CAREER POINT

JEE Advance Exam 2015 (Solution)

Date : 24 / 05 / 2015

Part I - PHYSICS

SECTION – 1 (Maximum Marks : 32)

- This Section contains EIGHT questions
- The answer to each question is a **SINGLE DIGIT INTEGER** ranging from 0 to 9, both inclusive.
- For each question, darken the bubble corresponding to the correct integer in the ORS
- Marking scheme :

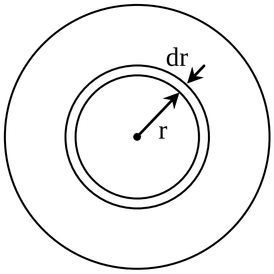
  + 4 If the bubble corresponding to the answer is darkened

  0 In all other cases

Q.1 The densities of two solid spheres A and B of the same radii R vary with radial distance r as $\rho_A(r) = k\left(\frac{r}{R}\right)$ and $\rho_B(r) = k\left(\frac{r}{R}\right)^5$, respectively, where k is a constant. The moments of inertia of the individual spheres about axes passing through their centres are I_A and I_B, respectively. If $\frac{I_B}{I_A} = \frac{n}{10}$, the value of n is

Ans. **[6]**

Sol.



Consider a shell of radius r and thickness dr

mass of element = $\rho \times 4\pi r^2 dr$

Moment of inertial of shell = $dI = \frac{2}{3}\rho \times 4\pi r^2 dr\, r^2$

$$dI = \frac{2}{3} \times \rho \times 4\pi r^4 dr$$

$$dI = \frac{2}{3} \times \frac{kr}{R} \times 4\pi r^4\, dr$$

$$dI = \frac{2}{3}\ \frac{k}{R} \times 4\pi r^5\, dr$$

$$I = \frac{2}{3}\ \frac{k}{R} \times\ 4\pi\int_0^R [r^5 dr]$$

$$I_A = \frac{2}{3}\ \frac{k}{R} \times \frac{4\pi R^6}{6} = \frac{2}{3}\ k \times 4\pi\ \frac{R^5}{6}$$

For $I_2 = \frac{2}{3} \times \frac{kr^5}{R^5} \times 4\pi r^4 dr$

$$I_2 \Rightarrow \frac{2}{3} \times \frac{k}{R^5} \times 4\pi \int_0^R r^9 dr$$

$$I_B = \frac{2}{3} \times \frac{k}{R^5} \times \frac{4\pi R^{10}}{10} = \frac{2}{3} \times k \times \frac{4\pi R^5}{10}$$

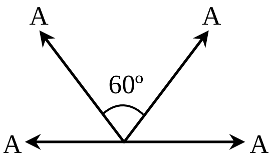
$$\frac{I_B}{I_A} = \frac{6}{10}$$

Q.2 Four harmonic waves of equal frequencies and equal intensities I_0 have phase angles 0, $\pi/3$, $2\pi/3$ and π. When they are superposed, the intensity of the resulting wave is nI_0. The value of n is

Ans. **[3]**

Sol.

$I_0 = kA^2$



Resultant amplitude = $\sqrt{3}$ A

$I_R = k3A^2$

$I_R = 3I_0$

n = 3

Q.3 For a radioactive material, its activity A and rate of change of its activity R are defined as $A = -\frac{dN}{dt}$ and $R = -\frac{dA}{dt}$, where N(t) is the number of nuclei at time t. Two radioactive sources P (mean life τ) and Q (mean life 2τ) have the same activity at t = 0. Their rates of change of activities at t = 2τ are R_P and R_Q, respectively. If $\frac{R_P}{R_Q} = \frac{n}{e}$, then the value of n is

Ans. [2]

Sol. $R = -\dfrac{dA}{dt}$

$A = -\dfrac{dN}{dt}$

At time t

$A = A_0 e^{-\lambda t}$

$R = -\dfrac{dA}{dt} = -A_0 e^{-\lambda t}(-\lambda)$

$R = A_0 \lambda e^{-\lambda t}$

$\lambda_P = \dfrac{1}{\tau} \qquad \lambda_Q = \dfrac{1}{2\tau}$

$R_P = A_0\, \lambda_P\, e^{-\frac{1}{\tau}2\tau} \Rightarrow A_0 \lambda_P e^{-2}$

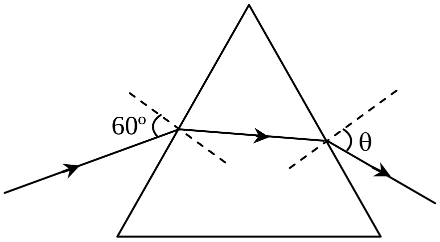
$R_Q = A_0 \lambda_Q\, e^{-\frac{1}{2\tau}\times 2\tau} = A_0 \lambda_Q e^{-1}$

$\dfrac{R_P}{R_Q} = \dfrac{\lambda_P}{\lambda_Q}\, \dfrac{e^{-2}}{e^{-1}}$

$\Rightarrow \dfrac{2}{e}$

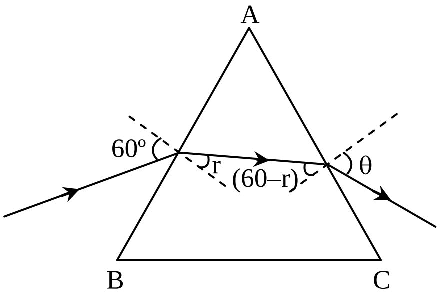
$\therefore \quad n = 2$

Q.4 A monochromatic beam of light is incident at 60º on one face of an equilateral prism of refractive index n and emerges from the opposite face making an angle θ(n) with the normal (see the figure). For $n = \sqrt{3}$ the value of θ is 60º and $\frac{d\theta}{dn} = m$. The value of m is



Ans. **[2]**

Sol.



Applying snell law at face AB.

$\sin 60^\circ = n \times \sin r$

$\sin r = \frac{\sqrt{3}}{2n}$ ....(1)

Applying snell law at face AC

$n \times \sin(60 - r) = 1 \times \sin\theta$ ....(2)

if $n = \sqrt{3}$, $\theta = 60^\circ$

$\sqrt{3}\sin(60 - r) = \sin 60$

$\sin(60 - r) = \frac{1}{2}$

$60 - r = 30$

$r = 30^\circ$

from (1) $\sin 30^\circ = \frac{\sqrt{3}}{2n}$

$n = \sqrt{3}$

differentiating (2) with respect to n

$$1 \times \sin(60 - r) + n \times \cos(60 - r)\left[-\frac{dr}{dn}\right] = \cos\theta\,\frac{d\theta}{dn} \qquad ....(3)$$

differentiating (1) with respective to n

$$\cos r\,\frac{dr}{dn} = \frac{\sqrt{3}}{2}\left[\frac{-1}{n^2}\right] \qquad ....(4)$$

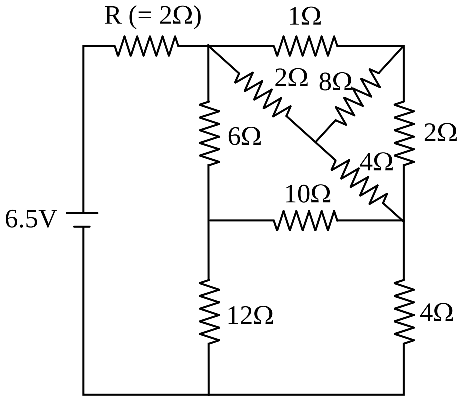
when n = $\sqrt{3}$ r = 30

$$\cos 30^o\,\frac{dr}{dn} = \frac{\sqrt{3}}{2}\left[-\frac{1}{3}\right] \qquad \because \quad -\frac{dr}{dn} = \frac{1}{3}$$

Put r = 30º, θ = 60º and $-\frac{dr}{dn} = \frac{1}{3}$ in equation (3)

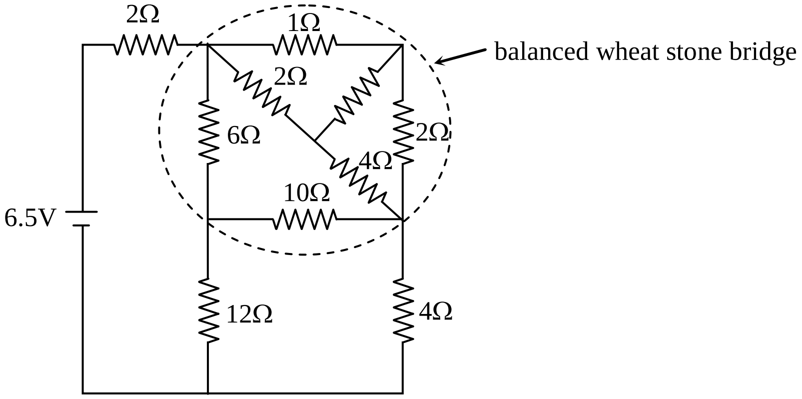
$$\sin(60 - 30) + \sqrt{3} \times \cos(60 - 30)\left[\frac{1}{3}\right] = \frac{1}{2}\,\frac{d\theta}{dn} \qquad \therefore \qquad \frac{d\theta}{dn} = 2$$

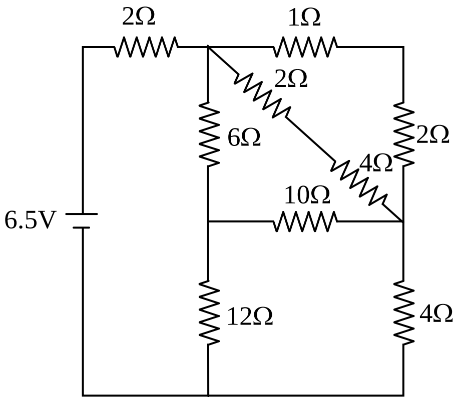
Q.5 In the following circuit, the current through the resistor R (= 2Ω) is I Amperes. The value of I is

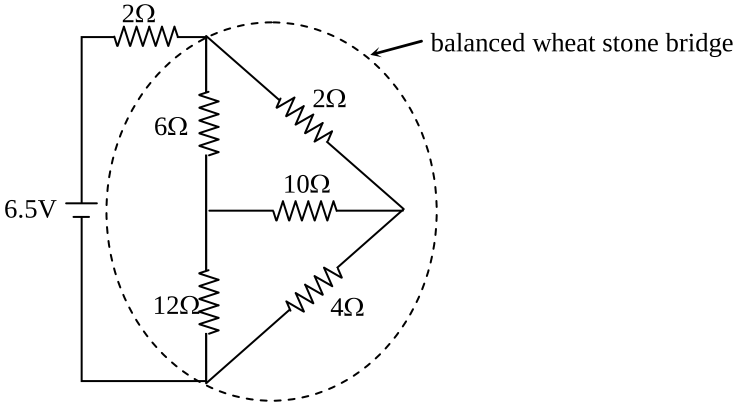


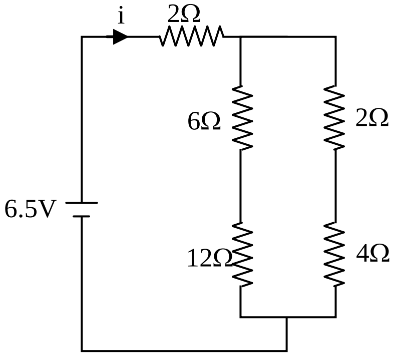
Ans. **[1]**

Sol.






balanced wheat stone bridge



$R_{eq.} = 6.5\Omega$

$$i = \frac{6.5}{6.5} = 1 \text{ Amp.}$$

Q.6 An electron in an excited state of Li^{2+} ion has angular momentum $3h/2\pi$. The de Broglie wavelength of the electron in this state is $p\pi a_0$ (where a_0 is the Bohr radius). The value of p is

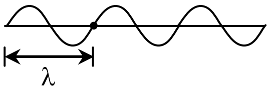
Ans. **[2]**

Sol. $L = \frac{nh}{2\pi}$

$\frac{3h}{2\pi} = \frac{nh}{2\pi}$

$\therefore \quad n = 3$

if $\quad n = 3$



(λ = de Broglie wavelength)

3λ = length of orbit

length of orbit $= 2\pi r_n$

$= \frac{2\pi a_0 n^2}{Z} = \frac{2\pi a_0 3^2}{3}$

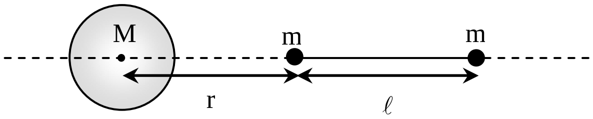
$= 6\pi a_0$

$3\lambda = 6\pi a_0$

$\lambda = 2\pi a_0$

So value of p = 2

Q.7 A large spherical mass M is fixed at one position and two identical point masses m are kept on a line passing through the centre of M (see figure). The point masses are connected by a rigid massless rod of length ℓ and this assembly is free to move along the line connecting them. All three masses interact only through their mutual gravitational interaction. When the point mass nearer to M is at a distance $r = 3\ell$ from M, the tension in the rod is zero for $m = k\left(\frac{M}{288}\right)$. The value of k is



Ans. **[7]**

Sol. $F_1 = \frac{GMm}{9\ell^2} - \frac{Gm^2}{\ell^2}$ (Force between M and middle m)

$F_2 = \frac{GMm}{16\ell^2} + \frac{Gm^2}{\ell^2}$ (Force between M and extreme m)

$F_1 = F_2$

$$\frac{GMm}{9\ell^2} - \frac{Gm^2}{\ell^2} = \frac{GMm}{16\ell^2} + \frac{Gm^2}{\ell^2}$$

$$M\left[\frac{1}{9} - \frac{1}{16}\right] = 2m$$

$$M\left[\frac{16-9}{144}\right] = 2m$$

$$m = \frac{7}{288}M$$

m = 7

Q.8 The energy of a system as a function of time t is given as $E(t) = A^2 \exp(-\alpha t)$, where $\alpha = 0.2\ s^{-1}$. The measurement of A has an error of 1.25 %. If the error in the measurement of time is 1.50 %, the percentage error in the value of E(t) at t = 5 s is

Ans. **[4]**

Sol. $E = A^2e^{-\alpha t}$

$E = A^2e^{-0.2t}$

$\ell n\,E = 2\ell nA - 0.2t$

$$\frac{dE}{E} = 2\frac{dA}{A} - 0.2\ dt$$

$$\frac{dE}{E} = 2\frac{dA}{A} - 0.2\ dt$$

$\frac{dt}{t} = 1.5\ \%$ $\quad\therefore\quad$ $dt = 1.5 \times 5\ \% = 1.5 \times 5 = 7.5$

$= 2 \times 1.25 + 0.2 \times 7.5$

$= 2.5 + 1.5$

$= 4\ \%$

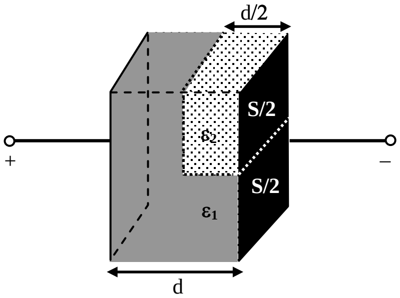
SECTION – 2 (Maximum Marks : 32)

- This section contains **EIGHT** questions
- Each questions has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four options(s) is(are) correct
- For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS
- Marking scheme :

+4 If only the bubble(s) corresponding to all the correct option(s) is (are) darkened

0	If none of the bubbles is darkened
–2	In all other cases

Q.9 A parallel plate capacitor having plates of area S and plate separation d, has capacitance C_1 in air. When two dielectrics of different relative permittivities ($\varepsilon_1 = 2$ and $\varepsilon_2 = 4$) are introduced between the two plates as shown in the figure, the capacitance becomes C_2. The ratio $\frac{C_2}{C_1}$ is

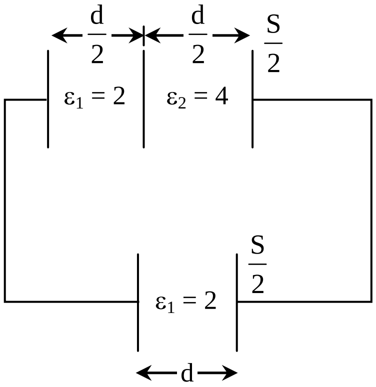


(A) 6/5 (B) 5/3 (C) 7/5 (D) 7/3

Ans. **[D]**

Sol. When no dielectric is filled then $C_1 = \frac{\varepsilon_0 S}{d}$

when dielectric is filled



$$C_2 = \frac{\varepsilon_0 \varepsilon_1 S}{2d} + \frac{\varepsilon_0 S}{2\left[\frac{d}{2\varepsilon_1} + \frac{d}{2\varepsilon_2}\right]}$$

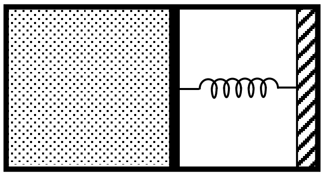
$$= \frac{\varepsilon_0 2 \times S}{2d} + \frac{\varepsilon_0 S}{2\left[\frac{d}{2\times 2} + \frac{d}{2\times 4}\right]}$$

$$\Rightarrow \quad \frac{\varepsilon_0 S}{d} + \frac{\varepsilon_0 S \times 8}{2 \times 3d}$$

$$\Rightarrow \quad \frac{\varepsilon_0 S}{d}\left[\frac{14}{6}\right] \Rightarrow \frac{\varepsilon_0 S}{d}\left[\frac{7}{3}\right]$$

$$\frac{C_2}{C_1} = \frac{7}{3}$$

Q.10 An ideal monoatomic gas is confined in a horizontal cylinder by a spring loaded piston (as shown in the figure). Initially the gas is at temperature T_1, pressure P_1 and volume V_1 and the spring is in its relaxed state. The gas is then heated very slowly to temperature T_2, pressure P_2 and volume V_2. During this process the piston moves out by a distance x. Ignoring the friction between the piston and the cylinder. the correct statements (s) is (are)



(A) If $V_2 = 2V_1$ and $T_2 = 3T_1$, then the energy stored in the spring is $\frac{1}{4}P_1V_1$

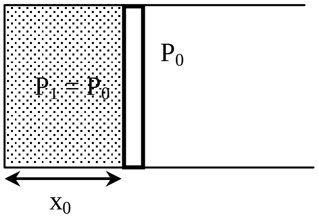
(B) If $V_2 = 2V_1$ and $T_2 = 3T_1$, then the change in internal energy is $3P_1V_1$

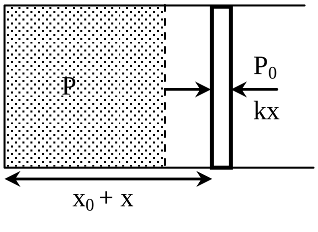
(C) If $V_2 = 3V_1$ and $T_2 = 4T_1$, then the work done by the gas is $\frac{7}{3}P_1V_1$

(D) If $V_2 = 3V_1$ and $T_2 = 4T_1$, then the heat supplied to the gas is $\frac{17}{6}P_1V_1$

Ans. **[A,B,C]**

Sol.





(A) $$P = P_1 + \frac{Kx}{A}$$

$$P_2 = \frac{3}{2}P_1 \quad \Rightarrow \quad x = \frac{V_1}{A}$$

$$\frac{3P_1}{2} = P_1 + \frac{Kx}{A}$$

$Kx = \frac{P_1A}{2}$

Energy of spring

$\frac{1}{2} Kx^2 = \frac{P_1A}{4} x = \frac{P_1V_1}{4}$ Ans. A

(B) $\Delta U = \frac{f}{2} (P_2V_2 - P_1V_1)$

$= 3P_1V_1$ **Ans. B**

(C) $P_f = \frac{4P_1}{3}$ $KX = \frac{P_1}{3} A$, $X = \frac{2V_1}{A}$

$W_{gas} = -(W_{P_{atm}} + W_{spring})$

$= (P_1Ax + \frac{1}{2}Kx . x)$

$= + \left(P_1A.\frac{2V_1}{A} + \frac{1}{2}.\frac{P_1A}{3}.\frac{2V_1}{A}\right)$

$= 2P_1V_1 + \frac{P_1V_1}{3} = \frac{7P_1V_1}{3}$

(D) $\Delta Q = W + \Delta U$

$= \frac{7P_1V_1}{3} + \frac{3}{2} (P_2V_2 - P_1V_1)$

$= \frac{7P_1V_1}{3} + \frac{3}{2} \left(\frac{4}{3}P_1.3V_1 - P_1V_1\right)$

$= \frac{7P_1V_1}{3} + \frac{9}{2}P_1V_1 = \frac{41P_1V_1}{6}$

Q.11 A fission reaction is given by ${}^{236}_{92}U \rightarrow {}^{140}_{54}Xe + {}^{94}_{38}Sr + x + y$, where x and y are two particles. Considering ${}^{236}_{92}U$ to be at rest, the kinetic energies of the products are denoted by K_{Xe} K_{Sr}, K_x(2MeV) and K_y(2MeV), respectively. Let the binding energies per nucleon of ${}^{236}_{92}U$, ${}^{140}_{54}Xe$ and ${}^{94}_{38}Sr$ be 7.5 MeV, 8.5 MeV and 8.5 MeV, respectively. Considering different conservation laws, the correct option (s) is (are)

(A) x = n , y = n, K_{Sr} = 129 MeV, K_{Xe} = 86 MeV

(B) x = p , y = e^-, K_{Sr} = 129 MeV, K_{Xe} = 86MeV

(C) x = p , y = n, K_{Sr} = 129 MeV, K_{Xe} = 86 MeV

(D) x = n , y = n, K_{Sr} = 86 MeV, K_{Xe} = 129 MeV

Ans. **[A]**

Sol. Q value of reaction, $K_x + K_y + K_{Xe} + K_{Sr} - K_{{}^{236}_{92}U}$

$Q = 2 + 2 + K_{Xe} + K_{Sr} – 0$

$Q = 4 + K_{Xe} + K_{Sr}$

$Q = 8.5 \times 140 + 8.5 \times 94 – 7.5 \times 236$

$Q = 219$ MeV

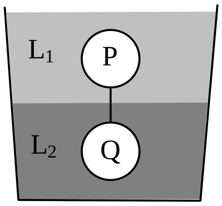
$219 – 4 = K_{Xe} + K_{Sr}$

$K_{Xe} + K_{Sr} = 215$ MeV

From momentum conservation

heavy particle must have small kinetic energy.

Q.12 Two spheres P and Q of equal radii have densities ρ_1 and ρ_2, respectively. The spheres are connected by a massless string and placed in liquids L_1 and L_2 of densities σ_1 and σ_2 and viscosities η_1 and η_2, respectively. They float in equilibrium with the sphere P in L_1 and sphere Q in L_2 and the string being taut (see figure). If sphere P alone in L_2 has terminal velocity $\vec{V}_P$ and Q alone in L_1 has terminal velocity $\vec{V}_Q$, then



(A) $\dfrac{|\vec{V}_P|}{|\vec{V}_Q|} = \dfrac{\eta_1}{\eta_2}$

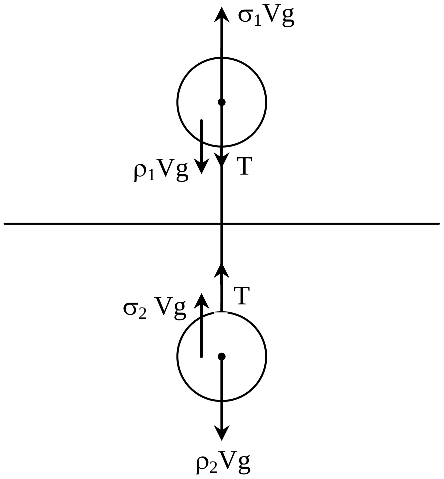
(B) $\dfrac{|\vec{V}_P|}{|\vec{V}_Q|} = \dfrac{\eta_2}{\eta_1}$

(C) $\vec{V}_P . \vec{V}_Q > 0$

(D) $\vec{V}_P . \vec{V}_Q < 0$

Ans. **[A,D]**

Sol.



$(\sigma_1 + \sigma_2) Vg = (\rho_1 + \rho_2) Vg$

$\sigma_1 + \sigma_2 = \rho_1 + \rho_2$

$\rho_1 - \sigma_2 = \sigma_1 - \rho_2$

If $\rho_1 > \sigma_2$ then $\sigma_1 > \rho_2$

In separate liquid one body will get terminal velocity upward then other one will get downward terminal velocity.

$\vec{V}_P \cdot \vec{V}_Q < 0$ (angle between $\vec{V}_p$ and $\vec{V}_Q$ is π)

& for terminal velocity $V \propto \frac{1}{\eta}$

$\frac{|\vec{V}_P|}{|\vec{V}_Q|} = \left(\frac{\eta_1}{\eta_2}\right)$ [because P in liquid L_2 and Q in liquid L_1]

Q.13 In terms of potential difference V, electric current I, permittivity ε_0, permeability μ_0, and speed of light c, the dimensionally correct equation (s) is (are)

(A) $\mu_0 I^2 = \varepsilon_0 V^2$ (B) $\varepsilon_0 I = \mu_0 V$ (C) $I = \varepsilon_0 cV$ (D) $\mu_0 cI = \varepsilon_0 V$

Ans. **[A,C]**

Sol. $W = qV$

$ML^2T^{-2} = ATV$

$V = M^1L^2T^{-3}A^{-1}$

$I \Rightarrow A$

$F = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{r^2}$

$\varepsilon_0 = \frac{q^2}{Fr^2}$

$\varepsilon_0 = \frac{A^2T^2}{MLT^{-2} \times L^2}$

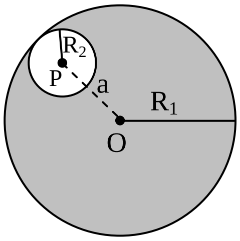
$\varepsilon_0 = M^{-1}L^{-3}T^4A^2$

$c = LT^{-1}$ Similarly $\mu_0 I^2 = \varepsilon_0 V^2$

$I = \varepsilon_0 cV$

$A = M^0L^0T^0A^1$

Q.14 Consider a uniform spherical charge distribution of radius R_1 centred at the origin O. In this distribution, a spherical cavity of radius R_2, centred at P with distance OP = a = $R_1 - R_2$ (see figure) is made. If the electric field inside the cavity at position $\vec{r}$ is $\vec{E}(\vec{r})$, then the correct statement (s) is (are)



(A) $\vec{E}$ is uniform, its magnitude is independent of R_2 but its direction depends on $\vec{r}$

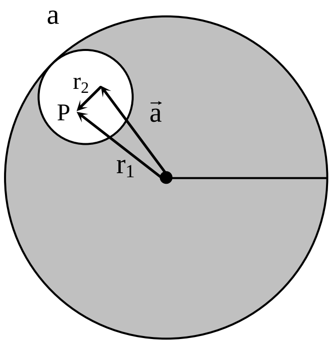
(B) $\vec{E}$ is uniform, its magnitude depends on R_2 and its direction depends on $\vec{r}$

(C) $\vec{E}$ is uniform, its magnitude is independent of a but its direction depends on $\vec{a}$

(D) $\vec{E}$ is uniform and both its magnitude and direction depend on $\vec{a}$

Ans. **[D]**

Sol.



Strength of electric field at point P is given by $\vec{E} = \dfrac{\rho}{3\varepsilon_0}(\vec{r}_1 - \vec{r}_2)$ .... (1)

because in solid sphere $\vec{E} = \dfrac{\rho}{3\varepsilon_0}\vec{r}$

$\because \vec{a} = \vec{r}_1 - \vec{r}_2$

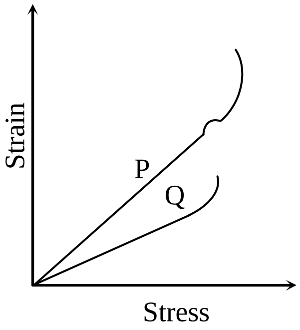
$\therefore$ by eq(1)

$\vec{E} = \dfrac{\rho}{3\varepsilon_0}\vec{a}$

$\because |\vec{a}| = R_1 - R_2 = \text{constant}$

$\therefore$ magnitude of $\vec{E}$ is constant and direction of $\vec{E}$ is depends on direction of $\vec{a}$

Q.15 In plotting stress versus strain curves for two materials P and Q, a student by mistake puts strain on the y-axis and stress on the x-axis as shown in the figure. Then the correct statements (s) is (are)



(A) P has more tensile strength than Q

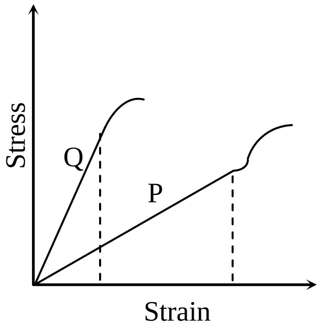
(B) P is more ductile than Q

(C) P is more brittle than Q

(D) The Young's modulus of P is more than that of Q

Ans. **[A,B]**

Sol. Actual graph is



∵ Length of linear part of P is large then Q so P has more tensile strength than Q.

∵ Curve part of P is large the curve part of Q so P is more ductile than Q.

∵ $(\text{Slope})_Q > (\text{Slope})_P$

∴ Young's modulus of Q > Young's modulus of P

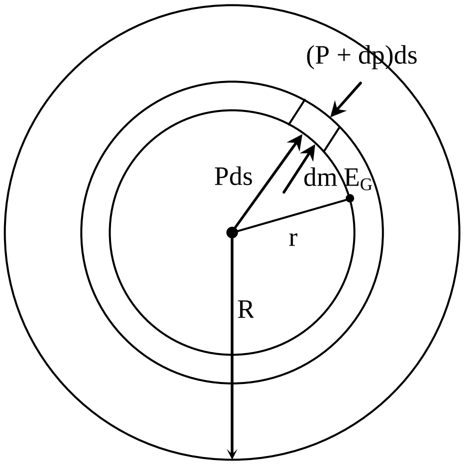
Q.16 A spherical body of radius R consists of a fluid of constant density and is in equilibrium under its own gravity. If P(r), is the pressure at r(r < R), then the correct option (s) is (are)

(A) $P(r = 0) = 0$ (B) $\frac{P(r = 3R/4)}{P(r = 2R/3)} = \frac{63}{80}$

(C) $\frac{P(r=3R/5)}{P(r=2R/5)} = \frac{16}{21}$ (D) $\frac{P(r=R/2)}{P(r=R/3)} = \frac{20}{27}$

Ans. **[B,C]**

Sol.



Gravitational field

$$E_G = \frac{4}{3}\pi\rho G r$$

$E_G \propto r$ s

$E_G = Cr$ $\left[\begin{matrix} C = \frac{4}{3}\pi\rho G \\ C = \text{const.}\end{matrix}\right]$

$(P + dp)\, ds + dm\, E_G = Pds$

$dpds = -\, dm\, E_G$

$dpds = -\, \rho\, dsdr\, .\, E_G$

$dp = -\, \rho E_G dr$

$$\int_0^P dp = \int_R^r -\rho C r\, dr$$ [pressure at surface is zero]

$$P = \rho C\left[\frac{-r^2}{2}\right]_R^r$$

$$P = \frac{\rho C}{2}[R^2 - r^2]$$

(1) $P(r = 0) \neq 0$

(ii) $$\frac{P\left(r=\frac{3R}{4}\right)}{P\left(r=\frac{2R}{3}\right)} = \left[\frac{R^2-\left(\frac{3R}{4}\right)^2}{R^2-\left(\frac{2R}{3}\right)^2}\right] = \frac{63}{80}$$

(iii) $$\frac{P\left(r=\frac{3R}{5}\right)}{P\left(r=\frac{2R}{5}\right)} = \frac{R^2-\left(\frac{3R}{5}\right)^2}{R^2-\left(\frac{2R}{5}\right)^2} = \frac{16}{21}$$

(iv) $$\frac{P\left(r=\frac{R}{2}\right)}{P\left(r=\frac{R}{3}\right)} = \frac{R^2-\left(\frac{R}{2}\right)^2}{R^2-\left(\frac{R}{3}\right)^2} = \frac{27}{32}$$

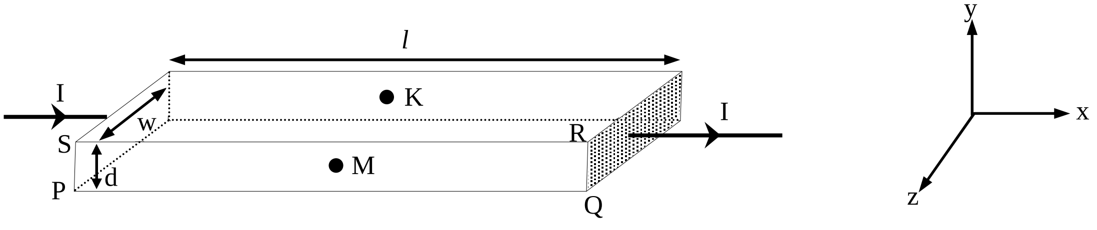
SECTION – 3 (Maximum Marks : 16)

- This section contains **TWO** paragraphs
- Based on each paragraph, there will be **TWO** questions
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is(are) correct
- For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS
- Marking scheme :

+4 If only the bubble(s) corresponding to all the correct option(s) is(are) darkened
0 If none of the bubbles is darkened
–2 In all other cases

PARAGRAPH 1

In a thin rectangular metallic strip a constant current I flows along the positive x-direction, as shown in the figure. The length, width and thickness of the strip are *l*, w and d, respectively. A uniform magnetic field $\vec{B}$ is applied on the strip along the positive y-direction. Due to this, the charge carriers experience a net deflection along the z-direction. This results in accumulation of charge carriers on the surface PQRS and appearance of equal and opposite charges on the face opposite to PQRS. A potential difference along the z-direction is thus developed. Charge accumulation continues until the magnetic force is balanced by the electric force. The current is assumed to be uniformly distributed on the cross section of the strip and carried by electrons.



Q.17 Consider two different metallic strips (1 and 2) of the same material. Their lengths are the same, widths are w_1 and w_2 and thicknesses are d_1 and d_2, respectively. Two points K and M are symmetrically located on the opposite faces parallel to the x-y plane (see figure). V_1 and V_2 are the potential differences between K and M in strips 1 and 2, respectively. Then, for a given current I flowing through them in a given magnetic field strength B, the correct statement(s) is(are)

(A) If $w_1 = w_2$ and $d_1 = 2d_2$, then $V_2 = 2V_1$

(B) If $w_1 = w_2$ and $d_1 = 2d_2$, then $V_2 = V_1$

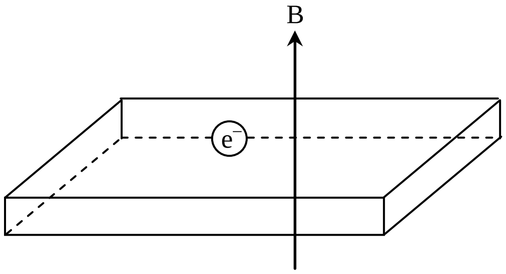
(C) If $w_1 = 2w_2$ and $d_1 = d_2$, then $V_2 = 2V_1$

(D) If $w_1 = 2w_2$ and $d_1 = d_2$, then $V_2 = V_1$

Ans. **[A,D]**

Sol. $i = neAV_d$

$$V_d = \frac{i}{neA}$$



Magnetic force on e = eV_dB

due to magnetic force e drift towards face M. due to which an electric field is produce between two faces.

$$V_d = \frac{E}{B}$$

$$E = V_dB$$

Potential difference between k & M = V

$$V = EW$$

$$V = V_dBW$$

$$V = \frac{iBW}{neA} \Rightarrow \frac{iBW}{neWd} = \frac{iB}{ned}$$

$$V \propto \frac{1}{d}$$

$\therefore$ if $d_1 = 2d_2$ then $V_1 = \frac{V_2}{2}$ or $V_2 = 2V_1$ Ans. (A)

if $d_1 = d_2$ then $V_1 = V_2$ Ans. (D)

Q.18 Consider two different metallic strips (1 and 2) of same dimensions (length *l*, width w and thickness d) with carrier densities n_1 and n_2, respectively. Strip 1 is placed in magnetic field B_1 and strip 2 is placed in magnetic field B_2, both along positive y-directions. Then V_1 and V_2 are the potential differences developed between K and M in strips 1 and 2, respectively. Assuming that the current I is the same for both the strips, the correct option(s) is(are)

(A) If $B_1 = B_2$ and $n_1 = 2n_2$, then $V_2 = 2V_1$

(B) If $B_1 = B_2$ and $n_1 = 2n_2$, then $V_2 = V_1$

(C) If $B_1 = 2B_2$ and $n_1 = n_2$, then $V_2 = 0.5V_1$

(D) If $B_1 = 2B_2$ and $n_1 = n_2$, then $V_2 = V_1$

Ans. **[A,C]**

Sol. $V = \frac{iB}{ned}$

d is same for both strip

$V \propto \frac{B}{n}$

if $B_1 = B_2$

$V \propto \frac{1}{n}$

$n_1 = 2n_2$

$V_1 = \frac{V_2}{2}$

$V_2 = 2V_1$ Ans. (A)

if $n_1 = n_2$

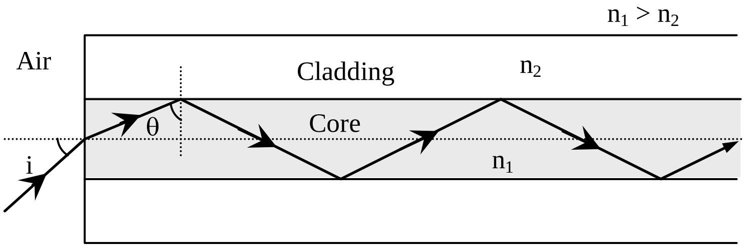
$V \propto B$

if $B_1 = 2B_2$ then $V_1 = 2V_2$

$V_2 = 0.5\ V_1$ Ans. (C)

PARAGRAPH 2

Light guidance in an optical fiber can be understood by considering a structure comprising of thin solid glass cylinder of refractive index n_1 surrounded by a medium of lower refractive index n_2. The light guidance in the structure takes place due to successive total internal reflections at the interface of the media n_1 and n_2 as shown in the figure. All rays with the angle of incidence i less than a particular value i_m are confined in the medium of refractive index n_1. The numerical aperture (NA) of the structure is defined as sin i_m.



Q.19 For two structures namely S_1 with $n_1 = \sqrt{45}/4$ and $n_2 = 3/2$, and S_2 with $n_1 = 8/5$ and $n_2 = 7/5$ and taking the refractive index of water to be 4/3 and that of air to be 1, the correct option(s) is(are)

(A) NA of S_1 immersed in water is the same as that of S_2 immersed in a liquid of refractive index $\frac{16}{3\sqrt{15}}$

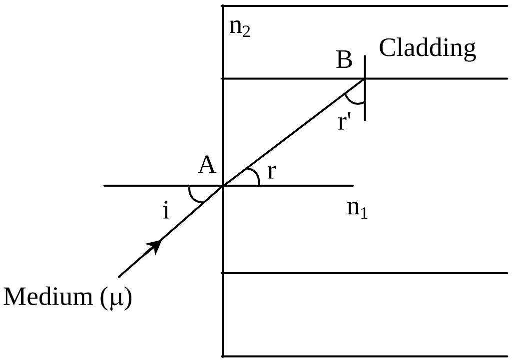
(B) NA of S_1 immersed in liquid of refractive index $\frac{16}{\sqrt{15}}$ is the same as that of S_2 immersed in water

(C) NA of S_1 placed in air is the same as that of S_2 immersed in liquid of refractive index $\frac{4}{\sqrt{15}}$

(D) NA of S_1 placed in air is the same as that of S_2 placed in water

Ans. **[A,C]**

Sol.



at B

$$\sin C = \frac{n_2}{n_1}$$

$\cos C = \dfrac{\sqrt{n_1^2 - n_2^2}}{n_1}$

For TIR at B –

$r' \geq C$ & $r + r' = 90°$

or $\cos r' \leq \cos C$ $r = 90 - r'$

$\cos r' \leq \dfrac{\sqrt{n_1^2 - n_2^2}}{n_1}$ $\sin r = \cos r'$

Now at A – (using snell's Laws)

$\mu \times \sin i = n_1 \sin r$

$= n_1 \cos r'$

$\sin i \leq \dfrac{\sqrt{n_1^2 - n_2^2}}{\mu}$

i.e. $\sin i_m = \dfrac{\sqrt{n_1^2 - n_2^2}}{\mu}$ & $i \leq i_m$

i.e. $NA = \dfrac{\sqrt{n_1^2 - n_2^2}}{\mu}$

using this relation we get

Ans. A & C

Q.20 If two structures of same cross-sectional area, but different numerical apertures NA_1 and NA_2 ($NA_2 < NA_1$) are joined longitudinally, the numerical aperture of the combined structure is

(A) $\dfrac{NA_1\,NA_2}{NA_1 + NA_2}$ (B) $NA_1 + NA_2$ (C) NA_1 (D) NA_2

Ans. **[D]**

Sol. On the basis of discussion in Q.19, we can conclude that

if $NA_2 < NA_1$ then $i_{m_2} < i_{m_1}$

for transmission ($i \leq i_m$)

hence $i \leq i_{m_2}$

i.e. NA should be NA_2 for combined structure.

Part II - CHEMISTRY

SECTION – 1 (Maximum Marks: 32)

- This Section contains **EIGHT** questions
- This answer to each questions is a **SINGLE DIGIT INTEGER** ranging from 0 to 9. both inclusive
- For each questions, darken the bubble corresponding to the correct integer in the ORS
- Marking scheme :

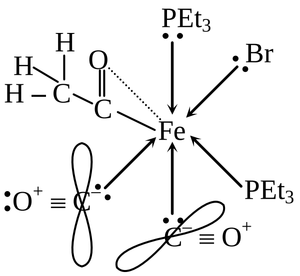
  + 4 If the bubbler corresponding to the answer is darkened

  0 In all other cases

Q.21 In the complex acetylbromidodicarbonylbis(triethylphosphine)iron(II), the number of Fe – C bond (s) is

Ans. **[3]**

Sol. $[\overset{+2}{Fe}(CH_3\overset{|}{C}O)(Br)(CO)_2(PEt_3)_2]^{+1}$



Q.22 Among the complex ions, $[Co(NH_2 - CH_2 - CH_2 - NH_2)_2Cl_2]^+$, $[CrCl_2(C_2O_4)_2]^{3-}$, $[Fe(H_2O)_4(OH)_2]^+$, $[Fe(NH_3)_2(CN)_4]^-$, $[Co(NH_2 - CH_2 - CH_2 - NH_2)_2(NH_3)Cl]^{2+}$ and $[Co(NH_3)_4(H_2O)Cl]^{2+}$, the number of complex ion(s) that show(s) *cis-trans* isomerism is

Ans. **[6]**

Sol. $[Co(NH_2 - CH_2 - CH_2 - NH_2)_2Cl_2]^+$ shows one cis and one trans isomer

$[CrCl_2(C_2O_4)_2]^{3-}$ shows one cis and one trans isomer

$[Fe(H_2O)_4(OH)_2]^+$ shows one cis and one trans isomer

$[Fe(NH_3)_2(CN)_4]^-$ shows one cis and one trans isomer

$[Co(NH_2 - CH_2 - CH_2 - NH_2)_2(NH_3)Cl]^{2+}$ shows one cis and one trans isomer

$[Co(NH_3)_4(H_2O)Cl]^{2+}$ shows one cis and one trans isomer

Q.23 Three moles of B_2H_6 are completely reacted with methanol. The number of moles of boron containing product formed is

Ans. **[6]**

Sol. $3B_2H_6 + 18MeOH \rightarrow 6B(OMe)_3 + 18H_2$

Q.24 The molar conductivity of a solution of a weak acid HX (0.01 M) is 10 times smaller than the molar conductivity of a solution of a weak acid HY (0.10 M). If $\lambda^0_{X^-} \approx \lambda^0_{Y^-}$, the difference in their pK_a values, $pK_a(HX) - pK_a(HY)$, is (consider-degree of ionization of both acids to be << 1)

Ans. **[3]**

Sol. Degree of ionization of weak acid HX(α_{HX}) = $\frac{\Lambda_{HX}}{\Lambda^{\circ}_{HX}}$

Degree of ionization of weak acid HY(α_{HY}) = $\frac{\Lambda_{HY}}{\Lambda^{\circ}_{HY}}$

$$\therefore \qquad \alpha_{HX} = \frac{1}{10}\,\alpha_{HY} \qquad \left\{\because \Lambda_{HX} = \frac{\Lambda_{HY}}{10}\right\}$$

$$K_a = \alpha^2 C \qquad \{\text{for } \alpha << 1\}$$

$$K_{a_{HX}} = \alpha^2_{HY} \times \frac{1}{100} \times C_1 = \alpha^2_{HY} \times \frac{1}{100} \times 0.01 \ \{\because \ C_1 = 0.01\text{ M}\}$$

$$K_{a_{HY}} = \alpha^2_{HY} \times C_2 = \alpha^2_{HY} \times 0.1 \qquad \{\because \ C_2 = 0.1\text{ M}\}$$

$$K_{a_{HX}} = \frac{1}{1000}\,K_{a_{HY}}$$

$$pK_{a_{HY}} = -\log K_{a_{HX}} = -\log\frac{K_{a_{HY}}}{1000} = 3 - \log K_{a_{HY}}$$

$$pK_{a_{HY}} = -\log K_{a_{HY}} = -\log K_{a_{HY}}$$

$$pK_{ka_{HY}} - pK_{a_{HY}} = 3$$

Q.25 A closed vessel with rigid walls contains 1 mole of ${}^{238}_{92}U$ and 1 mole of air at 298 K. Considering complete decay of ${}^{238}_{92}U$ to ${}^{206}_{82}Pb$, the ratio of the final pressure to the initial pressure of the system at 298 K is

Ans. **[9]**

Sol. Initial mole of gas $n_i = 1$

$${}_{92}U^{238} \rightarrow {}_{82}Pb^{206} + 8\ {}_{2}He^{4} + 6\ {}_{-1}e^{0}$$

Final mole of gas $n_f = 8 + 1 = 9$

At const T, & V ; $p \propto n$

$$\frac{P_f}{P_i} = \frac{n_f}{n_i} = \frac{9}{1} \qquad \therefore \text{ Answer is 9}$$

Q.26 In dilute aqueous H_2SO_4, the complex diaquodioxalatoferrate(II) is oxidized by MnO_4^-, For this reaction, the ratio of the rate of change of $[H^+]$ to the rate of change of $[MnO_4^-]$ is

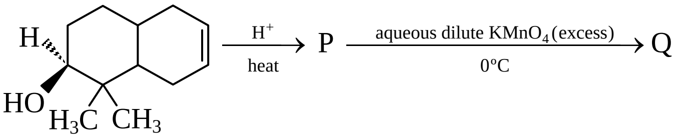
Ans. **[8]**

Sol. $$1\overset{(+7)}{Mn}O_4^- + 1[\overset{(+2)}{Fe}(H_2O)_2(\overset{+3}{OX})_2]^{2-} + 8H^+ \rightarrow 1\overset{+2}{Mn}{}^{2+} + 1\overset{+3}{Fe}{}^{3+} + 4\overset{+4}{C}O_2 + 6H_2O$$

$$5 \times 1 = \underset{1}{\cancel{5}} \qquad (1 \times 1) + (1 \times 4) = \underset{1}{\cancel{5}}$$

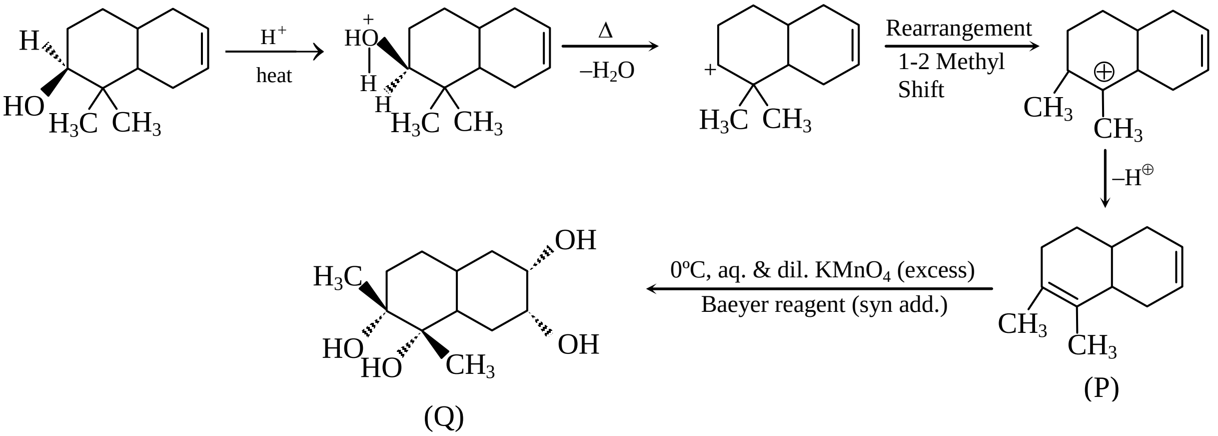
$$\therefore \frac{\text{rate of change of }[H^+]}{\text{rate of change of }[MnO_4^-]} = \frac{8}{1}$$

Q.27 The number of hydroxyl group(s) in **Q** is



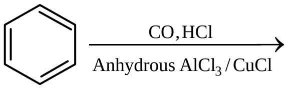
Ans. **[4]**

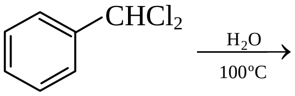
Sol. First reaction involve dehydration of alcohol and second reaction is formation of syndiol rearrangement



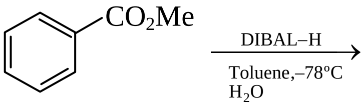
Total number of hydroxyl groups in Q ⇒ 4

Q.28 Among the following, the number of reaction(s) that produce(s) benzaldehyde is

I. 

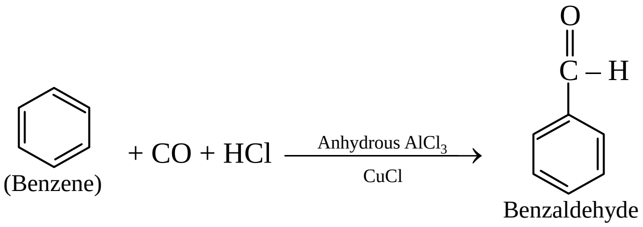
II. 

III. 

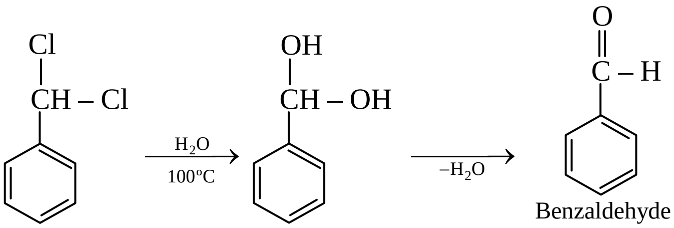
IV 

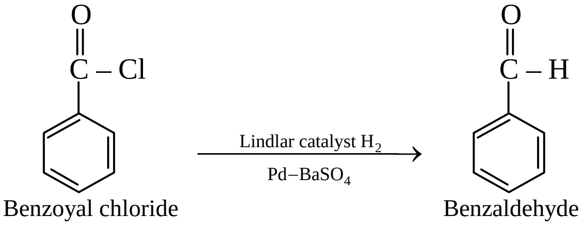
Ans. **[4]**

Sol. All four reaction produce benzaldehyde

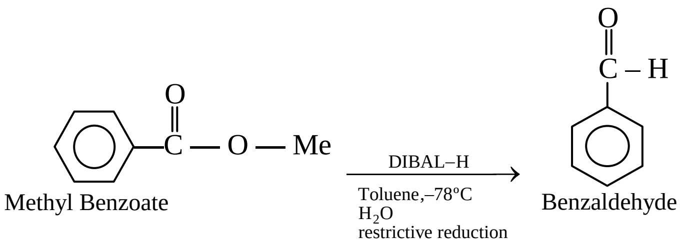
I. 

(Gatter mann's koch aldehyde syn.)

II. 

III. 

[Rosenmund reduction]

IV. 

SECTION – 2 (Maximum Marks: 32)

- This Section contains **EIGHT** questions
- Each question has **FOUR** options (A), (B), (C) and (D) **ONE OR MORE THAN ONE** of these four option (s) is (are) correct
- For each questions, darken the bubble (s) corresponding to all the correct option (s) in the **ORS**
- Marking scheme :

  +4 If only the bubble (s) corresponding to all the correct option (s) is (are) darkened

  0 If none of the bubbles is darkened

  –2 In all other cases

Q.29 Under hydrolytic conditions, the compounds used for preparation of linear polymer and for chain termination, respectively, are

(A) CH_3SiCl_3 and $Si(CH_3)_4$

(B) $(CH_3)_2SiCl_2$ and $(CH_3)_3SiCl$

(C) $(CH_3)_2SiCl_2$and CH_3SiCl_3

(D) $SiCl_4$ and $(CH_3)_3SiCl$

Ans. **[B]**

Sol. (i) $(CH_3)_2SiCl_2$ undergoes hydrolysis by two sides so it form linear polymer

(ii) $(CH_3)_3$ SiCl undergoes hydrolysis by one side so it is chain termination.

Q.30 When O_2 is adsorbed on a metallic surface, electron transfer occurs from the metal to O_2. The **TRUE** statement(s) regarding this adsorption is(are)

(A) O_2 is physisorbed

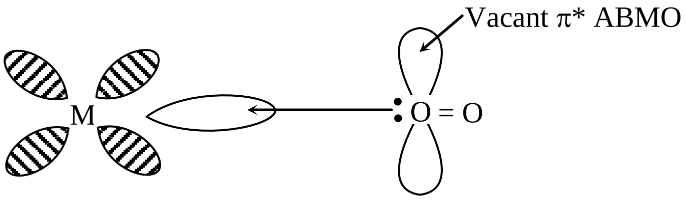
(B) heat is released

(C) occupancy of π^*_{2p} of O_2 is increased

(D) bond length of O_2 is increased

Ans. **[B,C,D]**

Sol.



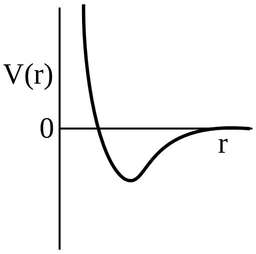
(i) Due to bonding, energy is released

(ii) Due to back donation bond length of O_2 is increased

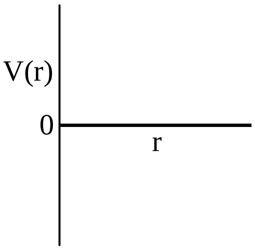
(iii) Due to presence of vacant π^*2p orbital it accept some electron cloud from metal.

Q.31 One mole of a monoatomic real gas satisfies the equation $p(V - b) = RT$ where b is a constant. The relationship of interatomic potential V(r) and interatomic distance r for the gas is given by-

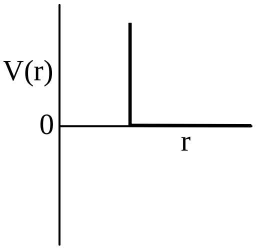
(A)



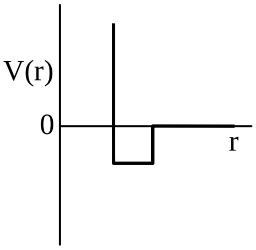
(B)



(C)



(D)

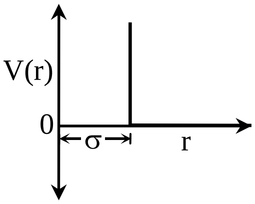


Ans. **[C]**

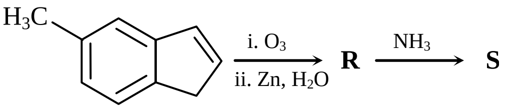
Sol. when only repulsive force act.

$V(r) = \infty \qquad r < \sigma$

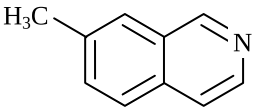
& $V(r) = 0 \qquad r > \sigma$



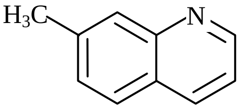
Q.32 In the following reaction, the product **S** is –



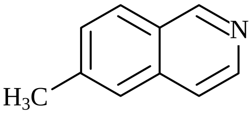
(A)



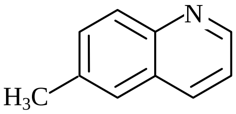
(B)



(C)

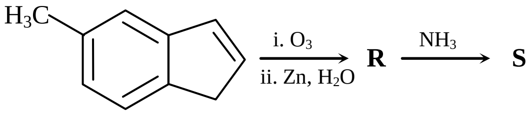


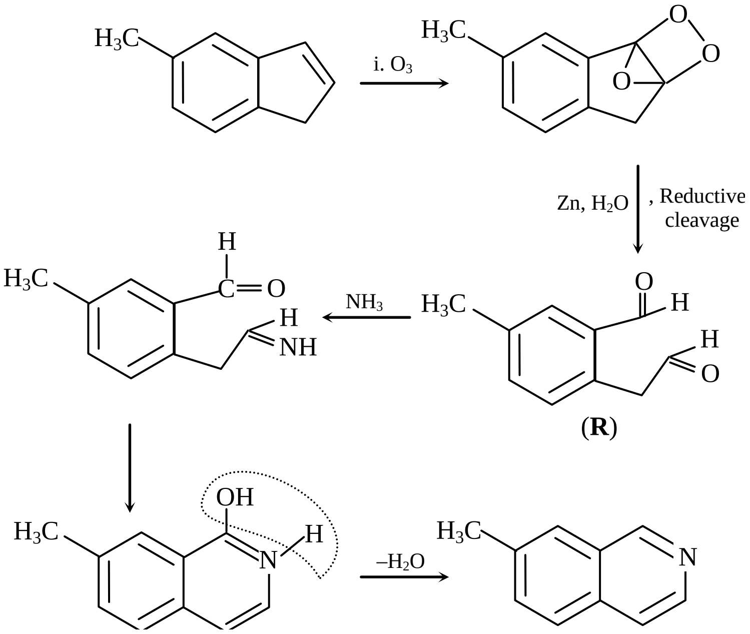
(D)



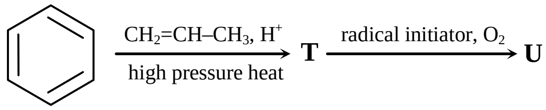
Ans. **[A]**

Sol.

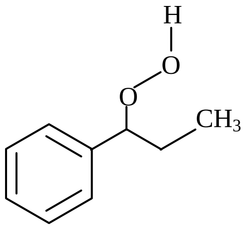




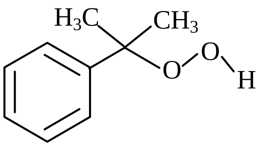
Q.33 The major product **U** in the following reactions is –



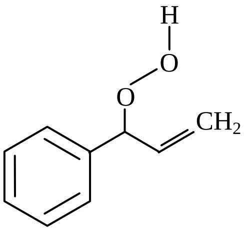
(A)



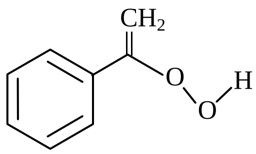
(B)



(C)

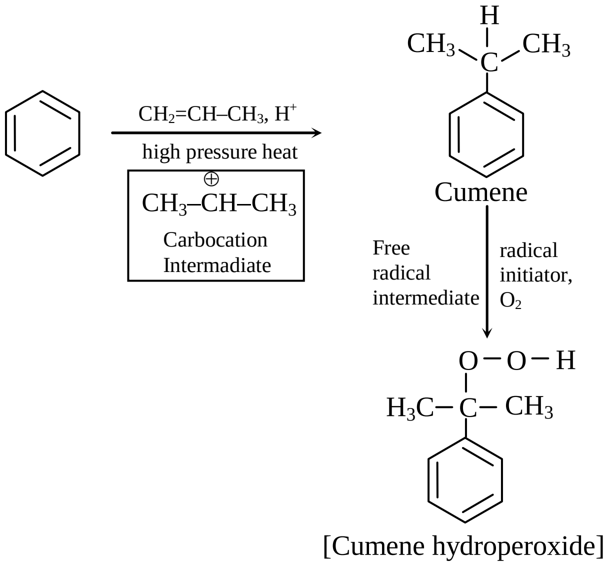


(D)

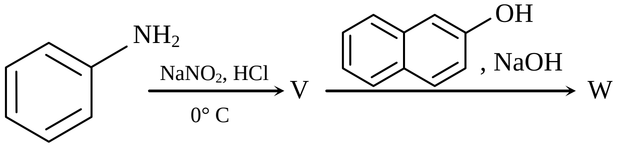


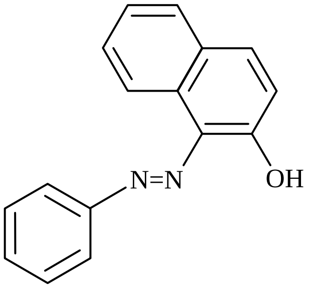
Ans. **[B]**

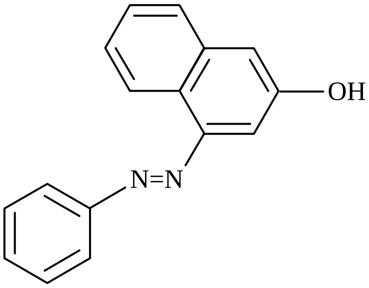
Sol. This reaction is a part of formation of phenol

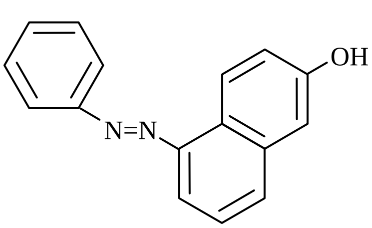


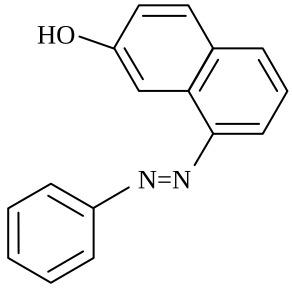
Q.34 In the following reactions, the major product **W** is



(A)


(B)


(C)


(D)


Ans. **[A]**

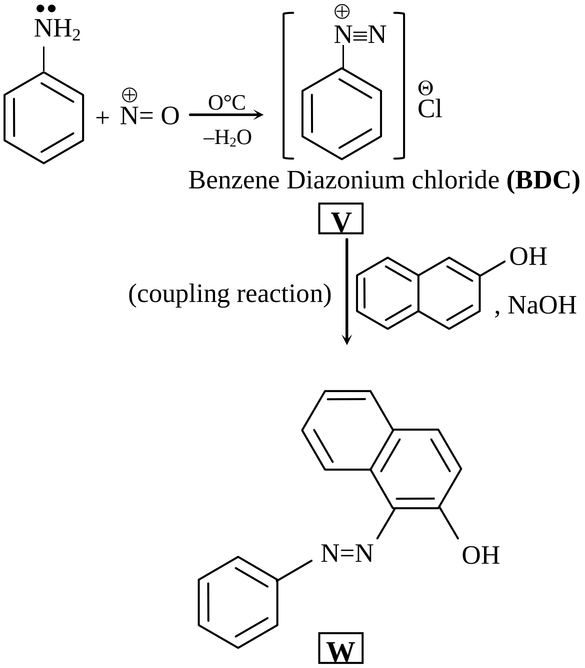
Sol.

$NaNO_2 + HCl \longrightarrow HO\text{–}N\text{=}O + NaCl$

$\downarrow$

$\overset{\oplus}{N}\text{= }O$

(Attacking species)
[Nitrosonium ion]



In coupling reaction if para position is blocked then attack occur at ortho position

Q.35 The correct statement(s) regarding, (i) HClO, (ii) $HClO_2$, (iii) $HClO_3$, and (iv) $HClO_4$, is (are)

(A) The number of Cl = O bonds in (ii) and (iii) together is two

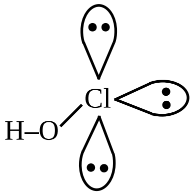
(B) The number of lone pairs of electrons on Cl in (ii) and (iii) together is three

(C) The hybridization of Cl in (iv) is sp^3

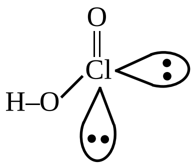
(D) Amongst (i) to (iv), the strongest acid is (i)

Ans. **[B, C]**

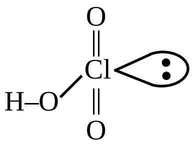
Sol. (i)



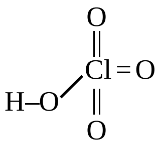
(ii)



(iii)



(iv)



sp^3, hybridization
strongest acid

Q.36 The pair(s) of ions where BOTH the ions are precipitated upon passing H_2S gas in presence of dilute HCl, is(are) -

(A) Ba^{2+}, Zn^{2+} (B) Bi^{3+}, Fe^{3+}

(C) Cu^{2+}, Pb^{2+} (D) Hg^{2+}, Bi^{3+}

Ans. **[C, D]**

Sol. Group number (I) and (II) radicals give precipitate

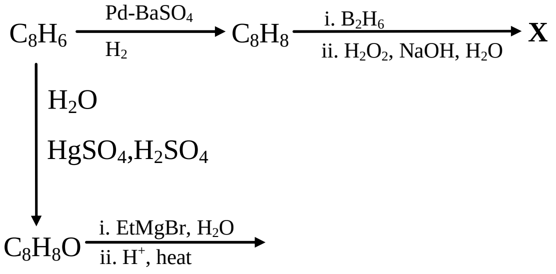
SECTION – 3 (Maximum Marks : 16)

- This section contains **TWO** Paragraphs
- Based on each paragraph, there will be **TWO** questions
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct
- Four each question, darken the bubble(s) corresponding to all the correct option(s) in the **ORS**.
- Marking scheme :

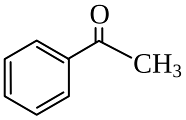
+ 4 If only the bubble(s) corresponding to all the correct option(s) is (are) darkened
0 If none of the bubbles is darkened
–2 In all other cases

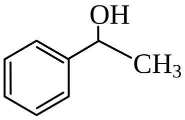
PARAGRAPH 1

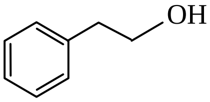
In the following reactions

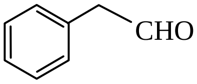


Q.37 Compound **X** is -

(A) 

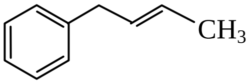
(B) 

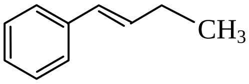
(C) 

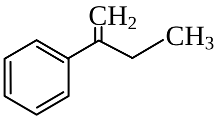
(D) 

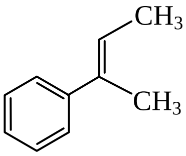
Ans. **[C]**

Q.38 The major compound **Y** is -

(A) 

(B) 

(C) 

(D) 

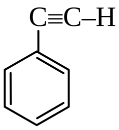
Ans. **[D]**

Sol. The following reactions

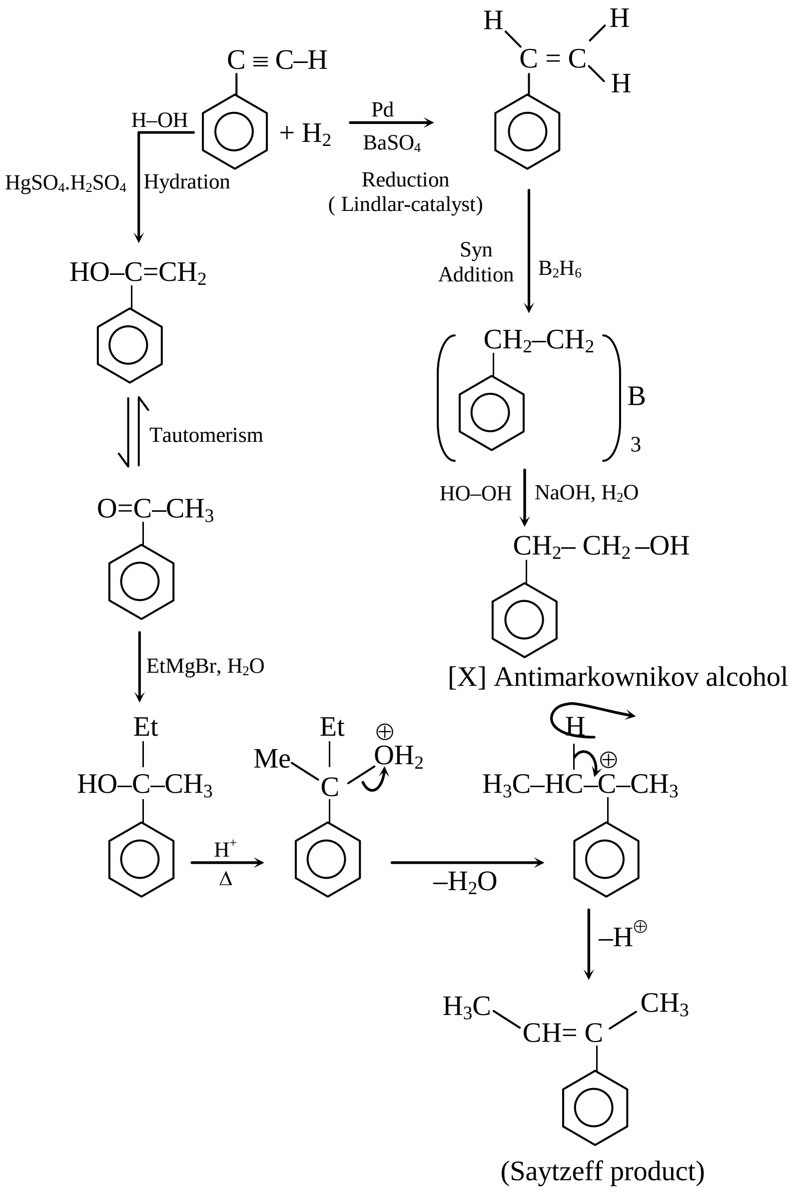
$$C_8H_6 \xrightarrow[H_2]{Pd-BaSO_4} C_8H_8$$

$\uparrow$

DBE = (x + 1) –1/2
= (8 + 1) –6/2
= 9 – 3 = 6

Compound - C_8H_6 is 

Now,



PARAGRAPH 2

When 100 mL of 1.0 M HCl was mixed with 100 mL of 1.0 M NaOH in an insulated beaker at constant pressure, a temperature increase of 5.7ºC was measured for the beaker and its contents (**Expt.1**). Because the enthalpy of neutralization of a strong acid with a strong base is a constant (–57.0 kJ mol^{-1}), this experiment could be used to measure the calorimeter constant. In a second experiment (**Expt. 2**), 100 mL of 2.0 M acetic acid ($K_a = 2.0 \times 10^{-5}$) was mixed with 100 mL of 1.0 M NaOH (under identical conditions to **Expt. 1**) where a temperature rise of 5.6 ºC was measured.

(Consider heat capacity of all solutions as 4.2 J g^{-1} K^{-1} and density of all solutions as 1.0 g mL^{-1})

Q.39 Enthalpy of dissociation (in kJ mol^{-1}) of acetic acid obtained from the **Expt. 2** is -

(A) 1.0 (B) 10.0 (C) 24.5 (D) 51.4

Ans. **[A]**

Sol. After experiment 1

mass of solution = 200× 1.0 = 200 g {because density of all solution = 1g/ml}

let C is calorimetric constant

$Q = m.s.\Delta T + C.\Delta T.$

$$5.7 = \frac{200 \times 4.2 \times 5.7}{1000} + C \times 5.7 \ \{\text{for 0.1 mol}\}$$

$\Rightarrow C = 0.16$

After experiment 2

$$\frac{x}{10} = \frac{200 \times 4.2 \times 5.6}{1000} + 0.16 \times 5.6$$

$x = 56$

$\Delta H_{N(WASB)} = \Delta H_{N(SASB)} + \Delta H_{diss.\ of\ WA}$

$57 = 56 + \Delta H_{diss.\ of\ WA}$ {for 1 mol}

$\Delta H_{diss.\ of\ WA} =$ 1 KJ /mol.

Q.40 The pH of the solution after **Expt. 2** is -

(A) 2.8 (B) 4.7 (C) 5.0 (D) 7.0

Ans. **[B]**

Sol.

$$CH_3COOH + NaOH \rightleftharpoons CH_3COONa + \cancel{H_2O}$$

Initial	0.2	0.1	0
Final	0.1	0	0.1

$$pH = pK_a + \log \frac{[\text{conjugate base}]}{[\text{acid}]}$$

$pH = pK_a = 4.7$

Part III - MATHEMATICS

SECTION – 1 (Maximum Marks: 32)

- This section contains **EIGHT** questions
- The answer to each questions is a **SINGLE DIGIT INTEGER** ranging from 0 to 9, both inclusive
- For each questions, darken the bubble corresponding to the correct integer in the ORS
- Marking scheme :

+ 4 If the bubble corresponding to the answer is darkened

0 In all other cases

Q.41 The coefficient of x^9 in the expansion of $(1 + x)(1 + x^2)(1 + x^3) \ldots\ldots (1 + x^{100})$ is

Ans. **[8]**

Sol. Required will be same as that of coefficient of x^9 in $(1 + x)(1 + x^2) \ldots\ldots(1 + x^9)$

Possible cases are

(i) $x^0\, x^9$

(ii) $x^1\, x^8$

(iii) $x^2\, x^7$

(iv) $x^3\, x^6$

(v) $x^4\, x^5$

(vi) $x^1\, x^2\, x^6$

(vii) $x^2\, x^3\, x^4$

(viii) $x^1\, x^3\, x^5$

Each give (+1) to coefficient of $x^9 = 8$

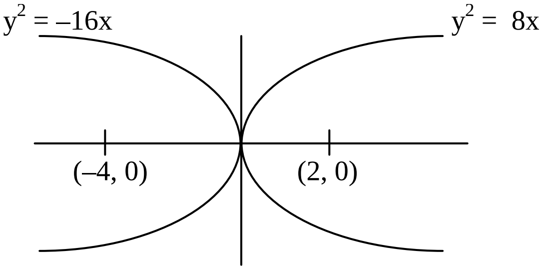
Q.42 Suppose that the foci of the ellipse $\frac{x^2}{9} + \frac{y^2}{5} = 1$ are $(f_1, 0)$ and $(f_2, 0)$ where $f_1 > 0$ and $f_2 < 0$. Let P_1 and P_2 be two parabolas with a common vertex at (0, 0) and with foci at $(f_1, 0)$ and $(2f_2, 0)$, respectively. Let T_1 be a tangent to P_1 which passes through $(2f_2, 0)$ and T_2 be a tangent to P_2 which passes through $(f_1, 0)$. If m_1 is the slope of T_1 and m_2 is the slope of T_2, then the value of $\left(\frac{1}{m_1^2} + m_2^2\right)$ is

Ans. [4]

Sol. Foci of the ellipse are (2, 0) and (–2, 0)

So $f_1 = 2$, $f_2 = -2$

Equation of T_1 :



$$T_1 : y = m_1x + \frac{2}{m_1}$$

$\because$ T_1 passes through (– 4, 0)

$$\text{so } m_1^2 = \frac{1}{2}$$

$$T_2 : y = m_2x - \frac{4}{m_2}$$

T_2 passes through (2, 0)

$$\therefore\ m_2^2 = 2$$

$$\therefore\ \frac{1}{m_1^2} + m_2^2 = 4$$

Q.43 Let m and n be two positive integers greater than 1.

$$\text{If } \lim_{\alpha \to 0}\left(\frac{e^{\cos(\alpha^n)} - e}{\alpha^m}\right) = -\left(\frac{e}{2}\right)$$

then the value of $\frac{m}{n}$ is

Ans. [2]

Sol.
$$\lim_{\alpha \to 0}\left(\frac{e^{\cos\alpha^n} - e}{\alpha^m}\right)$$

$$= e\left[\frac{e^{\cos\alpha^n - 1} - 1}{\cos\alpha^n - 1}\right] . \frac{\cos\alpha^n - 1}{(\alpha^n)^2}\ \frac{(\alpha^n)^2}{\alpha^m}$$

$$= e(\ell ne)\left(\frac{-1}{2}\right)\ a^{2n-m} = \frac{-e}{2}$$

So $2n - m = 0$

$m = 2n$

$$\therefore\ \frac{m}{n} = 2$$

Q.44 If

$$\alpha = \int_0^1 (e^{9x+3\tan^{-1}x})\left(\frac{12+9x^2}{1+x^2}\right)dx$$

where $\tan^{-1}x$ takes only principal values, then the value of $\left(\log_e |1+\alpha| - \frac{3\pi}{4}\right)$ is

Ans. **[9]**

Sol. Let x = tan θ

$dx = \sec^2\theta\, d\theta$

$$\alpha = \int_0^{\pi/4} e^{(3\theta+9\tan\theta)}.(12+9\tan^2\theta)d\theta$$

$$\alpha = \int_0^{\pi/4} e^{(3\theta+9\tan\theta)}.(9\sec^2\theta+3)d\theta$$

Let 3θ + 9tanθ = t

$(3 + 9\sec^2\theta)\, d\theta = dt$

$$\alpha = \int_0^{9+\frac{3\pi}{4}} e^t dt = \left[e^{9+\frac{3\pi}{4}} - 1\right]$$

$$\alpha + 1 = e^{9+\frac{3\pi}{4}}$$

$$\ell n\,|1+\alpha| - \frac{3\pi}{4} = 9$$

Q.45 Let f : R → R be a continuous odd function, which vanishes exactly at one point and $f(1) = \frac{1}{2}$. Suppose that $F(x) = \int_{-1}^{x} f(t)dt$ for all $x \in [-1, 2]$ and $G(x) = \int_{-1}^{x} t\,|f(f(t))|\,dt$ for all $x \in [-1, 2]$. If $\lim_{x\to1}\frac{F(x)}{G(x)} = \frac{1}{14}$, then the value of $f\left(\frac{1}{2}\right)$ is

Ans. **[7]**

Sol. As f(x) is continuous odd function

So f(0) = 0

Given $f(1) = \frac{1}{2}$

$$F(x) = \int_{-1}^{x} f(t)dt \qquad \forall\ x \in [-1, 2]$$

$$G(x) = \int_{-1}^{x} t\,|f(f(t))|\,dt \qquad \forall\ x \in [-1, 2]$$

$$\lim_{x\to 1}\frac{F(x)}{G(x)} = \frac{1}{14}$$

$$\Rightarrow \lim_{x\to 1}\frac{\int_{-1}^{x} f(t)dt}{\int_{-1}^{x} t\,|f(f(t))|\,dt} = \lim_{x\to 1}\frac{f(x)}{x\,|f(f(x))|} \qquad \{\text{Using L' Hospital's Rule}\}$$

$$\Rightarrow \frac{\frac{1}{2}}{f\left(\frac{1}{2}\right)} = \frac{1}{14}$$

$$\Rightarrow f\left(\frac{1}{2}\right) = 7$$

Q.46 Suppose that $\vec{p}, \vec{q}$ and $\vec{r}$ are three non-coplanar vectors in R^3. Let the components of a vector $\vec{s}$ along $\vec{p}, \vec{q}$ and $\vec{r}$ be 4, 3 and 5, respectively. If the components of this vector $\vec{s}$ along $(-\vec{p}+\vec{q}+\vec{r})$, $(\vec{p}-\vec{q}+\vec{r})$ and $(-\vec{p}-\vec{q}+\vec{r})$ are x, y and z, respectively, then the value of 2x + y + z is

Ans. **[9]**

Sol. $\vec{s} = 4\vec{p} + 3\vec{q} + 5\vec{r}$

$\vec{s} = x(-\vec{p}+\vec{q}+\vec{r}) + y(\vec{p}-\vec{q}+\vec{r}) + z(-\vec{p}-\vec{q}+\vec{r})$

$\vec{s} = (-x+y-z)\vec{p} + (x-y-z)\vec{q} + (x+y+z)\vec{r}$

$\therefore -x + y - z = 4$ ….(1)

$x - y - z = 3$ ….(2)

$x + y + z = 5$ ….(3)

Solving (1), (2) & (3)

$x = 4$; $y - z = 8$ and $y + z = 1$

$\therefore\ 2x + y + z = 8 + 1 = 9$

Q.47 For any integer k, let $\alpha_k = \cos\left(\frac{k\pi}{7}\right) + i\sin\left(\frac{k\pi}{7}\right)$, where $i = \sqrt{-1}$. The value of the expression

$$\frac{\sum_{k=1}^{12}|\alpha_{k+1} - \alpha_k|}{\sum_{k=1}^{3}|\alpha_{4k-1} - \alpha_{4k-2}|}$$ is

Ans. **[4]**

Sol. $\alpha_k = \cos\left(\frac{k\pi}{7}\right) + i\sin\left(\frac{k\pi}{7}\right) = e^{i\frac{k\pi}{7}}$

Note :

$$|\alpha_{n+1} - \alpha_n| = \left|e^{i(n+1)\frac{\pi}{7}} - e^{i\frac{n\pi}{7}}\right|$$

$$= \left|e^{i\frac{n\pi}{7}}\left(e^{i\frac{\pi}{7}} - 1\right)\right|$$

$$= \left|e^{i\frac{n\pi}{7}}.e^{i\frac{\pi}{14}}\left(e^{i\frac{\pi}{14}} - e^{-i\frac{\pi}{14}}\right)\right|$$

$$= \left|e^{i\left(\frac{n\pi}{7}+\frac{\pi}{14}\right)}\right| \left|2\sin\frac{\pi}{14}i\right|$$

$$= 2\sin\frac{\pi}{14}$$

$$\therefore \frac{\sum_{K=1}^{12}|\alpha_{k+1} - \alpha_k|}{\sum_{K=1}^{3}|\alpha_{4k-1} - \alpha_{4k-2}|} = \frac{12\cdot 2\sin\frac{\pi}{14}}{3\cdot 2\sin\frac{\pi}{14}} = 4$$

Q.48 Suppose that all the terms of an arithmetic progression (A.P.) are natural numbers. If the ratio of the sum of the first seven terms to the sum of the first eleven terms is 6 : 11 and the seventh term lies in between 130 and 140, then the common difference of this A.P. is

Ans. **[9]**

Sol. An A.P. of natural numbers

$$\frac{S_7}{S_{11}} = \frac{6}{11} \Rightarrow \frac{\frac{7}{2}[2a+6d]}{\frac{11}{2}[2a+10d]} = \frac{6}{11}$$

$$\Rightarrow \frac{7(2a+6d)}{11(2a+10d)} = \frac{6}{11}$$

$\Rightarrow$ 14a + 42d = 12a + 60 d

$\Rightarrow$ 2a = 18 d

$\Rightarrow$ a = 9d

and $130 < a_7 < 140$

$\Rightarrow$ 130 < a + 6d < 140

$\Rightarrow$ 130 < 15 d < 140

$$\Rightarrow \frac{26}{3} < d < \frac{28}{3}$$

$\Rightarrow$ d = 9 (as 'd' is a natural number)

SECTION – 2 (Maximum Marks : 32)

- This Section contains **EIGHT** questions
- Each question has **FOUR** options (A), (B), (C) and (D) **ONE OR MORE THAN ONE** of these four option(s) is (are) correct
- For each questions, darken the bubble (s) corresponding to all the correct option (s) in the **ORS**
- Marking scheme :

  +4 If only the bubble (s) corresponding to all the correct option (s) is (are) darkened

  0 If none of the bubbles is darkened

  –2 In all other cases

Q.49 Let f, g : [–1, 2] → R be continuous functions which are twice differentiable on the interval (–1, 2). Let the values of f and g at the points – 1, 0 and 2 be as given in the following table :

	x = – 1	x = 0	x = 2
f(x)	3	6	0
g(x)	0	1	–1

In each of the intervals (–1, 0) and (0, 2) the function (f – 3g)" never vanishes. Then the correct statement(s) is (are)

(A) f'(x) – 3g'(x) = 0 has exactly three solutions in (– 1, 0) ∪ (0, 2)

(B) f'(x) – 3g'(x) = 0 has exactly one solution in (– 1, 0)

(C) f'(x) – 3g'(x) = 0 has exactly one solution in (0, 2)

(D) f'(x) – 3g'(x) = 0 has exactly two solutions in (– 1, 0) and exactly two solutions in (0, 2)

Ans. **[B, C]**

Sol. f(–1) = 3 f(0) = 6 f(2) = 0

g(–1) = 0 g(0) = 1 g(2) = – 1

h(x) = f(x) – 3g(x)

h (–1) = f(–1) – 3g (–1) = 3 – 3 (0) = 3

h (0) = f(0) – 3g (0) = 6 – 3 (1) = 3

h (2) = f(2) – 3g (2) = 0 – 3 (–1) = 3

By rolles theorem atleast one solution for h'(x) = 0 in (–1, 0)

& atleast one solution for h'(x) = 0 in (0, 2)

Now given at (–1, 0) & (0, 3)

(f – 3g)" = h"(x) ≠ 0

h'(x) is monotonic

so exactly one solution in (–1, 0)

& exactly one solution in (0, 2)

Q.50 Let f(x) = 7 $\tan^8 x + 7\tan^6 x - 3\tan^4 x - 3\tan^2 x$ for all $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$. Then the correct expression(s) is (are)

(A) $\int_0^{\pi/4} x f(x)dx = \frac{1}{12}$

(B) $\int_0^{\pi/4} f(x)dx = 0$

(C) $\int_0^{\pi/4} x f(x)dx = \frac{1}{6}$

(D) $\int_0^{\pi/4} f(x)dx = 1$

Ans. **[A, B]**

Sol. $\int_0^{\pi/4} f(x)dx$

$$= \int_0^{\pi/4} 7\tan^6 x \sec^2 xdx - \int_0^{\pi/4} 3\tan^2 x.\sec^2 xdx$$

Let tan x = t , $\sec^2 xdx = dt$

$$= \int_0^1 7t^6 dt - \int_0^1 3t^2 dt$$

$$= [t^7]_0^1 - [t^3]_0^1 = 1 - 1 = 0$$

Now

$$\int_0^{\pi/4} x f(x)dx$$

$$= \int_0^{\pi/4} \underset{I}{x}(\underset{II}{7\tan^6 x \sec^2 x})dx - \int_0^{\pi/4} \underset{I}{x}.(\underset{II}{3\tan^2 x \sec^2 x})dx$$

$$= [x.\tan^7 x]_0^{\pi/4} - \int_0^{\pi/4} \tan^7 xdx - [x \tan^3 x]_0^{\pi/4} + \int_0^{\pi/4} \tan^3 xdx$$

$$= \frac{\pi}{4} - \int_0^{\pi/4} (\tan^7 x - \tan^3 x)dx - \frac{\pi}{4}$$

$$= -\int_0^{\pi/4} \tan^3 x(\tan^4 x - 1)dx$$

$$-\int_0^{\pi/4} \tan^3 x(\tan^2 x - 1)\sec^2 xdx$$

Let tan x = t

$\therefore \sec^2 xdx = dt$

$$= -\int_0^1 t^3(t^2-1)dt$$

$$= -\left[\frac{t^6}{6}-\frac{t^4}{4}\right]_0^1$$

$$= -\left[\frac{1}{6}-\frac{1}{4}\right] = \frac{1}{12}$$

So options (A, B) are correct

Q.51 Let $f'(x) = \dfrac{192x^3}{2+\sin^4 \pi x}$ for all $x \in R$ with $f\left(\dfrac{1}{2}\right) = 0$. If $m \le \displaystyle\int_{1/2}^{1} f(x)\,dx \le M$, then the possible values of m and M are -

(A) m = 13, M = 24

(B) $m = \dfrac{1}{4}$, $M = \dfrac{1}{2}$

(C) m = –11, M = 0

(D) m = 1, M = 12

Ans. **[D]**

Sol. $f'(x) = \dfrac{192x^3}{2+\sin^4 \pi x} > 0 \quad x \in \left(\dfrac{1}{2}, 1\right)$

f(x) monotonic increasing

$\min f'(x) = \dfrac{192x^3}{2+1} = \dfrac{192x^3}{3}$ (when $\sin^4 \pi x = 1$)

$\min f(x) = \displaystyle\int 64\, x^3\, dx = \dfrac{64x^4}{4} = 16x^4$ at $x = \dfrac{1}{2}$

$\max f'(x) = \dfrac{192x^3}{2+0} = 96x^3$ (when $\sin^4 \pi x = 0$)

$\max f(x) = \dfrac{96}{4}x^4 = 24x^4$ at x = 1

$$16\frac{1}{2^4}\int_{1/2}^{1} dx \le \int_{1/2}^{1} f(x)\,dx \le \int_{1/2}^{1} 24\,dx$$

$$\left(1-\frac{1}{2}\right) \le \int_0^1 f(x)\,dx \le 24\left(1-\frac{1}{2}\right)$$

$$\frac{1}{2} \le \int_{1/2}^{1} f(x)\,dx \le 12$$

Q.52 Let S be the set of all non-zero real numbers α such that the quadratic equation $\alpha x^2 - x + \alpha = 0$ has two distinct real roots x_1 and x_2 satisfying the inequality $|x_1 - x_2| < 1$. Which of the following intervals is(are) a subset(s) of S ?

(A) $\left(-\frac{1}{2}, -\frac{1}{\sqrt{5}}\right)$

(B) $\left(-\frac{1}{\sqrt{5}}, 0\right)$

(C) $\left(0, \frac{1}{\sqrt{5}}\right)$

(D) $\left(\frac{1}{\sqrt{5}}, \frac{1}{2}\right)$

Ans. **Ans. [A, D]**

Sol. Roots are real so

$D > 0$

$1 - 4\alpha^2 > 0$

$\Rightarrow -\frac{1}{2} < \alpha < \frac{1}{2}$ ...(i)

also, $x_1 + x_2 = \frac{1}{\alpha}$, $x_1x_2 = 1$

Given $|x_1 - x_2| < 1$

$\Rightarrow \sqrt{(x_1 + x_2)^2 - 4x_1x_2} < 1$

$\Rightarrow \sqrt{\frac{1}{\alpha^2} - 4} < 1$

$\Rightarrow \frac{1}{\alpha^2} - 4 < 1$

$\Rightarrow \frac{1}{\alpha^2} - 5 < 0$

$\Rightarrow (5\alpha^2 - 1) > 0$

$(\sqrt{5}\,\alpha - 1)(\sqrt{5}\,\alpha + 1) > 0$

$\Rightarrow \alpha < \frac{-1}{\sqrt{5}}$ or $\alpha > \frac{1}{\sqrt{5}}$ ...(ii)

from (i) & (ii)

$\alpha \in \left(\frac{-1}{2}, \frac{-1}{\sqrt{5}}\right) \cup \left(\frac{1}{\sqrt{5}}, \frac{1}{2}\right)$

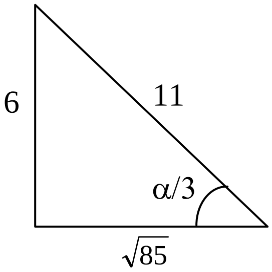
Q.53 If $\alpha = 3\sin^{-1}\left(\frac{6}{11}\right)$ and $\beta = 3\cos^{-1}\left(\frac{4}{9}\right)$, where the inverse trigonometric functions take only the principal values, then the correct option(s) is(are) -

(A) $\cos\beta > 0$ (B) $\sin\beta < 0$

(C) $\cos(\alpha + \beta) > 0$ (D) $\cos\alpha < 0$

Ans. **[B,C,D]**

Sol.



$$\frac{\alpha}{3} = \tan^{-1}\frac{6}{\sqrt{85}}$$

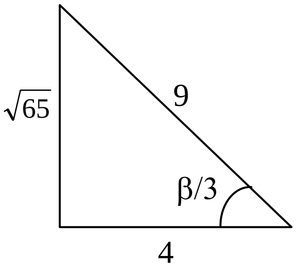
as $\frac{1}{\sqrt{3}} < \frac{6}{\sqrt{85}} < 1$

$$30^o < \frac{\alpha}{3} < 45^o$$

$\Rightarrow 90^o < \alpha < 135^o$

So, $\cos\alpha < 0$ (Option (D) is correct)

Also



$$\Rightarrow \frac{\beta}{3} = \tan^{-1}\frac{\sqrt{65}}{4}$$

as $\frac{\sqrt{65}}{4} > \sqrt{3}$

So, $60^o < \frac{\beta}{3} < 90^o$

$\therefore$ $180^o < \beta < 270^o$

$\therefore$ $\cos\beta < 0, \sin\beta < 0$

so option (A) is incorrect and option (B) is correct.

$(\alpha + \beta)$ will lie in 4^{th} quadrant.

So, $\cos(\alpha + \beta) > 0$

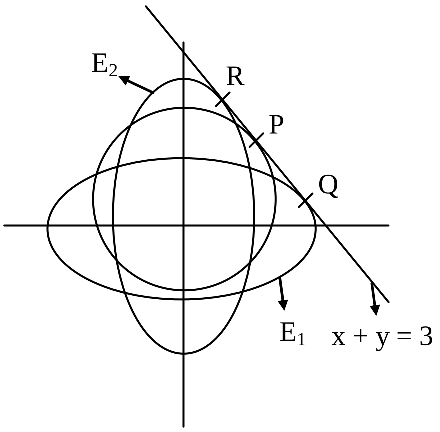
Hence option (C) is correct.

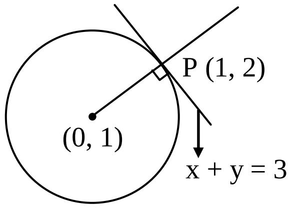
Q.54 Let E_1 and E_2 be two ellipses whose centers are at the origin. The major axes of E_1 and E_2 lie along the x-axis and the y-axis, respectively. Let S be the circle $x^2 + (y - 1)^2 = 2$. The straight line $x + y = 3$ touches the curves S, E_1 and E_2 at P, Q and R, respectively. Suppose that $PQ = PR = \frac{2\sqrt{2}}{3}$. If e_1 and e_2 are the eccentricites of E_1 and E_2, respectively, then the correct expression(s) is (are) -

(A) $e_1^2 + e_2^2 = \frac{43}{40}$ (B) $e_1e_2 = \frac{\sqrt{7}}{2\sqrt{10}}$ (C) $|e_1^2 - e_2^2| = \frac{5}{8}$ (D) $e_1e_2 = \frac{\sqrt{3}}{4}$

Ans. **[A, B]**

Sol.





Given $PQ = PR = \frac{2\sqrt{2}}{3}$

$\therefore$ using parametric equation of line

$$\therefore \quad Q = \left(1 + \frac{2\sqrt{2}}{3} \times \frac{1}{\sqrt{2}}, 2 - \frac{2\sqrt{2}}{3} \times \frac{1}{\sqrt{2}}\right)$$

$$\Rightarrow Q = \left(\frac{5}{3}, \frac{4}{3}\right)$$

Similarly; $R = \left(\frac{1}{3}, \frac{8}{3}\right)$

Now :

Let $\quad E_1 : \frac{x^2}{a_1^2} + \frac{y^2}{b_1^2} = 1$

Let point of contact is $Q(x_1, y_1)$ then equation of tangent is $\frac{xx_1}{a_1^2} + \frac{yy_1}{b_1^2} = 1$

Comparing it with x + y = 3

$Q(x_1, y_1) = \left(\frac{a_1^2}{3}, \frac{b_1^2}{3}\right) \equiv \left(\frac{5}{3}, \frac{4}{3}\right)$

$a_1^2 = 5,\ b_1^2 = 4$

$\Rightarrow \qquad e_1 = \frac{1}{\sqrt{5}}$

Similarly let $E_2 : \frac{x^2}{a_2^2} + \frac{y^2}{b_2^2} = 1$

Then $\quad R\left(\frac{a_2^2}{3}, \frac{b_2^2}{3}\right) = \left(\frac{1}{3}, \frac{8}{3}\right)$

$a_2^2 = 1,\ \ b_2^2 = 8$

$\Rightarrow \qquad e_2 \ \frac{\sqrt{7}}{2\sqrt{2}}$

Now checking option

(A) $e_1^2 + e_2^2 = \frac{1}{5} + \frac{7}{8} = \frac{43}{40}$ (Correct)

(B) $e_1e_2 = \frac{\sqrt{7}}{2\sqrt{10}}$ (Correct) & (D) Incorrect

(C) $|\ e_1^2 - e_2^2| = \left|\frac{1}{5} - \frac{7}{8}\right| = \frac{27}{40}$ (Incorrect)

Q.55 Consider the hyperbola H : $x^2 - y^2 = 1$ and a circle S with center $N(x_2, 0)$. Suppose that H and S touch each other at a point $P(x_1, y_1)$ with $x_1 > 1$ and $y_1 > 0$. The common tangent to H and S at P intersects the x-axis at point M. If (l, m) is the centroid of the triangle ΔPMN, then the correct expression(s) is(are) -

(A) $\frac{dl}{dx_1} = 1 - \frac{1}{3x_1^2}$ for $x_1 > 1$

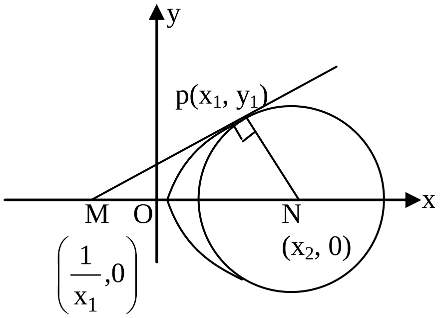
(B) $\frac{dm}{dx_1} = \frac{x_1}{3\left(\sqrt{x_1^2 - 1}\right)}$ for $x_1 > 1$

(C) $\frac{dl}{dx_1} = 1 + \frac{1}{3x_1^2}$ for $x_1 > 1$

(D) $\frac{dm}{dy_1} = \frac{1}{3}$ for $y_1 > 0$

Ans. **[A,B,D]**

Sol. PN is normal to hyperbola & circle



Now normal at $P(x_1, y_1)$ on hyperbola is

$$y - y_1 = \frac{-y_1}{x_1}(x - x_1)$$

put y = 0

$x = 2x_1$

$\therefore\ x_2 = 2x_1$

$N(2x_1, 0)$

Equation of tangent at P (x_1, y_1) on hyperbola is $xx_1 - yy_1 = 1$

Put y = 0

$$x = \frac{1}{x_1}$$

Point $M\left(\frac{1}{x_1}, 0\right)$

$$\therefore\ \ell = \frac{2x_1 + x_1 + \frac{1}{x_1}}{3} \ \&\ m = \frac{0 + y_1 + 0}{3}$$

$$\frac{dm}{dy_1} = \frac{1}{3}\ (x_1 > 1)$$

$$\ell = x_1 + \frac{1}{3x_1}$$

$$\frac{d\ell}{dx_1} = 1 - \frac{1}{3x_1^2}\ (x_1 > 1)$$

Also $m = \frac{1}{3}\sqrt{x_1^2 - 1}$

$$\therefore\ \frac{dm}{dx_1} = \frac{x_1}{3\left(\sqrt{x_1^2 - 1}\right)} \text{ if } x_1 > 1$$

Q.56 The option(s) with the values of a and L that satisfy the following equation is(are)

$$\frac{\int_0^{4\pi} e^t(\sin^6 at + \cos^4 at)dt}{\int_0^{\pi} e^t(\sin^6 at + \cos^4 at)dt} = L\ ?$$

(A) a = 2, L = $\frac{e^{4\pi}-1}{e^{\pi}-1}$ (B) a = 2, L = $\frac{e^{4\pi}+1}{e^{\pi}+1}$

(C) a = 4, L = $\frac{e^{4\pi}-1}{e^{\pi}-1}$ (D) a = 4, L = $\frac{e^{4\pi}+1}{e^{\pi}+1}$

Ans. **[A,C]**

Sol. $$\underbrace{\int_0^{\pi} e^t(\sin^6 at + \cos^4 at)\ dt}_{I_1} + \underbrace{\int_{\pi}^{2\pi} e^t(\sin^6 at + \cos^4 at)\ dt}_{I_2} + \underbrace{\int_{2\pi}^{3\pi} e^t(\sin^6 at + \cos^4 at)\ dt}_{I_3} + \underbrace{\int_{3\pi}^{4\pi} e^t(\sin^6 at + \cos^4 at)\ dt}_{I_4}$$

I_2 put t = π + α

I_3 put t = 2π + α

I_4 put t = 3π + α

Numerator =

$$\int_0^{\pi} e^t(\sin^6 at + \cos^4 at)\ dt + e^{\pi}\int_0^{\pi} e^t(\sin^6 at + \cos^4 at)\ dt + e^{2\pi}\int_0^{\pi} e^t(\sin^6 at + \cos^4 at)\ dt + e^{3\pi}\int_0^{\pi} e^t(\sin^6 at + \cos^4 at)\ dt$$

$$= (1 + e^{\pi} + e^{2\pi} + e^{3\pi})\int_0^{\pi} e^t(\sin^6 at + \cos^4 at)\ dt$$

Now, $$\frac{(1+e^{\pi}+e^{2\pi}+e^{3\pi})\int_0^{\pi} e^t(\sin^6 at + \cos^4 at)\ dt}{\int_0^{\pi} e^t(\sin^6 at + \cos^4 at)\ dt}$$

$L = 1.\ \frac{e^{4\pi}-1}{e^{\pi}-1}$, a = 2, 4

SECTION – 3 (Maximum Marks: 16)

- This Section contains **Two** paragraphs
- Based on each paragraph, there will be **Two** questions
- Each question has **Four** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct
- For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS
- Marking scheme :

+4 If only the bubble(s) corresponding to all the correct option(s) is (are) darkened
0 If none of the bubbles is darkened
–2 In all other cases

Paragraph # 1 (Questions 57 & 58)

Let n_1 and n_2 be the number of red and black balls, respectively, in box I. Let n_3 and n_4 be the number of red and black balls, respectively, in box II.

Q.57 One of the two boxes, box I and box II, was selected at random and a ball was drawn randomly out of this box. The ball was found to be red. If the probability that this red ball was drawn from box II is $\frac{1}{3}$, then the correct option(s) with the possible values of n_1, n_2, n_3 and n_4 is(are) -

(A) $n_1 = 3, n_2 = 3, n_3 = 5, n_4 = 15$

(B) $n_1 = 3, n_2 = 6, n_3 = 10, n_4 = 50$

(C) $n_1 = 8, n_2 = 6, n_3 = 5, n_4 = 20$

(D) $n_1 = 6, n_2 = 12, n_3 = 5, n_4 = 20$

Ans. **[A, B]**

Sol.

Box I	Box II
Red → n_1	Red → n_3
Black → n_2	Black → n_4

Event A = Red ball is drawn.

E_1 : Box I selected

E_2 : Box II selected

$$P\left(\frac{E_2}{A}\right) = \frac{P(E_2)P\left(\frac{A}{E_2}\right)}{P(E_1)P\left(\frac{A}{E_1}\right)+P(E_2)P\left(\frac{A}{E_2}\right)}$$

$$\frac{1}{3} = \frac{\frac{1}{2}.\frac{n_3}{n_3+n_4}}{\frac{1}{2}\frac{n_1}{n_1+n_2}+\frac{1}{2}\frac{n_3}{n_3+n_4}}$$

$$\frac{n_1}{n_1+n_2}+\frac{n_3}{n_3+n_4} = \frac{3n_3}{n_3+n_4}$$

(A) $\frac{3}{6}+\frac{5}{20}=\frac{1}{2}+\frac{1}{4}=\frac{3.5}{20}=\frac{3}{4}$ correct

(B) $\frac{3}{9}+\frac{10}{60}=\frac{3(10)}{60} \Rightarrow \frac{1}{3}+\frac{1}{6}=\frac{1}{2} \Rightarrow \frac{1}{2}=\frac{1}{2}$ correct

(C) $\frac{8}{14}+\frac{5}{25}=\frac{3.5}{25} \Rightarrow \frac{4}{7}+\frac{1}{5}\neq\frac{3}{5}$ incorrect

(D) $\frac{6}{18}+\frac{5}{25}=\frac{3.5}{25} \Rightarrow \frac{1}{3}+\frac{1}{5}\neq\frac{3}{5}$ incorrect

Q.58 A ball is drawn at random from box I and transferred to box II. If the probability of drawing a red ball from box I, after this transfer, is $\frac{1}{3}$, then the correct option(s) with the possible values of n_1 and n_2 is (are) -

(A) $n_1 = 4$ and $n_2 = 6$ (B) $n_1 = 2$ and $n_2 = 3$ (C) $n_1 = 10$ and $n_2 = 20$ (D) $n_1 = 3$ and $n_2 = 6$

Ans. **[C, D]**

Sol. Case (I) : Red ball is transferred

Red → $n_1 - 1$

Black → n_2

Case (II) : Black ball is transferred

Red → n_1

Black → $n_2 - 1$

$$P = \underbrace{\left(\frac{n_1}{n_1+n_2}\right)}_{\substack{\downarrow\\ \text{Probability}\\ \text{of Red ball}\\ \text{transferred}}}\underbrace{\left(\frac{n_1-1}{n_1+n_2-1}\right)}_{\substack{\downarrow\\ \text{Probability}\\ \text{of selecting}\\ \text{Red ball}}} + \underbrace{\left(\frac{n_2}{n_1+n_2}\right)}_{\substack{\downarrow\\ \text{Probability}\\ \text{of Black ball}\\ \text{transferred}}}\underbrace{\left(\frac{n_1}{n_1+n_2-1}\right)}_{\substack{\downarrow\\ \text{Probability}\\ \text{of selecting}\\ \text{Red ball}}}$$

$$P = \frac{n_1^2 - n_1 + n_1n_2}{(n_1+n_2)(n_1+n_2-1)} = \frac{n_1(n_1+n_2-1)}{(n_1+n_2)(n_1+n_2-1)} = \frac{n_1}{(n_1+n_2)} = \frac{1}{3}$$

(A) $\frac{4}{10} \neq \frac{1}{3}$ Incorrect

(B) $\frac{2}{5} \neq \frac{1}{3}$ Incorrect

(C) $\frac{10}{30} = \frac{1}{3}$ Correct

(D) $\frac{3}{9} = \frac{1}{3}$ Correct

Paragraph For Questions 59 and 60

Let F : R → R be a thrice differentiable function. Suppose that $F(1) = 0$, $F(3) = -4$ and $F'(x) < 0$ for all $x \in (1/2, 3)$. Let $f(x) = xF(x)$ for all $x \in R$.

Q.59 The correct statement(s) is(are) -

(A) $f'(1) < 0$ (B) $f(2) < 0$

(C) $f'(x) \neq 0$ for any $x \in (1, 3)$ (D) $f'(x) = 0$ for some $x \in (1, 3)$

Ans. **[A,B,C]**

Sol. $F(1) = 0$ $F(3) = –4$ $F'(x) < 0,\ x \in \left(\frac{1}{2}, 3\right)$

$F(x)$ is decreasing in $\left(\frac{1}{2}, 3\right)$

$f(x) = xF(x)$

$f'(x) = F(x) + xF'(x) < 0, x \in (1, 3)$

so $f(x)$ is decreasing

(A) $f'(1) = F(1) + F'(1) = F'(1) < 0$

So option (A) is correct

(B) $f(1) = 1F(1) = 0$

$f(3) = 3F(3) = –12$

$f(2) < 0$ correct

(C) $f'(x) < 0$ so $f'(x) \neq 0$ any $x \in (1, 3)$ correct

Q.60 If $\int_1^3 x^2F'(x)dx = –12$ and $\int_1^3 x^3F''(x)dx = 40$, then the correct expression(s) is(are) -

(A) $9f'(3) + f'(1) – 32 = 0$

(B) $\int_1^3 f(x)dx = 12$

(C) $9f'(3) – f'(1) + 32 = 0$

(D) $\int_1^3 f(x)dx = –12$

Ans. **[C,D]**

Sol. $\int_1^3 x^3F''(x)dx = x^3F'(x)\Big|_1^3 – \int_1^3 3x^2F'(x)dx$

$40 = 27F'(3) – F'(1) – 3(–12)$

$27F'(3) – F'(1) = 4$ … (1)

$f'(x) = F(x) + xF'(x)$

$f'(1) = F(1) + F'(1) \Rightarrow F'(1) = f'(1)$

$f'(3) = F(3) + 3F'(3)$

$f'(3) = – 4 + 3F'(3) \Rightarrow 3F'(3) = f'(3) + 4$

Eq^n (1) $9(f'(3) + 4) – f'(1) = 4$

$\Rightarrow 9f'(3) – f'(1) + 32 = 0$ so option (C) is correct

$$\int_1^3 f(x)\,dx = \int_1^3 xF(x)\,dx = \frac{x^2}{2}F(x)\Big|_1^3 - \int_1^3 \frac{x^2}{2}F'(x)\,dx$$

$$= \frac{9}{2}F(3) - \frac{F(1)}{2} - \frac{1}{2}\int_1^3 x^2F'(x)\,dx$$

$$= \frac{9}{2}(-4) - 0 - \frac{1}{2}(-12)$$

$= -18 + 6 = -12$ so option (D) is correct