CAREER POINT

JEE Main Exam 2017
(Paper & Solution)

Code – A Date : 02-04-2017

Part A – PHYSICS

Q.1 A man grows into a giant such that his linear dimensions increase by a factor of 9. Assuming that his density remains same, the stress in the leg will change by a factor of-

(1) 9 (2) $\frac{1}{9}$ (3) 81 (4) $\frac{1}{81}$

Ans. **[1]**

Sol. Let volume of man is Lbh

As all dimension increases by a factor (K = 9) keeping the density constant

Stress on his legs = $\frac{\text{weight}}{\text{area}} = \frac{V\rho g}{A}$

Initial stress = $\text{Stress}_1 = \frac{V\rho g}{A}$

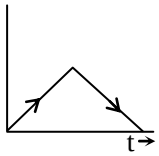
Final stress = $\text{Stress}_2 = \frac{K^3 V\rho g}{K^2 A}$

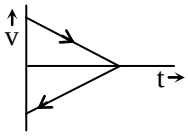
$\boxed{\text{Stress}_2 = K\ \text{Stress}_1}$

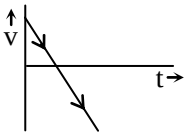
Where (K = 9)

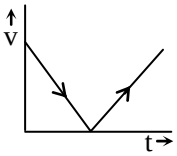
So stress is changed by a factor 9.

Q.2 A body is thrown vertically upwards. Which one of the following graphs correctly represent the velocity *vs* time ?

(1) 

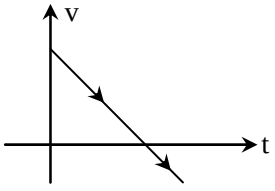
(2) 

(3) 

(4) 

Ans. **[3]**

Sol. v = u – gt (straight line graph)



Q.3 A body of mass $m = 10^{-2}$ kg is moving in a medium and experiences a frictional force $F = -kv^2$. Its initial speed is $v_0 = 10\ ms^{-1}$. If, after 10 s, its energy is $\frac{1}{8}mv_0^2$, the value of k will be-

(1) 10^{-3} kg m^{-1} (2) 10^{-3} kg s^{-1} (3) 10^{-4} kg m^{-1} (4) 10^{-1} kg m^{-1} s^{-1}

Ans. **[3]**

Sol. $a = -\frac{kv^2}{m}$

$$\frac{dv}{dt} = -\frac{kv^2}{m}$$

$$\int_{10}^{v}\frac{dv}{v^2} = -\frac{k}{m}\int_0^{10}dt$$

$$\left[-\frac{1}{v}\right]_{10}^{v} = -\frac{k}{m}\times 10$$

$$-\left[\frac{1}{v}-\frac{1}{10}\right] = \frac{k}{10^{-2}}\times 10$$

$$-\frac{1}{v}+\frac{1}{10} = -k\times 1000$$

According to question

$$KE = \frac{1}{2}mv^2 = \frac{1}{8}mv_0^2$$

$$v = \frac{v_0}{2} = \frac{10}{2}$$

$$-\frac{1}{10}\times 2+\frac{1}{10} = -k\times 1000$$

$$\frac{1}{10} = k\times 1000$$

$$k = 10^{-4}$$

Q.4 A time dependent force $F = 6t$ acts on a particle of mass 1 kg. If the particle starts from rest, the work done by the force during the first 1 sec. will be-

(1) 4.5 J (2) 22 J (3) 9 J (4) 18 J

Students may find similar question in CP exercise sheet :
[JEE Advance, Chapter : Work-Power-Energy, Ex.2, Page No.25, Q. No.13]

Ans. **[1]**

Sol. $a = \frac{6t}{1} = 6t$

$$\frac{dv}{dt} = 6t$$

$$v = \left[\frac{6t^2}{2}\right]_0^1 = 3\times 1^2 = 3$$

$$KE = \frac{1}{2}\times 1\times 3^2 = 4.5$$

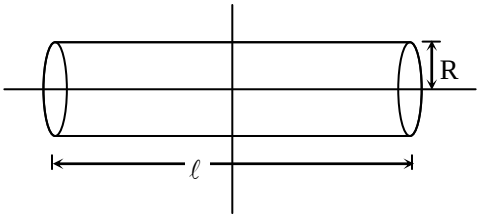
$W = \Delta KE = 4.5 - 0 = 4.5$ Joule

Q.5 The moment of inertia of a uniform cylinder of length *l* and radius R about its perpendicular bisector is I. What is the ratio *l*/R such that the moment of inertia is minimum ?

(1) $\sqrt{\frac{3}{2}}$ (2) $\frac{\sqrt{3}}{2}$ (3) 1 (4) $\frac{3}{\sqrt{2}}$

Ans. **[1]**

Sol.



Moment of inertia of cylinder about perpendicular bisector is I

$$I = M\left[\frac{L^2}{12}+\frac{R^2}{4}\right]$$

For given mass and density

$M = \pi R^2 L\rho$

$$R^2 = \frac{M}{\pi L\rho}$$

$$I = M\left[\frac{L^2}{12}+\frac{M}{4\pi L\rho}\right]$$

For maxima or minima of I

$$\frac{dI}{dL} = 0$$

$$\frac{dI}{dL} = M\left[\frac{2L}{12}-\frac{M}{4\pi L^2\rho}\right] = 0$$

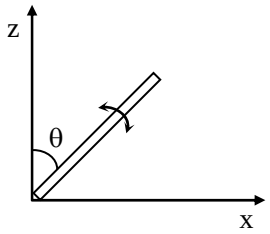
$$\frac{L}{6} = \frac{M}{4\pi L^2\rho}$$

$$\frac{L}{6} = \frac{\pi R^2 L\rho}{4\pi L^2\rho}$$

$$\frac{L^2}{R^2} = \frac{3}{2}$$

$$\boxed{\frac{L}{R} = \sqrt{\frac{3}{2}}}$$

Q.6 A slender uniform rod of mass M and length *l* is pivoted at one end so that is can rotate in vertical plane (see figure). There is negligible friction at the pivot. The free end is held vertically above the pivot and then released. The angular acceleration of the rod when it makes an angle θ with the vertical is-



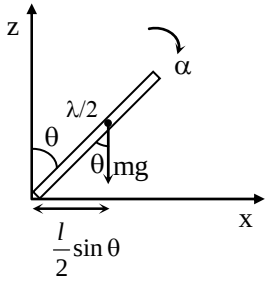
(1) $\frac{3g}{2l}\sin\theta$ (2) $\frac{2g}{3l}\sin\theta$ (3) $\frac{3g}{2l}\cos\theta$ (4) $\frac{2g}{3l}\cos\theta$

Students may find similar question in CP exercise sheet :

[JEE Advance, Chapter : Rotational motion, Ex.3, Page No., 36, Q. No.11]

Ans. **[1]**

Sol.



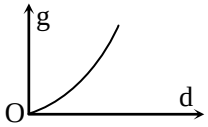
$\tau = mg\frac{l}{2}\sin\theta$

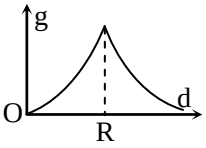
$\tau = I\alpha$

$mg\frac{l}{2}\sin\theta = \frac{ml^2}{3}\alpha$

$\alpha = \frac{3}{2}\frac{g}{l}\sin\theta$

Q.7 The variation of acceleration due to gravity g with distance d from centre of the earth is best represented by (R = Earth's radius)-

(1)


(2)


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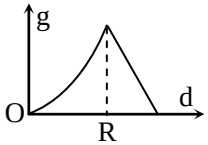
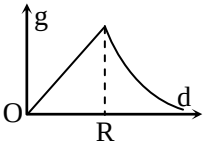


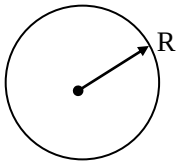
(4)


Students may find similar question in CP exercise sheet :

[JEE Main, Chapter : Gravitation, Ex.4, Page No., 42, Q. No.20]

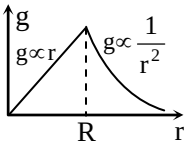
Ans. **[4]**

Sol.



$g = \frac{GM}{r^2}$ when $r > R$

$g = \frac{GMr}{R^3}$ when $r < R$



Q.8 A copper ball of mass 100 gm is at a temperature T. It is dropped in a copper calorimeter of mass 100 gm, filled with 170 gm of water at room temperature. Subsequently, the temperature of the system is found to be 75ºC. T is given by- (Given : room temperature = 30ºC, specific heat of copper = 0.1 cal/gmºC)

(1) 800ºC (2) 885ºC (3) 1250ºC (4) 825ºC

Ans. **[2]**

Sol. Total heat gain = Total heat loss

$100 \times 0.1 (75 - 30) + 170 \times 1 \times (75 - 30) = 100 \times 0.1 (T - 75)$

$10 \times 45 + 170 \times 45 = 10T - 750$

$1200 + 7650 = 10\ T$

$T = 885^{\circ}C$

Q.9 An external pressure P is applied on a cube at 0ºC so that it is equally compressed from all sides. K is the bulk modulus of the material of the cube and α is its coefficient of linear expansion. Suppose we want to bring the cube to its original size by heating. The temperature should be raised by-

(1) $\frac{P}{3\alpha K}$ (2) $\frac{P}{\alpha K}$ (3) $\frac{3\alpha}{PK}$ (4) $3PK\alpha$

Ans. **[1]**

Sol. $B = \frac{-P}{\frac{dV}{V}}$

$\frac{dV}{V} = \frac{-P}{B}$

$dV = \frac{-PV}{B}$

By heating we have to increase the volume by $\frac{PV}{B}$

$$\Delta V = V\gamma\Delta T = V \times 3\alpha\ \Delta T$$

$$V \times 3\alpha\Delta T = \frac{PV}{B}$$

$$\Delta T = \frac{P}{3\alpha B}$$

Here B = K

$$\therefore\ \Delta T = \frac{P}{3\alpha B}$$

Q.10 C_p and C_v are specific heats at constant pressure and constant volume respectively. It is observed that

$C_p - C_v = a$ for hydrogen gas

$C_p - C_v = b$ for nitrogen gas

The correct relation between a and b is-

(1) $a = \frac{1}{14}b$ (2) $a = b$ (3) $a = 14\ b$ (4) $a = 28\ b$

Students may find similar question in CP exercise sheet :

[*JEE Main, Chapter : KTG, Ex.1, Page No.17, Q. No.35*]

[*JEE Advance, Chapter : KTG, Ex.1, Page No.25,Q. No.22*]

Ans. **[3]**

Sol. $C_p - C_v = R$

If C_p and C_v are molar specific heat

But if C_p and C_v are specific heat i.e. gram specific heat then

$C = MS_g$ $\{Q = 1C\Delta T \Rightarrow Q = MS_g\Delta T \Rightarrow C = MS_g\}$

$$S_g = \frac{C}{M}$$

$$MS_{gp} - MS_{gv} = R$$

$$S_{gp} - S_{gv} = \frac{R}{M}$$

$$\frac{R}{2} = a$$

$$\frac{R}{28} = b$$

$$14 = \frac{a}{b}$$

$$a = 14\ b$$

Q.11 The temperature of an open room of volume 30 m^3 increases from 17°C to 27°C due to the sunshine. The atmospheric pressure in the room remains 1×10^5 Pa. In n_i and n_f are the number of molecules in the room before and after heating, the $n_f - n_i$ will be :

(1) -1.61×10^{23} (2) 1.38×10^{23} (3) 2.5×10^{25} (4) -2.5×10^{25}

Students may find similar question in CP exercise sheet :

[JEE Advance, Chapter : KTG, Ex.3, Page No.27, Q. No.5]

Ans. **[4]**

Sol. $PV = nRT$

$$n = \frac{PV}{RT}$$

$T_i = 273 + 17$

$= 290$ K

$T_f = 273 + 27$

$= 300$ K

$$n_f - n_i = \frac{10^5 \times 30}{8.314}\left[\frac{1}{300} - \frac{1}{290}\right] \times 6.023 \times 10^{23}$$

$$= \frac{3\times 10^6}{8.314}\left[\frac{-10}{300\times 290}\right] \times 6.023 \times 10^{23}$$

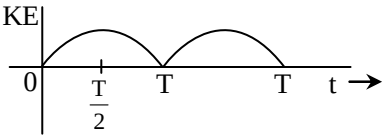
$$= -\frac{3\times 10^{27} \times 6.023}{8.314\times 3\times 29}$$

$$= -\frac{-6.023\times 10^{27}}{8.314\times 29}$$

$$= -0.025 \times 10^{27}$$

$$= -2.5 \times 10^{25}$$

Q.12 A particle is executing simple harmonic motion with a time period T. At time t = 0 it is at its position of equilibrium. The kinetic energy - time graph of the particle will look like :

(1)


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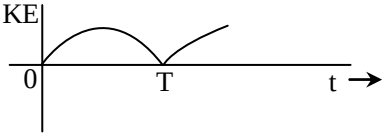
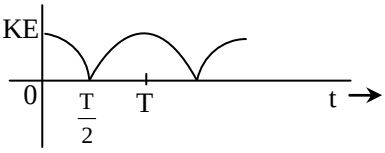


(3)


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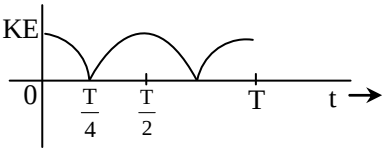


Students may find similar question in CP exercise sheet :

[JEE Main, Chapter : SHM, Ex.1, Page No.27, Q. No.33]

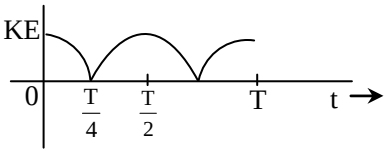
Ans. **[4]**

Sol. $x = A \sin \omega t$

$v = \frac{dx}{dt} = A\omega \cos \omega t$

$KE = \frac{1}{2} mA^2\omega^2\cos^2\omega t$

$= \frac{1}{2} mA^2\omega^2\cos^2 \frac{2\pi}{T} t$



Q.13 An observer is moving with half the speed of light towards a stationary microwave source emitting waves at frequency 10 GHz. What is the frequency of the microwave measured by the observer ? (speed of light = 3×10^8 ms^{-1})

(1) 10.1 GHz (2) 12.1 GHz (3) 17.3 GHz (4) 15.3 GHz

Ans. **[3]**

Sol. According to theory of relativity

$$f_{app} = \left[\frac{1+\frac{v}{c}}{\sqrt{1-\frac{v^2}{c^2}}}\right] f \quad \text{(for approach)}$$

$$= \frac{1+\frac{c/2}{c}}{\sqrt{1-\left(\frac{c/2}{c}\right)^2}} \times 10 \text{ GHz}$$

$$= \frac{\frac{3}{2}}{\sqrt{\frac{3}{4}}} \times 10 \text{ GHz}$$

$$= \sqrt{3} \times 10 \text{ GHz}$$

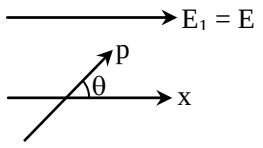
$f_{app} = 17.32$ GHz

Q.14 An electric dipole has fixed dipole moment $\vec{p}$, which makes angle θ with respect to x-axis . When subjected to an electric field $\vec{E}_1 = E\hat{i}$, it experience a torque $\vec{T}_1 = \tau\hat{k}$. When subjected to another electric field $\vec{E}_2 = \sqrt{3}\ E_1\hat{j}$ it experiences a torque $\vec{T}_2 = -\vec{T}_1$. The angle θ is.

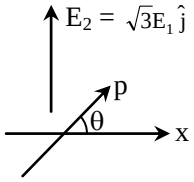
(1) 30° (2) 45° (3) 60° (4) 90°

Ans. **[3]**

Sol.



$\tau = \vec{T}_1 = PE\sin\theta\ \hat{k}$



$\tau = \vec{T}_2 = P\sqrt{3}E\cos\theta(-\hat{k})$

$PE\sin\theta(\hat{k}) = -P\sqrt{3}E\cos\theta(-\hat{k})$

$PE\sin\theta = \sqrt{3}E\cos\theta$

$\tan\theta = \sqrt{3}$

$\theta = 60°$

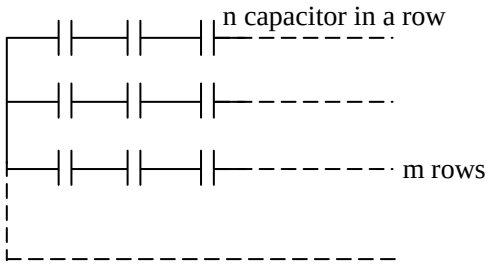
Q.15 A capacitance of 2 μF is required in an electrical circuit across a potential difference of 1.0 kV. A large number of 1μF capacitors are available which can withstand a potential difference of not more than 300 V. The minimum number of capacitors required to achieve this is :

(1) 2 (2) 16 (3) 24 (4) 32

Students may find similar question in CP exercise sheet :
[JEE Main, Chapter : Capacitance, Ex.3, Page No.47, Q. No.12]

Ans. **[4]**

Sol.



Potential on each capacitor $V = \frac{1000}{n}$

$\frac{1000}{n} = 300$

$n = \frac{10}{3} \approx 4$

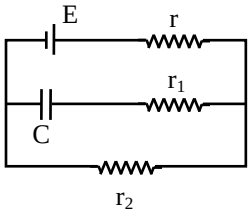
$C_{eq} = \frac{C}{n} \times m$

$\frac{1}{n} \times m = 2$

$m = n \times 2 = 4 \times 2 = 8$

Minimum number of capacitor = $8 \times 4 = 32$

Q.16 In the given circuit diagram when the current reaches steady state in the circuit, the charge on the capacitor of capacitance C will be:

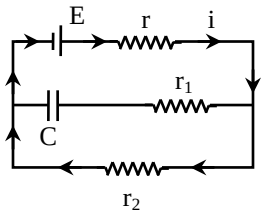


(1) CE (2) CE $\frac{r_1}{(r_2+r)}$ (3) CE $\frac{r_2}{(r+r_2)}$ (4) CE $\frac{r_1}{(r_1+r)}$

Students may find similar question in CP exercise sheet :
[JEE Main, Chapter : Capacitance, Ex.2, Page No.44, Q. No.25]

Ans. **[3]**

Sol.



At steady state current through capacitor branch become zero.

$i = \frac{E}{r+r_2}$

Potential difference across capacitor ΔV

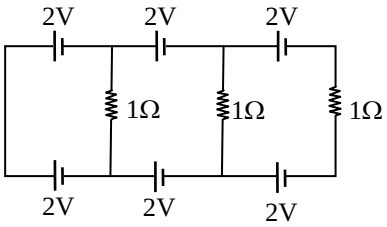
$\Delta V = ir_2$

$\Delta V = \left(\frac{E}{r+r_2}\right) r_2$

charge on capacitor = C ΔV

$= CE\left(\frac{r_2}{r+r_2}\right)$

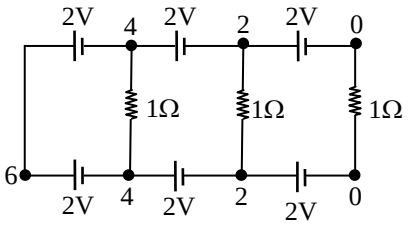
Q.17 In the above circuit the current in each resistance is :



(1) 1A (2) 0.25 A (3) 0.5 A (4) 0 A

Ans. **[4]**

Sol.



Potential difference across each resistor is zero so current in each resistor also zero.

Q.18 A magnetic needle of magnetic moment 6.7×10^{-2} Am^2 and moment of inertia 7.5×10^{-6} kg m^2 is performing simple harmonic oscillations in a magnetic field of 0.01 T. Time taken for 10 complete oscillations is :

(1) 6.65 s (2) 8.89 s (3) 6.98 s (4) 8.76 s

Ans. **[1]**

Sol. $\left|\vec{M}\right| = 6.7 \times 10^{-2}$ Am^2

$I = 7.5 \times 10^{-6}$ kg m^2, B = 0.01 T

$\tau = -$ MB sin θ

$I\alpha = -$ MB θ (for small oscillations)

$$\alpha = \left(\frac{MB}{I}\right)\theta \quad \Rightarrow \omega = \sqrt{\frac{MB}{I}} \Rightarrow T = 2\pi\sqrt{\frac{I}{MB}}$$

$$T = 2\pi\sqrt{\frac{7.5\times10^{-6}}{6.7\times10^{-2}\times0.01}}$$

$\Rightarrow T = 0.6644$ sec

Time for 10 oscillation $\Delta t = 10\ T \Rightarrow \Delta t = 6.65$ sec

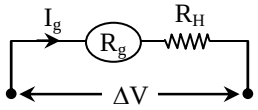
Q.19 When a current of 5 mA is passed through a galvanometer having a coil of resistance 15Ω, it shows full scale deflection. The value of the resistance to be put in series with the galvanometer to convert it into a voltmeter of range 0 – 10 V is

(1) $1.985 \times 10^3\ \Omega$ (2) $2.045 \times 10^3\ \Omega$ (3) $2.535 \times 10^3\ \Omega$ (4) $4.005 \times 10^3\ \Omega$

Ans. **[1]**

Sol. $I_{g\,max} = 5$ mA , $R_g = 15\ \Omega$

Range of voltmeter = 10 volt



$\Delta V = I_g (R_g + R_H)$

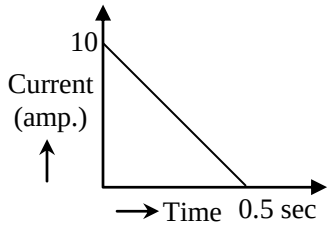
Range $\Delta V_{max} = I_{g\,max} (R_g + R_H)$

$10 = 5 \times 10^{-3} (15 + R_H)$

$R_H = 1985\ \Omega$

$R_H = 1.985 \times 10^3\ \Omega$

Q.20 In a coil of resistance 100 Ω, a current is induced by changing the magnetic flux through it as shown in the figure. The magnitude of change in flux through the coil is :

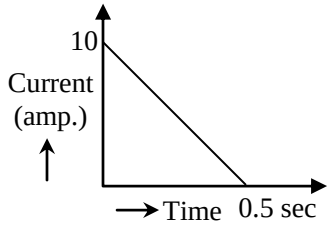


(1) 200 Wb (2) 225 Wb (3) 250 Wb (4) 275 Wb

Students may find similar question in CP exercise sheet :
[JEE Advance, Chapter : EMI, Ex.1, Page No.31, Q. No.1]

Ans. **[3]**

Sol.



Emf = iR

$$-\frac{d\phi}{dt} = iR$$

$$\int(-d\phi) = \int(idt)\ R$$

$$(-\Delta\phi) = R\int_0^{0.5} idt$$

$$|(-\Delta\phi)| = R \text{ (area of i – t curve)}$$

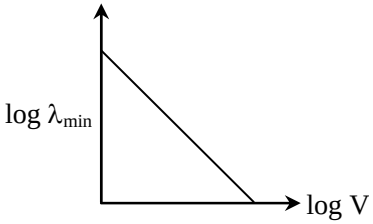
$$\Delta\phi = R\left(\frac{1}{2} \times 0.5 \times 10\right)$$

$$\Delta\phi = 100\ (2.5)$$

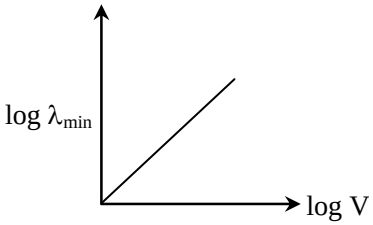
$$\Delta\phi = 250 \text{ Wb}$$

Q.21 An electron beam is accelerated by a potential difference V to hit a metallic target to produce X-ray. It produces continuous as well as characteristic X-rays. If λ_{min} is the smallest possible wavelength of X-ray in the spectrum, the variation of log λ_{min} with log V is correctly represented in -

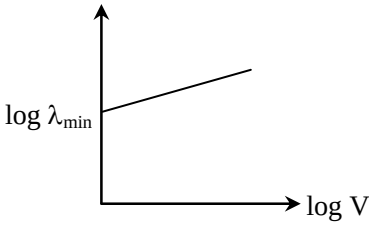
(1)



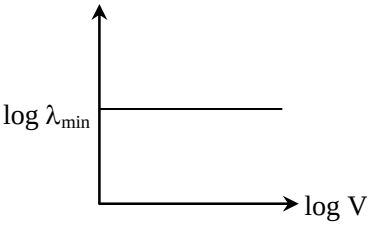
(2)



(3)

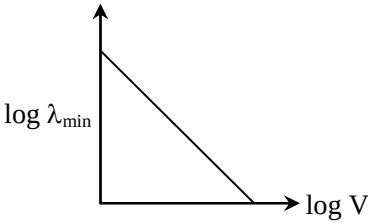


(4)



Ans. **[1]**

Sol.



K.E. = eV (K.E. = kinetic energy of electron)

$E_{P_{max}}$ = K.E.

$\frac{hc}{\lambda_{min}} = eV \Rightarrow \lambda_{min} V = \left(\frac{hc}{e}\right) = \text{constant}$

$\ell n\, \lambda_{min} + \ell n\, V = \ell n$ constant

$\ell n\, \lambda_{min} = -\ell n\, V + \ell n$ (constant)

Straight line of – ve slope

Q.22 A diverging lens with magnitude of focal length 25 cm is placed at a distance of 15 cm from a converging lens of magnitude of focal length 20 cm. A beam of parallel light falls on the diverging lens. The final image formed as -

(1) real and at a distance of 40 cm from convergent lens

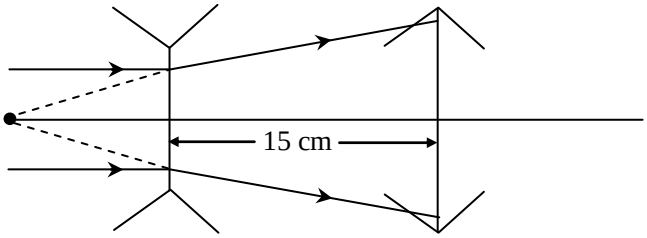
(2) virtual and at a distance of 40 cm from convergent lens

(3) real and at a distance of 40 cm from the divergent lens

(4) real and at a distance of 6 cm from the convergent lens

Ans. **[1]**

Sol.



f = 25 cm f = 20 cm

Image form by diverging is at the focus of diverging lens.

Now image form by diverging act as a source for converging lens.

For converging lens object real at a distance 40 cm from it which is at $(2f)$.

$u = -2f, f = +f, v = +2f$ $\left(\frac{1}{v} - \frac{1}{u} = \frac{1}{f}\right)$

Final image real at a distance $2f$ = 40 cm from converging lens.

Q.23 In a Young's double slit experiment, slits are separated by 0.5 mm, and the screen is placed 150 cm away. A beam of light consisting of two wavelengths, 650 nm and 520 nm, is used to obtain interference fringes on the screen. The least distance from the common central maximum to the point where the bright fringes due to both the wavelengths coincide is -

(1) 1.56 mm (2) 7.8 mm (3) 9.75 mm (4) 15.6 mm

Students may find similar question in CP exercise sheet :

[JEE Advance, Chapter : Wave nature of light : Interference, Ex.6, Page No.43, Q. No.23]

Ans. **[2]**

Sol. d = 0.5 mm, D = 1.5 m, λ_1 = 650 nm, λ_2 = 520 nm

$$\beta_1 = \frac{\lambda_1 D}{d} = \frac{650\times10^{-9}\times1.5}{0.5\times10^{-3}} = 1.95 \text{ mm}$$

$$\beta_2 = \frac{\lambda_2 D}{d} = \frac{520\times10^{-9}\times1.5}{0.5\times10^{-3}} = 1.56 \text{ mm}$$

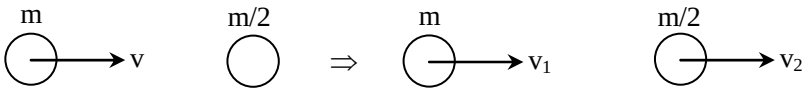
Least distance where their maxima again coincides from central maxima is = LCM of β_1 & β_2 = 7.8 mm

Q.24 A particle A of mass m and initial velocity v collides with a particle B of mass $\frac{m}{2}$ which is at rest. The collision is head on, and elastic. The ratio of the de-Broglie wavelengths λ_A to λ_B after the collision is -

(1) $\frac{\lambda_A}{\lambda_B}=\frac{1}{3}$ (2) $\frac{\lambda_A}{\lambda_B}=2$ (3) $\frac{\lambda_A}{\lambda_B}=\frac{2}{3}$ (4) $\frac{\lambda_A}{\lambda_B}=\frac{1}{2}$

Ans. **[2]**

Sol.



$$v_1 = \frac{m - m/2}{m + m/2}v + 0$$

$$v_1 = \frac{m/2}{3m/2}v$$

$$v_1 = \frac{v}{3}$$

Similarly

$$v_2 = 0 + \frac{2m}{m + m/2}\times v$$

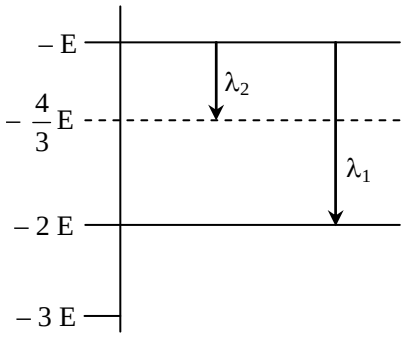
$$v_2 = \frac{2m}{3m/2}v$$

$$v_2 = \frac{4}{3}v$$

$$\because \lambda_1 = \frac{h}{m_1 v_1}, \lambda_2 = \frac{h}{m_2 v_2}$$

$$\frac{\lambda_1}{\lambda_2} = \frac{m_2 v_2}{m_1 v_1} = \frac{m/2}{m}\cdot\frac{4v/3}{v/3} = 2$$

Q.25 Some energy levels of a molecule are shown in the figure. The ratio of the wavelength $r = \frac{\lambda_1}{\lambda_2}$, is given by -



(1) $r = \frac{4}{3}$ (2) $r = \frac{2}{3}$ (3) $r = \frac{3}{4}$ (4) $r = \frac{1}{3}$

Students may find similar question in CP exercise sheet :
[JEE Main, Chapter : Atomic Structure, Ex.5, Q. No.27]

Ans. **[4]**

Sol. $\frac{hc}{\lambda_1} = (-E) - (-2E)$

$\frac{hc}{\lambda_1} = E$ ….(i)

$\frac{hc}{\lambda_2} = (-E) - \left(-\frac{4E}{3}\right) = -E + \frac{4E}{3} = \frac{-3E + 4E}{3} = \frac{E}{3}$ ….(ii)

By (i) & (ii)

$r = \frac{\lambda_1}{\lambda_2} = \frac{\frac{hc}{E}}{\frac{hc}{E/3}} = \frac{1}{3}$

Q.26 A radioactive nucleus A with a half life T, decays into a nucleus B. At t = 0, there is no nucleus B. At sometime t, the ratio of the number of B to that of A is 0.3. Then, t is given by -

(1) $t = \frac{T}{2}\frac{\log 2}{\log 1.3}$ (2) $t = T\frac{\log 1.3}{\log 2}$ (3) $t = T \log (1.3)$ (4) $t = \frac{T}{\log(1.3)}$

Ans. **[2]**

Sol. $A \longrightarrow B$

$A = A_0 e^{-\lambda t}$; $B = A_0 (1 - e^{-\lambda t})$; $\frac{B}{A} = \frac{A_0(1 - e^{-\lambda t})}{A_0 e^{-\lambda t}}$

$0.3 = e^{\lambda t} - 1$

$e^{\lambda t} = 1.3$

$\lambda t = \ln (1.3)$

$\frac{\ln(2)}{T} t = \ln (1.3)$

$t = T \frac{\ln(1.3)}{\ln(2)} \Rightarrow t = \frac{T \log(1.3)}{\log(2)}$

Q.27 In a common emitter amplifier circuit using an n-p-n transistor, the phase difference between the input and the output voltages will be -

(1) 45º (2) 90º (3) 135º (4) 180º

Ans. **[4]**

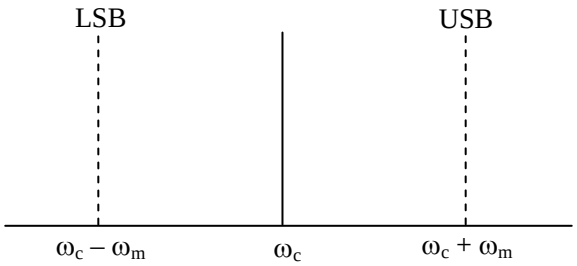
Sol. In C-E amplifier phase difference between input-output voltage is 180º.

Q.28 In amplitude modulation, sinusoidal carrier frequency used is denoted by ω_c and the signal frequency is denoted ω_m. The bandwidth ($\Delta\omega_m$) of the signal is such that $\Delta\omega_m << \omega_c$. Which of the following frequencies is not contained in the modulated wave?

(1) ω_m (2) ω_c (3) $\omega_m + \omega_c$ (4) $\omega_c - \omega_m$

Ans. **[1]**

Sol.



because $\Delta\omega_m << \omega_c$

$\therefore$ ω_m is not present in modulated wave.

Q.29 Which of the following statements is **false** ?

(1) Wheatstone bridge is the most sensitive when all the four resistances are of the same order of magnitude

(2) In a balanced Wheatstone bridge if the cell and the galvanometer are exchanged, the null point is disturbed

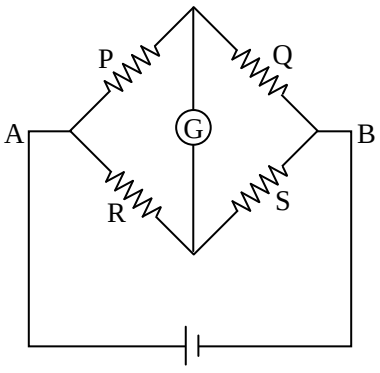
(3) A rheostat can be used as a potential divider

(4) Kirchhoff's second law represents energy conservation

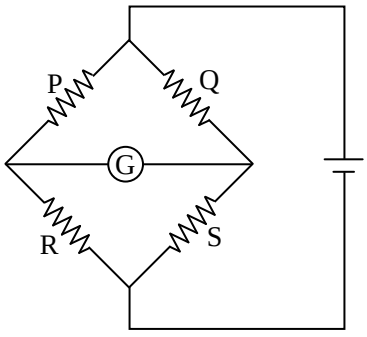
Ans. **[2]**

Sol.

(I)



(II)



In I[st] case for balance

$\frac{P}{R} = \frac{Q}{S}$

$\frac{P}{Q} = \frac{R}{S}$ ….(i)

In IInd case for balance

$\frac{P}{Q} = \frac{R}{S}$ ….(ii)

In both case

Null point is same.

Q.30 The following observations were taken for determining surface tension T of water by capillary method :

diameter of capillary, D = 1.25×10^{-2} m

rise of water, h = 1.45×10^{-2} m.

Using g = 9.80 m/s^2 and the simplified relation T = $\frac{rhg}{2} \times 10^3$ N/m, the possible error in surface tension is closest to -

(1) 0.15% (2) 1.5% (3) 2.4% (4) 10%

Students may find similar question in CP exercise sheet :

[JEE Advance, Chapter : Practical Physics, Ex.2, Page No.70, Q. No.7]

Ans. **[2]**

Sol. D = 1.25×10^{-2} m, $\Delta D = 0.01 \times 10^{-2}$ m, h = 1.45×10^{-2} m, $\Delta h = 0.01 \times 10^{-2}$ m

g = 9.80 m/s^2

$T = \frac{rhg}{2} \times 10^3$ N/m

$Tr^{-1}h^{-1} = \frac{g}{2} \times 10^3$

$Tr^{-1}h^{-1}$ = constant

$\frac{\Delta T}{T} - \frac{\Delta r}{r} - \frac{\Delta h}{h} = 0$

$\frac{\Delta T}{T} = \left(\frac{\Delta r}{r} + \frac{\Delta h}{h}\right)$

$\frac{\Delta T}{T} = \left(\frac{0.01\times10^{-2}}{1.25\times10^{-2}}\right) + \left(\frac{0.01\times10^{-2}}{1.45\times10^{-2}}\right) \left\{\begin{matrix} D = 2r \\ \Delta D = 2\Delta r \\ \frac{\Delta D}{D} = \frac{\Delta r}{r} \end{matrix}\right\}$

% error = $\frac{\Delta T}{T} \times 100$

$\approx$ 1.5 %

Part B – CHEMISTRY

Q.31 Given

$C_{(graphite)} + O_2(g) \longrightarrow CO_2(g)$;

$\Delta_r H° = -393.5$ kJ mol^{-1}

$H_2(g) + \frac{1}{2}O_2(g) \longrightarrow H_2O(1)$;

$\Delta_r H° = -285.8$ kJ mol^{-1}

$CO_2(g) + 2H_2O(1) \longrightarrow CH_4(g) + 2O_2(g)$;

$\Delta_r H° = +890.3$ kJ mol^{-1}

Based on the above thermochemical equations, the value of $\Delta_r H°$ at 298 K for the reaction

$C_{(graphite)} + 2H_2(g) \longrightarrow CH_4(g)$ will be ;

(1) –74.8 kJ mol^{-1} (2) –144.0 kJ mol^{-1} (3) +74.8 kJ mol^{-1} (4) +144.0 kJ mol^{-1}

Students may find similar question in CP exercise sheet :

[*JEE Main, Chapter : Chemical Energetic ,Exercise-1 Q. No.12*]

[*JEE Advance, Chapter : Chemical Energetic ,Exercise-4 Q. No.11*]

Ans. **[1]**

Sol. $C_{(graphite)} + O_2(g) \longrightarrow CO_2(g)$; $\Delta_r H° = -393.5$ kJ .... (i)

$H_2(g) + \frac{1}{2}O_2(g) \longrightarrow H_2O(1)$; $\Delta_r H° = -285.8$ kJ .... (ii)

$CO_2(g) + 2H_2O(1) \longrightarrow CH_4(g) + 2O_2(g)$; $\Delta_r H° = +890.3$ ..... (iii)

$C_{(graphite)} + 2H_2(g) \longrightarrow CH_4(g)$; $\Delta_r H° = ?$ .... (iv)

eq. (iv) = eq. (i) + 2 × eq (ii) + eq. (iii)

= –393. 5 + 2 (–285.8) + 890.3

= –74.8 kJ/mol

Q.32 1 gram of a carbonate (M_2CO_3) on treatment with excess HCl produces 0.01186 mole of CO_2. The molar mass of M_2CO_3 in g mol^{-1} is -

(1) 118.6 (2) 11.86 (3) 1186 (4) 84.3

Ans. **[4]**

Sol. $\underset{1gm}{M_2CO_3} + HCl \longrightarrow \underset{.01186}{CO_2}$

POAC on carbon

$\frac{1}{x} = \frac{.01186}{1} \Rightarrow x = 84.3$

Q.33 ΔU is equal to -

(1) Adiabatic work (2) Isothermal work (3) Isochoric work (4) Isobaric work

Ans. **[1]**

Sol. FLOT (According to first law of thermodynamics)

$\Delta E = q + w$

If $q = 0$, $\Delta E = w$ $\therefore$ adiabatic process

Q.34 The Tyndall effect is observed only when following conditions are satisfied -

(a) The diameter of the dispersed particles is much smaller than the wavelength of the light used

(b) The diameter of the dispersed particle is not much smaller than the wavelength of the light used

(c) The refractive indices of the dispersed phase and dispersion medium are almost similar in magnitude

(d) The refractive indices of the dispersed phase and dispersion medium differ greatly in magnitude.

(1) (a) and (c) (2) (b) and (c) (3) (a) and (d) (4) (b) and (d)

Students may find similar question in CP exercise sheet :

[*JEE Advance, Chapter : Surface Chemistry, Key Concept, Optical Properties*]

Ans. **[4]**

Sol. Facts

Q.35 A metal crystallises in the face centred cubic structure. if the edge length of its unit cell is 'a', the closest approach between two atoms in metallic crystal will be :

(1) $\sqrt{2}a$ (2) $\frac{a}{\sqrt{2}}$ (3) $2a$ (4) $2\sqrt{2}a$

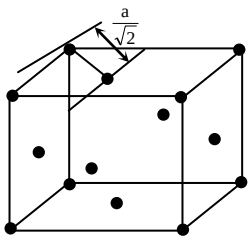
Students may find similar question in CP exercise sheet :

[*JEE Main, Chapter : Solid State, Solved Example, Q. No.21*]

[*JEE Advance, Chapter : Solid State, Solved Exercise-4, Q. No.4*]

Ans. **[2]**

Sol. $\therefore$ nearest distance = $\frac{a}{\sqrt{2}}$



fcc

Q.36 Given

$E^{\circ}_{Cl_2/Cl^-} = 1.36V$, $E^{\circ}_{Cr^{3+}/Cr} = -0.74V$

$E^{\circ}_{Cr_2O_7^{2-}/Cr^{3+}} = 1.33V$, $E^{\circ}_{MnO_4^-/Mn^{2+}} = 1.51V$,

Among the following, the strongest reducing agent is -

(1) Cr^{3+} (2) Cl^- (3) Cr (4) Mn^{2+}

Students may find similar question in CP exercise sheet :

[*JEE Main, Chapter : Electro Chemistry, Exercise-4, Q. No.26*]

[*JEE Advance, Chapter : Electro Chemistry, Exercise-5, Section [A], Q. No.11*]

Ans. **[3]**

Sol. Less is the SRP, more is the reducing power & strongest is the reducing agent.

Q.37 The freezing point of benzene decreases by 0.45°C when 0.2 g of acetic acid is added to 20 g of benzene. If acetic acid associates to form a dimer in benzene, percentage association of acetic acid in benzene will be

(K_f for benzene = 5.12 K kg mol^{-1})

(1) 74.6% (2) 94.6% (3) 64.6% (4) 80.4%

Students may find similar question in CP exercise sheet :

[*JEE Main, Chapter : Solution, Solved Example Q. No.40*]

[*JEE Advance, Chapter : Solution, Exercise-5, Section [B], Q. No.2*]

Ans. **[2]**

Sol. $\Delta T_f = i \times K_f \times m$

$$0.45 = \frac{i \times 5.12 \times 0.2 \times 1000}{60 \times 20}$$

$i = .527$

$$\beta = \frac{1-i}{1-1/n} = \frac{1-.527}{1-1/2} = .946 \text{ or } 94.6\ \%$$

Q.38 The radius of the second Bohr orbit for hydrogen atom is –

(Planck's Const. h = 6.6262 × 10^{-34} Js; mass of electron = 9.1091 × 10^{-31} kg; charge of electron e = 1.60210 ×10^{-19} C; permittivity of vaccum $\in_0$ = 8.854185 ×10^{-12} $kg^{-1}m^{-3}A^2$)

(1) 0.529 Å (2) 2.12 Å (3) 1.65 Å (4) 7.76 Å

Students may find similar question in CP exercise sheet :

[*JEE Main, Chapter : Atomic Structure, Exercise-1, Q. No.14*]

[*JEE Advance, Chapter : Atomic Structure, Exercise-1, Q. No.21*]

Ans. **[2]**

Sol. $r = 0.529 \times \frac{n^2}{z}$ Å

$= 0.529 \times \frac{(2)^2}{1} = 2.116$ Å $\cong$ 2.12 Å

Q.39 Two reactions, R_1 and R_2 have identical pre-exponential factors. Activation energy of R_1 exceeds that of R_2 by 10 kJ mol^{-1} . If k_1 and k_2 are rate constants for reactions R_1 and R_2 respectively at 300 K, then $\ln(k_2/k_1)$ is equal to. ($R = 8.314$ J mol^{-1} K^{-1}).

(1) 6 (2) 4 (3) 8 (4) 12

Students may find similar question in CP exercise sheet :
[JEE Main, Chapter : Chemical Kinetics, Exercise-5, Section [B], Q. No.27]
[JEE Advance, Chapter : Chemical Kinetics, Exercise-4, Q. No.25]

Ans. **[2]**

Sol.

R_1	R_2
A	A
$E_a + 10$	E_a
k_1	k_2

$k_1 = A\, e^{-(Ea + 10)/RT}$

$k_2 = A\, e^{-Ea/RT}$

$$\frac{k_2}{k_1} = e^{(-Ea + Ea + 10)/RT}$$

$$\frac{k_2}{k_1} = e^{10/RT} = e^{10\times10^3/8.314\times300}$$

$$= e^{10000/2494.2} = e^4$$

$$\boxed{\ln\frac{k_2}{k_1} = 4}$$

Q.40 pk_a of a weak acid (HA) and pk_b of a weak base (BOH) are 3.2 and 3.4, respectively. The pH of their salt (AB) solution is-

(1) 7.0 (2) 1.0 (3) 7.2 (4) 6.9

Students may find similar question in CP exercise sheet :
[JEE Main, Chapter : Ionic Equilibrium, Exercise-4, Q. No.8]
[JEE Advance, Chapter : Ionic Equilibrium, Exercise-1, Q. No.28]

Ans. **[4]**

Sol. $$pH = \frac{1}{2}pK_w + \frac{1}{2}pK_a - \frac{1}{2}pK_b$$

$$= \frac{1}{2}\times14 + \frac{1}{2}\times3.2 - \frac{1}{2}\times3.4$$

$= 7 + 1.6 - 1.7$

$= 6.9$

Q.41 Both lithium and magnesium display several similar properties due to the diagonal relationship; however, the one which is incorrect, is :

(1) Both form nitrides
(2) Nitrates of both Li and Mg yield NO_2 and O_2 on heating
(3) Both form basic carbonates
(4) Both form soluble bicarbonates

Ans. **[3]**

Sol. It is the best possible option but, it should be bonus because Li_2CO_3 is less basic and $MgCO_3$ is basic

Q.42 Which of the following species is not paramagnetic ?

(1) O_2 (2) B_2 (3) NO (4) CO

Students may find similar question in CP exercise sheet :
[JEE Main, Chapter : Chemical bonding, Exercise # 1, Q. No.92]
[JEE Advance, Chapter : Chemical bonding, Exercise # 2, Q. No.43]

Ans. **[4]**

Sol. CO is diamagnetic because all electrons are paired in it.

Q.43 Which of the following reactions is an example of a redox reaction ?

(1) $XeF_6 + H_2O \rightarrow XeOF_4 + 2HF$ (2) $XeF_6 + 2H_2O \rightarrow XeO_2F_2 + 4HF$

(3) $XeF_4 + O_2F_2 \rightarrow XeF_6 + O_2$ (4) $XeF_2 + PF_5 \rightarrow [XeF]^+ PF_6^-$

Ans. **[3]**

Sol. $\overset{+4}{Xe}F_4 + \overset{+1}{O_2}F_2 \rightarrow \overset{+6}{Xe}F_6 + \overset{0}{O_2}$

This reaction is a redox reaction

Q.44 A water sample has ppm level concentration of following anions

$F^- = 10$; $SO_4^{2-} = 100$; $NO_3^- = 50$

The anion / anions that make / makes the water sample unsuitable for drinking is / are

(1) Only F^- (2) Only SO_4^{2-}

(3) Only NO_3^- (4) Both SO_4^{2-} and NO_3^-

Ans. **[1]**

Sol. F^- ion concentration above 2 ppm causes brown mottling of teeth

Q.45 The group having isoelectronic species is

(1) O^{2-}, F^-, Na, Mg^{2+} (2) O^-, F^-, Na^+, Mg^{2+} (3) O^{2-}, F^-, Na^+, Mg^{2+} (4) O^-, F^-, Na, Mg^+

Students may find similar question in CP exercise sheet :
[JEE Main, Chapter : Periodic Table, Exercise # 4, Q. No.5]
[JEE Advance, Chapter : Periodic Table, Exercise # 2, Q. No.3]

Ans. **[3]**

Sol. O^{2-}, F^-, Na^+, Mg^{2+} are isoelectronic species. They all have $10e^-$

Q.46 The products obtained when chlorine gas reacts with cold and dilute aqueous NaOH are

(1) Cl^- and ClO^- (2) Cl^- and ClO_2^- (3) ClO^- and ClO_3^- (4) ClO_3^- and ClO_3^-

Students may find similar question in CP exercise sheet :
[JEE Main, Chapter : p-Block elements, Exercise # 2, Q. No. 39]
[JEE Advance, Chapter : p-Block elements (Halogens), Exercise # 2, Q. No. 9]

Ans. **[1]**

Sol. $Cl_2 + NaOH_{(cold/dilute)} \rightarrow NaCl + NaOCl + H_2O$

Q.47 In the following reactions, ZnO is respectively acting as a / an

(a) $Zn + Na_2O \rightarrow Na_2ZnO_2$

(b) $Zn + CO_2 \rightarrow ZnCO_3$

(1) acid and acid (2) acid and base (3) base and acid (4) base and base

Ans. **[2]**

Sol. $ZnO + Na_2O \rightarrow Na_2ZnO_2$

acid

$ZnO + CO_2 \rightarrow ZnCO_3$

Base

ZnO is an amphoteric oxide

Q.48 Sodium salt of an organic acid 'X' produces effervescence with conc. H_2SO_4. 'X' reacts with the acidified aqueous $CaCl_2$ solution to give a white precipitate which decolourises acidic solution of $KMnO_4$. 'X' is -

(1) CH_3COONa (2) $Na_2C_2O_4$ (3) C_6H_5COONa (4) HCOONa

Students may find similar question in CP exercise sheet :

[JEE Advance, Chapter : Salt analysis, Exercise # 1, Q. No. 19]

Ans. **[2]**

Sol. [X] $\xrightarrow{\text{Conc. } H_2SO_4}$ $Na_2CO_3 + H_2C_2O_4$

$Na_2C_2O_4$

(Oxalate-ion)

$\downarrow CaCl_2$

$CaC_2O_4\downarrow + 2NaCl$

White ppt

$H_2C_2O_4 \rightarrow CO_2\uparrow$

Effervesce

$KMnO_4 + C_2O_4^{-2} \xrightarrow{H^+} CO_2\uparrow + Mn^{+2}$

Pink Colour — Oxalate ion — Colourless

Q.49 The most abundant elements by mass in the body of a healthy human adult are : Oxygen (61.4%), Carbon (22.9 %), Hydrogen (10.0%) and Nitrogen (2.6%). The weight which a 75kg person would gain if all ^1H atoms are replaced by ^2H atoms is

(1) 7.5 kg (2) 10 kg (3) 15 kg (4) 37.5 kg

Ans. **[1]**

Sol. Weight of ^1_1H in person = $75 \times \frac{10}{100} = 7.5$ kg

Now if $_1H^1$ is replaced by $_1H^2$

Than person will gain weight by = 7.5 kg

Q.50 On treatment of 100 mL of 0.1 M solution of $CoCl_3.6H_2O$ with excess $AgNO_3$; 1.2×10^{22} ions are precipitated. The complex is :

(1) $[Co(H_2O)_6]Cl_3$ (2) $[Co(H_2O)_5Cl]Cl_2.H_2O$

(3) $[Co(H_2O)_4Cl_2]Cl.2H_2O$ (4) $[Co(H_2O)_3Cl_3].3H_2O$

Ans. **[2]**

Sol. $CoCl_3.6H_2O + AgNO_3$ (excess) $\longrightarrow$ AgCl

100 ml, 0.1 M 1.2×10^{22} ion

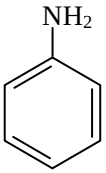
No. of moles = 0.01 mol $\frac{1.2\times10^{22}}{6\times10^{23}} = 0.02$ mol

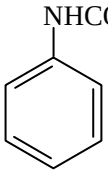
$\because$ 0.01 mol of $CoCl_3.6H_2O$ produce 0.02 mol of AgCl

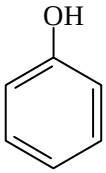
$\therefore$ 1 mol of $CoCl_3.6H_2O$ produce 2 mol of AgCl

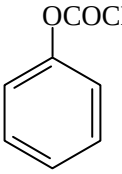
$\therefore$ Correct complex is $[Co(H_2O)_5Cl]Cl_2.H_2O$

Q.51 Which of the following compounds will form significant amount of meta product during mono-nitration reaction?

(1) 

(2) 

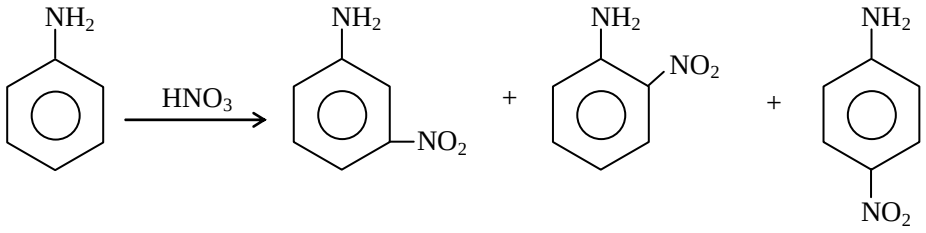
(3) 

(4) 

Students may find similar question in CP exercise sheet :

[*JEE Advance, Chapter : Amines and Nitrogen Compounds, Example-4*]

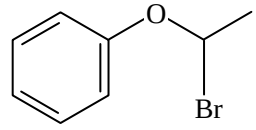
Ans. **[1]**

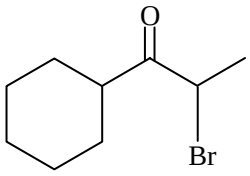
Sol. 

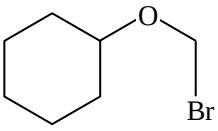
[47%] [2%] [51%]

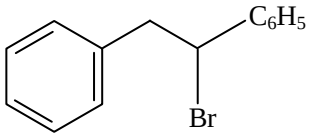
In aniline protonation of NH_2 Group make it deactivating and meta directing. So meta product is significant.

Q.52 Which of the following, upon treatment with tert-BuONa followed by addition of bromine water, fails to decolourize the colour of bromine?

(1) 

(2) 

(3) 

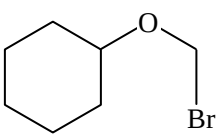
(4) 

Students may find similar question in CP exercise sheet :

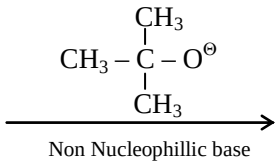
[JEE Advance, Chapter : Halogen Derivatives, Exercise # 2, Q. No.6]

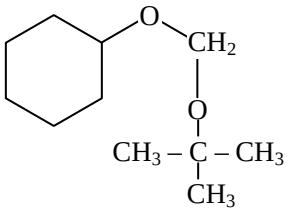
Ans. **[3]**

Sol.



Elimination reaction not possible





It do not give unsaturation test

Q.53 The formation of which of the following polymers involves hydrolysis reaction?

(1) Nylon 6, 6 (2) Terylene (3) Nylon 6 (4) Bakelite

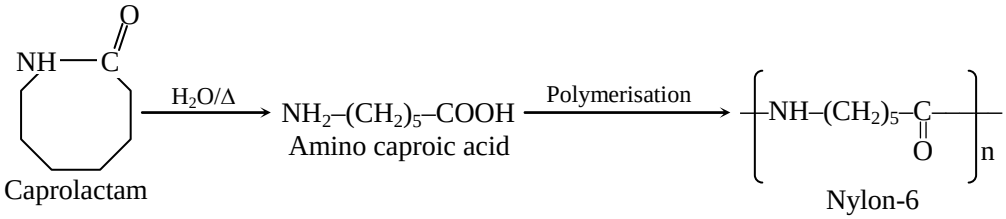
Students may find similar question in CP exercise sheet :

[JEE Main, Chapter : Polymer, Exercise # 5, Q. No.1]

[JEE Advance, Chapter : Carbohydrate, Amino acid, Protein & Polymer, Example-21]

Ans. **[3]**

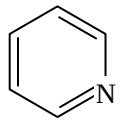
Sol. Nylon-6 is formed by monomer which is obtain by hydrolysis of CAPROLACTAM

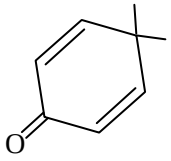


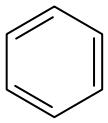
Caprolactam

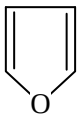
Nylon-6

Q.54 Which of the following molecules is least resonance stabilized?

(1) 

(2) 

(3) 

(4) 

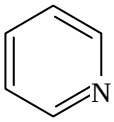
Students may find similar question in CP exercise sheet :

[JEE Main, Chapter : GOC, Page 51, Q. No.32]

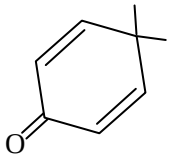
[JEE Advance, Chapter :GOC, Exercise # 1, Q. No.54]

Ans. **[2]**

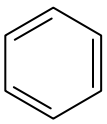
Sol.



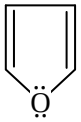
Aromatic



Non Aromatic



Aromatic



Aromatic

2 is least stable as other are aromatic and 2 is non aromatic

Q.55 The increasing order of the reactivity of the following halides for the S_N1 reaction is :

$CH_3\underset{\underset{Cl}{|}}{C}HCH_2CH_3$ (I) $\quad CH_3CH_2CH_2Cl$ (II) $\quad p - H_3CO - C_6H_4 - CH_2Cl$ (III)

(1) (I) < (III) < (II) (2) (II) < (III) < (I) (3) (III) < (II) < (I) (4) (II) < (I) < (III)

Students may find similar question in CP exercise sheet :
[*JEE Main, Chapter : Haloalkane, page 28, Q. No.17*]
[*JEE Advance, Chapter : Halogen Derivatives, Exercise # 1, Q. No.14*]

Ans. **[4]**

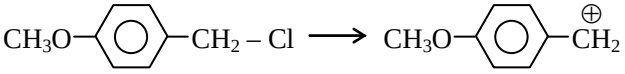
Sol. Rate of SN_1 reaction $\propto$ stability of carbocation

$CH_3\underset{\underset{Cl}{|}}{C}HCH_2CH_3 \longrightarrow CH_3 - \overset{\oplus}{C}H - CH_2 - CH_3 + Cl^{\ominus}$

(I) 2° Carbocation

$CH_3 - CH_2 - CH_2 - Cl \longrightarrow CH_3 - CH_2 - \overset{\oplus}{C}H_2$

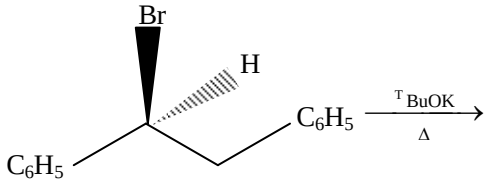
1° Carbocation



Resonance Stabilize

Order of SN_1 = II < I < III

Q.56 The major product obtained in the following reaction is :



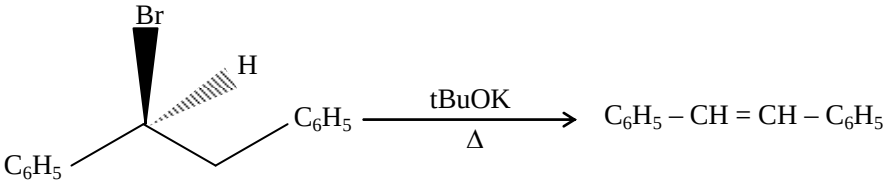
(+)

(1) (+)$C_6H_5CH(O^tBu)CH_2C_6H_5$
(2) (–)$C_6H_5CH(O^tBu)CH_2C_6H_5$
(3) (±)$C_6H_5CH(O^tBu)CH_2C_6H_5$
(4) $C_6H_5CH = CHC_6H_5$

Students may find similar question in CP exercise sheet :
[*JEE Main, Chapter : Hydrocarbon, page 37, Q. No. 31*]
[*JEE Advance, Chapter : Hydrocarbon, Exercise # 2, Q. No. 19*]

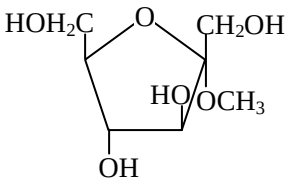
Ans. **[4]**

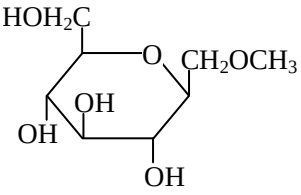
Sol.

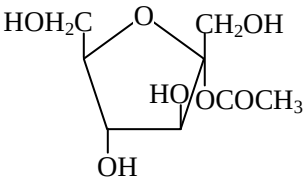


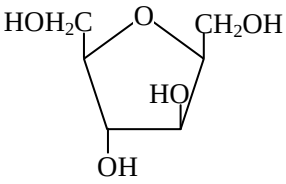
It is example of E_2 elimination as t butoxide is stronger base and heating is also used.

Q.57 Which of the following compounds will behave as a reducing sugar in an aqueous KOH solution?

(1) 

(2) 

(3) 

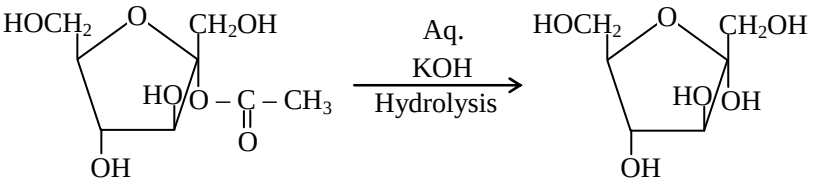
(4) 

Students may find similar question in CP exercise sheet :

[*JEE Main, Chapter : Carbohydrate, page 74, Q. No. 9*]

[*JEE Advance, Chapter : Carbohydrate, Protein & Polymer, Exercise # 5, Q. No. 2*]

Ans. **[3]**

Sol. 

It is hemiacetal that is why it can behave as reducing sugar.

Q.58 3-Methyl-pent-2-ene on reaction with HBr in presence of peroxide forms an addition product. The number of possible stereoisomers for the product is :

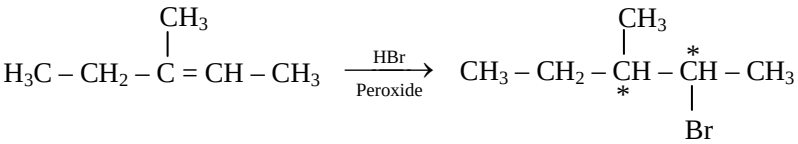
(1) Two (2) Four (3) Six (4) Zero

Students may find similar question in CP exercise sheet :

[*JEE Main, Chapter : Hydrocarbon, page 30, Q. No.50*]

[*JEE Advance, Chapter : Hydrocarbon, Example-14, Q. No.50*]

Ans. **[2]**

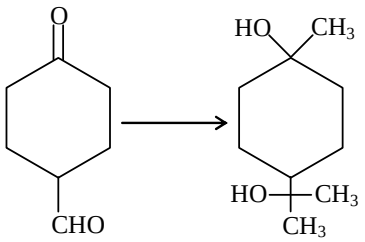
Sol. 

Major Product

Two new chiral center are formed so total 4 stereoisomer

This HBr/Peroxide gives antimarkownikov product as major product

Q.59 The correct sequence of reagents for the following conversion will be :

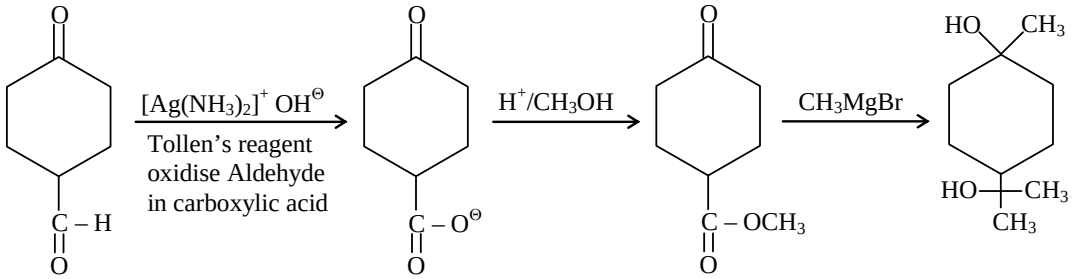


(1) CH_3MgBr, $[Ag(NH_3)_2]^+OH^-$, H^+/CH_3OH (2) $[Ag(NH_3)_2]^+OH^-$, CH_3MgBr, H^+/CH_3OH

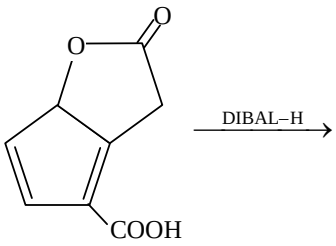
(3) $[Ag(NH_3)_2]^+OH^-$, H^+/CH_3OH, CH_3MgBr (4) CH_3MgBr, H^+/CH_3OH, $[Ag(NH_3)_2]^+OH^-$

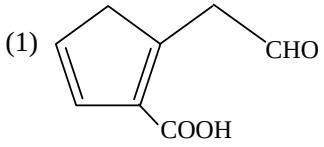
Ans. **[3]**

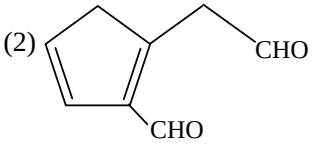
Sol.

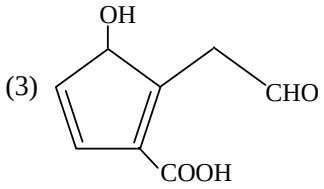


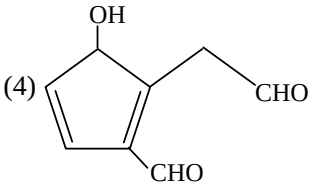
Q.60 The major product obtained in the following reaction is :



(1) 

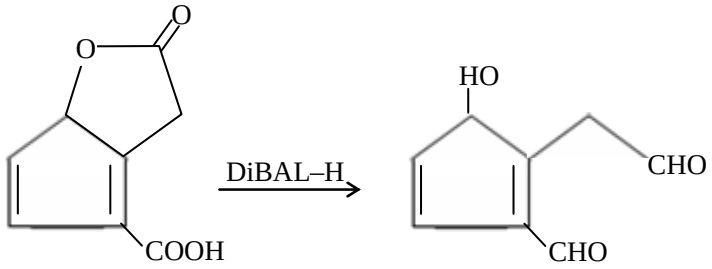
(2) 

(3) 

(4) 

Ans. **[4]**

Sol.



DiBAL-H is selective reducing agent which reduce carboxylic acid and it's derivative up to aldehyde only [further reduction not possible]

Part C – MATHS

Q.61 The function $f : R \to \left[-\frac{1}{2}, \frac{1}{2}\right]$ defined as $f(x) = \frac{x}{1+x^2}$, is

(1) injective but not surjective

(2) surjective but not injective

(3) neither injective nor surjective

(4) invertible

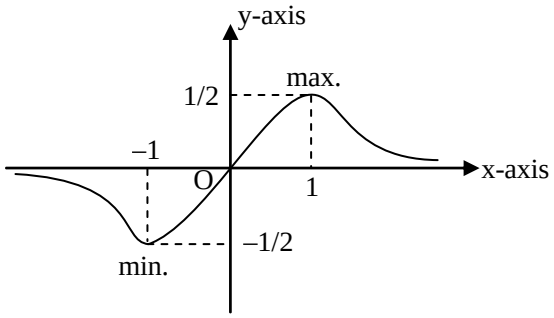
Students may find similar question in CP exercise sheet :

[JEE Main, Chapter : Function, Page 55, Ex. 5A, Q. No. 23]

[JEE Advance, Chapter : Function, Page 20, Ex. 1,]

Ans. **[2]**

Sol.

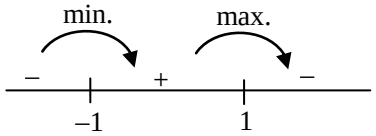


$f(x) = \frac{x}{1+x^2}$ (odd)

(symmetry about origin)

$$\frac{dy}{dx} = \frac{(1+x^2)\cdot 1 - x(0+2x)}{(1+x^2)^2}$$

$$= \frac{1-x^2}{(1+x^2)^2} = 0 \Rightarrow x = 1, -1$$



Line parallel to x-axis cuts the graph more than one points hence function is many one.

Range $= \left[-\frac{1}{2}, \frac{1}{2}\right]$ = codomain hence function is onto

Q.62 If, for a positive integer n, the quadratic equation, $x(x+1) + (x+1)(x+2) + ... + (x + \overline{n-1})(x+n) = 10n$ has two consecutive integral solutions, then n is equal to

(1) 9 (2) 10 (3) 11 (4) 12

Ans. **[3]**

Sol. $x(x+1)+(x+1)(x+2)+....+(x+(n-1))(x+n)=10n$

After simplify

$nx^2+(1+3+5+7+.....+(2n-1)x+(0\cdot1+1\cdot2+2\cdot3+...+(n-1)n)=10n$

$nx^2+n^2x+\frac{n(n^2-1)}{3}-10n=0$

$x^2+nx+\frac{n^2-1-30}{3}=0$

$x^2+nx+\frac{n^2-31}{3}=0$

Put $n=11$ (where $n\in I^+$)

$x^2+11x+\frac{121-31}{3}=0$

$x^2+11x+30=0$

$(x+6)(x+5)=0$

i.e. $x=-5,-6$ (Two consecutive integral solutions)

So, $n=11$

Q.63 Let ω be a complex number such that $2\omega+1=z$ where $z=\sqrt{-3}$. If $\begin{vmatrix}1 & 1 & 1\\ 1 & -\omega^2-1 & \omega^2\\ 1 & \omega^2 & \omega^7\end{vmatrix}=3k$, then k is equal to

(1) z (2) –1 (3) 1 (4) –z

Students may find similar question in CP exercise sheet :

[JEE Advance, Chapter : Complex Number, Page 29, Ex. 3]

Ans. **[4]**

Sol. Apply operation $C_1=C_1+C_2+C_3$

$\begin{vmatrix}3 & 1 & 1\\ 0 & -(1+\omega^2) & \omega^2\\ 0 & \omega^2 & \omega\end{vmatrix}=3k$

$\begin{vmatrix}3 & 1 & 1\\ 0 & \omega & \omega^2\\ 0 & \omega^2 & \omega\end{vmatrix}=3k$ (Because $1+\omega+\omega^2=0$)

open by C_1

$3(\omega^2-\omega^4)=3k$

$3(\omega^2-\omega)=3k$

$3(-1-\omega-\omega)=3k$

$-3(1+2\omega)=3k$ Given that $2\omega+1=z$

$-3z=3k$

$k=-z$

Q.64 If $A = \begin{bmatrix} 2 & -3 \\ -4 & 1 \end{bmatrix}$, then adj $(3A^2 + 12A)$ is equal to

(1) $\begin{bmatrix} 51 & 63 \\ 84 & 72 \end{bmatrix}$ (2) $\begin{bmatrix} 51 & 84 \\ 63 & 72 \end{bmatrix}$ (3) $\begin{bmatrix} 72 & -63 \\ -84 & 51 \end{bmatrix}$ (4) $\begin{bmatrix} 72 & -84 \\ -63 & 51 \end{bmatrix}$

Ans. **[1]**

Sol. $A = \begin{bmatrix} 2 & -3 \\ -4 & 1 \end{bmatrix}$

$A^2 = \begin{bmatrix} 2 & -3 \\ -4 & 1 \end{bmatrix}\begin{bmatrix} 2 & -3 \\ -4 & 1 \end{bmatrix} = \begin{bmatrix} 16 & -9 \\ -12 & 13 \end{bmatrix}$

$3A^2 + 12A = \begin{bmatrix} 48 & -27 \\ -36 & 39 \end{bmatrix} + \begin{bmatrix} 24 & -36 \\ -48 & 12 \end{bmatrix}$

$= \begin{bmatrix} 72 & -63 \\ -84 & 51 \end{bmatrix}$

adj $(3A^2 + 12A) = \begin{bmatrix} 51 & 63 \\ 84 & 72 \end{bmatrix}$

Q.65 If S is the set of distinct values of 'b' for which the following system of linear equations

$x + y + z = 1$

$x + ay + z = 1$

$ax + by + z = 0$

has no solution, then S is

(1) an infinite set

(2) a finite set containing two or more elements

(3) a singleton

(4) an empty set

Ans. **[3]**

Sol. $\Delta = 0$ and at the one of Δ_1 or Δ_2 or $\Delta_3 \neq 0$

$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & a & 1 \\ a & b & 1 \end{vmatrix} = 0$

$1[a - b] - 1[1 - a] + 1[b - a^2] = 0$

$2a - b - 1 + b - a^2 = 0$

$a^2 - 2a + 1 = 0$

$a = 1$

$x + y + z = 1$

$x + y + z = 1$

$x + by + z = 0$

only one value of b, S is singleton set

Q.66 A man X has 7 friends, 4 of them are ladies and 3 are men. His wife Y also has 7 friends, 3 of them are ladies and 4 are men. Assume X and Y have no common friends. Then the total number of ways in which X and Y together can throw a party inviting 3 ladies and 3 men, so that 3 friends of each of X and Y are in this party, is

(1) 468 (2) 469 (3) 484 (4) 485

Students may find similar question in CP exercise sheet :
[JEE Main, Chapter :P & C, Page 16, Ex. 1, Q. No. 29]

Ans. **[4]**

Sol.

X Y

7 friends 7 friends

3M 4L 4M 3L

Case I : 3L from X side and 3M from Y side

$^4C_3 \times {}^4C_3 = 4 \times 4 = 16$

Case II : 3M from X side and 3L from Y side

$^3C_3 \times {}^3C_3 = 1 \times 1 = 1$

Case III : 2L and 1M from X side and 2M and 1L from Y side

$(^4C_2 \times {}^3C_1) \times (^4C_2 \times {}^3C_1) = (6 \times 3) \times (6 \times 3) = 18 \times 18 = 324$

Case IV : 2M and 1L from X side and 1M and 2L from Y side

$(^3C_2 \times {}^4C_1) \times (^4C_1 \times {}^3C_2) = (3 \times 4) \times (4 \times 3) = 12 \times 12 = 144$

Total number of ways = Case I + Case II + Case III + Case IV

$= 16 + 1 + 324 + 144$

$= 485$

Q.67 The value of $(^{21}C_1 - {}^{10}C_1) + (^{21}C_2 - {}^{10}C_2) + (^{21}C_3 - {}^{10}C_3) + (^{21}C_4 - {}^{10}C_4) + ... + (^{21}C_{10} - {}^{10}C_{10})$ is

(1) $2^{21} - 2^{10}$ (2) $2^{20} - 2^9$ (3) $2^{20} - 2^{10}$ (4) $2^{21} - 2^{11}$

Ans. **[3]**

Sol. $(^{21}C_1 + {}^{21}C_2 + ... + {}^{21}C_{10}) - (^{10}C_1 + {}^{10}C_2 + ... + {}^{10}C_{10})$

$= \frac{1}{2}\left[2 \times {}^{21}C_1 + 2 \times {}^{21}C_2 + ... + 2 \times {}^{21}C_{10}\right] - (2^{10} - 1)$

$= \frac{1}{2}\left[{}^{21}C_0 + {}^{21}C_1 + {}^{21}C_2 + ... + {}^{21}C_{10} + {}^{21}C_{11} + ... + {}^{21}C_{20} + {}^{21}C_{21} - ({}^{21}C_0 + {}^{21}C_{21})\right] - (2^{10} - 1)$

$= \frac{1}{2}(2^{21} - 2) - (2^{10} - 1)$

$= 2^{20} - 1 - 2^{10} + 1$

$= 2^{20} - 2^{10}$

Q.68 For any three positive real numbers a, b and c, $9(25a^2 + b^2) + 25(c^2 - 3ac) = 15b(3a + c)$. Then

(1) b, c and a are in A.P. (2) a, b and c are in A.P.

(3) a, b and c are in G.P. (4) b, c and a are in G.P.

Students may find similar question in CP exercise sheet :

[JEE Main, Chapter : Progression, Page 27, Ex. 4, Q. No. 19]

[JEE Advance, Chapter : Progression, Page 20, Ex. 2, Q. No. 26]

Ans. **[1]**

Sol. $225a^2 + 9b^2 + 25c^2 - 75ac - 45ab - 15bc = 0$

$450a^2 + 18b^2 + 50c^2 - 150ac - 10ab - 30bc = 0$

$(15a - 3b)^2 + (3b - 5c)^2 + (15a - 5c)^2 = 0$

$(15a - 3b)^2 = 0,\ (3b - 5c)^2 = 0,\ (15a - 5c)^2 = 0$

$15a = 3b,\ 3b = 5c$

$15a = 3b = 5c$

$\frac{a}{1} = \frac{b}{5} = \frac{c}{3} = k$ (let)

$a = k, b = 5k, c = 3k$

Then a, c and b are in A.P.

Q.69 Let $a, b, c \in R$. If $f(x) = ax^2 + bx + c$ is such that $a + b + c = 3$ and $f(x + y) = f(x) + f(y) + xy, \forall\ x, y \in R$, then $\sum_{n=1}^{10} f(n)$ is equal to

(1) 165 (2) 190 (3) 255 (4) 330

Ans. **[4]**

Sol. As $a + b + c = 3$

So, $f(1) = 3$

$f(x + y) = f(x) + f(y) + xy$

Put $x = 1, y = 1$

$f(2) = 2f(1) + 1 = 7$

Put $x = 2, y = 1$

$f(3) = f(2) + f(1) + 2 = 12$

Put $x = 2,\ y = 2$

$f(4) = 2f(2) + 4 = 18$

So, $\sum_{n=1}^{10} f(n) = f(1) + f(2) + f(3) + ... + f(10)$

Let $S = 3 + 7 + 12 + 18 + ... + f(n)$

$S = 3 + 7 + 12 + ... + f(n)$

$0 = 3 + 4 + 5 + 6 + ... - f(n)$

$f(n) = 3 + 4 + 5 + 6 + ... = \frac{n}{2}[6 + (n - 1)]$

$$f(n) = \frac{n(n+5)}{2}$$

$$\therefore \quad \sum_{n=1}^{10} f(n) = \frac{1}{2}\sum_{n=1}^{10} n^2 + \frac{5}{2}\sum_{n=1}^{10} n$$

$$= \frac{1}{2} \times \frac{10\times11\times21}{6} + \frac{5}{2} \times \frac{10\times11}{2}$$

$$= 330$$

Q.70 $\lim\limits_{x\to\frac{\pi}{2}} \frac{\cot x - \cos x}{(\pi - 2x)^3}$ equals :

(1) $\frac{1}{16}$ (2) $\frac{1}{8}$ (3) $\frac{1}{4}$ (4) $\frac{1}{24}$

Students may find similar question in CP exercise sheet :
[JEE Main, Chapter :Limit, Page 20, Ex. 2, Q. No. 30]
[JEE Advance, Chapter :Limit, Page 16, Ex. 1, Q. No. 18]

Ans. **[1]**

Sol. Put $x = \frac{\pi}{2} + h$

$$\lim_{h\to0} \frac{\cot\left(\frac{\pi}{2}+h\right) - \cos\left(\frac{\pi}{2}+h\right)}{\left(\pi - 2\left(\frac{\pi}{2}+h\right)\right)^3}$$

$$\lim_{h\to0} \frac{-\tan h + \sin h}{-8h^3}$$

$$\lim_{h\to0} \frac{\tan h - \sin h}{8h^3}$$

by expansion method

$$\lim_{h\to0} \frac{\left(h + \frac{h^3}{3} + \frac{2}{15}h^5 ....\right) - \left(h - \frac{h^3}{3!} + \frac{h^5}{5!} ...\right)}{8h^3}$$

$$\lim_{h\to0} \frac{h^3\left(\frac{1}{3}+\frac{1}{3!}\right) + h^5\left(\frac{2}{15} - \frac{1}{5!}\right) + ...}{8h^3}$$

$$= \frac{\frac{1}{3}+\frac{1}{6}+0}{8} = \frac{1}{16}$$

Q.71 If for $x \in \left(0, \frac{1}{4}\right)$, the derivative of $\tan^{-1}\left(\frac{6x\sqrt{x}}{1-9x^3}\right)$ is $\sqrt{x} \cdot g(x)$, then g(x) equals :

(1) $\frac{3x\sqrt{x}}{1-9x^3}$ (2) $\frac{3x}{1-9x^3}$ (3) $\frac{3}{1+9x^3}$ (4) $\frac{9}{1+9x^3}$

Students may find similar question in CP exercise sheet :
[JEE Advance, Chapter : Differentiation, Page 25, Ex. 2, Q. No. 26]

Ans. **[4]**

Sol. $y = \tan^{-1}\left(\frac{6x\sqrt{x}}{1-9x^3}\right)$

$= \tan^{-1}\left(\frac{2 \cdot 3x\sqrt{x}}{1-(3x\sqrt{x})^2}\right)$

$= 2\tan^{-1}(3x\sqrt{x})$

$\frac{dy}{dx} = \frac{2}{1+(3x\sqrt{x})^2} \cdot 3\left(x \cdot \frac{1}{2\sqrt{x}} + \sqrt{x} \cdot 1\right)$ (differentiating w.r.t. x)

$= \frac{6}{1+9x^3}\left(\frac{\sqrt{x}}{2} + \sqrt{x}\right)$

$= \frac{9\sqrt{x}}{1+9x^3}$

$= \sqrt{x} \cdot \frac{9}{1+9x^3}$

Q.72 The normal to the curve $y(x-2)(x-3) = x+6$ at the point where the curve intersects the y-axis passes through the point :

(1) $\left(\frac{1}{2}, \frac{1}{2}\right)$ (2) $\left(\frac{1}{2}, -\frac{1}{3}\right)$ (3) $\left(\frac{1}{2}, \frac{1}{3}\right)$ (4) $\left(-\frac{1}{2}, -\frac{1}{2}\right)$

Ans. **[1]**

Sol. $y(x-2)(x-3) = x+6$

at y axis, $x = 0$

$y(-2)(-3) = 0+6$

$y = 1$

Now $y(x^2-5x+6) = x+6$

$y = \frac{x+6}{x^2-5x+6}$

$\frac{dy}{dx} = \frac{(x^2-5x+6)\cdot 1-(x+6)(2x-5)}{(x^2-5x+6)^2}$

at $x = 0, y = 1$ $= \frac{6-(6)(-5)}{6^2} = 1$

equation of normal

$$y - 1 = -1(x - 0)$$

$$x + y = 1$$

passes $\left(\frac{1}{2}, \frac{1}{2}\right)$ (by option)

Q.73 Twenty meters of wire is available for fencing off a flower-bed in the form of a circular sector. Then the maximum area (in sq. m) of the flower-bed, is :

(1) 10 (2) 25 (3) 30 (4) 12.5

Students may find similar question in CP exercise sheet :

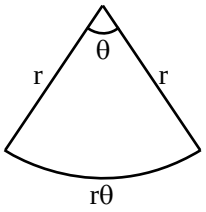
[JEE Main, Chapter :Maxima & Minima, Page 96, Ex. 2, Q. No. 13]

Ans. **[2]**

Sol. Given

$r + r + r\theta = 20$

$$\theta = \frac{20 - 2r}{r}$$



$$\text{Area} = \frac{1}{2}r^2\theta$$

$$= \frac{1}{2}r^2 \cdot \left(\frac{20 - 2r}{r}\right)$$

$$z = \frac{1}{2}(20r - 2r^2)$$

$$\frac{dz}{dr} = \frac{1}{2}(20 - 4r) = 0 \Rightarrow r = 5$$

at $r = 5$, $\theta = 2$, $\frac{d^2z}{dr^2} < 0$ (hence maxima)

maximum area

$$z = \frac{1}{2}r^2\theta$$

$$= \frac{1}{2} \times 5^2 \times 2 = 25m^2$$

Q.74 Let $I_n = \int \tan^n x\, dx, (n > 1)$. If $I_4 + I_6 = a \tan^5 x + bx^5 + C$, where C is a constant of integration, then the ordered pair (a, b) is equal to :

(1) $\left(\frac{1}{5}, 0\right)$ (2) $\left(\frac{1}{5}, -1\right)$ (3) $\left(-\frac{1}{5}, 0\right)$ (4) $\left(-\frac{1}{5}, 1\right)$

Students may find similar question in CP exercise sheet :
[JEE Main, Chapter : Indefinite Integration, Page 33, Ex. 3, Q. No. 2]
[JEE Advance, Chapter : Indefinite Integration, Page 35, Ex. 4, Q. No. 1]

Ans. **[1]**

Sol. $I_n = \int \tan^n x\, dx$

$I_4 + I_6 = \int (\tan^4 x + \tan^6 x)\, dx$

$= \int \tan^4 x (1 + \tan^2 x)\, dx$

$= \int \tan^4 x \cdot \sec^2 x\, dx$ put $t = \tan x$

$= \int t^4 \cdot dt$

$= \frac{t^5}{5} + C$

$= \frac{\tan^5 x}{5} + C$

On comparison, we get

$a = \frac{1}{5}, \quad b = 0$

Q.75 The integral $\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{dx}{1+\cos x}$ is equal to :

(1) 2 (2) 4 (3) – 1 (4) – 2

Students may find similar question in CP exercise sheet :
[JEE Main, Chapter :Definite Integration, Page 42, Ex. 5A, Q. No. 17]

Ans. **[1]**

Sol. $I = \int_{\pi/4}^{3\pi/4} \frac{dx}{1+\cos x}$

$I = \int_{\pi/4}^{3\pi/4} \frac{dx}{1+\cos(\pi - x)}$ by using $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$

$$2I = \int_{\pi/4}^{3\pi/4}\left(\frac{1}{1+\cos x}+\frac{1}{1-\cos x}\right)dx = \int_{\pi/4}^{3\pi/4}\left(\frac{2}{1-\cos^2 x}\right)dx$$

$$2I = 2\int_{\pi/4}^{3\pi/4}\text{cosec}^2 x\,dx$$

$$I = -(\cot x)_{\pi/4}^{3\pi/4}$$

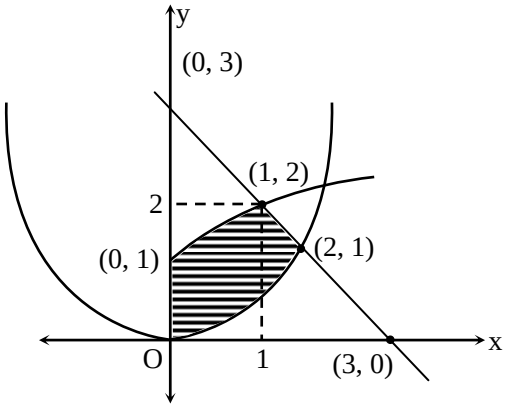
$$-(-1-1) = 2$$

Q.76 The area (in sq. units) of the region $\{(x, y) : x \ge 0, x + y \le 3, x^2 \le 4y \text{ and } y \le 1 + \sqrt{x}\}$ is :

(1) $\frac{3}{2}$ (2) $\frac{7}{3}$ (3) $\frac{5}{2}$ (4) $\frac{59}{12}$

Ans. **[3]**

Sol.



$$\text{Required Area} = \int_0^1\left(1+\sqrt{x}-\frac{x^2}{4}\right)dx + \int_1^2\left(3-x-\frac{x^2}{4}\right)dx$$

$$= \left[x+\frac{x^{3/2}}{3/2}-\frac{x^3}{12}\right]_0^1 + \left[3x-\frac{x^2}{2}-\frac{x^3}{12}\right]_1^2$$

$$= \frac{19}{12}+\frac{11}{12} = \frac{5}{2}$$

Q.77 If $(2 + \sin x)\frac{dy}{dx} + (y+1)\cos x = 0$ and $y(0) = 1$, then $y\left(\frac{\pi}{2}\right)$ is equal to :

(1) $-\frac{2}{3}$ (2) $-\frac{1}{3}$ (3) $\frac{4}{3}$ (4) $\frac{1}{3}$

Students may find similar question in CP exercise sheet :
[JEE Main, Chapter : Differential Equation, Page 70, Ex. 5A, Q. No. 6]

Ans. **[4]**

Sol. $(2+\sin x)\frac{dy}{dx}+(y+1)\cos x=0$

$$\frac{dy}{dx}=\frac{-(y+1)\cos x}{2+\sin x}$$

$$\Rightarrow \int\frac{dy}{y+1}=-\int\left(\frac{\cos x}{2+\sin x}\right)dx$$

$\Rightarrow \log(y+1) = -\log(2+\sin x) + \log c$

$$\Rightarrow y+1=\frac{c}{2+\sin x} \qquad \ldots(1)$$

Given that $y(0) = 1$

$$\therefore \quad 1+1=\frac{c}{2} \Rightarrow c=4$$

$\therefore$ Equation of curve

$$y+1=\frac{4}{2+\sin x}$$

at $x=\frac{\pi}{2} \Rightarrow y+1=\frac{4}{2+1}$

$$\Rightarrow y=\frac{4}{3}-1$$

$$y=\frac{1}{3}$$

Q.78 Let k be an integer such that the triangle with vertices (k, –3k), (5, k) and (–k, 2) has area 28 sq. units. Then the orthocentre of this triangle is at the point :

(1) $\left(1,\frac{3}{4}\right)$ (2) $\left(1,-\frac{3}{4}\right)$ (3) $\left(2,\frac{1}{2}\right)$ (4) $\left(2,-\frac{1}{2}\right)$

Ans. **[3]**

Sol. Area

$$\frac{1}{2}\begin{vmatrix} k & -3k & 1 \\ 5 & k & 1 \\ -k & 2 & 1 \end{vmatrix}=\pm 28$$

$k(k-2) + 3k(5+k) + 1(10+k^2) = \pm 56$

$k^2 - 2k + 15k + 3k^2 + 10 + k^2 = \pm 56$

$5k^2 + 13k + 10 = \pm 56$

$5k^2 + 13k + 66 = 0$

$D = 169 - 4 \times 5 \times 66 < 0$

No solution

$5k^2 + 13k - 46 = 0$

$5k^2 + 23k - 10k - 46 = 0$

$k(5k+23) - 2(5k+23) = 0$

$(5k+23)(k-2) = 0$

$k = 2$ (k is integer)

Hence co-ordinate

(2, –6) (5, 2) (–2, 2)

E(2, β)

AD ⊥ BC

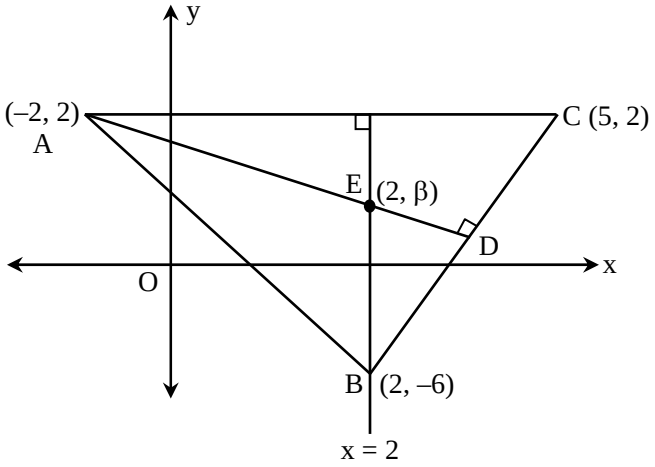
$\frac{\beta-2}{2+2}\times\frac{8}{3}=-1$

$\frac{\beta-2}{4}=\frac{-3}{8}$

$\beta-2=-\frac{3}{2}$

$\beta=2-\frac{3}{2}=\frac{1}{2}$

$\left(2,\frac{1}{2}\right)$



Q.79 The radius of a circle, having minimum area, which touches the curve $y = 4 - x^2$ and the lines, $y = |x|$ is :

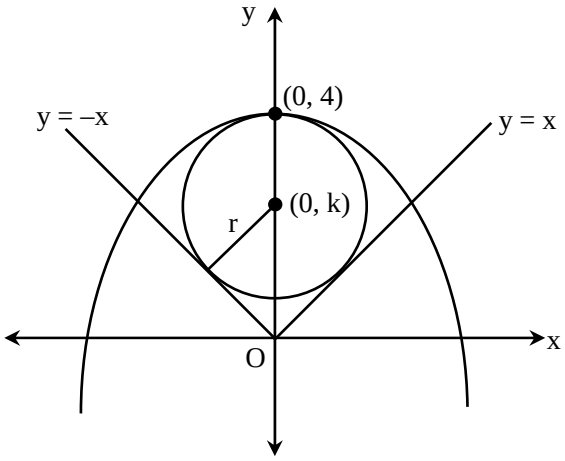
(1) $2(\sqrt{2}-1)$ (2) $4(\sqrt{2}-1)$ (3) $4(\sqrt{2}+1)$ (4) $2(\sqrt{2}+1)$

Students may find similar question in CP exercise sheet :

[JEE Advance, Chapter : Area under the curve, Ex. 3]

Ans. **[2]**

Sol.



Circle touches the line

By graph radius = 4 – k

Perpendicular distance from centre = radius

$\Rightarrow \quad 4-k=\left|\frac{0-k}{\sqrt{2}}\right|$

$\Rightarrow \quad 16+k^2-8k=\frac{k^2}{2}$

$\Rightarrow \quad k^2-16k+32=0$

$\Rightarrow \quad k=\frac{16\pm\sqrt{256-4(32)}}{2}$

$\Rightarrow \quad k=\frac{16\pm\sqrt{128}}{2}$

$\Rightarrow \quad k=\frac{16\pm8\sqrt{2}}{2}$

$\Rightarrow \quad k=8\pm4\sqrt{2}$

$\Rightarrow \quad k=8-4\sqrt{2}$ (k should be $0 < k < 4$)

Radius $= 4-k$

$= 4-(8-4\sqrt{2})$

$= 4(\sqrt{2}-1)$

Q.80 The eccentricity of an ellipse whose centre is at the origin is $\frac{1}{2}$. If one of its directrices is x = – 4, then the equation of the normal to it at $\left(1, \frac{3}{2}\right)$ is :

(1) $4x-2y=1$ (2) $4x+2y=7$ (3) $x+2y=4$ (4) $2y-x=2$

Students may find similar question in CP exercise sheet :
[JEE Main, Chapter : Ellipse, Page 58, Ex. 3, Q. No. 3]
[JEE Advance, Chapter : Ellipse, Page 24, Ex. 3, Q. No. 13]

Ans. **[1]**

Sol. Let the equation of ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

given that $e=\frac{1}{2}$

and directrix $\left\{\begin{array}{c} x=-a/e \\ \text{or} \\ x=-4 \end{array}\right\}$

$\Rightarrow \frac{a}{e} = 4$

$\Rightarrow a = 2$

Now, $b^2 = a^2 (1 - e^2)$

$$b^2 = 4\left(1 - \frac{1}{4}\right) = 3$$

Equation of ellipse

$$\frac{x^2}{4} + \frac{y^2}{3} = 1$$

diff. w.r.t. x

$$\frac{2x}{4} + \frac{2y}{3}\frac{dy}{dx} = 0$$

$$\left(\frac{dy}{dx}\right)_{(1, 3/2)} = -\frac{1}{2}$$

$\therefore$ Equation of normal at $\left(1, \frac{3}{2}\right)$ is

$$y - y_1 = -\frac{1}{\left(\frac{dy}{dx}\right)}(x - x_1)$$

$\Rightarrow \quad y - \frac{3}{2} = 2(x - 1)$

$\Rightarrow \quad 2y - 3 = 4x - 4$

$\Rightarrow \quad 4x - 2y = 1$

Q.81 A hyperbola passes through the point P($\sqrt{2}$, $\sqrt{3}$) and has foci at (±2, 0). Then the tangent to this hyperbola at P also passes through the point :

(1) ($2\sqrt{2}$, $3\sqrt{3}$) (2) ($\sqrt{3}$, $\sqrt{2}$) (3) ($-\sqrt{2}$, $-\sqrt{3}$) (4) ($3\sqrt{2}$, $2\sqrt{3}$)

Ans. **[1]**

Sol. Let the equation of hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

given that foci (±ae, 0) = (±2, 0)

$\Rightarrow \quad ae = 2$

Hyperbola passes through P($\sqrt{2}$, $\sqrt{3}$)

$\therefore \frac{2}{a^2} - \frac{3}{b^2} = 1$

$\Rightarrow \frac{2}{a^2} - \frac{3}{a^2(e^2 - 1)} = 1$

$\Rightarrow \frac{2}{a^2} - \frac{3}{4-a^2} = 1$

$\Rightarrow 8 - 2a^2 - 3a^2 = 4a^2 - a^4$

$\Rightarrow a^4 - 9a^2 + 8 = 0$

$\Rightarrow (a^2 - 8)(a^2 - 1) = 0$

$\Rightarrow a^2 = 8$ and $a^2 = 1$

$b^2 = a^2(e^2 - 1)$	$b^2 = a^2(e^2 - 1)$
$b^2 = 4 - 8$	$b^2 = 4 - 1 = 3$
$b^2 = -4$ (not possible)	$\therefore$ Equation of hyperbola
	$\frac{x^2}{1} - \frac{y^2}{3} = 1$
	tangent at P($\sqrt{2}$, $\sqrt{3}$)
	T = 0
	$\sqrt{2}x - \frac{y}{\sqrt{3}} = 1$

By option it passes through ($2\sqrt{2}$, $3\sqrt{3}$)

Q.82 The distance of the point (1, 3, –7) from the plane passing through the point (1, –1, –1), having normal perpendicular to both the lines $\frac{x-1}{1} = \frac{y+2}{-2} = \frac{z-4}{3}$ and $\frac{x-2}{2} = \frac{y+1}{-1} = \frac{z+7}{-1}$, is :

(1) $\frac{10}{\sqrt{83}}$ (2) $\frac{5}{\sqrt{83}}$ (3) $\frac{10}{\sqrt{74}}$ (4) $\frac{20}{\sqrt{74}}$

Students may find similar question in CP exercise sheet :

[JEE Advance, Chapter : 3D, Page 110, Ex. 2, Q. No. 17]

Ans. **[1]**

Sol. $\hat{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 3 \\ 2 & -1 & -1 \end{vmatrix} = 5\hat{i} - \hat{j}(-7) + 3\hat{k}$

Equation of plane $5(x - 1) + 7(y + 1) + 3(z + 1) = 0$

$5x + 7y + 3z + 5 = 0$

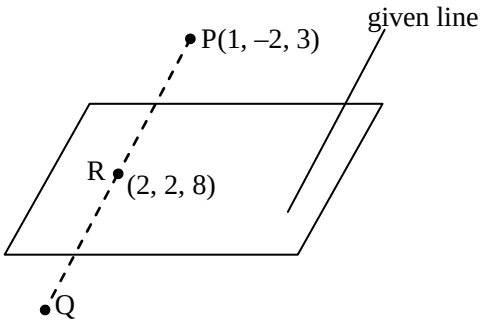
Perpendicular distance of the plane from (1, 3, –7) is = $\frac{|5 + 21 - 21 + 5|}{\sqrt{25 + 49 + 9}} = \frac{10}{\sqrt{83}}$

Q.83 If the image of the point P(1, –2, 3) in the plane, $2x + 3y - 4z + 22 = 0$ measured parallel to the line, $\frac{x}{1} = \frac{y}{4} = \frac{z}{5}$ is Q, then PQ is equal to :

(1) $2\sqrt{42}$ (2) $\sqrt{42}$ (3) $6\sqrt{5}$ (4) $3\sqrt{5}$

Ans. **[1]**

Sol.



The equation of line PR

$\frac{x-1}{1} = \frac{y+2}{4} = \frac{z-3}{5} = k$

let R(k + 1, 4k – 2, 5k + 3)

it lies on the plane $2x + 3y - 4z + 22 = 0$

$\therefore 2(k + 1) + 3(4k – 2) – 4(5k + 3) + 22 = 0$

$\Rightarrow – 6k + 6 = 0$

$\Rightarrow k = 1$

$\therefore$ R(2, 2, 8)

Image of P in the plane is (R is the mid-point of PQ)

$\therefore$ Q(3, 6, 13)

$PQ = \sqrt{4 + 64 + 100} = \sqrt{168} = 2\sqrt{42}$

Q.84 Let $\vec{a} = 2\hat{i} + \hat{j} - 2\hat{k}$ and $\vec{b} = \hat{i} + \hat{j}$. Let $\vec{c}$ be a vector such that $|\vec{c} - \vec{a}| = 3$, $|(\vec{a} \times \vec{b}) \times \vec{c}| = 3$ and the angle between $\vec{c}$ and $\vec{a} \times \vec{b}$ be 30º. Then $\vec{a} \cdot \vec{c}$ is equal to :

(1) 2 (2) 5 (3) $\frac{1}{8}$ (4) $\frac{25}{8}$

Students may find similar question in CP exercise sheet :
[JEE Main, Chapter : Vector, Page 55, Ex. 5A, Q. No. 10]
[JEE Advance, Chapter : Vector, Page 38, Ex. 4, Q. No. 35]

Ans. **[1]**

Sol. $|\vec{a} \times \vec{b}| = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -2 \\ 1 & 1 & 0 \end{vmatrix}$

$|\vec{a} \times \vec{b}| = 2\hat{i} - 2\hat{j} + \hat{k}$

$|\vec{a} \times \vec{b}| = 3$

given $|(\vec{a} \times \vec{b}) \times \vec{c}| = 3$

$|\vec{a} \times \vec{b}||\vec{c}| \sin 30^o = 3$

$\Rightarrow \quad 3|\vec{c}| \cdot \frac{1}{2} = 3$

$\Rightarrow \quad |\vec{c}| = 2$

Now $|\vec{c} - \vec{a}| = 3$

$|\vec{c}|^2 + |\vec{a}|^2 - 2\vec{a} \cdot \vec{c} = 9$

$\Rightarrow \quad 4 + 9 - 2\vec{a} \cdot \vec{c} = 9$

$\Rightarrow \quad \vec{a} \cdot \vec{c} = 2$

Q.85 A box contains 15 green and 10 yellow balls. If 10 balls are randomly drawn, one-by-one, with replacement, then the variance of the number of green balls drawn is :

(1) 6 (2) 4 (3) $\frac{6}{25}$ (4) $\frac{12}{5}$

Ans. **[4]**

Sol. Total no. of balls = 25

(15 green and 10 yellow balls)

(Varience) $\sigma^2 = npq$

where $n \rightarrow$ No. of Trial

$p \rightarrow$ Probability of happening of that event

$q \rightarrow$ Probability of not happening of that event

$n = 10, p = \frac{15}{25} = \frac{3}{5}, q = \frac{10}{25} = \frac{2}{5}$

So, $\sigma^2 = 10 \times \frac{3}{5} \times \frac{2}{5}$

$\sigma^2 = \frac{60}{25} = \frac{12}{5}$

Q.86 For three events A, B and C,

P (Exactly one of A or B occurs) = P(Exactly one of B or C occurs) = P(Exactly one of C or A occurs) = $\frac{1}{4}$

and P(All the three events occur simultaneously) = $\frac{1}{16}$.

Then the probability that at least one of the events occurs, is :

(1) $\frac{7}{16}$ (2) $\frac{7}{64}$ (3) $\frac{3}{16}$ (4) $\frac{7}{32}$

Students may find similar question in CP exercise sheet :
[JEE Main, Chapter : Probability, Page 35, Ex. 3, Q. No. 6]

Ans. **[1]**

Sol. P(Exactly one of A or B occurs) = P(A) + P(B) – 2P(A ∩ B) = $\frac{1}{4}$ ... (1)

P(Exactly one of B or C occurs) = P(B) + P(C) – 2P(B ∩ C) = $\frac{1}{4}$ ... (2)

P(Exactly one of C or A occurs) = P(C) + P(A) – 2P(C ∩ A) = $\frac{1}{4}$ ... (3)

Adding (1), (2) and (3)

$2[P(A) + P(B) + P(C) – P(A \cap B) – P(B \cap C) – P(C \cap A)] = \frac{3}{4}$

$P(A) + P(B) + P(C) – P(A \cap B) – P(B \cap C) – P(C \cap A) = \frac{3}{8}$

$P(\text{All the three events occurs simultaneously}) = P(A \cap B \cap C) = \frac{1}{16}$

$P(\text{Atleast one of the events occurs}) = P(A \cup B \cup C) = \frac{3}{8} + \frac{1}{16} = \frac{7}{16}$

Q.87 If two different numbers are taken from the set {0, 1, 2, 3, ……, 10}, then the probability that their sum as well as absolute difference are both multiple of 4, is :

(1) $\frac{12}{55}$ (2) $\frac{14}{45}$ (3) $\frac{7}{55}$ (4) $\frac{6}{55}$

Ans. **[4]**

Sol. $n(S) = {}^{11}C_2 = 55$

Favorable events $\begin{cases} (0, 4) \\ (0, 8) \\ (2, 6), (2, 10) \\ (4, 8), (6, 10) \end{cases}$

So, required probability = $\frac{\text{Fav. Events}}{\text{Total Events}} = \frac{6}{55}$

Q.88 If $5(\tan^2x - \cos^2x) = 2\cos2x + 9$, then the value of $\cos4x$ is :

(1) $\frac{1}{3}$ (2) $\frac{2}{9}$ (3) $-\frac{7}{9}$ (4) $-\frac{3}{5}$

Ans. **[3]**

Sol. $5(\tan^2x - \cos^2x) = 2\cos2x + 9$

$\Rightarrow 5\left[\frac{\sin^2 x}{\cos^2 x} - \cos^2 x\right] = 2(2\cos^2x - 1) + 9$

$\Rightarrow 5[(1 - \cos^2x) - \cos^4x] = 4\cos^4x - 2\cos^2x + 9\cos^2x$

$\Rightarrow 9\cos^4x + 12\cos^2x - 5 = 0$

$\Rightarrow 9\cos^4x + 15\cos^2x - 3\cos^2x - 5 = 0$

$\Rightarrow 3\cos^2x\,(3\cos^2x + 5) - (3\cos^2x + 5) = 0$

$\Rightarrow \cos^2x = \frac{1}{3}$

$\Rightarrow \cos2x = 2\cos^2x - 1$

$\Rightarrow \cos2x = \frac{2}{3} - 1 = \frac{-1}{3}$

Now $\cos4x = 2\cos^2 2x - 1$

$\cos4x = 2\left(\frac{1}{9}\right) - 1$

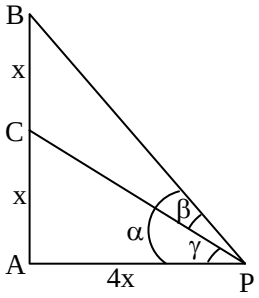
$\cos4x = \frac{-7}{9}$

Q.89 Let a vertical tower AB have its end A on the level ground. Let C be the mid-point of AB and P be a point on the ground such that AP = 2AB. If $\angle BPC = \beta$, then $\tan\beta$ is equal to :

(1) $\frac{1}{4}$ (2) $\frac{2}{9}$ (3) $\frac{4}{9}$ (4) $\frac{6}{7}$

Ans. **[2]**

Sol.



$\beta = \alpha - \gamma$

$\tan\beta = \frac{\tan\alpha - \tan\gamma}{1 + \tan\alpha.\tan\gamma} = \frac{\frac{1}{2} - \frac{1}{4}}{1 + \frac{1}{8}} = \frac{2}{9}$

Q.90 The following statement $(p \rightarrow q) \rightarrow [(\sim p \rightarrow q) \rightarrow q]$ is :

(1) equivalent to $\sim p \rightarrow q$ (2) equivalent to $p \rightarrow \sim q$ (3) a fallacy (4) a tautology

Ans. **[4]**

Sol.

q	p	~p	$(p \rightarrow q)$	$(\sim p \rightarrow q)$	$(\sim p \rightarrow q) \rightarrow q$	$(p \rightarrow q) \rightarrow [(\sim p \rightarrow q) \rightarrow q]$
T	T	F	T	T	T	T
F	T	F	F	T	F	T
T	F	T	T	T	T	T
F	F	T	T	F	T	T

It is a tautology.