Date : 02 / 06 / 2013

# Part I - PHYSICS

## SECTION – 1 (Only One option correct Type)

This section contains 10 **multiple choice questions**. Each question has 4 choices (A), (B), (C) and (D), out of which **ONLY ONE** is correct.

**Q.1** A particle of mass m is projected form the ground with an initial speed $\mu_0$ at an angle $\alpha$ with the horizontal. At the highest point of its trajectory, it makes a completely inelastic collision with another identical particle, which was thrown vertically upward from the ground with the same initial speed $u_0$. The angle that the composite system makes with the horizontal immediately after the collision is

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{4}+\alpha$ (C) $\frac{\pi}{2}-\alpha$ (D) $\frac{\pi}{2}$

**Ans.** **[A]**

**Sol.** At highest point of projectile, velocity of particle is $(u_0 \cos\alpha)\,\hat{i}$

max. height attained by particle $H = \frac{u_0 \sin^2\alpha}{2g}$

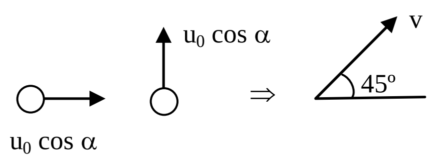
For another particle thrown vertically upward

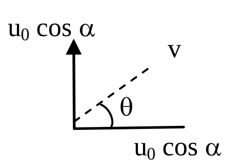
$$v_y^2 = u_0^2 - 2gH$$

$$v_y^2 = u_0^2 - 2g \times \frac{u_0^2 \sin^2\alpha}{2g}$$

$$v_y = u_0 \cos\alpha$$

at the instant of collision




$\tan\theta = 1$

$\theta = 45^\circ$

**Q.2** The image of an object, formed by a plano-convex lens at a distance of 8m behind the lens, is real and is one third the size of the object. The wavelength of light inside the lens is 2/3 times the wavelength in free space. The radius of the curved surface of the lens is -

(A) 1m (B) 2m (C) 3m (D) 6m

**Ans.** **[C]**

**Sol.** Given that

position of image v = 8,

magnification $m = -\frac{1}{3} = \frac{f-v}{f}$

$\frac{4}{3}f = 8$

f = 6

here $\lambda' = \frac{2}{3}\lambda_0$

refractive index $\mu \propto \frac{1}{v}$

$\mu \propto \frac{1}{\lambda}$

$\mu = \frac{3}{2}$

we have

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R} - \frac{1}{\infty}\right)$$

R = 3 m

**Q.3** The diameter of a cylinder is measured using a Vernier callipers with no zero error. It is found that the zero of the Vernier scale lies between 5.10 cm and 5.15 cm of the main scale. The Vernier scale has 50 divisions equivalent to 2.45 cm. The $24^{th}$ division of the Vernier scale exactly coincides with one of the main scale divisions. The diameter of the cylinder is

(A) 5.112 cm (B) 5.124 cm (C) 5.136 cm (D) 5.148 cm

**Ans.** **[B]**

**Sol.** Here

1 MSD = 0.05 cm

$1 \text{ VSD} = \frac{2.45}{50} = 0.049$ cm

Least count = 1 MSD – 1VSD

= 0.001 cm

diameter = 5.10 + (0.001) × 24

diameter = 5.124 cm

**Q.4** The work done on a particle of mass m by a force, $K\left[\frac{x}{(x^2+y^2)^{3/2}}\hat{i}+\frac{y}{(x^2+y^2)^{3/2}}\hat{j}\right]$ (K being a constant of appropriate dimensions), when the particle is taken from the point (a, 0) to the point (0, a) along a circular path of radius *a* about the origin in the x-y plane is

(A) $\frac{2K\pi}{a}$ (B) $\frac{K\pi}{a}$ (C) $\frac{K\pi}{2a}$ (D) 0

**Ans.** **[D]**

**Sol.** Here $\frac{\partial F_x}{\partial y} = \frac{\partial F_y}{\partial x}$

that's why force is conservative and particle is moving on circle i.e. $x^2 + y^2 = a^2$

hence $W = \int_a^0 F_x\, dx + \int_a^0 F_y\, dy$

$= \frac{K}{a^3}\int_a^0 x\, dx + \frac{K}{a^3}\int_0^a y\, dy$

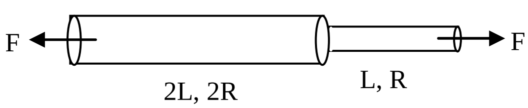
$= 0$

**Q.5** One end of a horizontal thick copper wire of length 2L and radius 2R is welded to an end of another horizontal thin copper wire of length L and radius R. When the arrangement is stretched by applying forces at two ends, the ratio of the elongation in the thin wire to that in the thick wire is

(A) 0.25 (B) 0.50 (C) 2.00 (D) 4.00

**Ans.** **[B]**

**Sol.**




We have, change in length

$\Delta\ell = \frac{F_{rest}\ \ell}{yA}$

Since the two rods are in series hence

$(F_1)_{rest} = (F_2)_{rest}$

$\boxed{\Delta\ell \propto \frac{\ell}{A}}$ or $\Delta\ell = \frac{\ell}{R^2}$ $(\because A = \pi R^2)$

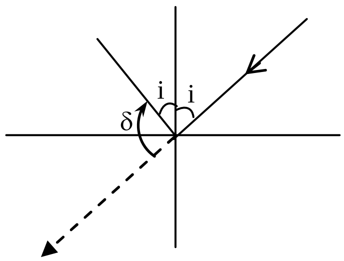
$\therefore \frac{\Delta\ell_1}{\Delta\ell_2} = \frac{\ell_1}{\ell_2}\times\frac{R_2^2}{R_1^2}$

**Q.6** A ray of light travelling in the direction $\frac{1}{2}\left(\hat{i}+\sqrt{3}\hat{j}\right)$ is incident on a plane mirror. After reflection, it travels along the direction $\frac{1}{2}\left(\hat{i}-\sqrt{3}\hat{j}\right)$. The angle of incidence is

(A) 30º (B) 45º (C) 60º (D) 75º

**Ans.** **[A]**

**Sol.** Here




For angle between incident ray and reflected ray i.e. δ

$$\cos\delta = \frac{(i+\sqrt{3}\hat{j}).(\hat{i}-3\hat{j})}{2.2}$$

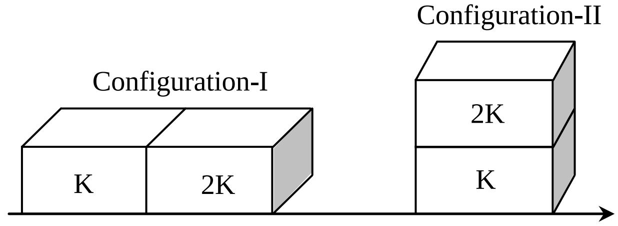
$$\cos\delta = -\frac{1}{2}$$

δ = 120º

i.e. 180 – 2i = 120

i = 30º

**Q.7** Two rectangular blocks , having identical dimensions, can be arranged either in configuration I or in configuration II as show in the figure. One of the blocks has thermal conductivity K and the other 2K. The temperature difference between the ends along the x-axis is the same in both the configurations. It takes 9s to transport a certain amount of heat from the hot end to the cold end in the configuration I. The time to transport the same amount of heat in the configuration II is




(A) 2.0 s (B) 3.0 s (C) 4.5 s (D) 6.0 s

**Ans.** **[A]**

**Sol.** In configuration 1

total thermal resistance $R_{eq} = \frac{\ell}{KA} + \frac{\ell}{2KA} = \frac{3}{2}\frac{\ell}{KA}$

In configuration 2

$$R'_{eq} = \left(\frac{1\times\frac{1}{2}}{1+\frac{1}{2}}\right)\frac{\ell}{KA} = \frac{1}{3}\frac{\ell}{KA}$$

We have

$$Q = \frac{(T_1 - T_2)}{R}t$$

$t \propto R$

$$\frac{t_2}{t_1} = \frac{R_2}{R_1} \Rightarrow t_2 = \frac{2}{9} \times 9 = 2 \text{ sec}$$

**Q.8** A pulse of light of duration 100 ns is absorbed completely by a small object initially at rest. Power of the pulse is 30 mW and the speed of light is $3 \times 10^8$ ms$^{-1}$. The final momentum of object is

(A) $0.3 \times 10^{-17}$ kg ms$^{-1}$ (B) $1.0 \times 10^{-17}$ kg ms$^{-1}$ (C) $3.0 \times 10^{-17}$ kg ms$^{-1}$ (D) $9.0 \times 10^{-17}$ kg ms$^{-1}$

**Ans.** **[B]**

**Sol.** Energy absorbed by the object, $E = P \times t$

$E = 3 \times 10^{-9}$ J

Linear momentum = $\frac{E}{C}$

Linear momentum = $\frac{3\times10^{-9}}{3\times10^{8}}$

Linear momentum = $1 \times 10^{-17}$ kg ms$^{-1}$

**Q.9** In the Young's double slit experiment using a monochromatic light of wavelength $\lambda$, the path difference (in terms of an integer n) corresponding to any point having half the peak intensity is

(A) $(2n + 1)\frac{\lambda}{2}$ (B) $(2n + 1)\frac{\lambda}{4}$ (C) $(2n + 1)\frac{\lambda}{8}$ (D) $(2n + 1)\frac{\lambda}{16}$

**Ans.** **[B]**

**Sol.** We have

$$I = 4I_0 \cos^2\frac{\phi}{2}$$

$$2I_0 = 4I_0 \cos^2\frac{\phi}{2}$$

$$\cos\frac{\phi}{2} = \frac{1}{\sqrt{2}}$$

$$\phi = \frac{\pi}{2}$$

Path difference $\Delta x = \frac{\lambda}{2\pi} \times \Delta\phi$

$\Delta x = \frac{\lambda}{4}$

Option B verifies the result hence option B is correct.

**Q.10** Two non-reactive monatomic ideal gases have their atomic masses in the ratio 2 : 3. The ratio of their partial pressures, when enclosed in a vessel kept at a constant temperature, is 4 : 3. The ratio of their densities is

(A) 1 : 4 (B) 1 : 2 (C) 6 : 9 (D) 8 : 9

**Ans.** **[D]**

**Sol.** We have

PV = nRT

or

$PM = \rho RT$

$$\frac{\rho_1}{\rho_2} = \left(\frac{P_1}{P_2}\right)\left(\frac{M_1}{M_2}\right)$$

$$\frac{\rho_1}{\rho_2} = \frac{2}{3} \times \frac{4}{3}$$

$$\frac{\rho_1}{\rho_2} = \frac{8}{9}$$

## SECTION – 2 (One or more options correct Type)

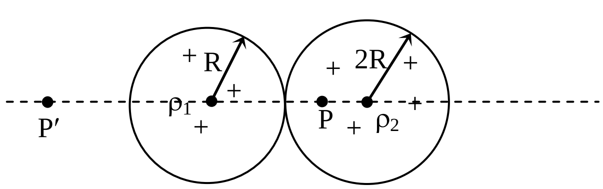
This section contains **5 multiple choice questions**. Each question has four choices (A), (B), (C) and (D), out of which **ONE or More** are correct.

**Q.11** Two non-conduction solid spheres of radii R and 2R, having uniform volume charge densities $\rho_1$ and $\rho_2$ respectively, touch each other. The net electric field at a distance 2R from the centre of the smaller sphere, along the line joining the centers of the spheres, is zero. The ratio $\frac{\rho_1}{\rho_2}$ can be

(A) –4 (B) $-\frac{32}{25}$ (C) $\frac{32}{25}$ (D) 4

**Ans.** **[B, D]**

**Sol.**




at point P and P′, E = 0

$$\vec{E}_{\text{due to } \rho_1} + \vec{E}_{\text{due to } \rho_2} = 0$$

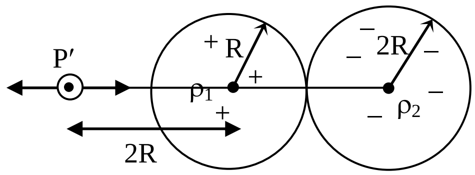
**For point P**

$$\frac{k\rho_1}{(2R)^2}\times\frac{4}{3}\pi R^3 = \frac{\rho_2\times R}{3\in_0}$$

$$\frac{\rho_1 R^3}{4R^2\times 3\in_0} = \frac{\rho_2\times R}{3\in_0}$$

$$\frac{\rho_1}{\rho_2} = 4 \text{ (option D)}$$

**For point P′**




$$\frac{k\rho_1}{4R^2}\times\frac{4}{3}\pi R^3 = \frac{k\rho_2}{(5R)^2}\times\frac{4}{3}\pi(8R^3)$$

$$\frac{\rho_1}{\rho_2} = -\frac{32}{25} \text{ (option B)}$$

**Q.12** A horizontal stretched string, fixed at two ends, is vibrating in fifth harmonic according to the equation, $y(x, t) = (0.01\text{ m})\sin[(62.8\text{ m}^{-1})x]\cos[(628\text{ s}^{-1})t]$ Assuming $\pi = 3.14$, the correct statement(s) is (are)

(A) The number of nodes is 5

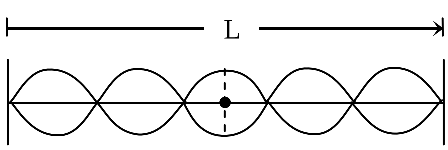
(B) The length of the string is 0.25 m.

(C) The maximum displacement of the midpoint of the string, from its equilibrium position is 0.01 m.

(D) The fundamental frequency is 100 Hz.

**Ans.** **[B, C]**

**Sol.**




$y = 0.01 \sin 6.2\,8x \cos 628t$

$$k = 62.8 = \frac{2\pi}{\lambda}$$

$$62.8 = \frac{2\times 3.14}{\lambda}$$

$$\lambda = \frac{6.28}{62.8} = \frac{1}{10}$$

$$\frac{5\lambda}{2} = L$$

$L = 5\times\frac{1}{20} \Rightarrow \frac{1}{4}$ meter $\Rightarrow 0.25$ m

Antinode is at mid point of string

Maximum displacement at this point = 0.01

Fundamental frequency = $\frac{v}{2\ell}$

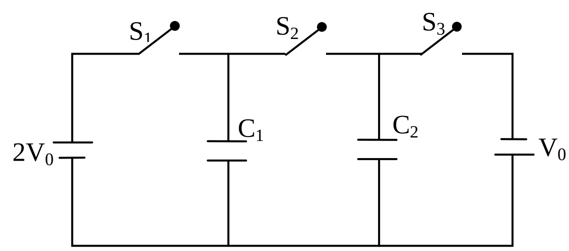
$v = \frac{\omega}{k} \Rightarrow \frac{6.28}{62.8} = 10$

Fundamental frequency $\Rightarrow \frac{10\times100}{2\times0.25}$ (V = fλ)

$\Rightarrow \frac{10}{0.5}$ $\left(v = \frac{\omega\lambda}{2\pi}\right)$

$\Rightarrow \frac{100}{5} \Rightarrow 20$Hz

**Q.13** In the circuit shown in the figure, there are two parallel plate capacitors each of capacitance C. The switch $S_1$ is pressed first to fully charge the capacitor $C_1$ and then released. The switch $S_2$ is then pressed to charge the capacitor $C_2$. After some time, $S_2$ is released and then $S_3$ is pressed. After some time,




(A) the charge on the upper plate of $C_1$ is $2CV_0$.

(B) the charge on the upper plate of $C_1$ is $CV_0$

(C) the charge on the upper plate of $C_2$ is 0

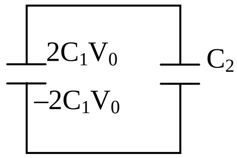
(D) the charge on the upper plate of $C_2$ is $-CV_0$

**Ans.** **[B, D]**

**Sol.** $S_1$ is closed $C_1$ get charged

Charge on $C_1 = C_1 \times 2V_0$

Now $S_2$ is pressed and $S_1$ is open




$V_{common} = \frac{2C_1V_0 + 0}{C_1 + C_2} \Rightarrow \frac{2C_1V_0}{C_1 + C_2}$

Charge on $C_2 = \frac{C_2 \times 2C_1V_0}{C_1 + C_2} \Rightarrow \frac{2C^2V_0}{2C} \Rightarrow$ $CV_0$ with upper plate positive.

Now $S_3$ is pressed and $S_2$ is open

$\therefore$ charge on $C_2 = CV_0$ with upper plate negative.

**Q.14** A particle of mass M and positive charge Q, moving with a constant velocity $\vec{u}_1 = 4\hat{i}\ ms^{-1}$ enters a region of uniform static magnetic field normal to the x-y plane. The region of the magnetic field extends from x = 0 to x = L for all values of y . After passing through this region, the particle emerges on the other side after 10 milliseconds with a velocity $\vec{u}_2 = 2(\sqrt{3}\hat{i} + \hat{j})\,ms^{-1}$. The correct statement(s) is (are)

(A) The direction of the magnetic field is –z direction

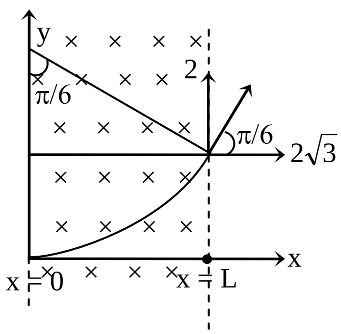
(B) The direction of the magnetic field is +z direction

(C) The magnitude of the magnetic field $\frac{50\pi M}{3Q}$ units

(D) The magnitude of the magnetic field $\frac{100\pi M}{3Q}$ units

**Ans.** **[A,C]**

**Sol.**




$\tan\theta = \frac{2}{2\sqrt{3}} = \frac{1}{\sqrt{3}}$

$\theta = \pi/6$

$t = \frac{\pi m}{6qB}$

$10 \times 10^{-3} = \frac{\pi \times M}{6QB}$

$B = \frac{\pi M}{6Q} \times 100$

$B = \frac{50\pi M}{3Q}$

**Q.15** A solid sphere of radius R and density ρ is attached to one end of a mass-less spring of force constant k. The other end of the spring is connected to another solid sphere of radius R and density 3ρ. The complete arrangement is placed in a liquid of density 2ρ and is allowed to reach equilibrium. The correct statement(s) is (are)

(A) the net elongation of the spring is $\frac{4\pi R^3 \rho g}{3k}$.

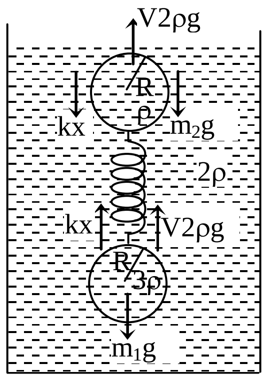
(B) the net elongation of the spring is $\frac{8\pi R^3 \rho g}{3k}$.

(C) the light sphere is partially submerged.

(D) the light sphere is completely submerged.

**Ans.** **[A,D]**

**Sol.**




Weight of system $\Rightarrow \left[\rho \times \frac{4}{3}\pi R^3 + 3\rho \times \frac{4}{3}\pi R^3\right]g$

$\Rightarrow \frac{4}{3}\pi R^3 \times 4\rho \times g \Rightarrow \frac{16\rho g \pi R^3}{3}$

Buoyancy Force = V.2ρg

$V.2\rho g = \frac{16\rho g \pi R^3}{3}$

$V \Rightarrow \frac{8\pi R^3}{3}$

Submerged volume = $\frac{8\pi R^3}{3}$, it mean both sphere are submerged completely

as total volume of both sphere is $\frac{4}{3}\pi R^3 + \frac{4}{3}\pi R^3 = \frac{8}{3}\pi R^3$

for spring elongation

$kx + \frac{4}{3}\pi R^3 \times 2\rho g = 3\rho \times \frac{4}{3}\pi R^3 \times g$

$kx = \rho g \times \frac{4}{3}\pi R^3$

$x = \rho g \times \frac{4}{3}\pi \frac{R^3}{k}$

# SECTION – 3 (Integer value correct Type)

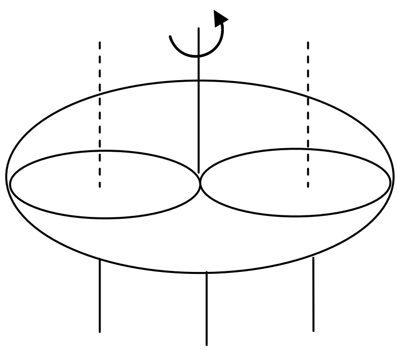
This section contains **5 questions**. The answer to each question is **single digit integer**, ranging from 0 to 9 (both inclusive).

**Q.16** A uniform circular disc of mass 50 kg and radius 0.4 m is rotating with a angular velocity of 10 rad $s^{-1}$ about its own axis, which is vertical. Two uniform circular rings, each of mass 6.25 kg and radius 0.2 m, are gently placed symmetrically on the disc in such a manner that they are touching each other along the axis of the disc and are horizontal. Assume that the friction large enough such that the axis of the rings are at rest relative to the disc and the system rotates about the original axis. The new angular velocity (in rad $s^{-1}$) of the system is

**Ans.** **[8]**

**Sol.** Conservation of angular momentum

$I_1\omega_1 = I_2\omega_2$



$$\frac{50\times(0.4)^2}{2}\times 10 = \left[\frac{50\times(0.4)^2}{2}+[6.25\times(0.2)^2]\times 4\right]\omega'_2$$

$\omega_2 = 8$

**Q.17** The work functions of Silver and Sodium are 4.6 and 2.3 eV, respectively. The ratio of the slope of the stopping potential versus frequency plot for Silver to that Sodium is

**Ans.** **[1]**

**Sol.** $hf = KE_{max} + \phi$

$hf = eV_s + \phi$

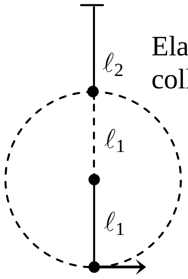
$f = \frac{e}{h}V_s + \phi$

slope is $\frac{e}{h}$, slope does not depend on work function and it is same for all metals. So answer is 1.

**Q.18** A bob a mass *m*, suspended by a string of length $\ell_1$, is given a minimum velocity required to complete a full circle in the vertical plane. At the highest point, it collides elastically with another bob of mass *m* suspended by a string of length $\ell_2$, which is initially at rest. Both the strings are mass-less and inextensible. If the second bob, after collision acquired the minimum speed required to complete a full circle in the vertical plane, the ratio $\frac{\ell_1}{\ell_2}$ is

**Ans.** **[5]**

**Sol.**




$U = V_{min} = \sqrt{5g\ell_1}$

At the highest point velocity is

$\Rightarrow \sqrt{u^2 - 2g \times 2\ell_1}$

$\Rightarrow \sqrt{5g\ell_1 - 2g(2\ell_1)}$

$\Rightarrow \sqrt{g\ell_1}$

After collision at highest point.

As collision is elastic and both bodies have same mass so velocities of both get exchange.

$\therefore$ body velocity = $\sqrt{g\ell_1}$

$\sqrt{g\ell_1} = V_{min}$ required to complete circle

$\sqrt{g\ell_1} = \sqrt{5g\ell_2}$

$\frac{\ell_1}{\ell_2} = 5$

**Q.19** A particle of mass 0.2 kg is moving in one dimension under a force that delivers a constant power 0.5 W to the particle. If the initial speed (in $ms^{-1}$) of the particle is zero, the speed (in $ms^{-1}$) after 5 s is

**Ans.** **[5]**

**Sol.** P = 0.5 watt.

in 5 sec., W = Pt

$\Rightarrow 0.5 \times 5$

work $\Rightarrow$ 2.5 joule

$W = \Delta KE$ (work energy theorem)

$$2.5 = \frac{1}{2} \times 0.2[v^2 - 0]$$

$v = 5\ ms^{-1}$

**Q.20** A freshly prepared sample of a radioisotope of half-life 1386 s has activity $10^3$ disintegrations per second. Given that $\ell n\ 2 = 0.693$, the fraction of the initial number of nuclei (expressed in nearest integer percentage) that will decay in the first 80 s after preparation of the sample is

**Ans.** **[4]**

**Sol.** $T_{1/2} = \frac{\ell n 2}{\lambda}$

$$1386 = \frac{0.693}{\lambda}$$

$$\lambda = \frac{0.693}{1386 \times 1000} \Rightarrow \frac{1}{2000}$$

$10^3 = \lambda N_0$

$$N_0 = \frac{10^3}{\lambda} = 1000 \times 2000 \Rightarrow 2 \times 10^6$$

$N = N_0 e^{-\lambda t}$

$N_0 - N_0 e^{-\lambda t}$ = No. of decayed nuclei

$$\text{\% of decayed nuclei} \Rightarrow \frac{N_0 - N_0 e^{-\lambda t}}{N_0} \times 100$$

$$\Rightarrow (1 - e^{-\lambda \times t}) \times 100$$

$$\Rightarrow (1 - e^{-\lambda \times 80}) \times 100$$

$$\Rightarrow (1 - e^{-\frac{80}{2000}}) \times 100$$

$= 3.9\% \Rightarrow 4\%$

# Part II - CHEMISTRY

**Q.21** The standard enthalpies of formation of $CO_2(g)$, $H_2O(\ell)$ and glucose (s) at 25ºC are –400 kJ/mol, –300 kJ/mol and –1300 kJ/mol, respectively. The standard enthalpy of combustion per gram of glucose at 25ºC is -

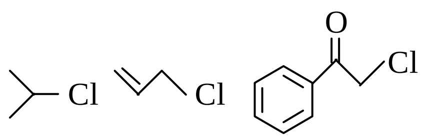
(A) + 2900 kJ (B) –2900 kJ (C) –16.11 kJ (D) +16.11 J

**Ans.** **[C]**

**Sol.** $C_6H_{12}O_6(s) + 6O_2(g) \rightarrow 6CO_2(g) + 6H_2O(\ell)$

$\Delta H = 6 \times (-400) + 6(-300) - (-1300)$
$= -2900$ kJ/mol
$= -16.11$ kJ/g

**Q.22** KI in acetone, undergoes $S_N2$ reaction with each of P, Q, R and S. The rates of the reaction vary as

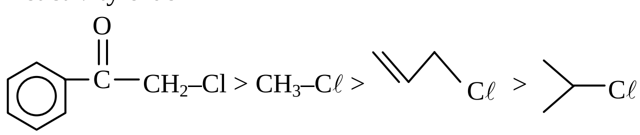
$H_3C–Cl$ 

P Q R S

(A) P > Q > R > S (B) S > P > R > Q (C) P > R > Q > S (D) R > P > S > Q

**Ans.** **[B]**

**Sol.** Reactivity order




**Q.23** The compound that does NOT liberate $CO_2$, on treatment with aqueous sodium bicarbonate solution, is -

(A) Benzoic acid (B) Benzensulphonic acid
(C) Salicylic acid (D) Carbolic acid (Phenol)

**Ans.** **[D]**

**Sol.** Phenol is less acidic than carbonic acid ($H_2CO_3$) so it does not release $CO_2$ with $NaHCO_3$.

**Q.24** Consider the following complex ions, P, Q and R.

$P = [FeF_6]^{3-}$ $Q = [V(H_2O)_6]^{2+}$ and $R = [Fe(H_2O)_6]^{2+}$

The correct order of the complex ions, according to their spin-only magnetic moment values (in B.M.) is -

(A) R < Q < P (B) Q < R < P (C) R < P < Q (D) Q < P < R

**Ans.** **[B]**

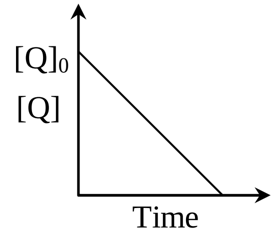
**Sol.** Q < R < P

$P = [FeF_6]^{-3}$
$Fe^{+3} = [Ar]\ 4s^o\ 3d^5$
5 unpaired $e^-$
$Q = [V(H_2O)_6]^{+2}$
$V^{+2} = [Ar]\ 4s^o\ 3d^3$ 3 unpaired $e^-$
$R = [Fe(H_2O)_6]^{+2}$
$Fe^{+2} = [Ar]\ 4s^o\ 3d^6$
4 unpaired $e^-$

**Q.25** In the reaction,

$P + Q \longrightarrow R + S$

The time taken for 75 % reaction of P is twice the time taken for 50 % reaction of P. The concentration of Q varies with reaction time as shown in the figure. The overall order of the reaction is -




(A) 2 (B) 3 (C) 0 (D) 1

**Ans.** **[D]**

**Sol.** $P + Q \rightarrow R + S$

For I[st] order reaction $t_{75\%} = 2 \times t_{1/2}$

$\therefore$ order w.r.t. P is 1.

For zero order reaction

Integrated rate law is

$(a_0 - x) = -kt + a_0$

$\therefore$ Q follows zero order

Hence overall order is 1.

**Q.26** Concentrated nitric acid, upon long standing, turns yellow-brown due to the formation of -

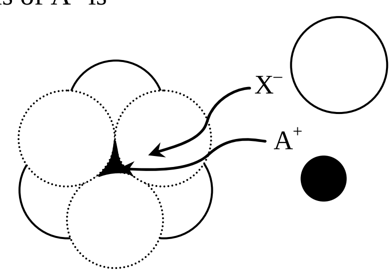
(A) NO (B) $NO_2$ (C) $N_2O$ (D) $N_2O_4$

**Ans.** **[B]**

**Sol.** $NO_2$

$4HNO_{3(\ell)} \xrightarrow{hv} 4NO_{2(g)}\uparrow + O_{2(g)}\uparrow + 2H_2O$

**Q.27** The arrangement of $X^-$ ions around $A^+$ ion in solid AX is given in the figure (not drawn to scale). If the radius of $X^-$ is 250 pm, the radius of $A^+$ is -




(A) 104 pm (B) 125 pm (C) 183 pm (D) 57 pm

**Ans.** **[A]**

**Sol.** $\frac{r_+}{r_-} = 0.414$ for octahedral void.

$r_+ = 0.414 \times 250 = 103.5 \cong 104$ pm

**Q.28** Upon treatment with ammoniacal $H_2S$, the metal ion that precipitates as a sulfide is -

(A) Fe(III) (B) Aℓ(III) (C) Mg(II) (D) Zn(II)

**Ans.** **[D]**

**Sol.** Zn(II)

Group 4 cations give precipitate with ammonical $H_2S$.

**Q.29** Methylene blue, from its aqueous solution, is adsorbed on activated charcoal at 25ºC. For this process, the correct statement is -

(A) The adsorption requires activation at 25ºC.

(B) The adsorption is accompanied by a decrease in enthalpy.

(C) The adsorption increases with increase of temperature.

(D) The adsorption is irreversible.

**Ans.** **[B]**

**Sol.** This is a process of physical adsorption it results in release of energy.

**Q.30** Sulfide ores are common for the metals -

(A) Ag, Cu and Pb (B) Ag, Cu and Sn (C) Ag, Mg and Pb (D) Al, Cu and Pb

**Ans.** **[A]**

**Sol.** Ag, Cu and Pb

$Ag \rightarrow Ag_2S$

$Cu \rightarrow CuFeS_2$

$Pb \rightarrow PbS$

**Q.31** Benzene and nepthalene form an ideal solution at room temperature. For this process, the true statement(s) is(are) -

(A) $\Delta G$ is positive (B) $\Delta S_{system}$ is positive (C) $\Delta S_{surroundings} = 0$ (D) $\Delta H = 0$

**Ans.** **[B, C, D]**

**Sol.** Condition for ideal solution

$\Delta G < 0$, $\Delta S > 0$ & $\Delta H = 0$

**Q.32** The pair(s) of coordination complexes/ions exhibiting the same kind of isomerism is(are) -

(A) $[Cr(NH_3)_5Cl]Cl_2$ and $[Cr(NH_3)_4Cl_2]Cl$ (B) $[Co(NH_3)_4Cl_2]^+$ and $[Pt(NH_3)_2(H_2O)Cl]^+$

(C) $[CoBr_2Cl_2]^{2-}$ and $[PtBr_2Cl_2]^{2-}$ (D) $[Pt(NH_3)_3(NO_3)]Cl$ and $[Pt(NH_3)_3Cl]Br$

**Ans.** **[B, D]**

**Sol.** **[B]** $[Co(NH_3)_4Cl_2]^2$ $Ma_4b_2$

$[Pt(NH_3)_2(H_2O)Cl]^+$ $Ma_2bc$

Both can show geometrical isomerism

**[D]** $[Pt(NH_3)_2(NO_3)]Cl$

$[Pt(NH_3)_3Cl]Br$

Both can show ionization isomerism

**Q.33** The initial rate of hydrolysis of methyl acetate (1M) by a weak acid (HA, 1M) is 1/100$^{th}$ of that of a strong acid (HX, 1M), at 25ºC. The $K_a$ of HA is -

(A) $1 \times 10^{-4}$ (B) $1 \times 10^{-5}$ (C) $1 \times 10^{-6}$ (D) $1 \times 10^{-3}$

**Ans.** **[A]**

**Sol.** $CH_3COOCH_3 + H_2O \xrightarrow{H^+} CH_3COOH + CH_3OH$

Hydrolysis of ester

rate $r_1 = k[CH_3COOCH_3][H^+]_{SA}$

rate $r_2 = k[CH_3COOCH_3][H^+]_{WA}$

$$\frac{r_1}{r_2} = \frac{[H^+]_{SA}}{[H^+]_{WA}}$$

$$\frac{100}{1} = \frac{1}{[H^+]_{WA}}$$

$[H^+]_{WA} = 0.01$

We know $[H^{+1}] = \alpha C = \sqrt{k_a C}$

or $\sqrt{k_a C} = 0.01$

$k_a \times 1 = 10^{-4}$

$k_a = 1 \times 10^{-4}$

**Q.34** The hyperconjugative stabilities of tert-butyl cation and 2-butene, respectively, are due to -

(A) $\sigma \to$ p (empty) and $\sigma \to \pi^*$ electron delocalisations

(B) $\sigma \to \sigma^*$ and $\sigma \to \pi$ electron delocalisations

(C) $\sigma \to$ p (filled) and $\sigma \to \pi$ electron delocalisations

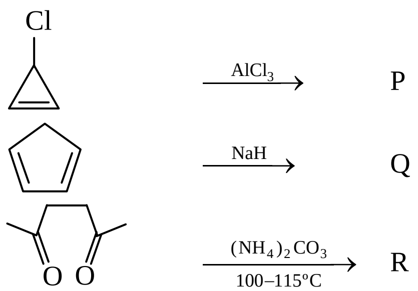
(D) p (filled) $\to \sigma \to \pi^*$ electron delocalisations

**Ans.** **[A]**

**Sol.** $\sigma - \pi$ empty and $\sigma - \pi^*$ electron delocalization.

**Q.35** Among P, Q, R and S, the aromatic compound(s) is/are -

(A) P (B) Q (C) R (D) S



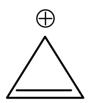
**Ans.** **[A, B, C, D]**

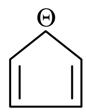
**Sol.** A = P Aromatic

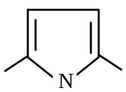
B = Q Aromatic

C = R Aromatic

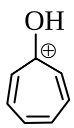
D = S Aromatic

P = 

Q = 

R = 

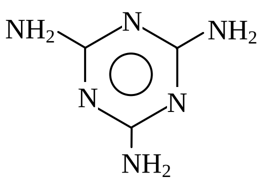
Aromatic

S = 

**Q.36** The total number of lone-pairs of electrons in melamine is -

**Ans.** **[6]**

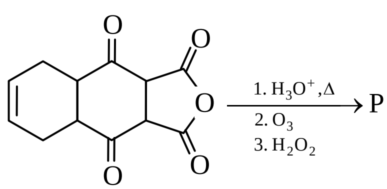
**Sol.**



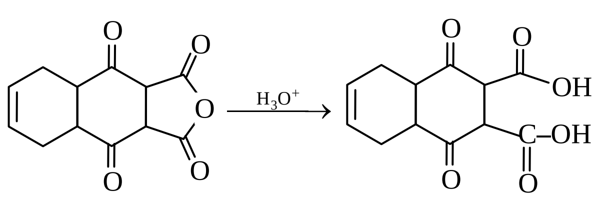
Melamine

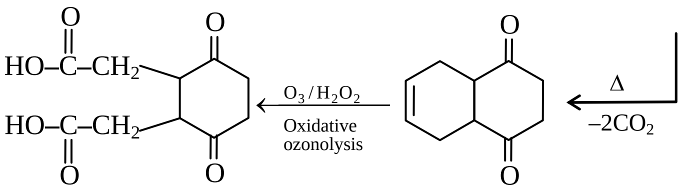
6 lone pairs are present.

**Q.37** The total number of carboxylic acid groups in the product P is -



**Ans.** **[2]**

**Sol.** 




**Q.38** The atomic masses of He and Ne are 4 and 20 a.m.u., respectively. The value of the de Broglie wavelength of He gas at – 73ºC is "M" times that of the de Broglie wavelength of Ne at 727ºC. M is -

**Ans.** **[5]**

**Sol.** $$\frac{\lambda_{He} = \dfrac{h}{m_{He}V_{He}}}{\lambda_{Ne} = \dfrac{h}{m_{Ne}V_{Ne}}} = \frac{m_{Ne}V_{Ne}}{m_{He}V_{He}}$$

$$M = \frac{20}{4} \times \frac{\sqrt{T/M}}{\sqrt{T/M}}$$

$$= \frac{20}{4} \times \frac{\sqrt{1000/20}}{\sqrt{200/4}}$$

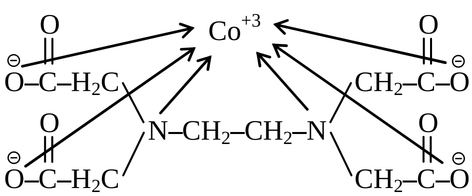
$$= 5 \times \sqrt{\frac{50}{50}}$$

$$= 5$$

**Q.39** $EDTA^{4-}$ is ethylenediaminetetraacetate ion. The total number of N–Co–O bond angles in $[Co(EDTA)]^{1-}$ complex ion is -

**Ans.** **[8]**

**Sol.**




**Q.40** A tetrapeptide has –COOH group on alanine. This produces glycine (Gly), valine (Val), phenyl (Phe) and alanine (Ala), on complete hydrolysis. For this tetrapeptide, the number of possible sequences (primary structures) with $-NH_2$ group attached to a chiral center is -

**Ans.** **[4]**

**Sol.** Tetrapeptide has 4 amino acid and –COOH group at alamine that means it should be at one end.

→ So possible 1º structure are

| I | II | III | IV | V | VI |
|---|---|---|---|---|---|
| G | V | P | V | G | P |
| V | G | V | P | P | G |

| | | | | | |
|---|---|---|---|---|---|
| P | P | G | G | V | V |
| A | A | A | A | A | A |

In glycine $-NH_2$ group is not attach chiral centre in other $-NH_2$ is attach at chiral centre.

So II, III, IV & VI structure have $-NH_2$
Group attach at chiral centre.

# Part III - Mathematics

**CODE : 3** **02 / 06 / 2013**

## SECTION – 1 (Only One option correct Type)

This section contains 10 **multiple choice questions**. Each question has four choices (A), (B), (C) and (D), out of which **ONLY ONE** is correct.

**Q.41** The value of $\cot\left(\sum_{n=1}^{23}\cot^{-1}\left(1+\sum_{k=1}^{n}2k\right)\right)$ is -

(A) $\frac{23}{25}$ (B) $\frac{25}{23}$ (C) $\frac{23}{24}$ (D) $\frac{24}{23}$

**Ans.** **[B]**

**Sol.** $\cot\left(\sum_{n=1}^{23}\cot^{-1}\left(1+\sum_{k=1}^{n}2k\right)\right)$

$= \cot\left(\sum_{n=1}^{23}\cot^{-1}(1+n(n+1))\right)$

$= \cot\left(\sum_{n=1}^{23}\cot^{-1}\left(\frac{1+n(n+1)}{(n+1)-n}\right)\right)$

$= \cot\left(\sum_{n=1}^{23}(\cot^{-1}(n+1)-\cot^{-1}(n))\right)$

$= \cot(\cot^{-1}2 - \cot^{-1}1 + \cot^{-1}3 - \cot^{-1}2 + \ldots + \cot^{-1}24 - \cot^{-1}23)$

$= \cot(\cot^{-1}24 - \cot^{-1}1)$

$= \frac{1+\cot(\cot^{-1}24)\cot(\cot^{-1}1)}{\cot(\cot^{-1}24)-\cot(\cot^{-1}1)}$

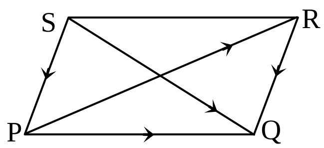
$= \frac{1+24\times 1}{24-1} = \frac{25}{23}$

**Q.42** Let $\overrightarrow{PR} = 3\hat{i} + \hat{j} - 2\hat{k}$ and $\overrightarrow{SQ} = \hat{i} - 3\hat{j} - 4\hat{k}$ determine diagonals of a parallelogram PQRS and $\overrightarrow{PT} = \hat{i} + 2\hat{j} + 3\hat{k}$ be another vector. Then the volume of the parallelepiped determined by the vectors $\overrightarrow{PT}, \overrightarrow{PQ}$ and $\overrightarrow{PS}$ is -

(A) 5 (B) 20 (C) 10 (D) 30

**Ans.** **[C]**

**Sol.**




$\overrightarrow{PQ} + \overrightarrow{QR} = 3\hat{i} + \hat{j} - 2\hat{k}$ ….(i)

$\overrightarrow{PQ} + \overrightarrow{RQ} = \hat{i} - 3\hat{j} - 4\hat{k}$ ….(ii)

$\overrightarrow{PQ} - \overrightarrow{QR} = \hat{i} - 3\hat{j} - 4\hat{k}$ ….(iii)

$2\,\overrightarrow{QR} = 4\hat{i} - 2\hat{j} - 6\hat{k}$

$\overrightarrow{QR} = 2\hat{i} - \hat{j} - 3\hat{k}$ ….(iv)

$\overrightarrow{PQ} = (3\hat{i} + \hat{j} - 2\hat{k}) - (2\hat{i} - \hat{j} - 3\hat{k})$

$\overrightarrow{PQ} = \hat{i} + 2\hat{j} + \hat{k}$

$\overrightarrow{PT} = \hat{i} + 2\hat{j} + 3\hat{k}$ (given)

and

$\overrightarrow{PQ} = \hat{i} + 2\hat{j} + \hat{k}$

$\overrightarrow{QR} = 2\hat{i} - \hat{j} - 3\hat{k}$

$$\therefore \text{Volume} = \begin{vmatrix} 1 & 2 & 3 \\ 1 & 2 & 1 \\ 2 & -1 & -3 \end{vmatrix}$$

$= 1(-6 + 1) - 2(-3 - 2) + 3(-1 - 4)$

$= -5 + 10 - 15 = -10$

$= 10$

**Q.43** Let complex numbers $\alpha$ and $\frac{1}{\overline{\alpha}}$ lie on circles $(x - x_0)^2 + (y - y_0)^2 = r^2$ and $(x - x_0)^2 + (y - y_0)^2 = 4r^2$, respectively. If $z_0 = x_0 + iy_0$ satisfies the equation $2\,|z_0|^2 = r^2 + 2$, then $|\alpha| =$

(A) $\frac{1}{\sqrt{2}}$ (B) $\frac{1}{2}$ (C) $\frac{1}{\sqrt{7}}$ (D) $\frac{1}{3}$

**Ans.** **[C]**

**Sol.** $\alpha$ lies on $|z - z_0| = r$

So $|\alpha - z_0| = r \Rightarrow |\alpha - z_0|^2 = r^2$ …(i)

$\frac{1}{\overline{\alpha}}$ lies on $|z - z_0| = 2r$

So $\left|\frac{1}{\overline{\alpha}} - z_0\right| = 2r$

$|1 - \overline{\alpha}z_0| = 2r|\overline{\alpha}|$

$|1 - \overline{\alpha}z_0| = 2r|\alpha|$

$\Rightarrow |1 - \overline{\alpha}z_0|^2 = 4r^2|\alpha|^2$ …(ii)

Subtract (ii) from (i)

$|1 - \overline{\alpha}z_0|^2 - |\alpha - z_0|^2 = r^2(4|\alpha|^2 - 1)$

$\Rightarrow 1 + |\alpha|^2|z_0|^2 - |\alpha|^2 - |z_0|^2 = r^2(4|\alpha|^2 - 1)$

$\Rightarrow (1 - |\alpha|^2)(1 - |z_0|^2) - r^2(4|\alpha|^2 - 1) = 0$

$\Rightarrow (1 - |\alpha|^2)(1 - |z_0|^2) + 2(1 - |z_0|^2)(4|\alpha|^2 - 1) = 0$

$\Rightarrow (1 - |z_0|^2)(1 - |\alpha|^2 + 8|\alpha|^2 - 2) = 0$

$\Rightarrow (1 - |z_0|^2)(7|\alpha|^2 - 1) = 0$

$\Rightarrow |\alpha|^2 = 1/7$

$\Rightarrow |\alpha| = \frac{1}{\sqrt{7}}$

**Q.44** For a > b > c > 0, the distance between (1, 1) and the point of intersection of the lines ax + by + c = 0 and bx + ay + c = 0 is less than $2\sqrt{2}$. Then -

(A) a + b – c > 0 (B) a – b + c < 0 (C) a – b + c > 0 (D) a + b – c < 0

**Ans.** **[A]**

**Sol.** ax + by + c = 0

bx + ay + c = 0

Intersection point

$\left(-\frac{c}{a+b}, -\frac{c}{a+b}\right)$

Distance

$\left(1+\frac{c}{a+b}\right)^2 + \left(1+\frac{c}{a+b}\right)^2 < 8$

$2(a + b + c)^2 < 8(a + b)^2$

$(a + b + c)^2 < (2a + 2b)^2$

$(2a + 2b)^2 - (a + b + c)^2 > 0$

$(a + b - c)(3a + 3b + c) > 0$

so, $(a + b - c) > 0$

**Q.45** Perpendiculars are drawn from points on the line $\frac{x+2}{2} = \frac{y+1}{-1} = \frac{z}{3}$ to the plane x + y + z = 3. The feet of perpendiculars lie on the line -

(A) $\frac{x}{5} = \frac{y-1}{8} = \frac{z-2}{-13}$

(B) $\frac{x}{2} = \frac{y-1}{3} = \frac{z-2}{-5}$

(C) $\frac{x}{4} = \frac{y-1}{3} = \frac{z-2}{-7}$

(D) $\frac{x}{2} = \frac{y-1}{-7} = \frac{z-2}{5}$

**Ans.** **[D]**

**Sol.** Let point lies on given line is

(– 2, –1, 0)

Line ⊥ to plane and passing through (–2, –1, 0) is

$\frac{x+2}{1} = \frac{y+1}{1} = \frac{z}{1} = \lambda$

General point on above line is

A(λ – 2, λ – 1, λ)

Now this point lies on plane so put point A in equation of plane so we get λ = 2

Point A (0, 1, 2)

Let second point on line is (0, –2, 3)

Let line ⊥ to plane and passing through point (0, –2, 3) is $\frac{x}{1} = \frac{y+2}{1} = \frac{z-3}{1} = \lambda$

General point on above line is B(λ, λ – 2, λ + 3)

Now this point lies on plane so we get λ = 2/3

So point B (2/3, –4/3, 11/3)

Clearly drs of line join foot of ⊥ i.e. A and B is (2/3, –7/3, 5/3) or (2, –7, 5)

**Q.46** Four persons independently solve a certain problem correctly with probabilities $\frac{1}{2}, \frac{3}{4}, \frac{1}{4}, \frac{1}{8}$. Then the probability that the problem is solved correctly by at least one of them is -

(A) $\frac{235}{256}$ (B) $\frac{21}{256}$ (C) $\frac{3}{256}$ (D) $\frac{253}{256}$

**Ans.** **[A]**

**Sol.** $P(A) = \frac{1}{2}$, $P(B) = \frac{3}{4}$, $P(C) = \frac{1}{4}$, $P(D) = \frac{1}{8}$

Required probability = $1 - P(\bar{A})P(\bar{B})P(\bar{C})P(\bar{D})$

$= 1 - \frac{1}{2} \times \frac{1}{4} \times \frac{3}{4} \times \frac{7}{8}$

$= 1 - \frac{21}{256}$

$= \frac{235}{256}$

**Q.47** The area enclosed by the curves y = sin x + cos x and y = | cos x – sin x | over the interval $\left[0, \frac{\pi}{2}\right]$ is -

(A) $4(\sqrt{2}-1)$ (B) $2\sqrt{2}(\sqrt{2}-1)$ (C) $2(\sqrt{2}+1)$ (D) $2\sqrt{2}(\sqrt{2}+1)$

**Ans.** **[B]**

**Sol.** Area = $\int_0^{\pi/2} ((\sin x + \cos x) - |\cos x - \sin x|)\, dx$

$= \int_0^{\pi/2} (\sin x + \cos x)\, dx - \int_0^{\pi/4} (\cos x - \sin x)\, dx - \int_{\pi/4}^{\pi/2} (\sin x - \cos x)\, dx$

$= [-\cos x + \sin x]_0^{\pi/2} - [\sin x + \cos x]_0^{\pi/4} - [-\cos x - \sin x]_{\pi/2}^{\pi/4}$

$= (1 + 1) - (\sqrt{2} - 1) - (-1 + \sqrt{2})$

$= 2 - \sqrt{2} + 1 + 1 - \sqrt{2}$

$= 4 - 2\sqrt{2}$

$= 2\sqrt{2}\ (\sqrt{2} - 1)$

**Q.48** A curve passes through the point $\left(1, \frac{\pi}{6}\right)$. Let the slope of the curve at each point (x, y) be $\frac{y}{x} + \sec\left(\frac{y}{x}\right)$, x > 0.

Then the equation of the curve is -

(A) $\sin\left(\frac{y}{x}\right) = \log x + \frac{1}{2}$ (B) $\operatorname{cosec}\left(\frac{y}{x}\right) = \log x + 2$

(C) $\sec\left(\frac{2y}{x}\right) = \log x + 2$ (D) $\cos\left(\frac{2y}{x}\right) = \log x + \frac{1}{2}$

**Ans.** **[A]**

**Sol.** $\frac{dy}{dx} = \frac{y}{x} + \sec\frac{y}{x}$

y = vx

$\frac{dy}{dx} = v + x\frac{dv}{dx}$

$v + x\frac{dv}{dx} = v + \sec v$

$\int \cos v\,dv = \int \frac{dx}{x}$

$\sin v = \ell n|x| + c$

$\sin \frac{y}{x} = \ell n|x| + c$

As curve passes through $\left(1, \frac{\pi}{6}\right)$

So $\sin \frac{\pi}{6} = 0 + c \Rightarrow c = \frac{1}{2}$

So $\sin \frac{y}{x} = \log x + \frac{1}{2}$

**Q.49** Let $f : \left[\frac{1}{2}, 1\right] \to R$ (the set of all real numbers) be a positive, non-constant and differentiable function such that $f'(x) < 2f(x)$ and $f\left(\frac{1}{2}\right) = 1$. Then the value of $\int_{1/2}^{1} f(x)dx$ lies in the interval -

(A) $(2e - 1, 2e)$

(B) $(e - 1, 2e - 1)$

(C) $\left(\frac{e-1}{2}, e-1\right)$

(D) $\left(0, \frac{e-1}{2}\right)$

**Ans.** **[D]**

**Sol.** $f'(x) - 2f(x) < 0$ …(i)

Multiply equation (i) by $e^{-2x}$

$f'(x)\,e^{-2x} - 2e^{-2x} f(x) < 0$

$\frac{d}{dx}(f(x)\,e^{-2x}) < 0$

So $f(x)\,e^{-2x}$ decreases

So $f(x)\,e^{-2x} < f(1/2)\,e^{-1}$ ; for $x \in [1/2, 1]$

$f(x)\,e^{-2x} < \frac{1}{e}$ ; for $x \in [1/2, 1]$

$f(x) < e^{+2x-1}$ ; for $x \in [1/2, 1]$

since $f(x) > 0$ (given)

so $0 < \int_{1/2}^{1} f(x)\,dx < \int_{1/2}^{1} e^{2x-1}\,dx$

so $0 < \int_{1/2}^{1} f(x)\,dx < \left(\frac{e^{2x-1}}{2}\right)_{1/2}^{1} dx$

so, $0 < \int_{1/2}^{1} f(x)\,dx < \frac{e-1}{2}$

**Q.50** The number of points in $(-\infty, \infty)$, for which $x^2 - x \sin x - \cos x = 0$, is -

(A) 6 (B) 4 (C) 2 (D) 0

**Ans.** **[C]**

**Sol.** Let $f(x) = x^2 - x \sin x - \cos x$

$f'(x) = 2x - x \cos x$

$f'(x) = x(2 - \cos x)$

as for $x < 0$, $f'(x) < 0$ so $f(x)$ is decreasing.

and for $x > 0$, $f'(x) > 0$ so $f(x)$ is increasing.

So, $f(x)$ will be zero at 2 points.

## SECTION – 2 (One or more options correct Type)

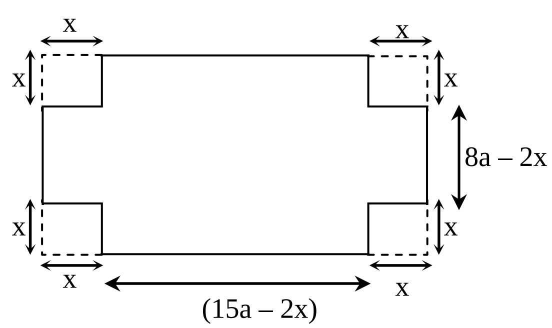
This section contains 5 **multiple choice questions**. Each question has four choices (A), (B), (C) and (D), out of which **ONE or MORE** are correct.

**Q.51** A rectangular sheet of fixed perimeter with sides having their lengths in the ratio 8 : 15 is converted into an open rectangular box by folding after removing squares of equal area from all four corners. If the total area of removed squares is 100, the resulting box has maximum volume. Then the lengths of the sides of the rectangular sheet are -

(A) 24 (B) 32 (C) 45 (D) 60

**Ans.** **[A,C]**

**Sol.**




$V = (15a - 2x)(8a - 2x)x$

$V = 4x^3 - 46ax^2 + 120a^2x$

$\frac{dV}{dx} = 12x^2 - 92ax + 120a^2$

$= 4(3x^2 - 23ax + 30a^2)$

at x = 5, $\frac{dV}{dx} = 0$

$30a^2 - 115a + 75 = 0$

$\Rightarrow 6a^2 - 23a + 15 = 0$

$\Rightarrow (a - 3)(6a - 5) = 0$

$\Rightarrow$ So, a = 3 or a = $\frac{5}{6}$

Now $\frac{d^2V}{dx^2} = 24x - 92a$

For a = 3, $\frac{d^2V}{dx^2} < 0$

So, V is maximum for a = 3.

Hence lengths are 24 and 45.

**Q.52** Let $S_n = \sum_{k=1}^{4n} (-1)^{\frac{k(k+1)}{2}} k^2$. Then $S_n$ can take value(s)

(A) 1056 (B) 1088 (C) 1120 (D) 1332

**Ans.** **[A,D]**

**Sol.** $\therefore S_n = -1^2 - 2^2 + 3^2 + 4^2 - 5^2 - 6^2 + 7^2 + 8^2 \ldots\ldots\ldots\ldots + 4n^2$

$= [3^2 - 1^2 + 7^2 - 5^2 \ldots\ldots\ldots 2n \text{ terms}] + [4^2 - 2^2 + 8^2 - 6^2 \ldots\ldots\ldots\ldots 2n \text{ terms}]$

$= 2[1 + 3 + 5 + 7 \ldots\ldots\ldots 2n \text{ terms}] + 2[2 + 4 + 6 + 8 \ldots\ldots\ldots 2n \text{ terms}]$

$= 2[2n/2\ [2 + (2n - 1)2] + 2[2n/2\ (4 + (2n - 1)2]$

$= 2n[4n] + 2n[4n + 2]$

$= 8n^2 + 8n^2 + 4n$

$= 16n^2 + 4n$

$S_n = 4n(4n + 1)$

which gives option A and D for n = 8, 9

**Q.53** A line $\ell$ passing through the origin is perpendicular to the lines

$\ell_1 : (3 + t)\hat{i} + (-1 + 2t)\hat{j} + (4 + 2t)\hat{k}, -\infty < t < \infty$

$\ell_2 : (3 + 2s)\hat{i} + (3 + 2s)\hat{j} + (2 + s)\hat{k}, -\infty < s < \infty$

Then the coordinates(s) of the point(s) on $\ell_2$ at a distance of $\sqrt{17}$ from the point of intersection of $\ell$ and $\ell_1$ is(are)

(A) $\left(\frac{7}{3}, \frac{7}{3}, \frac{5}{3}\right)$ (B) (–1, –1, 0) (C) (1, 1, 1) (D) $\left(\frac{7}{9}, \frac{7}{9}, \frac{8}{9}\right)$

**Ans.** **[B,D]**

**Sol.** $\ell_1 = (3\hat{i} + \hat{j} + 4\hat{k}) + t(\hat{i} + 2\hat{j} + 2\hat{k})$

$\ell_2 = (3\hat{i} + 3\hat{j} + 2\hat{k}) + s(2\hat{i} + 2\hat{j} + \hat{k})$

Drs of line $\perp$ to both lines (2, –3, 2)

So line $\ell$ is $\frac{x}{2} = \frac{y}{-3} = \frac{z}{2}$

Intersection point of line $\ell$ and $\ell_1$ is A (2, –3, 2)

General point on $\ell_2$ is B (2k + 3, 2k + 3, k + 2)

Distance between A and B = $\sqrt{17}$

$\sqrt{(2k+1)^2 + (2k+6)^2 + k^2} = \sqrt{17}$

k = –2 and k = $-\frac{10}{9}$

So point if k = – 2, is (–1, –1, 0)

if k = $-\frac{10}{9}$ is $\left(\frac{7}{9}, \frac{7}{9}, \frac{8}{9}\right)$

**Q.54** Let f(x) = x sin πx, x > 0. Then for all natural numbers n, f '(x) vanishes at -

(A) a unique point in the interval $\left(n, n + \frac{1}{2}\right)$

(B) a unique point in the interval $\left(n + \frac{1}{2}, n + 1\right)$

(C) a unique point in the interval (n, n + 1)

(D) two points in the interval (n, n + 1)

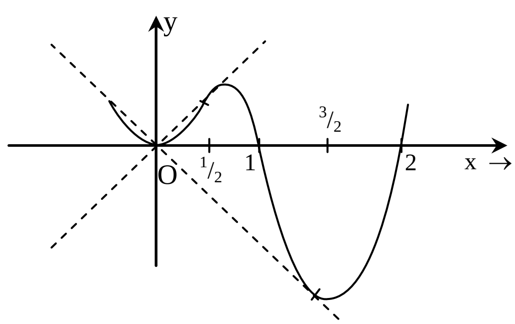
**Ans.** **[B, C]**

**Sol.** f(x) = x sin πx

f ′(x) = sin πx + πx cos πx

$f'\left(n + \frac{1}{2}\right)$ = 1; n ∈ even

= –1; n ∈ odd




So $f'(x) = 0$ between $\left(n + \frac{1}{2}, n + 1\right)$

So correct options are B & C

**Q.55** For 3 × 3 matrices M and N, which of the following statement(s) is (are) NOT correct ?

(A) $N^TMN$ is symmetric or skew symmetric, according as M is symmetric or skew symmetric

(B) MN – NM is skew symmetric for all symmetric matrices M and N

(C) MN is symmetric for all symmetric matrices M and N

(D) (adj M) (adj N) = adj (MN) for all invertible matrices M and N

**Ans.** **[C,D]**

**Sol.** For option (A)

$(N^TMN)^T = N^TM^T(N^T)^T$

$= N^TM^TN$

if M is symmetric then $M^T = M$

so $N^TMN$ is also symmetric

if M is skew symmetric then $M^T = -M$

So, $N^TMN$ is also skew symmetric

So (A) is correct

For option (B)

$M^T = M$, $N^T = N$

$(MN - NM)^T = (MN)^T - (NM)^T$

$= N^TM^T - M^TN^T = NM - MN$

$= -(MN - NM)$

So, option (B) is correct.

For option (C)

$M^T = M$, $N^T = N$

$(MN)^T = N^TM^T = NM$

So, option (C) is not correct.

For option (D)

(adjM) (adjN) = adj(NM)

so, option (D) is not correct.

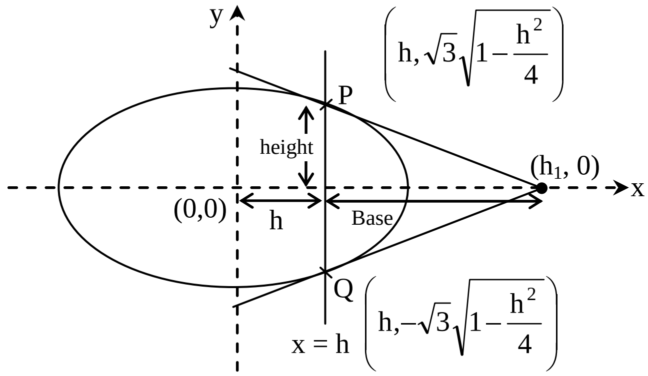
## SECTION – 3 (Integer value correct Type)

This section contains **5 questions**. The answer to each question is a **single digit integer**, ranging from 0 to 9 (both inclusive)

**Q.56** A vertical line passing through the point (h, 0) intersects the ellipse $\frac{x^2}{4} + \frac{y^2}{3} = 1$ at the points P and Q. Let the tangents to the ellipse at P and Q meet at the point R. If $\Delta(h)$ = area of the triangle PQR, $\Delta_1 = \max\limits_{1/2 \le h \le 1} \Delta(h)$ and $\Delta_2 = \min\limits_{1/2 \le h \le 1} \Delta(h)$, then $\frac{8}{\sqrt{5}}\Delta_1 - 8\Delta_2$ =

**Ans.** **[9]**

**Sol.**




Line PQ is chord of contact

$\Rightarrow \frac{xh_1}{4} + 0 = 1$ ….(1)

$x = h$ …..(2)

Compare (1) & (2)

$h_1 = \frac{4}{h}$

So area = $\left(\frac{4}{h} - h\right) \times \sqrt{3}\sqrt{1-\frac{h^2}{4}}$

$= \frac{\sqrt{3}}{2}\frac{(4-h^2)^{3/2}}{h}$ regular decreasing

$(\text{Area}_{\max})_{h=1/2} = \frac{\sqrt{3}}{2}\frac{\left(4-\frac{1}{h}\right)^{3/2}}{1/2}$, $(\text{Area})_{\min\, h=1} = \frac{\sqrt{3}}{2}(3)^{3/2}$

$= \frac{\sqrt{3}}{2}(\sqrt{15})^{3/2}$

So, $\frac{8}{\sqrt{5}}\Delta_1 - 8\Delta_2 = \frac{8}{\sqrt{5}} \times \frac{\sqrt{3}}{8} \times (\sqrt{15})^{3/2} - 8 \times \frac{\sqrt{3}}{2}(3)^{3/2}$

$= 5 \times 9 - 4 \times 9$

$= 45 - 36$

$= 9$

**Q.57** The coefficients of three consecutive terms of $(1 + x)^{n+5}$ are in the ratio 5 : 10 : 14. Then n =

**Ans.** **[6]**

**Sol.** $\frac{^{n+5}C_r}{^{n+5}C_{r+1}} = \frac{5}{10} \Rightarrow n - 3r = -3$ …(1)

$\frac{^{n+5}C_{r+1}}{^{n+5}C_{r+2}} = \frac{10}{14} \Rightarrow 5n - 12r = -6$ …(2)

solve (1) and (2)

r = 3

n = 6

**Q.58** Consider the set of eight vectors $V = \{a\hat{i} + b\hat{j} + c\hat{k} : a, b, c \in \{-1, 1\}\}$. Three non-coplanar vectors can be chosen from V in $2^p$ ways. Then p is

**Ans.** **[5]**

**Sol.** Total no. of vectors = $^8C_3 = 56$

Let consider following pairs of vectors

(i) $\hat{i} + \hat{j} + \hat{k}$ and $-\hat{i} - \hat{j} - \hat{k}$

(ii) $-\hat{i} + \hat{j} + \hat{k}$ and $\hat{i} - \hat{j} - \hat{k}$

(iii) $\hat{i} + \hat{j} - \hat{k}$ and $-\hat{i} - \hat{j} + \hat{k}$

(iv) $\hat{i} - \hat{j} + \hat{k}$ and $-\hat{i} + \hat{j} - \hat{k}$

If we select any one pair out of these pairs and one vector from remaining 6 vectors then these 3 vectors will be coplanar.

So, total no. of coplanar vectors = $^4C_1 \times {}^6C_1 = 24$

So, total no. of non coplanar vectors = 56 – 24

= $32 = 2^5$

$\therefore$ p = 5

**Q.59** Of the three independent events $E_1$, $E_2$ and $E_3$, the probability that only $E_1$ occurs is $\alpha$, only $E_2$ occurs is $\beta$ and only $E_3$ occurs is $\gamma$. Let the probability p that none of events $E_1$, $E_2$ or $E_3$ occurs satisfy the equations $(\alpha - 2\beta)\, p = \alpha\beta$ and $(\beta - 3\gamma)\, p = 2\beta\gamma$. All the given probabilities are assumed to lie in the interval (0, 1).

Then $\dfrac{\text{Probability of occurrence of } E_1}{\text{Probability of occurrence of } E_3} =$

**Ans.** **[6]**

**Sol.** Let probabilities of $E_1$, $E_2$ and $E_3$ are $p_1$, $p_2$ and $p_3$ respectively.

Given, $p_1(1 - p_2)(1 - p_3) = \alpha$

and $p_2(1 - p_1)(1 - p_3) = \beta$

and $(1 - p_1)(1 - p_2)p_3 = \gamma$

also $(1 - p_1)(1 - p_2)(1 - p_3) = p$

so $\dfrac{\alpha}{p} = \dfrac{p_1}{1-p_1}$, $\dfrac{\beta}{p} = \dfrac{p_2}{1-p_2}$, $\dfrac{\gamma}{p} = \dfrac{p_3}{1-p_3}$

also given that

$(\alpha - 2\beta)\, p = \alpha\beta$

$\Rightarrow \left(\frac{p}{\beta} - \frac{2p}{\alpha}\right) = 1$ …(i)

also $(\beta - 3\gamma)\, p = 2\beta\gamma$

$\frac{p}{\gamma} - \frac{3p}{\beta} = 2$ …(ii)

from (i) and (ii)

$\frac{p}{\gamma} - 3 - \frac{6p}{\alpha} = 2$

$\frac{p}{\gamma} - \frac{6p}{\alpha} = 5$

$\frac{1-p_3}{p_3} - \frac{6(1-p_1)}{p_1} = 5$

$\Rightarrow \frac{1}{p_3} - 1 - \frac{6}{p_1} + 6 = 5$

$\Rightarrow \frac{p_1}{p_3} = 6$

**Q.60** A pack contains n cards numbered from 1 to n. Two consecutive numbered cards are removed from the pack and the sum of the numbers on the remaining cards is 1224. If the smaller of the numbers on the removed cards is k, then k – 20 =

**Ans.** **[5]**

**Sol.** Sum of n cards = 1 + 2 …….. + n = $\frac{n(n+1)}{2}$

$\therefore \frac{n(n+1)}{2} - (k + k + 1) = 1224$

$\frac{n(n+1)}{2} = 1224 + 2k + 1$ …(1)

$\therefore$ as $k \geq 1$

so, $\frac{n(n+1)}{2} \geq 1224 + 1 + 2$

$\Rightarrow n^2 + n \geq 2448 + 6$

$\Rightarrow \left(\frac{n+1}{2}\right)^2 \geq 2448 + 6 + \frac{1}{4}$

$\left(n + \frac{1}{2}\right)^2 \geq (49.5)^2$

$n \geq 49$ …(2)

also $k \leq n$

$\therefore \frac{n(n+1)}{2} \leq 1224 + n + n + 1$

$n^2 + n \leq 2448 + 4n + 2$

$n^2 - 3n \leq 2450$

$\left(n - \frac{3}{2}\right)^2 \leq 2450 + \frac{9}{4} < 50^2$

$n - \frac{3}{2} < 50$

$n < 51.5$ ….(3)

from (2) and (3)

n can be 49, 50, 51

put n = 49 in (1) we get

$49 \times 25 = 1224 + 2k + 1$

$\Rightarrow k = 0$ not possible

At n = 50 we get

$25.5 = 1224 + 2k + 1$

$k = 25$

At n = 51 we get

$51.26 = 1224 + 2k + 1$

$102 = 2k + 1$

$\therefore k \notin I$

$\therefore k = 25 \Rightarrow k - 20 = 5$