JEE MAIN ONLINE PAPER 2018

Held on April 15, 2018 (Morning)

Instructions

1. This test will be a 3 hours Test.
2. This test consists of Physics, Chemistry and Mathematics questions with equal weight age of 120 marks.
3. Each question is of 4 marks.
4. There are three parts in the question paper consisting of Physics (Q. no. 1 to 30), Chemistry (Q. no. 31 to 60) and Mathematics (Q. no. 61 to 90).
5. There will be only one correct choice in the given four choices. For each question 4 marks will be awarded for correct choice, 1 mark will be deducted for incorrect choice and zero mark will be awarded for not attempted question.
6. Any textual, printed or written material, mobile phones, calculator etc. is not allowed for the students appearing for the test.
7. All calculations / written work should be done in the rough sheet provided.

PHYSICS

Q.1 The relative error in the determination of the surface area of a sphere is α. Then the relative error in the determination of its volume is

[JEE-Main On line-2018]

(A) $\frac{2}{3}\alpha$ (B) $\frac{1}{3}\alpha$ (C) $\frac{3}{2}\alpha$ (D) α

Q.2 A thin uniform tube is bent into a circle of radius r in the virtical plane. Equal volumes of two immiscible liquids, whose densities are ρ_1 and ρ_2 ($\rho_1 > \rho_2$) fill half the circle. The angle q between the radius vector passing through the common interface and the vertical is

[JEE-Main On line-2018]

(A) $\theta = \tan^{-1}\left[\frac{\pi}{2}\left(\frac{\rho_1-\rho_2}{\rho_1+\rho_2}\right)\right]$

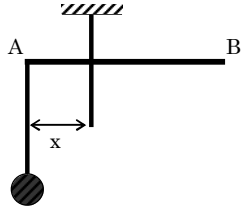
(B) $\theta = \tan^{-1}\frac{\pi}{2}\left(\frac{\rho_1-\rho_2}{\rho_1+\rho_2}\right)$

(C) $\theta = \tan^{-1}\pi\left(\frac{\rho_1}{\rho_2}\right)$

(D) $\theta = \tan^{-1}\frac{\pi}{2}\left(\frac{\rho_2}{\rho_1}\right)$

Q.3 A uniform rod AB is suspended from a point X, at a variable distance from x from A, as shown. To make the rod horizontal, a mass m is suspended from its end A. A set of (m, x) values is recorded. The appropriate variable that give a straight line, when plotted, are:

[JEE-Main On line-2018]



(A) m, $\frac{1}{x}$ (B) m, $\frac{1}{x^2}$ (C) m, x (D) m, x^2

Q.4 An ideal capacitor of capacitance 0.2 μF is charged to a potential difference of 10V. The charging battery is then disconnected. The capacitor is then connected to an ideal inductor of self inductance 0.5 mH. The

current at a time when the potential difference across the capacitor is 5V, is:

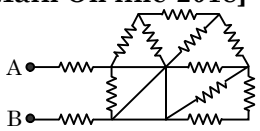
[JEE-Main On line-2018]

(A) 0.17A (B) 0.15A

(C) 0.34A (D) 0.25A

Q.5 In the given circuit all resistances are of value R ohm each. The equivalent resistance between A and B is:

[JEE-Main On line-2018]



(A) 2 R (B) $\frac{5R}{2}$ (C) $\frac{5R}{3}$ (D) 3 R

Q.6 In a common emitter configuration with suitable bias, it is given than R_L is the load resistance and R_{BE} is small signal dynamic resistance (input side). Then, voltage gain, current gain and power gain are given, respectively, by :

[β is current gain, I_B, I_C, I_E are respectively base, collector and emitter currents]

[JEE-Main On line-2018]

(A) $\beta\frac{R_L}{R_{BE}},\frac{\Delta I_E}{\Delta I_B},\beta^2\frac{R_L}{R_{BE}}$

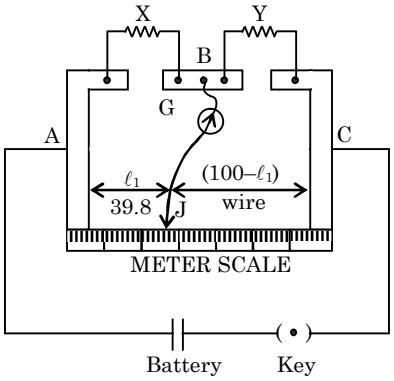
(B) $\beta^2\frac{R_L}{R_{BE}},\frac{\Delta I_C}{\Delta I_B},\beta\frac{R_L}{R_{BE}}$

(C) $\beta^2\frac{R_L}{R_{BE}},\frac{\Delta I_E}{\Delta I_B},\beta^2\frac{R_L}{R_{BE}}$

(D) $\beta\frac{R_L}{R_{BE}},\frac{\Delta I_C}{\Delta I_B},\beta^2\frac{R_L}{R_{BE}}$

Q.7 In a meter bridge, as shown in the figure, it is given that resistance Y=12.5 Ω and that the balance is obtained at a distance 39.5 cm from end A (by jockey J). After interchanging the resistances X and Y, a new balance point is found at a distance ℓ_2 from end A. What are the values of X and l_2 ?

[JEE-Main On line-2018]



(A) 19.15 Ω and 39.5 cm

(B) 8.16 Ω and 60.5 cm

(C) 19.15 Ω and 60.5 cm

(D) 5.16 Ω and 39.5 cm

Q.8 Two electrons are moving with non-relativistic speeds perpendicular to each other. If corresponding de Broglie wavelengths are λ_1 and l_2, their de Broglie wavelength in the frame of reference attached to their centre of mass is:

[JEE-Main On line-2018]

(A) $\lambda_{CM}=\lambda_1=\lambda_2$ (B) $\frac{1}{\lambda_1}=\frac{1}{\lambda_1}+\frac{1}{\lambda_2}$

(C) $\lambda_{CM}=\frac{2\lambda_1\lambda_2}{\sqrt{\lambda_1^2+\lambda_2^2}}$ (D) $\lambda_{CM}=\left(\frac{\lambda_1\lambda_2}{2}\right)$

Q.9 A tuning fork vibrates with frequency 256 Hz and gives one best per second with the third normal mode of vibration of an open pipe. What is the length of the piper? (Speed of sound of air is 340 ms^{-1})

[JEE-Main On line-2018]

(A) 190 cm (B) 180 cm

(C) 220 cm (D) 200 cm

Q.10 The number of amplitude modulated broadcast stations that can be accommodated in a 300 kHz band width for the highest modulating frequency 15 kHz will be:

[JEE-Main On line-2018]

(A) 20 (B) 10 (C) 8 (D) 15

Q.11 A body of mass m is moving in a circular orbit of radius R about a planet of mass M. At some instant, it splits into two equal masses. The first mass moves in a circular orbit of radius R/2, and the other mass, in a circular orbit of radius 3R/2. The difference between the final and initial total energies is: **[JEE-Main On line-2018]**

(A) $-\frac{GMm}{2R}$ (B) $+\frac{GMm}{6R}$

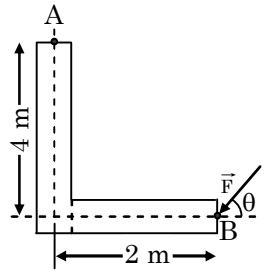
(C) $-\frac{GMm}{6R}$ (D) $\frac{GMm}{2R}$

Q.12 Light of wavelength 550 nm falls normally on a slit of width 22.0×10^{-5} cm. The angular position of the second minima from the central maximum will be (in radians) **[JEE-Main On line-2018]**

(A) $\frac{\pi}{8}$ (B) $\frac{\pi}{12}$

(C) $\frac{\pi}{4}$ (D) $\frac{\pi}{6}$

Q.13 A force of 40 N acts on a point B at the end of an L-shaped object, as shown in the figure. The angle θ that will produce maximum moment of the force about point A is given by: **[JEE-Main On line-2018]**



(A) $\tan\theta = \frac{1}{4}$ (B) $\tan\theta = 2$

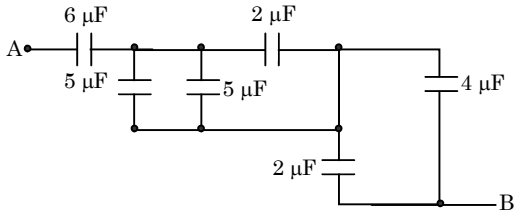
(C) $\tan\theta = \frac{1}{2}$ (D) $\tan\theta = 4$

Q.14 A given object takes n times more times to slide down a 45° rough inclined plane as it takes to slide down a perfectly smooth 45° incline. The coefficient of kinetic friction between the object and the incline is : **[JEE-Main On line-2018]**

(A) $\sqrt{1-\frac{1}{n^2}}$ (B) $1-\frac{1}{n^2}$

(C) $\frac{1}{2-n^2}$ (D) $\sqrt{\frac{1}{1-n^2}}$

Q.15 The equivalent capacitance between A and B in the circuit given below is : **[JEE-Main On line-2018]**



(A) 4.9 μF (B) 3.6 μF

(C) 5.4 μF (D) 2.4 μF

Q.16 A Carnot's engine works as a refrigerator between 250 K and 300 K. It receives 500 cal heat from the reservoir at the lower temperature. The amount of work done in each cycle to operate the refrigerator is: **[JEE-Main On line-2018]**

(A) 420 J (B) 2100 J

(C) 772 J (D) 2520 J

Q.17 One mole of an ideal monoatomic gas is compressed isothermally in a rigid vessel to double its pressure eat room temperature, 27°C. The work done on the gas will be: **[JEE-Main On line-2018]**

(A) 300R ℓn 6 (B) 300R

(C) 300R ℓn 7 (D) 300R ℓn 2

Q.18 An automobile, travelling at 40 km/h, can be stopped at a distance of 40 m by applying brakes. If the same automobile is travelling at 80 km/h, the minimum stopping distance, in metres, is (assume no skidding) **[JEE-Main On line-2018]**

(A) 75 m (B) 160 m

(C) 100 m (D) 150 m

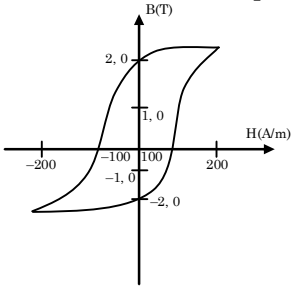
Q.19 Take the mean distance of the moon and the sun from the earth to be 0.4×10^6 km and 150×10^6 km respectively. Their masses are 8×10^{22} kg and 2×10^{30} kg respectively. The radius of the earth is 6400 km. Let ΔF_1 be the difference in the forces exerted by the moon at the nearest and farthest points on the earth and ΔF_2 be the difference in the force exerted by the sun at the nearest and farthest points on the earth. The, the number closest to $\frac{\Delta F_1}{\Delta F_2}$ is:

[JEE-Main On line-2018]

(A) 2 (B) 6 (C) 10^{-2} (D) 0.5

Q.20 The B-H curve for a ferromagnet is shown in the figure. The ferromagnet is placed inside a long solenoid with 1000 turns/cm. The current that should be passed in the solenoid to demagnetise the ferromagnet completely is:

[JEE-Main On line-2018]



(A) 2 mA (B) 1 mA

(C) 40 μA (D) 20 μA

Q.21 A planoconvex lens becomes an optical system of 28 cm focal length when its plane surface is silvered and illuminated from left to right as shown in Fig-A. If the same lens is instead silvered on the curved surface and illuminated from other side as in Fig. B, it acts like an optical system of focal length 10 cm. The refractive index of the material of lens is

[JEE-Main On line-2018]

Fig. A Fig. B

(A) 1.50 (B) 1.55 (C) 1.75 (D) 1.51

Q.22 A body of mass M and charge q is connected to a spring of spring constant k. It is oscillating along x-direction about its equilibrium position, taken to be at x = 0, with an amplitude A. An electric field E is applied along the x-direction. Which of the following statements is correct?

[JEE-Main On line-2018]

(A) The total energy of the system is $\frac{1}{2}m\omega^2A^2+\frac{1}{2}\frac{q^2E^2}{k}$

(B) The new equilibrium position is at a distance : $\frac{2qE}{k}$ from x = 0

(C) The new equilibrium position is at a distance: $\frac{qE}{2k}$ from x = 0

(D) The total energy of the system is $\frac{1}{2}m\omega^2A^2-\frac{1}{2}\frac{q^2E^2}{k}$

Q.23 In a screw gauge, 5 complete rotations of the screw cause it to move a linear distance of 0.25 cm. There are 100 circular scale divisions. The thickness of a wire measured by this screw gauge gives a reading of 4 main scale divisions and 30 circular scale divisions. Assuming negligible zero error, the thickness of the wire is:

[JEE-Main On line-2018]

(A) 0.0430 cm (B) 0.3150 cm

(C) 0.4300 cm (D) 0.2150 cm

Q.24 A solution containing active cobalt $^{60}_{27}Co$ having activity of 0.8 μCi and decay constant λ is injected in an animal's body. If $1cm^3$ of blood is drawn from the animal's body after 10 hrs of injection, the activity found was 300 decays per minute. What is the volume of blood that is flowing in the body? ($1Ci = 3.7\times10^{10}$ decay per second and at t = 10 hrs $e^{-\lambda t} = 0.84$)

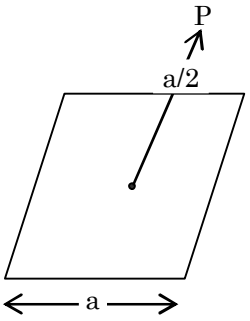
[JEE-Main On line-2018]

(A) 6 litres (B) 7 litres

(C) 4 litres (D) 5 litres

Q.25 A charge Q is placed at a distance a/2 above the centre of the square surface of edge a as shown in the figure. The electric flux through the square surface is:

[JEE-Main On line-2018]



(A) $\frac{Q}{3\varepsilon_0}$ (B) $\frac{Q}{6\varepsilon_0}$ (C) $\frac{Q}{2\varepsilon_0}$ (D) $\frac{Q}{\varepsilon_0}$

Q.26 The energy required to remove the electron from a singly ionized Helium atom is 2.2 times the energy required to remove an electron from Helium atom. The total energy required to ionize the Helium atom completely is:

[JEE-Main On line-2018]

(A) 20 eV (B) 79 eV
(C) 109 eV (D) 34 eV

Q.27 A monochromatic beam of light has a frequency $v = \frac{3}{2\pi} \times 10^{12}$ Hz and is propagating along the direction $\frac{\hat{i}+\hat{j}}{\sqrt{2}}$. It is polarized along the $\hat{k}$ direction. The acceptable form for the magnetic field is:

[JEE-Main On line-2018]

(A) $\frac{E_0}{C}\left(\frac{\hat{i}-\hat{j}}{\sqrt{2}}\right)\cos\left[10^4\left(\frac{\hat{i}-\hat{j}}{\sqrt{2}}\right)\cdot\vec{r}-(3\times10^{12})t\right]$

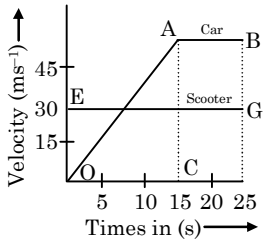
(B) $\frac{E_0}{C}\left(\frac{\hat{i}-\hat{j}}{\sqrt{2}}\right)\cos\left[10^4\left(\frac{\hat{i}+\hat{j}}{\sqrt{2}}\right)\cdot\vec{r}-(3\times10^{12})t\right]$

(C) $\frac{E_0}{C}\vec{k}\cos\left[10^4\left(\frac{\hat{i}+\hat{j}}{\sqrt{2}}\right)\cdot\vec{r}+(3\times10^{12})t\right]$

(D) $\frac{E_0}{C}\frac{(\hat{i}+\hat{j}+\vec{k})}{\sqrt{3}}\cos\left[10^4\left(\frac{\hat{i}+\hat{j}}{\sqrt{2}}\right)\cdot\vec{r}+(3\times10^{12})t\right]$

Q.28 The velocity-time graphs of a car and a scooter are shown in the figure. (i) the difference between the distance travelled by the car and the scooter in 15 s and (ii) the time at which the car will catch up with the scooter are, respectively

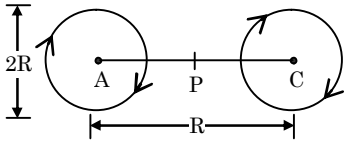
[JEE-Main On line-2018]



(A) 337.5m and 25 s (B) 225.5m and 10s
(C) 112.5m and 22.5s (D) 112.5m and 15s

Q.29 A Helmholtz coil has pair of loops, each with N turns and radius R. They are placed coaxially at distance R and the same current I flows through the loops in the same direction. The magnitude of magnetic field at P, midway between the centres A and C, is given by (Refer to figure):

[JEE-Main On line-2018]



(A) $\frac{4N\mu_0 I}{5^{3/2}R}$ (B) $\frac{8N\mu_0 I}{5^{3/2}R}$
(C) $\frac{4N\mu_0 I}{5^{1/2}R}$ (D) $\frac{8N\mu_0 I}{5^{1/2}R}$

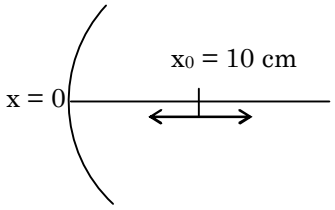
Q.30 A particle is oscillating on the X-axis with an amplitude 2 cm about the point x_0 = 10 cm with a frequency ω. A concave mirror of focal length 5 cm is placed at the origin (see figure)

Identify the correct statements:

[JEE-Main On line-2018]

(a) The image executes periodic motion
(b) The image executes non-periodic motion
(c) The turning points of the image are asymmetric w.r.t the image of the point at x = 10 cm

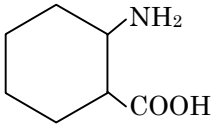
(d) The distance between the turning points of the oscillation of the image is $\frac{100}{21}$

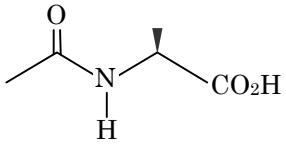


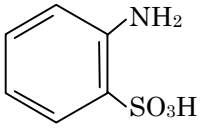
(A) (b), (d) (B) (b), (c)
(C) (a), (c), (d) (D) (a), (d)

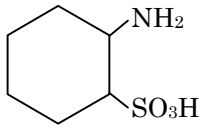
CHEMISTRY

Q.31 Which of the following will not exist in zwitter ionic at pH = 7 ?
[JEE-Main On line-2018]

(A) 

(B) 

(C) 

(D) 

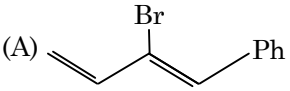
Q.32 A sample of $NaClO_3$ is converted by heat to NaCl with a loss of 0.16 g of oxygen. The residue is dissolved in water and precipitated as AgCl. The mass of AgCl (in g) obtained will be :
(Given: Molar mass of AgCl = 143.5 g mol^{-1})
[JEE-Main On line-2018]
(A) 0.35 (B) 0.54
(C) 0.41 (D) 0.48

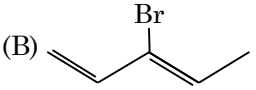
Q.33 For which of the following reactions, ΔH is equal to ΔU?
[JEE-Main On line-2018]
(A) $N_2(g) + 3H_2(g) \rightarrow 2NH_3(g)$
(B) $2HI(g) \rightarrow H_2(g) + I_2(g)$
(C) $2SO_2(g) + O_2(g) \rightarrow 2SO_3(g)$
(D) $2NO_2(g) \rightarrow N_2O_4(g)$

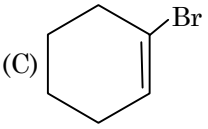
Q.34 N_2O_5 decomposes to NO_2 and O_2 and follows first order kinetics. After 50 minutes, the pressure inside the vessel increases from 50 mm Hg to 87.5 mm Hg. The pressure of the gaseous mixture after 100 minutes at constant temperature will the _____.
[JEE-Main On line-2018]
(A) 136.25 mm Hg (B) 106.25 mm Hg
(C) 175.0 mm Hg (D) 116.25 mm Hg

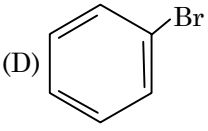
Q.35 In the molecular orbital diagram for the molecular ion, N_2^+, the number of electrons in the σ_{2p} molecular orbital is :
[JEE-Main On line-2018]
(A) 0 (B) 2
(C) 3 (D) 1

Q.36 Which of the following will most readily give the dehydrohalogenation product?
[JEE-Main On line-2018]

(A) 

(B) 

(C) 

(D) 

Q.37 Identify the pair in which the geometry of the species is T-shape and square pyramidal, respectively
[JEE-Main On line-2018]

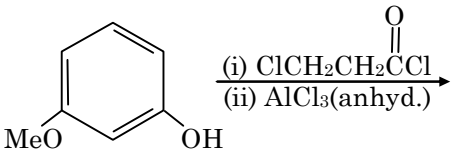
(A) ICl_2^- and ICl_5

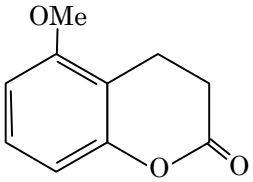
(B) IO_3^- and $IO_2F_2^-$

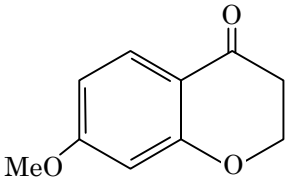
(C) ClF_3 and IO_4^-

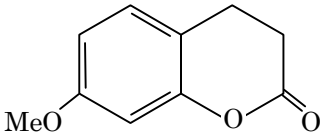
(D) $XeOF_2$ and $XeOF_4$

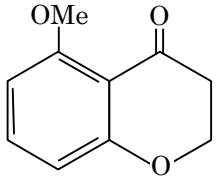
Q.38 The major product of the following reaction is **[JEE-Main On line-2018]**



(A) 

(B) 

(C) 

(D) 

Q.39 (I) (II)

H – N ---- N ---- N

In hydrogen azide, the bond orders of bonds (I) and (II) are _____.

[JEE-Main On line-2018]

(A) I<2, II>2 (B) I>2, II>2

(C) I>2, II<2 (D) I<2, II<2

Q.40 For Na^+, Mg^{2+}, F^- and $O2^-$; the correct order of increasing Ionic radii is:

[JEE-Main On line-2018]

(A) $O^{2-} < F^- < Na^+ < Mg^{2+}$

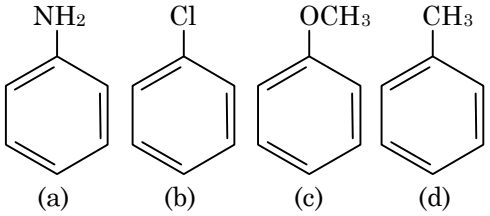
(B) $Na^+ < Mg^{+2} < F^- < O^{2-}$

(C) $Mg^{2+} < Na^+ < F^- < O^{2-}$

(D) $Mg^{2+} < O^{2-} < Na^+ < F^-$

Q.41 The increasing order of nitration of the following compound is:

[JEE-Main On line-2018]



(A) (a) < (b) < (d) < (c)

(B) (a) < (b) < (c) < (d)

(C) (b) < (a) < (c) < (d)

(D) (b) < (a) < (d) < (c)

Q.42 The correct match between items of List-I and List-II is:

[JEE-Main On line-2018]

	List-I		**List-II**
i.	Coloured impurity	(p)	Steam distillation
ii.	Mixture of o-nitrophenol and p-nitophenol	(q)	Fractional distillation
iii.	Crude naptha	(r)	Charcoal treatment
iv.	Mixture of glycerol and sugars	(s)	Distillation under reduced pressure

(A) (i)-(r), (ii)-(s), (iii)-(p), (iv)-(q)

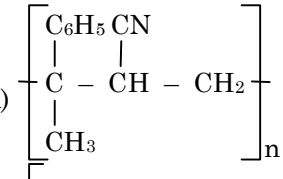
(B) (i)-(p), (ii)-(s), (iii)-(r), (iv)-(q)

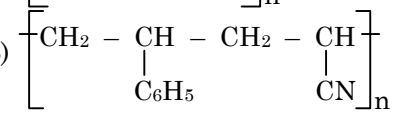
(C) (i)-(r), (ii)-(p), (iii)-(q), (iv)-(s)

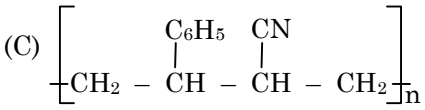
(D) (i)-(r), (ii)-(p), (iii)-(s), (iv)-(q)

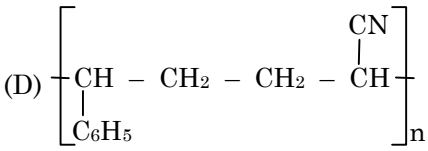
Q.43 The copolymer formed by addition polymerization of styrene and acrylonitrile in the presence of peroxide is :

[JEE-Main On line-2018]

(A) 

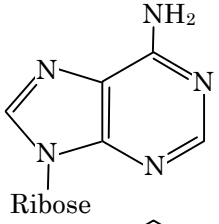
(B) 

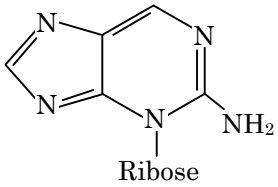
(C) 

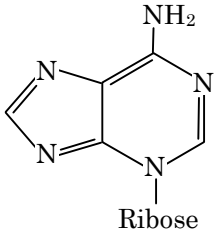
(D) 

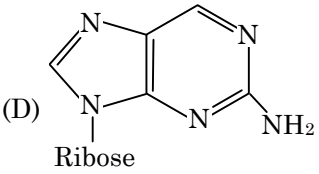
Q.44 Which of the following is a lewis acid?
[JEE-Main On line-2018]
(A) PH_3 (B) NF_3
(C) NaH (D) $B(CH_3)_3$

Q.45 Which of the following statements about colloids is false?
[JEE-Main On line-2018]
(A) When slilver nitrate solution is added to potassium iodide solution, a negatively charged colloidal solution is formed
(B) Freezing point of colloidal solution is lower than true solution at same concentration of a solute
(C) Colloidal particles can pass through ordinary filter paper
(D) When excess of electrolyte is added to colloidal solution, colloidal particle will be precipitated

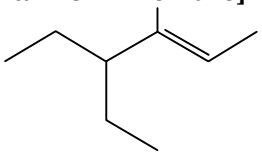
Q.46 Which of the following is the correct structure of adenosine?
[JEE-Main On line-2018]
(A) 

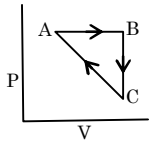
(B) 

(C) 

(D) 

Q.47 The correct combination is:
[JEE-Main On line-2018]
(A) $[NiCl_4]^{2-}$ -square-planar; $[Ni(CN)_4]^{2-}$ - paramagnetic
(B) $[Ni(CN)_4]^{2-}$ -tetrahedral; $[Ni(CO)_4]$ - paramagnetic
(C) $[NiCl_4]^{2-}$ -paramagnetic; $[Ni(CO)_4]$ - tetrahedral
(D) $[NiCl_4]^{2-}$ -dimagnetic; $[Ni(CO)_4]$ – square-planar

Q.48 The IUPAC name of the following compound is :
[JEE-Main On line-2018]

(A) 3-ethyl-4-methylhex-4-ene
(B) 4, 4-diethyl-3-methylbut-2-ene
(C) 4-methyl-3-ethylhex-4-ene
(D) 4-ethyl-3-methylhex-2-ene

Q.49 An ideal gas undergoes a cyclic process as shown in figure.
[JEE-Main On line-2018]

$\Delta U_{BC} = -5$ kJ mol^{-1}, $q_{AB} = 2$ kJ mol^{-1}
$W_{AB} = -5$ kJ mol^{-1}, $W_{CA} = 3$ kJ mol^{-1},
Heat absorbed by the system during process CA is:
(A) –5 kJ mol^{-1} (B) +5 kJ mol^{-1}
(C) 18 kJ mol^{-1} (D) –18 kJ mol^{-1}

Q.50 Ejection of the photoelectron from metal in the photoelectric effect experiment can be stopped by applying 0.5 V when the radiation of 250 nm is used. The work function of the metal is :
[JEE-Main On line-2018]
(A) 4 eV (B) 5.5 eV (C) 4.5 eV (D) 5 eV

Q.51 In graphite and diamond, the percentage of p-character of the hybrid orbitals in hybridisation are respectively:
[JEE-Main On line-2018]
(A) 33 and 25 (B) 67 and 75
(C) 50 and 75 (D) 33 and 75

Q.52 When an electric current is passes through acidified water, 112 mL of hydrogen gas at N.T.P. was collected at the cathode in 965 seconds. The current passed, in ampere, is:
[JEE-Main On line-2018]
(A) 2.0 (B) 0.1 (C) 0.5 (D) 1.0

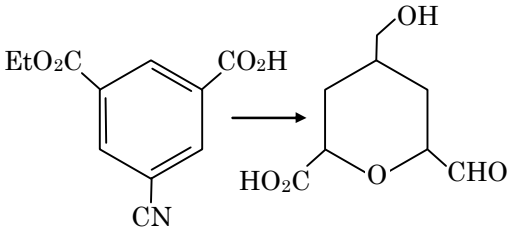
Q.53 The minimum volume of water required to dissolve 0.1 g lead (II) chloride to get a saturated solution (K_{SP} of $PbCl_2 = 3.2 \times 10^{-8}$; atomic mass of Pb = 207 u) is:
[JEE-Main On line-2018]
(A) 1.798 L (B) 0.36 L
(C) 17.95 L (D) 0.18 L

Q.54 In which of the following reactions, an increase in the volume of the container will favour the formation of products?
[JEE-Main On line-2018]
(A) $4NH_3(g) + 5O_2(g) \rightleftharpoons 4NO(g) + 6H_2O(1)$
(B) $2NO_2(g) \rightleftharpoons 2NO(g) + O_2$
(C) $3O_2 \rightleftharpoons 2O_3$
(D) $H_2(g) + I_2(g) \rightleftharpoons 2HI(g)$

Q.55 The decreasing order of bond angles in BF_3, NH_3, PF_3, and I_3^- is :
[JEE-Main On line-2018]
(A) $I_3^- > BF_3 > NH_3 > PF_3$
(B) $BF_3 > I_3^- > PF_3 > NH_3$
(C) $BF_3 > NH_3 > PF_3 > I_3^-$
(D) $I_3^- > NH_3 > PF_3 > BF_3$

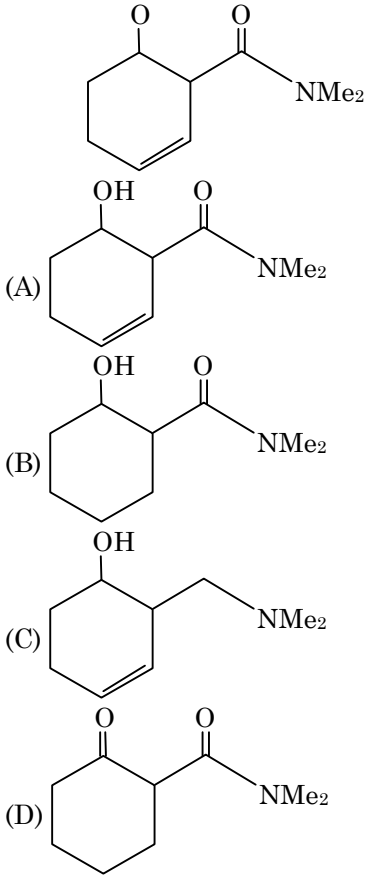
Q.56 Which of the following arrangements shown the schematic alignment of magnetic moments of antiferromagnetic substance?
[JEE-Main On line-2018]
(A) ↑ ↓ ↓ ↓ ↓ ↑
(B) ↑ ↑ ↑ ↑ ↑ ↑
(C) ↑ ↑ ↓ ↑ ↑ ↓
(D) ↑ ↓ ↑ ↓ ↑ ↓

Q.57 The reagent(s) required for the following conversion are:
[JEE-Main On line-2018]



(A) (i) $NaBH_4$, (ii) Raney NiH_2, (iii) H_3O^+
(B) (i) $LiAlH_4$, (ii) H_3O^+
(C) (i) B_2H_6, (ii) DIBAL-H, (iii) H_3O^+
(D) (i) B_2H_6, (ii) $SnCl_2$/HCl, (iii) H_3O^+

Q.58 The main reduction product of the following compound with $NaBH_4$ in methanol is:
[JEE-Main On line-2018]



Q.59 Xenon hexafluoride on partial hydrolysis produces compounds 'X' and 'Y'. Compounds 'X' and 'Y' and the oxidation state of Xe are respectively:
[JEE-Main On line-2018]
(A) $XeOF_4$(+6) and XeO_3(+6)

(B) XeO_2(+4) and XeO_3 (+6)
(C) $XeOF_4$(+6) and XeO_2F_2(+6)
(D) XeO_2F_4(+6) and XeO_2(+4)

Q.60 A white sodium salt dissolves readily in water to give a solution which is neutral to litmus. When silver nitrate solution is added to the aforementioned solution, a white precipitate is obtained which does not dissolve in dil. nitric acid. The anion is:
[JEE-Main On line-2018]
(A) CO_3^{2-} (B) SO_4^{2-} (C) S^{2-} (D) Cl^-

MATHEMATICS

Q.61 In a triangle ABC, coordianates of A are (1, 2) and the equations of the medians through B and C are x + y = 5 and x = 4 respectively. Then area of ΔABC (in sq. units) is
[JEE-Main On line-2018]
(A) 5 (B) 9 (C) 12 (D) 4

Q.62 If b is the first term of an infinite G.P. whose sum is five, then b lies in the interval
[JEE-Main On line-2018]
(A) $(-\infty, -10)$ (B) $(10, \infty)$
(C) (0, 10) (D) (–10, 0)

Q.63 A variable plane passes through a fixed point (3, 2, 1) and meets x, y and z axes at A, B and C respectively. A plane is drawn parallel to yz-plane through A, a second plane is drawn parallel zx-plane through B and a third plane is drawn parallel to xy-plane through C. Then the locus of the point of intersection of these three planes, is
[JEE-Main On line-2018]
(A) $x + y + z = 6$ (B) $\frac{x}{3}+\frac{y}{2}+\frac{z}{1}=1$
(C) $\frac{3}{x}+\frac{2}{y}+\frac{1}{z}=1$ (D) $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=\frac{11}{6}$

Q.64 If $f\left(\frac{x-4}{x+2}\right) = 2x + 1$, $(x \in R - \{1, -2\})$, then $\int f(x)\,dx$ is equal to (where C is a constant of integration)
[JEE-Main On line-2018]
(A) $12 \log_e |1 - x| - 3x + c$
(B) $-12 \log_e |1 - x| - 3x + c$
(C) $-12 \log_e |1 - x| + 3x + c$
(D) $12 \log_e |1 - x| + 3x + c$

Q.65 If $\lambda \in R$ is such that the sum of the cubes of the roots of the equation, $x^2 + (2 - \lambda)x + (10 - \lambda) = 10$ is minimum, then the magnitude of the difference of the roots of this equation is
[JEE-Main On line-2018]
(A) 20 (B) $2\sqrt{5}$
(C) $2\sqrt{7}$ (D) $4\sqrt{2}$

Q.66 Consider the following two binary relations on the set A = {a, b, c} : R_1 = {(c, a), (b, b), (a, c), (c, c), (b, c), (a, a)} and R_2 = {(a, b), (b, a), (c, c), (c, a), (a, a), (b, b), (a, c). Then
[JEE-Main On line-2018]
(A) R_2 is symmetric but it is not transitive
(B) Both R_1 and R_2 are transitive
(C) Both R_1 and R_2 are not symmetric
(D) R_1 is not symmetric but it is transitive

Q.67 If $x^2 + y^2 + \sin y = 4$, then the value of $\frac{d^2y}{dx^2}$ at the point (–2, 0) is
[JEE-Main On line-2018]
(A) –34 (B) –32 (C) –2 (D) 4

Q.68 A circle passes through the points (2, 3) and (4, 5). If its centre lies on the line, $y - 4x + 3 = 0$, then its radius is equal to
[JEE-Main On line-2018]
(A) $\sqrt{5}$ (B) 1 (C) $\sqrt{2}$ (D) 2

Q.69 If the tangent drawn to the hyperbola $4y^2 = x^2 + 1$ intersect the co-ordinate axes at the distinct points A and B, then the locus of the mid point of AB is
[JEE-Main On line-2018]
(A) $x^2 - 4y^2 + 16x^2y^2 = 0$
(B) $4x^2 - y^2 + 16x^2y^2 = 0$
(C) $4x^2 - y^2 - 16x^2y^2 = 0$
(D) $x^2 - 4y^2 - 16x^2y^2 = 0$

Q.70 Let A be a matrix such that A. $\begin{bmatrix}1 & 2\\ 0 & 3\end{bmatrix}$ is a scalar matrix and |3A| = 108. Then A^2 equals

[JEE-Main On line-2018]

(A) $\begin{bmatrix}4 & -32\\ 0 & 36\end{bmatrix}$ (B) $\begin{bmatrix}4 & 0\\ -32 & 36\end{bmatrix}$

(C) $\begin{bmatrix}36 & 0\\ -32 & 4\end{bmatrix}$ (D) $\begin{bmatrix}36 & -32\\ 0 & 4\end{bmatrix}$

Q.71 If $f(x) = \begin{vmatrix}\cos x & x & 1\\ 2\sin x & x^2 & 2x\\ \tan x & x & 1\end{vmatrix}$, then $\lim\limits_{x\to 0}\frac{f'(x)}{x}$

[JEE-Main On line-2018]

(A) exists and is equal to –2

(B) does not exist

(C) exist and is equal to 0

(D) exists and is equal to 2

Q.72 If $x_1, x_2, \ldots\ldots, x_n$ and $\frac{1}{h_1}, \frac{1}{h_2}\ldots\ldots\frac{1}{h_n}$ are two A.P's such that $x_3 = h_2 = 8$ and $x_8 = h_7 = 20$, then $x_5 . h_{10}$ equals

[JEE-Main On line-2018]

(A) 2560 (B) 2650 (C) 3200 (D) 1600

Q.73 The mean of a set of 30 observations is 75. If each other observation is multiplied by a non-zero number λ and then each of them is decreased by 25, their mean remains the same. Then λ is equal to

[JEE-Main On line-2018]

(A) $\frac{10}{3}$ (B) $\frac{4}{3}$ (C) $\frac{1}{3}$ (D) $\frac{2}{3}$

Q.74 If n is the degree of the polynomial.

$$\left[\frac{1}{\sqrt{5x^3+1}-\sqrt{5x^3-1}}\right]^8+\left[\frac{1}{\sqrt{5x^3+1}+\sqrt{5x^3-1}}\right]^8$$

and m is the coefficient of x^n in it, then the ordered pair (n, m) is equal to

[JEE-Main On line-2018]

(A) $(12, (20)^4)$ (B) $(8, 5(10)^4)$

(C) $(24, (10)^8)$ (D) $(12, 8(10)^4)$

Q.75 Let S be the set of all real values of k for which the system of linear equations

x + y + z = 2

2x + y – z = 3

3x + 2y + kz = 4

has a unique solution. Then S is

[JEE-Main On line-2018]

(A) an empty set (B) equal to R – {0}

(C) equal to {0} (D) equal to R

Q.76 The value of the integral

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\sin^4 x\left(1+\log\left(\frac{2+\sin x}{2-\sin x}\right)\right)dx \text{ is}$$

[JEE-Main On line-2018]

(A) $\frac{3}{16}\pi$ (B) 0 (C) $\frac{3}{8}\pi$ (D) $\frac{3}{4}$

Q.77 An angle between the plane x + y + z = 5 and the line of intersection of the planes, 3x + 4y + z – 1 = 0 and 5x + 8y + 2z + 14 = 0, is

[JEE-Main On line-2018]

(A) $\cos^{-1}\left(\frac{3}{\sqrt{17}}\right)$ (B) $\cos^{-1}\left(\sqrt{\frac{3}{17}}\right)$

(C) $\sin^{-1}\left(\frac{3}{\sqrt{17}}\right)$ (D) $\sin^{-1}\left(\sqrt{\frac{3}{17}}\right)$

Q.78 If a right circular cone having maximum volume, is inscribed in a sphere of radius 3 cm, then the curved surface area (in cm^2) of this cone is

[JEE-Main On line-2018]

(A) $8\sqrt{3}\,\pi$ (B) $6\sqrt{2}\,\pi$

(C) $6\sqrt{3}\,\pi$ (D) $8\sqrt{2}\,\pi$

Q.79 If tan A and tan B are the roots of the quadratic equation, $3x^2 – 10x – 25 = 10$ then the value of $3\sin^2(A + B) – 10\sin(A + B).\cos(A + B) – 25\cos^2(A + B)$ is

[JEE-Main On line-2018]

(A) 25 (B) –25 (C) –10 (D) 10

Q.80 Two parabolas with a common vertex and with axes along x-axis and y-axis, respectively, intersect each other in the first quadrant. If the length of the latus rectum of each parabola is 3, then the equation of the common tangent to the two parabolas is

[JEE-Main On line-2018]

(A) $3(x + y) + 4 = 0$ (B) $8(2x + y) + 3 = 0$

(C) $4(x + y) + 3 = 0$ (D) $x + 2y + 3 = 0$

Q.81 Let $y = y(x)$ be the solution of the differential equation $\frac{dy}{dx} + 2y = f(x)$, where

$$f(x) = \begin{cases} 1, & x \in [0,1] \\ 0, & \text{otherwise} \end{cases}$$

If $y(0) = 0$, then $y\left(\frac{3}{2}\right)$ is

[JEE-Main On line-2018]

(A) $\frac{e^2 - 1}{2e^3}$ (B) $\frac{e^2 - 1}{e^3}$

(C) $\frac{1}{2e}$ (D) $\frac{e^2 + 1}{2e^4}$

Q.82 The area (in sq. units) of the region $\{x \in R : x \geq 0, y \geq 0, y \geq x - 2 \text{ and } y \leq \sqrt{x}\}$, is

[JEE-Main On line-2018]

(A) $\frac{13}{3}$ (B) $\frac{10}{3}$ (C) $\frac{5}{3}$ (D) $\frac{8}{3}$

Q.83 If β is one of the angles between the normals to the ellipse, $x^2 + 3y^2 = 9$ at the points $(3\cos\theta, \sqrt{3}\sin\theta)$ and $(-3\sin\theta, \sqrt{3}\cos\theta)$; $\theta \in \left(0, \frac{\pi}{2}\right)$; then $\frac{2\cot\beta}{\sin 2\theta}$ is equal to

[JEE-Main On line-2018]

(A) $\sqrt{2}$ (B) $\frac{2}{\sqrt{3}}$ (C) $\frac{1}{\sqrt{3}}$ (D) $\frac{\sqrt{3}}{4}$

Q.84 If $(p \wedge \sim q) \wedge (p \wedge r) \rightarrow \sim p \vee q$ is false, then the truth values of p, q and r are respectively

[JEE-Main On line-2018]

(A) F, T, F (B) T, F, T

(C) F, F, F (D) T, T, T

Q.85 The set of all $\alpha \in R$, for which $w = \frac{1 + (1 - 8\alpha)z}{1 - z}$ is a purely imaginary number, for all $z \in C$ satisfying $|z| = 1$ and $\text{Re}(z) \neq 1$, is

[JEE-Main On line-2018]

(A) {0} (B) an empty set

(C) $\left\{0, \frac{1}{4}, -\frac{1}{4}\right\}$ (D) equal to R

Q.86 If $\vec{a}, \vec{b}$ and $\vec{c}$ are unit vectors such that $\vec{a} + 2\vec{b} + 2\vec{c} = \vec{0}$, then $\left|\vec{a} \times \vec{c}\right|$ is equal to

[JEE-Main On line-2018]

(A) $\frac{1}{4}$ (B) $\frac{\sqrt{15}}{4}$

(C) $\frac{15}{16}$ (D) $\frac{\sqrt{15}}{16}$

Q.87 An aeroplane flying at a constant speed, parallel to the horizontal ground, $\sqrt{3}$ km above it, is observed at an elevation of 60° from a point on the ground. If, after five seconds, its elevation from the same point, is 30°, then the speed (in km/hr) of the aeroplane is

[JEE-Main On line-2018]

(A) 1500 (B) 750

(C) 720 (D) 1440

Q.88 Let $S = \{(\lambda, \mu) \in R \times R :$ $f(t) = (|\lambda| e^{|t|} - \mu).\sin(2|t|)$, $t \in R$, is a differentiable function$\}$. Then S is a subset of **[JEE-Main On line-2018]**

(A) $R \times [0, \infty)$ (B) $(-\infty, 0) \times R$

(C) $[0, \infty) \times R$ (D) $R \times (-\infty, 0)$

Q.89 n-digit numbers are formed using only three digits 2, 5 and 7. The smallest value of n for which 900 such distinct numbers can be formed, is

[JEE-Main On line-2018]

(A) 6 (B) 8

(C) 9 (D) 7

Q.90 A box 'A' contains 2 white, 3 red and 2 black balls. Another box 'B' contains 4 white, 2 red and 3 black balls. If two balls are drawn at random, without replacement, from a randomly selected box and one ball turns out to be white while the other ball turns out to be red, then the probability that both balls are drawn from box 'B' is

[JEE-Main On line-2018]

(A) $\frac{7}{16}$ (B) $\frac{9}{32}$

(C) $\frac{7}{8}$ (D) $\frac{9}{16}$

ANSWERS

1. (3) **2.** (1) **3.** (1) **4.** (1) **5.** (2) **6.** (4) **7.** (2) **8.** (3) **9.** (4) **10.** (2) **11.** (3) **12.** (1) **13.** (3) **14.** (2)

15. (4) **16.** (1) **17.** (4) **18.** (2) **19.** (1) **20.** (2) **21.** (2) **22.** (1) **23.** (4) **24.** (4) **25.** (2) **26.** (2) **27.** (1) **28.** (3)

29. (2) **30.** (3) **31.** (2) **32.** (4) **33.** (2) **34.** (2) **35.** (4) **36.** (1) **37.** (4) **38.** (3) **39.** (1) **40.** (3) **41.** (1) **42.**(3)

43. (2) **44.** (4) **45.** (2) **46.** (1) **47.** (3) **48.** (4) **49.** (2) **50.** (3) **51.** (2) **52.** (4) **53.** (4) **54.** (2) **55.** (1) **56.** (4)

57. (4) **58.** (1) **59.** (3) **60.** (4) **61.** (2) **62.** (3) **63.** (3) **64.** (2) **65.** (2) **66.** (1) **67.** (1) **68.** (3) **69.** (4) **70.**(4)

71. (1) **72.** (1) **73.** (2) **74.** (4) **75.** (2) **76.** (3) **77.** (4) **78.** (1) **79.** (2) **80.** (3) **81.** (1) **82.** (2) **83.** (2) **84.** (2)

85. (1) **86.** (2) **87.** (4) **88.** (1) **89.** (4) **90.** (1)

Hints & Solutions

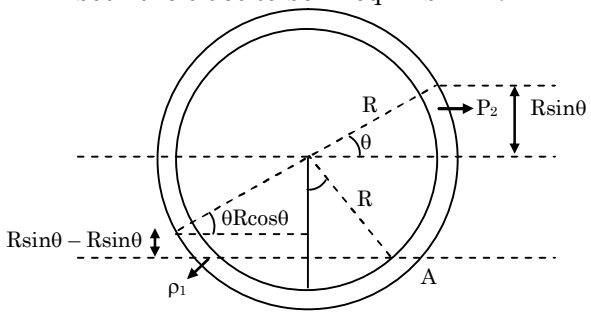
PHYSICS

1.[3] Relative error in surface area, $\frac{\Delta s}{s} = 2 \times \frac{\Delta r}{r} = \alpha$ and relative error in volume, $\frac{\Delta v}{v} = 3 \times \frac{\Delta r}{r}$

$\therefore$ Relative error in volume w.r.t. relative error in are,

$\frac{\Delta v}{v} = \frac{3}{2}\alpha$

2.[1] Pressure at interface A must be same from both the sides to be in equilibrium.



$\therefore$ $(R\cos\theta + R\sin\theta)\ \rho_2 g$

$= (R\cos\theta - R\sin\theta)\ \rho_1 g$

$\Rightarrow \frac{\rho_1}{\rho_2} = \frac{\cos\theta + \sin\theta}{\cos\theta - \sin\theta} = \frac{1 + \tan\theta}{1 - \tan\theta}$

$\Rightarrow \rho_1 - \rho_1 \tan\theta = \rho_2 + \rho_2 \tan\theta$

$\Rightarrow (\rho_1 + \rho_1)\tan\theta = \rho_1 - \rho_2$

$\therefore \theta = \tan^{-1}\left(\frac{\rho_1 - \rho_2}{\rho_1 + \rho_2}\right)$,

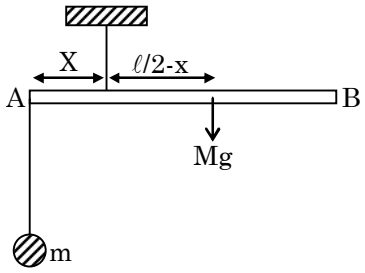
so closest answer is (1)

3.[1] Balancing torque w.r.t. point of suspension

$mg\,x = Mg\left(\frac{\ell}{2} - x\right)$

$\Rightarrow mx = M\,\frac{\ell}{2} - Mx$

$m = \left(M\frac{\ell}{2}\right)\frac{1}{x} - M$



$y = \alpha\frac{1}{x} - C$

Straight line equation.

4.[1] Given : Capacitance, $C = 0.2\ \mu F = 0.2 \times 10^{-6}$ F

Inductance $L = 0.5$ mH $= 0.5 \times 10^{-3}$ H

Current I = ?

Using energy conservation

$\frac{1}{2}CV^2 = \frac{1}{2}CV_1^2 + \frac{1}{2}LI^2$

$$\frac{1}{2}\times 0.2\times 10^{-6}\times 10^{2}+0$$

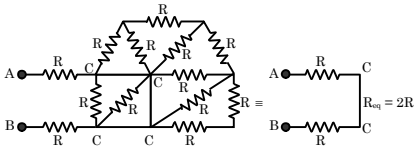
$$=\frac{1}{2}\times 0.2\times 10^{-6}\times 5^{2}+\frac{1}{2}\times 0.5\times 10^{-3}\,I^{2}$$

$$\therefore I=\sqrt{3}\times 10^{-1}\ A$$

$$=0.17\ A$$

5.[1] $R_{series} = R_1 + R_2 + ... + R_n$

$$\frac{1}{R_{parallel}}=\frac{1}{R_1}+\frac{1}{R_2}+.....+\frac{1}{R_n}$$



6.[4] Current gain $\beta = \frac{\Delta I_C}{\Delta I_B}$

Voltage gain A_v

$$= \text{Current gain} \times \text{Resistance gain} = \beta\frac{R_L}{R_{BE}}$$

Power gain A_p

$$= (\text{Current gain})^2 \times \text{Resistance gain} = \beta^2\frac{R_L}{R_{BE}}$$

7.[2] For a balanced meter bridge,

$$\frac{x}{39.5}=\frac{Y}{(100-39.5)}$$

$$\Rightarrow Y = 39.5 = X\times(100-39.5)$$

$$\text{or, } X=\frac{12.5\times 39.5}{60.5}=8.16\ \Omega$$

When X and Y are interchanged l_1 and $(100 - l_1)$ will also interchange so, $l_2 = 60.5$ cm

8.[3] Momentum (p) of each electron

$$\frac{h}{\lambda_1}\hat{i}\ \text{and}\ \frac{h}{\lambda_2}\hat{j}$$

velocity of centre of mass

$$V_{cm}=\frac{h}{2m\lambda_1}\hat{i}+\frac{h}{2m\lambda_2}\hat{j}\qquad(\because\ p = mv)$$

Velocity of I st particle about centre of mass

$$V_{1cm}=\frac{h}{2m\lambda_1}\hat{i}-\frac{h}{2m\lambda_2}$$

$$\lambda_{cm}=\frac{h}{\sqrt{\frac{h^2}{4\lambda_1^2}+\frac{h^2}{4\lambda_2^2}}}=\frac{2\lambda_1\lambda_2}{\sqrt{\lambda_1^2+\lambda_2^2}}\quad\left(\because\lambda=\frac{h}{p}\right)$$

9.[4] According to question, tuning fork gives 1 beat/second with (N) 3rd normal mode. Therefore, organ pipe will have frequency (256 ± 1) Hz. In open organ pipe, frequency

$$n=\frac{NV}{2\ell}$$

$$\text{or, } 255=\frac{3\times 340}{2\times\ell}\Rightarrow \ell = 2m = 200\ cm$$

10.[2] Given, modulating frequency $f_m = 15$ Hz

$\therefore$ Bandwidth of one channel $= 2f_m = 30$ kHz

$$\therefore \text{No of channels accommodate} = \frac{330\,kHz}{30\,kHz}$$

$$= 10$$

11.[3] Initial gravitational potential energy,

$$E_i = -\frac{GMm}{2R}$$

Final gravitational potential energy,

$$E_f = -\frac{GMm/2}{2\left(\frac{R}{2}\right)}-\frac{GMm/2}{2\left(\frac{3R}{2}\right)}$$

$$=-\frac{GMm}{2R}-\frac{GMm}{6R}$$

$$=-\frac{4GMm}{6R}-\frac{2GMm}{3R}$$

$\therefore$ Difference between initial and final energy,

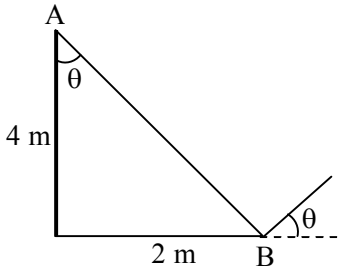
$$E_f - E_i = \frac{GMm}{R}\left(\frac{-2}{3}+\frac{1}{3}\right)=-\frac{GMm}{6R}$$

12.[1] If angular position of 2nd maximum from central maxima is θ then

$$\sin\theta=\frac{(2n-1)\lambda}{2a}=\frac{3\lambda}{2a}=\frac{3\times 550\times 10^{-9}}{2\times 22\times 10^{-7}}$$

$$\therefore\quad \theta=\frac{\pi}{8}\ rad$$

13.[3] To produce maximum moment of force line of action of force must be perpendicular to line AB.



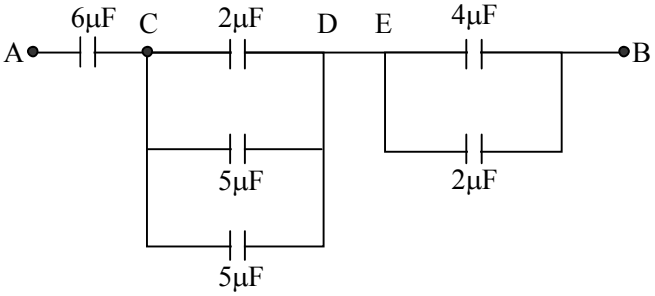
$\therefore \tan\theta = \frac{2}{4} = \frac{1}{2}$

14.[2] The coefficients of kinetic friction between the object and the incline

$\mu = \tan\theta\left(1 - \frac{1}{n^2}\right)$

$\Rightarrow \mu = 1 - \frac{1}{n^2}$ $(\because \theta = 45°)$

15.[4] The simplified circuit of the circuit given in question a follows :



The equivalent capacitance between C & D capacitors of 2 μF, 5 μF and 5 μF are in parallel.

$\therefore C_{CD} = 2+5+5=12$ μF ($\because$ In parallel grouping $C_{eq} = C_1+C_2+.....+C_n$)

Similarly equivalent capacitance beteen E & B

$C_{EB} = 4 + 2 = 6\mu F$

Now equivalent capacitance between A & B

$\frac{1}{C_{eq}} = \frac{1}{6} + \frac{1}{12} + \frac{1}{6} = \frac{5}{12}$

$\Rightarrow C_{eq} = \frac{12}{5} = 2.4$ μF ($\because$ In series grouping, $\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + ... + \frac{1}{C_n}$)

16.[1] Given : Temperature of cold body, $T_2 = 250$ K temperature of hot body; $T_1 = 300$ K

Heat received, $Q_2 = 500$ cal work done, $W = ?$

$\text{Efficiency} = 1 - \frac{T_2}{T_1} = \frac{W}{Q_2 + W}$

$\Rightarrow 1 - \frac{250}{300} = \frac{W}{Q_2 + W}$

$W = \frac{Q_2}{5} = \frac{500 \times 4.2}{5}\text{ J} = 420\text{ J}$

17.[4] Work done on gas $= nRT\,\ell n\left(\frac{p_f}{p_1}\right)$

$= R(300)\,\ell n(2)$ $\left(\because \frac{p_f}{p_i} = 2 \text{ given}\right)$

18.[2] According to question, $\mu_1 = 40$ km/h, $v_1 = 0$ and $s_1 = 40$ m

using $v^2 - u^2 = 2as$; $0^2 - 40^2 = 2a \times 40$ ....(i)

Again, $0^2 - 80^2 = 2as$ ...(ii)

From eqn. (i) and (ii)

Stopping distance, $s = 160$ m

19.[1] As we know, Gravitational force of attraction,

$F = \frac{GMm}{R^2}$

$F_1 = \frac{GM_e m}{r_1^2}$ and $F_2 = \frac{GM_e M_s}{r_2^2}$

$\Delta F_1 = \frac{2GM_e m}{r_1^3}\Delta r_1$ and $\Delta F_2 = \frac{2GM_e m}{r_2^3}\Delta r_2$

$\frac{\Delta F_1}{\Delta F_2} = \frac{m\Delta r_1}{r_1^3}\frac{r_2^3}{M_s \Delta r_2} = \left(\frac{m}{M_s}\right)\left(\frac{r_2^3}{r_1^3}\right)\left(\frac{\Delta r_1}{\Delta r_2}\right)$

Using $\Delta r_1 = \Delta r_2 = 2\,R_{earth}$; $m = 8 \times 10^{22}$ kg; $M_s = 2 \times 10^{30}$ kg

$r_1 = 0.4 \times 10^6$ km and $r_2 = 150 \times 10^6$ km

$\frac{\Delta F_1}{\Delta F_2} = \left(\frac{8 \times 10^{22}}{2 \times 10^{30}}\right)\left(\frac{150 \times 10^6}{0.4 \times 10^6}\right) \times 1 \cong 2$

20.[2] Given Number of turns,

$n = 1000$ turns/cm $= 1000 \times 100$ turns/m

Coercivity of ferromagnet, $H = 100$ A/m

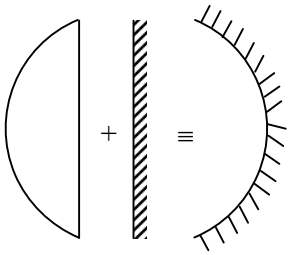
Current to demagnetise the ferromagnet, $I = ?$

Using, $H = nI$

or, $100 = 10^5 \times 1$

$\therefore I = \frac{100}{10^5} = 1$ mA

21.[2] **Case-I**

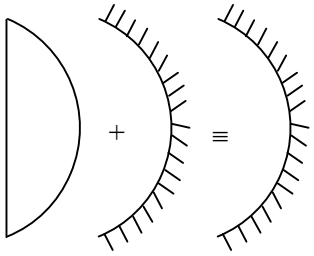


$\frac{1}{f_1} = \left(\frac{\mu - 1}{R}\right)$ $\quad f = -28$

$P = 2P_1 + P_2 \Rightarrow \frac{1}{28} 2\left(\frac{\mu - 1}{R}\right)$

($\because$ Power, $P = \frac{1}{f}$ & $f_{\text{plane mirror}} = \infty$)

Case-II



$\frac{1}{f_1}\left(\frac{\mu - 1}{R}\right)$, $f_2 = \frac{R}{2}$ $\quad f = -10$ cm

$P = 2P_1 + P_2$

$\Rightarrow \frac{1}{10} = \left(\frac{\mu - 1}{2}\right) + \frac{2}{R}$

or, $\frac{1}{10} = \frac{1}{28} + \frac{2}{R} \Rightarrow \frac{2}{R} = \frac{1}{10} - \frac{2}{28} = \frac{18}{280}$

or, $R = \frac{280}{9}$ cm

or, $\frac{1}{28} = 2\left(\frac{\mu - 1}{280}\right)9$

$\Rightarrow \mu - 1 = \frac{5}{9}$

$\therefore \mu = 1 + \frac{5}{9} = \frac{14}{9} = 1.55$

22.[1] Equilibrium position will shift to point where resultant force = 0

$kx_{eq} = qE \Rightarrow x_{eq} = \frac{qE}{k}$

Total energy = $\frac{1}{2} m\omega^2 A^2 + \frac{1}{2} kx_{eq}^2$

Total energy = $\frac{1}{2} m\omega^2 A^2 + \frac{1}{2} \frac{q^2 E^2}{k}$

23.[4] Least count

$= \frac{\text{Value of 1 part on main scale}}{\text{Number of parts on vernier scale}}$

$= \frac{0.25}{5 \times 100}$ cm $= 5 \times 10^{-4}$ cm

Reading = 4×0.05 cm + $30 \times 5 \times 10^{-4}$ cm

= (0.2 + 0.0150) cm

= 0.2150 cm (Thickness of wire)

24.[4] Let initial activity = No = 0.8 μ ci

$0.8 \times 3.7 \times 10^4$ dps

Activity in 1 cm^3 of blood at t = 10 hr,

$n = \frac{300}{60}$ dps = 5 dps

N = Activity of whole blood at time t = 10 hr.

Total volume of the blood in the person,

$V = \frac{N}{n}$

$= \frac{N_0 e - \lambda t}{n} = \frac{0.8 \times 3.7 \times 10^4 \times 0.7927}{5} \cong 5$ litres

25.[2] When cube is of side a and point charge Q is at the centre of the cube then the total electric flux due to this charge will pass evenly through the six faces of the cube. So, the electric flux through one face will be equal to 1/6 of the total electric flux due to this charge.

Flux through 6 faces = $\frac{Q}{\epsilon_o}$

$\therefore$ Flux through 1 face, = $\frac{Q}{6\,\epsilon_o}$

26.[2] Energy required to remove e^- from singly ionized helium atom = $\frac{(13.6)z^2}{1^2} = 54.4$ eV

($\because$ Z = 2)

Energy required to remove e^- from helium atom = xeV

According to question, 54.4 eV = 2.2x

$\Rightarrow$ x = 24.73 eV

Therefore, energy required to ionize helium atom = (54.4 + 24.73) eV = 79.12 eV

27.[1]

28.[3] Using equation, $\alpha = \frac{v - u}{t}$ and

$S = ut + \frac{1}{2} at^2$

Distance travelled by car in 15 sec

$= \frac{1}{2} \frac{(45)}{15} (15)^2$

$= \frac{675}{2}$ m

Distance traveled by scooter in 15 seconds $= 30 \times 15 = 450$ ($\because$ distance = speed × time)

Distance between distance travelled by car and scooter in 15 sec, $450 - 337.5 = 112.5$ m Let car catches scooter in time t;

$\frac{675}{2} + 45(t - 15) = 30\ t$

$337.5 + 45t - 675 = 30t$

$\Rightarrow \quad 15t = 337.5$

$\Rightarrow \quad t = 22.5$ sec

29.[2] Point *P* is situated at the mid-point of the line joining the centres of the circular of the circular wires which have same radii (R). The magnetic fields ($\vec{B}$) at P due to the currents in the wires are in same direction.

Magnitude of magnetic field at point, *P*

$$B = 2\left\{\frac{\mu_0 NIR^2}{2\left(R^2 + \frac{R^2}{4}\right)}\right\}$$

$$= \frac{\mu_0 NIR^2}{\frac{5^{3/2}}{8}} = \frac{8\mu_0 NI}{5^{3/2} R}$$

30.[3] When object is at 8 cm

Image $V_1 = \frac{f \times u}{u - f} = \frac{5 \times 8}{8 - 5} = -\frac{40}{3}$ cm

When object is at 12 cm

Image $V_2 = \frac{f \times u}{u - f} = \frac{5 \times 12}{12 - 5} = -\frac{60}{7}$

Separation = $|V_1 - V_2| = \frac{40}{3} - \frac{60}{7} = \frac{100}{21}$ cm

So A, C and D are correct satements.

CHEMISTRY

31.[2] The N atom of amide is not basic.

32.[4] No. of moles of oxygen in 0.16 g of oxygen molecule

$= \frac{0.16\text{g}}{32\text{g/mol}} = 0.005$ mole

$2NaClO_3 \xrightarrow{\Delta} 2NaCl + 3O_2$

According to the reaction,

3 moles of O_2 = 2 moles of NaCl = 2 moes of AgCl

kMolar mass of AgCl = 143.5 g/mol

0.005 moles of O_2 will ppt.

$= 0.005 \times \frac{2}{3}$ moles AgCl

= 0.0033 moles of AgCl

$\therefore$ Mass of AgCl (in g) obtained will be

= 143.5 g/mol × 0.0033 moles

= 0.48 g.

33.[2] $\Delta H = \Delta U + \Delta n_g RT$

$2HI(g) \rightarrow H_2(g) + I_2(g)$; $\Delta n_g = (1 + 1) - 2 = 0$

$\therefore \Delta H = \Delta U + 0$

34.[2]

	N_2O_5	$\rightarrow$	$2NO_2$	+	$\frac{1}{2}O_2$
At t=0	50		0		0
At t=50 mint	50-p_1		$2p_1$		$\frac{p_1}{2}$

Total pressure at 50 minutes

$= 50 - p_1 + 2p_1 + \frac{p_1}{2} = 87.5$

$50 + \frac{3p_1}{2} = 87.5$

$\frac{3p_1}{2} = 37.5$

$\therefore p_1 = \frac{37.5 \times 2}{3} = 25$

At t = 100 min	$50 - p_2$		$2p_2$		$\frac{p_2}{2}$

50 minutes is half life period

For 100 minutes i.e. for 2 half lives $50 - p_2 = 12.5$

$\therefore p_2 = 37.5$ mm of Hg

Total pressure at 100 minutes

$= 50 - p_2 + 2p_2 + \frac{p_2}{2}$

$= 50 + \frac{3p_2}{2}$

$= 50 + \frac{3}{2} \times 37.5$

$= 50 + 56.25$

$= 106.25$ mm of Hg

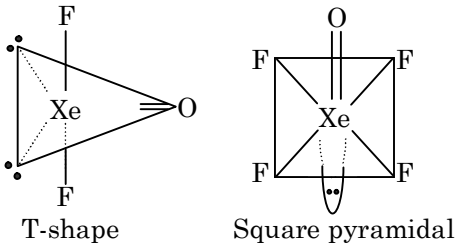
35.[4] Total electrons in $N_2^+ = (7 \times 2) - 1 = 13$

$N_2^+ \rightarrow \sigma_{1s^2}, \overset{*}{\sigma}_{1s^2}, \sigma_{2s^2}, \overset{*}{\sigma}_{1s^2}, \left[\pi^2_{2px} = \pi^2_{2py}\right] \sigma^1_{2pz}$

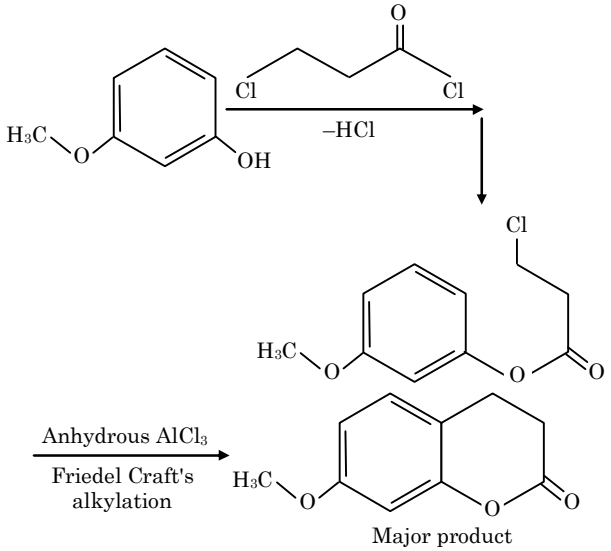
Number of electron in σ_{2pz} is 1

36.[1] Here dehydrohalogenation goes by EIcB and most stable carbanion formation is favoured in (a).

37.[4] $XeOF_2$ $XeOF_4$



38.[3] The major product of the reaction is given by the option (c).
Since acid chloride is more reaction than alkyl halide – COCl group will react first.
In the second step, friedel Craft's alkylation occurs in a position that is ortho to alkoxy group and para to methoxy group. Both methoxy and alkoxy groups are ortho para directing groups.



39.[1] As in the resonance structure of hydrogen azide, it can be seen that number of N-N bonds is ≤ 2

$$H-\overset{(I)}{N^-}-\overset{(II)}{N}\equiv N^+ \leftrightarrow H-\overset{(I)}{N}=\overset{(II)}{N^+}=N^- \leftrightarrow H-\overset{(I)}{N}-\overset{(II)}{N}=N^-$$

Hence for bond (I), bond order will be < 2 whereas for bond (II), number of bond ≥ 2. Thus its bond order will be > 2.

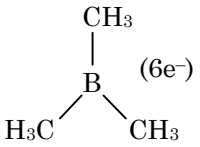
40.[3] All species are isoelectronic ($10e^-$).
isoelectronic series, when negative charge increases the radius of ion increases.
$\therefore Mg^{2+} < Na^+ < F^- < O^{2-}$

41.[1] In the given substituted benzene rings, the substitutents methoxy ($-OCH_3$) and amino ($-NH_2$) are strongly activating groups while methyl ($-CH_3$) is weakly activating and chloro (–Cl) is a deavtivating group towards electrophilic aromatic substituation reaction. Since among methyl and methoxy group is more reactive than methyl group, (c) is more reactive than (d).
Although amino group is strongly activating group, it gets protonated in presence of acid to form anilinium ion $(-\overset{+}{N}H_3)$ which is strongly deactivating Hence, (a) is less reactive than (c) and (d).
Chloro group is also deactivating group but less deactivating than $(-\overset{+}{N}H_3)$. Thus order is (a) < (b) < (d) < (c).
Note:
The activating groups increases the electron density on benzene ring and increases the rate of electrophilic aromatic substitution reaction. The deactivating groups decreases the electron density on benzene ring and decreases the rate of electrophilic aromatic substitution reaction.

42.[3] (a) Charcoal treatment removes coloured impurity through adsorption.
(b) Steam distillation separates the mixture of o-nitrophenol and p-nitrophenol. The o-nitrophenol is steam volatile (due to intramolecular hydrogen bonding), and the para isomer is not volatile.
(c) Fractional distillation separates crude naphtha. Naphtha is a flammable liquid hydrocarbon mixture.
(d) Distillation under reduced pressure separates mixture of glycerol and sugars. Vacuum distillation lowers the

boiling point and prevents decomposition.

44.[4]



45.[2] (a) Colloidal particles are so small that they can pass through ordinary filter paper. Also, they cannot be seen with ordinary microscope.

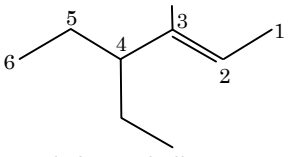
(b) Freezing point of colloidal solution is same as that of true solution at same concentration of a solute. The depression in freezing point is a colloidal property and independent of size or shape of solute particles.

(c) When silver nitrate solution is added to poteassium iodide solution, a neatively selective adsorption of I^- ion from the dispersion medium. However, if the order of addition is reversed, i.e, potassium iodide solution is added to silver nitrate solution, due to selective adsorption of Ag^+ ion from the dispersion medium, a positively charged colloidal solution is obtained.

(d) When excess of electrolyte is added to colloidal solution, colloidal particle will be precipitated. Although electrolytes in minute quantities are necessary for the stability of colloids, they cause coagulation of disperse phase if present in large quantities.

47.[3] $[Ni(CN)_4]^{2-}$ is square planar, diamagnetic (0 unpaired electrons) with dsp^2 hybridisation. $[Ni(CO)_4]$ – is tetrahedral, diamagnetic (0 unpaired electrons) with sp^3 hybridisation. $[NiCl_4]^{2-}$ is tetrahedral, paramagnetic (2 unpaired electrons) with sp^3 hybridisation. Hence, the option (c) is the ocrrect answer.

48.[4]



4-Ethyl-3-methylhex-2-ene

49.[2] $\Delta U_{AB} = q_{AB} + W_{AB} = 2 + (-5) = -3$kJ/mol

$\Delta U_{BC} = -5$kJ/mol

For cyclic process, $\Delta U = 0$

$\Delta U_{AB} + \Delta U_{BC} + \Delta U_{CA} = 0$

$\Delta U_{CA} + -\Delta U_{AB} - \Delta U_{BC}$

$\Delta U_{CA} = -(-3) - (-5) = 8$ kJ/mol

$\Delta U_{CA} = q_{CA} + W_{CA}$

$8 = q_{CA} + 3$

$q_{CA} = +5$kJ/mol

Heat absorbed has positive sign.

50.[3] $\lambda = 250$ nm $= 2500$ Å

$$E = \frac{hc}{\lambda} = \frac{12400}{2500} = 4.96 \text{ eV}$$

KE = stopping potential = 0.5 eV

$E = W_0 + K.E.$

$4.96 = W_0 + 0.5$

$W_0 = 4.46 \approx 4.5$ eV

51.[2] % of p-in graphite $(sp^2) = \frac{2}{3} \times 100 = 67\%$

% of p-in diamond $(sp^3) = \frac{3}{4} \times 100 = 75\%$

52.[4] Reduction at cathode:

$2e^- + 2H_2O \rightarrow H_2 + 2OH^-$

(valence factor) $H_2 = 2$

At NTP 22400 mL of H_2 = 1 mole of H_2 112 mL of

$$H_2 = \frac{1}{22400} \times 112 = 0.005 \text{ mole of } H_2$$

$$\text{Moles of } H_2 \text{ produced} = \frac{I \times t}{96500} \times \text{mole ratio}$$

$$0.005 = \frac{I \times 965}{96500} \times \frac{1 \text{ mole of } H_2}{2 \text{ mole of } e^-}$$

I = 1.0 A

53.[4] $(Ksp)_{pbCl_2} = 3.2 \times 10^{-8} = 32 \times 10^{-9}$

$$PbCl2 \rightleftharpoons \underset{s}{Pb^{2+}} + \underset{2s}{2Cl^-}$$

$K_{sp} = [Pb^{2+}][Cl^-]^2$

$K_{sp} = 4s^3 = 32 \times 10^{-9}$

$s^3 = 8 \times 10^{-9}$

$s = 2 \times 10^{-3}$ M

$$\frac{w}{M.W.} \times \frac{1}{V_L} = 2 \times 10^{-3}$$

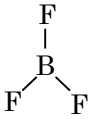
$$\frac{0.1}{278} \times \frac{1}{V_L} = 2 \times 10^{-3}$$

$$V_L = \frac{0.1 \times 1000}{278 \times 2} = 0.18 \text{ L}$$

54.[2] Volume ↑ P ↓, reaction proceeds in which direction where the number of mole of gases increases.

$2NO_2(g) \rightleftharpoons 2NO(g) + O_2(g)$

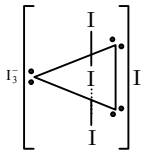
$\Delta n_g = (2 + 1) - 2 = 1$

55.[1]  B is sp^2. Bond angle = 120°

 N is sp^3. with 1 *l*p, Bond angle = 107°

 P is sp^3. with 1 *l*p, when central atom

size ↑, bond angle ↓, ∴ NH3 > PF_3

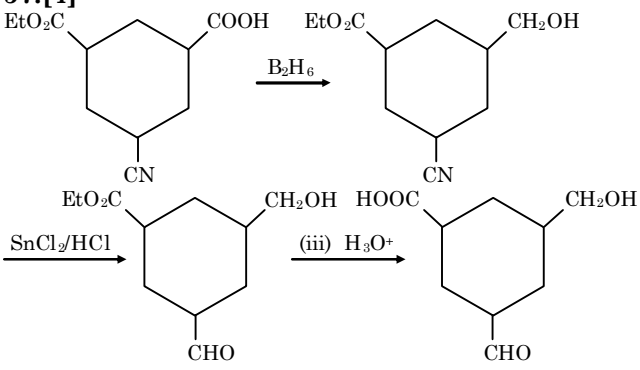
 I is sp^3d(linear), Bond angle = 180°

∴ Dereasing order of bond angle

$I_3^- > BF_3 > NH_3 > PF_3$

56.[4] When the magnetic moments align in a regular pattern with neighbouring spins pointing in opposite directions (as in d), the substance shows antiferromagnetism.

57.[4]



58.[1] Sodium borohydride reduces ketonic group to alcohol, but not the amide group and C = C double bond.

59.[3] $XeF_6 + H_2O \xrightarrow[\text{hydrolysis}]{\text{partial}} \overset{+6}{Xe}OF_4 + 2HF$ ('X')

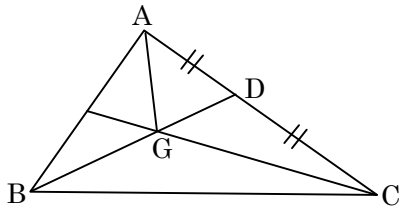
$XeOF_4 + H_2O \rightarrow \overset{+6}{Xe}O_2F_2 + 2HF$ ('X' → 'Y')

60.[4] $\underset{\text{(white ppt)}}{Cl^-} + Na^+ \rightarrow \underset{\text{(Neutral)}}{NaCl} \xrightarrow{AgNO_3} AgCl$ (insoluble in dilute HNO_3)

MATHEMATICS

61.[2] Median through C is x = 4

So the x coordinate of C is 4. Let C ≡ (4, y), then the midpoint of A (1, 2) and C(4, y) is D which lies on the median through B.



$\therefore D \equiv \left(\frac{1+4}{2}, \frac{2+y}{2}\right)$

Now, $\frac{1+4+2+y}{2} = 5 \Rightarrow y = 3.$

So, C ≡ (4, 3)

The centroid of the triangle is the intersection of the medians. Here the medians x = 4 and x + y = 5 intersect at G(4, 1).

The area of triangle ΔABC = 3 × ΔAGC

$= 3 \times \frac{1}{2}[1(1-3) + 4(3-2) + 4(2-1)] = 9.$

62.[3] First term = b and common ratio = r

For infinite series, Sum = $\frac{b}{1-r} = 5$

$\Rightarrow b = 5(1 - r)$

So, interval of b = (0, 10) as, $-1 < r < 1$ for infinite GP.

63.[3] If a, b, c are the intercepts of the variable plane on the x, y, z axes respectively, then the equation of the plane is $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1.$

And the point of intersection of the planes parallel to the xy, yz and zx planes is (a, b, c).

As the point (3, 2, 1) lies on the variable plane, so $\frac{3}{a} + \frac{2}{b} + \frac{1}{c} = 1$

Therefore, the required locus is $\frac{3}{x}+\frac{2}{y}+\frac{1}{z}=1$

64.[2] Suppose, $\frac{x-4}{x+2}=y$

$\Rightarrow x-4=y(x+2)$

$\Rightarrow x(1-y)=2y+4$

$\Rightarrow x=\frac{2y+4}{1-y}$

So, $f(y)=2\left(\frac{2y+4}{1-y}\right)+1$

Now, $f(x)=2\left(\frac{2x+4}{1-x}\right)+1=\frac{3x+9}{1-x}$

$=\frac{3(x+3)}{1-x}=\frac{3(x-1+4)}{1-x}=-3+\frac{12}{1-x}$

$\therefore \int f(x)dx=-12\log_e|1-x|-3x+c$

65.[2] Let, the roots of the equation,
$x^2+(2-\lambda)x+(10-\lambda)=0$ are α and β.
Also roots of the given equation are

$\frac{\lambda-2\pm\sqrt{4-4\lambda+\lambda^2-40+4\lambda}}{2}$

$=\frac{\lambda-2\pm\sqrt{\lambda^2-36}}{2}$

The magnitude of the difference of the roots is $\left|\sqrt{\lambda^2-36}\right|$

So, $\alpha^3+\beta^3=\frac{(\lambda-2)^3}{4}+\frac{3(\lambda-2)(\lambda^2-36)}{4}$

$=\frac{(\lambda-2)(4\lambda^2-4\lambda-104)}{4}$

$=(\lambda-2)(\lambda^2-\lambda-26)=f(\lambda)$

As $f(\lambda)$ attains its minimum value at $\lambda=4$.
Therefore, the magnitude of the difference of the roots is $\left|i\sqrt{20}\right|=2\sqrt{5}$

66.[1] Both R_1 and R_2 are symmetric as
For any $(x, y) \in R_1$, we have
$(y, x) \in R_1$ and similarly for R_2
Now, for R_2, $(b, a) \in R_2$, $(a, c) \in R_2$ but $(b, c) \notin R_2$.
Similarly, for R_1, $(b, c) \in R_1$, $(c, a) \in R_1$ but $(b, a) \notin R_1$.
Therefore, neither R_1 nor R_2 is transitive.

67.[1] Given, $x^2+y^2+\sin y=4$

After differentiating the above equation w.r.t. x we get

$2x+2y\frac{dy}{dx}+\cos y\frac{dy}{dx}=0$ .....(1)

$\Rightarrow 2x+(2y+\cos y)\frac{dy}{dx}=0$

$\Rightarrow \frac{dy}{dx}=\frac{-2x}{2y+\cos y}$

At $(-2, 0)$, $\left(\frac{dy}{dx}\right)_{(-2,0)}=\frac{4}{0+1}$

$\Rightarrow \left(\frac{dy}{dx}\right)_{(2,0)}=4$ .....(2)

Again differentiating equation (1) w.r. to x, we get

$2+2\left(\frac{dy}{dx}\right)^2+2y\frac{d^2y}{dx^2}-\sin y\left(\frac{dy}{dx}\right)^2+\cos y\frac{d^2y}{dx^2}=0$

$\Rightarrow 2+(-2\sin y)\left(\frac{dy}{dx}\right)^2+(2y+\cos y)\frac{d^2y}{dx^2}=0$

$\Rightarrow (2y+\cos y)\frac{d^2y}{dx^2}=-2-(2-\sin y)\left(\frac{dy}{dx}\right)^2$

$\Rightarrow \frac{d^2y}{dx^2}=\frac{-2-(2-\sin y)\left(\frac{dy}{dx}\right)^2}{2y+\cos y}$

So, at $(-2, 0)$,

$\frac{d^2y}{dx^2}=\frac{-2-(2-0)\times 4^2}{2\times 0+1}$

$\Rightarrow \frac{d^2y}{dx^2}=\frac{-2-2\times 16}{1}$

$\Rightarrow \frac{d^2y}{dx^2}=-34$

68.[3] Equation of the line passing through the points (2, 3) and (4, 5) is

$y-3=\left(\frac{5-3}{4-2}\right)x-2 \Rightarrow x-y+1=0$ ...(1)

Equation of the perpendicular line passing through the midpoint (3, 4) is

$x+y-7=0$ ...(2)

Lines (1) and (2) intersect at the center of the circle. So, the center of the circle is (3, 4)
Therefore, the radius is

$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$= \sqrt{(2-3)^2+(3-4)^2} = \sqrt{2}$ units

69.[4] Equation of hyperbola is :

$4y^2 = x^2 + 1 \Rightarrow -x^2 + 4y^2 = 1$

$\Rightarrow -\frac{x^2}{1^2} + \frac{y^2}{\left(\frac{1}{2}\right)^2} = 1$

$\therefore a = 1, b = \frac{1}{2}$

Now, tangent to the curve at point (x_1, y_1) is given by.

$4 \times 2y_1 \frac{dy}{dx} = 2x_1$

$\Rightarrow \frac{dy}{dx} = \frac{2x_1}{8y_1} = \frac{x_1}{4y_1}$

Equation of tangent at (x_1, y_1) is

$y = mx + c$

$\Rightarrow y = \frac{x_1}{4y_1} . x + c$

As tangent passes through (x_1, y_1)

$\therefore y_1 = \frac{x_1 x_1}{4y_1} + c$

$\Rightarrow c = \frac{4y_1^2 - x_1^2}{4y_1} = \frac{1}{4y_1}$

Therefore, $y = \frac{x_1}{4y_1}x + \frac{1}{4y_1}$

$\Rightarrow 4y_1 y = x_1 x + 1$

which intersects x-axis at $A\left(\frac{-1}{x_1}, 0\right)$ and y-axis at $B\left(0, \frac{1}{4y_1}\right)$

Let midpoint of AB is (h, k)

$\therefore h = \frac{-1}{2x_1}$

$\Rightarrow x_1 = \frac{-1}{2h}$ & $y_1 = \frac{1}{8k}$

Thus, $4\left(\frac{1}{8k}\right)^2 = \left(\frac{-1}{2h}\right)^2 + 1$

$\Rightarrow \frac{1}{16k^2} = \frac{1}{4h^2} + 1$

$\Rightarrow 1 = \frac{16k^2}{4h^2} + 16k^2$

$\Rightarrow h^2 = 4k^2 + 16h^2k.$

So, required equation is

$x^2 - 4y^2 - 16x^2y^2 = 0$

70.[4] Since

$A \cdot \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$ is a scalar matrix and $|3A| = 108$

suppose the scalar matrix is $\begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}$

$\therefore A \cdot \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}$

$\Rightarrow A = \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}^{-1}$

$[\because AB = C \Rightarrow ABB^{-1} = CB^{-1} \Rightarrow A = CB^{-1}]$

$\Rightarrow A = \frac{1}{3}\begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}\begin{bmatrix} 3 & -2 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow A = \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}\begin{bmatrix} 1 & -\frac{2}{3} \\ 0 & \frac{1}{3} \end{bmatrix}$

$\Rightarrow A = \begin{bmatrix} k & -\frac{2}{3}k \\ 0 & \frac{k}{3} \end{bmatrix}$ .....(i)

$\because |3A| = 108$

$\Rightarrow 108 = \begin{vmatrix} 3k & -2k \\ 0 & k \end{vmatrix}$

$\Rightarrow 3k^2 = 108 \Rightarrow k^2 = 36 \Rightarrow k = \pm 6$

For k = 6

$A = \begin{bmatrix} 6 & -4 \\ 0 & 2 \end{bmatrix}$ From (1)

$\Rightarrow A^2 = \begin{bmatrix} 36 & -32 \\ 0 & 4 \end{bmatrix}$

For k = –6

$\Rightarrow A = \begin{bmatrix} -6 & 4 \\ 0 & -2 \end{bmatrix}$ From (1)

$\Rightarrow A^2 = \begin{bmatrix} 36 & -32 \\ 0 & 4 \end{bmatrix}$

71.[1] $f(x) = \begin{vmatrix} \cos x & x & 1 \\ 2\sin x & x^2 & 2x \\ \tan x & x & 1 \end{vmatrix}$

$= \cos x (x^2 - 2x^2) - x(2 \sin x - 2x \tan x)$
$+1(2x \sin x - x^2 \tan x)$

$= -x^2\cos x - 2x\sin x + 2x^2\tan x$
$+2x\sin x - x^2\tan x$
$= -x^2\tan x - x^2\cos x = x^2(\tan x - \cos x)$
$\Rightarrow f'(x) = 2x(\tan x - \cos x) + x^2(\sec^2 x + \sin x)$
$\therefore \lim\limits_{x\to 0}\dfrac{f'(x)}{x}$
$= \lim\limits_{x\to 0}\dfrac{2x(\tan x - \cos x) + x^2(\sec^2 x + \sin x)}{x}$
$= \lim\limits_{x\to 0}(\tan x - \cos x) + x(\sec^2 x + \sin x)$
$= 2(0 - 1) + 0 = -2$
So, $\lim\limits_{x\to 0}\dfrac{f'(x)}{x} = -2$

72.[1] Suppose d_1 is the common difference of the A.P.
$x_1, x_2, \ldots\ldots x_n$ then
$\because x_8 - x_3 = 5d_1 = 12 \Rightarrow d_1 = \dfrac{12}{5} = 2.4$
$\Rightarrow x_5 = x_3 + 2d_1 = 8 + 2\times\dfrac{12}{5} = 12.8$
Suppose d_2 is the common difference of the A.P. $\dfrac{1}{h_1}, \dfrac{1}{h_2}, \ldots\ldots\dfrac{1}{h_n}$ then
$5d_2 = \dfrac{1}{20} - \dfrac{1}{8} = \dfrac{-3}{40} \Rightarrow d_2 = \dfrac{-3}{200}$
$\because \dfrac{1}{h_{10}} = \dfrac{1}{h_7} + 3d_2 = \dfrac{1}{200} \Rightarrow h_{10} = 200$
$\Rightarrow x_5.h_{10} = 12.8\times 200 = 2560$

73.[2] As mean is a linear operation, so if each observation is multiplied by λ and decreased by 25 then the mean becomes $75\lambda - 25$.
According to the question,
$75\lambda - 25 = 75 \Rightarrow \lambda = \dfrac{4}{3}$

74.[4]
$$\left[\frac{1}{\sqrt{5x^3+1}-\sqrt{5x^3-1}}\right]^8 + \left[\frac{1}{\sqrt{5x^3+1}+\sqrt{5x^3-1}}\right]^8$$
After rationalise the polynomial we get
$$\left[\frac{1}{\sqrt{5x^3+1}-\sqrt{5x^3-1}}\times\frac{\sqrt{5x^3+1}+\sqrt{5x^3-1}}{\sqrt{5x^3+1}+\sqrt{5x^3-1}}\right]^8$$
$$+\left[\frac{1}{\sqrt{5x^3+1}+\sqrt{5x^3-1}}\times\frac{\sqrt{5x^3+1}-\sqrt{5x^3-1}}{\sqrt{5x^3+1}-\sqrt{5x^3-1}}\right]^8$$
$$=\left[\frac{\sqrt{5x^3+1}+\sqrt{5x^3-1}}{(5x^3+1)-(5x^3-1)}\right]^8 + \left[\frac{\sqrt{5x^3+1}-\sqrt{5x^3-1}}{(5x^3+1)-(5x^3-1)}\right]^8$$
$$=\frac{1}{2^8}\left[\left(\sqrt{5x^3+1}+\sqrt{5x^3-1}\right)^8 + \left(\sqrt{5x^3+1}-\sqrt{5x^3-1}\right)^8\right]$$
$$=\frac{1}{2^8}\left[\begin{array}{l}{}^8C_0\left(\sqrt{5x^3+1}\right)^8 + {}^8C_2\left(\sqrt{5x^3+1}\right)^6\left(\sqrt{5x^3-1}\right)^2 \\ +\,{}^8C_4\left(\sqrt{5x^3+1}\right)^4\left(\sqrt{5x^3-1}\right)^4 + \\ {}^8C_6\left(\sqrt{5x^3+1}\right)^2\left(\sqrt{5x^3-1}\right)^6 + {}^8C_8\left(\sqrt{5x^3-1}\right)^8\end{array}\right]$$
$$=\frac{1}{2^8}\left[\begin{array}{l}{}^8C_0(5x^3+1)^4 + {}^8C_2(5x^3+1)^3(5x^3-1) \\ +\,{}^8C_4(5x^3+1)^2(5x^3-1)^2 + \\ {}^8C_6(5x^3+1)(5x^3-1)^3 + {}^8C_8(5x^3-1)^4\end{array}\right]$$
So, the degree of polynomial is 12,
Now, coefficient of
$x^{12} = [{}^8C_0 5^4 + {}^8C_2 5^4 + {}^8C_4 5^4 + {}^8C_6 5^4 + {}^8C_8 5^4]$
$= 5^4\times\dfrac{2^8}{2} = 5^4\times 2^4\times\dfrac{2^2}{2}$
$= 10^4\times 2^3 = 8(10)^4$

75.[2] The system of linear equations is :
$x + y + z = 2$
$2x + y - z = 3$
$3x + 2y + kz = 4$
As, system has unique solution.
So, $\begin{vmatrix}1 & 1 & 1\\ 2 & 1 & -1\\ 3 & 2 & k\end{vmatrix} \neq 0$
$\Rightarrow k + 2 - (2k + 3) + 1 \neq 0$
$\Rightarrow k \neq 0$
Hence, $k \in R - \{0\} \equiv S$

76.[3] Let
$$I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\sin^4 x\left(1+\log\left(\frac{2+\sin x}{2-\sin x}\right)\right)dx \quad ...(1)$$
$$\Rightarrow I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\sin^4(-x)\left(1+\log\left(\frac{2+\sin(-x)}{2-\sin(-x)}\right)\right).dx$$
$$\left[\because \int_a^b f(x).dx = \int_a^b f(a+b-x).dx\right]$$

$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (\sin^4 x)\left(1+\log\left(\frac{2-\sin x}{2+\sin x}\right)\right).dx$$

$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^4 x\left(1-\log\left(\frac{2+\sin x}{2-\sin x}\right)\right).dx \quad ....(2)$$

After adding equation (1) and (2) we get,

$$2I = 2\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^4 x\,.dx$$

$$2I = 4\int_{0}^{\frac{\pi}{2}} \sin^4 x\,.dx$$

$$I = 2\int_{0}^{\frac{\pi}{2}} \sin^4 x\,.dx = \frac{\frac{3}{2}\times\frac{1}{2}\times\pi}{2\times 2} = \frac{3\pi}{8}$$

[By Gamma function]

77.[4] Normal to $3x + 4y + z = 1$ is $3\hat{i}+4\hat{j}+\hat{k}$.

Normal to $5x + 8y + 2z = -14$ is $5\hat{i}+8\hat{j}+2\hat{k}$

The line of intersection of the planes is perpendicular to both normals, so, direction ratios of the intersection line are directly proportional to the cross product of the normal vectors.

Therefore the direction ratios of the line is $-\hat{j}+4\hat{k}$

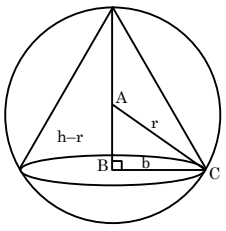
Hence the angle between the plane $x + y + z + 5 = 0$ and the intersection line is

$$\sin^{-1}\left(\frac{-1+4}{\sqrt{17}\sqrt{3}}\right) = \sin^{-1}\left(\sqrt{\frac{3}{17}}\right)$$

78.[1] Sphere of radius r = 3 cm

Let b, h be base radius and height of cone respectively.

So, volume of cone $= \frac{1}{2}\pi b^2 h$



In right angled ΔABC by Pythagoras Theorem $(h - r)^2 + b^2 = r^2$

$\Rightarrow b^2 = r^2 - (h - r)^2 = r^2 - (h^2 - 2hr + r^2)$

$= 2hr - h^2$

$\therefore$ Volume $(v) = \frac{1}{3}\pi h\,[2hr - h^2]$

$$= \frac{1}{3}\pi\,(2rh^2 - h^3)$$

$$\frac{dv}{dh} = \frac{1}{3}(4hr - 3h^2) = 0$$

$\Rightarrow h(4r - 3h) = 0$

$$\frac{d^2v}{dh^2} = \frac{1}{3}\left[4r - 6h\right]$$

$$\text{At } h = \frac{4r}{3}, \frac{d^2v}{dh^2} = \frac{1}{3}\left[4r - \frac{4r}{3}\times 6\right]$$

$$= \frac{1}{3}\,[4r - 8r] < 0$$

$\Rightarrow$ maximum volume occurs at

$$h = \frac{4r}{3} = \frac{4}{3}\times 3 = 4 \text{ cm}$$

As from (1),

$(h - r)^2 + b^2 = r^2$

$\Rightarrow b^2 = 2hr - h^2$

$$= 2.\ \frac{4r}{3}r - \frac{16r^2}{9}$$

$$= \frac{8r^2}{3} - \frac{16r^2}{9}$$

$$= \frac{(24-16)r^2}{9} = \frac{8r^2}{9}$$

$$\Rightarrow b = \frac{2\sqrt{2}}{3}, r = 2\sqrt{2} \text{ cm}$$

Therefore curved surface area

$$= \pi bl = \pi b\ \sqrt{h^2 + r^2}$$

$$= \pi 2\ \sqrt{2}\ \ \sqrt{4^2 + 8} = 8\sqrt{3}\,\pi \text{ cm}^2$$

79.[2] As tan A and tan B are the roots of $3x^2 - 10x - 25 = 0$,

$$\text{So, } \tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B} = \frac{\frac{10}{3}}{1 + \frac{25}{3}}$$

$$= \frac{10/3}{28/3} = \frac{5}{14}$$

Now, $\cos^2(A + B) = -1 + 2\cos^2(A + B)$

$$= \frac{1 - \tan^2(A+B)}{1 + \tan^2(A+B)} \Rightarrow \cos^2(A + B) = \frac{196}{221}$$

$\therefore 3\sin^2(A + B) - 10\sin(A + B)\cos(A + B) - 25\cos^2(A + B)$

$= \cos^2(A + B)[3\tan^2(A + B) - 10\tan(A + B) - 25]$

$$= \frac{75 - 700 - 4900}{196}\times\frac{196}{221} = -\frac{5525}{196}\times\frac{196}{221}$$

$= -25$

80.[3] As origin is the only common point to x-axis and y-axis, so, origin is the common vertex Let the equation of two of parabolas be $y^2 = 4ax$ and $x^2 = 4by$

Now latus rectum of both parabolas = 3

$\therefore 4a = 4b = 3$

$\Rightarrow a = b = \frac{3}{4}$

$\therefore$ Two parabolas are $y^2 = 3x$ and $x^2 = 3y$

Suppose $y = mx + c$ is the common tangent.

$\therefore y^2 = 3x \Rightarrow (mx + c)^2 = 3x$

$\Rightarrow m^2x^2 + (2mc - 3)x + c^2 = 0$

As, the tangent touches at one point only

So, $b^2 - 4ac = 0$

$\Rightarrow (2mc - 3)^2 - 4m^2c^2 = 0$

$\Rightarrow 4m^2c^2 + 9 - 12mc - 4m^2c^2 = 0$

$\Rightarrow c = \frac{9}{12m} = \frac{3}{4m}$ ...(i)

$\therefore x^2 = 3y \Rightarrow x^2 = 3(mx + c)$

$\Rightarrow x^2 - 3mx - 3c = 0$

Again, $b^2 - 4ac = 0$

$\Rightarrow 9m^2 - 4(1)(-3c) = 0$

$\Rightarrow 9m^2 = -12c$ ...(ii)

From (i) and (ii)

$m^2 = \frac{-4c}{3} = \frac{-4}{3}\left(\frac{3}{4m}\right)$

$\Rightarrow m^3 = -1$

$\Rightarrow m = -1$

$\Rightarrow c = \frac{-3}{4}$

Hence, $y = mx + c = -x - \frac{3}{4}$

$\Rightarrow 4(x + y) + 3 = 0$

81.[1] When $x \in [0, 1]$, then $\frac{dy}{dx} + 2y = 1$

$\Rightarrow y = \frac{1}{2} + C_1e^{-2x}$

$\because y(0) = 0 \Rightarrow y(x) = \frac{1}{2} - \frac{1}{2}e^{-2x}$

Here, $y(1) = \frac{1}{2} - \frac{1}{2}e^{-2} = \frac{e^2 - 1}{2e^2}$

When $x \notin [0, 1]$, than $\frac{dy}{dx} + 2y = 0$

$\Rightarrow y = c_2e^{-2x}$

$\because y(1) = \frac{e^2 - 1}{2e^2} \Rightarrow \frac{e^2 - 1}{2} = c^2e^{-2}$

$\Rightarrow C_2 = \frac{e^2 - 1}{2}$

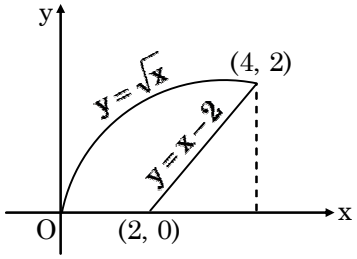
$\therefore y(x) \left(\frac{e^2 - 1}{2}\right)e^{-2x}$

$\Rightarrow y\left(\frac{3}{2}\right) = \frac{e^2 - 1}{2e^3}$

82.[2] The intersection point of $y = x - 2$ and $y = \sqrt{x}$ is (4, 2).

The required area

$= \int_0^4 \sqrt{x}dx - \frac{1}{2} \times 2 \times 2 = \frac{16}{3} - 2 = \frac{10}{3}$



83.[2] Since, $x^2 + 3y^2 = 9$

$\Rightarrow 2x + 6y\frac{dy}{dx} = 0$

$\Rightarrow \frac{dy}{dx} = \frac{-x}{3y}$

Slope of normal is $-\frac{dx}{dy} = \frac{3y}{x}$

$\Rightarrow \left(-\frac{dy}{dx}\right)_{(3\cos\theta, \sqrt{3}\sin\theta)} = \frac{3\sqrt{3}\sin\theta}{3\cos\theta}$

$= \sqrt{3}\tan\theta = m_1$

& $\left(-\frac{dx}{dy}\right)_{(-3\sin\theta m\sqrt{3}\cos\theta)} = \frac{3\sqrt{3}\cos\theta}{-3\sin\theta}$

$= -\sqrt{3}\cot\theta = m_2$

As, β is the angle between the normals to the given ellipse then

$\tan\beta = \left|\frac{m_1 - m_2}{1 + m_1m_2}\right|$

$= \left|\frac{\sqrt{3}\tan\theta + \sqrt{3}\cot\theta}{1 - 3\tan\theta\cot\theta}\right|$

$$= \left|\frac{\sqrt{3}\tan\theta + \sqrt{3}\cot\theta}{1-3}\right|$$

So, $\tan\beta = \frac{\sqrt{3}}{2}\,|\tan\theta + \cot\theta|$

$$\Rightarrow \frac{1}{\cot\beta} = \frac{\sqrt{3}}{2}\left|\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}\right|$$

$$\Rightarrow \frac{1}{\cot\beta} = \frac{\sqrt{3}}{2}\left|\frac{1}{\sin\theta\cos\theta}\right|$$

$$\Rightarrow \frac{1}{\cot\beta} = \frac{\sqrt{3}}{\sin 2\theta} \Rightarrow \frac{2\cos\beta}{\sin 2\theta} = \frac{2}{\sqrt{3}}$$

84.[2] As the truth table for the $(p \wedge \sim q) \vee (p \wedge r) \rightarrow \sim p \vee q$ is false, then only possible values of (p, q, r) is (T, F, T)

P	q	r	~q	p∧ ~q	p∧r	~p	~p ∨ q
T	T	T	F	F	T	F	T
T	F	T	T	T	T	F	F
T	T	F	F	F	F	F	T
F	T	T	F	F	F	T	T
F	F	T	T	F	F	T	T
F	T	F	F	F	F	T	T
T	F	F	T	T	F	F	F
F	F	F	T	F	F	T	T

$(p \vee \sim q) \wedge (p \wedge r)$	$(p \wedge \sim q) \wedge (p \wedge r) \rightarrow \sim p \vee q$
F	T
T	F
F	T
F	T
F	T
F	T
F	T
F	T

85.[1] $\because |z| = 1$ & Re $z \neq 1$

Suppose $z = x + iy \Rightarrow x^2 + y^2 = 1$ ...(i)

Now, $w = \frac{1+(1-8\alpha)z}{1-z}$

$$\Rightarrow w = \frac{1+(1-8\alpha)(x+iy)}{1-(x+iy)}$$

$$\Rightarrow w = \frac{1+(1-8\alpha)(x+iy)((1-x)+iy)}{1-(x+iy)((1-x)+iy)}$$

$$\Rightarrow w = \frac{[(1+x(1-8\alpha))(1-x)-(1-8)y^2]}{(1-x)^2+y^2} + i\,\frac{[(1+x(1-8\alpha))y-(1-8\alpha)y(1-x)]}{(1-x)^2+y^2}$$

If, w is purely imaginary. So,

$$\text{Re } w = \frac{[(1+x(1-8\alpha))(1-\alpha)-(1-8\alpha)y^2]}{(1-x)^2+y^2}$$

$= 0$

$\Rightarrow (1-x) + x(1-8\alpha)(1-x) = (1-8)y^2$

$\Rightarrow (1-x) + x(1-8\alpha) - x^2(1-8\alpha) = (1-8)y^2$

$\Rightarrow (1-x) + x(1-8\alpha) = 1 - 8\alpha$

[From (i), $x^2 + y^2 = 1$]

$\Rightarrow 1 - 8\alpha = 1$

$\Rightarrow \alpha = 0$

$\therefore \alpha \in \{0\}$

86.[2] $\because \vec{a} + 2\vec{b} + 2\vec{c} = \vec{0}$ [Given]

$\Rightarrow \vec{a} + 2\vec{c} = -2\vec{b}$

$\Rightarrow (\vec{a} + 2\vec{c}) \cdot (\vec{a} + 2\vec{c}) = (-2\vec{b})(-2\vec{b})$

$\Rightarrow \vec{a} \cdot \vec{a} + 4\vec{c} \cdot \vec{c} + 4\vec{a} \cdot \vec{c} = 4\vec{b} \cdot \vec{b}$

$\Rightarrow 1 + 4 + s$

$\Rightarrow \vec{a} \cdot \vec{c} = \frac{-1}{4}$

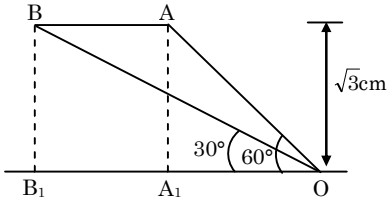
$\because \left|\vec{a} \cdot \vec{c}\right|^2 + \left|\vec{a} \times \vec{c}\right|^2 = 1$ ($\vec{a}$ is unit vector)

$$\frac{1}{16} + \left|\vec{a} \times \vec{c}\right|^2 = 1$$

$$\left|\vec{a} \times \vec{c}\right|^2 = \frac{15}{16}$$

$$\left|\vec{a} \times \vec{c}\right| = \frac{\sqrt{15}}{4}$$

87.[4] For ΔOA_1A, $OA_1 = \frac{\sqrt{3}}{\tan 60^\circ} = 1$ km.



For ΔOB_1B, $OB_1 = \frac{\sqrt{3}}{\tan 30^\circ} = 3$ km.

As, a distance of 3 – 1 = 2 km is covered in 5 seconds. Therefore the speed of the plane is $\frac{2\times3600}{5}$ = 1440 km/hr

88.[1] $S = \{\lambda, \mu) \in R \times R$:

$f(t) = (|\lambda|e^{|t|} - \mu) \sin(2|t|), t \in R$

$f(t) = (|\lambda|e^{|t|} - \mu) \sin(2|t|)$

$$\begin{cases}(|\lambda|e^{t} - \mu)\sin 2t, t > 0\\(|\lambda|e^{-t} - \mu)(-\sin 2t), \ t < 0\end{cases}$$

$f'(t) =$

$$\begin{cases}(|\lambda|e^{t})\sin 2t + (|\lambda|e^{t} - \mu)(2\cos 2t), t > 0\\|\lambda|e^{-t}\sin 2t + (|\lambda|e^{-t} - \mu)(-2\cos 2t), t < 0\end{cases}$$

As, $f(t)$ is differentiable

$\therefore$ LHD = RHD at t = 0

$|\lambda|.\sin 2(0) + (|\lambda|e^{0} - \mu)2\cos(0)$

$= |\lambda|e^{-0}.\sin 2(0) - 2\cos(0)(|\lambda|e^{-0} - \mu)$

$\Rightarrow \ 0 + (|\lambda| - \mu)2 = 0 - 2(|\lambda| - \mu)$

$\Rightarrow \ 4(|\lambda| - \mu) = 0$

$\Rightarrow \ |\lambda| = \mu$

So, $S \equiv (\lambda, \mu) = \{\lambda \in R \ \& \ \mu \in [0, \infty)\}$

Therefore set S is subset of $R \times [0, \infty)$

89.[4] Required n digit numbers is 3^n as each place can be filled by 2, 5, 7.

So smallest value of n such that $3^n > 900$.

Therefore n = 7.

90.[1] Probability of drawing a white ball and then a red ball from bag B is given by

$$\frac{{}^4C_1 \times {}^2C_1}{{}^9C_2} = \frac{2}{9}$$

Probability of drawing a white ball and then a red ball from bag A is given by

$$\frac{{}^2C_1 \times {}^3C_1}{{}^7C_2} = \frac{2}{7}$$

Hence, the probability of drawing a white ball and then a red ball from bag B

$$= \frac{\frac{2}{9}}{\frac{2}{7}+\frac{2}{9}} = \frac{2\times7}{18+14} = \frac{7}{16}.$$