JEE MAIN ONLINE PAPER 2018

Held on April 15, 2018 [Evening]

Instructions

1. This test will be a 3 hours Test.
2. This test consists of Physics, Chemistry and Mathematics questions with equal weight age of 120 marks.
3. Each question is of 4 marks.
4. There are three parts in the question paper consisting of Physics (Q. no. 1 to 30), Chemistry (Q. no. 31 to 60) and Mathematics (Q. no. 61 to 90).
5. There will be only one correct choice in the given four choices. For each question 4 marks will be awarded for correct choice, 1 mark will be deducted for incorrect choice and zero mark will be awarded for not attempted question.
6. Any textual, printed or written material, mobile phones, calculator etc. is not allowed for the students appearing for the test.
7. All calculations / written work should be done in the rough sheet provided.

PHYSICS

Q.1 A body of mass 2kg slides down with an acceleration of $3m/s^2$ on a rough inclined plane having a slope of 30^o. The external force required to take the same body up the plane with the same acceleration will be ($g = 10\ m/s^2$) **[JEE-Main On line-2018]**

(A) 4N (B) 14N

(C) 4N (D) 20N

Q.2 A plane polarized monochromatic EM wave is traveling a vacuum along z direction such that at $t = t_1$ it is found that the electric field is zero at a spatial point z_1. The next zero that occurs in its neighbourhood is at z_2. The frequency of the electromagnetic wave is -

[JEE-Main On line-2018]

(A) $\frac{3\times10^8}{|z_2 - z_1|}$ (B) $\frac{6\times10^8}{|z_2 - z_1|}$

(C) $\frac{1.5\times10^8}{|z_2 - z_1|}$ (D) $\frac{1}{t_1 + \frac{|z_2 - z_1|}{3\times10^8}}$

Q.3 A current of 1A is flowing on the sides of an equilateral triangle of side 4.5×10^{-2}m. The magnetic field at the center of the triangle will be -

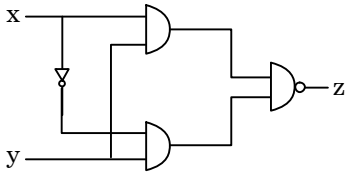
[JEE-Main On line-2018]

(A) 4×10^{-5} Wb/b^2 (B) Zero

(C) 2×10^{-5} Wb/m^2 (D) 8×10^{-5} Wb/m^2

Q.4 Truth table for the given circuit will be

[JEE-Main On line-2018]



(A)

x	y	z
0	0	1
0	1	1
1	0	1
1	1	0

(B)

x	y	z
0	0	0
0	1	0
1	0	0
1	1	1

	x	y	z
	0	0	1
(C)	0	1	1
	1	0	1
	1	1	1

	x	y	z
	0	0	0
(D)	0	1	1
	1	0	1
	1	1	1

Q.5 A constant voltage is applied between two ends of metallic wire. If the length is halved and the radius of the wire is doubled, the rate of heat developed in the wire will be -

[JEE-Main On line-2018]

(A) Increased 8 times
(B) Doubled
(C) Halved
(D) Unchanged

Q.6 The characteristic distance at which quantum gravitational effects are significant, the Planck length, can be determined from a suitable combination of the fundamental physical constants G, h and c. Which of the following correctly gives the Planck length ?

[JEE-Main On line-2018]

(A) G^2hc (B) $\left(\frac{Gh}{c^3}\right)^{\frac{1}{2}}$

(C) $G^{\frac{1}{2}}h^2c$ (D) Gh^2c^3

Q.7 As shown in the figure, forces of 10^5N each are applied in opposite directions, on the upper and lower faces of a cube of side 10 cm, shifting the upper face parallel to itself by 0.5 cm. If the side of another cube of the same material is, 20 cm then under similar conditions as above, the displacement will be - **[JEE-Main On line-2018]**

(A) 1.00 cm (B) 0.25 cm
(C) 0.37 cm (D) 0.75 cm

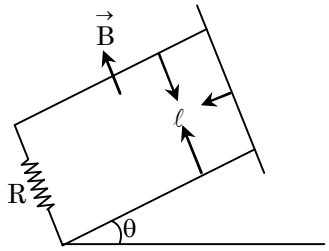
Q.8 The carrier frequency of a transmitter is provided by a tank circuit of a coil of inductance 49 μH and a capacitance of 2.5 μF. It is modulated by an audio signal of 12kHz. The frequency range occupied by the side bands is -

[JEE-Main On line-2018]

(A) 18kHz – 30 kHz
(B) 63 kHz – 75kHz
(C) 442kHz – 466 kHz
(D) 13482kHz – 13494kHz

Q.9 A copper rod of mass m slides under gravity on two smooth parallel rails, with separation ℓ and set at an angle of θ with the horizontal. At the bottom rails are joined by a resistance R. There is a uniform magnetic field B normal to the plane of the rails, as shown in the figure. The terminal speed of the copper rod is -

[JEE-Main On line-2018]

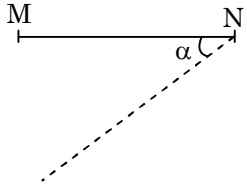


(A) $\frac{mgR\cos\theta}{B^2\ell^2}$ (B) $\frac{mgR\sin\theta}{B^2\ell^2}$

(C) $\frac{mgR\tan\theta}{B^2\ell^2}$ (D) $\frac{mgR\cot\theta}{B^2\ell^2}$

Q.10 A thin rod MN, free to rotate in the vertical plane about the fixed end N, is held horizontal. When the end M is released the speed of this end, when the rod makes an angle α with the horizontal, will be proportional to : (see figure)

[JEE-Main On line-2018]



(A) $\sqrt{\cos\alpha}$ (B) $\cos\alpha$

(C) $\sin\alpha$ (D) $\sqrt{\sin\alpha}$

Q.11 A parallel plate capacitor with area 200 cm^2 and separation between the plates 1.5 cm, is connected across a battery of emf V. If the force of attraction between the plates is 25×10^{-6} N, the value of V is approximately -

$\left(\varepsilon_0 = 8.85\times10^{-12}\,\frac{C^2}{N.m^2}\right)$

[JEE-Main On line-2018]

(A) 150 V (B) 100 V
(C) 250 V (D) 300 V

Q.12 A solid ball of radius R has a change density ρ given by $\rho = \rho_0\left(1-\frac{r}{R}\right)$ for $0 \le r \le R$. The electric field outside the ball is -

[JEE-Main On line-2018]

(A) $\frac{\rho_0 R^3}{\varepsilon_0 r^2}$ (B) $\frac{4\rho_0 R^3}{3\varepsilon_0 r^2}$
(C) $\frac{3\rho_0 R^3}{4\varepsilon_0 r^2}$ (D) $\frac{\rho_0 R^3}{12\varepsilon_0 r^2}$

Q.13 A proton of mass m collides elastically with a particle of unknown mass at rest. After the collision, the proton and the unknown particle are seen moving at an angle of 90º with respect to each other. The mass of unknown particle is -

[JEE-Main On line-2018]

(A) $\frac{m}{\sqrt{3}}$ (B) $\frac{m}{2}$ (C) 2m (D) m

Q.14 A disc rotates about its axis of symmetry in a horizontal plane at a steady rate of 3.5 revolutions per second. A coin placed at a distance of 1.25 cm from the axis of rotation remains at rest on the disc. The coefficient of friction between the coin and the disc is ($g = 10 m/s^2$)

[JEE-Main On line-2018]

(A) 0.5 (B) 0.7 (C) 0.3 (D) 0.6

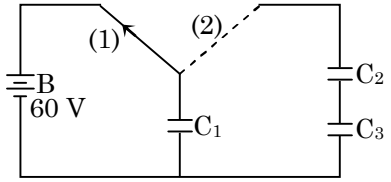
Q.15 At the centre of a fixed large circular coil of radius R, a much smaller circular coil of radius r is placed. The two coils are concentric and are in the same plane. The larger coil carries a current I. The smaller coil is set of rotate with a constant angular velocity ω about an axis along their common diameter. Calculate the emf induced in the smaller coil after a time of its start of rotation -

[JEE-Main On line-2018]

(A) $\frac{\mu_0 I}{2R}\,\omega r^2 \sin\omega t$ (B) $\frac{\mu_0 I}{4R}\,\omega\pi r^2 \sin\omega t$
(C) $\frac{\mu_0 I}{2R}\,\omega\pi r^2 \sin\omega t$ (D) $\frac{\mu_0 I}{4R}\,\omega r^2 \sin\omega t$

Q.16 A capacitor C_1 is charged up to a voltage V = 60 V by connecting it to battery B through switch (1). Now C_1 is disconnected from battery and connected to a circuit consisting of two uncharged capacitors C_2 = 3.0 μF and C_3 = 6.0 μF through a switch (2) as shown in the figure. The sum of final charges on C_2 and C_3 is -

[JEE-Main On line-2018]



(A) 36μC (B) 20μC
(C) 54μC (D) 40μC

Q.17 5 beats/second are heard when a turning fork is sounded with a sonometer wire under tension, when the length of the sonometer wire is either 0.95m or 1m. The frequency of the fork will be -

[JEE-Main On line-2018]

(A) 195Hz (B) 251Hz
(C) 150Hz (D) 300Hz

Q.18 Two simple harmonic motions, as shown, are at right angles. They are combined to form Lissajous figures.
$x(t) = A \sin(at + \delta)$
$y(t) = B \sin(bt)$
Identify the correct match below

[JEE-Main On line-2018]

(A) Parameters : A = B, a = 2b; $\delta = \frac{\pi}{2}$; Curve; Circle

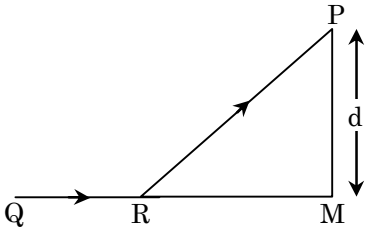
(B) Parameters : A = B, a = b ; $\delta = \frac{\pi}{2}$; Curve ; Line

(C) Parameters : $A \ne B$, a = b; $\delta = \frac{\pi}{2}$; Curve : Ellipse

(D) Parameters : $A \neq B$, $a = b$; $\delta = 0$; Curve : Parabola

Q.19 A man in a car at location Q on a straight highway is moving with speed v. He decides to reach a point P in a field at a distance d from highway (point M) as shown in the figure. Speed of the car in the field is half to that on the highway. What should be the distance RM, so that the time taken to reach P is minimum ?

[JEE-Main On line-2018]



(A) $\frac{d}{\sqrt{3}}$ (B) $\frac{d}{2}$ (C) $\frac{d}{\sqrt{2}}$ (D) d

Q.20 A copper rod of cross-sectional area A carries a uniform current I through it. At temperature T, if the volume charge density of the rod is ρ, how long will the charges take to travel a distance d ?

[JEE-Main On line-2018]

(A) $\frac{2\rho dA}{IT}$ (B) $\frac{2\rho dA}{I}$

(C) $\frac{\rho dA}{I}$ (D) $\frac{\rho dA}{IT}$

Q.21 Two Carnot engines A and B are operated in series. Engine A receives heat from a reservoir at 600 K and rejects heat to a reservoir at temperature T. Engine B receives heat rejected by engine A and in turn rejects it to a reservoir at 100 K. If the efficiencies of the two engines A and B are represented by η_A and η_B respectively, then what is the value of $\frac{\eta_A}{\eta_B}$

[JEE-Main On line-2018]

(A) $\frac{12}{7}$ (B) $\frac{12}{5}$ (C) $\frac{5}{12}$ (D) $\frac{7}{12}$

Q.22 A convergent doublet of separated lenses, corrected for spherical aberration, has resultant focal length of 10 cm. The separation between the two lenses is 2 cm. The focal lengths of the component lenses.

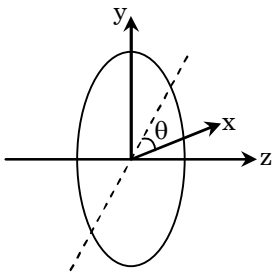
[JEE-Main On line-2018]

(A) 18 cm, 20 cm (B) 10cm, 12 cm

(C) 12 cm, 14 cm (D) 16 cm, 18 cm

Q.23 A plane polarized light is incident on a polarizer with its pass axis making angle θ with x-axis, as shown in the figure. At four different values of θ, $\theta = 8^o$, 38^o. 188^o and 218^o, the observed intensities are same. What is the angle between the direction of polarization and x-axis.

[JEE-Main On line-2018]



(A) 203^o (B) 45^o (C) 98^o (D) 128^o

Q.24 An unstable heavy nucleus at rest breaks into two nuclei which move away with velocities in the ratio of 8 : 27. The ratio of the radii of the nuclei (assumed to be spherical) is -

[JEE-Main On line-2018]

(A) 8 : 27 (B) 2 : 3

(C) 3 : 2 (D) 4 : 9

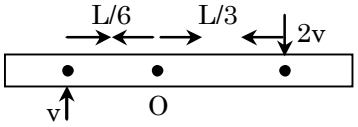
Q.25 A body takes 10 minutes to cook from 60^oC to 50^oC. The temperature of surroundings is constant at 25^oC. Then, the temperature of the body after next 10 minutes will be approximately -

[JEE-Main On line-2018]

(A) 43^oC (B) 47^oC (C) 41^oC (D) 45^oC

Q.26 A thin uniform bar of length L and mass 8m lies on a smooth horizontal table. Two point masses m and 2 m moving in the same horizontal plane from opposite sides of the bar with speeds 2 v and v respectively. The masses stick to the bar after collision at a distance $\frac{L}{3}$ and $\frac{L}{6}$ respectively from the centre of the bar. If the bar starts rotating about its center of mass as a result of collision, the angular speed of the bar will be :

[JEE-Main On line-2018]



(A) $\frac{v}{6L}$ (B) $\frac{6v}{5L}$ (C) $\frac{3v}{5L}$ (D) $\frac{v}{5L}$

Q.27 If the de Broglie wavelengths associated with a proton and an α-particle are equal, then the ratio of velocities of the proton and α-particle will be -

[JEE-Main On line-2018]

(A) 1 : 4 (B) 1 : 2 (C) 4 : 1 (D) 2 : 1

Q.28 When an air bubble of radius r rises from the bottom to the surface of a lake, its radius becomes $\frac{5r}{4}$. Taking the atmospheric pressure to be equal to 10m height of water column, the depth of the lake would approximately be (ignore the surface tension and the effect of temperature)

[JEE-Main On line-2018]

(A) 10.5m (B) 8.7m
(C) 11.2m (D) 9.5m

Q.29 Muon (μ^{-1}) is negatively charged ($|q| = |e|$) with a mass $m_\mu = 200\ m_e$, where m_e is the mass of the electron and e is the electronic charge. If μ^{-1} is bound to a proton to form a hydrogen like atom, identify the correct statements -

[JEE-Main On line-2018]

(a) Radius of the muonic orbit is 200 times smaller than that of the electron
(b) the speed of the μ^{-1} in the n^{th} orbit is $\frac{1}{200}$ times that of the electron in the n^{th} orbit
(c) The ionization energy of muonic atom is 200 times more than that of an hydrogen atom
(d) The momentum of the muon in the n^{th} orbit is 20 times more than that of the electron

(A) (a), (b), (d) (B) (b), (d)
(C) (c), (d) (D) (a), (c), (d)

Q.30 The value closed to the thermal velocity of a Helium atom at room temperature (300K) in ms^{-1} is -
[$k_B = 1.4 \times 10^{-23}$ J/K; $m_{He} = 7 \times 10^{-27}$ kg]

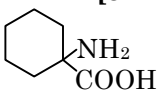
[JEE-Main On line-2018]

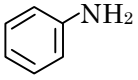
(A) 1.3×10^4 (B) 1.3×10^5
(C) 1.3×10^2 (D) 1.3×10^3

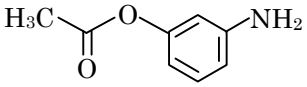
CHEMISTRY

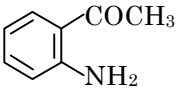
Q.31 The increasing order of diazotization of the of the following compounds is :

[JEE-Main On line-2018]

(a) 

(b) 

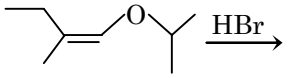
(c) 

(d) 

(A) (d) < (c) < (b) < (a)
(B) (a) < (d) < (b) (c)
(C) (a) < (b) < (c) < (d)
(D) (a) < (d) < (c) < (b)

Q.32 The total number of optically active compounds formed in the following reaction is –

[JEE-Main On line-2018]



(A) zero (B) six (C) four (D) two

Q.33 In KO_2, the nature of oxygen species and the oxidation state of oxygen atom are, respectively :

[JEE-Main On line-2018]

(A) Superoxide and –1
(B) Superoxide and –1/2
(C) Peroxide and –1/2
(D) Oxide and –2

Q.34 Δ_fG^o at 500 K for substance 'S' in liquid state and gaseous state are + 100.7 kcal mol^{-1} and + 103 kcal mol^{-1}, respectively. Vapour pressure of liquid 'S' at 500 K is approximately equal to -
($R = 2cal\ K^{-1}\ mol^{-1}$)

[JEE-Main On line-2018]

(A) 100 atm (B) 1 atm
(C) 10 atm (D) 0.1 atm

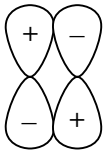
Q.35 In XeO_3F_2, the number of bond pair (s), π-bond (s) and lone pair (s) on Xe atom respectively are -

[JEE-Main On line-2018]

(A) 5, 3, 0 (B) 5, 2, 0
(C) 4, 2, 2 (D) 4, 4, 0

Q.36 Which of the following best describes the diagram of molecular orbital ?

[JEE-Main On line-2018]



(A) A bonding π orbital
(B) A non-bonding orbital
(C) An antibonding σ orbital
(D) An antibonding π orbital

Q.37 Following four solutions are prepared by mixing different volumes of NaOH and HCl of different concentrations. pH of which one of them will be equal to 1 ?

[JEE-Main On line-2018]

(A) 55 mL $\frac{M}{10}$ HCl + 45 mL $\frac{M}{10}$ NaOH

(B) 75 mL $\frac{M}{5}$ HCl + 25 mL $\frac{M}{5}$ NaOH

(C) 100 mL $\frac{M}{10}$ HCl + 100 mL $\frac{M}{10}$ NaOH

(D) 60 mL $\frac{M}{10}$ HCl + 40 mL $\frac{M}{10}$ NaOH

Q.38 Given
(i) $2Fe_2O_3(s) \rightarrow 4Fe(s) + 3O_2(g)$; $\Delta_rG^o = +1487.0\ kJ\ mol^{-1}$
(ii) $2CO(g) + O_2(g) \rightarrow 2CO_2(g)$; $\Delta_rG^o = -514.4\ kJ\ mol^{-1}$
Free energy change, Δ_rG^o for the reaction $2Fe_2O_3(s) + 6CO(g) \rightarrow 4Fe(s) + 6CO_2(g)$ will be - **[JEE-Main On line-2018]**

(A) –112.4 kJ mol^{-1} (B) –56.2 kJ mol^{-1}
(C) –208.0 kJ mol^{-1} (D) –168.2 kJ mol^{-1}

Q.39 At a certain temperature in a 5L vessel, 2 moles of carbon monoxide and 3 moles of chlorine were allowed to reach equilibrium according to the reaction, $CO + Cl_2 \rightleftharpoons COCl_2$. At equilibrium, if one mole of CO is present then equilibrium constant (K_c) for the reaction is -

[JEE-Main On line-2018]

(A) 2.5 (B) 4 (C) 2 (D) 3

Q.40 The correct order of spin-only magnetic moments among the following is : (Atomic number : Mn = 25, Co = 27, Ni = 28, Zn = 30) **[JEE-Main On line-2018]**

(A) $[ZnCl_4]^{2-} > [NiCl_4]^{2-} > [CoCl_4]^{2-} > [MnCl_4]^{2-}$
(B) $[CoCl_4]^{2-} > [MnCl_4]^{2-} > [NiCl_4]^{2-} > [ZnCl_4]^{2-}$
(C) $[NiCl_4]^{2-} > [CoCl_4]^{2-} > [MnCl_4]^{2-} > [ZnCl_4]^{2-}$
(D) $[MnCl_4]^{2-} > [CoCl_4]^{2-} > [NiCl_4]^{2-} > [ZnCl_4]^{2-}$

Q.41 When 2-butyne is treated with H_2/Lindlar's catalyst, compound X is produced as the major product and when treated with Na/liq. NH_3 it produces Y as the major product. Which of the following statement is correct ? **[JEE-Main On line-2018]**

(A) Y will have higher dipole moment and higher boiling point that X
(B) Y will have higher dipole moment and lower boiling point that X
(C) X will have lower dipole moment and lower boiling point than Y

(D) X will have higher dipole moment and higher boiling point than Y

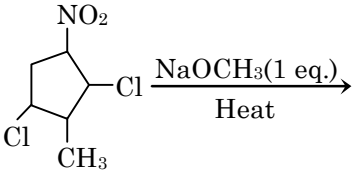
Q.42 For a first order reaction, A → P, $t_{1/2}$ (half-life) is 10 days. The time required for $\frac{1}{4}^{th}$ conversion of A(in days) is –
(ln2 = 0.693, ln 3 = 1.1)

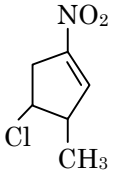
[JEE-Main On line-2018]

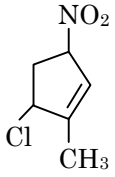
(A) 3.2 (B) 2.5 (C) 4.1 (D) 5

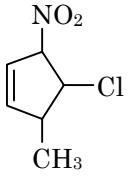
Q.43 The major product formed in the following reaction is -

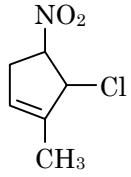
[JEE-Main On line-2018]



(A) 

(B) 

(C) 

(D) 

Q.44 The de-Broglie's wavelength of electron present in first Bohr orbit of 'H' atom is -

[JEE-Main On line-2018]

(A) 4 × 0.529 Å (B) 2π × 0.529 Å

(C) $\frac{0.529}{2\pi}$ Å (D) 0.529 Å

Q.45 Lithium aluminium hydride reacts with silicion tetrachloride to form -

[JEE-Main On line-2018]

(A) LiCl, AlH_3 and SiH_4
(B) LiCl, $AlCl_3$ and SiH_4
(C) LiH, $AlCl_3$ and $SiCl_2$
(D) LiH, AlH_3 and SiH_4

Q.46 The correct order of electron affinity is -

[JEE-Main On line-2018]

(A) O > F > Cl (B) F > O > Cl
(C) F > Cl > O (D) Cl > F > O

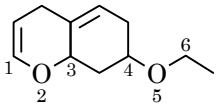
Q.47 Two 5 molal solutions are prepared by dissolving a non-electrolyte, non-volatile solute separately in the solvents X and Y. The molecular weights of the solvents are M_X and M_Y, respectively where $M_X = \frac{3}{4}M_Y$. The relative lowering of vapour pressure of the solution in X is "m" times that of the solution in Y. Given that the number of moles of solute is very small in comparison to that of solvent, the value of "m" is -

[JEE-Main On line-2018]

(A) $\frac{3}{4}$ (B) $\frac{1}{2}$ (C) $\frac{1}{4}$ (D) $\frac{4}{3}$

Q.48 On the treatment of the following compound with a strong acid, the most susceptible site for bond cleavage is -

[JEE-Main On line-2018]



(A) O2 – C3 (B) O5 – C6
(C) C4 – O5 (D) C1 – O2

Q.49 All of the following share the same crystal structure except -

[JEE-Main On line-2018]

(A) RbCl (B) NaCl
(C) CsCl (D) LiCl

Q.50 The total number of possible isomers for square-planar $[Pt(Cl)(NO_2)(NO_3)(SCN)]^{2-}$ is- **[JEE-Main On line-2018]**

(A) 16 (B) 12 (C) 8 (D) 24

Q.51 Two compounds I and II are eluted by column chromato-graphy(adsorption of I > II). Which one fo the following is a correct statement ?

[JEE-Main On line-2018]

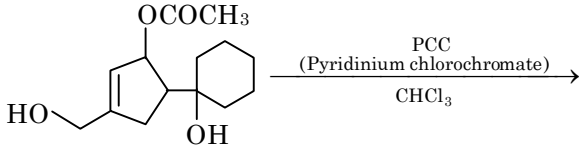
(A) II moves slower and has higher R_f value than I
(B) II moves faster and has higher R_f value than I
(C) I moves faster and has higher R_f value than II
(D) I moves slower and has higher R_f value than II

Q.52 The number of P–O bonds in P_4O_6 is -

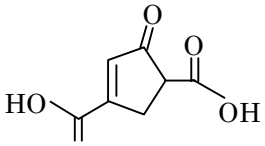
[JEE-Main On line-2018]

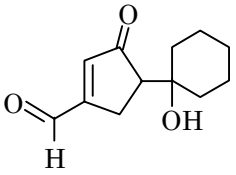
(A) 9 (B) 6 (C) 12 (D) 18

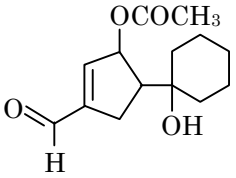
Q.53 The major product formed in the following reaction is -

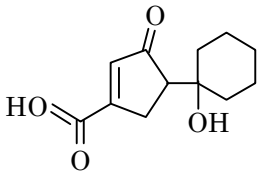


[JEE-Main On line-2018]

(A) 

(B) 

(C) 

(D) 

Q.54 For per gram of reactant, maximum quantity of N_2 gas is produced in which of the following thermal decomposition reactions ? (Given : Atomic wt. : Cr = 52 u, Ba = 137 u)

[JEE-Main On line-2018]

(A) $Ba(N_3)_2(s) \rightarrow Ba(C) + 3N_2(g)$

(B) $(NH_4)_2Cr_2O_7(s) \rightarrow N_2(g) + 4H_2O(g) + Cr_2O_3(s)$

(C) $2NH_3(g) \rightarrow N_2(g) + 3H_2(g)$

(D) $2NH_4NO_3(s) \rightarrow 2N_2(g) + 4H_2O(g) + O_2(g)$

Q.55 If x gram of gas is adsorbed by m gram of adsorbent at pressure P, the plot of $\log \frac{x}{m}$ versus log P is linear. The slope of the plot is : (n and k are constants and n > 1)

[JEE-Main On line-2018]

(A) log k (B) $\frac{1}{n}$ (C) 2k (D) n

Q.56 Biochemical oxygen demand (BOD) value can be a measure of water pollution caused by the organic matter. Which of the following statements is correct ?

(A) Polluted water has BOD value higher than 10 ppm

(B) Aerobic bacteria decreases the BOD value

(C) Anaerobic bacteria increases the BOD value

(D) Clean water has BOD value higher than 10 ppm

Q.57 In the leaching method, bauxite ore is digested with a concentrated solution of NaOH that produces 'X', when CO_2 gas is passed through the aqueous solution of 'X', a hydrated compound 'Y' is precipitated. 'X' and 'Y' respectively are -

[JEE-Main On line-2018]

(A) $Na[Al(OH)_4]$ and $Al_2(CO_3)_3 . xH_2O$

(B) $Al(OH)_3$ and $Al_2O_3 . xH_2O$

(C) $NaAlO_2$ and $Al_2(CO_3)_3 . xH_2O$

(D) $Na[Al(OH)_4]$ and $Al_2O_3 . xH_2O$

Q.58 Which of the following statements is not true ? **[JEE-Main On line-2018]**

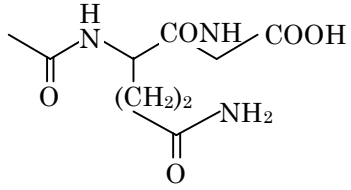
(A) Chain growth polymerization involves homopoly-merisation only

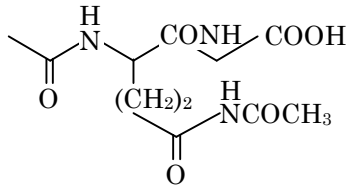
(B) Chain growth polymerization includes both homo-polymerisation and copolymerization

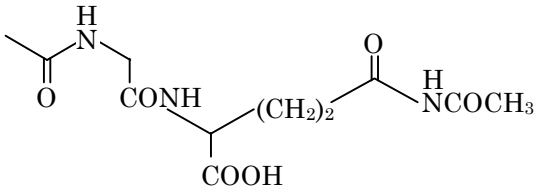
(C) Nylon 6 is an example of step-growth polyerisation

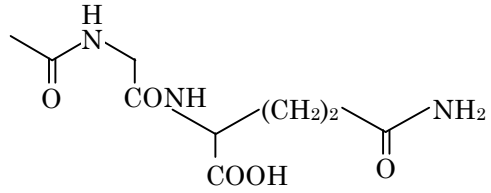
(D) Step growth polymerization requires a bifunctional monomer

Q.59 The dipeptide, Gln-Gly, on treatment with CH_3COCl followed by aqueous work up gives. **[JEE-Main On line-2018]**

(A) 

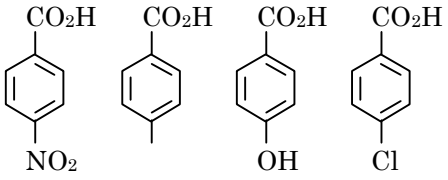
(B) 

(C) 

(D) 

Q.60 The increasing order of the acidity of the following carboxylic acids is -

[JEE-Main On line-2018]



I II III IV

(A) III < II < IV < I (B) I < III < II < IV

(C) IV < II < III < I (D) II < IV < III < I

MATHEMATICS

Q.61 Let $f : A \to B$ be a function defined as $f(x) = \frac{x-1}{x-2}$, where $A = R - \{2\}$ and $B = R - \{1\}$. Then f is -

[JEE-Main On line-2018]

(A) invertible and $f^{-1}(y) = \frac{2y+1}{y-1}$

(B) invertible and $f^{-1}(y) = \frac{3y-1}{y-1}$

(C) no invertible

(D) invertible and $f^{-1}(y) = \frac{2y-1}{y-1}$

Q.62 The coefficient of x^{10} in the expansion of $(1+x)^2(1+x^2)^3(1+x^3)^4$ is equal to -

[JEE-Main On line-2018]

(A) 52 (B) 44 (C) 50 (D) 56

Q.63 If the system of linear equations

$x + ay + z = 3$

$x + 2y + 2z = 0$

$x + 5y + 3z = b$

has no solution, then

[JEE-Main On line-2018]

(A) $a = 1, b \neq 9$ (B) $a \neq -1, b = 9$

(C) $a = -1, b = 9$ (D) $a = -1, b \neq 9$

Q.64 If f(x) is a quadratic expression such that $f(1) + f(2) = 0$, and -1 is a root of $f(x) = 0$, then the other root of $f(x) = 0$ is -

[JEE-Main On line-2018]

(A) $-\frac{5}{8}$ (B) $-\frac{8}{5}$ (C) $\frac{5}{8}$ (D) $\frac{8}{5}$

Q.65 The number of four letter words that can be formed using the letters of the word BARRACK is -

[JEE-Main On line-2018]

(A) 144 (B) 120 (C) 264 (D) 270

Q.66 The number of solutions of $\sin 3x = \cos 2x$, in the interval $\left(\frac{\pi}{2}, \pi\right)$ is -

[JEE-Main On line-2018]

(A) 3 (B) 4 (C) 2 (D) 1

Q.67 The curve satisfying the differential equation, $(x^2 - y^2)\,dx + 2xydy = 0$ and passing through the point (1, 1) is -

[JEE-Main On line-2018]

(A) a circle of radius two

(B) a circle of radius one

(C) a hyperbola

(D) an ellipse

Q.68 A player X has a biased coin whose probability of showing heads is p and a player Y has a fair coin. They start playing a game with their own coins and play alternately. The player who throws a head first is a winner. If X starts the game, and the probability of winning the game by both the players is equal, then the value of 'p' is -

[JEE-Main On line-2018]

(A) $\frac{1}{3}$ (B) $\frac{1}{5}$ (C) $\frac{1}{4}$ (D) $\frac{2}{5}$

Q.69 Consider the following two statements.

Statement p :

The value of sin 120° can be divided by taking $\theta = 240°$ in the equation

$2\sin\frac{\theta}{2} = \sqrt{1+\sin\theta} - \sqrt{1-\sin\theta}$.

Statement q :

The angles A, B, C and D of any quadrilateral ABCD satisfy the equation

$\cos\left(\frac{1}{2}(A+C)\right) + \cos\left(\frac{1}{2}(B+D)\right) = 0$

Then the truth values of p and q are respectively. **[JEE-Main On line-2018]**

(A) F, T (B) T, T (C) F, F (D) T, F

Q.70 $\int \frac{2x+5}{\sqrt{7-6x-x^2}}\,dx$

$= A\sqrt{7-6x-x^2} + B\sin^{-1}\left(\frac{x+3}{4}\right) + C$

(where C is a constant of integration), then the ordered pair (A, B) is equal to -

[JEE-Main On line-2018]

(A) (–2, –1) (B) (2, –1)

(C) (–2, 1) (D) (2, 1)

Q.71 A plane bisects the line segment joining the points (1, 2, 3) and (–3, 4, 5) at right angles. Then this plane also passes through the point **[JEE-Main On line-2018]**

(A) (–3, 2, 1) (B) (3, 2, 1)

(C) (1, 2, –3) (D) (–1, 2, 3)

Q.72 If $|z - 3 + 2i| \le 4$ then the difference between the greatest value and the least value of $|z|$ is -

[JEE-Main On line-2018]

(A) $\sqrt{13}$ (B) $2\sqrt{13}$ (C) 8 (D) $4+\sqrt{13}$

Q.73 If the position vectors of the vertices A, B and C of a ΔABC are respectively $4\hat{i}+7\hat{j}+8\hat{k}$, $2\hat{i}+3\hat{j}+4\hat{k}$ and $2\hat{i}+5\hat{j}+7\hat{k}$, then the position vector of the point, where the bisector of $\angle A$ meets BC is -

[JEE-Main On line-2018]

(A) $\frac{1}{2}(4\hat{i}+8\hat{j}+11\hat{k})$

(B) $\frac{1}{3}(6\hat{i}+13\hat{j}+18\hat{k})$

(C) $\frac{1}{4}(8\hat{i}+14\hat{j}+9\hat{k})$

(D) $\frac{1}{3}(6\hat{i}+11\hat{j}+15\hat{k})$

Q.74 The foot of the perpendicular drawn from the origin, on the line, $3x + y = \lambda(\lambda \ne 0)$ is P. If the line meets x-axis at A and y-axis at B, then the ratio BP : PA is -

[JEE-Main On line-2018]

(A) 9 : 1 (B) 1 : 3 (C) 1 : 9 (D) 3 : 1

Q.75 If $f(x) = \sin^{-1}\left(\frac{2\times 3^x}{1+9^x}\right)$, then $f'\left(-\frac{1}{2}\right)$ equals

[JEE-Main On line-2018]

(A) $\sqrt{3}\log_e\sqrt{3}$ (B) $-\sqrt{3}\log_e\sqrt{3}$

(C) $-\sqrt{3}\log_e 3$ (D) $\sqrt{3}\log_e 3$

Q.76 Let $A_n = \left(\frac{3}{4}\right) - \left(\frac{3}{4}\right)^2 + \left(\frac{3}{4}\right)^3 - \ldots + (-1)^{n-1}\left(\frac{3}{4}\right)^n$ and $B_n = 1 - A_n$. Then, the least odd natural number p, so that $B_n > A_n$, for all $n \ge p$ is -

[JEE-Main On line-2018]

(A) 5 (B) 7 (C) 11 (D) 9

Q.77 A normal to the hyperbola, $4x^2 - 9y^2 = 36$ meets the co-ordinate axes x and y at A and B, respectively. If the parallelogram OABP (O being the origin) is formed, then the locus of P is -

[JEE-Main On line-2018]

(A) $4x^2 - 9y^2 = 121$ (B) $4x^2 + 9y^2 = 121$

(C) $9x^2 - 4y^2 = 169$ (D) $9x^2 + 4y^2 = 169$

Q.78 Let f(x) be a polynomial of degree 4 having extreme values at x = 1 and x = 2.

If $\lim_{x\to 0}\left(\frac{f(x)}{x^2}+1\right) = 3$ then f(–1) is equal to -

[JEE-Main On line-2018]

(A) $\frac{1}{2}$ (B) $\frac{3}{2}$ (C) $\frac{5}{2}$ (D) $\frac{9}{2}$

Q.79 If the mean of the data : 7, 8, 9, 7, 8, 7, λ, 8 is 8, then the variance of this data is -

(A) $\frac{9}{8}$ (B) 2 (C) $\frac{7}{8}$ (D) 1

Q.80 An angle between the lines whose direction cosines are given by the equations, $\ell + 3m + 5n = 0$ and $5\ell m - 2mn + 6n\ell = 0$, is -

[JEE-Main On line-2018]

(A) $\cos^{-1}\left(\frac{1}{8}\right)$ (B) $\cos^{-1}\left(\frac{1}{6}\right)$

(C) $\cos^{-1}\left(\frac{1}{3}\right)$ (D) $\cos^{-1}\left(\frac{1}{4}\right)$

Q.81 The tangent to the circle $C_1 : x^2 + y^2 - 2x - 1 = 0$ at the point (2, 1) cuts off a chord of length 4 from a circle C_2 whose center is (3, –2). The radius of C_2 is -

[JEE-Main On line-2018]

(A) $\sqrt{6}$ (B) 2 (C) $\sqrt{2}$ (D) 3

Q.82 Suppose A is any 3×3 non-singular matrix and $(A - 3I)(A - 5I) = O$, where $I = I_3$ and $O = O_3$. If $\alpha A + \beta A^{-1} = 4I$, then $\alpha + \beta$ is equal to -

[JEE-Main On line-2018]

(A) 8 (B) 12 (C) 13 (D) 7

Q.83 The value of integral $\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{x}{1+\sin x}\,dx$ is -

[JEE-Main On line-2018]

(A) $\frac{\pi}{2}(\sqrt{2}+1)$ (B) $\pi(\sqrt{2}-1)$

(C) $2\pi(\sqrt{2}-1)$ (D) $\pi\sqrt{2}$

Q.84 A tower T_1 of height 60 m is located exactly opposite to a tower T_2 of height 80 m on a straight road. From the top of T_1, if the angle of depression of the foot of T_2 is twice the angle of elevation of the top of T_2, then the width (in m) of the road between the feet of the towers T_1 and T_2 is -

[JEE-Main On line-2018]

(A) $20\sqrt{2}$ (B) $10\sqrt{2}$ (C) $10\sqrt{3}$ (D) $20\sqrt{3}$

Q.85 If $I_1 = \int_0^1 e^{-x}\cos^2 x\,dx$; $I_2 = \int_0^1 e^{-x^2}\cos^2 x\,dx$ and $I_3 = \int_0^1 e^{-x^3}\,dx$; then -

[JEE-Main On line-2018]

(A) $I_2 > I_3 > I_1$ (B) $I_3 > I_1 > I_2$

(C) $I_2 > I_1 > I_3$ (D) $I_3 > I_2 > I_1$

Q.86 The sides of a rhombus ABCD are parallel to the lines, $x - y + 2 = 0$ and $7x - y + 3 = 0$. If the diagonals of the rhombus intersect at P(1, 2) and the vertex A (different from the origin) is on the y-axis, then the ordinate of A is - **[JEE-Main On line-2018]**

(A) 2 (B) $\frac{7}{4}$ (C) $\frac{7}{2}$ (D) $\frac{5}{2}$

Q.87 $\lim_{x\to 0} \frac{x\tan 2x - 2x\tan x}{(1-\cos 2x)^2}$ equals -

[JEE-Main On line-2018]

(A) 1 (B) $-\frac{1}{2}$ (C) $\frac{1}{4}$ (D) $\frac{1}{2}$

Q.88 Let $f(x) = \begin{cases} (x-1)^{\frac{1}{2-x}}, & x > 1, x \neq 2 \\ k, & x = 2 \end{cases}$

The value of k for which f is continuous at x = 2 is - **[JEE-Main On line-2018]**

(A) e^{-2} (B) e (C) e^{-1} (D) 1

Q.89 If a, b, c are in A.P. and a^2, b^2, c^2 are in G.P. such that $a < b < c$ and $a + b + c = \frac{3}{4}$, then the value of a is -

[JEE-Main On line-2018]

(A) $\frac{1}{4} - \frac{1}{3\sqrt{2}}$ (B) $\frac{1}{4} - \frac{1}{4\sqrt{2}}$

(C) $\frac{1}{4} - \frac{1}{\sqrt{2}}$ (D) $\frac{1}{4} - \frac{1}{2\sqrt{2}}$

Q.90 Tangents drawn from the point (–8, 0) to the parabola $y^2 = 8x$ touch the parabola at P and Q. If F is the focus of the parabola, then the area of the triangle PFQ (in sq. units) is equal to -

[JEE-Main On line-2018]

(A) 48 (B) 32 (C) 24 (D) 64

ANSWERS

1. (4) **2.** (1) **3.** (1) **4.** (3) **5.** (1) **6.** (2) **7.** (2) **8.** (3) **9.** (2) **10.** (1) **11.** (3) **12.** (4) **13.** (4) **14.** (4)

15. (3) **16.** (1) **17.** (1) **18.** (3) **19.** (1) **20.** (3) **21.** (4) **22.** (1) **23.** (1) **24.** (3) **25.** (1) **26.** (1) **27.** (3) **28.** (4)

29. (4) **30.** (4) **31.** (2) **32.** (3) **33.** (2) **34.** (3) **35.** (1) **36.** (4) **37.** (2) **38.** (2) **39.** (1) **40.** (4) **41.** (4) **42.**(3)

43. (1) **44.** (2) **45.** (2) **46.** (4) **47.** (1) **48.** (2) **49.** (4) **50.** (2) **51.** (2) **52.** (3) **53.** (2) **54.** (3) **55.** (2) **56.** (1)

57. (4) **58.** (2) **59.** (1) **60.** (1) **61.** (4) **62.** (1) **63.** (4) **64.** (4) **65.** (4) **66.** (4) **67.** (2) **68.** (1) **69.** (1) **70.**(1)

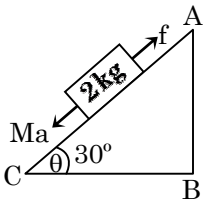
71. (1) **72.** (2) **73.** (2) **74.** (4) **75.** (1) **76.** (2) **77.** (3) **78.** (4) **79.** (4) **80.** (2) **81.** (1) **82.** (1) **83.** (1) **84.** (4)

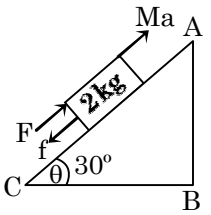
85. (4) **86.** (4) **87.** (4) **88.** (3) **89.** (4) **90.** (1)

Hints & Solutions

PHYSICS

1.[4] Equation of motion when the mass slides down





$Mg \sin\theta - f = Ma$

$\Rightarrow \quad 10 - f = 6$

$(M = 2kg, a = 3\ m/s^2, \theta = 30^o$ given)

$\therefore \quad f = 4N$

Equation of motion when the block is pushed up

Let the external force required to take the block up the plane with same acceleration be F

$F - Mg \sin\theta - f = Ma$

$\Rightarrow \quad F - 10 - 4 = 6$

$F = 20\ N$

2.[1] Using $E = E_0 - e^{i(kz - \omega t)}$

Given, at $t = t_1, z = z_1, E = 0$

The next zero that occurs in it's neighbourhood is at z_2, the frequency of the electromagnetic wave at t_2

$$e^{i(kz_1 - \omega t_1)} = e^{i(kz_2 - \omega t_2)}$$

$$kz_1 - \omega t_1 = kz_2 - \omega t_2$$

$$(t_2 - t_1)\,\omega = k(z - z_1)$$

Where $k = \dfrac{2\pi}{\lambda} = 2\pi v$

$$(t_2 - t_1) = \frac{2\pi}{\lambda \times 2\pi v}(z_2 - z_1)$$

$$= \frac{1}{x \times v}(z_2 - z_1)$$

$$\Rightarrow \quad \lambda \times v = \frac{(z_2 - z_1)}{(t_2 - t_1)} = C$$

$$(t_2 - t_1) = \frac{(z_2 - z_1)}{C}$$

Frequency is $f \propto \dfrac{1}{t}$ then $\dfrac{1}{(t_2 - t_1)} = \dfrac{C}{(z_2 - z_1)}$

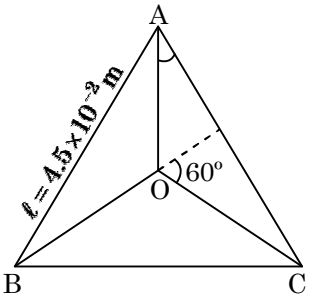
$\therefore$ Frequency, $f = \dfrac{3 \times 10^8}{(z_2 - z_1)}$

3.[1] Here, side of the triangle, $\ell = 4.5 \times 10^{-2}$ m, current, I = 1A

Magnetic field at the centre of the triangle 'O' B = ?

From figure, $\tan 60^o = \sqrt{3} = \dfrac{1}{2d}$

$\Rightarrow$ $d = \frac{1}{2\sqrt{3}}\left(\frac{4.5\times10^{-2}}{2\sqrt{3}}\right)m$



Magnetic field, $B = \frac{\mu_0 i}{4\pi d}(\cos\theta_1 + \cos\theta_2)$

Putting value of $\mu = 4\pi \times 10^{-7}$ and θ_1 and θ_2 we will get net magnetic field

$= 3 \times B = 4 \times 10^{-5}$ Wb/m^2

4.[3] Truth table of the circuit is as follows

x	y	$\bar{x}$	a = x.y	b – $\bar{x}$.y	z = $\overline{ab}$
0	0	1	0	0	1
0	1	1	0	1	1
1	0	0	0	0	1
1	1	0	1	0	1

5.[1] Rate of heat i.e., Power developed in the wire $= P = \frac{V^2}{R}$

Resistance of the wire of length, L

$$R_1 = \frac{\rho L}{A} = \frac{\rho L}{\pi r^2}$$

$\therefore$ Power, $P_1 = \frac{V^2}{R_1}$

Resistance of the wire when length is halved i.e., L/2

$$R_2 = \frac{\rho\frac{L}{2}}{\pi(2r)^2} = \frac{\rho L}{\pi 8r^2} = \frac{R_1}{8}$$

$\therefore$ Power, $P_2 = \frac{V}{\frac{R_1}{8}} = \frac{8V}{R_1}$

Or, $P_2 = 8P_1$ i.e., power increased 8 times of previous or original wire.

6.[2] Plank length is a limit of length,

$\ell_p = 1.616229 \times 10^{-35}$ m

$$\ell_p = \sqrt{\frac{hG}{c^3}}$$

7.[2] For same material the ratio of stress to strain is same

For first cube

$$\text{Stress}_1 = \frac{\text{force}_1}{\text{area}_1} = \frac{10^5}{(0.1)^2}$$

$$\text{Strain}_1 = \frac{\text{change in legnth}_1}{\text{orignal length}_1}$$

$$= \frac{0.5\times10^{-7}}{0.1}$$

For second block,

$$\text{Stress}_2 = \frac{\text{force}_2}{\text{area}_2} = \frac{10^5}{(0.2)^2}$$

$$\text{Strain}_2 = \frac{\text{change in length}_2}{\text{original length}_2} = \frac{x}{0.2}$$

x is the displacement for second block.

For same material, $\frac{\text{stress}_1}{\text{strain}_1} = \frac{\text{stress}_2}{\text{strain}_2}$

or, $\frac{\frac{10.5}{(0.1)^2}}{\frac{0.5\times10^{-2}}{0.1}} = \frac{\frac{10^5}{(0.2)^2}}{\frac{x}{0.2}}$

Solving we get, x = 0.25 cm

8.[3] Given : Inductance, L = 49 μH = 49 × 10^{-6}H, capacitance C = 2.5 nF = 2.5 × 10^{-9}F

Using $\omega = \frac{1}{\sqrt{LC}}$

$$= \frac{1}{\sqrt{49\times10^{-6}\times\frac{2.5}{10}\times10^{-9}}}$$

$$= \frac{1}{7\times5\times10^{-8}} = \frac{10^8}{7\times5}$$

or, $\frac{10^8}{7\times5} = 2\pi\times f$

$$= 2\times\frac{22}{7}\times f \quad (\because\ \omega = 2\pi f)$$

or, $f = \frac{10^7}{22} = \frac{10^4}{22}$ kHz = 454.54 kHz

Therefore frequency range 454.54 ± 12kHz

i.e., 442 kHz – 466 kHz

9.[2] From Faraday's law of electromagnetic induction,

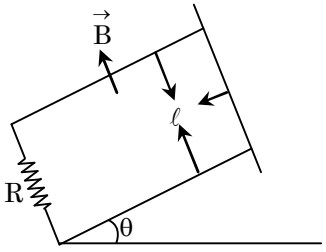
$$e = \frac{d\phi}{dt} = \frac{d(BA)}{dt} = \frac{d(B\ell\ell)}{dt}$$

$$= \frac{Bd\ell \times \ell)}{dt} = Bv\ell$$

Also, $F = i\ell B = \left(\frac{BV}{R}\right)(\ell^2 B)$

$$= \frac{B^2\ell^2 V}{R}$$

At equilibrium



$$mg \sin\theta = \frac{B^2\ell^2 V}{R}$$

$$\Rightarrow \quad V = \frac{mgR\sin\theta}{B^2\ell^2}$$

10.[1] When the rod makes an angle α

Displacement of centre of mass = $\frac{\ell}{2}\cos\alpha$

$$mg\ \frac{\ell}{2}\cos\alpha = \frac{\ell}{2} I\omega^2$$

$$mg\ \frac{\ell}{2}\cos\alpha = \frac{m\ell^2}{6}\ \omega^2$$

(∵ M.I. of thin uniform rod about an axis passing through its center of mass and perpendicular to the rod $I = \frac{m\ell^2}{12}$)

$$\Rightarrow \quad \omega = \sqrt{\frac{3g\cos\alpha}{\ell}}$$

Speed of end = $\omega \times \ell = \sqrt{3g\cos\alpha\ell}$

i.e., speed of end, $\omega \propto \sqrt{\cos\alpha}$

11.[3] Given area of parallel plate capacitor,

$A = 200\ cm^2$

Separation between the plates,

$d = 1.5\ cm$

Force of attraction between the plates,

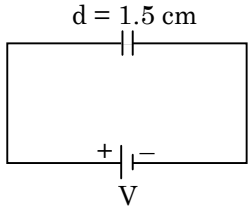
$F = 25 \times 10^{-6}\ N$

$F = QE$

$$F = \frac{Q^2}{2A\in_0}$$

(E due to parallel plate = $\frac{\sigma}{2\in_0} = \frac{Q}{A2\in_0}$)

But $Q = CV = \frac{\in_0 A(V)}{d}$



$$\therefore \quad F = \frac{(\in_0 AV^2)}{d^2 \times 2A\in_0}$$

$$= \frac{(\in_0 A)^2 \times V^2}{d^2 \times 2 \times (A\in_0)}$$

$$= \frac{(\in_0 A) \times V^2}{d^2 \times 2}$$

or, $$25 \times 10^{-6} = \frac{(8.85\times10^{-12}) \times 2.25\times10^{-4} \times V^2}{2.25\times10^{-4} \times 2}$$

$$\Rightarrow V = \frac{25\times10^{-6} \times 2.25\times10^{-4} \times 2}{8.85\times10^{-12} \times 200\times10^{-4}} \approx 250\ V$$

12.[4] Charge density, $\rho = \rho_0\left(1 - \frac{r}{R}\right)$

$$dq = \rho dv$$

$$q_{in} = \int dq = \rho dv$$

$$= \rho_0\left(1 - \frac{r}{R}\right)4\pi r^2 dr \qquad (\because\ dv = 4\pi r^2 dr)$$

$$= 4\pi\rho_0 \int_0^R \left(1 - \frac{r}{R}\right) r^2 dr$$

$$= 4\pi\rho_0 \int_0^R r^2 dr - \frac{r^2}{R} dr$$

$$= 4\pi\rho_0 \left[\left[\frac{r^3}{3}\right]_0^R - \left[\frac{r^4}{4R}\right]_0^R\right]$$

$$= 4\pi\rho_0 \left[\frac{R^3}{3} - \frac{R^4}{4R}\right]$$

$$= 4\pi\rho_0\left[\frac{R^3}{3} - \frac{R^3}{4}\right]$$

$$= 4\pi\rho_0\left[\frac{R^3}{12}\right]$$

$$q = \frac{\pi\rho_0 R^3}{3}$$

$$E \cdot 4\pi\rho^2 = \left(\frac{\pi\rho_0 R^3}{3\epsilon_0}\right)$$

$\therefore$ Electric field outside the ball,

$$E = \frac{\rho_0 R^3}{12\epsilon_0 r^2}$$

13.[4] Apply principle of conservation of momentum along x-direction,

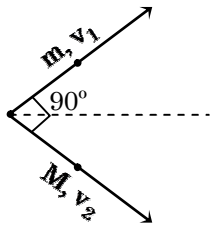
$$mu = mv_1 \cos 45^\circ + Mv_2 \cos 45^\circ$$

$$mu = \frac{1}{\sqrt{2}}(mv_1 + Mv_2) \quad \ldots(i)$$

Along y-direction

$$o = mv_1 \sin 45^\circ - Mv_2 \sin 45^\circ$$

$$o = (mv_1 - Mv_2)\frac{1}{\sqrt{2}} \quad \ldots(ii)$$



m, $u_1 = u$ M, $u_2 = 0$

Proton Unknown mass After collision

Before collision

Coefficient of restitution e = 1

$$= \frac{v_2 - v_1\cos 90^\circ}{u\cos 45^\circ} \quad (\because \text{ collision is elastic})$$

$$\Rightarrow \frac{v_2}{\frac{u}{\sqrt{2}}} = 1$$

$$\Rightarrow \quad u = \sqrt{2}\, v_2 \quad \ldots(iii)$$

Solving eqs(i), (ii), & (iii), we get mass of unknown particle, M = m

14.[4] Using, $\mu mg = \frac{mv^2}{r} = mr\omega^2$

$\omega = 2\pi n$

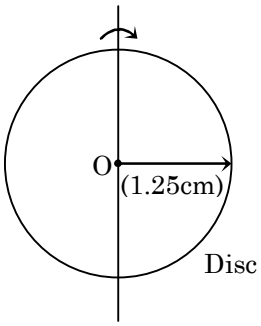
$= 2\pi \times 3.5$

$= 7\pi$ rad/sec

Radius, r = 1.25 cm = 1.25×10^{-2} m

Coefficient of friction, μ = ?

$$\mu mg = \frac{m(r\omega)^2}{r} \quad (\because v = r\omega)$$



$\mu mg = mr\omega^2$

$$\Rightarrow \quad \mu = \frac{r\omega^2}{g} = \frac{1.25\times10^{-2}\times\left(7\times\frac{22}{7}\right)^2}{10}$$

$$= \frac{1.25\times10^{-2}\times22^2}{10} = 0.6$$

15.[3] According to Faraday's law of electromagnetic induction,

$$e = \frac{d\phi}{dt} \text{ and } \phi = BA\cos\omega t$$

$$= B\pi r^2 \cos\omega t$$

$$\Rightarrow \quad e = -\frac{d}{dt}(\pi r^2 B\cos\omega t) = \pi r^2 B\sin\omega t$$

(ω)

$$\therefore \quad e = \frac{\mu_0 I}{2R}\pi\omega r^2 \sin\omega t \quad \left(\because \quad B = \frac{\mu_0 I}{2R}\right)$$

16.[1] The sum of final charges on C_2 and C_3 is 36 μC.

17.[1] Probable frequencies of tuning fork be $n \pm 5$.

Frequency of sonometer wire, $n \propto \frac{1}{\ell}$

$$\therefore \quad \frac{n+5}{n-5} = \frac{100}{95}$$

$$\Rightarrow \quad 95(n+5) = 100(n-5)$$

or, $95n + 475 = 100n - 500$

or, $5n = 975$

or, $n = \frac{975}{5} = 195$ Hz

18.[3] From the two mutually perpendicular S.H.M. 's, the general equation of Lissajous figure,

$$\frac{x^2}{A^2} + \frac{y^2}{B^2} - \frac{2xy}{AB}\cos\delta = \sin^2\delta$$

$x = A \sin(at + \delta)$

$y = B \sin(bt + r)$

Clearly $A \neq B$ hence ellipse.

19.[1] Let the car turn of the highway at a distance 'x' from the point M. So, RM = x And if speed of car in field is v, then time taken by the car to cover the distance QR = QM – x on the highway.

$$t_1 = \frac{QM - x}{2v} \quad \ldots(i)$$

Time taken to travel the distance 'RP' in the field

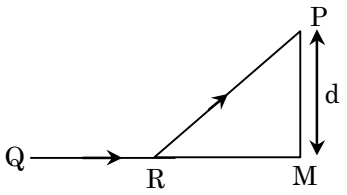
$$t_2 = \frac{\sqrt{d^2 + x^2}}{v} \quad \ldots(ii)$$

Total time elapsed to move the car from Q to P

$$t = t_1 + t_2$$

$$= \frac{QM - x}{2v} + \frac{\sqrt{d^2 + x^2}}{v}$$

For 't' to be minimum $\frac{dt}{dx} = 0$



$$\frac{1}{v}\left[-\frac{1}{2} + \frac{x}{\sqrt{d^2 + x^2}}\right] = 0$$

or $x = \frac{d}{\sqrt{2^2 - 1}} = \frac{d}{\sqrt{3}}$

20.[3] Charge density

$$\rho = \frac{\text{charge}}{\text{volume}} = \frac{q}{Ad}$$

$\Rightarrow q = \rho Ad$

Also, $q = IT$

$$\Rightarrow T = \frac{q}{I} = \frac{\rho Ad}{I}$$

21.[4] Efficiency of engine A, $\eta_A = \frac{T_1 - T_2}{T_1}$

and $\eta_B = \frac{T_2 - T_3}{T_2}$;

$$T_2 = \frac{T_1 + T_3}{2} = 350 \text{ K}$$

or $\frac{\eta_A}{\eta_B} = \frac{\frac{600 - 350}{600}}{\frac{350 - 100}{350}} = \frac{7}{12}$

22.[1] For minimum spherical aberration separation,

$d = f_1 - f_2 = 2$ cm

Resultant focal length = F = 10 cm

Using $\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$ and solving, we get f_1, f_2 18 cm and 20 cm respectively.

23.[1]

24.[3] Let heavy nucleus breaks into two nuclei of mass m_1 and m_2 and move away with velocities V_1 and V_2 respectively.

According to question, $\frac{V1}{V_2} = \frac{8}{27}$

$m_1V_1 = m_2V_2$ (Law of momentum conservation)

$$\Rightarrow \frac{m_1}{m_2} = \frac{V_2}{V_1} = \frac{27}{8}$$

$$\frac{\rho \times \frac{4}{3}\pi R_1^3}{\rho \times \frac{4}{3}\rho R_2^3} \left(\because \text{density } \rho = \frac{\text{mass}}{\text{volume}}\right)$$

$$\Rightarrow \left(\frac{R_1}{R_2}\right) = \left(\frac{27}{8}\right)^{\frac{1}{3}} = \left(\frac{3}{2}\right)^{3 \times \frac{1}{3}}$$

$$\therefore \frac{R_1}{R_2} = \frac{3}{2}$$

25.[1] According to Newtoni's law of colling,

$$\left(\frac{\theta_1 - \theta_2}{t}\right) = K\left(\frac{\theta_1 + \theta_2}{2} - \theta_0\right)$$

$$\left(\frac{60 - 50}{10}\right) = K\left(\frac{60 + 50}{2} - 25\right) \quad \ldots(i)$$

and, $\left(\frac{50-\theta}{10}\right) = K\left(\frac{50+\theta}{2}-25\right)$ ...(ii)

Dividing eq. (i) by (ii),

$\frac{10}{(50-\theta)} = \frac{60}{\theta} \Rightarrow \theta = 42.85^{\circ}C \cong 43^{\circ}C$

26.[1]

27.[3] According to question, $\lambda_p = \lambda_\alpha$

Using, $\lambda = \frac{h}{p} = \frac{h}{mv}$

So, $\frac{h}{m_p \times v_p} = \frac{h}{m_\alpha \times v_\alpha}$

$\Rightarrow \frac{v_p}{v_\alpha} = \frac{m_\alpha}{m_p} = \frac{4m_p}{m_p}$

($\because$ mass of α-particle is 4 times of mass of proton)

So, $\frac{v_p}{v_\alpha} = \frac{4}{1}$; i.e., 4 : 1

28.[4] Using $P_1V_1 = P_2V_2$

$(P_1)\ \frac{4}{3}\pi r^3 = (P_2)\ \frac{4}{3}\pi\ \frac{125r^3}{64}$

$\frac{\rho g(10)+\rho gh}{\rho g(10)} = \frac{125}{64}$

$640 + 64\ h = 1250$

On solving we get h = 9.5 m

29.[4] (A) Radius of muon = $\frac{\text{Radius of hydrogen}}{200}$

Radius of atom = $r = \frac{\epsilon_0\ n^2h^2}{\pi me^2}$

Radius of muon = $r_\mu = \frac{\epsilon_0\ n^2h^2}{\pi \epsilon 200me^2}$

$r_\mu = \frac{r}{100}$

(B) Velocity relation given is wrong

(C) Ionization energy in e–H atom

$E = \frac{+me^4}{8\epsilon_0^2\ n^2h^2}$

$E_\mu = \frac{200me^4}{8\epsilon_0^2\ n^2h^2} = 200\ E$

$mvr = \frac{nh}{2\pi}$

Hence (A), (C), (D) are correct.

30.[4] $v = \sqrt{3k_BT/m_{He}}$

$= \sqrt{\frac{3\times1.4\times10^{-23}\times300}{7\times10^{-27}}}$

$= 1.34 \times 10^3$ m/s

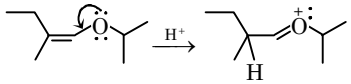
CHEMISTRY

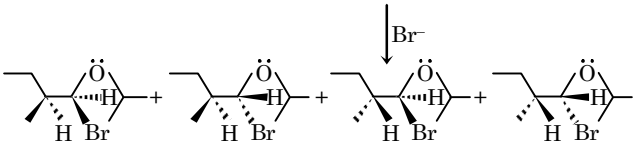
31.[2] Aromatic diazonium salts are more stable than aliphatic diazonium salts. The higher stability of aryl diazonium salts is due to resonance. Electron donating substituents increase electron density on benzene ring. Hereby they increase the stability of diazonium salts. Electron withdrawing substituents decrease electron density on benzene ring. Hereby they decrease the stability of diazonium salts. Electron withdrawing substituents decrease electron density on benzene ring. Hereby they decrease the stability of diazonium salts. $-COCH_3$ group is electron withdrawing and hence, diazonium salts from (D) is less stable than that from (B). Although $-O-COCH_3$ is electron donating substituent, but it is presenting meta position. Hence, it wil not have significant effect on stability.

The increasing order of diazotisation is

(A) < (D) < (B) < (C).

32.[3] The total number of optically active compounds formed is four. The product has two chiral C atoms. Thus, it has $2^n = 2^2 = 4$ stereoisomers.





33.[2] In KO_2, the nature of oxygen species and the oxidation state of oxygen atom are superoxide (superoxide ion is O_2^-) and –1/2 respectively.

Let x be oxidation state of oxygen. The oxidation state of K is + 1. Hence

$+1 + 2\ (x) = 0$

$2x = -1$

$$x = -\frac{1}{2}$$

34.[3] $\Delta G^{o}_{r\times n} = \Delta_f G^{o}$ (vapour) – $\Delta_f G^{o}$(liquid)

$\Delta G^{o}_{r\times n} = 103 - 100.7 = 2.3$ kcal/mol

$= 2300$ cal/mol

$\Delta G^{o}_{r\times n} = -RT \ln K$

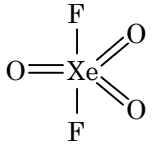
2300 cal/mol= –2 cal/mol K × 500 K × ln K

$\ln K = 2.3$

K = 10 atm = Vapour pressure of liquid 'S'

∴ Vapour pressure of liquid 'S' at 500 K is approximately equal to 10 atm.

35.[1] Structure of XeO_3F_2



36.[4] An antibonding π orbital best describes the diagram of a molecular orbital. Two orbital laterally overlap to form pi bond. Out of phase combination, of these two p-orbitals, give p^* MO.

37.[2] 75 mL $\frac{M}{5}$ HCl + 25 mL $\frac{M}{5}$ NaOH

25 mL $\frac{M}{5}$ NaOH will neutralize 25 mL $\frac{M}{5}$ HCl

$75 - 25 = 50$ mL $\frac{M}{5}$ HCl will remain.

Total volume will be 75 + 25 = 100 mL

50 mL $\frac{M}{5}$ HCl is diluted to 100 mL

$$[H^+] = [HCl] = \frac{M}{5}\times\frac{50}{100} = \frac{M}{10}$$

$$pH = -\log_{10}[H^+] = -\log_{10}\frac{M}{10} = 1$$

38.[2] (i) $2Fe_2O_3(s) \rightarrow 4Fe(s) + 3O_2(g)$;

$\Delta_r G^{o} = +1487.0$ kJ mol^{-1}

(ii) $2CO(g) + O_2(g) \rightarrow 2CO_2(g)$

$\Delta_r G^{o} = -514.4$ kJ mol^{-1}

Multiple reaction (ii) with 3, we get

(iii) $6CO(g)\ \ 3O_2(g) \rightarrow 6CO_2(g)$;

$\Delta_r G^{o} = 3 \times -514.4 = -1543.2$ kJmol^{-1}

When we add reaction (i) and reaction (iii), we get reaction (iv)

(iv) $2Fe_2O_3(s) + 6CO(g) \rightarrow 4Fe(s) + 6CO_2(g)$

Free energy change, $\Delta_r G^{o}$ for the reaction will be

$1487.0 - 1543.2 = -56.2$ kJ mol^{-1}

39.[1] Initially 2 moles of CO are present

At equilibrium, 1 mole of CO is present

Hence, 2 – 1 – 1 moles of CO has reacted. 1 mole of CO will react with 1 mole of Cl_2 to form 1 mole of $COCl_2$.

3 – 1 = 2 moles of Cl_2 remains at equilibrium \. The equilibrium constant

$$K_c = \frac{[COCl_2]}{[CO][Cl_2]} = \frac{\frac{1\text{ mol}}{5L}}{\frac{1\text{ mol}}{5L}\times\frac{2\text{ mol}}{5L}} = 2.5$$

40.[4] The complex having higher number of unpaired electrons will have higher value of spin only magnetic moment.

In all these complexes, the central metal ion is in +2 oxidation state.

Zn^{2+} has $3d^{10}$ outer electronic configuration with 0 unpaired electron.

Ni^{2+} has $3d^8$ outer electronic configuration with 2 unpaired electrons.

Co^{2+} has $3d^7$ outer electronic configuration with 3 unpaired electrons.

Mn^{2+} has $3d^5$ outer electronic configuration with 5 unpaired electrons

Hence the correct order of spin-only magnetic moments is

$[MnCl_4]^{2-} > [CoCl_4]^{2-} > [NiCl_4]^{2-} > [ZnCl_4]^{2-}$.

41.[4] When 2-butyne is treated with H_2/Lindlar's catalyst, compound X (cis-2-butene) is produced as the major product : and when treated with Na/liq NH_3 it produces Y (trans-2-butene) as the major product. Cis-isomer (X) will have higher dipole moment and higher boiling point than trans(Y).

42.[3] The half life $t_{1/2} = 10$ days.

The decay constant

$$k = \frac{0.693}{t_{1/2}} = \frac{0.693}{10\text{days}}$$

$= 0.0693$ days^{-1}

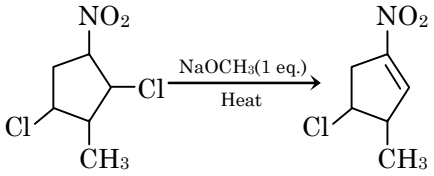
The time required for one fourth conversion

$$t = \frac{2.303}{k} \log_{10} \frac{a}{a-x}$$

$$= \frac{2.303}{0.0693\ day^{-1}} \log_{10} \frac{1}{1-(1/4)}$$

$$= 4.1 \text{ days}$$

43.[1] Note : In the given reaction a molecule of HCl is lost and C = C double bond is formed. Thus it is dehydrohalogenation reaction.
Nitro group is electron withdrawing group. Hence increase the acidity of H atom (attached to C atom bearing nitro group) which is removed easily. Further the newly formed double bond is in conjugation with nitro group.



44.[2] First Bohr orbit of H atom has radius

$$r = 0.529\ Å$$

Also, the angular momentum is quantized.

$$mvr = \frac{h}{2\pi}$$

$$2\pi r = \frac{h}{mv} = \lambda$$

$$\lambda = 2\pi \times 0.529\ Å$$

45.[2] $SiCl_4 + LiAlH_4 \rightarrow LiCl + AlCl_3 + SiH_4$

46.[4] On moving from left to right across a period, the electron affinity becomes more negative. On moving from top to bottom in a group, the electron affinity becomes less negative.
Chlorine ahs more negative electron affinity than fluorine. Because adding an electron to fluorine (2p orbital) causes greater repulsion than adding an electron to chlorine (3p orbital) which is larger in size.

47.[1] The relationship between molar masses of the two solvents is

$$M_X = \frac{3}{4} M_Y \qquad \ldots(i)$$

The relative lowering of vapour pressure of the two solutions is

$$\left(\frac{\Delta P}{P}\right)_X = m \left(\frac{\Delta P}{P}\right)_Y$$

But, the relative lowering of vapour pressure of solutions is directly proportional to the mole fraction of solute.
Given 5 molal solution, means 5 mole of solute are dissolved in 1 kg (or 1000 g) of solvent.

The number of moles of solvent = $\frac{1000g}{M}$

The mole fraction of solute = $\frac{5}{1000/M}$

$$= M \times \frac{5}{1000}$$

Hence

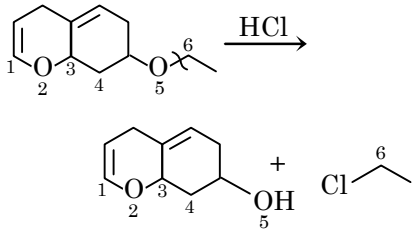
$$M_X \times \frac{5}{1000} = m \times M_Y \times \frac{5}{1000} \qquad \ldots(ii)$$

Substitute equation (i) in equation (ii)

$$\frac{3}{4} \times M_Y \times \frac{5}{1000} = m \times M_Y \times \frac{5}{1000}$$

$$m = \frac{3}{4}$$

48. [2] The lone pair of electrons on O2 is involved in resonance with C = C. Hence O2 will not be protonated.
The lone pair of electrons on O5 is not involved in resonance with C = C. Hence, O5 will be protonated. Chloride ion will then attack least substituted C atom (C6)



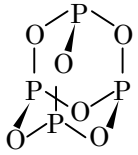
49.[4] RbCl , NaCl and CsCl share the same crystal structure except LiCl.
LiCl is deliquescent. It crystalises as a hydrated LiCl . $2H_2O$. Other alkali metal chlorides do not form hydrates.

50.[2] The square planar complex of the type $[Mabcd]^{n\pm}$, where all four ligands are different, has 3 geometrical isomers. But if one of the ligands is abidentate, then 2 × 3 – 6 geometrical isomers are possible. But if two ligands are ambidentate, then 4 × 3 = 12 geometrical isomers are possible.

In the given example, NO_2^- and SCN^- are ambidentate ligands.

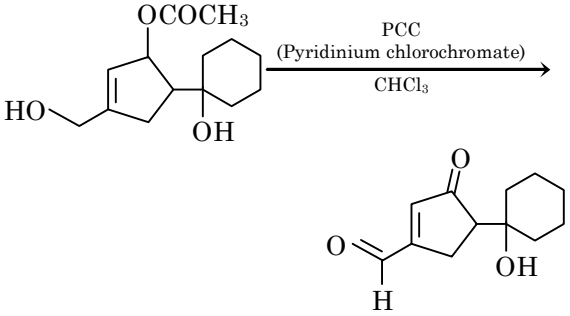
51.[2] Since, adsorption of 1 > II, I is firmly attached to column (stationary phase). Hence, it will move slowly and will move little distance. Also II is loosely attached to column (stationary phase). Hence, it will move faster and will move large distance.

52.[3] The number of P–O bonds in P_4O_6 = 12



53.[2] PCC oxidizes primary alcohols to aldehydes and secondary alcohols to ketones.

IN the above reaction, $-OCOCH_3$ group is hydrolyzed to secondary alcohol which is then oxidized (with PCC) to ketone.



54.[3] (a) Molar mass of $Ba(N_3)_2(s)$ = 221 g/mol

1 mole of $Ba(N_3)_2(s)$ will give 3 moles of N_2

Hence $\frac{1g}{221g/mol}$ moles of $Ba(N_3)_2(s)$ will give $3 \times \frac{1}{221} = 0.014$ moles of N_2

(b) Molar mass of $(NH_4)_2Cr_2O_7$ = 252 g/mol.

1 mole of $(NH_4)_2Cr_2O_7$ will give 1 mole of N_2

Hence $\frac{1g}{252g/mol}$ moles of $(NH_4)_2Cr_2O_7$ will give $1 \times \frac{1}{252} = 0.0039$ moles of N_2

(c) Molar mass of NH_3 = 17 g/mol

2 mole of NH_3 will give 1 mole of N_2

Hence $\frac{1g}{17g/mol}$ moles of NH_3 will give $\frac{1}{2\times17} = 0.0297$ moles of N_2.

(d) Molar mass of NH_4NO_3 = 80 g/mol.

1 mole of NH_4NO_3 will give 1 mole of N_2

Hence $\frac{1g}{80g/mol}$ moles NH_4NO_3 will give

$1 \times \frac{1}{80} = 0.0125$ moles of N_2

Hence thermal decomposition of NH_3 will produce maximum amount of N_2.

55.[2] According to Freundlich adsorption isotherm.

$$\frac{x}{m} = kP^{\frac{1}{n}}$$

$$\log_{10}\frac{x}{m} = \frac{1}{n}\log_{10}P + \log_{10}k$$

This is the equation of straight line of type

$$y = mx + c.$$

Hence slope is 1/n(m) and intercept is $\log_e k$.

56.[1] Clean water has BOD value less than 5 ppm. Polluted water has BOD value higher than 10 ppm.

57.[4] $\underset{\text{Alumina}}{Al_2O_3(s)} + 2NaOH(aq) + 3H_2O(l) \xrightarrow[35-36bar]{473-523K} \underset{\substack{\text{Sodium aluminate}\\(X)}}{2Na[Al(OH)_4](aq)}$

$2Na[Al(OH)_4](aq) + CO_2(g) \rightarrow \underset{\substack{\text{Hydrated alumina}\\(Y)}}{Al_2O_3xH_2O(s)} + 2NaHCO_3(aq)$

58.[2] The statement (2) is not true. Chain growth polymerization (or addition polymerisation) involves homopolymerisation only examples of such polymers include polythene, orlon and teflon.

59.[1] Amino group of glutamine is acetylated. Amide group of glutamine is not acetylated.

Note :

Acetalation of amide requires activation of amides and / or acyl donors, since the

nitrogen atom of amides is less basic than that of the corresponding amines due to amide resonance.

60.[1] The increasing order of the acidity of the carboxylic acids is III < II < IV < I. In aromatic acids, electron withdrawing groups like – Cl, – Cn, – NO_2 increases the acidity, whereas electron releasing groups like – CH_3, – OH, –OCH_3, – NH_2 decreases the acidity.

MATHEMATICS

61.[4] Suppose $y = f(x)$

$$\Rightarrow \quad y = \frac{x-1}{x-2}$$

$$\Rightarrow \quad yx - 2y = x - 1$$

$$\Rightarrow \quad (y-1)\,x = 2y - 1$$

$$\Rightarrow \quad x = f^{-1}(y) = \frac{2y-1}{y-1}$$

As the function is invertible on the given domain and its inverse can be obtained as above.

62.[1] $\because \quad (1+x)^2 = 1 + 2x + x^2$,

$(1+x^2)^3 = 1 + 3x^2 + 3x^4 + x^4$,

and $(1+x^3)^4 = 1 + 4x^3 + 6x^6 + 4x^9 + x^{12}$

So, the possible combinations for x^{10} are

$x \cdot x^9$, $x \cdot x^6 \cdot x^3$, $x^2 \cdot x^2 \cdot x^6$, $x^4 \cdot x^6$

Corresponding coefficients are 2×4, $2 \times 1 \times 4$, $1 \times 3 \times 6$, 3×6 or 8, 8, 18, 18.

$\therefore$ Sum of the coefficient is

$8 + 8 + 18 + 18 = 52$

Therefore, the coefficient of x^{10} in the expansion of $(1+x)^2 (1+x^2)^3 (1+x^3)^4$ is 52.

63.[4] As the system of equations has no solution then Δ should be zero and at least one of Δ_1, Δ_2 and Δ_3 should not be zero.

$$\therefore \quad \Delta = \begin{vmatrix} 1 & a & 1 \\ 1 & 2 & 2 \\ 1 & 5 & 3 \end{vmatrix} = 0$$

$$\Rightarrow \quad -a - 1 = 0 \Rightarrow a = -1$$

$$\Delta_2 = \begin{vmatrix} 1 & 3 & 1 \\ 1 & 6 & 2 \\ 1 & b & 3 \end{vmatrix} \neq 0$$

$$\Rightarrow \quad b \neq 9$$

64.[4] If a and –1 are the roots of the polynomial then we get

$$f(x) = x^2 + (1-a)\,x - a.$$

$\therefore \quad f(1) = 2 - 2a$

and $\quad f(2) = 6 - 3a$

As, $f(1) + f(2) = 0$

$$\Rightarrow \quad 2 - 2a + 6 - 3a = 0 \Rightarrow a = \frac{8}{5}$$

Therefore, the other root is $\frac{8}{5}$

65.[4] If all four letters are different then the number of words $^5C_4 \times 4! = 120$

If two letters are R and other two different letters are chosen from B, A, C, K then the number of words $= {}^4C_2 \times \frac{4!}{2!} = 72$

If two letters are A and other two different letters are chosen from B, R, C< K then the number of words $= {}^4C_2 \times \frac{4!}{2!} = 72$

If word is formed using two R's and two A's then the number of words $= \frac{4!}{2!\,2!} = 6.$

Therefore, the number of four-letter words that can be formed = 120 + 72 + 72 + 6 = 270.

66.[4] $\sin 3x = \cos 2x$

$$\Rightarrow \quad 3\sin x - 4\sin^3 x = 1 - 2\sin^2 x$$

$$\Rightarrow \quad 4\sin^3 x - 2\sin^2 x - 3\sin x + 1 = 0$$

$$\Rightarrow \quad \sin x = 1, \frac{-2 \pm 2\sqrt{5}}{8}$$

In the interval $\left(\frac{\pi}{2}, \pi\right)$, $\sin x = \frac{-2+2\sqrt{5}}{8}$

So, there is only one solution.

67.[2] $(x^2 - y^2)\,dx + 2xy\,dy = 0$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{y^2 - x^2}{2xy}$$

Let $y = vx$

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$\Rightarrow \quad v + x\frac{dv}{dx} = \frac{v^2x^2 - x^2}{2vx^2}$$

$\Rightarrow v + x\frac{dv}{dx} = \frac{v^2-1}{2v}$

$\Rightarrow x\frac{dv}{dx} = \frac{-v^2-1}{2v}$

$\Rightarrow \frac{2vdv}{v^2+1} = -\frac{dx}{x}$

After integrating, we get

$\ell n\,|v^2+1| = -\ell n\,|x| + \ell nc$

$\frac{y^2}{x^2} + 1 = \frac{c}{x}$

As curve passes through the point (1, 1), so $1 + 1 = c \Rightarrow c = 2$

$x^2 + y^2 - 2x = 0$, which is a circle of radius one.

68.[1] If the outcome is one of the following :
H, TTH, TTTTH, … , then X wins
As subsequent tosses are independent, so the probability that X wins is

$$p + \frac{p}{4} + \frac{p}{16} + \ldots = \frac{4p}{3}$$

Similarly Y wins if the outcome is one of the following : TH, TTTH, TTTTTH, …
Therefore, the probability that Y wins is

$$\frac{1-p}{2} + \frac{1-p}{8} + \frac{1-p}{32} = \frac{2(1-p)}{3}$$

Since, the probability of winning the game by both the players is equal then, we have

$\frac{4p}{3} = \frac{2(1-p)}{3} \Rightarrow p = \frac{1}{3}$.

69.[1] **Statement p :**

$\sin 120^\circ = \cos 30^\circ = \frac{\sqrt{3}}{2} \Rightarrow 2\sin 120^\circ = \sqrt{3}$

So, $\sqrt{1+\sin 240^\circ} - \sqrt{1-\sin 240^\circ}$

$= \sqrt{\frac{1-\sqrt{3}}{2}} - \sqrt{\frac{1+\sqrt{3}}{2}} \neq \sqrt{3}$

Statement q :

So, $A + B + C + D = 2\pi$

$\Rightarrow \frac{A+C}{2} + \frac{B+D}{2} = \pi$

$\Rightarrow \cos\left(\frac{A+C}{2}\right) + \cos\left(\frac{B+D}{2}\right)$

$\Rightarrow \cos\left(\frac{A+C}{2}\right) - \cos\left(\frac{A+C}{2}\right) = 0$

Therefore, statement p is false and statement q is true.

70.[1] $\because\ 7 - 6x - x^2 = 16 - (x+3)^2$

and $\frac{d}{dx}(7 - 6x - x^2) = -2x - 6$

So, $\int \frac{2x+5}{\sqrt{7-6x-x^2}}\,dx$

$= \int \frac{2x+6}{\sqrt{7-6x-x^2}}\,dx - \int \frac{1}{\sqrt{16-(x+3)^2}}\,dx$

$= -2\sqrt{7-6x-x^2} - \sin^{-1}\left(\frac{x+3}{4}\right) + C$

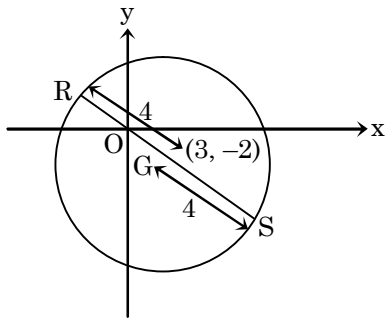
Therefore, A = –2, & B = –1.

71.[1] Since the plane bisects the line joining the points (1, 2, 3) and (–3, 4, 5) then the plane passes through the midpoint of the line which is :

$\left(\frac{1-3}{2}, \frac{2+4}{2}, \frac{5+3}{2}\right) \equiv \left(\frac{-2}{2}, \frac{6}{2}, \frac{8}{2}\right) \equiv (-1, 3, 4)$.

As plane cuts the line segment at right angle, so the direction cosines of the normal of the plane are (–3 – 1, 4 – 2, 5 – 3) = (–4, 2, 2).
So the equation of the plane is
$-4x + 2y + 2z = \lambda$
As plane passes though (–1, 3, 4) so
$-4(-1) + 2(3) + 2(4) = \lambda \Rightarrow \lambda = 18$
Therefore, equation of plane is :
$-4x + 2y + 2z = 18$
Now, only (–3, 2, 1) satisfies the given plane as – 4(–3) + 2 (2) + 2(1) = 18.

72.[2] $|z - (3 - 2i)| \leq 4$ represents a circle whose center is (3, –2) and radius = 4.
$|z| = |z - 0|$ represents the distance of point 'z' from origin (0, 0)



Suppose RS is the normal of the circle passing through origin 'O' and G is its center (3, –2).

Here, OR is the least distance
and OS is the greatest distance
OR = RG – OG and OS = OG + GS ...(1)
As, RG = GS = 4

$OG = \sqrt{3^2+(-2)^2} = \sqrt{9+4} = \sqrt{13}$

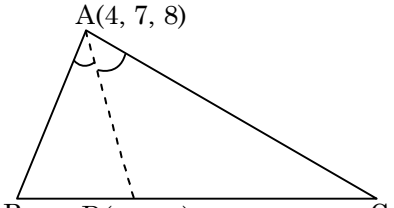
From (1), OR = $4-\sqrt{13}$ and OS = $4+\sqrt{13}$
So, required difference
= $(4+\sqrt{13})-(4-\sqrt{13})$
= $\sqrt{13}+\sqrt{13} = 2\sqrt{13}$

73.[2] Suppose angular bisector of A meets BC at D(x, y, z)
Using angular bisector theorem,

$$\frac{AB}{AC} = \frac{BD}{DC}$$

$$\frac{BD}{DC} = \frac{\sqrt{(4-2)^2+(7-3)^2+(8-4)^2}}{\sqrt{(4-2)^2+(7-5)^2+(8-7)^2}}$$

$$= \frac{\sqrt{2^2+4^2+4^2}}{\sqrt{2^2+2^2+1^2}} = \frac{6}{3} = 2$$



So,
D(x, y, z)

$$= \left(\frac{(2)(2)+(1)(2)}{2+1}, \frac{(2)(5)+(1)(3)}{2+1}, \frac{(2)(7)+(1)(4)}{2+1}\right)$$

$$D(x, y, z) = \left(\frac{6}{3}, \frac{13}{3}, \frac{18}{3}\right)$$

Therefore, position vector of point

$$P = \frac{1}{3}(6i + 13j + 18k)$$

74.[4] Equation of the line, which is perpendicular to the line, $3x + y = \lambda (\lambda \neq 0)$ and passing through origin, is given by

$$\frac{x-0}{3} = \frac{x-0}{3} = r$$

For foot of perpendicular

$$r = \frac{-((3\times 0)+(1\times 0)-\lambda)}{3^2+1^2} = \frac{\lambda}{10}$$

So, foot of perpendicular $P = \left(\frac{3\lambda}{10}, \frac{\lambda}{10}\right)$

Given the line meets X-axis at $A = \left(\frac{\lambda}{3}, 0\right)$

and meets Y-axis at $B = (0, \lambda)$
So,

$$BP = \sqrt{\left(\frac{3\lambda}{10}\right)^2 + \left(\frac{\lambda}{10}-\lambda\right)^2}$$

$$\Rightarrow \quad BP = \sqrt{\frac{9\lambda^2}{100} + \frac{81\lambda^2}{100}}$$

$$\Rightarrow \quad BP = \sqrt{\frac{90\lambda^2}{100}}$$

Now,

$$PA = \sqrt{\left(\frac{\lambda}{3}-\frac{3\lambda}{10}\right)^2 + \left(0-\frac{\lambda}{10}\right)^2}$$

$$\Rightarrow \quad PA = \sqrt{\frac{\lambda^2}{900} + \frac{\lambda^2}{100}}$$

$$\Rightarrow \quad PA = \sqrt{\frac{10\lambda^2}{900}}$$

Therefore BP : PA = 3 : 1

75.[1] Since $f(x) = \sin^{-1}\left(\frac{2\times 3^x}{1+9^x}\right)$

Suppose $3^x = \tan t$

$$\Rightarrow \quad f(x) = \sin^{-1}\left(\frac{2\tan t}{1+\tan^2 t}\right)$$

$= \sin^{-1}(\sin 2t) = 2t = 2\tan^{-1}(3^x)$

So, $f'(x) = \frac{2}{1+(3^x)^2} \times 3^x \cdot \log_e 3$

$$\therefore \quad f'\left(-\frac{1}{2}\right) = \frac{2}{1+\left(3^{-\frac{1}{2}}\right)^2} \times 3^{-\frac{1}{2}} \cdot \log_e 3$$

$$= \frac{1}{2} \times \sqrt{3} \times \log_e 3$$

$$= \sqrt{3} \times \log_e \sqrt{3}$$

76.[2] $A_n = \left(\frac{3}{4}\right) - \left(\frac{3}{4}\right)^2 + \left(\frac{3}{4}\right)^3 - \ldots + (-1)^{n-1}\left(\frac{3}{4}\right)^n$

Which is a G.P. with $a = \frac{3}{4}$, $r = \frac{-3}{4}$ and number of terms = n

$$\therefore A_n = \frac{\frac{3}{4}\times\left(1-\left(\frac{-3}{4}\right)^n\right)}{1-\left(\frac{-3}{4}\right)} = \frac{\frac{3}{4}\times\left(1-\left(\frac{-3}{4}\right)^n\right)}{\frac{7}{4}}$$

$$\Rightarrow A_n = \frac{3}{7}\left[1-\left(\frac{-3}{4}\right)^n\right] \quad \ldots(1)$$

For least odd natural number p, such that $B_n > A_n$

$\Rightarrow 1 - A_n > A_n$

$\Rightarrow 1 > 2 \times A_n$

$\Rightarrow A_n < \frac{1}{2}$

From eqn. (1), we get

$$\frac{3}{7}\times\left[1-\left(\frac{-3}{4}\right)^n\right] < \frac{1}{2}$$

$$\Rightarrow 1-\left(\frac{-3}{4}\right)^n < \frac{7}{6}$$

$$\Rightarrow 1-\frac{7}{6} < \left(\frac{-3}{4}\right)^n$$

$$\Rightarrow \frac{-1}{6} < \left(\frac{-3}{4}\right)^n$$

As n is odd, then $\left(\frac{-3}{4}\right)^n = -\frac{3^n}{4}$

So $\frac{-1}{6} < -\left(\frac{3}{4}\right)^n$

$$\Rightarrow \frac{1}{6} > \left(\frac{3}{4}\right)^n$$

$$\log\left(\frac{1}{6}\right) > n\log\left(\frac{3}{4}\right)$$

$\Rightarrow 6.228 < n$

Hence, n should be 7.

77.[3] Given, $4x^2 - 9y^2 = 36$

After differentiating w.r.t. x, we get

$$4.2.x \times 9.2.y.\frac{dy}{dx} = 0$$

$$\Rightarrow \text{slope of tangent} = \frac{dy}{dx} = \frac{4x}{9y}$$

So, slope of normal $= \frac{-9y}{4x}$

Now, equation of normal at point (x_0, y_0) is given by

$$y - y_0 = \frac{-9y_0}{4x_0}(x - x_0)$$

As normal intersects X axis at A, then

$$A \equiv \left(\frac{13x_0}{9}, 0\right)$$

and $$B \equiv \left(0, \frac{13y_0}{4}\right)$$

As OABP is a parallelogram

$$\therefore \text{ midpoint of OB} \equiv \left(0, \frac{13y_0}{8}\right) \equiv \text{Midpoint of AP.}$$

So, $$P(x, y) \equiv \left(\frac{-13x_0}{9}, \frac{13y_0}{4}\right) \quad \ldots(i)$$

$\because$ (x_0, y_0) lies on hyperbola, therefore

$4(x_0)^2 - 9(y_0)^2 = 36$ $\quad \ldots(ii)$

From equation (i), $x_0 = \frac{-9x}{13}$ and $y_0 = \frac{4y}{13}$

From equation (ii), we get

$9x^2 - 4y^2 = 169$

Hence, locus of point P is : $9x^2 - 4y^2 = 169$

78.[4] $\because$ f(x) has extremum values at x = 1 and x = 2

$\because$ $f'(1) = 0$ and $f'(2) = 0$

As, f(x) is a polynomial of degree 4.

Suppose $f(x) = Ax^4 + Bx^3 + Cx^2 + Dx + E$

$$\because \lim_{x\to 0}\left(\frac{f(x)}{x^2}+1\right) = 3$$

$$\Rightarrow \lim_{x\to 0}\left(\frac{Ax^4+Bx^3+Cx^2+Dx+E}{x^2}+1\right) = 3$$

$$\Rightarrow \lim_{x\to 0}\left(Ax^2+Bx+C+\frac{D}{x}+\frac{E}{x^2}+1\right) = 3$$

As limit has finite value, so D = 0 and E = 0

Now $A(0)^2 + B(0) + C + 0 + 0 + 1 = 3$

$\Rightarrow$ $c + 1 = 3 \Rightarrow c = 2$

$f'(x) = 4Ax^3 + 3Bx^2 + 2Cx + D$

$f'(1) = 0 \Rightarrow 4A(1) + 3B(1) + 2C(1) + D = 0$

$\Rightarrow$ $4A + 3B = -4$ $\quad \ldots(1)$

$f'(2) = 0 \Rightarrow 4A(8) + 3B(4) + 2C(2) + D = 0$

$\Rightarrow$ $8A + 3B = -2$ $\quad \ldots(2)$

From equations (1) and (2), we get

$$A = \frac{1}{2} \text{ and } B = -2$$

So, $$f(x) = \frac{x^4}{2} - 2x^3 + 2x^2$$

Therefore, $f(-1) = \frac{(-1)^4}{2} - 2(-1)^3 + 2(-1)^2$

$= \frac{1}{2} + 2 + 2 = \frac{9}{2}$

Hence $f(-1) = \frac{9}{2}$

79.[4] $\bar{x} = \frac{7+8+9+7+8+7+\lambda+8}{8} = 8$

$\Rightarrow \frac{54+\lambda}{8} = 8 \Rightarrow \lambda = 10$

Now variance = σ^2

$$= \frac{(7-8)^2 + (8-8)^2 + (9-8)^2 + (7-8)^2 + (8-8)^2 + (7-8)^2 + (10-8)^2 + (8-8)^2}{8}$$

$\Rightarrow \sigma^2 = \frac{1+0+1+1+0+1+4+0}{8} = \frac{8}{8} = 1$

Hence, the variance is 1.

80.[2] Given

$\ell + 3m + 5n = 0$ ...(1)

and $5\ell m - 2mn + 6n\ell = 0$ ...(2)

From eq. (1) we have

$\ell = -3m - 5n$

Put the value of ℓ in eq. (2), we get ;

$5(-3m - 5n)\, m - 2mn + 6n\,(-3m - 5n) = 0$

$\Rightarrow 15m^2 + 45\, mn + 30n^2 = 0$

$\Rightarrow m^2 + 3mn + 2n^2 = 0$

$\Rightarrow m^2 + 2mn + mn + 2n^2 = 0$

$\Rightarrow (m + n)(m + 2n) = 0$

$\therefore$ $m = -n$ or $m = -2n$

For $m = -n$, $\ell = -2n$

And for $m = -2n$, $\ell = n$

$\therefore$ $(\ell, m, n) = (-2n, -n, n)$

or $(\ell, m, n) = (n, -2n, n)$

$\Rightarrow$ $(\ell, m, n) = (-2, -1, 1)$ or

$(\ell, m, n) = (1, -2, 1)$

Therefore, angle between the lines is given as :

$$\cos(\theta) = \frac{(-2)(1) + (-1).(-2) + (1)(1)}{\sqrt{6}\,.\,\sqrt{6}}$$

$\Rightarrow \cos(\theta) = \frac{1}{6} \Rightarrow \theta = \cos^{-1}\left(\frac{1}{6}\right)$

81.[1] Here, equation of tangent on C_1 at (2, 1) is :

$2x + y - (x + 2) - 1 = 0$

or $x + y = 3$

If it cuts off the chord of the circle C_2 then the equation of the chord is :

$x + y = 3$

$\therefore$ distance of the chord from (3, –2) is :

$$d = \left|\frac{3 - 2 - 3}{\sqrt{2}}\right| = \sqrt{2}$$

Also, length of the chord is $\ell = 4$

$\therefore$ radius of $C_2 = r = \sqrt{\left(\frac{\ell}{2}\right)^2 + d^2}$

$= \sqrt{(2)^2 + (\sqrt{2})^2} = \sqrt{6}$

82.[1] We have

$(A - 3I)(A - 5I) = O$

$\Rightarrow A^2 - 8A + 15I = O$

Multiplying both sides by A^{-1}, we get;

$A^{-1} A.\, A - 8A^{-1}A + 15\, A^{-1} I = A^{-1}O$

$\Rightarrow A - 8I + 15A^{-1} = O$

$A + 15A^{-1} = 8I$

$\frac{A}{2} + \frac{15A^{-1}}{2} = 4I$

$\therefore \alpha + \beta = \frac{1}{2} + \frac{15}{2} = \frac{16}{2} = 8$

83.[1] Let $I = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{x}{1 + \sin x}\, dx$

Also let $K = \frac{x}{1 + \sin x}$

Multiplying numerator and denominator by

$(1 - \sin x)$, wet get;

$$K = \frac{x(1 - \sin x)}{1 - (\sin x)^2} = \frac{x(1 - \sin x)}{(\cos x)^2}$$

$= x(1 - \sin x)\sec^2 x$

$= x\sec^2 x - x \sin x \sec^2 x$

$= x\sec^2 x - x \tan x \sec x$

Now, $I = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} x\sec^2 x\, dx - \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} x \sec x \tan x\, dx$

$$= \left[x\tan x - \int \frac{dx}{dx}\tan x\,dx\right]_{\frac{\pi}{4}}^{\frac{3\pi}{4}}$$

$$- \left[x\sec x - \int \frac{dx}{dx}\sec x\,dx\right]_{\frac{\pi}{4}}^{\frac{3\pi}{4}}$$

$$= [x\tan x - \ln|\sec x|]_{\frac{\pi}{4}}^{\frac{3\pi}{4}}$$

$$- [x\sec x - \ln|\sec x + \tan x|]_{\frac{\pi}{4}}^{\frac{3\pi}{4}} + c$$

$$\Rightarrow I = \left\{\left[\frac{3\pi}{4}\tan\frac{3\pi}{4} - \ln\left|\sec\frac{3\pi}{4}\right|\right]\right.$$

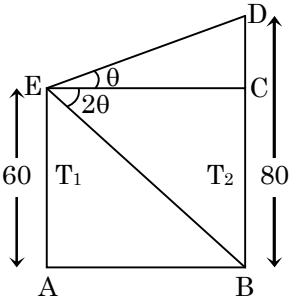
$$\left. - \left[\frac{3\pi}{4}\sec\frac{3\pi}{4} - \ln\left|\sec\frac{3\pi}{4} + \tan\frac{3\pi}{4}\right|\right]\right\}$$

$$- \left\{\left[\frac{\pi}{4}\tan\frac{\pi}{4} - \ln\left|\sec\frac{\pi}{4}\right|\right]\right.$$

$$\left. - \left[\frac{\pi}{4}\sec\frac{\pi}{4} - \ln\left|\sec\frac{\pi}{4} + \tan\frac{\pi}{4}\right|\right]\right\}$$

$$= \frac{\pi}{2}(\sqrt{2} + 1)$$

84.[4] Let the distance between T_1 and T_2 be x



From the figure

$EA = 60$ m (T_1)

and $DB = 80$ m (T_2)

$\angle DEC = \theta$

and $\angle BEC = 2\theta$

Now in ΔDEC,

$$\tan\theta = \frac{DC}{AB} = \frac{20}{x}$$

and in ΔBEC,

$$\tan 2\theta = \frac{BC}{CE} = \frac{60}{x}$$

We know that

$$\tan 2\theta = \frac{2\tan\theta}{1-(\tan\theta)^2}$$

$$\Rightarrow \frac{60}{x} = \frac{2\left(\frac{20}{x}\right)}{1-\left(\frac{20}{x}\right)^2}$$

$$\Rightarrow x^2 = 1200 \Rightarrow x = 20\sqrt{3}$$

85.[4] Given :

$$I_1 = \int_0^1 e^{-x}\cos^2 x\,dx\ ;$$

$$I_2 = \int_0^1 e^{-x^2}\cos^2 x\,dx \text{ and}$$

$$I_3 = \int_0^1 e^{-x^3}dx$$

For $x \in (0, 1)$

$\Rightarrow x > x^2$ or $-x < -x^2$

And $x^2 > x^3$ or $-x^2 < -x^3$

$\therefore\ e^{-x^2} < e^{-x^3}$ and $e^{-x} < e^{-x^2}$

$\Rightarrow e^{-x} < e^{-x^2} < e^{-x^3}$

$\Rightarrow e^{-x^3} > e^{-x^2} > e^{-x}$

$\Rightarrow I_3 > I_2 > I_1$

86.[4] Let the coordinate A be (0, c)

Equations of the given lines are

$x - y + 2 = 0$ and

$7x - y + 3 = 0$

We know that the diagonals of the rhombus will be parallel to the angle bisectors of the two given lines;

$y = x + 2$ and $y = 7x + 3$.

$\therefore$ equation of angle bisectors is given as :

$$\frac{x-y+2}{\sqrt{2}} = \pm\frac{7x-y+3}{5\sqrt{2}}$$

$5x - 5y + 10 = \pm(7x - y + 3)$

$\therefore$ Parallel equations of the diagonals are

$2x + 4y - 7 = 0$ and $12x - 6y + 13 = 0$

$\therefore$ slopes of diagonals are $\frac{-1}{2}$ and 2.

Now, slope of the diagonal from A(0, c) and passing through P(1, 2) is $(2 - c)$

$\therefore\ 2 - c = 2 \Rightarrow c = 0$ (not possible)

$\therefore\ 2 - c = \frac{-1}{2} \Rightarrow c = \frac{5}{2}$

$\therefore$ ordinate of A is $\frac{5}{2}$.

87.[4] Let,

$$L = \lim_{x\to 0} \frac{(x\tan 2x - 2x\tan x)}{(1-\cos 2x)^2} = \lim_{x\to 0} K$$

(say)

$$\Rightarrow K = \frac{x\left[\frac{2\tan x}{1-(\tan x)^2}\right] - 2x\tan x}{(1-(1-2\sin^2 x))^2}$$

$$= \frac{2x\tan x - [2x\tan x - 2x\tan^3 x]}{4\sin^4 x \times (1-\tan^2 x)}$$

$$= \frac{2x\tan^3 x}{4\sin^4 x \times (1-\tan^2 x)}$$

$$= \frac{2x\tan^3 x}{4\sin^4 x \times \left(\frac{\cos^2 x - \sin^2 x}{\cos^2 x}\right)}$$

$$= \frac{2x\frac{\sin^3 x}{\cos^2 x}}{4\sin^4 x \times \left(\frac{\cos^2 x - \sin^2 x}{\cos^2 x}\right)}$$

$$\Rightarrow K = \frac{x}{2\sin x \times (\cos^2 x - \sin^2 x)\cos x}$$

$$\therefore L = \lim_{x\to 0} \frac{x}{2\sin x} \times \lim_{x\to 0} \frac{1}{\cos x(\cos^2 x - \sin^2 x)}$$

$$= \lim_{x\to 0} \frac{x}{2\sin x} \times \lim_{x\to 0} \frac{1}{\cos x(\cos^2 x - \sin^2 x)}$$

$$= \frac{1}{2}$$

88.[3] Since f(x) is continuous at x = 2.

$\therefore \quad \lim_{x\to 2} f(x) = f(2)$

$$\Rightarrow \lim_{x\to 2} (x-1)^{\frac{1}{2-x}} = k \qquad (1^\infty \text{ form})$$

$$\therefore \quad e^\ell = k$$

Where

$$\ell = \lim_{x\to 2} (x-1-1) \times \frac{1}{2-x}$$

$$= \lim_{x\to 2} \frac{x-2}{2-x}$$

$$= \lim_{x\to 2} \left(\frac{x-2}{x-2}\right)$$

$\Rightarrow \quad k = e^{-1}$

89.[4] $\because$ a, b, c are in A.P. then

$$a + c = 2b$$

also it is given that,

$$a + b + c = \frac{3}{4} \qquad \ldots(1)$$

$$\Rightarrow 2b + b = \frac{3}{4} \Rightarrow b = \frac{1}{4} \qquad \ldots(2)$$

Again it is given that, a^2, b^2, c^2 are in G.P. then

$$(b^2)^2 = a^2c^2 \Rightarrow ac = \pm\frac{1}{16} \qquad \ldots(3)$$

From (1), (2) and (3), we get;

$$a \pm \frac{1}{16a} = \frac{1}{2} \Rightarrow 16a^2 - 8a \pm 1 = 0$$

Case I : $16a^2 - 8a + 1 = 0$

$\Rightarrow a = \frac{1}{4}$ (not possible as a < b)

Case II : $16a^2 - 8a - 1 = 0$

$$\Rightarrow a = \frac{8 \pm \sqrt{128}}{32}$$

$$\Rightarrow a = \frac{1}{4} \pm \frac{1}{2\sqrt{2}}$$

$$\therefore a = \frac{1}{4} - \frac{1}{2\sqrt{2}} \quad (\because a < b)$$

90.[1] Equation of the chord of contact PQ is given by :

$$T = 0$$

or $T = yy_1 - 4(x + x_1)$

where $(x_1, y_1) \equiv (-8, 0)$

$\therefore$ Equation becomes : x = 8

& Chord of contact is x = 8

$\therefore$ Coordinates of point P and Q are (8, 8) and (8, –8)

and focus of the parabola is F (2, 0)

$\therefore$ Area of triangle PQF $= \frac{1}{2} \times (8-2) \times (8+8)$

= 48 sq. units