JEE MAIN ONLINE PAPER 2018

Held on April 16, 2018

Instructions

1. This test will be a 3 hours Test.
2. This test consists of Physics, Chemistry and Mathematics questions with equal weight age of 120 marks.
3. Each question is of 4 marks.
4. There are three parts in the question paper consisting of Physics (Q. no. 1 to 30), Chemistry (Q. no. 31 to 60) and Mathematics (Q. no. 61 to 90).
5. There will be only one correct choice in the given four choices. For each question 4 marks will be awarded for correct choice, 1 mark will be deducted for incorrect choice and zero mark will be awarded for not attempted question.
6. Any textual, printed or written material, mobile phones, calculator etc. is not allowed for the students appearing for the test.
7. All calculations / written work should be done in the rough sheet provided.

PHYSICS

Q.1 A charge q is spread uniformly over an insulated loop of radius r. If it is rotated with an angular velocity ω with respect to normal axis then the magnetic moment of the loop is -

[JEE-Main On line-2018]

(A) $\frac{1}{2}q\omega r^2$ (B) $\frac{4}{3}q\omega r^2$

(C) $\frac{3}{2}q\omega r^2$ (D) $q\omega r^2$

Q.2 A heating element has a resistance of 100Ω at room temperature. When it is connected to a supply of 220 V, a steady current of 2 A passes in it and temperature is 500°C more than room temperature. What is the temperature coefficient of resistance of the heating element ?

[JEE-Main On line-2018]

(A) $1 \times 10^{-4}\ °C^{-1}$ (B) $5 \times 10^{-4}\ °C^{-1}$

(C) $2 \times 10^{-4}\ °C^{-1}$ (D) $0.5 \times 10^{-4}\ °C^{-1}$

Q.3 Let $\vec{A} = (\hat{i} + \hat{j})$ and $\vec{B} = (\hat{i} - \hat{j})$. The magnitude of a coplanar vector $\vec{C}$ such that $\vec{A}.\vec{C} = \vec{B}.\vec{C} = \vec{A}.\vec{B}$ is given by

[JEE-Main On line-2018]

(A) $\sqrt{\frac{5}{9}}$ (B) $\sqrt{\frac{10}{9}}$ (C) $\sqrt{\frac{20}{9}}$ (D) $\sqrt{\frac{9}{12}}$

Q.4 Two particles of the same mass are moving in circular orbits because of force, given by

$f(r) = \frac{-16}{r} - r^3$

The first particle is at a distance r = 1, and the second, at r = 4. The best estimate for the ratio of kinetic energies of the first and the second particle is closest to -

[JEE-Main On line-2018]

(A) 10^{-1} (B) 6×10^{-2}

(C) 6×10^{2} (D) 3×10^{-3}

Q.5 At some instant, a radioactive sample S_1 having an activity 5μCi has twice the number of nuclei as another sample S_2 which has an activity of 10μCi. The half lives of S_1 and S_2 are -

[JEE-Main On line-2018]

(A) 10 years and 20 years, respectively

(B) 5 years and 20 years, respectively

(C) 20 years and 10 years, respectively

(D) 20 years and 5 years, respectively

Q.6 In a circuit for finding the resistance of a galvanometer by half deflection method, a 6 V battery and high resistance of 11 kΩ are used. The figure of merit of the galvanometer 60μA/division. In the absence of shunt resistance, the galvanometer produces a deflection of θ = 9 divisions when current flows in the circuit. The value of the shunt resistance that can cause the deflection of θ/2, is closed to -

[JEE-Main On line-2018]

(A) 55Ω (B) 110Ω

(C) 220Ω (D) 550Ω

Q.7 A body of mass m starts moving from rest along x-axis so that its velocity varies as $v = a\sqrt{s}$ where a is a constant and s is the distance covered by the body. The total work done by all the forces acting on the body in the first second after the start of the motion is -

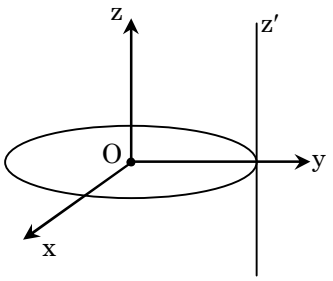
[JEE-Main On line-2018]

(A) $\frac{1}{8}$ ma^4t^2 (B) $4ma^4t^2$

(C) $8ma^4t^2$ (D) $\frac{1}{4}$ ma^4t^2

Q.8 A thin circular disc is in the xy plane as shown in the figure. The ratio of its moment of inertia about z and z' axes will be

[JEE-Main On line-2018]



(A) 1 : 2 (B) 1 : 4 (C) 1 : 3 (D) 1 : 5

Q.9 A small soap bubble of radius 4 cm is trapped inside another bubble of radius 6 cm without any contact. Let P_2 be the pressure inside the inner bubble and P_0, the pressure outside the outer bubble. Radius of another bubble with pressure difference $P_2 - P_0$ between its inside the outside would be -

[JEE-Main On line-2018]

(A) 6 cm (B) 12 cm

(C) 4.8 cm (D) 2.4 cm

Q.10 The de-Broglie wavelength (λ_B) associated with the electron orbiting in the second excited state of hydrogen atom is related to that in the ground state (λ_G) by -

[JEE-Main On line-2018]

(A) $\lambda_B = \lambda_{G/3}$ (B) $\lambda_B = \lambda_{G/2}$

(C) $\lambda_B = 2\lambda_G$ (D) $\lambda_B = 3\lambda_G$

Q.11 A coil of cross-sectional area A having n turns is placed in a uniform magnetic field B. When it is rotated with an angular velocity ω, the maximum e.m.f. induced in the coil will be -

[JEE-Main On line-2018]

(A) nBAω (B) $\frac{3}{2}$ nBAω

(C) 3nBAω (D) $\frac{1}{2}$ nBAω

Q.12 Unpolarized light of intensity I is incident on a system of two polarizers, A followed by B. The intensity of emergent light is $\frac{I}{2}$. If a third polarizer C is placed between A and B, the intensity of emergent light is reduced to $\frac{I}{3}$. The angle between the polarizers A and C is θ. Then -

[JEE-Main On line-2018]

(A) $\cos\theta = \left(\frac{2}{3}\right)^{1/4}$ (B) $\cos\theta = \left(\frac{1}{3}\right)^{1/4}$

(C) $\cos\theta = \left(\frac{1}{3}\right)^{1/2}$ (D) $\cos\theta = \left(\frac{2}{3}\right)^{1/2}$

Q.13 A power transmission line feeds input power at 2300 V to a step down transformer with its primary windings having 4000 turns, giving the output power at 230 V. If the current in the primary of the transformer is 5A, and its efficiency is 90 %, the output current would be -

[JEE-Main On line-2018]

(A) 20 A (B) 40 A (C) 45 A (D) 25 A

Q.14 The percentage errors in quantities P,Q,R and S are 0.5 %, 1%, 3% and 1.5 % respectively in the measurement of a physical quantity $A = \frac{P^3Q^2}{\sqrt{RS}}$. The maximum percentage error in the value of A will be -

[JEE-Main On line-2018]

(A) 8.5 % (B) 6.0 %

(C) 7.5 % (D) 6.5 %

Q.15 An oscillator of mass M is at rest in its equilibrium position in a potential $V = \frac{1}{2}k(x - X)^2$. A particle of mass m comes from right with speed u and collides completely inelastically with M and stickes to it. This process repeats every time the oscillator crosses its equilibrium position. The amplitude of oscillations after 13 collisions is : (M = 10, m = 5, u = 1, k = 1)

[JEE-Main On line-2018]

(A) $\frac{1}{2}$ (B) $\frac{1}{\sqrt{3}}$ (C) $\frac{2}{3}$ (D) $\sqrt{\frac{3}{5}}$

Q.16 Both the nucleus and the atom of some element are in their respective first excited states. They get de-excited by emitting photons of wavelengths λ_N, λ_A respectively. The ratio $\frac{\lambda_N}{\lambda_A}$ is closest to -

[JEE-Main On line-2018]

(A) 10^{-6} (B) 10 (C) 10^{-1} (D) 10^{-10}

Q.17 The end correction of a resonance column is 1 cm. If the shortest length resonating with the tuning is 10cm, the next resonating length should be -

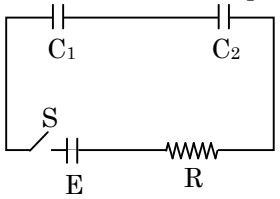
[JEE-Main On line-2018]

(A) 32 cm (B) 40 cm

(C) 28 cm (D) 36 cm

Q.18 In the following circuit, the switch S is closed at t = 0. The charge on the capacitor C_1 as a function of time will be given by $\left(C_{eq} = \frac{C_1C_2}{C_1 + C_2}\right)$

[JEE-Main On line-2018]



(A) $C_{eq}E[1 - \exp(-t/RC_{eq})]$

(B) $C_1E[1 - \exp(-tR/C_1)]$

(C) $C_2E[1 - ex(-t/RC_2)]$

(D) $C_{eq}E \exp(-t/RC_{eq})$

Q.19 The relative uncertainty in the period of a satellite orbiting around the earth is 10^{-2}. If the relative uncertainty in the radius of the orbit is negligible, the relative uncertainty in the mass of the earth I -

[JEE-Main On line-2018]

(A) 3×10^{-2} (B) 10^{-2}

(C) 2×10^{-2} (D) 6×10^{-2}

Q.20 Two moles of helium are mixed with n with moles of hydrogen. If $\frac{C_P}{C_V} = \frac{3}{2}$ for the mixture, then the value of n is -

[JEE-Main On line-2018]

(A) 3/2 (B) 2 (C) 1 (D) 3

Q.21 A particle executes simple harmonic motion and is located at x = a, b and c at times t_0 $2t_0$ and $3t_0$ respectively. The frequency of the oscillation is -

[JEE-Main On line-2018]

(A) $\frac{1}{2\pi t_0}\cos^{-1}\left(\frac{a+b}{2c}\right)$

(B) $\frac{1}{2\pi t_0}\cos^{-1}\left(\frac{a+b}{3c}\right)$

(C) $\frac{1}{2\pi t_0}\cos^{-1}\left(\frac{2a+3c}{b}\right)$

(D) $\frac{1}{2\pi t_0}\cos^{-1}\left(\frac{a+c}{2b}\right)$

Q.22 A galvanometer with its coil resistance 25Ω requires at current of 1mA for its full deflection. In order to construct an ammeter to read up to a current of 2A, the approximate value of the shunt resistance should be -

[JEE-Main On line-2018]

(A) $2.5 \times 10^{-2}\ \Omega$ (B) $1.25 \times 10^{-3}\Omega$

(C) $2.5 \times 10^{-3}\ \Omega$ (D) $1.25 \times 10^{-2}\ \Omega$

Q.23 Two sitar strings, A and B, playing the note 'Dha' are slightly out of tune and produce beats and frequency 5 Hz. The tension of the string B is slightly increased and the beat frequency is found to decrease by 3 Hz. If the frequency of A is 425 Hz, the original frequency of B is -

[JEE-Main On line-2018]

(A) 430 Hz (B) 428 Hz

(C) 422 Hz (D) 420 Hz

Q.24 Suppose that the angular velocity of rotation of earth is increased. Then, as a consequence.

[JEE-Main On line-2018]

(A) There will be no change in weight anywhere on the earth

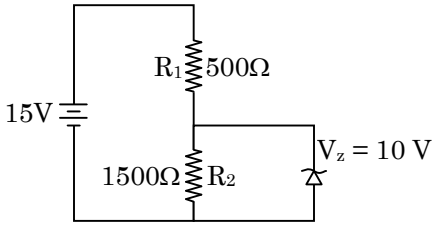
(B) Weight of the object, everywhere on the earth, will decrease

(C) Weight of the object, everywhere on the earth, will increase

(D) Except at poles, weight of the object on the earth will decrease

Q.25 In the given circuit, the current through Zener diode is -

[JEE-Main On line-2018]



(A) 2.5 mA (B) 3.3 mA

(C) 5.5 mA (D) 6.7 mA

Q.26 Two identical conducting spheres, A and B, carry equal charge. They are separated by a distance much larger than their diameter, and the force between them is F. A third identical conducting sphere, C, is uncharged. Sphere C is first touched to A, then to B, and then removed. As a result, the force between A and B would be equal to -

[JEE-Main On line-2018]

(A) $\frac{3F}{4}$ (B) $\frac{F}{2}$ (C) F (D) $\frac{3F}{8}$

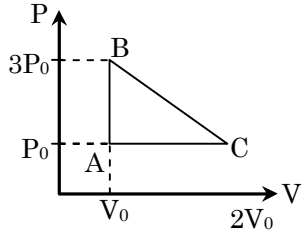
Q.27 A ray of light is incident at an angle of 60º on one face of a prism 30º. The emergent ray of light makes an angle of 30º with incident ray. The angle made by the emergent ray with second face of prism will be -

[JEE-Main On line-2018]

(A) 30º (B) 90º (C) 0º (D) 45º

Q.28 One mole of an ideal monoatomic gas is taken along the path ABCA as shown in the PV diagram. The maximum temperature attained by the gas along the path BC is given by

[JEE-Main On line-2018]



(A) $\frac{25}{8}\frac{P_0V_0}{R}$ (B) $\frac{25}{4}\frac{P_0V_0}{R}$

(C) $\frac{25}{16}\frac{P_0V_0}{R}$ (D) $\frac{5}{8}\frac{P_0V_0}{R}$

Q.29 A carrier wave of peak voltage 14 V is used for transmitting a message signal. The peak voltage of modulating signal given to achieve a modulation index of 80 % will be-

[JEE-Main On line-2018]

(A) 11.2 V (B) 7 V

(C) 22.4 V (D) 28 V

Q.30 A plane electromagnetic wave of wavelength λ has an intensity I. It is propagating along the positive Y-direction. The allowed expressions for the electric and magnetic fields are given by -

[JEE-Main On line-2018]

(A) $\vec{E}=\sqrt{\frac{I}{\varepsilon_0 C}}\cos\left[\frac{2\pi}{\lambda}(y-ct)\right]\hat{i}\,;\ \vec{B}=\frac{1}{c}E\hat{k}$

(B) $\vec{E}=\sqrt{\frac{I}{\varepsilon_0 C}}\cos\left[\frac{2\pi}{\lambda}(y-ct)\right]\hat{k}\,;\ \vec{B}=-\frac{1}{c}E\hat{i}$

(C) $\vec{E}=\sqrt{\frac{2I}{\varepsilon_0 C}}\cos\left[\frac{2\pi}{\lambda}(y-ct)\right]\hat{k}\,;\ \vec{B}=+\frac{1}{c}E\hat{i}$

(D) $\vec{E}=\sqrt{\frac{2I}{\varepsilon_0 C}}\cos\left[\frac{2\pi}{\lambda}(y+ct)\right]\hat{k}\,;\ \vec{B}=\frac{1}{c}E\hat{i}$

CHEMISTRY

Q.31 For standardizing NaOH solution, which of the following is used as a primary standard?

[JEE-Main On line-2018]

(A) Sodium tetraborate

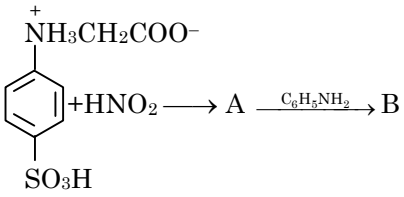
(B) Ferrous ammonium sulfate

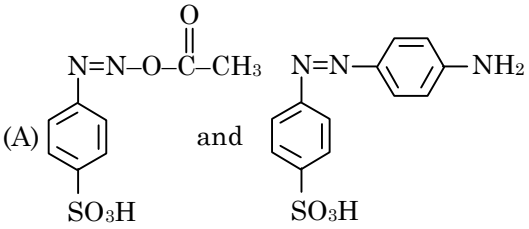
(C) Oxalic acid

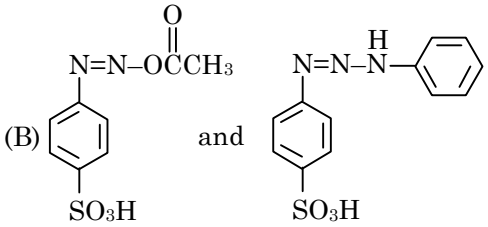
(D) dil. HCl

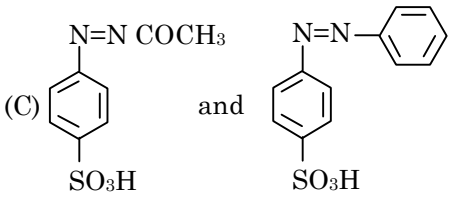
Q.32 Products A and B formed in the following reactions are respectively :

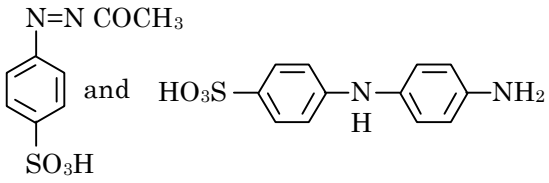
[JEE-Main On line-2018]



(A)  and

(B)  and

(C) 

(D) 

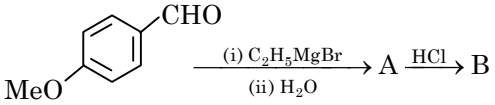
Q.33 When XO_2 is fused with an alkali metal hydroxide in presence of an oxidizing agent such as KNO_3, a dark green product is formed which disproportionate in acidic solution to afford a dark purple solution, X is -

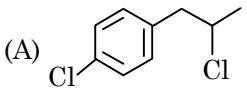
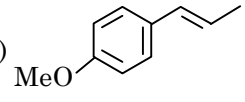
[JEE-Main On line-2018]

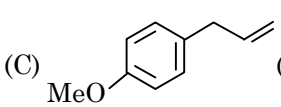
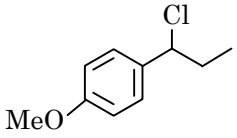
(A) Mn (B) Cr (C) V (D) Ti

Q.34 The major product B formed in the following reaction sequence is -

[JEE-Main On line-2018]



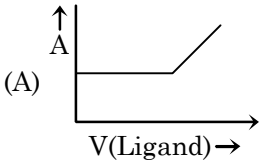
(A)  (B) 

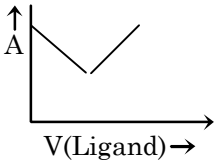
(C)  (D) 

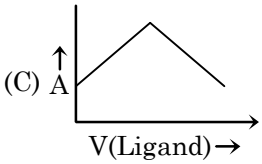
Q.35 In a complexometric tritration of metal ion with ligand M(Metal ion) + L(Ligand) → C(complex) end point is estimated spectrophotometrically (through light absorption). If 'M' and 'C' do not absorb

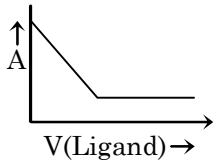
light and only 'L' absorbs, then the titration plot between absorbed light (A) versus volume of ligand 'L' (V) would look like :

[JEE-Main On line-2018]

(A) 

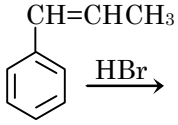
(B) 

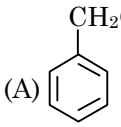
(C) 

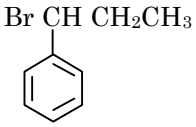
(D) 

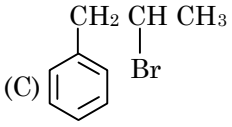
Q.36 The major product of the following reaction is :

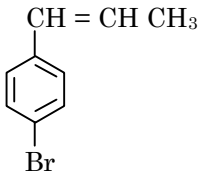
[JEE-Main On line-2018]



(A) 

(B) 

(C) 

(D) 

Q.37 Among the following the incorrect statement is -

[JEE-Main On line-2018]

(A) Cellulose and amylase have 1,4-glycosidic linkage
(B) Lactose contains β-D-galactose and B-D-glucose
(C) Maltose and lactose have 1,4-glycosidic linkage
(D) Sucrose and amylase have 1,2-glycosidic linkage

Q.38 In the extraction of copper from its sulphide ore, metal is finally obtained by the oxidation of cuprous sulphide with -

[JEE-Main On line-2018]

(A) SO_2 (B) Fe_2O_3
(C) Cu_2O (D) CO

Q.39 Among the oxides of nitrogen : N_2O_3, N_2O_4 and N_2O_5; the molecule (s) having nitrogen-nitrogen bond is/are -

[JEE-Main On line-2018]

(A) N_2O_3 and N_2O_4 (B) N_2O_4 and N_2O_5
(C) N_2O_3 and N_2O_5 (D) Only N_2O_5

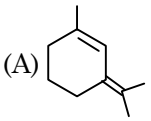
Q.40 Which of the following conversions involves change in both shape and hybridization ?

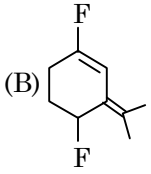
[JEE-Main On line-2018]

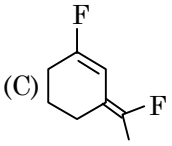
(A) $H_2O \rightarrow H_3O^+$ (B) $BF_3 \rightarrow BF_4^-$
(C) $CH_4 \rightarrow C_2H_6$ (D) $NH_3 \rightarrow NH_4^+$

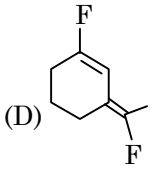
Q.41 The most polar compound among the following is -

[JEE-Main On line-2018]

(A) 

(B) 

(C) 

(D) 

Q.42 In Wilkinson's catalyst, the hybridization of central metal ion and its shape are respectively -

[JEE-Main On line-2018]

(A) sp^3d, trigonal bipyramidal
(B) d^2sp^3, octahedral
(C) dsp^2, square planar
(D) sp^3, tetrahedral

Q.43 At 320 K, a gas A_2 is 20 % dissociated to A(g). The standard free energy change at 320 K and 1 atm in J mol^{-1} is approximately :

(R = 8.314 JK^{-1} mol^{-1}; ln 2 = 0.693; ln 3 = 1.098)

[JEE-Main On line-2018]

(A) 1844 (B) 2068 (C) 4281 (D) 4763

Q.44 Which of the following complexes will show geometrical isomerism ?

[JEE-Main On line-2018]

(A) Potassium tris(oxalate)-chromate (III)
(B) Pentaaquachlorochromium (III) chloride

(C) Aquachlorobis (ethylenediamine) cobalt (II) chloride
(D) Potassium amminetrichloroplatinate (II)

Q.45 Which of the following is false ?
[JEE-Main On line-2018]
(A) Splitting of spectral lines in electrical field is called Stark effect.
(B) Frequency of emitted radiation from a black body goes from a lower wavelength to higher wavelength as the temperature increases
(C) Photon has momentum as well as wavelength
(D) Rydberg constant has unit of energy

Q.46 When 9.65 ampere current was passed for 1.0 hour into nitrobenzene in acidic medium, the amount of p-amino-phenol produced is …. .
[JEE-Main On line-2018]
(A) 109.0 g (B) 98.1 g
(C) 9.81 g (D) 10.9 g

Q.47 For which of the following processes, ΔS is negative ?
[JEE-Main On line-2018]
(A) C(diamond) $\rightarrow$ C(graphite)
(B) $N_2(g, 1atm) \rightarrow N_2(g. 5atm)$
(C) $N_2(g, 273K) \rightarrow N_2(g, 300K)$
(D) $H_2(g) \rightarrow 2H(g)$

Q.48 An unknown chlorohydrocarbon has 3.55 % of chlorine. If each molecule of the hydrocarbon has one chlorine atom only, chlorine atoms present in 1 g of chlorohydrocarbon are :
(Atomic wt. of Cl = 35.5u ; Avogadro constant = 6.023×10^{23} mol^{-1})
[JEE-Main On line-2018]
(A) 6.023×10^9 (B) 6.023×10^{23}
(C) 6.023×10^{21} (D) 6.023×10^{20}

Q.49 The incorrect statement out of the following is -
[JEE-Main On line-2018]
(A) Cu^{2+} ion gives chocolate coloured precipitate with potassium ferrocyanide solution
(B) Cu^{2+} and Ni^{2+} ions give black precipitate with H_2S in presence of HCl solution
(C) Ferric ion gives blood red colour with potassium thiocynate
(D) Cu^{2+} salts give red coloured borax bead test in reducing flame.

Q.50 The mass of non-volatile, non-electrolyte solute (molar mass = 50 g mol^{-1}) needed to be dissolved in 114 g octane to reduce its vapour pressure to 75 %, is …… .
[JEE-Main On line-2018]
(A) 37.5 g (B) 75 g
(C) 150 g (D) 50 g

Q.51 The incorrect geometry is represented by …… .
[JEE-Main On line-2018]
(A) NF_3-trigonal planar
(B) BF_3-trigonal planar
(C) AsF_5-trigonal bipyramidal
(D) H_2O-bent

Q.52 Assuming ideal gas behaviour, the ratio of density of ammonia to that of hydrogen chloride at same temperature and pressure is : (Atomic wt. of Cl = 35.5 u)
[JEE-Main On line-2018]
(A) 1.46 (B) 1.64
(C) 0.46 (D) 0.64

Q.53 The correct match between items of List-I and List-II is –
[JEE-Main On line-2018]

List-I	List-II
(A) Phenelzine	(p) Pyrimidine
(B) Chloroxylenol	(q) Furan
(C) Uracil	(r) Hydrazine
(D) Ranitidine	(s) Phenol

(A) (A)-(s); (B)-(r); (C)-(q); (D)-(p)
(B) (A)-(r); (B)-(s); (C)-(p); (D)-(q)
(C) (A)-(r); (B)-(s); (C)-(q); (D)-(p)
(D) (A)-(s); (B)-(r); (C)-(p); (D)-(q)

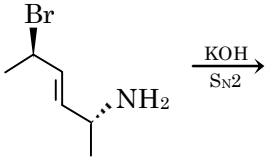
Q.54 The gas phase reaction $2NO_2(g) \rightarrow N_2O_4(g)$ is an exothermic reaction. The decomposition of N_2O_4, in equilibrium mixture of $NO_2(g)$ and $N_2O_4(g)$, can be increased by :
[JEE-Main On line-2018]

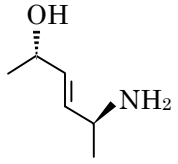
(A) addition of an inert gas at constant pressure
(B) lowering the temperature
(C) increasing the pressure
(D) addition of an inert gas at constant volume

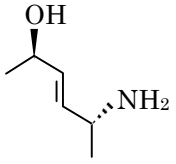
Q.55 Which one of the following is not a property of physical adsorption ?
[JEE-Main On line-2018]
(A) Higher the pressure, more the adsorption
(B) Greater the surface area, more the adsorption
(C) Lower the temperature, more the adsorption
(D) Unilayer adsorption occurs

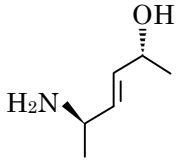
Q.56 A group 13 element 'X' reacts with chlorine gas to produce a compound XCl_3. XCl_3 is electron deficient and easily reacts with NH_3 to form $Cl_3X \leftarrow NH_3$ adduct, however, XCl_3 does not dimerize, X is -
[JEE-Main On line-2018]
(A) B (B) Al (B) ln (D) Ga

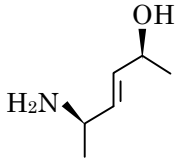
Q.57 The major product of the following reaction is :
[JEE-Main On line-2018]



(A) 

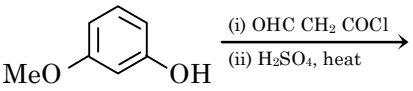
(B) 

(C) 

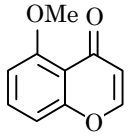
(D) 

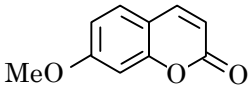
Q.58 If 50 % of a reaction occurs in 100 seconds and 75 % of the reaction occurs in 200 seconds, the order of this reaction is -
[JEE-Main On line-2018]
(A) 2 (B) 3 (C) zero (D) 1

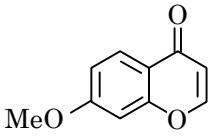
Q.59 The major product of the given reaction is :

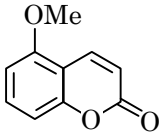


[JEE-Main On line-2018]

(A) 

(B) 

(C) 

(D) 

Q.60 Which of the following compounds will most readily be dehydrated to give alkene under acidic condition ?
[JEE-Main On line-2018]
(A) 4-Hydroxypentan-2-one
(B) 3-Hydroxypentan-2-one
(C) 1-Pentanol
(D) 2-Hydroxycyclopentanone

MATHEMATICS

Q.61 Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$ and $B = A^{20}$. Then the sum of the elements of the first column of B is ? **[JEE-Main On line-2018]**
(A) 211 (B) 210 (C) 231 (D) 251

Q.62 A man on the top of a vertical tower observes a car moving at a uniform speed towards the tower on a horizontal road. If it takes 18 min, for the angle of depression of the car to change from 30^o to 45^o, then after this, the time taken (in min) by the car to reach the foot of the tower, is -
[JEE-Main On line-2018]
(A) $9(1 + \sqrt{3})$ (B) $\frac{9}{2}(\sqrt{3} - 1)$
(C) $18(1 + \sqrt{3})$ (D) $18(\sqrt{3} - 1)$

Q.63 Let $\frac{1}{x_1}, \frac{1}{x_2}, \frac{1}{x_3}, \ldots, (x_i \neq 0 \text{ for } i = 1, 2, \ldots, n)$ be in A.P. such that $x_1 = 4$ and $x_{21} = 20$. If n is the least positive integer for which $x_n > 50$, then $\sum_{i=1}^{n}\left(\frac{1}{x_i}\right)$ is equal to -

[JEE-Main On line-2018]

(A) 3 (B) $\frac{13}{8}$ (C) $\frac{13}{4}$ (D) $\frac{1}{8}$

Q.64 The locus of the point of intersection of the lines, $\sqrt{2}x - y + 4\sqrt{2}k = 0$ and $\sqrt{2}kx + ky - 4\sqrt{2} = 0$ (k is any non-zero real parameter) is -

[JEE-Main On line-2018]

(A) A hyperbola with length of its transverse axis $8\sqrt{2}$

(B) An ellipse with length of its major axis $8\sqrt{2}$

(C) An ellipse whose eccentricity is $\frac{1}{\sqrt{3}}$

(D) A hyperbola whose eccentricity is $\sqrt{3}$

Q.65 The number of numbers between 2,000 and 5,000 that can be formed with the digits 0, 1, 2, 3, 4, (repetition of digits is not allowed) and are multiple of 3 is ?

[JEE-Main On line-2018]

(A) 30 (B) 48 (C) 24 (D) 36

Q.66 Let p, q and r be real numbers ($p \neq q$, $r \neq 0$), such that the roots of the equation $\frac{1}{x+p} + \frac{1}{x+q} = \frac{1}{r}$ are equal in magnitude but opposite in sign, then the sum of squares of these roots is equal to -

[JEE-Main On line-2018]

(A) $p^2 + q^2 + r^2$ (B) $p^2 + q^2$

(C) $2(p^2 + q^2)$ (D) $\frac{p^2+q^2}{2}$

Q.67 If $p \rightarrow (\sim p \vee \sim q)$ is false, then the truth values of p and q are respectively -

[JEE-Main On line-2018]

(A) T, F (B) F, F (C) F, T (D) T, T

Q.68 If an angle A of a ΔABC satisfies $5\cos A + 3 = 0$, then the roots of the quadratic equation, $9x^2 + 27x + 20 = 0$ are -

[JEE-Main On line-2018]

(A) sin A, sec A (B) sec A, tan A

(C) tan A, cos A (D) sec A, cot A

Q.69 If $x = \sqrt{2^{\text{cosec}^{-1}t}}$ and $y = \sqrt{2^{\sec^{-1}t}}$ ($|t| \geq 1$), then $\frac{dy}{dx}$ is equal to -

[JEE-Main On line-2018]

(A) $\frac{y}{x}$ (B) $-\frac{y}{x}$ (C) $-\frac{x}{y}$ (D) $\frac{x}{y}$

Q.70 The coefficient of x^2 in the expansion of the product $(2 - x^2) \cdot ((1 + 2x + 3x^2)^6 + (1 - 4x^2)^6)$ is - **[JEE-Main On line-2018]**

(A) 106 (B) 107 (C) 155 (D) 108

Q.71 The sum of the intercepts on the coordinate axes of the plane passing through the point (–2, –2, 2) and containing the line joining the points (1, –1, 2) and (1, 1, 1) is -

[JEE-Main On line-2018]

(A) 12 (B) –8 (C) –4 (D) 4

Q.72 The differential equation representing the family of ellipse having foci either on the x-axis or on the y-axis centre at the origin and passing through the point (0, 3) is -

[JEE-Main On line-2018]

(A) $xyy' + y^2 - 9 = 0$

(B) $x + yy'' = 0$

(C) $xyy'' + x(y')^2 - yy' = 0$

(D) $xyy' - y^2 + 9 = 0$

Q.73 Let A, B and C be three events, which are pair-wise independence and $\overline{E}$ denotes the complement of an event E. If $P(A \cap B \cap C) = 0$ and $P(C) > 0$, then $P[(\overline{A} \cap \overline{B}) | C]$ is equal to - **[JEE-Main On line-2018]**

(A) $P(A) + P(\overline{B})$ (B) $P(\overline{A}) - P(\overline{B})$

(C) $P(\overline{A}) - P(B)$ (D) $P(\overline{A}) + P(\overline{B})$

Q.74 The number of values of k for which the system of linear equations, $(k + 2)x + 10y = k$, $kx + (k + 3)y = k - 1$ has no solution, is- **[JEE-Main On line-2018]**

(A) Infinitely many (B) 3

(C) 1 (D) 2

Q.75 If the angle between the lines, $\frac{x}{2} = \frac{y}{2} = \frac{z}{1}$ and $\frac{5-x}{-2} = \frac{7y-14}{p} = \frac{z-3}{4}$ is $\cos^{-1}\left(\frac{2}{3}\right)$, then p is equal to - **[JEE-Main On line-2018]**

(A) $-\frac{7}{4}$ (B) $\frac{2}{7}$ (C) $-\frac{4}{7}$ (D) $\frac{7}{2}$

Q.76 Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{c} = \hat{j} - \hat{k}$ and a vector $\vec{b}$ be such that $\vec{a} \times \vec{b} = \vec{c}$ and $\vec{a} \cdot \vec{b} = 3$. Then $|\vec{b}|$ equals ? **[JEE-Main On line-2018]**

(A) $\sqrt{\frac{11}{3}}$ (B) $\frac{\sqrt{11}}{3}$ (C) $\frac{11}{\sqrt{3}}$ (D) $\frac{11}{3}$

Q.77 The least positive integer n for which $\left(\frac{1+i\sqrt{3}}{1-i\sqrt{3}}\right)^n = 1$, is ? **[JEE-Main On line-2018]**

(A) 2 (B) 6 (C) 5 (D) 3

Q.78 $\lim\limits_{x\to 0} \frac{(27+x)^{\frac{1}{3}} - 3}{9 - (27+x)^{\frac{2}{3}}}$ equals - **[JEE-Main On line-2018]**

(A) $-\frac{1}{3}$ (B) $\frac{1}{6}$ (C) $-\frac{1}{6}$ (D) $\frac{1}{3}$

Q.79 If $f(x) = \int_0^x t(\sin x - \sin t)\,dt$ then **[JEE-Main On line-2018]**

(A) $f'''(x) + f'(x) = \cos x - 2x \sin x$

(B) $f'''(x) + f''(x) - f'(x) = \cos x$

(C) $f'''(x) - f''(x) = \cos x - 2x \sin x$

(D) $f'''(x) + f''(x) = \sin x$

Q.80 Let M and m be respectively the absolute maximum and the absolute minimum values of the function, $f(x) = 2x^3 - 9x^2 + 12x + 5$ in the interval [0, 3]. Then M – m is equal to - **[JEE-Main On line-2018]**

(A) 1 (B) 5 (C) 4 (D) 9

Q.81 If a circle C, whose radius is 3, touches externally the circle, $x^2 + y^2 + 2x - 4y - 4 = 0$ at the point (2, 2), then the length of the intercept cut by circle C, on the x-axis is equal to - **[JEE-Main On line-2018]**

(A) $\sqrt{5}$ (B) $2\sqrt{3}$ (C) $3\sqrt{2}$ (D) $2\sqrt{5}$

Q.82 If the area of the region bounded by the curves, $y = x^2$, $y = \frac{1}{x}$ and the lines $y = 0$ and $x = t$ $(t > 1)$ is 1 sq. unit, then t is equal to - **[JEE-Main On line-2018]**

(A) $\frac{4}{3}$ (B) $e^{2/3}$ (C) $\frac{3}{2}$ (D) $e^{3/2}$

Q.83 Let P be a point on the parabola, $x^2 = 4y$. If the distance of P from the centre of the circle $x^2 + y^2 + 6x + 8 = 0$ is minimum, then the equation of the tangent to the parabola at P, is - **[JEE-Main On line-2018]**

(A) $x + 4y - 2 = 0$ (B) $x + 2y = 0$

(C) $x + y + 1 = 0$ (D) $x - y + 3 = 0$

Q.84 If the function f defined as $f(x) = \frac{1}{x} - \frac{k-1}{e^{2x}-1}$, $x \neq 0$, is continuous at $x = 0$, then the ordered pair (k, f(0)) is equal to ? **[JEE-Main On line-2018]**

(A) (3, 1) (B) (3, 2)

(C) $\left(\frac{1}{3}, 2\right)$ (D) (2, 1)

Q.85 The mean and the standard deviation (s.d.) of five observations are 9 and 0, respectively. If one of the observations is changed such that the mean of the new set of five observations becomes 10, then their s.d. is ?

[JEE-Main On line-2018]

(A) 0 (B) 4 (C) 2 (D) 1

Q.86 Let N denote the set of all natural numbers. Define two binary relations on N as $R_1 = \{(x, y) \in N \times N: 2x + y = 10\}$ and $R_2 = \{(x, y) \in N \times N: x + 2y = 10\}$. Then -

[JEE-Main On line-2018]

(1) Both R_1 and R_2 are transitive relations

(2) Both R_1 and R_2 are symmetric relations

(3) Range of R_2 is {1, 2, 3, 4}

(4) Range of R_1 is {2, 4, 8}

Q.87 The sum of the first 20 terms of the series $1 + \frac{3}{2} + \frac{7}{4} + \frac{15}{8} + \frac{31}{16} + \ldots$ is ?

[JEE-Main On line-2018]

(A) $38 + \frac{1}{2^{20}}$ (B) $39 + \frac{1}{2^{19}}$

(C) $39 + \frac{1}{2^{20}}$ (D) $38 + \frac{1}{2^{19}}$

Q.88 If the length of the latus rectum of an ellipse is 4 units and the distance between a focus and its nearest vertex on the major axis is $\frac{3}{2}$ units, then its eccentricity is ?

[JEE-Main On line-2018]

(A) $\frac{1}{2}$ (B) $\frac{2}{3}$ (C) $\frac{1}{9}$ (D) $\frac{1}{3}$

Q.89 Two different families A and B are blessed with equal number of children. There are 3 tickets to be distributed amongst the children of these families so that no child gets more than one ticket. If the probability that all the tickets go to the children of the family B is $\frac{1}{12}$, then the number of children in each family is ?

[JEE-Main On line-2018]

(A) 4 (B) 6 (C) 3 (D) 5

Q.90 If $\int \frac{\tan x}{1 + \tan x + \tan^2 x}\,dx$

$= x - \frac{K}{\sqrt{A}} \tan^{-1}\left(\frac{K\tan x + 1}{\sqrt{A}}\right) + C,$

(C is a constant of integration), then the ordered pair (K, A) is equal to -

[JEE-Main On line-2018]

(A) (2, 3) (B) (2, 1) (C) (–2, 1) (D) (–2, 3)

ANSWERS

1. (1) **2.** (3) **3.** (1) **4.** (2) **5.** (2) **6.** (2) **7.** (1) **8.** (3) **9.** (4) **10.** (4) **11.** (1) **12.** (1) **13.** (3) **14.** (4)

15. (2) **16.** (1) **17.** (1) **18.** (1) **19.** (3) **20.** (2) **21.** (4) **22.** (4) **23.** (4) **24.** (4) **25.** (2) **26.** (4) **27.** (3) **28.** (1)

29. (1) **30.** (3) **31.** (3) **32.** (1) **33.** (1) **34.** (4) **35.** (1) **36.** (2) **37.** (4) **38.** (3) **39.** (1) **40.** (2) **41.** (3) **42.**(3)

43. (3) **44.** (3) **45.** (2,4) **46.** (3) **47.** (2) **48.** (4) **49.** (2) **50.** (3) **51.** (1) **52.** (3) **53.** (2) **54.** (1) **55.** (4) **56.** (1)

57. (3) **58.** (4) **59.** (2) **60.** (1) **61.** (3) **62.** (1) **63.** (3) **64.** (1) **65.** (1) **66.** (2) **67.** (4) **68.** (2) **69.** (2) **70.**(1)

71. (3) **72.** (3) **73.** (3) **74.** (3) **75.** (4) **76.** (1) **77.** (4) **78.** (3) **79.** (1) **80.** (1) **81.** (4) **82.** (2) **83.** (3) **84.** (1)

85. (3) **86.** (3) **87.** (4) **88.** (4) **89.** (4) **90.** (1)

Hints & Solutions

PHYSICS

1.[1] Magnetic moment,

$$\mu = 1A = \frac{qv}{2\pi r}(\pi r^2)$$

or, $$\mu = \frac{qv\omega}{2\pi r}(\pi r^2) = \frac{1}{2} qr^2 \omega$$

2.[3] Resistance after temperature increases by 500°C i.e., $R_t = \frac{V}{I} = \frac{200}{2} = 110\Omega$

$R_0 = 100$ (given) temperature coefficient of resistance, $\alpha = ?$

Using $R_t = R_0(1 + \alpha t)$

$$110 = 100\,(1 + \alpha 500)$$

$$\alpha = \frac{10}{100 \times 500}$$

or, $\alpha = 2 \times 10^{-4}\ ^\circ C^{-1}$

3.[1] If $\vec{C} = a\hat{i} + b\hat{j}$

then $\vec{A}.\vec{C} = \vec{A}.\vec{B}$

$a + b = 1$ ...(i)

$\vec{B}.\vec{C} = \vec{A}.\vec{B}$

$2a - b = 1$ ...(ii)

Solving equation (i) and (ii) we get

$$a = \frac{1}{3}, b = \frac{2}{3}$$

$\therefore$ Magnitude of coplanar vector,

$$|\vec{C}| = \sqrt{\frac{1}{9} + \frac{4}{9}} = \sqrt{\frac{5}{9}}$$

4.[2] As the particles moving in circular orbits, So

$$\frac{mv^2}{r} = \frac{16}{r} + r^3$$

Kinetic energy,

$$KE_0 = \frac{1}{2} mv^2 = \frac{1}{2}[16 + r^4]$$

For first particle, $r = 1$, $K_1 = \frac{1}{2}(16 + 1)$

Similarly, for second particle, $r = 4$,

$$K_2 = \frac{1}{2}(16 + 256)$$

$\therefore$ $$\frac{K_1}{K_2} = \frac{\frac{16+1}{2}}{\frac{16+256}{2}} = \frac{17}{272} = 6 \times 10^{-2}$$

5.[2] Given : $N_1 = 2N_2$

Activity of radioactive substance $= \lambda N$

Half life period $t = \frac{\ln 2}{\lambda}$ or, $\lambda = \frac{\ln 2}{t}$

$$L_1N_1 = \frac{\ln 2}{t_1} \times N_1 = 5\mu C_i \quad ...(i)$$

$$\lambda_2N_2 = \frac{\ln 2}{t_2} \times N_2 = 10\mu C_i \quad ...(ii)$$

Dividing equation (ii) by (i)

$$\frac{t_2}{t_1} \times \frac{N_1}{N_2} = \frac{1}{2}$$

$$\frac{t_2}{t_1} = \frac{1}{4} \Rightarrow t_1 = 4t_2$$

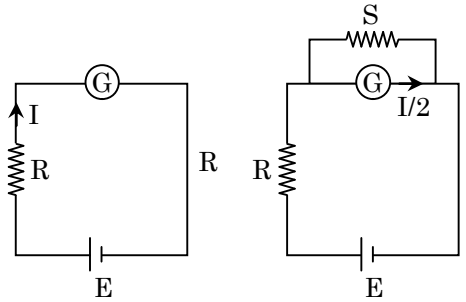
i.e., Half life of S_1 is four times of sample S_2. Hence 5 years and 20 years.

6.[2] Figure of merit of a galvanometer is the current required to produce a deflection of one division in the galvanometer i.e., figure of merit $= \frac{1}{\theta}$

$$I = \frac{\varepsilon}{R+G} \qquad G = \frac{1}{9}\,k\Omega$$

$$\frac{1}{2} = \frac{\varepsilon}{R + \frac{GS}{G+S}} \times \frac{S}{S+G} \Rightarrow \frac{1}{2} = \frac{\varepsilon S}{R(S+G)+GS}$$

$$S = \frac{RG \times \frac{1}{2}}{\varepsilon - \frac{(R+G)I}{2}}$$



$$S = \frac{11\times10^3 \times \frac{1}{2} \times 10^2 \times 270 \times 10^{-6}}{6 - \left(\frac{6}{2}\right)} = 110W$$

7.[1] From question, $v = a\sqrt{s} = \frac{ds}{dt}$

or, $$2\sqrt{2} = at \Rightarrow a = \frac{a^2t^2}{4}$$

$$F = m \times \frac{a^2}{2}$$

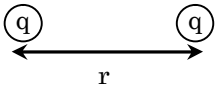
Work done $= \frac{ma^2}{2} \times \frac{a^2t^2}{4} = \frac{1}{8}\,ma^4t^2$

8.[3] As we know, moment of inertia of a disc about an axis passing through C.G. and perpendicular to its plane.

$$I_z = \frac{mR^2}{2}$$

Moment of inertia of a disc about a tangential axis perpendicular to its own plane,

$$I_Z' = \frac{3}{2}\,mR^2$$



$$\therefore \quad I_Z / I_Z' = \frac{mR^2}{2} \Big/ \frac{3mR^2}{2} = 1/3$$

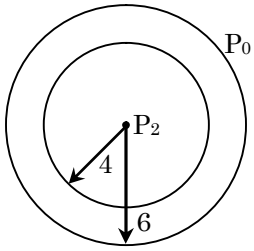
9.[4] Clearly from figure,

$$P_2 = P_0 + \frac{4T}{6} + \frac{4T}{4}$$

or, $$P_2 - P_0 = \frac{4T}{6} + \frac{4T}{4} \qquad \ldots(i)$$

Let r be the radius of bubble with pressure difference $P_2 - P_0$ so,

$$P_2 - P_0 = \frac{4T}{4} \qquad \ldots(ii)$$



From eqn (i) and (ii),

$$\frac{4T}{r} = \frac{4T}{6} + \frac{4T}{4}$$

$$\Rightarrow \quad \frac{1}{r} = \frac{1}{6} + \frac{1}{4}$$

$$\Rightarrow \quad r = 2.4 \text{ cm}$$

10.[4] de-Broglie wavelength, $\lambda = \frac{h}{P}$

$$\frac{\lambda_B}{\lambda_G} = \frac{P_G}{P_B} = \frac{mv_G}{mv_B}$$

Speed of electron $v \propto \frac{z}{n}$

So, $$\frac{\lambda_B}{\lambda_G} = \frac{n_B}{n_G} = \frac{3}{1}$$

$$\Rightarrow \quad \lambda_B = 3\lambda_G$$

11.[1] Induced emf in a coil,

$$e = -\frac{d\phi}{dt} = NBA\omega \sin \omega t$$

Also, $e = e_0 \sin \omega t$

$\therefore$ Maximum emf induced, $e_0 = nBA\omega$

12.[1] Polariser A and B have same alignment of transmission axis.

Lets assume polarizer C is introduced at θ angle.

$$\frac{1}{2}\cos^2\theta \times \cos^2\theta = \frac{1}{3}$$

or, $$\cos^4\theta = \frac{2}{3} \Rightarrow \cos\theta = \left(\frac{2}{3}\right)^{1/4}$$

13.[3] Given :

$$V_P = 2300\ V,\ V_s = 230\ V,\ I_P = 5A,$$
$$\eta = 90\% = 0.9$$

Efficiency $$\eta = 0.9 = \frac{P_s}{P_P} \Rightarrow P_s = 0.9\ P_P$$

$$V_sI_s = 0.9 \times V_PI_P\ (\because\ P = VI)$$

$$I_s = \frac{0.9 \times 2300 \times 5}{230} = 45A$$

14.[4] Maximum percentage error in A

= 3(% error in P) + 2(% error in Q)

$$+ \frac{1}{2}\ (\%\ \text{error in R}) + 1(\%\ \text{error in S})$$

$$= 3 \times 0.5 + 2 \times 1 + \frac{1}{2} \times 3 + 1 \times 5\ 1.5$$

$$= 1.5 + 2 + 1.5 + 1.5$$

$$= 6.5\ \%$$

15.[2] In first collision mu momentum will be imparted to system, in second collision when momentum of (M + m) is in opposite direction mu momentum of particle will make its momentum zero.

On 13th collision,

$$mu = (M + 13m)\ v$$

$$\Rightarrow \quad v = \frac{mu}{M + 13m} = \frac{u}{15}$$

$$v = \omega A$$

$$\Rightarrow \quad \frac{u}{15} = \sqrt{\frac{K}{M - 13m}} \times A$$

Putting value of M, m, u and K we get amplitude

$$A = \frac{1}{15}\sqrt{\frac{75}{1}} = \frac{1}{\sqrt{3}}$$

16.[1] Energy $$E = \frac{hc}{\lambda};$$

$$= \frac{\lambda_N}{\lambda_A} = \frac{E_A}{E_N}$$

Where E_A and E_N are energies of photons from atom and nucleus respectively. E_N is of the order of MeV = 10^6 eV and E_A in few eV.

So $$\frac{\lambda_N}{\lambda_A} = 10\text{–}6$$

17.[1] For first resonance, $\frac{\lambda}{4} = \ell_1 + e = 11$ cm

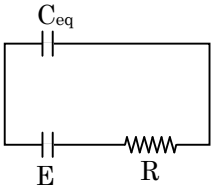
($\because$ end correction e = 1 cm given)

For second resonance, $\frac{3\lambda}{4} = \ell_2 + e$

$\Rightarrow \quad \ell_2 = 3 \times 11 - 1 = 32$ cm

18.[1] During charging charge on the capacitor increase with time. Charge on the capacitor C_1 as a function of time, $Q = Q_0(1 - e^{-t/RC})$

$Q = C_{eq}\ E\ [1 - e^{-t/RC_{eq}}]$



($\because\ Q_0 = C_{eq}E$)

Both capacitor will have charge as they are connected in series.

19.[3] From Kepler's law, time period of a satellite,

$$T = 2\pi\sqrt{\frac{r^3}{Gm}}$$

$$T^2 = \frac{4\pi^2}{GM}r^3$$

Relative uncertainty in the mass of the earth $$\left|\frac{\Delta M}{M}\right| = 2\frac{\Delta T}{T} = 2 \times 10^{-2}$$

($\because$ 4π & G constant and relative uncertainty in radius $\frac{\Delta r}{r}$ negligible)

20.[2] Using formula,

$$\gamma_{mixtuer} = \left(\frac{C_p}{C_v}\right)_{mix} = \frac{\frac{n_1\gamma_1}{\gamma_1 - 1} + \frac{n_2\gamma_2}{\gamma_2 - 1}}{\frac{n_1}{\gamma_1 - 1} + \frac{n_2}{\gamma_2 - 1}}$$

Putting the value of $n_1 = 2$, $n_2 = n$,

$$\left(\frac{C_p}{C_v}\right)_{mix} = \frac{3}{2}$$

$\gamma_1 = \frac{5}{3}$, $\gamma_2 = \frac{7}{5}$ and solving we get, $n = 2$

21.[4] Using $y = A \sin \omega t$

$a = A \sin \omega t_0$

$b = A \sin 2wt_0$

$c = A \sin 3\omega t_0$

$a + c = A [\sin \omega t_0 + \sin 3\omega t_0]$

$= 2A \sin 2\omega t_0 \cos \omega t_0$

$$\frac{a+c}{b} = 2\cos \omega t_0$$

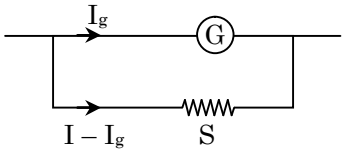
$$\Rightarrow \quad \omega = \frac{1}{t_0} \cos^{-1}\left(\frac{a+c}{2b}\right)$$

$$\Rightarrow \quad f = \frac{1}{2\pi t_0} \cos^{-1}\left(\frac{a+c}{2b}\right)$$

22.[4] According to question, current through galvanometer, $I_g = 1mA$

Current through shut $(I - I_g) = 2A$

Galvanometer resistance $R_g = 25\Omega$



Resistance of shunt, $S = ?$

$I_g R_g = (1 - I_g)S$

$$\Rightarrow \quad S = \frac{10^{-3} \times 25}{2}$$

$S \cong 1.25 \times 10^{-2}\ \Omega$

23.[4] $n_A = 425$ Hz, $n_B = ?$

Beat frequency $x = 5$Hz which is decreasing $(5 \rightarrow 3)$ after increasing the tension of the string B.

Also tension of string B increasing so

$n_B \uparrow (\because \quad n \propto \sqrt{T})$

Hence, $n_A - n_B \uparrow = x \uparrow \longrightarrow$ correct

$n_B \uparrow - n_A = x \downarrow \longrightarrow$ incorrect

$\therefore \quad n_B = n_A - x = 425 - 5 = 420$ Hz

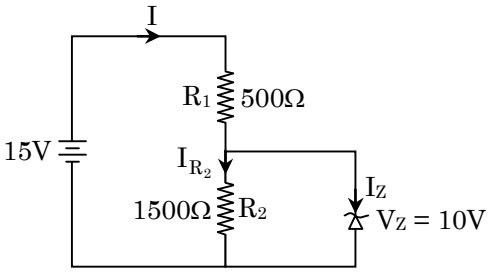
24.[4] With rotation of earth or latitude, acceleration due to gravity vary as

$$g' = g - \omega^2 R \cos^2 \phi$$

Where ϕ is latitude, there will be no change in gravity at poles as $\phi = 90^o$

At all other points as to increases g′ will decrease hence, weight, $W = mg$ decreases.

25.[2]



The voltage drop across R_2 is

$$V_{R_2} = V_Z = 10\ V$$

The current through R_2 is

$$I_{R_2} = \frac{V_{R_2}}{R_2} = \frac{10V}{1500\Omega} = 0.667 \times 10^{-2} A$$

$= 6.67 \times 10^{-3}$ A $= 6.67$ mA

The voltage drop across R_1 is

$$V_{R_1} = 15V - V_{R_2} = 15V - 10V = 5V$$

The current through R_1 is

$$I_{R_1} = \frac{V_{R_1}}{R_1} = \frac{5V}{500\Omega} = 10^{-2}\ A$$

$= 10 \times 10^{-3}$ A $= 10$ mA

The current through the zener diode is

$I_Z = I_{R_1} - I_{R_2} = (10 - 6.67)$ mA

$= 3.3$ mA

26.[4] Spheres A and B carry equal charge say 'q'

$\therefore$ Force between them, $F = \frac{kqq}{r^2}$

When A and C are touched, charge on both

$$q_A = q_C = \frac{q}{2}$$

Then when B and C are touched, charge on B

$$q_B = \frac{\frac{q}{2} + q}{2} = \frac{3q}{4}$$

Now, the force between charge q_A and q_B

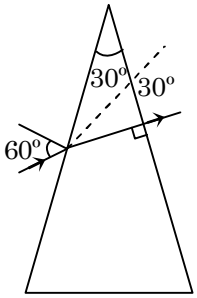
$$f' = \frac{kq_Aq_B}{r^2} = \frac{k \times \frac{q}{2} \times \frac{3q}{4}}{r^2} = \frac{3}{8}\frac{kq^2}{r^2} = \frac{3}{8}\ F$$

27.[3] Angle of prism, A = 30°, i = 60°, angle of deviation, δ = 30° Using formula,

$$\delta = i + e - A$$

$$\Rightarrow \quad e = \delta + A - i$$

$$= 30^\circ + 30^\circ - 60^\circ = 0^\circ$$



∴ Emergent ray will be perpendicular to the face.

So it will make angle 90° with the face through which it emerges.

28.[1] Equation of the BC

$$P = P_0 - \frac{2P_0}{V_0}(V - 2V_0)$$

Using PV = nRT

$$\text{Temperature, } T = \frac{P_0V - \frac{2P_0V^2}{V_0} + 4P_0V}{1 \times R}$$

(∵ n = 1 mole given)

$$T = \frac{P_0}{R}\left[5V - \frac{2V^2}{V_0}\right]$$

$$\frac{dT}{dV} = 0$$

$$\Rightarrow \quad 5 - \frac{4V}{V_0} = 0$$

$$\Rightarrow \quad V = \frac{5}{4}V_0$$

$$T = \frac{P_0}{R}\left[5 \times \frac{5V_0}{4} - \frac{2}{V_0} \times \frac{25}{16}V_0^2\right] = \frac{25}{8}\frac{P_0V_0}{R}$$

29.[1] Given : Modulation index m = 80 % = 0.8

$$E_c = 14V,\ E_m = ?$$

Using, $m = \frac{E_m}{E_c}$

$$\Rightarrow \quad Em = m \times E_c$$

$$= 0.8 \times 14$$

$$= 11.2\ V$$

30.[3] If E_0 is magnitude of electric field then

$$\frac{1}{2}\varepsilon_0E_0^2 \times C = I \Rightarrow E_0 = \sqrt{\frac{2I}{C\varepsilon_0}}$$

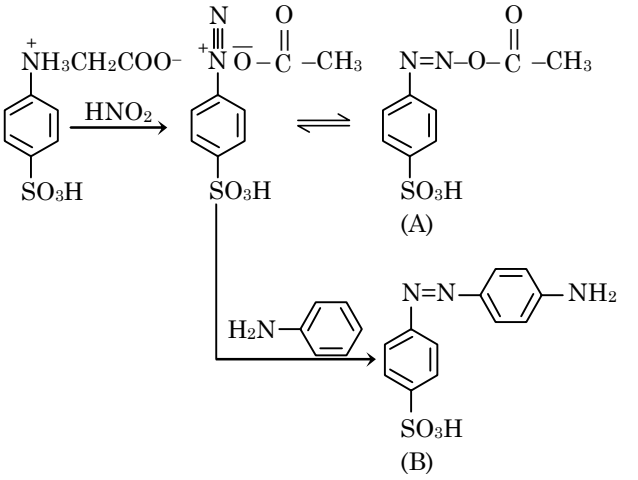
$$B_0 = \frac{E_0}{C}$$

Direction of $\vec{E} \times \vec{B}$ will be along + $\hat{j}$.

CHEMISTRY

31.[3] Oxalic acid is used as a primary standard for NaOH standardizing.

32.[1]

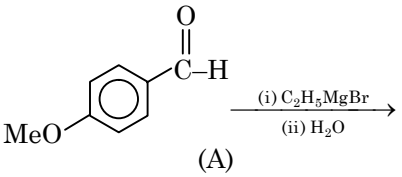


33.[1]

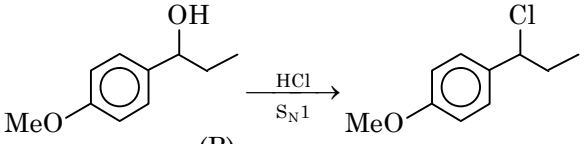
$$MnO_2 + KOH + KNO_3 \rightarrow \underset{\text{(dark green)}}{K_2MnO_4} + KNO_2 + H_2O$$

$$MnO_4^{2-} + 4H^+ \xrightarrow{\text{disproportionattion}} \underset{\text{(dark purple)}}{2MnO_4^-} + MnO_2 + 2H_2O$$

34.[4]



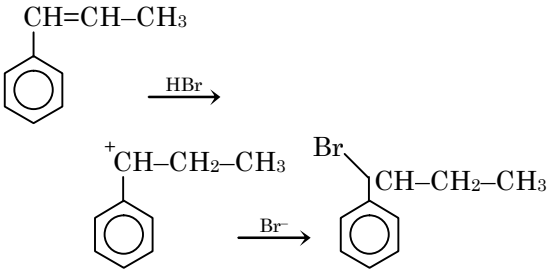
(A)



(B)

35.[1] Initially ligand is consumed by metal due to formation of complex. So absorbed light (A) remain constant, after complex formation is completed, extra volume of ligand solution increases ligand concentration and also increases absorbed light.

36.[2]



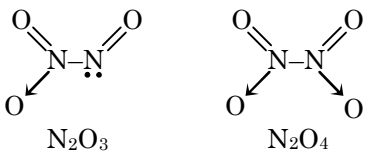
Benzal carbocation

37.[4] In amylase 1,4-α-glycosidic linkage is present.

38.[3] $Cu_2S + 2Cu_2O \longrightarrow 6Cu + SO_2$

It is an example of auto-reduction, CuO_2 is reduced to copper metal.

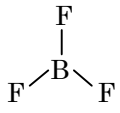
39.[1]



N_2O_3 N_2O_4

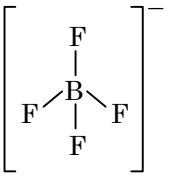
40.[2]

$BF_3 \longrightarrow BF_4^-$



sp^2

Trigonal planar



sp^3

Tetrahedral

41.[3] Among the substituents attached to the given compounds, fluorine has maximum electronegatiyity. So it will push electron pair towards itself. In option (2), the two F groups are attached opposite to each other thus net dipole moment will cancel each other and reduce its polarity. In option (4), the F groups are attached in slightly opposite direction thus this also decreases its polarity. But in option (3), the compound has thee two F groups along same direction. Thus net dipole moment will increase in this direction thus it will have maximum polarity. Thus the compound in option (3) has maximum polarity.

42.[3] Wilkinson catalyst is $[RhCl(PPh_3)_3]$

43.[3] In the reaction $A_2 \rightleftharpoons 2A$

Initially, Let $[A_2] = 1M$ and $[A] = 0$ M

After 20 % dissociation, 80 % of A_2 remains.

$$[A_2] = 1 \times \frac{80}{100} = 0.8 \text{ M}$$

20 % of 1M is

$$1 \times \frac{20}{100} = 0.2 \text{ . } [A] = 2 \times 0.2 = 0.4 \text{ M}$$

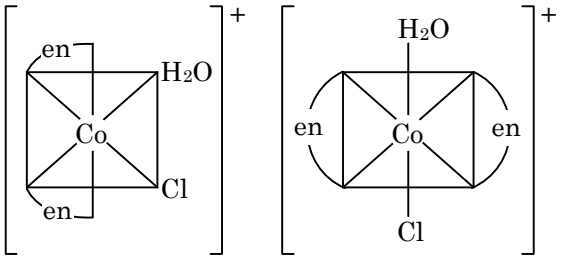
The equilibrium constant

$$K = \frac{[A]^2}{[A_2]} \text{ ; } K = \frac{[0.4]^2}{[0.8]} = 0.2$$

$\Delta G^o = RT \ln K = -8.314 \text{ JK}^{-1} \text{ mol}^{-1}$

$\times 320 \text{ K} \times \ln 0.2 = 4281 \text{ J/mol}$

44.[3] $[Co(H_2O)Cl(en)_2]Cl$



(Geometrical isomers)

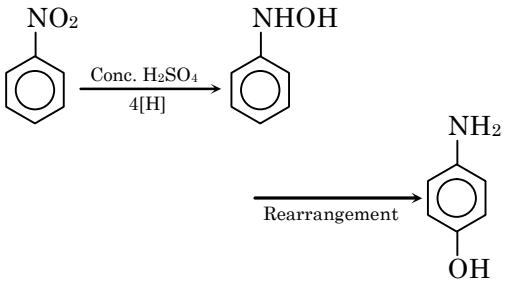
45.[2] and (4**]**

When temperature is increased, black body emit high energy radiation, from higher wavelength to lower wavelength. Rydberg constant has unit of length i.e. cm^{-1}

46.[3] 9.65 ampere current was passed for 1.0 hour (3600 seconds)

Number of moles of electrons passed

$$= \frac{I(A) \times t(s)}{96500} = \frac{9.65A \times 3600s}{96500} = 0.36 \text{ moles}$$



$C_6H_5NO_2 + 4e^- + 4H^+ \longrightarrow$ p-Aminophenol + H_2O

4 moles of electrons will reduce 1 mole of nitrobenzene to p-aminophenol.

0.36 moles of electrons will reduce $\frac{0.36}{4}$ = 0.09 moles of nitrobenzene to p-aminophenol.

p-aminophenol molar mass = 109.14 g/mol

Mass of p-aminophenol obtained = 109.14 g/mol × 0.09 mol = 9.81 g

47.[2] (1) When diamond is converted into graphite when it is heated to 1500°C and entropy is increased so $\Delta S > 0$.

(2) When pressure increases then molecules of gas will come closer and intermolecular distance decreases so entropy will also decrease and $\Delta S < 0$.

(3) When we increase the temperature of a gas then randomness is increased due to kinetic energy gained by molecules. So, $\Delta S > 0$.

(4) H_2 molecule is converted into atoms, the no. of particles increases. Thus entropy will increase. So $\Delta S > 0$.

48.[4] Given percentage of chlorine in an hydrocarbon = 3.55 % i.e.,

100 g of chlorohydrocarbon has 3.55 g of chlorine.

1 g of chlorohydrocarbon will have

$$\frac{3.55}{100} = 0.0355 \text{ g of chlorine.}$$

Atomic wt. of Cl = 35.5 g/mol

Number of moles of Cl

$$= \frac{0.0355 \text{ g}}{35.5 \text{ g/mol}} = 0.001 \text{ mole}$$

Number of atoms of

$$Cl = 0.001 \text{ mole} = 6.023 \times 10^{23} \text{ mol}^{-1}$$
$$= 6.023 \times 10^{20}$$

49.[2] Due to common ion effect insufficient concentration of sulphide ion (S^{2-}) is produced. Thus the ionic product of Ni^{2+} and S^{2-} ions is less than the K_{sp} of NiS, so NiS will not precipitate out.

50.[3] Molar mass of octane = 114 g/mol

From the lowering of vapour pressure we have

$$\frac{\Delta P}{P} = \frac{\frac{W_2}{M_2}}{\frac{W_2}{M_2} + \frac{W_1}{M_1}}$$

$$\frac{75}{100} = \frac{\frac{W_2}{50 \text{g/mol}}}{\frac{W_2}{50 \text{g/mol}} + \frac{114 \text{g}}{114 \text{g/mol}}}$$

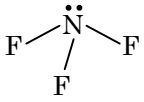
$$0.75 = \frac{\frac{W_2}{50}}{\frac{W_2}{50} + 1}$$

$$\frac{W_2}{50} + 1 = \frac{W_2}{50 \times 0.75}$$

$$W_2 = 150 \text{ g}$$

Note : W_2 and M_2 are mass and molar mass of solute and W_1 and M_1 are mass and molar mass of octane.

51.[1]



NF_3 has trigonal pyramidal geometry N atom has one lone pair and three bond pairs of electrons. The electron pair geometry is tetrahedral and molecular geometry is trigonal pyramidal. The bond angles are lower than tetrahedral bond angles are lower than tetrahedral bond angles due to lone pair – lone pair and lone pair – bond pair repulsions. N atom is sp^3 hybridised.

52.[3] From ideal gas equation -

$$PV = nRT$$

Where $n = m/M$

So, $PV = mRT/M$

$$P = mRT/MV$$

$$P = dRT/M$$

At constant temperature and pressure

$$d \propto M$$

$$d_1/d_2 = M_1/M_2$$

(Here d_1 and M_1 are density and molecular massof ammonia and d_2 and M_2 are density and molecular mass of hydrogen chloride)

$$d_1/d_2 = 17/36.5$$

$$d_1/d_2 = 0.46$$

53.[2] → Phenelzine contains hydrazine
→ Chloroxylenol contains phenol
→ Uracil is the pyrimidine base
→ Ranitidine contains furan ring

54.[1] Reaction at equilibrium

$N_2O_4 \rightleftharpoons 2NO_2$

According to Le chateller's principle.

(1) addition of an inert gas at constant pressure will increase volume and equilibrium shifts towards more number of molecules.

(2) Decomposition of N_2O_4 will be forward reaction when temperature is increased. So, It is incorrect. It will not effect reaction (volume is constant)

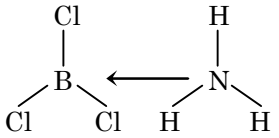
(3) Increasing the pressure on a gas reaction shifts the position of equilibrium towards the side with fewer molecules.

So, It will move in backward direction which leads to formation of N_2O_4 from NO_2.

55.[4] The physical adsorption is temperature and pressure dependent. It increases with increase in pressure and decreases with increase in temperature. Also, when there is more surface area then physical adsorption will be more. Physical adsorption forms multiple layer of adsorption. Thus, statement given in option (4) is incorrect.

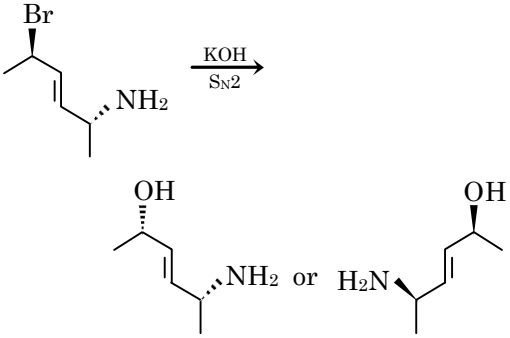
56.[1] BCl_3

$$B + Cl_2 \longrightarrow \underset{\substack{\text{[does not dimerise due to}\\ (p\pi-p\pi)\text{ back bonding]}}}{BCl_2} \xrightarrow{NH_3}$$



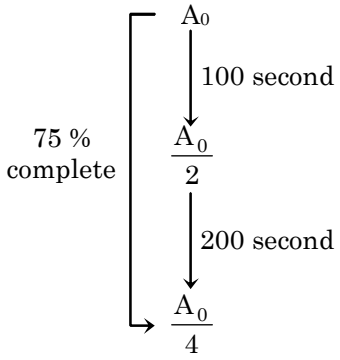
BCl_3 is electron deficient but it does not form dimmer like Al, Ga or ln because its electron deficiency is complemented by the formation of co-ordinate bond between line pair of electron of chlorine and empty unhybridized p-orbital of boron forming pπ-pπ bonding.

57.[3]



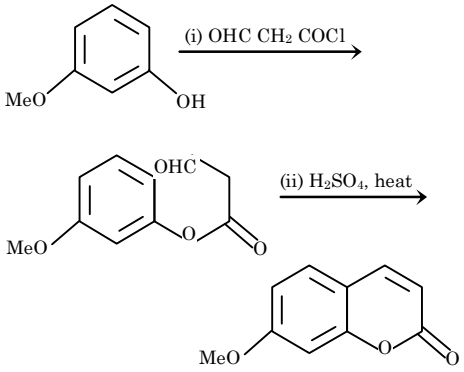
Inversion takes place at the carbon containing bromine atom.

58.[4]

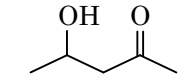


First order reaction as half life is constant.

59.[2] Acid chloride is more reactive than aldehyde. Hence, phenoilic – OH will react with –COCl group first to form ester. This is followed by cyclisation in presence of conc. Sulfuric acid.

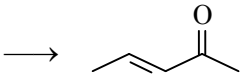


60.[1]



4-Hydroxpentan-2one

will most readily be dehydrated to give unsaturated ketone



α,β-Unsaturated ketone

MATHEMATICS

61.[3] Here $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$

$\therefore \quad A^2 = A \cdot A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix}$

Also $A^3 = A^2 \cdot A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 6 & 3 & 1 \end{bmatrix}$

and, $A^4 = A^3 \cdot A = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 6 & 3 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 10 & 4 & 1 \end{bmatrix}$

On observing the pattern, we come to a conclusion that,

$A^n = \begin{bmatrix} 1 & 0 & 0 \\ n & 1 & 0 \\ \frac{n(n+1)}{2} & n & 1 \end{bmatrix}$

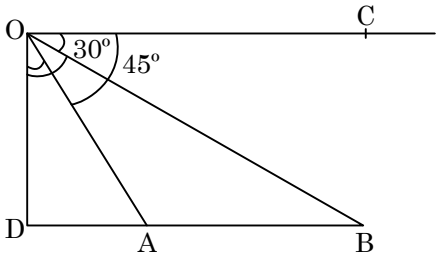
$\therefore \quad A^{20} = \begin{bmatrix} 1 & 0 & 0 \\ 20 & 1 & 0 \\ 210 & 20 & 1 \end{bmatrix}$

Therefore, sum of first column of

$A^{20} = [1 + 20 + 210] = 231$

62.[1] Here, $\angle DOA = 45^\circ$; $\angle DOB = 60^\circ$

Now let height of tower = h.



In ΔDOA, $\tan(\angle DOA) = \frac{DA}{OD}$

$\Rightarrow \quad \tan 45^\circ = \frac{DA}{h}$

$\Rightarrow \quad h = DA$

Now, in ΔDOB

$\tan(\angle DOB) = \frac{BD}{OD}$

$\Rightarrow \tan 60^o = \frac{BD}{h}$

$\Rightarrow BD = \sqrt{3}\, h.$

$\therefore$ speed for the distance

$BA = \frac{BD - AD}{18} = \frac{(\sqrt{3}-1)h}{18}$

$\therefore$ required time taken

$= \frac{AD}{\text{speed}} = \frac{h \times 18}{(\sqrt{3}-1)h}$

$= \frac{18}{\sqrt{3}-1} = 9(\sqrt{3}+1)$

63.[3] $\because \frac{1}{x_1}, \frac{1}{x_2}, \frac{1}{x_3}, ..., \frac{1}{x_n}$ are in A.P.

$x_1 = 4$ and $x_{21} = 20$

Let 'd' be the common difference of this A.P.

$\therefore$ its 21st term, $\frac{1}{x_{21}} = \frac{1}{x_1} + [(21-1) \times d]$

$\Rightarrow d = \frac{1}{20} \times \left(\frac{1}{20} - \frac{1}{4}\right)$

$\Rightarrow d = -\frac{1}{100}$

Also, $x_n > 50$ (given)

$\therefore \frac{1}{x_n} = \frac{1}{x_1} + [(n-1) \times d]$

$\Rightarrow x_n = \frac{x_1}{1 + (n-1) \times d \times x_1}$

$\therefore x_n = \frac{x_1}{1 + (n-1) \times d \times x_1} > 50$

$\Rightarrow \frac{4}{1 + (n-1) \times \left(-\frac{1}{1000}\right) \times 4} > 50$

$\Rightarrow 1 + (n-1) \times \left(-\frac{1}{100}\right) \times 4 < \frac{4}{50}$

$\Rightarrow -\frac{1}{100}(n-1) < -\frac{23}{100}$

$\Rightarrow n - 1 > 23$

$\Rightarrow n > 24$

Therefore, n = 25.

$\Rightarrow \sum_{i=1}^{25} \frac{1}{x_i} = \frac{25}{2}\left[\left(2 \times \frac{1}{4}\right) + (25-1) \times \left(-\frac{1}{100}\right)\right]$

$= \frac{13}{4}$

64.[1] Here, lines are :

$\sqrt{2}\, x - y + 4\sqrt{2}\, k = 0$

$\Rightarrow \sqrt{2}\, x + 4\sqrt{2}\, k = y$ ....(i)

and $\sqrt{2}\, kx + ky - 4\sqrt{2} = 0$ ...(ii)

Put the value of y from (i) in (ii) we get;

$\Rightarrow 2\sqrt{2}\, kx + 4\sqrt{2}\,(k^2 - 1) = 0$

$\Rightarrow x = \frac{2(1-k^2)}{k}, y = \frac{2\sqrt{2}(1+k^2)}{k}$

$\therefore \left(\frac{y}{4\sqrt{2}}\right)^2 - \left(\frac{x}{4}\right)^2 = 1$

$\therefore$ length of transverse axis.

$2a = 2 \times 4\sqrt{2} = 8\sqrt{2}$

Hence, the locus is a hyperbola with length of its transverse axis equal to $8\sqrt{2}$.

65.[1] The thousands place can only be filled with 2, 3 or 4, since the number is greater than 2000.

For the remaining 3 places, we have pick out digits such that the resultant number is divisible by 3.

If the sum of digits of the number is divisible by 3, then the number itself is divisible by 3.

Case I : If we take 2 at thousands place.

The remaining digits can be filled as :

0, 1 and 3 as 2 + 1 + 0 + 3 = 6 is divisible by 3.

0, 3 and 4 as 2 + 3 + 0 + 4 = 9 is divisible by 3.

In both the above combinations the remaining three digits can be arranged in 3! ways.

$\therefore$ Total number of numbers in this case $= 2 \times 3! = 12$.

Case 2 : If we take 3 at thousand place.

The remaining digits can be filled as :

0, 1 and 2 as 3 + 1 + 0 + 2 = 6 is divisible by 3.

0, 2 and 4 as 3 + 2 + 0 + 4 = 9 is divisible by 3.

In both the above combinations, the remaining three digits can be arranged in 3! ways. Total number of numbers in this case $= 2 \times 3! = 12$.

Case 3 : If we take 4 at thousands place.

The remaining digits can be filled as :

0, 2 and 3 as 4 + 2 + 0 + 3 = 9 is divisible by 3.

In the above combination, the remaining three digits can be arranged in 3! ways.

$\therefore$ Total number of numbers in this case $= 3! = 6$.

$\therefore$ Total number of numbers between 2000 and 5000 divisible by 3 are $12 + 12 + 6 = 30$.

66.[2] $\frac{1}{x+p} + \frac{1}{x+q} = \frac{1}{r}$

$\frac{x+p+x+q}{(x+p)(x+q)} = \frac{1}{r}$

$(2x + p + q)r = x^2 + px + qx + pq$

$x^2 + (p + q - 2r)x + pq - pr - qr = 0$

Let α and β be the roots.

$\therefore$ $\alpha + \beta = -(p + q - 2r)$ ...(i)

$\alpha\beta = pq - pr - qr$ ...(ii)

$\because$ $\alpha = -\beta$ (given)

$\therefore$ in eq. (1), we get

$\Rightarrow -(p + q - 2r) = 0$

Now, $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$= (-(p + q - 2r))^2 - 2(pq - pr - qr)$ ... (from (i) and (ii))

$= p^2 + q^2 + 4r^2 + 2pq - 4pr - 4qr - 2pq + 2pr + 2qr$

$= p^2 + q^2 + 4r^2 - 2pr - 2qr$

$= p^2 + q^2 + 2r(2r - p - q)$ ...(from (iii))

$= p^2 + q^2 + 0$

$= p^2 + q^2$

67.[4]

p	q	~ p	~ q	~ p∨ ~ q	p → (~ p∨ ~ q)
T	T	F	F	F	F
T	F	F	T	T	T
F	T	T	F	T	T
F	F	T	T	T	T

From the truth table,

$p \rightarrow (\sim p \vee \sim q)$ is false only when p and q both are true.

68.[2] Here, $9x^2 + 27x + 20 = 0$

$\therefore$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\Rightarrow$ $x = \frac{-27 \pm \sqrt{27^2 - 4 \times 9 \times 20}}{2 \times 9}$

$\Rightarrow$ $x = -\frac{4}{3}, -\frac{5}{3}$

Given, $\cos A = -\frac{3}{5}$

$\therefore$ $\sec A = \frac{1}{\cos A} = -\frac{5}{3}$

Here, A is an obtuse angle.

$\therefore$ $\tan A = -\sqrt{\sec^2 A - 1} = -\frac{4}{3}$.

Hence, roots of the equation are sec A and tan A.

69.[2] Here,

$\frac{dx}{dt} = \frac{1}{2\sqrt{2^{\operatorname{cosec}^{-1}t}}} 2^{\operatorname{cosec}^{-1}t} \log 2. \frac{-1}{x\sqrt{x^2 - 1}}$

$\frac{dy}{dt} = \frac{1}{2\sqrt{2^{\sec^{-1}t}}} 2^{\sec^{-1}t} \log 2. \frac{1}{x\sqrt{x^2 - 1}}$

$\therefore$ $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{-\sqrt{2^{\operatorname{cosec}^{-1}t}}}{\sqrt{2^{\sec^{-1}t}}} \frac{2^{\sec^{-1}t}}{2^{\operatorname{cosec}^{-1}t}}$

$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = -\sqrt{\frac{2\sec^{-1}t}{2\operatorname{cosec}^{-1}t}} = \frac{-y}{x}$

70.[1] Let $a = ((1 + 2x + 3x^2)^6 + (1 - 4x^2)^6)$

$\therefore$ Coefficient of x^2 in the expansion of the product

$(2 - x^2)((1 + 2x + 3x^2)^6 + (1 - 4x^2)^6)$

$= 2$(Coefficient of x^2 in a) $- 1$ (Constant of expansion)

In the expansion of

$((1 + 2x + 3x^2)^6 + (1 - 4x^2)^6)$

Constant $= 1 + 1 = 2$

Coefficient of x^2 = [Coefficient of x^2 in $({}^6C_0(1 + 2x)^6 (3x^2)^0)$] + [Coefficient of x^2 in $({}^6C_1(1 + 2x)^5 (3x^2)^1)$] $-$ [$^6C_1(4x^2)$]

$= 60 + 6 \times 3 - 24 = 54$

$\therefore$ The coefficient of x^2 in $(2 - x^2)$ $((1 + 2x + 3x^2)^6 + (1 - 4x^2)^6)$

$= 2 \times 54 - 1(2) = 108 - 2 = 106$.

71.[3] Equation of plane passing through three given points is -

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix} = 0$$

$\Rightarrow$ $$\begin{vmatrix} x + 2 & y + 2 & z - 2 \\ 1 + 2 & -1 + 2 & 2 - 2 \\ 1 + 2 & 1 + 2 & 1 - 2 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} x+2 & y+2 & z-2 \\ 3 & 1 & 0 \\ 3 & 3 & -1 \end{vmatrix} = 0$$

$$\Rightarrow -x + 3y + 6z - 8 = 0$$

$$\Rightarrow \frac{x}{8} - \frac{3y}{8} - \frac{6z}{8} + \frac{8}{8} = 0$$

$$\Rightarrow \frac{x}{8} - \frac{y}{\frac{8}{3}} - \frac{z}{\frac{8}{6}} = -1$$

$$\Rightarrow \frac{x}{-8} + \frac{y}{\frac{8}{3}} + \frac{z}{\frac{8}{6}} = 1$$

$\therefore$ Sum of intercepts $= -8 + \frac{8}{3} + \frac{8}{6} = -4$

72.[3] $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Since, it passes, through (0, 3)

$\therefore \quad \frac{0}{a^2} + \frac{9}{b^2} = 1$

$\Rightarrow \quad b^2 = 9$

$\therefore$ eq. of ellipse becomes :

$$\frac{x^2}{a^2} + \frac{y^2}{9} = 1$$

differential w.r.t. x, we get;

$$\frac{2x}{a^2} + \frac{2y}{9}\frac{dy}{dx} = 0$$

$$\Rightarrow \quad \frac{y}{x}\left(\frac{dy}{dx}\right) = \frac{-9}{a^2}$$

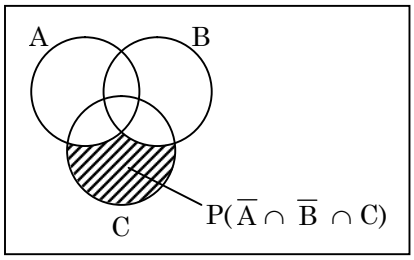
Again differentiating w.r.t. x, we get ;

$$\frac{y}{x}\frac{d^2y}{dx^2} + \frac{x\frac{dy}{dx} - y}{x^2}\frac{dy}{dx} = 0$$

$\Rightarrow \quad xyy'' + x(y')^2 - yy' = 0$

73.[3] Here, $P(\overline{A} \cap \overline{B})\,|C| = \frac{P(\overline{A} \cap \overline{B} \cap C)}{P(C)}$

$$= \frac{P(C) - P(A \cap C - P(B \cap C) + P(A \cap B \cap C))}{P(C)}$$



$$= 1 - \frac{P(A).P(C) + P(B).P(C)}{P(C)}$$

$(\because P(A \cap B \cap C) = 0)$

$= 1 - P(A) - P(B)$

$= P(\overline{A}) - P(B)$

74.[3] Here, the equations are :

$(k + 2)\,x + 10y = k$

& $kx + (k + 3)y = k - 1$

These equations can be written in the form of Ax = B as

$$\begin{bmatrix} k+2 & 10 \\ k & k+3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} k \\ k-1 \end{bmatrix}$$

For the system to have no solution

$|A| = 0$

$$\Rightarrow \begin{bmatrix} k+2 & 10 \\ k & k+3 \end{bmatrix} = 0$$

$\Rightarrow (k + 2)(k + 3) - k \times 10 = 0$

$\Rightarrow k^2 - 5k + 6 = (k - 2)(k - 3) = 0$

$\therefore \quad k = 2, 3$

For k = 2, equations become :

$4x + 100y = 2$ & $2x + 5y = 1$

& hence infinite number of solutions.

For k = 3, equations becomes :

$5x + 10y = 3$

$3x + 6y = 2$

Hence no solution

$\therefore$ req. number of values of k is 1.

75.[4] Let θ be the angle between the two lines

Here direction cosines of

$$\frac{x}{2} = \frac{y}{2} = \frac{z}{1} \text{ are } 2, 2, 1$$

Also second line can be written as :

$$\frac{x-5}{2} = \frac{y-2}{\frac{P}{7}} = \frac{z-3}{4}$$

$\therefore$ Its direction cosines are $2, \frac{P}{7}, 4$

Also, $\cos\theta = \frac{2}{3}$ (Given)

$\because \quad \cos\theta = \left|\frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}\right|$

$$\Rightarrow \frac{2}{3} = \left|\frac{(2\times 2) - \left(2\times\frac{P}{7}\right) + (1\times 4)}{\sqrt{2^2+2^2+1^2}\sqrt{2^2+\frac{P^2}{49}+4^2}}\right|$$

$$= \frac{4 + \frac{2P}{7} + 4}{3\times\sqrt{2^2+\frac{P^2}{49}+4^2}}$$

$$\Rightarrow \left(4+\frac{P}{7}\right)^2 = 20 + \frac{P^2}{49}$$

$$\Rightarrow 16 + \frac{8P}{7} + \frac{P^2}{49} = 20 + \frac{P^2}{49}$$

$$\Rightarrow \frac{8P}{7} - 4$$

$$\Rightarrow P = \frac{7}{2}$$

76.[1] $\because \quad \vec{a} = \hat{i} + \hat{j} + \hat{k} \Rightarrow |\vec{a}| = \sqrt{3}$

$\vec{c} = \hat{j} - \hat{k} \Rightarrow$ (Given) $|\vec{c}| = \sqrt{2}$

Now, $\vec{a} \times \vec{b} = \vec{c}$

$\Rightarrow |\vec{a}||\vec{b}|\sin\theta = |\vec{c}|$

$\Rightarrow |\vec{a}||\vec{b}|\sin\theta = \sqrt{2}$ ...(i)

Also $\vec{a}\,.\,\vec{b} = 3$

$\Rightarrow |\vec{a}||\vec{b}|\sin\theta = 3$ ...(ii)

Dividing (i) by (ii), we get

$$\tan\theta = \frac{\sqrt{2}}{3}$$

$$\therefore \quad \sin\theta = \frac{\sqrt{2}}{\sqrt{11}}$$

Substituting value of sinθ in (i) we get

$$\sqrt{3}\,|\vec{b}|\,\frac{\sqrt{2}}{\sqrt{11}} = \sqrt{2}$$

$$\Rightarrow |\vec{b}| = \frac{\sqrt{11}}{\sqrt{3}}$$

77.[4] Let $\ell = \left(\frac{1+i\sqrt{3}}{1-i\sqrt{3}}\right)$

$$\therefore \quad \ell = \left(\frac{1+i\sqrt{3}}{1-i\sqrt{3}}\right)\times\left(\frac{1+i\sqrt{3}}{1+i\sqrt{3}}\right)$$

$$= \left(\frac{-2+i2\sqrt{3}}{4}\right)\times\left(\frac{1-i\sqrt{3}}{-2}\right)$$

Also, $\ell = \left(\frac{1+i\sqrt{3}}{1-i\sqrt{3}}\right)\times\left(\frac{1-i\sqrt{3}}{1-i\sqrt{3}}\right)$

Now,

$$\left(\frac{1+i\sqrt{3}}{1-i\sqrt{3}}\right)^3 = \left(\frac{1+i\sqrt{3}}{1-i\sqrt{3}}\right)\times\left(\frac{1+i\sqrt{3}}{1-i\sqrt{3}}\right)\times\left(\frac{1+i\sqrt{3}}{1-i\sqrt{3}}\right)$$

$$= \left(\frac{1+i\sqrt{3}}{1-i\sqrt{3}}\right)\times\left(\frac{-2}{1+i\sqrt{3}}\right)\times\left(\frac{1-i\sqrt{3}}{-2}\right)$$

$= 1$

$\therefore$ least positive integer n is 3.

78.[3] Let $L = \lim\limits_{x\to 0}\frac{(27+x)^{\frac{1}{3}} - 3}{9 - (27+x)^{\frac{2}{3}}}$

Here 'L' is in the indeterminate form i.e., $\frac{0}{0}$

$\therefore$ using the L' Hospital rule we get :

$$L = \lim_{x\to 0}\frac{\frac{1}{3}(27+x)^{\frac{-2}{3}}}{-\frac{2}{3}(27+3)^{\frac{-1}{3}}} = \frac{\frac{1}{3}\times(27)^{\frac{-2}{3}}}{-\frac{2}{3}\times 27^{\frac{-1}{3}}} = -\frac{1}{6}$$

79.[1] $$f(x) = \int_0^x t(\sin x - \sin t)\,.\,dt$$

$$= \sin x\int_0^x t\,.\,dt - \int_0^x t\,.\sin t\,.\,dt$$

$$= \frac{x^2}{2}\sin x + [t\cos t]_0^x + \sin x$$

$$\Rightarrow f(x) = \frac{x^2}{2}\sin x + x\cos x + \sin x$$

$$f'(x) = \frac{x^2}{2}\cos x + 2\cos x$$

$$f''(x) = x\cos x - \frac{x^2}{2}\sin x - 2\sin x$$

$$f'''(x) = \cos x - 2x\sin x - \frac{x^2}{2}\cos x - 2\cos x$$

$\therefore$ $f'''(x) + f'(x) = \cos x - 2x \sin x$

80.[1] Here, $f(x) = 2x^3 - 9x^2 + 12x + 5$

$\Rightarrow$ $f'(x) - 6x^2 - 18x + 12 = 0$

For maxima or minima put $f'(x) = 0$

$\Rightarrow$ $x^2 - 3x + 2 = 0$

$\Rightarrow$ $x = 1$ or $x = 2$

Now, $f''(x) = 12x - 18$

$\Rightarrow$ $f''(1) = 12(1) - 18 = -6 < 0$

Hence, f(x) has maxima at x = 1

$\therefore$ maximum value

$= M = f(1) = 2 - 9 + 12 + 5 = 10.$

And, $f''(2) = 12\,(2) - 18 = 6 > 0$

Hence, f(x) has minima at x = 2.

$\therefore$ minimum value

$= m = f(2) = 2(8) - 9\,(4) + 12(2) + 5 = 9$

$\therefore$ $M - m = 10 - 9 = 1$

81.[4] Given circle is :

$x^2 + y^2 + 2x - 4y - 4 = 0$

$\therefore$ its centre is (–1, 2) and radius is 3 units.

Let A = (x, y) be the centre of the circle C

$\therefore$ $\frac{x-1}{2} = 2 \Rightarrow x = 5$ and $\frac{y+2}{2} = 2$

$\Rightarrow$ $y = 2$

So the centre of C is (5, 2) and its radius is 3

$\therefore$ equation of centre C is :

$x^2 + y^2 - 10x - 4y + 20 = 0$

$\therefore$ The length of the intercept it cuts on the x-axis $= 2\sqrt{g^2 - c} = 2\sqrt{25 - 20} = 2\sqrt{5}$

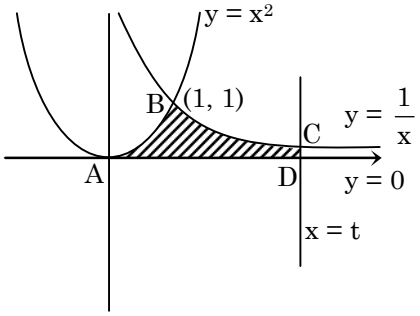
82.[2] The intersection point of

$y = x^2$ and $y = \frac{1}{x}$ is (1, 1)

Area bounded by the curves is the region ABCDA is given as :

$$\text{Area} = \int_0^1 x^2 dx + \int_1^t \frac{1}{x} dx$$

$$= \left[\frac{x^3}{3}\right]_0^1 + [\ln(x)]_1^t = \frac{1}{3} + \ln(t)$$



$\because$ area = 1

$\Rightarrow$ $\frac{1}{3} + \ln(t) = 1$

$\Rightarrow$ $\ln(t) = \frac{2}{3} \Rightarrow t = e^{\frac{2}{3}}$

83.[3] Let $P(2t, t^2)$ be any point on the parabola.

Centre of the given circle

$C = (-g, -f) = (-3, 0)$

For PC to be minimum, it must be the normal to the parabola at P.

Slope of line PC $= \frac{y_2 - y_1}{x_2 - x_1} = \frac{t^2 - 0}{2t + 3}$

Also, slope of tangent to parabola at

$$P = \frac{dy}{dx} = \frac{x}{2} = t$$

$\therefore$ Slope of normal $= \frac{-1}{t}$

$\therefore$ $\frac{t^2 - 0}{2t + 3} = \frac{-1}{t}$

$\Rightarrow$ $t^3 + 2t + 3 = 0$

$\Rightarrow$ $(t + 1)(t^2 - t + 3) = 0$

$\therefore$ Real roots of above equation is

$t = -1$

Coordinate of $P = (2t, t^2) = (-2, 1)$

Slope of tangent to parabola at P = $t = -1$.

Therefore, equation of tangent is :

$(y - 1) = (-1)(x + 2)$

$\Rightarrow$ $x + y + 1 = 0$

84.[1] If the function is continuous at x = 0, then $\lim_{x\to 0} f(x)$ will exist

and $f(0) = \lim_{x\to 0} f(x)$

Now, $\lim_{x\to 0} f(x) = \lim_{x\to 0} \left(\frac{1}{x} - \frac{k-1}{e^{2x} - 1}\right)$

$$= \lim_{x\to 0}\left(\frac{e^{2x}-1-kx+x}{(x)(e^{2x}-1)}\right)$$

$$= \lim_{x\to 0}\left[\frac{\left(1+2x+\frac{(2x)^2}{2!}+\frac{(2x)^3}{3!}+\ldots\right)-1-kx+x}{(x)\left(\left(1+2x+\frac{(2x)^2}{2!}+\frac{(2x)^3}{3!}+\ldots\right)-1\right)}\right]$$

$$= \lim_{x\to 0}\frac{(3-k)x+\frac{4x^2}{2!}+\frac{8x^3}{3!}+\ldots}{\left(2x^2+\frac{4x^3}{2!}+\frac{8x^4}{3!}+\ldots\right)}$$

For the limit to exist, power of x in the numerator should be greater than or equal to the power of x in the denominator. Therefore, coefficient of x in numerator is equal to zero.

$\Rightarrow 3 - k = 0$

$\Rightarrow k = 3$

$$= \lim_{x\to 0}\frac{(x^2)\left(\frac{4}{2!}+\frac{8x}{3!}+\ldots\right)}{(x^2)\left(2+\frac{4x}{2!}+\frac{8x^2}{3!}+\ldots\right)}$$

$$= \lim_{x\to 0}\frac{\frac{4}{2!}+\frac{8x}{3!}+\ldots}{2+\frac{4x}{2!}+\frac{8x^2}{3!}+\ldots} = 1$$

Hence, $f(0) = 1$

85.[3] Here mean $= \bar{x} = 9$

$$\Rightarrow \bar{x} = \frac{\Sigma x_i}{n} = 9$$

$$\Rightarrow \Sigma x_i = 9 \times 5 = 45$$

Now, standard deviation = 0

$\therefore$ all the five terms are same i.e.; 9.

Now, for changed observation

$$\bar{x}_{new} = \frac{36 + x_5}{5} = 10$$

$$\Rightarrow x_5 = 14$$

$$\therefore \sigma_{new} = \sqrt{\frac{\Sigma(x_i - \bar{x}_{new})^2}{n}}$$

$$= \sqrt{\frac{4(9-10)^2 + (14-10)^2}{5}}$$

$$= 2$$

86.[3] Here,

$R_1 = \{(x, y) \in N \times N : 2x + y = 10\}$

and $R_2 = \{(x, y) \in N \times N; x + 2y = 10\}$

For R_1 ; $2x + y = 10$ and $x, y \in N$

So, possible values for x and y are

$x = 1, y = 8$ i.e. (1,8);

$x = 2, y = 6$ i.e. (2, 6);

$x = 3, y = 4$ i.e. (3, 4) and

$x = 4, y = 2$ i.e. (4, 2)

$R_1 = \{(1, 8), (2, 6), (3, 4), (4, 2)\}$

Therefore, Range of R_1 is {2, 4, 6, 8}

R_1 is not symmetric

Also, R_1 is not transitive because

$(3, 4), (4, 2) \in R_1$ but $(3, 2) \notin R_1$

Thus, options A, B and D are incorrect.

For R_2 : $x + 2y = 10$ and $x, y \in N$

So, possible values for x and y are :

$x = 8, y = 1$ i.e. (8, 1) ;

$x = 6, y = 2$ i.e. (6, 2);

$x = 4, y = 3$ i.e. (4, 3) and

$x = 2, y = 4$ i.e. (2, 4)

$R_2 = \{(8, 1), (6, 2), (4, 3), (2, 4)\}$

Therefore, Range of R_2 is {1, 2, 3, 4}

R_2 is not symmetric and transitive.

87.[4] The general term of the given series

$$= \frac{2\times 2^r - 1}{2^r},$$

Where $r \geq 0$

$$\therefore \text{ req. sum} = 1 + \sum_{r=1}^{19}\frac{2\times 2^r - 1}{2^r}$$

Now,

$$\sum_{r=1}^{19}\left(\frac{2\times 2^r - 1}{2^r}\right) = \sum_{r=1}^{19}\left(2 - \frac{1}{2^r}\right)$$

$$= 2(19) - \frac{\frac{1}{2}\left(1-\left(\frac{1}{2}\right)^{19}\right)}{1-\frac{1}{2}}$$

$$= 38 + \frac{\left(\frac{1}{2}\right)^{19} - 1}{1}$$

$$= 38 + \left(\frac{1}{2}\right)^{19} - 1$$

$= 37 + \left(\frac{1}{2}\right)^{19}$

$\therefore$ req. sum $= 1 + 37 + \left(\frac{1}{2}\right)^{19} = 38 + \left(\frac{1}{2}\right)^{19}$

88.[4] Let for ellipse coordinates of focus and vertex are (ae, 0) and (a, 0) respectively.

$\therefore$ Distance between focus and vertex

$$= a(1 - e) = \frac{3}{2} \quad \text{(given)}$$

$$\Rightarrow \quad a - \frac{3}{2} = ae$$

$$\Rightarrow \quad a^2 + \frac{9}{4} - 3a = a^2e^2 \quad \ldots(i)$$

Length of latus rectum $= \frac{2b^2}{a} = 4$

$$\Rightarrow \quad b^2 = 2a \quad \ldots(ii)$$

$$e^2 = 1 - \frac{b^2}{a^2}$$

$$\Rightarrow \quad e^2 = 1 - \frac{2a}{a^2} \quad \text{(from (ii))}$$

$$\Rightarrow \quad e^2 = 1 - \frac{2}{a} \quad \ldots(iii)$$

Substituting the value of e^2 in eq. (i) we get ;

$$\Rightarrow \quad a^2 + \frac{9}{4} - 3a = a^2\left(1 - \frac{2}{a}\right)$$

$$\Rightarrow \quad a = \frac{9}{4}$$

$\therefore$ from eq.(iii) we get ;

$$e^2 = 1 - \frac{2}{a} = 1 - \frac{8}{9} = \frac{1}{9}$$

$$\Rightarrow \quad e = \frac{1}{3}$$

89.[4] Let the number of children in each family be x.

Thus the total number of children in both the families are 2x.

Now, it is given that 3 tickets are distributed amongst the children of these two families. Thus, the probability that all the three tickets go to the children in family B

$$= \frac{{}^{x}C_3}{{}^{2x}C_3} = \frac{1}{12}$$

$$\Rightarrow \quad \frac{x(x-1)(x-2)}{2x(2x-1)(2x-2)} = \frac{1}{12}$$

$$\Rightarrow \quad \frac{(x-2)}{(2x-1)} = \frac{1}{6}$$

$$\Rightarrow \quad x = 5$$

Thus, the number of children in each family is 5.

90.[1] Let $I = \int \frac{\tan x}{1 + \tan x + \tan^2 x}\,dx$

$$\Rightarrow I = \int \frac{\tan x + 1 + \tan^2 x}{\tan x + 1 + \tan^2 x}\,dx - \int \frac{(1 + \tan^2 x)}{1 + \tan x + \tan^2 x}$$

$$\Rightarrow \quad I = x - \int \frac{\sec^2 x\,dx}{1 + \tan x + \tan^2 x}$$

Put $\tan x = t \Rightarrow \sec^2 x \,.\, dx = dt$

$$\therefore \quad I = x - \int \frac{dt}{t^2 + t + \frac{1}{4} + 1 - \frac{1}{4}}$$

$$= x - \int \frac{dt}{\left(t+\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$$

$$\Rightarrow \quad I = x - \frac{2}{\sqrt{3}} \tan^{-1}\left(\frac{t+\frac{1}{2}}{\frac{\sqrt{3}}{2}}\right) + C$$

$$\Rightarrow \quad I = x - \frac{2}{\sqrt{3}} \tan^{-1}\left(\frac{2\tan x + 1}{\sqrt{3}}\right) + C$$

$\therefore$ $A = 3$ and $K = 2$.