CAREER POINT

JEE Advance Exam 2014 (Solution)

Part I - PHYSICS

Date : 25 / 05 / 2014

SECTION – 1 (One or More Than One Option Correct Type)

This section contains 10 **multiple choice questions**. Each question has 4 choices (A), (B), (C) and (D), out of which **ONE or MORE THAN ONE** are correct.

Q.1 A student is performing an experiment using a resonance column and tuning fork of frequency 244 s^{-1}. He is told that the air in the tube has been replaced by another gas (assume that the column remains filled with gas). If the minimum height at which resonance occurs is (0.350 ± 0.005) m, the gas in the tube is

(Useful information : $\sqrt{167RT} = 640\ J^{1/2}\ mole^{-1/2}$; $\sqrt{140RT} = 590\ J^{1/2}\ mole^{-1/2}$. The molar masses M in grams are given in the options. Take the values of $\sqrt{\frac{10}{M}}$ for each gas as given there.)

(A) Neon $\left(M = 20, \sqrt{\frac{10}{20}} = \frac{7}{10}\right)$

(B) Nitrogen $\left(M = 28, \sqrt{\frac{10}{28}} = \frac{3}{5}\right)$

(C) Oxygen $\left(M = 32, \sqrt{\frac{10}{32}} = \frac{9}{16}\right)$

(D) Argon $\left(M = 36, \sqrt{\frac{10}{36}} = \frac{17}{32}\right)$

Ans. **[D]**

Sol. $\ell = (0.350 \pm 0.005)$ m

$\lambda = (4\ell) = 4\,(0.350 \pm 0.005)$

$\lambda = (1.4 \pm 0.020)$

$1.38\text{ m} \le \lambda \le 1.42\text{ m}$

$v = n\lambda \Rightarrow \lambda = \frac{v}{n}$

$$\lambda = \frac{\sqrt{\frac{\gamma RT}{M}}}{n}$$

$$\lambda = \frac{1}{n}\sqrt{\frac{\gamma RT}{M\times10^{-3}}} \qquad M \text{ (in gm)}$$

$$\lambda = \frac{1}{n}\sqrt{100\gamma RT}\sqrt{\frac{10}{M}}$$

(A) Neon $\left(\gamma = \frac{5}{3}\right)$ M = 20

$$\lambda = \frac{1}{244}\times\sqrt{100\left(\frac{5}{3}\right)RT}\sqrt{\frac{10}{20}} \quad\Rightarrow\quad \lambda = \frac{640}{244}\times\left[\frac{7}{10}\right] = 1.83 \text{ m}$$

(B) Nitrogen N_2 $\left(\gamma = \frac{7}{5}\right)$, M = 28

$$\lambda = \frac{1}{244}\times\sqrt{100\frac{7}{5}RT}\times\sqrt{\frac{10}{28}} \quad\Rightarrow\quad \lambda = \frac{590}{244}\times\frac{3}{5} = 1.45 \text{ m}$$

(C) Oxygen (O_2) (γ= 7/5), M = 32

$$\lambda = \frac{1}{244}\times\sqrt{100\times\frac{7}{5}RT}\times\left(\sqrt{\frac{10}{32}}\right) \quad\Rightarrow\quad \lambda = \frac{1}{244}\times590\times\frac{9}{16} = 1.36 \text{ m}$$

(D) Argon (M = 36) (γ = 5/3)

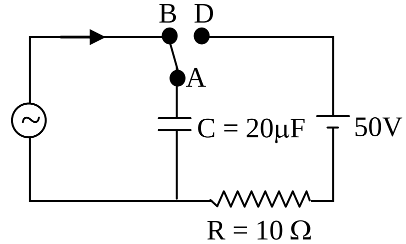
$$\lambda = \frac{1}{244}\times\sqrt{100\left(\frac{5}{3}\right)RT}\times\sqrt{\frac{10}{36}} \quad\Rightarrow\quad \lambda = \frac{1}{244}\times640\times\frac{17}{32}$$

λ = 1.39 m

only wavelength in (D) option lies b/w 1.38 m to 1.42 m

Ans. $\Rightarrow$ option (D)

Q.2 At time t = 0, terminal A in the circuit shown in the figure is connected to B by a key and an alternating current I (t) = I_0 cos (ωt), with I_0 = 1A and ω = 500 rad s^{-1} starts flowing in it with the initial direction shown in the figure . At t = $\frac{7\pi}{6\omega}$, the key is switched from B to D. Now onwards only A and D are connected. A total charge Q flows from the battery to charge the capacitor fully. If C = 20 μF, R = 10 Ω and the battery is ideal with emf of 50 V, identify the correct statement (s).



(A) Magnitude of the maximum charge on the capacitor before t = $\frac{7\pi}{6\omega}$ is 1×10^{-3} C.

(B) The current in the left part of the circuit before circuit just before t = $\frac{7\pi}{6\omega}$ is clockwise.

(C) Immediately after A is connected to D, the current in R is 10 A.

(D) Q = 2×10^{-3} C.

Ans. **[C, D]**

Sol. given that-

$I = I_0 \cos \omega t$

$$\frac{dq}{dt} = I_0 \cos \omega t$$

$$q = \int_0^{t_0} I_0 \cos\omega t \Rightarrow q = \frac{I_0}{\omega}\left[\sin \omega t\right]_0^{t_0} = \frac{7\pi}{6\omega} \Rightarrow q = \frac{1}{500} \sin \frac{7\pi}{6}$$

$q = 10^{-3}$ C

But it is not maximum charge.

Maximum charge is

$$q = \frac{I_0}{\omega} = \frac{1}{500} \text{ C}$$

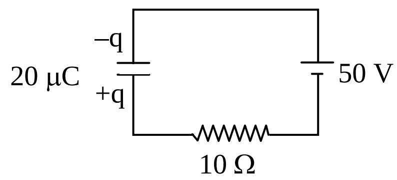
So, (A) is incorrect

(B) $I = I_0 \cos \omega t$

$$I_0 = I_0 \cos \omega \frac{7\pi}{6\omega} = I_0 \cos(\pi + \pi/3)$$

$I_0 = -I_0 \frac{\sqrt{3}}{2}$ opposite to initial direction and Anticlockwise

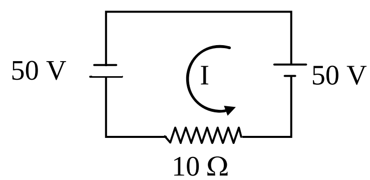
(C) Immediately after A is connected to D the current in R is 10 A.



$q = 10^{-3}$ C

$\therefore$ Potential difference across capacitor = $\frac{q}{C} = \frac{10^{-3}}{20\times10^{-6}}$ = 50 Volt .

circuit is

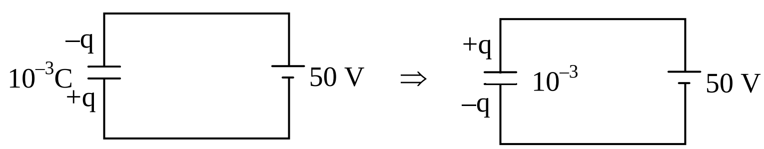


$I = \frac{50+50}{10} = \frac{100}{10} = 10$ A

So, C is correct

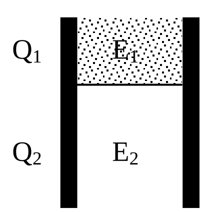
(D) Final charge on capacitor q = CV

$q = 20 \times 10^{-6} \times 50 = 10^{-3}$ C



Net charge flow = 2×10^{-3} C So D is correct

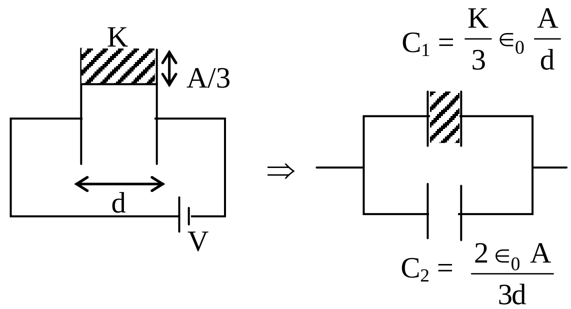
Q.3 A parallel plate capacitor has a dielectric slab of dielectric constant K between its plates that covers 1/3 of the area of its plates, as shown in the figure. The total capacitance of the capacitor is C while that of the portion with dielectric in between is C_1. When the capacitor is charged, the plate are covered by the dielectric gets charge Q_1 and the rest of the area charge Q_2. The electric field in the dielectric is E_1 and that in the other portion is E_2. Choose the correct option/ options, ignoring edge effects.



(A) $\frac{E_1}{E_2} = 1$ (B) $\frac{E_1}{E_2} = \frac{1}{K}$ (C) $\frac{Q_1}{Q_2} = \frac{3}{K}$ (D) $\frac{C}{C_1} = \frac{2+K}{K}$

Ans. **[A, D]**

Sol.



$C_1 = \frac{K\in_0 A}{3d} \Rightarrow C = \frac{K\in_0 A}{3d} + \frac{2\in_0 A}{3d} \Rightarrow C = (K+2)\in_0 \frac{A}{3d}$

$q_1 = C_1 V = \frac{K\in_0 AV}{3d}$; $q_2 = C_2 V = \frac{2\in_0 AV}{3d}$

$E_1 = \frac{q_1}{K\in_0 \frac{A}{3}} = \frac{V}{d}$; $E_2 = \frac{q_2}{\in_0\left(\frac{2}{3}A\right)} = \frac{V}{d}$

here $\frac{C}{C_1} = \frac{2+K}{K}$; $\frac{q_1}{q_2} = \frac{K}{2}$; $\frac{E_1}{E_2} = 1$

Q.4 One end of a taut string of length 3m along the x axis is fixed at x = 0. The speed of the waves in the string is 100 ms^{-1}. The other end of the string is vibrating in the y direction so that stationary waves are set up in string. The possible waveforms (s) of these stationary waves is (are)

(A) $y(t) = A\sin\frac{\pi x}{6}\cos\frac{50\pi t}{3}$

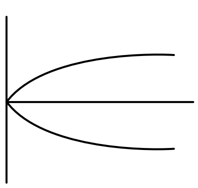
(B) $y(t) = \sin\frac{\pi x}{3}\cos\frac{100\pi t}{3}$

(C) $y(t) = A\sin\frac{5\pi x}{6}\cos\frac{250\pi t}{3}$

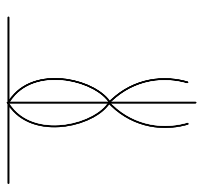
(D) $y(t) = A\sin\frac{5\pi x}{3}\cos 250\pi t$

Ans. **[A,C,D]**

Sol.



$\ell = \frac{\lambda}{4}$; $\lambda = 4 \times 3 = 12$



$\frac{3\lambda}{4} = \ell$; $\ell = 4$

<u>**Similarly**</u>

$\frac{15\lambda}{4} = \ell$

$\frac{2\pi}{\lambda} = \frac{\pi}{6}$ $\qquad$ $\frac{2\pi}{\lambda} = \frac{5\pi}{6}$

$\boxed{\lambda = 12}$ $\qquad$ $\boxed{\lambda = \frac{12}{5}}$

$\frac{2\pi}{\lambda} = \frac{5\pi}{6}$; $\lambda = \left(\frac{12}{5}\right)$; $\ell = \frac{5}{4} \times \frac{12}{5}$; $\ell = 3$

Alternative Solution

String in fixed at $x = 0$

And the other is vibrating so that the stationary wave is developed, that means source is at other end.

At source position there will be node

That implies both end is node.

V = 100 m/s.

$\ell = 3$ m

Option (A) $\Rightarrow K = \frac{\pi}{6} \Rightarrow \lambda = 12$ m

$\ell = \frac{\lambda}{4}$ one node other antinode

Option (B) $\Rightarrow K = \frac{\pi}{3} \Rightarrow \frac{2\pi}{\lambda} = \frac{\pi}{3} \Rightarrow \lambda = 6$ m.

$\ell = \frac{\lambda}{2}$ both end node.

Option (C) $\Rightarrow K = \frac{5\pi}{6} \Rightarrow \frac{2\pi}{\lambda} = \frac{5\pi}{6} \Rightarrow \lambda = \frac{12}{5}$

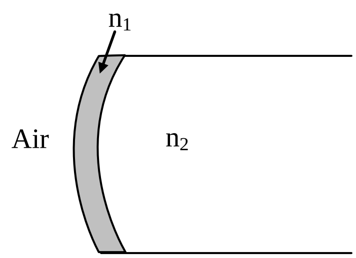
$\ell = \frac{5\lambda}{4}$ one node other antinodes

Option (D) $K = \frac{5\pi}{2} \Rightarrow \frac{2\pi}{\lambda} = \frac{5\pi}{2} \Rightarrow \lambda = \frac{4}{5}$

$\ell = \frac{15}{4}\lambda$, one node other antinodes

Answer (B) option

Q.5 A transparent thin film of uniform thickness and refractive index $n_1 = 1.4$ is coated on the convex spherical surface of radius R at one end of a long solid glass cylinder of refractive index $n_2 = 1.5$, as shown in figure. Rays of light parallel to the axis of the cylinder traversing through the film from air to glass get focused at distance f_1 from the film, while rays of light traversing from glass to air get focused at distance f_2 from the film. Then



(A) $|f_1| = 3R$ (B) $|f_1| = 2.8R$ (C) $|f_2| = 2R$ (D) $|f_1| = 1.4$ R

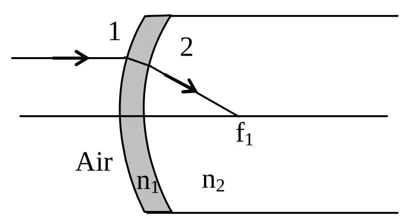
Ans. **[A, C]**

Sol. For 1st Refraction

$$\frac{n_1}{v_1} - \frac{1}{\infty} = \frac{n_1 - 1}{R} \Rightarrow \frac{n_1}{v_1} = \frac{1.4-1}{R}$$

$$v_1 = \frac{1.4R}{0.4} \Rightarrow v_1 = \frac{7R}{2}$$

Now for 2nd Refraction-



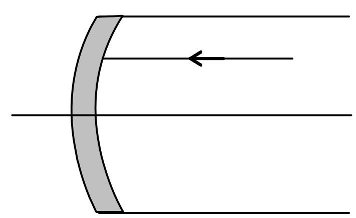
$$\frac{1.5}{f_1} - \frac{1.4}{\frac{7R}{2}} = \frac{1.5-1.4}{R}$$

$$\frac{1.5}{f_1} = \frac{1}{10R} + \frac{2.8}{7R} = \frac{1}{2R}$$

$f_1 = 3R$

For 2nd Case:

For 1st refraction-



$$\frac{n_1}{v_1} - \frac{n_2}{\infty} = \frac{n_1 - n_2}{-R}$$

$$\frac{1.4}{v_1} = \frac{1.4-1.5}{-R}$$

$v_1 = 14$ R

For 2nd Refraction-

$$\frac{1}{f_2} - \frac{1.4}{14R} = \frac{1-1.4}{-R}$$

$$\frac{1}{f_2} = \frac{4}{10R} + \frac{1}{10R} = \frac{1}{2R}$$

$f_2 = 2R$

Q.6 Heater of an electric kettle is made of a wire of length L and d. it takes 4 minutes to raise the temperature of 0.5 kg water by 40 K. This heater is replaced by a new heater having two wires of the same material each of length L and diameter 2d. The way these wires are connected is given in the options. How much time in minutes will it take to raise the temperature of the same amount of water by 40 K ?

(A) 4 if wires are in parallel
(B) 2 if wires are in series
(C) 1 if wires are in the series
(D) 0.5 if wires are in parallel

Ans. **[B, D]**

Sol. Resistance of wire = $\dfrac{\rho L}{\frac{\pi}{4}d^2}$ = RΩ

Power produced = $\dfrac{V^2}{R}$ watt

Energy in 4 min = $\dfrac{V^2}{R} \times 4 \times 60 = \dfrac{240V^2}{R}$ Joule

this energy is used to raise the temperature of 0.5 kg water by 40k

$$\frac{240V^2}{R} = 0.5\ S \times \Delta T$$

$$\frac{240V^2}{R} = 0.5 \times S \times 40 \qquad \ldots (1)$$

New wire have length L & diameter = 2d

$\therefore$ New resistance of wire = R′ = $\dfrac{\rho L}{\frac{\pi}{4} \times (2d)^2} = \dfrac{R}{4}\ \Omega$

if these wire are connected in parallel then equivalent resistance $R_{eq} = \dfrac{R}{8}\ \Omega$

then to raise the temperature of water

$$\frac{V^2}{\frac{R}{8}} \times t \times 60 = 0.5 \times S \times 40 \qquad \ldots (2)$$

where t is in minute

$$480\ \frac{V^2}{R}\ t = 0.5 \times S \times 40$$

from (1) & (2) $480\dfrac{V^2}{R}\ t = 240\dfrac{V^2}{R}$

$t = \frac{1}{2} = 0.5$ min. Ans. (D)

if these wire are connected in series $R_{eq} = \frac{R}{4} + \frac{R}{4} = \frac{R}{2}$

then to raise the temperature of water

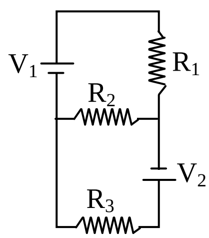
$$\frac{V^2}{\frac{R}{2}} t \times 60 = 0.5S \times 40$$

$$120\frac{V^2}{R} t = 0.5 \times S \times 40 \quad \ldots (3)$$

from (1) & (3)

$$120\frac{V^2}{R} t = 240\frac{V^2}{R} \therefore t = 2 \text{ min Ans. (B)}$$

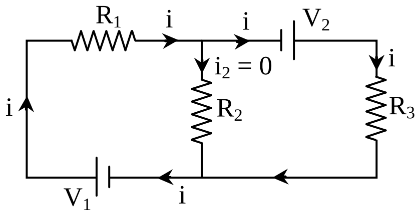
Q.7 Two ideal batteries of emf V_1 and V_2 and three resistance R_1, R_2 and R_3 connected as shown in the figure. The current in resistance R_2 would be zero if



(A) $V_1 = V_2$ and $R_1 = R_2 = R_3$
(B) $V_1 = V_2$ and $R_1 = 2R_2 = R_3$
(C) $V_1 = 2V_2$ and $2R_1 = 2R_2 = R_3$
(D) $2V_1 = V_2$ and $2R_1 = R_2 = R_3$

Ans. **[A,B,D]**

Sol.



$$i = \frac{V_1 + V_2}{R_1 + R_3} = \frac{V_1}{R_1} = \frac{V_2}{R_3}$$

or $\boxed{\frac{V_1}{V_2} = \frac{R_1}{R_3}}$

Q.8 Let $E_1(R)$ and E_2 (r) and E_3 (r) be the respective electric fields at a distance r from a point charge Q, an infinitely long wire with constant linear charge density λ, and an infinite plane with uniform surface charge density σ . If E_1 (r_0) = E_2 (r_0) = E_3 (r_0) at a given distance r_0, then

(A) $Q = 4\sigma\pi r_0^2$

(B) $r_0 = \frac{\lambda}{2\pi\sigma}$

(C) E_1 $(r_0 / 2) = 2E_2$ $(r_0 /2)$

(D) E_2 $(r_0 /2) = 4E_3$ $(r_0 /2)$

Ans. **[C]**

Sol. $E_1(r_0) = E_2(r_0) = E_3(r_0)$

$$\frac{1}{4\pi\in_0}\frac{Q}{r_0^2} = \frac{\lambda}{2\pi\in_0 r_0} = \frac{\sigma}{2\in_0}$$

i.e. $\boxed{Q = 2\pi\sigma r_0^2 = 2\lambda r_0}$

and $r_0 = \boxed{\frac{\lambda}{\pi\sigma}}$

Q.9 A light source, which emits two wavelength λ_1 = 400 nm and λ_2 = 600 nm, is used in a Young's double slit experiment. If recorded fringe widths for λ_1 and λ_2 are β_1 and β_2 and the number of fringes for them within a distance y on one side of the central maximum are m_1 and m_2 respectively, then -

(A) $\beta_2 > \beta_1$

(B) $m_1 > m_2$

(C) from the central maximum, 3rd maximum of λ_2 overlaps with 5th minimum of λ_1

(D) the angular separation of fringes for λ_1 is greater than λ_2

Ans. **[A, B, C]**

Sol. * Fringe width $\beta = \dfrac{\lambda D}{d}$

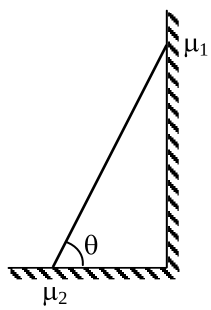
$\because\ \lambda_2 > \lambda_1$

$\therefore\ \beta_2 > \beta_1$

* $m_1\beta_1 = m_2\beta_2$

* $(2\times5-1)\dfrac{\lambda_1}{2}\dfrac{D}{d} = 3\lambda_2\dfrac{D}{d}$

Q.10 In the figure, a ladder of mass m is shown leaning against a wall. It is in static equilibrium making an angle θ with the horizontal floor. The coefficient of friction between the wall and the ladder is μ_1 and that between the floor and the ladder is μ_2. The normal reaction of the wall on the ladder is N_1 and that of the floor is N_2. If the ladder is about to slip, then -



(A) $\mu_1 = 0$; $\mu_2 \neq 0$ and $N_2 \tan\theta = \dfrac{mg}{2}$

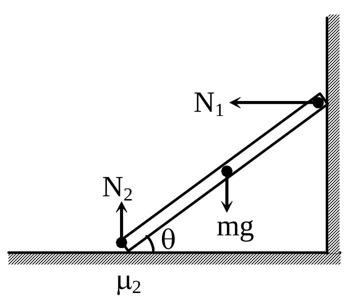
(B) $\mu_1 \neq 0$; $\mu_2 = 0$ and $N_1 \tan\theta = \dfrac{mg}{2}$

(C) $\mu_1 \neq 0$; $\mu_2 \neq 0$ and $N_2 = \dfrac{mg}{1+\mu_1\mu_2}$

(D) $\mu_1 = 0$; $\mu_2 \neq 0$ and $N_1 \tan\theta = \dfrac{mg}{2}$

Ans. **[C,D]**

Sol. When $\mu_1 = 0$ $\mu_2 \neq 0$



$mg = N_2$

$N_1 = \mu N_2$

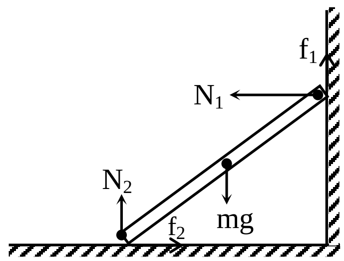
$$mg\frac{\ell}{2}\cos\theta = N_1\ell\sin\theta$$

$$N_1 \tan\theta = \frac{mg}{2}$$

$\therefore$ (D) is correct

(A) is incorrect

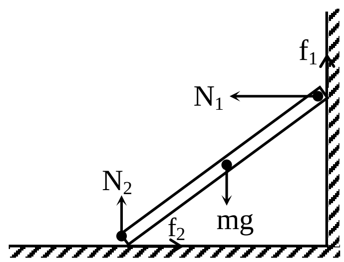
when $\mu_1 \neq 0$, $\mu_2 = 0$



$\because f_2 = 0 \quad \therefore N_1 = 0$

(B) is wrong

When $\mu_1 \neq 0 \quad \mu_2 \neq 0$



$N_2 + f_1 = mg$

$f_1 = \mu_1 N_1$

$\therefore N_2 + \mu_1 N_1 = mg$ ....(i)

$N_1 = f_2$

$N_1 = \mu N_2$ ....(ii)

$N_2 + \mu_1\mu_2 N_2 = mg$

$$N_2 = \frac{mg}{1+\mu_1\mu_2}$$

(C) is correct

SECTION – 2 (ONE Integer Value Correct Type)

This section contains **10 questions**. Each question, when worked out will result in one integer from 0 to 9 (both inclusive)

Q.11 During Searle's experiment, zero of the vernier scale lies between 3.20×10^{-2} m and 3.25×10^{-2} m of the main scale. The 20^{th} devision of the vernier scale exactly coincides with one of the main scale division. When an additional load of 2 kg is applied to the wire, the zero of the vernier scale still lies between 3.20×10^{-2} m and 3.25×10^{-2} m of the main scale but now the 45^{th} division of vernier scale concides with one of the main scale divisions. The length of the thin metallic wire is 2 m and its cross-sectional area is 8×10^{-7} m^2. The least count of the vernier scale is 1.0×10^{-5} m. The maximum percentage error in the Young's modulus of the wire is.

Ans. **[4]**

Sol. $\frac{Mg}{AY} = \frac{\text{change in length}}{\text{length}} = \frac{x}{\ell}$

$\Rightarrow \quad Y = \frac{Mg\ell}{Ax}$

$\Rightarrow \quad \frac{dY}{Y} = \frac{dx}{x}$

dx is the maximum error which is equal to least count = 1.0×10^{-5} metre $\Rightarrow$ 0.001 cm

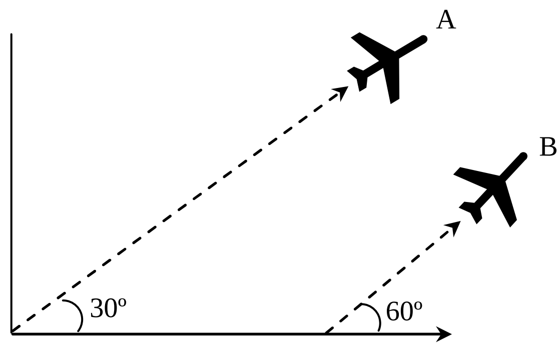
$\Rightarrow \quad x = (45 - 20) \times L.C$

$= 25 \times 0.001$

$\Rightarrow \quad 0.025$ cm

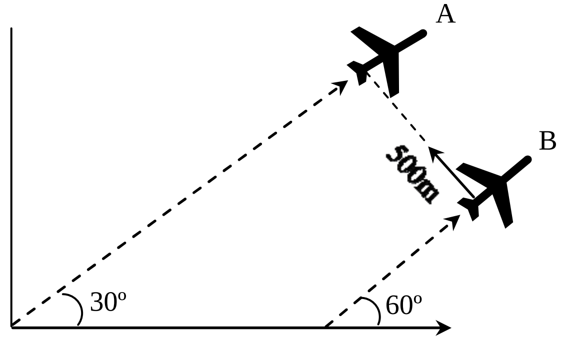
$\Rightarrow \quad \text{error } \% \ \frac{dY}{Y} = \frac{0.001}{0.025} \times 100 = 4\ \%$

Q.12 Airplanes A and B are flying with constant velocity in the same vertical plane at angles 30º and 60º with respect to the horizontal respectively as shown in figure. The speed of A is $100\sqrt{3}$ ms^{-1}. At time t = 0 s, an observer in A finds B at a distance of 500 m. This observer sees B moving with a constant velocity perpendicular to the line of motion of A. If at $t = t_0$, A just escapes being hit by B, t_0 in seconds is.



Ans. **[5]**

Sol.



Plane are collide at t_0

$$500 = \left|(\vec{V}_A - \vec{V}_B)\right| \times t_0 \qquad ....(1)$$

$$\vec{V}_A = 100\sqrt{3}\,[\cos 30\,\hat{i} + \sin 30\,\hat{j}] = 100\sqrt{3}\left[\frac{\sqrt{3}}{2}\hat{i} + \frac{1}{2}\hat{j}\right] = 50\sqrt{3}[\sqrt{3}\,\hat{i} + \hat{j}]$$

$$\vec{V}_B = V_0\left[\frac{1}{2}\hat{i} + \frac{\sqrt{3}}{2}\hat{j}\right]$$

It is given

$$\left(\frac{\sqrt{3}}{2}\hat{i} + \frac{1}{2}\hat{j}\right) . [\vec{V}_B - \vec{V}_A] = 0$$

$$\left(\frac{\sqrt{3}}{2}\hat{i} + \frac{1}{2}\hat{j}\right) . \left[V_0\left(\frac{1}{2}\hat{i} + \frac{\sqrt{3}}{2}\hat{j}\right) - 50\sqrt{3}(\sqrt{3}\,\hat{i} + \hat{j})\right] = 0$$

$$\left(\frac{\sqrt{3}}{2}\hat{i} + \frac{1}{2}\hat{j}\right)\left[\left(\frac{V_0}{2} - 150\right)\hat{i} + \left(V_0\frac{\sqrt{3}}{2} - 50\sqrt{3}\right)\hat{j}\right] = 0$$

$$\frac{\sqrt{3}}{2}\left(\frac{V_0}{2} - 150\right) + \frac{1}{2}\left(V_0\frac{\sqrt{3}}{2} - 50\sqrt{3}\right) = 0$$

$$\frac{\sqrt{3}}{4}V_0 - 75\sqrt{3} + V_0\frac{\sqrt{3}}{4} - 25\sqrt{3} = 0 \Rightarrow \qquad \frac{\sqrt{3}}{2}V_0 = 100\sqrt{3} \Rightarrow V_0 = 200$$

$$\vec{V}_B = 200\left[\frac{1}{2}\hat{i} + \frac{\sqrt{3}}{2}\hat{j}\right]$$

by equation (1)

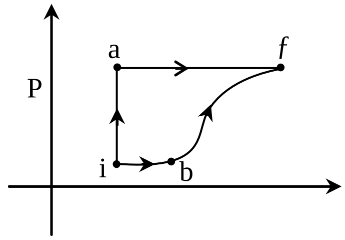
$$500 = |100\,\hat{i} + 100\sqrt{3}\,\hat{j} - 150\,\hat{i} - 50\sqrt{3}\,\hat{j}|\,t$$

$$500 = |-50\,\hat{i} + 50\sqrt{3}\,\hat{j}| \times t$$

$$500 = \sqrt{(50)^2 + (50\sqrt{3})^2}\ t$$

$$500 = 50\sqrt{1+3}\,t \Rightarrow t = \frac{10}{2} = 5 \text{ sec}$$

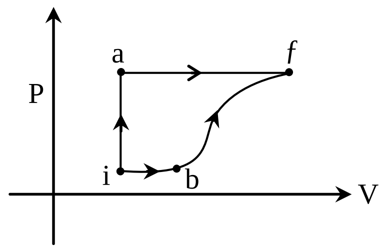
Q.13 A thermodynamic system is taken from an initial state i with internal energy $U_i = 100$ J to the final state f along two different paths ia$f$ and ib$f$, as schematically shown in the figure. The work done by the system along the paths af, ib and bf are $W_{af} = 200$ J, $W_{ib} = 50$ J and $W_{bf} = 100$ J respectively. The heat supplied to the system along the path ia$f$, ib and bf are Q_{iaf}, Q_{ib} and Q_{bf} respectively. If the internal energy of the system in the state b is $U_b = 200$ J and $Q_{iaf} = 500$ J the ratio Q_{bf}/Q_{ib} is.



Ans. **[2]**

Sol.

$$\left.\begin{array}{ll} W_{af} = 200\text{ J} & U_b = 200\text{ J} \\ W_{ib} = 50\text{ J} & Q_{iaf} = 500\text{ J} \\ W_{bf} = 100\text{ J} & U_i = 100\text{ J} \end{array}\right] \text{ Given}$$



For process iaf

$Q_{iaf} = W_{af} + U_f - U_i$

$500 = 200 + U_f - 100$

$U_f = 400$ J

For process ib

$Q_{ib} = W_{ib} + U_b - U_i = 50 + 200 - 100$

$Q_{ib} = 150$ J

For process bf

$Q_{bf} = W_{bf} + U_f - U_b = 100 + 400 - 200$

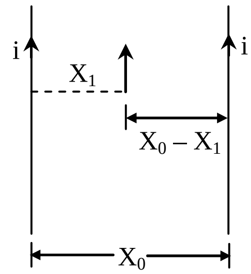
$Q_{bf} = 300$ J

$$\frac{Q_{bf}}{Q_{ib}} = \frac{300}{150} = 2$$

Q.14 Two parallel wires in the plane of the paper are distance X_0 apart. A point charge is moving with speed u between the wires in he same plane at a distance X_1 from one of the wires. When the wires carry current of magnitude I in the same direction, the radius of curvature of the path of the point charge is R_1. In contrast, if the currents I in the two wires have directions opposite to each other, the radius of curvature of the path is R_2. If $\frac{X_0}{X_1} = 3$, the value of $\frac{R_1}{R_2}$ is.

Ans. **[3]**

Sol.



Radius of curvature of path = R = $\frac{mv}{qB}$

$\Rightarrow \quad R_1 = \frac{mv}{qB_1}$

$\Rightarrow \quad R_2 = \frac{mv}{qB_2}$

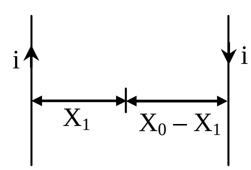
$\Rightarrow \quad \frac{R_1}{R_2} = \frac{B_2}{B_1}$ ….(i)

$\Rightarrow \quad B_1 = \frac{\mu_0 i}{2\pi X_1} - \frac{\mu_0 i}{2\pi(X_0 - X_1)}$

Given $\frac{X_0}{X_1} = 3 \quad \therefore X_0 = 3X_1$

$B_1 = \frac{\mu_0 i}{2\pi X_1} - \frac{\mu_0 i}{2\pi(3X_1 - X_1)} \Rightarrow \frac{\mu_0 i}{4\pi X_1}$ ….(ii)

When current in wire is in opposite direction



Then , $B_2 = \frac{\mu_0 i}{2\pi X_1} + \frac{\mu_0 i}{2\pi(X_0 - X_1)} = \frac{\mu_0 i}{2\pi X_1} + \frac{\mu_0 i}{2\pi(3X_1 - X_1)}$

$\Rightarrow \quad B_2 = \frac{\mu_0 i}{2\pi X_1}\left[1 + \frac{1}{2}\right] = \frac{3\mu_0 i}{4\pi X_1}$ ....(iii)

from (i), (ii) & (iii)

$\Rightarrow \quad \frac{R_1}{R_2} = \frac{B_2}{B_1} = 3$

Q.15 To find the distance d over which a signal can be seen clearly in foggy conditions, a railways engineer uses dimensional analysis and assumes that the distance depends on the mass density ρ of the fog, intensity (power/area) S of the light from the signal and its frequency f. The engineer finds that d is proportional to $S^{1/n}$. The value of n is.

Ans. **[3]**

Sol. $d = \rho^a S^b f^c$

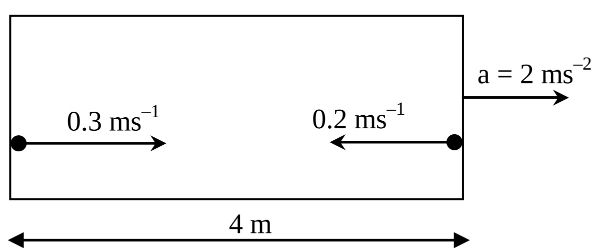
$d' = (ML^{-3})^a \left(\frac{M^1L^1T^{-2}LT^{-1}}{L^2}\right)^b (T^{-1})^c$

$M^0L^1T^0 = M^{a+b}L^{-3a}T^{-3b-c}$

$a + b = 0;$ $\quad - 3a = 1, a = -\frac{1}{3}$ $\quad ;$ $\quad - 3b - c = 0$

$b = -a \Rightarrow b = \frac{1}{3} \Rightarrow n = 3$

Q.16 A rocket is moving in a gravity free space with a constant acceleration of 2 ms^{-2} along +x direction (see figure). The length of a chamber inside the rocket is 4 m. A ball is thrown from the left end of the chamber in +x direction with a speed of 0.3 ms^{-1} relative to the rocket. At the same time, another ball is thrown in –x direction with a speed of 0.2 ms^{-1} from its right end relative to the rocket. The time in seconds when the two balls hit each other is.



Ans. **[8, 2]**

Sol. (i) $S_{rel} = u_{rel} \times t + \frac{1}{2} a_{rel} \times t^2$

$4 = (0.2 + 0.3)\, t + 0 \;\Rightarrow\; t = \dfrac{4}{0.5}$

$\boxed{t = 8\ s}$

(ii) Maximum displacement of (A)

$S_1 = \dfrac{\mu^2}{2a} = \dfrac{(0.3)^2}{2\times 2} = 0.0225$

$S_1 <<< 4$

So total displacement traveld by (B) only

$S = \mu t + \dfrac{1}{2}at^2$

$4 = 0.2 \times t + \dfrac{1}{2}(2)t^2$

by solving t = 2s

Q.17 A galvanometer gives full scale deflection with 0.006 A current. By connecting it to a 4990 Ω resistance, it can be converted into a voltmeter of range 0-30 V. If connected to a $\dfrac{2n}{249}$ Ω resistance, it becomes an ammeter of range 0-1.5 A. The value of n is.

Ans. **[5]**

Sol. $I_g = 0.006$ amp (Given)

to convert into a voltmeter of range V = 30 volt connect 4990 in series with galvanometer

$\Rightarrow \quad I_g (G + R) = V$

$\Rightarrow \quad 0.006\,[G + 4990] = 30$

$\Rightarrow \quad G + 4990 = \dfrac{30}{6} \times 1000$

$\Rightarrow \quad G = 5000 - 4990 = 10\ \Omega$

To convert galvanometer into ammeter of range I = 1.5 amp connect a resistance in parallel i.e. SΩ

$\Rightarrow \quad I_g \times G = (I - I_g) \times S$

$\Rightarrow \quad 0.006 \times 10 = (1.5 - 0.006) \times S$

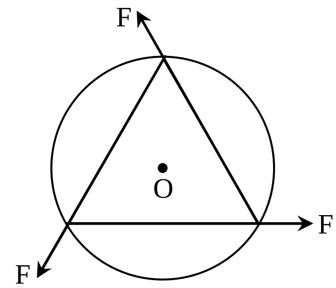
$\Rightarrow \quad 0.06 = 1.494 \times S$

$\Rightarrow \quad S = \dfrac{0.06}{1.494} \quad \Rightarrow \quad S = \dfrac{2\times 5}{249}$

$\Rightarrow \quad S = \dfrac{2n}{249}$ given

$\therefore$ n = 5

Q.18 A uniform circular disc of mass 1.5 kg and radius 0.5 m is initially at rest on a horizontal frictionless surface. Three forces of equal magnitude F = 0.5 N are applied simultaneously along the three sides of an equilateral triangle XYZ with its vertices on the perimeter of the disc (see figure). One second after applying the forces, the angular speed of the disc in rad s^{-1} is.

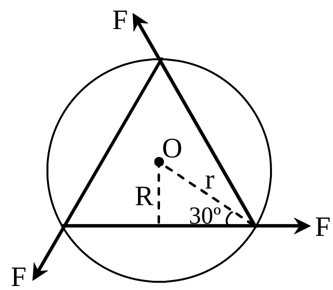


Ans. **[2]**

Sol. F = 0.5 N

F_{Net} on the disc = 0

Each force F produce torque and the disc rotate about its centre



r = 0.5 m (Given)

$\Rightarrow$ $\sin 30^\circ = \frac{1}{2}$

$\Rightarrow$ $\frac{R}{r} = \sin 30^\circ$

$\Rightarrow$ $R = r \times \sin 30^\circ = \frac{0.5}{2}$

$\Rightarrow$ τ about O $= 3 \times F \times R = 3 \times 0.5 \times \frac{0.5}{2} = \frac{3 \times 0.25}{2}$ N-m

$\Rightarrow$ I = moment of inertia about O of disc $= \frac{Mr^2}{2}$

$\Rightarrow$ M = 1.5 kg ; r = 0.5 (Given)

$\Rightarrow$ $I = \frac{1.5 \times (0.5)^2}{2} = \frac{1.5 \times 0.25}{2}$

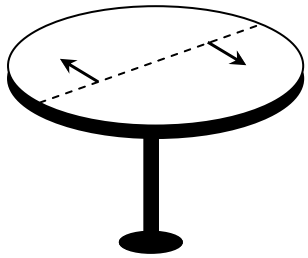
$\Rightarrow$ $\tau = I\alpha$

$\Rightarrow \quad \alpha = \frac{\tau}{I} = \frac{3\times0.25\times2}{2\times1.5\times0.25} = 2\ \text{rad/s}^2$

$\Rightarrow \quad W = \alpha t$

$\Rightarrow \quad 2 \times 1 = 2$ rad/sec

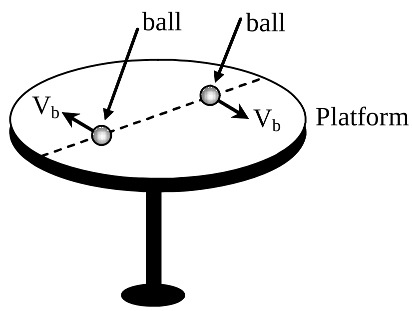
Q.19 A horizontal circular platform of radius 0.5 m and mass 0.45 kg is free to rotate about its axis. Two massless spring toy-guns, each carrying a steel ball of mass 0.05 kg are attached to the platform at a distance 0.25 m from the centre on its either sides along its diameter (see figure). Each gun simultaneously fires the balls horizontally and perpendicular to the diameter in opposite directions. After leaving the platform, the balls have horizontal speed of 9 ms^{-1} with respect to the ground. The rotational speed of the platform in rad s^{-1} after the balls leave the platform is.



Ans. **[4]**

Sol. mass of ball = m_b = 0.05 kg

v_b = 9 m/s



Mass of platform = m_p = 0.45 kg

Radius of platform = r_p = 0.5 metre

r = 0.25 m

Applying angular momentum conservation

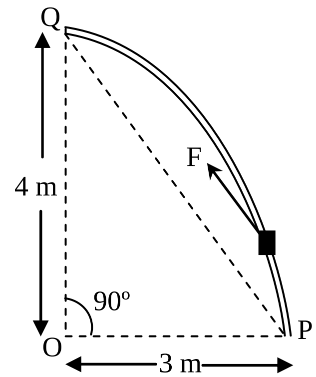
$\Rightarrow \quad L_i = L_f$

$\Rightarrow \quad 0 = (m_b \times v_b \times r)2 - I\omega$

$\Rightarrow \quad 0 = \frac{(m_b \times 9 \times r)2}{I} - \frac{m_p r_p^2 \omega}{2}$

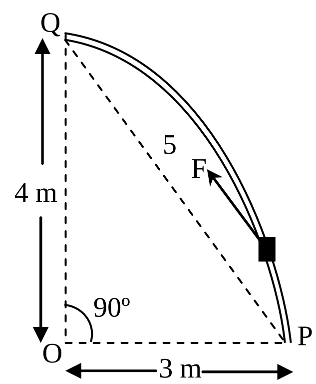
$\Rightarrow \quad \omega = \frac{(m_b \times 9 \times r)2}{\frac{m_p r_p^2}{2}} \quad \Rightarrow \frac{0.05\times9\times0.25\times2\times2}{0.45\times(0.5)^2} = 4$ rad/sec

Q.20 Consider an elliptically shaped rail PQ in the vertical plane with OP = 3 m and OQ = 4 m. A block of mass 1 kg pulled along the rail from P to Q with a force of 18 N, which is always parallel to line PQ (see figure). Assuming no frictional losses, the kinetic energy of the block when it reaches Q is (n × 10) Joules. The value of n is (taken acceleration due to gravity = 10 ms^{-2}).



Ans. **[5]**

Sol.



Work energy theorem

⇒ $W_{total} = \Delta KE$

⇒ W by gravity + W by F = KE – 0

⇒ $-mg \times h + \vec{F}.\vec{S} = KE$

given m = 1 kg; h = 4; $|\vec{F}|$ = 18 N; $|\vec{S}| = \sqrt{4^2+3^2} = 5$

⇒ $-1 \times 10 \times 4 + 18 \times 5 \times \cos 0 = KE$

⇒ KE = 90 – 40 = 50

⇒ $\therefore \boxed{n = 5}$

Part II - CHEMISTRY

SECTION – 1 (One or More Than One Options Correct Type)

This section contains 10 multiple choice type questions. Each questions has four choices (A), (B), (C) and (D) out of which ONE or MORE THAN ONE are correct.

Q.21 The correct combination of names for isomeric alcohols with molecular formula $C_4H_{10}O$ is/are :

(A) tert-butanol and 2-methylpropan-2-ol

(B) tert-butanol and 1, 1-dimethylethan-1-ol

(C) n-butanol and butan-1-ol

(D) isobutyl alcohol and 2-methylpropan-1-ol

Ans. **[A, C, D]**

Sol. $C_4H_{10}O$

(A) tert-butanol and 2 methyl propan-2-ol

$$\begin{array}{c} \quad {}_{3}CH_3 \\ \quad | \\ CH_3-\underset{2}{C}-OH \\ \quad | \\ \quad {}_{1}CH_3 \end{array}$$

Correct name

(B) wrong name

(C) n butanol and butan-1-ol

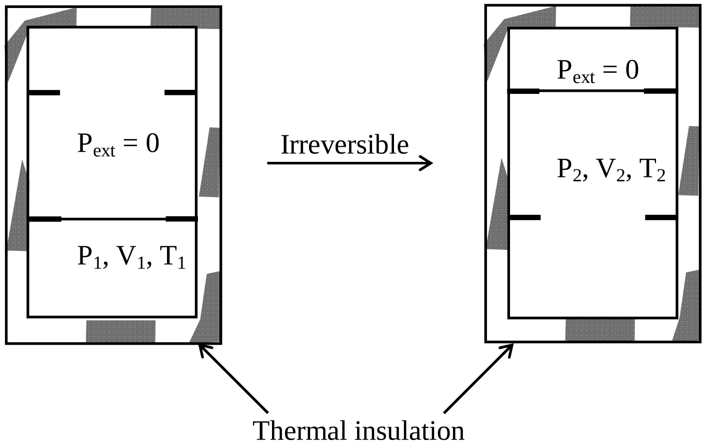
$\overset{4}{C}H_3-\overset{3}{C}H_2-\overset{2}{C}H_2-\overset{1}{C}H_2-OH$ Correct name

(D) Isobutyl alcohol and 2 methyl propan-1-ol

$$\begin{array}{c} \quad CH_3 \\ \quad | \\ \underset{3}{CH_3}-\underset{2}{CH}-\underset{1}{CH_2}-OH \end{array}$$

Correct name

Q.22 An ideal gas in a thermally insulated vessel at internal pressure = P_1, volume = V_1 and absolute temperature = T_1 expands irreversibly against zero external pressure, as shown in the diagram. The final internal pressure, volume and absolute temperature of the gas are P_2, V_2 and T_2, respectively. For this expansion,



(A) $q = 0$ (B) $T_2 = T_1$ (C) $P_2V_2 = P_1V_1$ (D) $P_2V_2^{\gamma} = P_1V_1^{\gamma}$

Ans. **[A, B, C]**

Sol. Thermally insulated means $q = 0$ & from 1st law, $\Delta E = q + w$ & free expansion means, $w = 0$

$\therefore\ \Delta E = 0$

Q.23 Hydrogen bonding plays a central role in the following phenomena :

(A) Ice floats in water

(B) Higher Lewis basicity of primary amines than tertiary amines in aqueous solutions

(C) Formic acid is more acidic than acetic acid.

(D) Dimerisation of acetic acid in benzene

Ans. **[A, B, D]**

Sol. (A) Ice float in water because due to H-bonding ice has cage like structure so it's density is less than water.

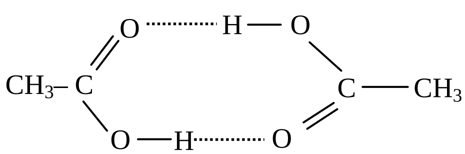
(B) $R - NH_2 + H^+ \longrightarrow R\overset{\oplus}{N}H_3$: H……….OH_2, H…….OH_2, H……….OH_2 More H-bonding

1° Amine
More basic

$R_3N + H^+ \longrightarrow R_3\overset{\oplus}{N}–H$ …… OH_2 Less H-bonding

3° Amine
less basic

(C) Acetic acid is less acidic due to +I effect of methyl not by H-bonding

(D) 

Dimer of Acetic acid

Q.24 In a galvanic cell, the salt bridge

(A) does not participate chemically in the cell reaction

(B) stops the diffusion of ions from one electrode to another

(C) is necessary for the occurrence of the cell reaction

(D) ensures mixing of the two electrolytic solutions

Ans. **[A]**

Sol. By definition, electrolyte to be used in salt bridge must not take part in cell reaction.

Q.25 For the reaction

$$I^- + ClO_3^- + H_2SO_4 \rightarrow Cl^- + HSO_4^- + I_2$$

The correct statement(s) in the balanced equation is/are :-

(A) Stoichiometric coefficient of HSO_4^- is 6

(B) Iodide is oxidized

(C) Sulphur is reduced

(D) H_2O is one of the products

Ans. **[A, B, D]**

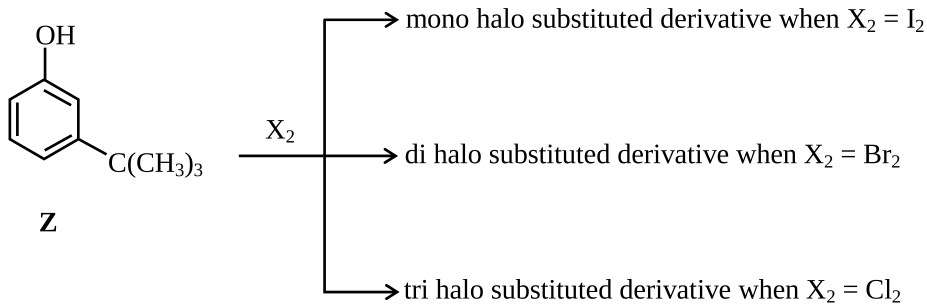
Sol. Given

$I^- + ClO_3^- + H_2SO_4 \longrightarrow Cl^- + HSO_4^- + I_2$

on balancing

$6I^- + 1ClO_3^- + 6H_2SO_4 \longrightarrow 1Cl^- + 6HSO_4^- + 3I_2 + 3H_2O$

Q.26 The reactivity of compound Z with different halogens under appropriate conditions is given below



Z

The observed pattern of electrophilic substitution can be explained by :

(A) the steric effect of the halogen

(B) the steric effect of the tert-butyl group

(C) the electronic effect of the phenolic group

(D) the electronic effect of the tert-butyl group

Ans. **[A, B, C]**

Sol. (A) Steric effect of halogen because as size of halogen increase no. of halogen at benzene ring decrease.

(B) The steric effect of tert butyl group that is why in bromination dihalo substituted product is obtained

(C) The electronic effect of phenolic group i.e. why poly halo product are formed

(D) It is negligible compare to phenolic group

Q.27 The correct statement(s) for orthoboric acid is/are :

(A) It behaves as a weak acid in water due to self ionization

(B) Acidity of its aqueous solution increases upon addition of ethylene glycol

(C) It has a three dimensional structure due to hydrogen bonding

(D) It is a weak electrolyte in water

Ans. **[B, D]**

Sol. H_3BO_3 or $B(OH)_3$ is weak lewis acid but it is not a proton donor. Upon addition of ethylene glycol acidity of its aqueous solution increases.

$$B(OH)_3 + H_2O \rightleftharpoons [B(OH)_4]^- + H^+$$

Ethylene glycol combines with $[B(OH)_4]^-$. Thus, this reaction shifted in forward direction.

It has two dimensional sheet like structure due to hydrogen bonding.

It is weak electrolyte in water due to its weak acidic nature.

Q.28 Upon heating with Cu_2S, the reagent(s) that give copper metal is/are :

(A) $CuFeS_2$ (B) CuO (C) Cu_2O (D) $CuSO_4$

Ans. **[B, C, D]**

Sol. (B) $Cu_2S + 2CuO \rightarrow 4Cu + SO_2$ (C) $Cu_2S + 2Cu_2O \longrightarrow 6Cu + SO_2\uparrow$

(D) $Cu_2S + CuSO_4 \longrightarrow 3Cu + SO_2\uparrow$

Q.29 The pair(s) of reagents that yield paramagnetic species is/are :

(A) Na and excess of NH_3 (B) K and excess of O_2

(C) Cu and dilute HNO_3 (D) O_2 and 2-ethylanthraquinol

Ans. **[A, B, C]**

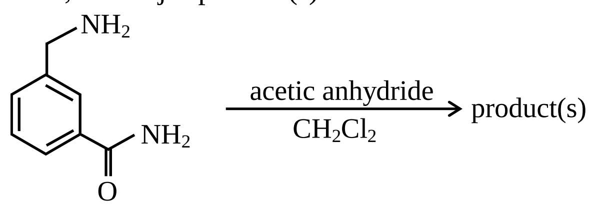
Sol. (A) $Na + (x + y)\ NH_3 \longrightarrow [Na(NH_3)_x]^+ + [e(NH_3)_y]^-$ (paramagnetic)

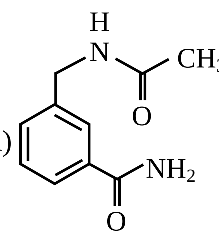
(B) $K + \text{excess } O_2 \longrightarrow KO_2$ (super oxide is paramagnetic)

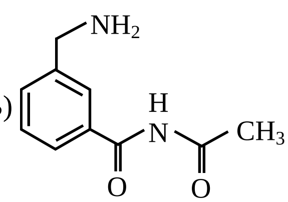
(C) $Cu + HNO_3\ (\text{dil.}) \longrightarrow Cu(NO_3)_2 + NO + H_2O$ (NO is paramagnetic)

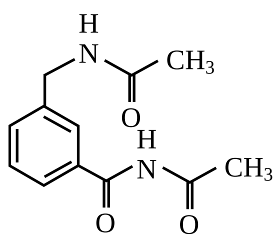
(D) O_2 + 2-ethylanthraquinol $\longrightarrow$ 2-ethylanthraquinone + H_2O_2 (diamagnetic)

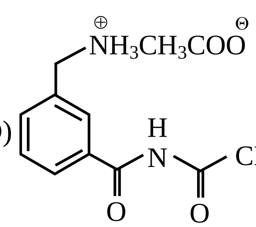
Q.30 In the reaction shown below, the major product(s) formed is/are :



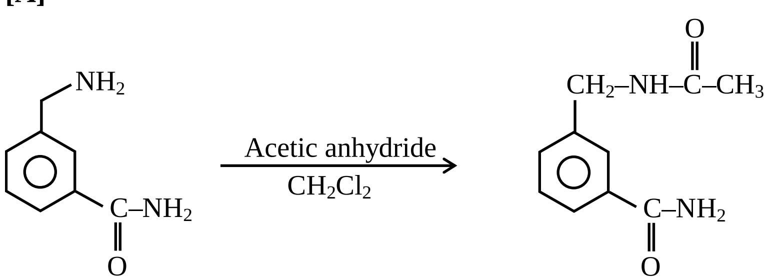
(A)  + CH_3COOH

(B)  + CH_3COOH

(C)  + H_2O

(D) 

Ans. **[A]**

Sol. 

It is example of nucleophilic substitution and amine is more nucleophilic than amide. That is why acylation take place on amine group. So Ans. Should be [A]

SECTION – 2 (Only Integer Value Correct Type)

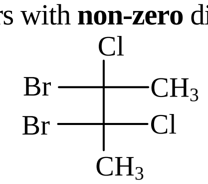
This section contains 10 questions. Each question, when worked out will result in one integer from 0 to 9 (both inclusive)

Q.31 Among PbS, CuS, HgS, MnS, Ag_2S, NiS, CoS, Bi_2S_3 and SnS_2, the total number of **BLACK** coloured sulphides is

Ans. **[6 or 7]**

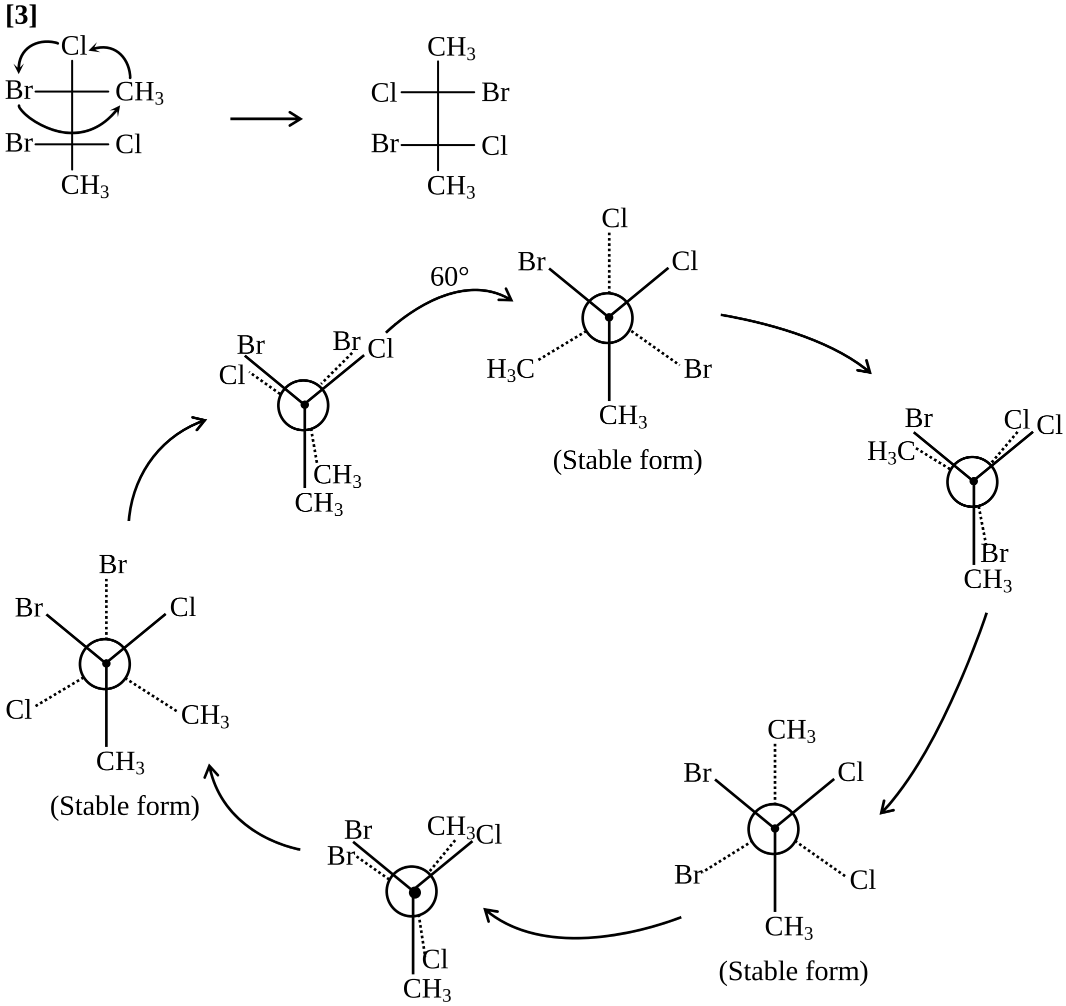
Sol. PbS → Black ; CuS → Black ; HgS → Black
MnS → Buff pink; Ag_2S → Black; NiS → Black
CoS → Black ; Bi_2S_3 → Black (in amorphous form Red colour); SnS_2 → Yellow

Q.32 The total number(s) of **stable** conformers with **non-zero** dipole moment for the following compound is (are)



Ans. **[3]**

Sol.



Total stable conformers with non zero dipole moment are = 3.

Q.33 Consider the following list of reagents :

Acidified $K_2Cr_2O_7$, alkaline $KMnO_4$, $CuSO_4$, H_2O_2, Cl_2, O_3, $FeCl_3$, HNO_3 and $Na_2S_2O_3$.

The total number of reagents that can oxidise aqueous iodide to iodine is

Ans. **[7]**

Sol. Acidified $K_2Cr_2O_7$ ✓

Alkaline $KMnO_4$ ×

$CuSO_4$ ✓

H_2O_2 ✓

Cl_2 ✓

O_3 ✓

$FeCl_3$ ✓

HNO_3 ✓

$Na_2S_2O_3$ ×

$K_2Cr_2O_7 + H^+ + I^- \longrightarrow Cr^{+3} + I_2 + H_2O$

$KMnO_4 + OH^- + I^- \longrightarrow K_2MnO_4 + KIO_3 + H_2O$

$CuSO_4 + I^- \longrightarrow Cu_2I_2 + I_2$

$H_2O_2 + I^- \longrightarrow I_2 + H_2O$

$Cl_2 + I^- \longrightarrow Cl^- + I_2$

$O_3 + I^- \longrightarrow O_2 + I_2$

$FeCl_3 + I^- \longrightarrow I_2 + FeCl_2$

$HNO_3 + I^- \longrightarrow I_2 + NO_2 + H_2O$

$Na_2S_2O_3 + I^- \longrightarrow$ No reaction

Q.34 A list of species having the formula XZ_4 is given below

XeF_4, SF_4, SiF_4, BF_4^-, BrF_4^-, $[Cu(NH_3)_4]^{2+}$, $[FeCl_4]]^{2-}$, $[CoCl_4]^{2-}$ and $[PtCl_4]^{2-}$

Defining shape on the basis of the location of X and Z atoms, the total number of species having a square planar shape is

Ans. **[4]**

Sol. $XeF_4 \rightarrow$ Square planar

$SF_4 \rightarrow$ Sea-saw shape

$SiF_4 \rightarrow$ Tetrahedral

$BF_4^- \rightarrow$ Tetrahedral

$BrF_4^- \rightarrow$ Square planar

$[Cu(NH_3)_4]^{2+} \rightarrow$ Square planar

$[FeCl_4]]^{2-} \rightarrow$ Tetrahedral

$[CoCl_4]^{2-} \rightarrow$ Tetrahedral

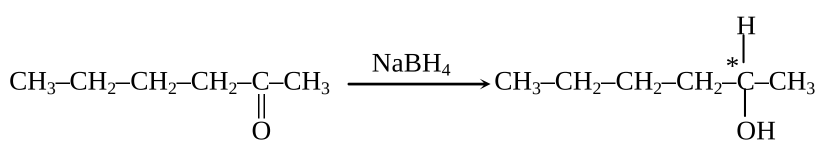
$[PtCl_4]^{2-} \rightarrow$ Square planar

Q.35 Consider all possible isomeric ketones, including stereoisomers of MW = 100. All these isomers are independently reacted with $NaBH_4$ (**NOTE :** stereoisomers are also reacted separately). The total number of ketones that give a racemic product(s) is / are

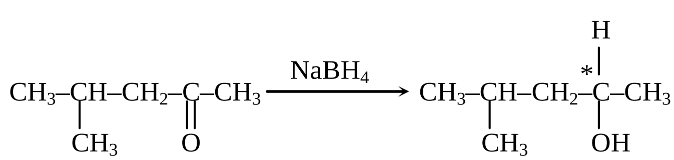
Ans. **[5]**

Sol. Ketone with mol. weight = 100

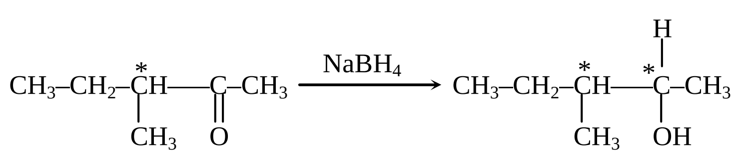
Having molecular formula = $C_6H_{12}O$

(1) 

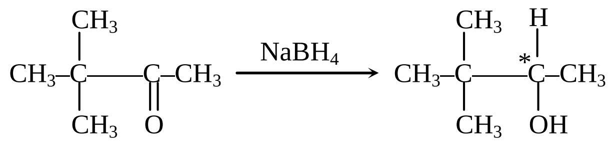
(Racemic)

(2) 

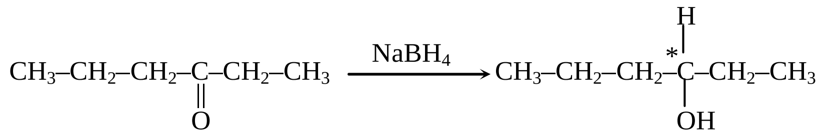
(Racemic)

(3) 

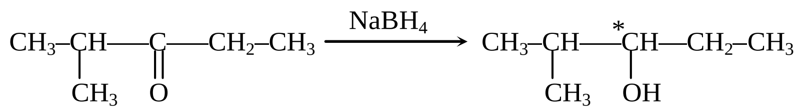
(Diastereomer)

(4) 

(Racemic)

(5) 

(Racemic)

(6) 

(Racemic)

Total number of ketone with molecular formula $C_6H_{12}O$ = 7

But when react with $NaBH_4$ total 5 ketone gives racemic products and example 3 already contain a chiral centre that is why diastereomer pair is formed.

Q.36 In an atom, the total number of electrons having quantum numbers n = 4, $|m_l|$ = 1 and m_s = – 1/2 is

Ans. **[6]**

Sol. $n = 4$; $m_\ell = \pm 1$ & $m_s = -\frac{1}{2}$

For n = 4, i.e. fourth shell

$\therefore$ 4s $\rightarrow$ do not contains $m_\ell = \pm 1$

$\left.\begin{matrix} 4p \\ 4d \\ 4f \end{matrix}\right\} \rightarrow$ All three contains two orbital one having $m_\ell = +1$ & other having $m_\ell = -1$ & each orbital contains one electron of $m_s = -1/2$

$\therefore$ Ans is 6

Q.37 If the value of Avogadro number is 6.023×10^{23} mol^{-1} and the value of Boltzmann constant is 1.380×10^{-23} J K^{-1}, then the number of significant digits in the calculated value of the universal gas constant is

Ans. **[4]**

Sol. R = (Avogadro number) × (Boltzmann constant)

Both contains four significant digits

$\therefore$ answer must contains four significant digits

Q.38 **MX_2** dissociates into **M^{2+}** and **X^-** ions in an aqueous solution, with a degree of dissociation (α) of 0.5. The ratio of the observed depression of freezing point of the aqueous solution to the value of the depression of freezing point in the absence of ionic dissociation is

Ans. **[2]**

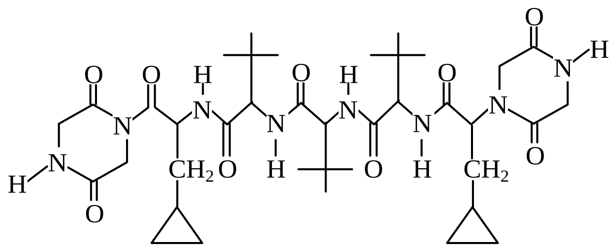
Sol. For MX_2 ;

$i = 1 + (n - 1)\alpha = 1 + (3 - 1)\, 0.5$

$= 1 + 1 = 2$

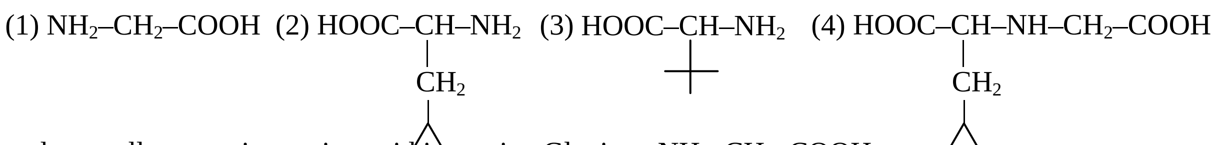
$$\frac{(\Delta T_f)_{obs.}}{(\Delta T_f)_{cal.}} = i = 2$$

Q.39 The total number of **<u>distinct naturally occurring amino acids</u>** obtained by complete acidic hydrolysis of the peptide shown below is



Ans. **[1]**

Sol. On hydrolysis of this peptide 4 different type of amino acids are obtained



and naturally occurring amino acid is one i.e. Glycine $NH_2–CH_2–COOH$

Q.40 A compound **H_2X** with molar weight of 80 g is dissolved in a solvent having density of 0.4 g ml^{-1}. Assuming no change in volume upon dissolution, the **molality** of a 3.2 molar solution is

Ans. **[8]**

Sol. Assuming no change in volume upon dissolution,

we can say that mass of solvent in 1000 ml of solution = 400 g

& Molarity is 3.2 i.e. mole of solute in 1000 ml solution is 3.2

$$\therefore\ m = \frac{3.2}{0.4} = 8$$

Part III - MATHEMATICS

SECTION – 1 (One or More Than One Options Correct Type)

This section contains 10 **multiple choice questions**. Each question has four choices (A), (B), (C) and (D), out of which **ONE or MORE THAN ONE** are correct.

Q.41 Let f : (0, ∞) → R be given by $f(x) = \int_{1/x}^{x} e^{-\left(t+\frac{1}{t}\right)} \frac{dt}{t}$. Then

(A) f(x) is monotonically increasing on [1, ∞)

(B) f(x) is monotonically decreasing on (0, 1)

(C) $f(x) + f\left(\frac{1}{x}\right) = 0$, for all $x \in (0, \infty)$

(D) $f(2^x)$ is an odd function of x on R

Ans. **[A,C,D]**

Sol. $f(x) = \int_{1/x}^{x} e^{-\left(t+\frac{1}{t}\right)} \frac{dt}{t}$

$$f'(x) = \frac{e^{-\left(x+\frac{1}{x}\right)}}{x} - e^{-\left(x+\frac{1}{x}\right)} . x\left(-\frac{1}{x^2}\right)$$

$$= e^{-\left(x+\frac{1}{x}\right)}\left(\frac{1}{x}+\frac{1}{x}\right)$$

$$= 2\frac{e^{-\left(x+\frac{1}{x}\right)}}{x} > 0 \quad \forall\ x \in (0, \infty)$$

So option (A) is true.

Now $f(1/x) = \int_{x}^{1/x} e^{-\left(t+\frac{1}{t}\right)} \frac{dt}{t}$

put t = 1/u

$$dt = -\frac{1}{u^2}du$$

$$= -\int_{1/x}^{x} e^{-\left(u+\frac{1}{u}\right)} u. \left(-\frac{1}{u^2}\right)du$$

$$= -\int_{1/x}^{x} e^{-\left(u+\frac{1}{u}\right)} \frac{1}{u} du = -f(x)$$

$f(x) + f(1/x) = 0 \ \forall\ x \in (0, \infty)$

$$g(x) = f(2^x) = \int_{2^{-x}}^{2^x} e^{-\left(t+\frac{1}{t}\right)} \frac{dt}{t}$$

$$g(-x) = \int_{2^x}^{2^{-x}} e^{-\left(t+\frac{1}{t}\right)} \frac{dt}{t} = -g(x)$$

$\therefore$ g(x) is an odd function of x on R.

Q.42 Let $a \in R$ and let $f : R \to R$ be given by $f(x) = x^5 - 5x + a$. Then

(A) f(x) has three real roots if a > 4 (B) f(x) has only one real roots if a > 4

(C) f(x) has three real roots if a < – 4 (D) f(x) has three real roots if – 4 < a < 4

Ans. **[B, D]**

Sol. Let $g(x) = x^5 - 5x$

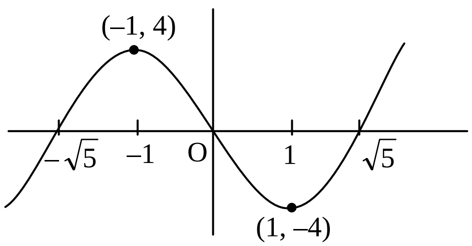
$g'(x) = 5x^4 - 5 = 5(x^4 - 1) = 0$

$\Rightarrow x = \pm 1$

Now $g(1) = -4$, $g(-1) = 4$

$g(x) = 0 \qquad \Rightarrow x = 0, \pm\sqrt{5}$

So graph of g(x) is



so f(x) has three real roots if $-4 < -a < 4 \qquad \Rightarrow -4 < a < 4$

f(x) has only one real root if $-a > 4$ or $-a < -4 \Rightarrow a < -4$ or $a > 4$

Q.43 For every pair of continuous functions $f, g : [0, 1] \to R$ such that

$\max\{f(x) : x \in [0, 1]\} = \max\{g(x) : x \in [0, 1]\}$, the correct statement(s) is(are) :

(A) $(f(c))^2 + 3f(c) = (g(c))^2 + 3g(c)$ for some $c \in [0, 1]$

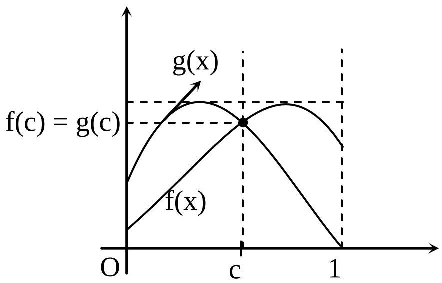
(B) $(f(c))^2 + f(c) = (g(c))^2 + 3g(c)$ for some $c \in [0, 1]$

(C) $(f(c))^2 + 3f(c) = (g(c))^2 + g(c)$ for some $c \in [0, 1]$

(D) $(f(c))^2 = (g(c))^2$ for some $c \in [0, 1]$

Ans. **[A, D]**

Sol.



max {f(x) : x $\in$ [0, 1]} = max {g(x) : x $\in$ [0, 1]}

Both reaches the same maximum value at some point in [0, 1]

Means they intersect at some point 'c' $\in$ [0, 1]

i.e., f(c) = g(c)

(A) Let $h(x) = (f(x))^2 - (g(x))^2 + 3[f(x) - g(x)]$

$\because h(c) = [f(c) - g(c)]\,[f(c) + g(c) + 3]$

$\because f(c) : g(c)$ so $h(c) = 0$

(D) Similarly

Let $\phi(x) = (f(x))^2 - (g(x))^2$

$\phi(c) = (f(c))^2 - (g(c))^2$

$= 0$

Q.44 A circle S passes through the point (0, 1) and is orthogonal to the circles $(x - 1)^2 + y^2 = 16$ and $x^2 + y^2 = 1$. Then

(A) radius of S is 8 (B) radius of S is 7

(C) centre of S is (–7, 1) (D) centre of S is (–8, 1)

Ans. **[B, C]**

Sol. Let equation of the circle is

$x^2 + y^2 + 2gx + 2fy + c = 0$

It passes through (0, 1)

$1 + 2f + c = 0$ … (1)

It is orthogonal to $x^2 + y^2 - 1 = 0$

So $2g(0) + 2f(0) = c - 1$

$\therefore c = 1$ … (2)

It is also orthogonal to $x^2 + y^2 - 2x - 15 = 0$

So $2g(-1) + 2f(0) = c - 15$

$-2g = c - 15$ … (3)

From (1), (2) and (3)

$g = 7, f = -1, c = 1,$

$\therefore$ equation of circle is $x^2 + y^2 + 14x - 2y + 1 = 0$

whose centre is (–7, 1) and radius = 7

Q.45 Let $\vec{x}$, $\vec{y}$ and $\vec{z}$ be three vectors each of magnitude $\sqrt{2}$ and the angle between each pair of them is $\frac{\pi}{3}$.

If $\vec{a}$ is a nonzero vector perpendicular to $\vec{x}$ and $\vec{y}\times\vec{z}$ and $\vec{b}$ is a nonzero vector perpendicular to $\vec{y}$ and $\vec{z}\times\vec{x}$, then

(A) $\vec{b} = (\vec{b}\,.\,\vec{z})(\vec{z}-\vec{x})$ (B) $\vec{a} = (\vec{a}\,.\,\vec{y})(\vec{y}-\vec{z})$

(C) $\vec{a}\,.\,\vec{b} = -(\vec{a}\,.\,\vec{y})(\vec{b}\,.\,\vec{z})$ (D) $\vec{a} = (\vec{a}\,.\,\vec{y})(\vec{z}-\vec{y})$

Ans. **[A, B, C]**

Sol. $\vec{a}$ is perpendicular to $\vec{x}$ and $\vec{y}\times\vec{z}$

So $\vec{a}$ is parallel to $\vec{x}\times(\vec{y}\times\vec{z})$

$\Rightarrow \vec{a} = \lambda\,(\vec{x}\times(\vec{y}\times\vec{z}))$

$\Rightarrow \vec{a} = \lambda\,(\vec{x}\,.\,\vec{z})\,\vec{y} - \lambda(\vec{x}\,.\,\vec{y})\,\vec{z}$

$\Rightarrow \vec{a} = \lambda\,(\vec{y}-\vec{z})$ … (1) $\{\vec{x}\,.\,\vec{z} = \vec{x}\,.\,\vec{y} = \vec{y}\,.\,\vec{z} = 1\}$

also $\vec{b}$ is perpendicular to $\vec{y}$ and $\vec{z}\times\vec{x}$

So $\vec{b}$ is parallel to $\vec{y}\times(\vec{z}\times\vec{x})$

$\Rightarrow \vec{b} = \mu\,(\vec{y}\times(\vec{z}\times\vec{x}))$

$\Rightarrow \vec{b} = \mu\,(\vec{z}-\vec{x})$ … (2) (Similarly as above)

Now,

$\vec{a}\,.\,\vec{y} = \lambda\,(2-1) = \lambda$

So from (1) (B) is correct

Similarly; $\vec{b}\,.\,\vec{z} = \mu$ (A) is correct

and $\vec{a}\,.\,\vec{b} = \lambda\mu\,(1-1-2+1)$

$= -\lambda\mu.$ (C) is correct

Q.46 From a point P(λ, λ, λ), perpendiculars PQ and PR are drawn respectively on the lines y = x, z = 1 and y = –x, z = –1. If P is such that ∠QPR is a right angle, then the possible value(s) of λ is(are) -

(A) $\sqrt{2}$ (B) 1 (C) – 1 (D) $-\sqrt{2}$

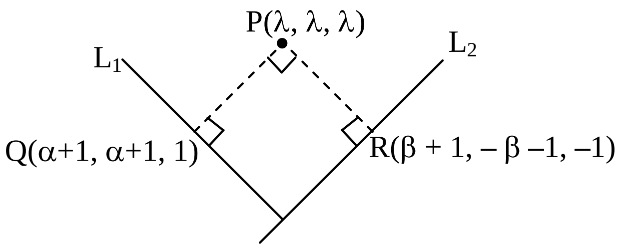
Ans. [C]

Sol. For first line assume two points (1, 1, 1); (0, 0, 1)

Line L_1 : $\frac{x-1}{1} = \frac{y-1}{1} = \frac{z-1}{0} = \alpha$

For second line assume Two points (1, –1, –1) & (0, 0, –1)

Line L_2 : $\frac{x-1}{1} = \frac{y+1}{-1} = \frac{z+1}{0} = \beta$



$\therefore$ PQ ⊥r $L_1 \Rightarrow (\alpha + 1 - \lambda)\, 1 + (\alpha + 1 - \lambda)\,(1) + (1 - \lambda)(0) = 0$

$\Rightarrow (\alpha + 1) = \lambda$ … (1)

& PR ⊥r $L_2 \Rightarrow (\beta + 1 - \lambda)(1) + (-\beta - 1 - \lambda)(-1) + (-1 - \lambda)(0) = 0$

$\Rightarrow \beta = -1$

$\therefore$ R = (0, 0, – 1)

PQ ⊥r to PR $\Rightarrow (\lambda - \alpha - 1)\,(\lambda) + (\lambda - \alpha - 1)\,(\lambda) + (\lambda - 1)\,(\lambda + 1) = 0$

$\Rightarrow 0 + 0 + \lambda^2 - 1 = 0$

$\Rightarrow \lambda = \pm 1$

$\because$ for λ = 1, the point P lies on line L_1 so λ = 1 is rejected.

Q.47 Let M be a 2 × 2 symmetric matrix with integer entries. Then M is invertible if -

(A) the first column of M is the transpose of the second row of M

(B) the second row of M is the transpose of the first column of M

(C) M is a diagonal matrix with nonzero entries in the main diagonal

(D) the product of entries in the main diagonal of M is not the square of an integer

Ans. **[C, D]**

Sol. Let $M = \begin{bmatrix} a & c \\ c & b \end{bmatrix}$

$| M | = ab - c^2$

Now;

(A) 1^{st} column = $(2^{nd}\text{ row})^T$

$\Rightarrow \begin{bmatrix} a \\ c \end{bmatrix} = [c\ \ b]^T = \begin{bmatrix} c \\ b \end{bmatrix}$

$\Rightarrow a = b = c$

not invertible

(B) $[c\ \ b] = \begin{bmatrix} a \\ c \end{bmatrix}^T \Rightarrow a = b = c$

again not invertible

(C) $M = \begin{bmatrix} a & 0 \\ 0 & a \end{bmatrix}$ is invertible

when $a \neq 0$

(D) ab = integer,

for being $| M | \neq 0$; $ab \neq c^2$

Q.48 Let M and N be two 3 × 3 matrices such that MN = NM. Further, if $M \neq N^2$ and $M^2 = N^4$, then

(A) determinant of $(M^2 + MN^2)$ is 0

(B) there is a 3 × 3 non-zero matrix U such that $(M^2 + MN^2)U$ is the zero matrix

(C) determinant of $(M^2 + MN^2) \geq 1$

(D) for a 3 × 3 matrix U, if $(M^2 + MN^2)U$ equals the zero matrix then U is the zero matrix

Ans. **[A,B]**

Sol. $MN = NM$; $M^2 = N^4$

(A) $|M^2 + MN^2| = |M|\ |M + N^2| = 0$

Note : $(M + N^2)(M - N^2) = M^2 + N^2M - MN^2 - N^4$

$= N.NM - MN.N$

$= NMN - NMN$

$= 0$

If $A.B = 0 \Rightarrow | A | = | B | = 0$

So $| M + N^2 | = 0$ and $| M - N^2 | = 0$

$\therefore | M^2 + MN^2 | = 0$

(B) $(M^2 + MN^2)U = M(M + N^2)U$

Yes there exist some U

(C) is obviously wrong.

(D) $(M^2 + MN^2)U = 0$ doest not imply that U is null matrix

Q.49 Let f : [a, b] → [1, ∞) be a continuous function and let g : R → R be defined as. Then

$$g(x) = \begin{cases} 0 & \text{if } x < a, \\ \int_a^x f(t)\,dt & \text{if } a \le x \le b, \text{ then} \\ \int_a^b f(t)\,dt & \text{if } x > b \end{cases}$$

(A) g(x) is continuous but not differentiable at a

(B) g(x) is differentiable on R

(C) g(x) is continuous but not differentiable at b

(D) g(x) is continuous and differentiable at either a or b but not both

Ans. **[A, C]**

Sol. (A) g(x) is continuous

$$g'(x) = \begin{cases} 0 & x < a \\ f(x) & a \le x \le b \\ 0 & x > b \end{cases} \text{ and } f : [a, b] \to [1, \infty],\ f(x) \ne 0$$

Not differentiable at a & b

Q.50 Let $f : \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \to R$ be given by $f(x) = (\log(\sec x + \tan x))^3$.

(A) f(x) is an odd function

(B) f(x) is a one-one function

(C) f(x) is an onto function

(D) f(x) is an even function

Ans. **[A,B,C]**

Sol. (A) $f(x) = (\ell n\,(\sec x + \tan x)^3$

$f(-x) = (\ell n\,(\sec x - \tan x))^3$

$$f(-x) = \left(\ell n\left(\frac{1}{\sec x + \tan x}\right)\right)^3$$

$f(-x) = -f(x)$ odd function

(B) Let $f(x) = (\log(\sec x + \tan x))^3$

$$f'(x) = 3[\log(\sec x + \tan x)^2] \times \frac{1}{(\sec x + \tan x)} \times (\sec x \tan x + \sec^2 x)$$

$f'(x) = [3(\log(\sec x + \tan x))^2 \times \sec x]$ which is always positive so f(x) is one-one

(C) $\lim\limits_{x \to \pi/2} (\sec x + \tan x) = \infty$

$\Rightarrow \log(\infty) = +\infty$

$$\lim_{x \to -\pi/2} (\sec x + \tan x) \Rightarrow \lim_{x \to -\pi/2} \frac{1 + \sin x}{\cos x} \cdot \left(\frac{0}{0}\right)$$

$$\Rightarrow \lim_{x \to -\pi/2} \frac{\cos x}{-\sin x} \to 0$$

$\Rightarrow \log(\to 0) \to -\infty$

So f(x) is onto function

SECTION – 2 (One Integer Value Correct Type)

This section contains 10 questions. Each question, when worked out will result in one integer from 0 to 9 (both inclusive).

Q.51 Let $n_1 < n_2 < n_3 < n_4 < n_5$ be positive integers such that $n_1 + n_2 + n_3 + n_4 + n_5 = 20$. Then the number of such distinct arrangements $(n_1, n_2, n_3, n_4, n_5)$ is

Ans. **[7]**

Sol. $n_1 = a \qquad a \geq 1$

$n_2 - n_1 = b \qquad b \geq 1$

$n_3 - n_2 = c \qquad c \geq 1$

$n_4 - n_3 = d \qquad d \geq 1$

$n_5 - n_4 = e \qquad e \geq 1$

$a + (a + b) + (a + b + c) + (a + b + c + d) + (a + b + c + d + e) = 20$

$5a + 4b + 3c + 2d + e = 20$

$1 \leq a \leq 2$

$1 \leq b \leq 2$

$1 \leq c \leq 2$

$1 \leq d \leq 3$

$1 \leq e \leq 6$

the number of distinct arrangements $(n_1, n_2, n_3, n_4, n_5)$ is coefficient of x^{20} in

$(x^5 + x^{10})(x^4 + x^8)(x^3 + x^6)(x^2 + x^4 + x^6)(x^1 + \ldots\ldots + x^6)$

$=$ Coefficient of x^5 in $(1 + x^5)(1 + x^4)(1 + x^3)(1 + x^2 + x^4)(1 + x + x^2 \ldots\ldots + x^5)$

$=$ Coefficient of x^5 in $(1 + x^4 + x^5 + x^9)(1 + x^2 + x^3 + x^4 + x^5)(1 + x + x^2 + \ldots + x^5)$

$= 7$

Q.52 Let $n \geq 2$ be an integer. Take n distinct points on a circle and join each pair of points by a line segment. Colour the line segment joining every pair of adjacent points by blue and the rest by red. If the number of red and blue line segments are equal, then the value of n is

Ans. **[5]**

Sol. Total line segments = ${}^nC_2$

red colour line segments = ${}^nC_2 - n$

blue colour line segments = n

So $n = {}^nC_2 - n$

$\Rightarrow 2n = \frac{n(n-1)}{2}$

$\Rightarrow n^2 - 5n = 0$

$\Rightarrow n = 0, 5$

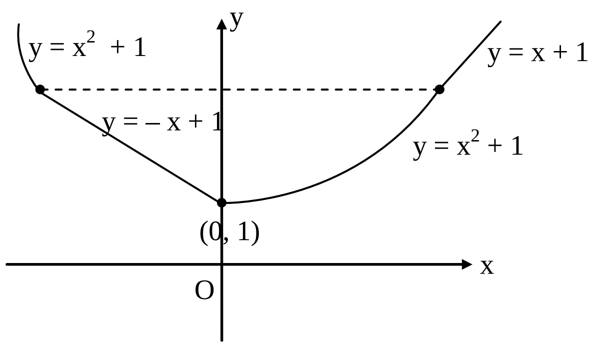
as $n \geq 2$, So n = 5

Q.53 Let $f : R \to R$ and $g : R \to R$ be respectively given by $f(x) = |x| + 1$ and $g(x) = x^2 + 1$. Define $h : R \to R$ by $h(x) = \begin{cases} \max.\{f(x), g(x)\} & \text{if } x \leq 0 \\ \min\{f(x), g(x)\} & \text{if } x > 0 \end{cases}$

The number of points at which h(x) is not differentiable is

Ans. **[3]**

Sol. Graph of h(x) is



No. of points of non-differentiability = 3

Q.54 Let a, b, c be positive integers such that $\frac{b}{a}$ is an integer. If a, b, c are in geometric progression and the arithmetic mean of a, b, c is b + 2, then the value of $\frac{a^2 + a - 14}{a + 1}$ is

Ans. **[4]**

Sol. $a, b, c \in I^+$

given $b^2 = ac$ ...(1)

& $\frac{a+b+c}{3} = b + 2$

$a + b + c = 3b + 6$

$c = 2b + 6 - a$

$\Rightarrow ar^2 = 2ar + 6 - a$ $\Rightarrow a(r-1)^2 = 6 \times 1$

If $r \in$ Integer & $a \in$ Integer then $a = 6$ & $r = 2$ ($\because r \neq 0$)

$\therefore \frac{a^2+a-14}{a+1} = \frac{6^2+6-14}{6+1} = 4$

Q.55 Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three non-coplanar unit vectors such that the angle between every pair of them is $\frac{\pi}{3}$. If $\vec{a} \times \vec{b} + \vec{b} \times \vec{c} = p\vec{a} + q\vec{b} + r\vec{c}$, where p, q and r are scalars, then the value of $\frac{p^2+2q^2+r^2}{q^2}$ is

Ans. **[4]**

Sol. $\vec{a}.\vec{b} = \frac{1}{2} = \vec{b}.\vec{c} = \vec{c}.\vec{a}$ and $\vec{a}.\vec{a} = \vec{b}.\vec{b} = \vec{c}.\vec{c} = 1$

$$[\vec{a}\ \vec{b}\ \vec{c}]^2 = \begin{vmatrix} \vec{a}.\vec{a} & \vec{a}.\vec{b} & \vec{a}.\vec{c} \\ \vec{a}.\vec{b} & \vec{b}.\vec{b} & \vec{b}.\vec{c} \\ \vec{a}.\vec{c} & \vec{b}.\vec{c} & \vec{c}.\vec{c} \end{vmatrix} = \begin{vmatrix} 1 & 1/2 & 1/2 \\ 1/2 & 1 & 1/2 \\ 1/2 & 1/2 & 1 \end{vmatrix} = \frac{1}{2}$$

Now,

$$\vec{a} \times \vec{b} + \vec{b} \times \vec{c} = p\vec{a} + q\vec{b} + r\vec{c}$$

Take dot product on both sides with $\vec{a}$

$\Rightarrow \quad \frac{1}{\sqrt{2}} = p + \frac{q}{2} + \frac{r}{2}$ … (1)

Dot product with $\vec{b}$,

$0 = \frac{p}{2} + q + \frac{r}{2}$ … (2)

Dot product with $\vec{c}$,

$\frac{1}{\sqrt{2}} = \frac{p}{2} + \frac{q}{2} + r$ … (3)

Solve (1), (2) & (3); $p = \frac{1}{\sqrt{2}}$; $q = \frac{-1}{\sqrt{2}}$; $r = \frac{1}{\sqrt{2}}$

$\therefore p = -q = r$

Q.56 The slope of the tangent to the curve $(y - x^5)^2 = x(1 + x^2)^2$ at the point (1, 3) is

Ans. **[8]**

Sol. $(y - x^5)^2 = x(1 + x^2)^2$

Differentiate both sides w.r.t. x

$$2(y - x^5)\left(\frac{dy}{dx} - 5x^4\right) = (1 + x^2)^2 + 4x^2(1 + x^2)$$

at (1, 3)

$$2(3 - 1)\left(\frac{dy}{dx} - 5\right) = 4 + 8$$

$$\Rightarrow \frac{dy}{dx} = 8$$

$\therefore$ slope of tangent at (1, 3) is 8.

Q.57 The value of $\int_0^1 4x^3\left\{\frac{d^2}{dx^2}(1-x^2)^5\right\}dx$ is

Ans. **[2]**

Sol. $\int_0^1 4x^3\left(\frac{d^2}{dx^2}(1-x^2)^5\right)dx$

$$= \int_0^1 4x^3\left(\frac{d}{dx}5(1-x^2)^4(-2x)\right)dx$$

$$= -40\int_0^1 x^3\left(\frac{d}{dx}(1-x^2)^4 x\right)dx$$

$$= -40\int_0^1 x^3\left[(1 - x^2)^4 + 4x(1 - x^2)^3(-2x)\right]dx$$

$$= -40\left[\int_0^1 x^3(1-x^2)^4dx - 8\int_0^1 x^5(1-x^2)^3dx\right]$$

Let $(1 - x^2) = t$

$-2x\,dx = dt$

$$= -40\left[-\frac{1}{2}\int_1^0(1-t)t^4dt + 4\int_1^0 t^3(1-t)^2dt\right]$$

$$= -40\left[\frac{1}{2}\int_0^1(t^4 - t^5)dt - 4\int_0^1 t^3(t^2+1-2t)dt\right]$$

$$= -40\left[\frac{1}{2}\left(\frac{1}{5}-\frac{1}{6}\right) - 4\left(\frac{1}{6}+\frac{1}{4}-\frac{2}{5}\right)\right]$$

$$= -40\left[\frac{1}{60}-\frac{4}{60}\right] = 2$$

Q.58 The largest value of the non-negative integer a for which

$$\lim_{x\to1}\left\{\frac{-ax+\sin(x-1)+a}{x+\sin(x-1)-1}\right\}^{\frac{1-x}{1-\sqrt{x}}} = \frac{1}{4} \text{ is}$$

Ans. **[0]**

Sol. $$\lim_{x\to1}\left\{\frac{-ax+\sin(x-1)+a}{x+\sin(x-1)-1}\right\}^{\frac{1-x}{1-\sqrt{x}}} = \frac{1}{4}$$

$$\Rightarrow \lim_{x\to1}\left\{\frac{-ax+\sin(x-1)+a}{x+\sin(x-1)-1}\right\}^{\lim\limits_{x\to1}(1+\sqrt{x})} = \frac{1}{4}$$

$$\Rightarrow \lim_{x\to1}\left\{\frac{-ax+\sin(x-1)+a}{x+\sin(x-1)-1}\right\}^{2} = \frac{1}{4}$$

Using L' Hospital's Rule

$$\lim_{x\to1}\left(\frac{-a+\cos(x-1)}{1+\cos(x-1)}\right)^{2} = \frac{1}{4}$$

$$\Rightarrow \left(\frac{-a+1}{2}\right)^{2} = \frac{1}{4}$$

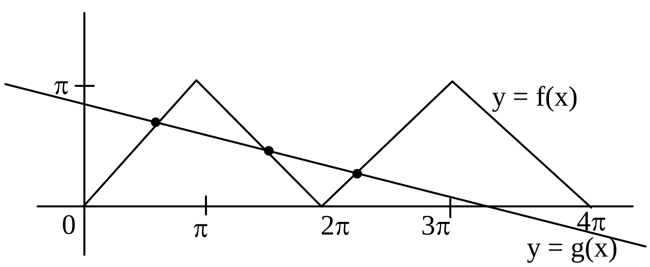
$$\Rightarrow \frac{-a+1}{2} = \pm\frac{1}{2} \qquad \Rightarrow a = 0, 2$$

Note : Since for a = 2, base will be negative.

Q.59 Let f : $[0, 4\pi] \to [0, \pi]$ be defined by $f(x) = \cos^{-1}(\cos x)$. The number of points $x \in [0, 4\pi]$ satisfying the equation $f(x) = \dfrac{10-x}{10}$ is

Ans. **[3]**

Sol. $\cos^{-1}(\cos x) = \dfrac{10-x}{10}$



Let $f(x) = \cos^{-1}\cos x$

$g(x) = \dfrac{10-x}{10}$

$\therefore$ there are three points of intersections.

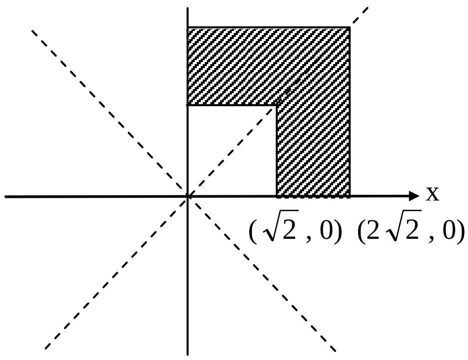
Q.60 For a point P in the plane, let $d_1(P)$ and $d_2(P)$ be the distance of the point P from the lines $x - y = 0$ and $x + y = 0$ respectively. The area of the region R consisting of all points P lying in the first quadrant of the plane and satisfying $2 \le d_1(P) + d_2(P) \le 4$, is

Ans. **[6]**

Sol. $2 \le d_1(P) + d_2(P) \le 4$

$$2 \le \frac{|x-y|}{\sqrt{2}} + \frac{|x+y|}{\sqrt{2}} \le 4$$

$$2\sqrt{2} \le |x-y| + |x+y| \le 4\sqrt{2}$$



Area of required region

$$= \frac{(4\sqrt{2})^2 - (2\sqrt{2})^2}{4}$$

$$= \frac{32-8}{4} = \frac{24}{4} = 6$$