(\frac{\pi}{6}\right) = \frac{\pi}{6}.\frac{1}{2} + \frac{c}{2} = -\frac{\pi}{\sqrt{2}} \Rightarrow c = 0$

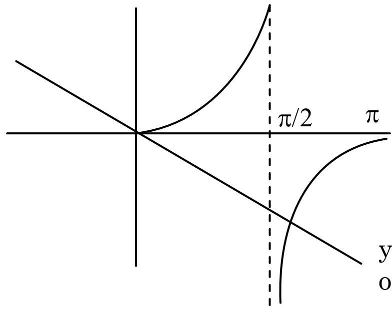
$f(x) = -x\sin x \Rightarrow f\left(\frac{\pi}{4}\right) = -\frac{\pi}{4\sqrt{2}} \Rightarrow f(x) < \frac{x^4}{6} - x^2$

$g(x) = -x\sin x + x^2 - \frac{x^4}{6} = -x\left(\sin x - x + \frac{x^3}{6}\right) < 0$ (True)

($\sin x - x + \frac{x^3}{6}$ → $\oplus$)

(C) $f'(x) = -\sin x - x\cos x = 0$

$\tan x = -x$



(D) $f''(x) = -\cos x - \cos x + x\sin x$

$f''\left(\frac{\pi}{2}\right) = 0 + \frac{\pi}{2}$ $\quad f\left(\frac{\pi}{2}\right) = -\frac{\pi}{2}$

$f''\left(\frac{\pi}{2}\right) + f\left(\frac{\pi}{2}\right) = 0$

SECTION – 2 (Maximum Marks : 24)

- This section contains **EIGHT (08)** questions. The answer to each question is a **NUMERICAL VALUE**.
- For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the **second decimal place; e.g.** 6.25, 7.00, –0.33, –.30, 30.27, –127.30) using the mouse and the onscreen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme.

Full Marks : +3 If ONLY the correct numerical value is entered as answer.

Zero Marks : 0 In all other cases.

Q.7 The value of the integral $\int_0^{\frac{1}{2}} \frac{1+\sqrt{3}}{((x+1)^2(1-x)^6)^{\frac{1}{4}}}dx$ is ________ .

Ans. **[2.00]**

Sol. $\int_0^{\frac{1}{2}} \frac{1+\sqrt{3}}{((x+1)^2(1-x)^6)^{\frac{1}{4}}}dx$

$= \int_0^{\frac{1}{2}} \frac{1+\sqrt{3}\,dx}{((x+1)^2\left(\frac{1+x}{1-x}\right)^{\frac{1}{2}}}$

$= \int_1^3 \frac{1+\sqrt{3}}{t^{\frac{1}{2}}}\frac{dt}{2} = \frac{1+\sqrt{3}}{2}\left.\frac{t^{\frac{1}{2}}}{1/2}\right|_1^3$ $\quad$ put $\frac{1+x}{1-x} = t \Rightarrow \frac{2}{1-x} - 1 = t$

$= (1 + \sqrt{3})(\sqrt{3} - 1) = 2$ $\quad \frac{-2}{(1-x)^2}(-dx) = dt$

Q.8 Let P be a matrix of order 3 × 3 such that all the entries in P are from the set {–1, 0, 1). Then, the maximum possible value of the determinant of P is ______________ .

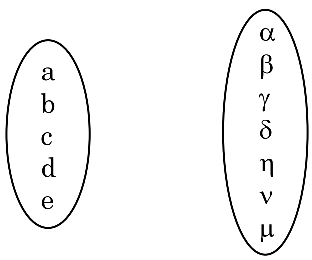
Ans. **[4.00]**

Sol. $P = \begin{vmatrix} 1 & -1 & -1 \\ 1 & 1 & -1 \\ 1 & -1 & 1 \end{vmatrix} = 1 + 1 + 1 - (-1 + 1 - 1) = 4$ maximum

Q.9 Let X be a set with exactly 5 elements and Y be a set with exactly 7 elements. If α is the number of one-one functions from X to Y and β is the number of onto functions from Y to X, then the value of $\frac{1}{5!}(\beta - \alpha)$ is __________ .

Ans. **[119.00]**

Sol.



$\alpha = {}^7C_5 \times 5! = \frac{7.6}{2}.5!$

$= 21 \times 5!$

3, 1, 1, 1, 1   2, 2, 1, 1, 1

$\beta = \left(\frac{7!}{3!(1!)^4} \times \frac{1}{4!} + \frac{7!}{(2!)^2(1!)^3} + \frac{1}{2!\,3!}\right) 5!$

$= \left(\frac{7.6.5}{6} + \frac{7.6.5.4}{8}\right) 5!$

$= (35 + 105)\, 5! = (140)\, 5!$

$\frac{\beta - \alpha}{5!} = (140 - 21) = 119$

Q.10 Let $f : R \to R$ be a differentiable function with $f(0) = 0$. If $y = f(x)$ satisfies the differential equation $\frac{dy}{dx} = (2 + 5y)(5y - 2)$, then the value of $\lim_{x \to -\infty} f(x)$ is __________ .

Ans. **[0.40]**

Sol. $\frac{dy}{dx} = 25y^2 - 4 \Rightarrow \frac{dy}{25y^2 - 4} = dx$

$\Rightarrow \frac{1}{25}\int \frac{dy}{y^2 - \left(\frac{2}{5}\right)^2} = \int dx$

$$\Rightarrow \frac{1}{25}\frac{1}{2\left(\frac{2}{5}\right)}\log\left(\frac{y-\frac{2}{5}}{y+\frac{2}{5}}\right) = x + c$$

$$\Rightarrow \log\left|\frac{y-\frac{2}{5}}{y+\frac{2}{5}}\right| = 20x + 20c$$

f(0) = 0 $\quad$ log(1) = 0 + 20c $\Rightarrow$ c = 0

$$\log\left(\frac{y-\frac{2}{5}}{y+\frac{2}{5}}\right) = 20x$$

$$\left(\frac{y-\frac{2}{5}}{y+\frac{2}{5}}\right) = e^{20x}$$

$$y - \frac{2}{5} = ye^{20x} + 0.40\ e^{20x}$$

$$f(x)\ y = \frac{0.40(1+e^{20x})}{(1-e^{20x})}$$

$$\lim_{x\to-\infty} = \frac{0.40(1+0)}{1-0} = 0.40$$

Q.11 Let $f : R \to R$ be a differentiable function with $f(0) = 1$ and satisfying the equation $f(x + y) = f(x)\, f'(y) + f'(x)\, f(y)$ for all $x, y \in R$. Then, the value of $\log_e (f(4))$ is ___________ .

Ans. **[2.00]**

Sol. $f(x + y) = f(x)\, f'(y) + f'(x)\, f(x)f(y)$ ....(1)

$x = y = 0$

$f(0) = f(0)f'(0) + f'(0)f(0)$

$2f'(0) = 1 \qquad \therefore f(0) = 1$

$$f'(0) = \frac{1}{2}$$

replace in Eq. (1)

$x = x : y = 0$

$f(x) = f'(x)\, f'(0) + f'(x)\, f(0)$

$$f(x) = \frac{f(x)}{2} + f'(x) \Rightarrow \frac{f(x)}{2} + f'(x) \Rightarrow \frac{dy}{dx} = \frac{y}{2} \Rightarrow \int\frac{dy}{dx} = \int\frac{dx}{2} \Rightarrow \log y = \frac{x}{2} + c \Rightarrow y = e^{\frac{x}{2}+c}$$

$$f(x) = ke^{\frac{x}{2}}$$

$f(0) = 1 = k$

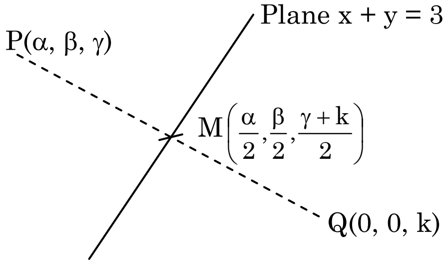
$$f(x) = e^{\frac{x}{2}}$$

$f(4) = e^2 \Rightarrow \log f(4) = 2$

Q.12 Let P be a point in the first octant, whose image Q in the plane x + y = 3 (that is, the line segment PQ is perpendicular to the plane x + y = 3 and the mid-point of PQ lies in the plane x + y = 3) lies on the z-axis. Let the distance of P from the x-axis be 5. If R is image of P in the xy-plane, then the length of PR is ______________ .

Ans. **[8.00]**

Sol. $P(\alpha, \beta, \gamma) \qquad \alpha, \beta, \gamma > 0$



$\because \sqrt{\beta^2 + \gamma^2} = 5$

$\beta^2 + \gamma^2 = 25$

$9 + \gamma^2 = 25$

$\gamma = 4$

$\frac{\alpha}{2} + \frac{\beta}{2} = 3$

$\alpha + \beta = 6$

$\alpha = \beta = 3$

direction ratio of PQ

$\alpha - 0, \beta - 0, \gamma - k$

1, 1, 0

$\alpha = \beta\ ; \quad \gamma = k$

P(3, 3, 4)

XY plane

Z = 0

$$\frac{x-3}{0} = \frac{y-3}{0} = \frac{z-4}{1} = -2\frac{(4)}{1} = -8$$

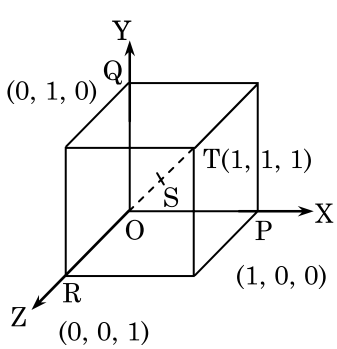
R(3, 3, –4)

PR = 8

Q.13 Consider the cube in the first octant with sides OP, OQ and OR of length 1, along the x-axis, y-axis and z-axis, respectively, where O (0, 0, 0) is the origin. Let $S\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right)$ be the centre of the cube and T be the vertex of the cube opposite to the origin O such that S lies on the diagonal OT. If $\vec{p} = \overrightarrow{SP}$, $\vec{q} = \overrightarrow{SQ}$, $\vec{r} = \overrightarrow{SR}$ and $\vec{t} = \overrightarrow{ST}$, then the value of $|(\vec{p} \times \vec{q}) \times (\vec{r} \times \vec{t})|$ is _________ .

Ans. **[0.50]**

Sol.



$\vec{p} = \vec{SP} = \hat{i} - \frac{\hat{i}}{2} - \frac{\hat{j}}{2} - \frac{\hat{k}}{2} = \frac{\hat{i}}{2} - \frac{\hat{j}}{2} - \frac{\hat{k}}{2}$

$\vec{q} = \overrightarrow{SQ} = \hat{j} - \frac{\hat{i}}{2} - \frac{\hat{j}}{2} - \frac{\hat{k}}{2} = -\frac{\hat{i}}{2} + \frac{\hat{j}}{2} - \frac{\hat{k}}{2}$

$\vec{r} = \overrightarrow{SR} = \hat{k} - \frac{\hat{i}}{2} - \frac{\hat{j}}{2} - \frac{\hat{k}}{2} = -\frac{\hat{i}}{2} - \frac{\hat{j}}{2} + \frac{\hat{k}}{2}$

$\vec{t} = \overrightarrow{ST} = \hat{i} + \hat{j} + \hat{k} - \frac{\hat{i}}{2} - \frac{\hat{j}}{2} - \frac{\hat{k}}{2} = \frac{\hat{i}}{2} + \frac{\hat{j}}{2} + \frac{\hat{k}}{2}$

$$\vec{p} \times \vec{q} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & +\frac{1}{2} & -\frac{1}{2} \end{vmatrix} = \hat{i}\left(\frac{1}{4} + \frac{1}{4}\right) - \hat{j}\left(-\frac{1}{4} - \frac{1}{4}\right) + \hat{k}\left(\frac{1}{4} - \frac{1}{4}\right) = \frac{\hat{i}}{2} + \frac{\hat{j}}{2}$$

$$\vec{r} \times \vec{t} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{vmatrix} = \hat{i}\left(-\frac{1}{4} - \frac{1}{4}\right) - \hat{j}\left(-\frac{1}{4} - \frac{1}{4}\right) + \hat{k}(0) = -\frac{\hat{i}}{2} + \frac{\hat{j}}{2}$$

$$(\vec{p} \times \vec{q}) \times (\vec{r} \times \vec{t}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{1}{2} & \frac{1}{2} & 0 \\ -\frac{1}{2} & \frac{1}{2} & 0 \end{vmatrix} = \left(\frac{1}{4} + \frac{1}{4}\right)\hat{k} = \frac{\hat{k}}{2}$$

$$\left|(\vec{p} \times \vec{q}) \times (\vec{r} \times \vec{t})\right| = \frac{1}{2}$$

Q.14 Let X = $({}^{10}C_1)^2 + 2({}^{10}C_2)^2 + 3({}^{10}C_3)^2 + ... + 10({}^{10}C_{10})^2$, where ${}^{10}C_r$, $r \in \{1, 2, ..., 10\}$ denote binomial coefficients. Then, the value of $\frac{1}{1430}X$ is ______________ .

Ans. **[646.00]**

Sol. $x = \sum_{r=1}^{10} r\left({}^{10}C_r\right)^2 = \sum_{r=1}^{10} r\frac{10}{r}\ {}^{9}C_{r-1}.{}^{10}C_r = 10 \sum_{r=1}^{10} {}^{9}C_{r-1}.{}^{10}C_r$

$= 10\ [{}^9C_0\ {}^{10}C_1 + {}^9C_1\ {}^{10}C_2 + ... + {}^9C_9\ {}^{10}C_{10}]$

$= 10$ [coeff of x^9 in $(1 + x)^9 (x + 1)^{10}$]

$= 10$ [coeff of x^9 in $(1 + x)^{19}$]

$= 10 \cdot {}^{19}C_9 = 10 \cdot \frac{19 \cdot 18 \cdot 17 \cdot 16 \cdot 15 \cdot 14 \cdot 13 \cdot 12 \cdot 11}{9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$

$X = 10 \cdot 19 \cdot 17 \cdot 13 \cdot 2 \cdot 11$

$\frac{X}{1430} = \frac{10 \cdot 19 \cdot 17 \cdot 13 \cdot 2 \cdot 11}{10 \times 143} = 38 \times 17 = 646$

SECTION – 3 (Maximum Marks : 12)

- This section contains **FOUR (04)** questions.
- Each question has **TWO (02)** matching lists : **LIST-I** and **LIST-II**.
- **FOUR** options are given representing matching of elements from **LIST-I** and **LIST-II**. **ONLY ONE** of these four options corresponds to a correct matching.
- For each question, choose the option corresponding to the correct matching.
- For each question, marks will be awarded according to the following marking scheme :

  Full Marks : +3 If ONLY the option corresponding to the correct matching is chosen.

  Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered.)

  Negative Marks : –1 in all other cases.

Q.15 Let $E_1 = \left\{x \in R : x \neq 1 \text{ and } \frac{x}{x-1} > 0\right\}$

and $E_2 = \left\{x \in E_1 : \sin^{-1}\left(\log_e\left(\frac{x}{x-1}\right)\right) \text{is a real number}\right\}$

$\left(\text{Here, the inverse trigonometric function } \sin^{-1} x \text{ assumes values in} \left[-\frac{\pi}{2}, \frac{\pi}{2}\right].\right)$

Let $f : E_1 \to R$ be the function defined by $f(x) = \log_e\left(\frac{x}{x-1}\right)$

and $g : E_2 \to R$ be the function defined by $g(x) = \sin^{-1}\left(\log_e\left(\frac{x}{x-1}\right)\right)$

List-I	List-II
(P) The range of f is	(1) $\left(-\infty, \frac{1}{1-e}\right] \cup \left[\frac{e}{e-1}, \infty\right)$
(Q) The range of g contains	(2) (0, 1)
(R) The domain of f contains	(3) $\left[-\frac{1}{2}, \frac{1}{2}\right]$
(S) The domain of g is	(4) $(-\infty, 0) \cup (0, \infty)$
	(5) $\left(-\infty, \frac{e}{e-1}\right]$
	(6) $(-\infty, 0) \cup \left(\frac{1}{2}, \frac{e}{e-1}\right]$

The correct option is :

(A) (P) → (4); (Q) → (2); (R) → (1); (S) → (1)

(B) (P) → (3); (Q) → (3); (R) → (6); (S) → (5)

(C) (P) → (4); (Q) → (2); (R) → (1); (S) → (6)

(D) (P) → (4); (Q) → (3); (R) → (6); (S) → (5)

Ans. **[A]**

Sol. $\frac{x}{x-1} > 0$ $\quad x \in (-\infty, 0) \cup (1, \infty) \equiv E_1$

$-1 \leq \log_e\left(\frac{x}{x-1}\right) \leq 1 \Rightarrow e^{-1} \leq \frac{x}{x-1} \leq e$

$\frac{1}{e} \le \frac{x}{x-1}$ f $\qquad \frac{x}{x-1} \le e$

$\frac{x}{x-1} - \frac{1}{e} \ge 0 \qquad \frac{x-ex+e}{x-1} \le 0$

$+ \quad \frac{-1}{e-1} \quad - \quad 1 \quad + \qquad\qquad - \quad 1 \quad + \quad \frac{e}{e-1} \quad -$

$$x \in \left(-\infty, \frac{-1}{e-1}\right] \cup (1, \infty) \cap x \in (-\infty, 1) \cup \left[\frac{e}{e-1}, \infty\right)$$

$$E_2 : x \in \left(-\infty, \frac{-1}{e-1}\right] \cup \left[\frac{e}{e-1}, \infty\right)$$

$$f(x) = \ln\left(\frac{x}{x-1}\right)$$

Domain of $f(x)$ $\qquad x \in (-\infty, 0) \cup (1, \infty)$

Range of $f(x)$ $\qquad y \in (-\infty, 0) \cup (0, \infty)$

$g(x) = \sin^{-1}\left(\ln\left(\frac{x}{x-1}\right)\right)$ $\qquad$ Domain of g(x)

$$x \in \left(-\infty, \frac{-1}{e-1}\right] \cup \left[\frac{e}{e-1}, \infty\right)$$

Range of g(x) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$

Q.16 In a high school, a committee has to be formed from a group of 6 boys $M_1, M_2, M_3, M_4, M_5, M_6$ and 5 girls G_1, G_2, G_3, G_4, G_5.

(i) Let α_1 be the total number of ways in which the committee can be formed such that the committee has 5 members, having exactly 3 boys and 2 girls.

(ii) Let α_2 be the total number of ways in which the committee can be formed such that the committee has at least 2 members, and having an equal number of boys and girls.

(iii) Let α_3 be the total numbers of ways in which the committee can be formed such that the committee has 5 members, at least 2 of them being girls.

(iv) Let α_4 be the total number of ways in which the committee can be formed such that the committee has 4 members, having at least 2 girls and such that both M_1 and G_1 are **NOT** in the committee together.

List-I	**List-II**
(P) The value of α_1 is	(1) 136
(Q) The value of α_2 is	(2) 189
(R) The value of α_3 is	(3) 192
(S) The value of α_4 is	(4) 200
	(5) 381
	(6) 461

The correct option is :

(A) (P) → (4); (Q) → (6); (R) → (2); (S) → (1)

(B) (P) → (1); (Q) → (4); (R) → (2); (S) → (3)

(C) (P) → (4); (Q) → (6); (R) → (5); (S) → (2)

(D) (P) → (4); (Q) → (2); (R) → (3); (S) → (1)

Ans. **[C]**

Sol. (i) $\alpha_1 = {}^6C_3 \times {}^5C_2 = 200$

(ii) $\alpha_2 = {}^6C_1 \times {}^5C_1 + {}^6C_2 \times {}^5C_2 + {}^6C_3 \times {}^5C_3 + {}^6C_4 \times {}^5C_4 + {}^6C_5 \times {}^5C_5 = {}^{11}C_5 - 1 = 461$

(iii) $\alpha_3 = {}^5C_2 \times {}^6C_3 + {}^5C_3 \times {}^6C_2 + {}^5C_4 \times {}^6C_1 + {}^5C_5 \times {}^6C_0 = 200 + 150 + 30 + 1 = 381$

(iv) $\alpha_4 = {}^5C_2 \times {}^6C_2 - {}^4C_1 \times {}^5C_1 + {}^5C_3 \times {}^6C_1 - {}^4C_2 \times {}^1C_1 + {}^5C_1 + {}^5C_4 = 150 - 20 + 60 - 6 + 5 = 189$

Q.17 Let H : $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, where a > b > 0, be a hyperbola in the xy-plane whose conjugate axis LM subtends an angle of 60° at one of its vertices N. Let the area of the triangle LMN be $4\sqrt{3}$.

List-I	**List-II**
(P) The length of the conjugate axis of H is	(1) 8
(Q) The eccentricity of H is	(2) $\frac{4}{\sqrt{3}}$
(R) The distance between the foci of H is	(3) $\frac{2}{\sqrt{3}}$
(S) The length of the latus rectum of H is	(4) 4

The correct option is :

(A) (P) → (4); (Q) → (2); (R) → (1); (S) → (3)

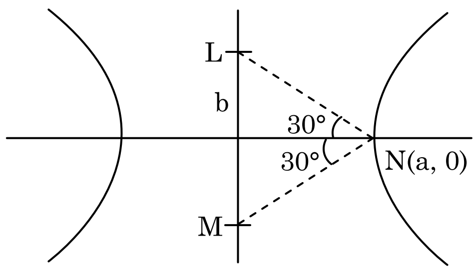
(B) (P) → (4); (Q) → (3); (R) → (1); (S) → (2)

(C) (P) → (4); (Q) → (1); (R) → (3); (S) → (2)

(D) (P) → (3); (Q) → (4); (R) → (2); (S) → (1)

Ans. **[B]**

Sol.



$\tan 30° = \frac{b}{a} = \frac{1}{\sqrt{3}} \Rightarrow a = \sqrt{3}\, b$

$$e = \sqrt{1 + \frac{b^2}{a^2}} = \sqrt{1 + \frac{1}{3}} = \frac{2}{\sqrt{3}}$$

Area of $\Delta LMN = 4\sqrt{3}$

$\frac{1}{2}(2b)\, a = 4\sqrt{3}$

$b\sqrt{3}\, b = 4\sqrt{3}$

$b = 2 \qquad a = 2\sqrt{3}$

(P) $\ell_{conjugate} = 2b = 4.$

(Q) $e = \frac{2}{\sqrt{3}}$

(R) distance between focii = $2ae = 4\sqrt{3}\left(\frac{2}{\sqrt{3}}\right) = 8$

(S) $\ell_{L.R} = \frac{2b^2}{a} = \frac{2 \times 4}{2\sqrt{3}} = \frac{4}{\sqrt{3}}$

Q.18 Let $f_1 : R \to R$, $f_2 : \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \to R$, $f_3 : (-1, e^{\frac{\pi}{2}} - 2) \to R$ and $f_4 : R \to R$ be functions defined by

(i) $f_1(x) = \sin(\sqrt{1-e^{-x^2}})$,

(ii) $f_2(x) = \begin{cases} \frac{|\sin x|}{\tan^{-1} x} & \text{if } x \neq 0 \\ 1 & \text{if } x = 0 \end{cases}$, where the inverse trigonometric function $\tan^{-1} x$ assumes values in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

(iii) $f_3(x) = [\sin(\log_e(x+2))]$, where, for $t \in R$, $[t]$ denotes the greatest integer less than or equal to t

(iv) $f_4(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$

List-I	List-II
(P) The function f_1 is	(1) **NOT** continuous at x = 0
(Q) The function f_2 is	(2) continuous at x = 0 and **NOT** differentiable at x = 0
(R) The function f_3 is	(3) differentiable at x = 0 and its derivative is **NOT** continuous at x = 0
(S) The function f_4 is	(4) differentiable at x = 0 and its derivative is continuous at x = 0

The correct option is :

(A) (P) → (2); (Q) → (3); (R) → (1); (S) → (4)

(B) (P) → (4); (Q) → (1); (R) → (2); (S) → (3)

(C) (P) → (4); (Q) → (2); (R) → (1); (S) → (3)

(D) (P) → (2); (Q) → (1); (R) → (4); (S) → (3)

Ans. **[D]**

Sol. (P) $f_1'(x) = \cos\sqrt{1-e^{-x^2}}\ \frac{1}{2\sqrt{1-e^{-x^2}}}(0 - e^{-x^2}(2x))$

at x = 0 $\quad f_1'(x)$ Does not exist

$P \to 2$

(Q) $f_2(0^+) = \lim_{h\to 0} \frac{|\sin h|}{\tan^{-1} h} = \lim_{h\to 0} \frac{\sin h}{h\frac{\tan^{-1} h}{h}} = 1$

$f_2(0^-) = \lim_{h\to 0} \frac{|-\sin h|}{-\tan^{-1} h} = \lim_{h\to 0} \frac{\sin h}{h\left(-\frac{\tan^{-1} h}{h}\right)} = -1$

discontinuous & not differentiable

(R) $F_3(x) = [\sin(\ln(x + 2))]$

$F_3(0^+)$ $\lim_{h\to 0}$ [sin ln (2 + h)] = [lies between 0 to 1] = 0

Same as

$F_3(0^-) = \lim_{h\to 0} [\sin(2 - h)] = 0$

$F_3(0) = [\sin(\ln 2)] = 0$

$f(x) = 0 \quad x \in [-h, h]$ h is infinitely small

So that function is differentiable at x = 0 and its derivative also continuous at x = 0

(S) $f(x) = \begin{Bmatrix} x^2 \sin\left(\frac{1}{x}\right) & x \neq 0 \\ 0 & x = 0 \end{Bmatrix}$

$$f_4'(0^+) = \lim_{h\to 0} \frac{f(0+h).f(0)}{h} = \lim_{h\to 0} \frac{h^2 \sin\left(\frac{1}{4}\right) - 0}{h} = \lim_{h\to 0} h \sin\left(\frac{1}{4}\right) = 0$$

$$f_4'(0^-) = \lim_{h\to 0} \frac{f(0-h) - f(0)}{-h} = \lim_{h\to 0} \frac{h^2 \sin\left(-\frac{1}{4}\right) - 0}{-h} = \lim_{h\to 0} h \sin\left(\frac{1}{4}\right) = 0$$

$f_4'(0^-) =$ Doesn't exist

So function is differentiable at x = 0 and its derivative is not continuous at x = 0.