JEE Main Online Exam 2024

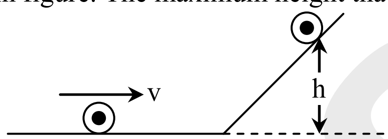
Questions & Solution

01st February 2024 | Evening

PHYSICS

Section-A: This section contains 20 multiple choice questions. Each question has 4 choices(1), (2), (3) and (4), out of which **ONLY ONE** is correct..

Q.1 A disc of radius R and mass M is rolling horizontally without slipping with speed v. It then moves up an inclined smooth surface as shown in figure. The maximum height that the disc can go up the incline is :



(1) $\frac{2}{3}\frac{v^2}{g}$ (2) $\frac{v^2}{g}$ (3) $\frac{3}{4}\frac{v^2}{g}$ (4) $\frac{1}{2}\frac{v^2}{g}$

Ans. **[4]**

Sol. $\frac{1}{2}Mv^2 = Mgh$ (No change in $KE_{rotational}$)

$$h = \frac{1}{2}\frac{v^2}{g}$$

Q.2 A light planet is revolving around a massive star in a circular orbit of radius R with a period of revolution T. If the force of attraction between planet and star is proportional to $R^{-3/2}$ then choose the correct option :

(1) $T^2 \propto R^{5/2}$ (2) $T^2 \propto R^3$ (3) $T^2 \propto R^{3/2}$ (4) $T^2 \propto R^{7/2}$

Ans. **[1]**

Sol. $F = \frac{mv^2}{R} \Rightarrow \frac{K}{R^{3/2}} = \frac{mv^2}{R} \Rightarrow v = \frac{K'}{R^{3/4}}$

$$T = \frac{2\pi R}{V} = \frac{2\pi R}{\frac{K}{R^{1/4}}} = \frac{2\pi}{K}R^{5/4}$$

$$T \propto R^{5/4} \Rightarrow T^2 \propto R^{5/2}$$

Q.3 Monochromatic light of frequency 6×10^{14} Hz is produced by a laser. The power emitted is 2×10^{-3} W. How many photons per second on an average, are emitted by the source? (Given $h = 6.63 \times 10^{-34}$ Js)

(1) 6×10^{15} (2) 5×10^{15} (3) 7×10^{16} (4) 9×10^{18}

Ans. **[2]**

Sol. $n = \frac{\text{Power}}{\text{Energy of one photon}} = \frac{P}{h\nu}$

$$n = \frac{2 \times 10^{-3}}{6.63 \times 10^{-34} \times 6 \times 10^{14}} = 5 \times 10^{15}$$

Q.4 Conductivity of a photodiode starts changing only if the wavelength of incident light is less than 660 nm. The band gap of photodiode is found to be $\left(\frac{X}{8}\right)$ eV. The value of X is

(Given, $h = 6.6 \times 10^{-34}$ Js, $e = 1.6 \times 10^{-19}$ C)

(1) 13 (2) 11 (3) 21 (4) 15

Ans. **[4]**

Sol. $E = \frac{hc}{\lambda} \Rightarrow \left(\frac{X}{8}\right) eV = \left(\frac{1240}{660}\right) eV$

$X = \frac{124 \times 8}{66} = 15$

Q.5 A diatomic gas ($\gamma = 1.4$) does 200 J of work when it is expanded isobarically. The heat given to the gas in the process is :

(1) 800 J (2) 600 J (3) 700 J (4) 850 J

Ans. **[3]**

Sol. $W = nR\Delta T \Rightarrow \Delta T = \frac{200}{nR}$

and $Q = U + W = \frac{F}{2} nR\Delta T + nR\Delta T = \left(\frac{F}{2}+1\right)nR\Delta T$

$\Rightarrow Q = \left(\frac{5}{2}+1\right) \times nR \times \frac{200}{nR} = 700$ J

Q.6 A big drop is formed by coalescing 1000 small droplets of water. The surface energy will become :

(1) 100 times (2) $\frac{1}{100}$th (3) $\frac{1}{10}$th (4) 10 times

Ans. **[3]**

Sol. $\therefore \frac{4}{3}\pi r^3 \times 1000 = \frac{4}{3}\pi R^3 \Rightarrow R = 10r$

Initial surface energy = $k.\pi r^2 \times 1000$

Final surface energy = $k.\pi(10r)^2$

$E_f = \frac{1}{10} E_i$

Q.7 A cricket player catches a ball of mass 120 g moving with 25 m/s speed. If the catching process is completed in 0.1 s, then the magnitude of force exerted by the ball on the hand of player will be (in SI unit)

(1) 24 (2) 25 (3) 30 (4) 12

Ans. **[3]**

Sol. Force exerted = $\frac{\Delta P}{\Delta t} = \frac{0.12 \times 25 - 0}{0.10}$

$= \frac{12 \times 25}{10}$ N

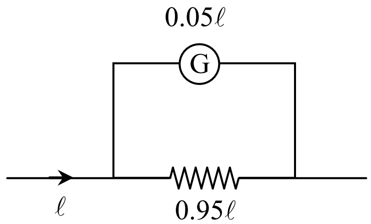
$= 30$N

Q.8 In an ammeter, 5% of the main current passes through the galvanometer. If resistance of the galvanometer is G, the resistance of ammeter will be :

(1) 200 G (2) $\frac{G}{199}$ (3) 199 G (4) $\frac{G}{200}$

Ans. **[None of these]**

Sol.



0.05I G = 0.95Ir

$r = \frac{G}{19}$

$R_A = G11\ \frac{G}{19} = \frac{G}{20}$

Q.9 A microwave of wavelength 2.0 cm falls normally on a slit of width 4.0 cm. The angular spread of the central maxima of the diffraction pattern obtained on a screen 1.5 m away from the slit, will be :

(1) 45º (2) 15º (3) 60º (4) 30º

Ans. **[3]**

Sol. Half angular spread : θ ; dsinθ = λ

$\sin\theta = \frac{\lambda}{d} = \frac{2}{4} = \frac{1}{2} \Rightarrow \theta = 30^{o}$

Angular spread = 2θ = 60°

Q.10 From the statements given below :

(A) The angular momentum of an electron in n^{th} orbit is an integral multiple of $\hbar$.

(B) Nuclear forces do not obey inverse square law.

(C) Nuclear forces are spin dependent.

(D) Nuclear forces are central and charge independent.

(E) Stability of nucleus is inversely proportional to the value of packing fraction.

Choose the correct answer from the options given below :

(1) (A), (B), (C), (D) only (2) (B), (C), (D), (E) only

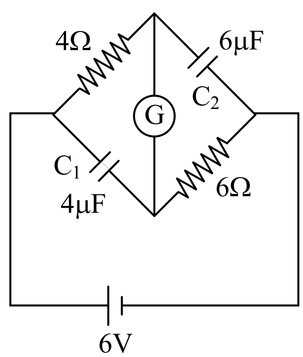
(3) (A), (C), (D), (E) only (4) (A), (B), (C), (E) only

Ans. **[4]**

Sol. (D) is incorrect. Rest all are correct.

Nuclear forces are non-central forces.

Q.11 A galvanometer (G) of 2 Ω resistance is connected in the given circuit. The ratio of charge stored in C_1 and C_2 is :



(1) 1 (2) $\frac{3}{2}$ (3) $\frac{2}{3}$ (4) $\frac{1}{2}$

Ans. **[4]**

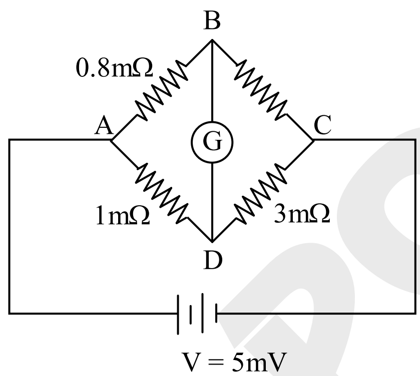
Sol. $i_{steady} = \frac{6}{12} = \frac{1}{2}$ A

$q_1 = C_1 \times \Delta V_1 = 6 \times 4 \times 3 = 12\ \mu C$

$q_2 = C_2 \times \Delta V_2 = 6 \times 4 = 24\ \mu C$

$\frac{q_1}{q_2} = \frac{1}{2}$

Q.12 To measure the temperature coefficient of resistivity α of a semiconductor, an electrical arrangement shown in the figure is prepared. The arm BC is made up of the semiconductor. The experiment is being conducted at 25°C and resistance of the semiconductor arm is 3 mΩ. Arm BC is cooled at a constant rate of 2°C/s. If the galvanometer G shows no deflection after 10 s, then α is :



(1) -1×10^{-2} °C^{-1} (2) -2.5×10^{-2} °C^{-1} (3) -1.5×10^{-2} °C^{-1} (4) -2×10^{-2} °C^{-1}

Ans. **[1]**

Sol. ∴ Wheatstone bridge : $0.8 \times 3 = 1 \times R$

$R = 2.4$ mr Ω

$R = R_0 (1 + \alpha\, \Delta T) \Rightarrow 2.4 = 3 (1 + 20\, \alpha)$

$\frac{2.4}{3} - 1 = 20\alpha \Rightarrow -\frac{0.6}{3 \times 20} = \alpha = -1 \times 10^{-2}$ °C^{-1}

Q.13 Train A is moving along two parallel rail tracks towards north with speed 72 km/h and train B is moving towards south with speed 108 km/h. Velocity of train B with respect to A and velocity of ground with respect to B are (in ms^{-1})

(1) –50 and –30 (2) 50 and –30 (3) –50 and 30 (4) –30 and 50

Ans. **[3]**

Sol. $\vec{V}_{BA} = \vec{V}_B - \vec{V}_A = -30 - 20 = -50$ m/s

$\vec{V}_{GB} = \vec{V}_G - \vec{V}_B = 0 - (-30) = 30$ m/s

Q.14 If frequency of electromagnetic wave is 60 MHz and it travels in air along z direction then the corresponding electric and magnetic field vectors will be mutually perpendicular to each other and the wavelength of the wave (in m) is :

(1) 2 (2) 2.5 (3) 10 (4) 5

Ans. **[4]**

Sol. $\lambda = \frac{c}{f} = \frac{3 \times 10^8}{60 \times 10^6} = \frac{30 \times 10^7}{6 \times 10^7}$

$= 5$ m

Q.15 A body of mass 4 kg experiences two forces

$\vec{F}_1 = 5\hat{i} + 8\hat{j} + 7\hat{k}$ and $\vec{F}_2 = 3\hat{i} - 4\hat{j} - 3\hat{k}$.

The acceleration acting on the body is :

(1) $-2\hat{i} - \hat{j} - \hat{k}$ (2) $2\hat{i} + \hat{j} + \hat{k}$ (3) $4\hat{i} + 2\hat{j} + 2\hat{k}$ (4) $2\hat{i} + 3\hat{j} + 3\hat{k}$

Ans. **[2]**

Sol. $$\vec{a} = \frac{\vec{F}_{net}}{m} = \frac{(5\hat{i}+8\hat{j}+7\hat{k})+(3\hat{i}-4\hat{j}-3\hat{k})}{4}$$

$$= 2\hat{i}+\hat{j}+\hat{k}$$

Q.16 Match List-I with List-II. :

List-I (Number)		List-II (Significant figure)	
(A)	1001	(I)	3
(B)	010.1	(II)	4
(C)	100.100	(III)	5
(D)	0.0010010	(IV)	6

Choose the correct answer from the options given below :

(1) (A)-(II), (B)-(I), (C)-(IV), (D)-(III) (2) (A)-(I), (B)-(II), (C)-(III), (D)-(IV)

(3) (A)-(III), (B)-(IV), (C)-(II), (D)-(I) (4) (A)-(IV), (B)-(III), (C)-(I), (D)-(II)

Ans. **[1]**

Sol. Significant figures are as

∴ 1001 : 4 ; 010.1 : 3; 100.100 : 6;
0.0010010 : 5
(Significant figures)

Q.17 If the root mean square velocity of hydrogen molecule at a given temperature and pressure is 2 km/s, the root mean square velocity of oxygen at the same condition in km/s is :

(1) 1.5 (2) 1.0 (3) 0.5 (4) 2.0

Ans. **[3]**

Sol. $$V_{RMS} \propto \frac{1}{\sqrt{M}} \Rightarrow \frac{V_{O_2}}{V_{H_2}} = \sqrt{\frac{M_{H_2}}{M_{O_2}}} \Rightarrow \frac{V_{O_2}}{2} = \sqrt{\frac{2}{32}}$$

$$\Rightarrow V_{O_2} = \frac{2}{4} = 0.5 \text{ km/s}$$

Q.18 In a metre-bridge when a resistance in the left gap is 2Ω and unknown resistance in the right gap, the balance length is found to be 40 cm. On shunting the unknown resistance with 2 Ω, the balance length changes by :

(1) 22.5 cm (2) 20 cm (3) 65 cm (4) 62.5 cm

Ans. **[1]**

Sol. ∴ Value of R : $\frac{2}{R} = \frac{40}{60} \Rightarrow R = 3\Omega$

After shunting : $\frac{2}{6/5} = \frac{\ell}{100-\ell} \Rightarrow \ell = 62.5$ cm

Shifting of length = 62.5 – 40 = 22.5 cm

Q.19 A transformer has an efficiency of 80% and works at 10 V and 4 kW. If the secondary voltage is 240 V, then the current in the secondary coil is :

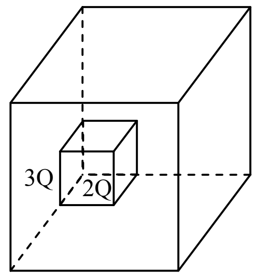
(1) 1.59 A (2) 15.1 A (3) 13.33 A (4) 1.33 A

Ans. **[3]**

Sol. $\eta = \frac{P_{output}}{P_{input}} = \frac{E_s.I_s}{P_{input}} \Rightarrow \frac{80}{100} = \frac{240 \times i_S}{4000}$

$i_S = \frac{40}{3}$ A = 13.33 A

Q.20 C_1 and C_2 are two hollow concentric cubes enclosing charges 2Q and 3Q respectively as shown in figure. The ratio of electric flux passing through C_1 and C_2 is :



(1) 2 : 3 (2) 5 : 2 (3) 2 : 5 (4) 3 : 2

Ans. **[3]**

Sol. $\varphi_{C_1} = \frac{2Q}{\varepsilon_0}$, $\varphi_{C_2} = \frac{2Q + 3Q}{\varepsilon_0}$

$\frac{\varphi_{C_1}}{\varphi_{C_2}} = \frac{2}{5}$; $\varphi_{C_1} : \varphi_{C_2} = 2 : 5$

Section-B: Numerical Value Type Questions: This section contains 10 Numerical based questions. Attempt any 5 questions out of 10. The answer to each question should be rounded-off to the nearest integer.

Q.21 A uniform rod AB of mass 2 kg and length 30 cm at rest on a smooth horizontal surface. An impulse of force 0.2 Ns is applied to end B. The time taken by the rod to turn through at right angles will be $\frac{\pi}{x}$ s, where x = _______.

Ans. **[4]**

Sol. Angular impulse $= I \cdot \frac{\ell}{2} = \frac{m\ell^2}{12} \cdot \omega$

$\Rightarrow \omega = \frac{6\ell}{m\ell}$

$\omega = 2$ rad/s; $\because \theta = \omega t$

$\Rightarrow \frac{\pi}{2} = 2 \cdot t$

$\Rightarrow t = \frac{\pi}{4}$

$\Rightarrow x = 4$

Q.22 Suppose a uniformly charged wall provides a uniform electric field of 2×10^4 N/C normally. A charged particle of mass 2 g being suspended through a silk thread of length 20 cm and remain stayed at a distance of 10 cm from the wall. Then the charge on the particle will be $\frac{1}{\sqrt{x}}\mu C$ where x = ……… . [Use g = 10 m/s^2]

Ans. **[3]**

Sol. $\sin\theta = \frac{1}{2}$

$\Rightarrow \tan\theta = \frac{1}{\sqrt{3}}$

$$\tan\theta = \frac{qE}{mg} = \frac{1}{\sqrt{3}} = \frac{1\times 2\times 10^4 \times 10^{-6}}{\sqrt{x}\times 20\times 10^{-3}} \Rightarrow x = 3$$

Q.23 One end of a metal wire is fixed to a ceiling and a load of 2 kg hangs from the other end. A similar wire is attached to the bottom of the load and another load of 1 kg hangs from this lower wire. Then the ratio of longitudinal stain of upper wire to that of the lower wire will be _______.
[Area of cross section of wire = 0.005 cm^2, Y = 2 × 10^{11} Nm^{-2} and g = 10 ms^{-2}]

Ans. **[3]**

Sol. Strain $= \frac{\text{Stress}}{Y} = \frac{F}{AY}$

$\Rightarrow$ Stress $\propto$ F

$$\frac{(\text{Stress})_U}{(\text{Stress})_L} = \frac{F_U}{F_L} = \frac{30}{10} = 3$$

Q.24 A particle initially at rest starts moving from reference point x = 0 along x-axis, with velocity v that varies as v= $4\sqrt{x}$ m/s. The acceleration of the particle is _______ ms^{-2}.

Ans. **[8]**

Sol. $a = v \cdot \frac{dv}{dx}$

$\Rightarrow a = 4\sqrt{x} \cdot \frac{4}{2\sqrt{x}}$

$\Rightarrow a = 8 \text{ m/s}^2$

Q.25 A mass m is suspended from a spring of negligible mass and the system oscillates with a frequency f_1.
The frequency of oscillations if a mass 9m is suspended from the same spring is f_2. The value of $\frac{f_1}{f_2}$ is _______.

Ans. **[3]**

Sol. $f = \frac{1}{2\pi}\sqrt{\frac{K}{m}}$

$\Rightarrow f \propto \frac{1}{\sqrt{m}}$

$$\frac{f_1}{f_2} = \sqrt{\frac{m_2}{m_1}} = \sqrt{\frac{9m}{m}} = 3$$

Q.26 In Young's double slit experiment, monochromatic light of wavelength 5000 Å is used. The slits are 1.0 mm apart and screen is placed at 1.0 m away from slits. The distance from the centre of the screen where intensity becomes half of the maximum intensity for the first time is __________ × 10^{-6} m.

Ans. **[125]**

Sol. $2I_0 = 4I_0 \cos^2\left(\frac{\Delta\varphi}{2}\right) \Rightarrow \cos\frac{\Delta\varphi}{2} = \frac{1}{\sqrt{2}} \Rightarrow \phi = \frac{\pi}{2}$

$$y = \frac{\Delta x . D}{d} = \frac{\lambda\varphi . D}{2\pi d} = \frac{\lambda D}{4d} = \frac{5\times 10^{-7}\times 1}{4\times 10^{-3}}$$

$y = 125 \times 10^{-6}$

Q.27 A particular hydrogen-like ion emits the radiation of frequency 3×10^{15} Hz when it makes transition from n = 2 to n = 1. The frequency of radiation emitted in transition from n = 3 to n = 1 is $\frac{x}{9} \times 10^{15}$ Hz, when x = _______.

Ans. **[32]**

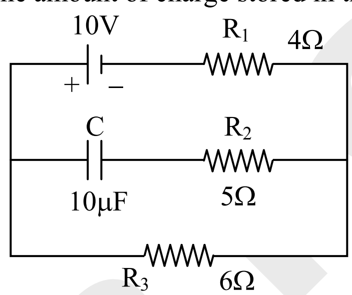
Sol. $\frac{1}{\lambda_1} = \frac{F_1}{C} = RZ^2\left(\frac{1}{1} - \frac{1}{4}\right) = \frac{3}{4} RZ^2 \Rightarrow F_1 = 3 \times 10^{15}$ Hz

$$\frac{3}{4} RZ^2 \cdot C = 3 \times 10^{15}$$

$\frac{1}{\lambda_2} = \frac{F_2}{C} = RZ^2\left(\frac{1}{1} - \frac{1}{9}\right) = \frac{8}{9} RZ^2 \Rightarrow F_2 = \frac{8}{9} RZ^2 \cdot C$

$F_2 = \frac{8}{9} \times \frac{4}{3} \times 3 \times 10^{15} = \frac{32}{9} \times 10^{15}$ Hz

Q.28 In an electrical circuit drawn below the amount of charge stored in the capacitor is _______ μC.



Ans. **[60]**

Sol. $i_{steady} = \frac{10}{10} = 1A$

$q = C.V_{Steady} = (10 \times 6)\ \mu C = 60\ \mu C$

Q.29 A moving coil galvanometer has 100 turns and each turn has an area of 2.0 cm^2. The magnetic field produced by the magnet is 0.01 T and deflection in the coil is 0.05 radian when a current of 10 mA is passed through it. The torsional constant of the suspension wire is $x \times 10^{-5}$ N-m/rad. The value of x is _______.

Ans. **[4]**

Sol. $i = \frac{C\theta}{NBA} \Rightarrow C = \frac{NBAi}{\theta} = \frac{100 \times 0.01 \times 2 \times 10^{-4} \times 10^{-2}}{0.05}$

$C = \frac{2 \times 10^{-6}}{50 \times 10^{-3}} = 4 \times 10^{-5}$ N-m/rad

$\Rightarrow \boxed{x = 4}$

Q.30 A coil of 200 turns and area 0.20 m^2 is rotated at half a revolution per second and is placed in uniform magnetic field of 0.01 T perpendicular to axis of rotation of the coil. The maximum voltage generated in the coil is $\frac{2\pi}{\beta}$ volt. The value of β is _______.

Ans. **[5]**

Sol. $T = 2$ sec $\Rightarrow \omega = \pi$ rad/s

$\phi = NBA \cos \omega t \Rightarrow \varepsilon = NBA\ \omega \sin \omega t$

$\varepsilon_0 = NBA\ \omega = 200 \times 0.01 \times 0.2 \times \pi = 0.4\pi\ \frac{2\pi}{\beta}$

$\boxed{\beta = 5}$

CHEMISTRY

Section-A: This section contains 20 multiple choice questions. Each question has 4 choices(1), (2), (3) and (4), out of which **ONLY ONE** is correct..

Q.31 Lassaigne's test is used for detection of :
(1) Nitrogen, Sulphur and Phosphorous only
(2) Nitrogen, Sulphur, Phosphorous and halogens
(3) Phosphorous and halogens only
(4) Nitrogen and Sulphur only

Ans. **[2]**

Sol. Nitrogen, sulphur, halogens and phosphorus present in an organic compound are detected by Lassaigne's test.

Q.32 Given below are two statements :
Statement (I): SiO_2 and GeO_2 are acidic while SnO and PbO are amphoteric in nature.
Statement (II): Allotropic forms of carbon are due to property of catenation and pπ-dπ bond formation.
In the light of the above statements, choose the **most appropriate** answer from the options given below :
(1) Both **statement I** and **statement II** are false
(2) Both **statement I** and **statement II** are true
(3) **Statement I** is false but **statement II** is true
(4) **Statement I** is true but **statement II** is false

Ans. **[4]**

Sol. SnO, PbO, SnO_2, PbO_2 : Amphoteric
CO_2, SiO_2, GeO_2, GeO : Acidic
CO : Neutral
pπ-dπ bonding is not possible between two carbon atoms hence **Statement I** is true but **Statement II** is false.

Q.33 The set of meta directing functional groups from the following sets is:
(1) $-NO_2$, $-NH_2$, –COOH, –COOR
(2) $-NO_2$, –CHO, $-SO_3H$, –COR
(3) –CN, $-NH_2$, –NHR, $-OCH_3$
(4) –CN, –CHO, $-NHCOCH_3$, –COOR

Ans. **[2]**

Sol. $-NO_2$, –CHO, $-SO_3H$, –COR, –CN, –COOR are having –R effect hence meta directing for incoming electrophilic $-NH_2$, $-NHCOCH_3$ are having +R effect.

Q.34 Given below are two statements: one is labeled as **Assertion (A)** and the other is labelled as **Reason (R)**.
Assertion (A): In aqueous solutions Cr^{2+} is reducing while Mn^{3+} is oxidising in nature.
Reason (R): Extra stability to half filled electronic configuration is observed than incompletely filled electronic configuration.
In the light of the above statements, choose the **most appropriate** answer from the options given below :
(1) Both **(A)** and **(R)** are true and **(R)** is the correct explanation of **(A)**.
(2) **(A)** is true but **(R)** is false.
(3) Both **(A)** and **(R)** are true but **(R)** is not the correct explanation of **(A)**.
(4) **(A)** is false but **(R)** is true.

Ans. **[1]**

Sol. Cr^{2+} and Mn^{3+} both has $3d^4 4s^0$ configuration.
Cr^{3+} is reducing as its configuration changes from
d^4 to d^3 having a half filled t_{2g} level and change from
Mn^{3+} to Mn^{2+} results in the half filled d^5 configuration
which has extra stability.
Hence **A** and **R** both are true and **R** is correct explanation of **A**.

Q.35 The strongest reducing agent among the following is:

(1) NH_3 (2) BiH_3 (3) PH_3 (4) SbH_3

Ans. **[2]**

Sol. Reducing character of the hydrides increases on moving down in the 15th group hence Ammonia is only a mild reducing agent while BiH_3 is strongest reducing agent.

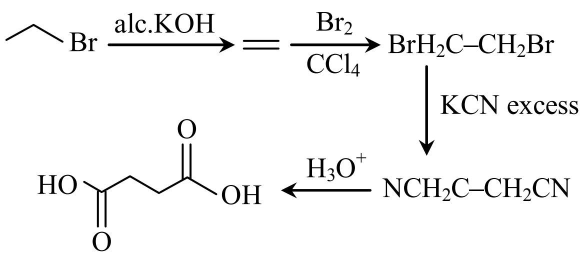
Q.36 $C_2H_5Br \xrightarrow{\text{alc. KOH}} A \xrightarrow[CCl_4]{Br_2} B \xrightarrow[\text{Excess}]{KCN} C \xrightarrow[\text{Excess}]{H_3O^+} D$

Acid D formed in above reaction is:

(1) Malonic acid (2) Oxalic acid (3) Gluconic acid (4) Succinic acid

Ans. **[4]**

Sol.



Succinic acid

Q.37 Given below are two statements :

Statement (I): A π bonding MO has lower electron density above and below the inter-nuclear axis.

Statement (II): The π* antibonding MO has a node between the nuclei.

In the light of the above statements, choose the **most appropriate** answer from the options given below:

(1) **Statement I** is true but **statement II** is false

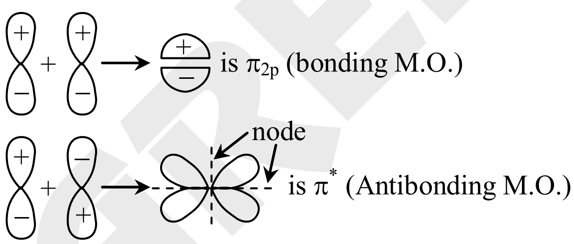
(2) Both **statement I** and **statement II** are true

(3) Both **statement I** and **statement II** are false

(4) **Statement I** is false but **statement II** is true

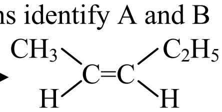
Ans. **[4]**

Sol.



Hence **statement I** is false and **statement II** is true

Q.38 In the given reactions identify A and B

$H_2 + A \xrightarrow{Pd/C}$ 

$CH_3 - C \equiv C - CH_3 + H_2 \xrightarrow{\text{Na/Liquid } NH_3}$ "B"

(1) A : n-Pentane B : trans-2-butene

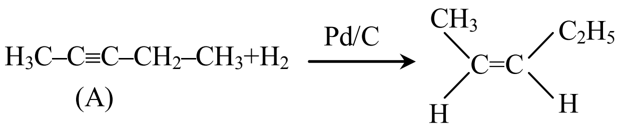
(2) A : n-Pentane B : Cis-2-butene

(3) A : 2-Pentyne B : trans-2-butene

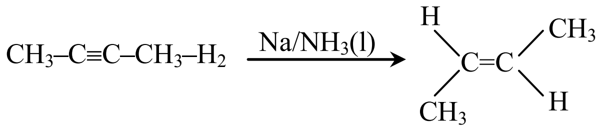
(4) A : 2-Pentyne B : Cis-2-butene

Ans. **[3]**

Sol.



(A)



(B)

Q.39 Given below are two statements:
Statement (I): Both metals and non-metals exist in p and d-block elements
Statement (II): Non-metals have higher ionization enthalpy and higher electronegativity than the metals.
In the light of the above statements, choose the **most appropriate** answer from the options given below:
(1) **Statement I** is false but **statement II** is true
(2) **Statement I** is true but **statement II** is false
(3) Both **statement I** and **statement II** are false
(4) Both **statement I** and **statement II** are true

Ans. **[1]**

Sol. d block does not contain any non-metal.
Non-metallic character, ionisation enthalpy, electronegativity increases on moving left to right in a period and decreases on moving down the group.

Q.40 Given below are two statements :
Statement I: Dimethyl glyoxime forms a six membered covalent chelate when treated with $NiCl_2$ solution in presence of NH_4OH.
Statement II: Prussian blue precipitate contains iron both in (+2) and (+3) oxidation states.
In the light of the above statements, choose the **most appropriate** answer from the options given below :
(1) **Statement I** is false but **Statement II** is true
(2) **Statement I** is true but **Statement II** is false
(3) Both **Statement I** and **Statement II** are false
(4) Both **Statement I** and **Statement II** are true

Ans. **[1]**

Sol. Dimethyl glyoxime forms a six-membered chelate via hydrogen bonding/noncovalent force when treated with $NiCl_2$ solution in presence of NH_4OH.
Prussian blue precipitate is $Fe_4[Fe(CN)_6]_3$ has Fe at +3 as well as +2 oxidation state. Where ionisation sphere is Fe^{3+} and coordination sphere has Fe^{2+}.

Q.41 Which of the following compounds show colour due to d-d transition?
(1) $CuSO_4.5H_2O$ (2) K_2CrO_4 (3) $KMnO_4$ (4) $K_2Cr_2O_7$

Ans. **[1]**

Sol. Cu^{2+}: $3d^9 4s^\circ$
$CuSO_4.5H_2O$ shows blue colour due to d–d transition.
Compounds given in option (2), (3) and (4) show colour due to LMCT (ligend to metal charge transfer).

Q.42 $[Co(NH_3)_6]^{3+}$ and $[CoF_6]^{3-}$ are respectively known as :
(1) Spin free Complex, Spin paired Complex
(2) Inner orbital Complex, Spin paired Complex
(3) Spin paired Complex, Spin free Complex
(4) Outer orbital Complex, inner orbital Complex

Ans. **[3]**

Sol. Co^{3+} : $3d^6 4s^\circ$
With NH_3, pairing will take place
$\Rightarrow$ Spin paired complex
With F^-, no paring will take place
$\Rightarrow$ Spin free complex.

Q.43 Match **List-I** with **List-II**.

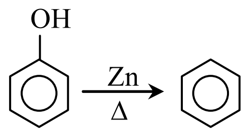
List-I Reactants		List-II Product	
(A)	Phenol, Zn/Δ	(i)	Salicylaldehyde
(B)	Phenol, $CHCl_3$, NaOH, HCl	(ii)	Salicylic acid
(C)	Phenol, CO_2, NaOH, HCl	(iii)	Benzene
(D)	Phenol, Conc, HNO_3	(iv)	Picric acid

Choose the **correct** answer from the option given below :

(1) (A)-(III), (B)-(IV), (C)-(I), (D)-(II)
(2) (A)-(III), (B)-(I), (C)-(II), (D)-(IV)
(3) (A)-(IV), (B)-(I), (C)-(II), (D)-(III)
(4) (A)-(IV), (B)-(II), (C)-(I), (D)-(III)

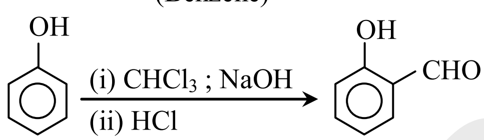
Ans. **[2]**

Sol. (A)



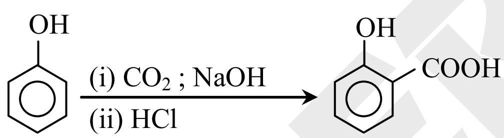
(Benzene)

(B)



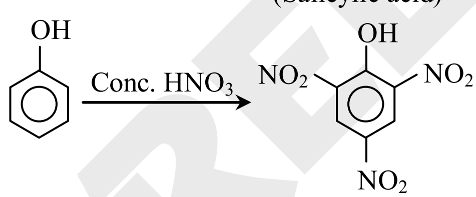
(Salicylaldehyde)

(C)



(Salicylic acid)

(D)



(Picric acid)

Q.44 Which among the following has highest boiling point?

(1) $CH_3CH_2CH_2CHO$
(2) $CH_3CH_2CH_2CH_2 - OH$
(3) $H_5C_2 - O - C_2H_5$
(4) $CH_3CH_2CH_2CH_3$

Ans. **[2]**

Sol. $CH_3 - CH_2 - CH_2 - CH_2 - OH$ will have highest boiling point due to intermolecular H-bonding.

Q.45 The transition metal having highest 3^{rd} ionization enthalpy is :

(1) Fe (2) Mn (3) Cr (4) V

Ans. **[2]**

Sol. Mn: $3d^5 4s^2$

Mn^{2+}: $3d^5 4s^\circ$

Electron removal will be most difficult from Mn^{2+} due to half-filled configuration of Mn^{2+} ions.

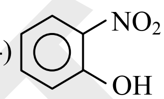
Q.46 The functional group that shows negative resonance effect is :

(1) —COOH (2) —NH_2 (3) —OR (4) —OH

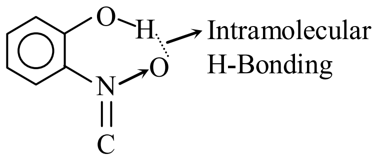
Ans. **[1]**

Sol. $-\overset{\overset{\displaystyle O}{\|}}{C}-OH$ group shows –M effect.

Q.47 Select the compound from the following that will show intramolecular hydrogen bonding.

(1) NH_3 (2) H_2O (3) C_2H_5OH (4) 

Ans. **[4]**

Sol. 

Q.48 Match **List-I** with **List-II**.

List-I Compound		List-II Use	
(A)	Carbon tetrachloride	(i)	Paint remover
(B)	Methylene chloride	(ii)	Refrigerators and air conditioners
(C)	DDT	(iii)	Fire extinguisher
(D)	Freons	(iv)	Non Biodegradable insecticide

Choose the **correct** answer from the options given below :

(1) (A)-(IV), (B)-(III), (C)-(II), (D)-(I)

(2) (A)-(II), (B)-(III), (C)-(I), (D)-(IV)

(3) (A)-(I), (B)-(II), (C)-(III), (D)-(IV)

(4) (A)-(III), (B)-(I), (C)-(IV), (D)-(II)

Ans. **[4]**

Sol. A : Carbon tetrachloride is used in fire extinguisher.

B : Methylene chloride is used in paint remover.

C : DDT is example of Non-biodegradable insecticide.

D : Freons are used in refrigerators and air conditioners.

Q.49 Solubility of calcium phosphate (molecular mass, M) in water is W_g per 100 mL at 25°C. Its solubility product at 25°C will approximately.

(1) $10^7\left(\frac{W}{M}\right)^3$ (2) $10^3\left(\frac{W}{M}\right)^5$ (3) $10^7\left(\frac{W}{M}\right)^5$ (4) $10^5\left(\frac{W}{M}\right)^5$

Ans. **[3]**

Sol. Solubility $=\left(\frac{10W}{M}\right)\frac{\text{mol}}{\text{lit}}$

$K_{sp} = 108(s)^5$

$K_{sp} = 108\,(10)^5 \times \left(\frac{W}{M}\right)^5 \approx 10^7\left(\frac{W}{M}\right)^5$

Q.50 The number of radial node/s for 3p orbital is :
(1) 4 (2) 3 (3) 1 (4) 2

Ans. **[3]**

Sol. No. of radial nodes $= n - l - 1 = 3 - 1 - 1 = 1$

Section-B: Numerical Value Type Questions: This section contains 10 Numerical based questions. Attempt any 5 questions out of 10. The answer to each question should be rounded-off to the nearest integer.

Q.51 Consider the following redox reaction :
$MnO_4^- + H^+ + H_2C_2O_4 \rightleftharpoons Mn^{2+} + H_2O + CO_2$
The standard reduction potentials are given below
(E^o_{red}) :
$E^o_{MnO_4^-/Mn^{2+}} = +1.51V$
$E^o_{CO_2/H_2C_2O_4} = -0.49V$
If the equilibrium constant of the above reaction is given as $K_{eq} = 10^x$, then the value of x = ______.
(nearest integer)

Ans. **[338]**

Sol. $E^o_{cell} = (1.51) + 0.49 = 2.0$ V

$$0 = 2 - \frac{.0591}{10}\log K$$

$\log K = 338.409$
$K = 10^{338.409}$
Nearest integer = 338

Q.52 Following Kjeldahl's method, 1g of organic compound released ammonia, that neutralised 10 mL of 2M H_2SO_4. The percentage of nitrogen in the compound is ______ %.

Ans. **[56]**

Sol. Moles of $H_2SO_4 = \frac{10 \times 2}{1000} = 0.02$

Moles of $NH_3 = 0.04$
Moles of $N = 0.04$
Mass of $N = 0.56$ gm
% by mass of $N = 56\%$

Q.53 10 mL of gaseous hydrocarbon of combustion gives 40 mL of $CO_2(g)$ and 50 mL of water vapour. Total number of carbon and hydrogen atoms in the hydrocarbon is ______.

Ans. **[14]**

Sol. Number of carbon atoms $= \frac{40}{10} = 4$

Number of H-atoms $= \frac{50 \times 2}{10} = 10$

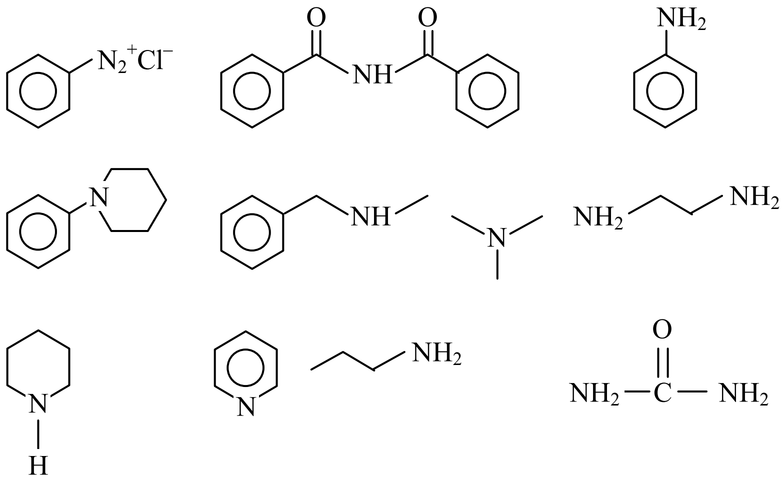
Total atoms = (4 + 10) = 14

Q.54 For a certain reaction at 300 K, K = 10, then ΔG° for the same reaction is –_____ $\times 10^{-1}$ kJ mol^{-1}. (Given R = 8.314 JK^{-1} mol^{-1})

Ans. **[57]**

Sol. $\Delta G° = -(2.303)(8.314)(300) \log K$
$= -(2.303)(8.314)(300)$
$= -5744$ J
$= -5.744$ kJ
$= -57.44 \times 10^{-1}$ kJ

Q.55 Number of compounds which give reaction with Hinsberg's reagent is _______.



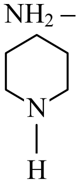
Ans. **[5]**

Sol. 1° and 2° amines will give reaction with Hinsberg reagent Compound which gives reaction with Hinsberg reagent are

(1) $Ph - NH_2$

(2) $Ph - CH_2 - NH - CH_3$

(3) $NH_2 - CH_2 - CH_2 - NH_2$

(4) 

(5) $CH_3 - CH_2 - CH_2 - NH_2$

Q.56 The amount of electricity in Coulomb required for the oxidation of 1 mol of H_2O to O_2 is _____ $\times 10^5$C.

Ans. **[2]**

Sol. $2H_2O \rightarrow O_2 + 4H^+ + 4e^-$

Mole of electron = 2

Charge = 2×96500 C

$= 1.93 \times 10^5$ C

Nearest integer = 2

Q.57 Total number of isomeric compounds (including stereoisomers) formed by monochlorination of 2-methylbutane is _____.

Ans. **[6]**

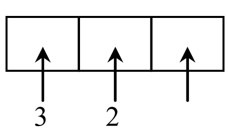
Sol. 4 structural isomers are obtained out of which 2 are optically active.

$\Rightarrow$ Total 6 isomers are obtained.

Q.58 The number of tripeptides formed by three different amino acids using each amino acid once is _____.

Ans. **[6]**

Sol.



Answer = $3 \times 2 \times 1$

$= 6$

Q.59 Mass of ethylene glycol (antifreeze) to be added to 18.6 kg of water to protect the freezing point at –24°C is ______ kg (Molar mass in g mol^{-1} for ethylene glycol 62, K_f of water = 1.86 K kg mol^{-1})

Ans. **[15]**

Sol. $\Delta T_f = K_f(m)$

$$24 = 1.86 \times \frac{\text{moles}}{18.6}$$

Moles = 240

Mass = 240 × 62 = 14880 gm = 14.88 kg

Nearest integer = 15

Q.60 The following data were obtained during the first order thermal decomposition of a gas A at constant volume :

$A(g) \rightarrow 2B(g) + C(g)$

S.No.	Time/s	Total pressure/(atm)
1.	0	0.1
2.	115	0.28

The rate constant of the reaction is ____ × 10^{-2} s^{-1} (nearest integer)

Ans. **[2]**

Sol. $$K = \frac{2.303}{115} \log\left(\frac{0.3-0.1}{0.3-0.28}\right)$$

$$K = \frac{2.303}{115} \log\left(\frac{0.2}{0.02}\right) = \frac{2.303}{115} \log 10$$

$$= \frac{2.303}{115} = 0.02002$$

$= 2 \times 10^{-2}$ sec^{-1}

Answer = 2

MATHEMATICS

Section-A: This section contains 20 multiple choice questions. Each question has 4 choices(1), (2), (3) and (4), out of which **ONLY ONE** is correct..

Q.61 Let Ajay will not appear in JEE exam with probability $p = \frac{2}{7}$, while both Ajay and Vijay will appear in the exam with probability $q = \frac{1}{5}$. Then the probability, that Ajay will appear in the exam and Vijay will not appear is :

(1) $\frac{9}{35}$ (2) $\frac{24}{35}$ (3) $\frac{3}{35}$ (4) $\frac{18}{35}$

Ans. **[4]**

Sol. $P(A \cap B) = \frac{1}{5}$

$P(A') = \frac{2}{7}$

$\therefore \quad P(A) = 1 - \frac{2}{7} = \frac{5}{7}$

$P(A \cap B') = P(A) - P(A \cap B)$

$= \frac{5}{7} - \frac{1}{5} = \frac{18}{35}$

Q.62 If the domain of the function $f(x) = \dfrac{\sqrt{x^2 - 25}}{(4 - x^2)} + \log_{10}(x^2 + 2x - 15)$ is $(-\infty, \alpha) \cup [\beta, \infty)$, then $\alpha^2 + \beta^3$ is equal to :

(1) 175 (2) 150 (3) 125 (4) 140

Ans. **[2]**

Sol. $f(x) = \dfrac{\sqrt{x^2 - 25}}{(4 - x^2)} + \log_{10}(x^2 + 2x - 15)$

$x^2 - 25 \geq 0$

$\Rightarrow x \in (-\infty, -5] \cup [5, \infty)$ ....(i)

$4 - x^2 \neq 0$

$\Rightarrow x \neq \pm 2$ ....(ii)

$x^2 + 2x - 15 > 0$

$(x - 3)(x + 5) > 0$

$x \in (-\infty, -5) \cup (3, \infty)$ ....(iii)

From (i), (ii) and (iii)

$x \in (-\infty, -5) \cup [5, \infty)$

$\therefore \alpha = -5, \beta = 5$

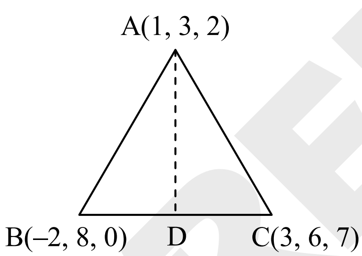
$\alpha^2 + \beta^3 = (-5)^2 + (5)^3 = 150$

Q.63 Consider a ΔABC where $A(1, 3, 2)$, $B(-2, 8, 0)$ and $C(3, 6, 7)$. If the angle bisector $\angle BAC$ meets the line BC at D, then the length of the projection of the vector $\overrightarrow{AD}$ on the vector $\overrightarrow{AC}$ is :

(1) $\dfrac{37}{2\sqrt{38}}$ (2) $\dfrac{39}{2\sqrt{38}}$ (3) $\sqrt{19}$ (4) $\dfrac{\sqrt{38}}{2}$

Ans. **[1]**

Sol.



D divides BC in ratio 1 : 1

$D : \left(\dfrac{1}{2}, 7, \dfrac{7}{2}\right)$

$\overrightarrow{AD} = \left(\dfrac{1}{2} - 1\right)\hat{i} + (7 - 3)\hat{j} + \left(\dfrac{7}{2} - 2\right)\hat{k}$

$= -\dfrac{1}{2}\hat{i} + 4\hat{j} + \dfrac{3}{2}\hat{k}$

$\Rightarrow \overrightarrow{AC} = 2\hat{i} + 3\hat{j} + 5\hat{k}$

Projection of $\overrightarrow{AD}$ on $\overrightarrow{AC}$

$= \dfrac{-1 + 12 + \dfrac{15}{2}}{\sqrt{4 + 9 + 25}} = \dfrac{37}{2\sqrt{38}}$

Q.64 Let S_n denote the sum of the first n terms of an arithmetic progression. If $S_{10} = 390$ and the ratio of the tenth the fifth terms is 15 : 7, then $S_{15} - S_5$ is equal to :

(1) 890 (2) 690 (3) 790 (4) 800

Ans. **[3]**

Sol. $S_{10} = 390$

$$\frac{a_{10}}{a_5} = \frac{15}{7}$$

$$\frac{a + 9d}{a + 4d} = \frac{15}{7}$$

$7a + 63d = 15a + 60d$

$8a - 3d = 0$ ….(1)

also, $S_{10} = 390$

$5[2a + 9d] = 390$

$2a + 9d = 78$ ….(2)

From equation (1) & (2)

$d = 8$

$\Rightarrow$ $a = 3$

So, $S_{15} - S_5 = \frac{15}{2}[2a + 14d] - \frac{5}{2}[2a + 4d]$

$= 5[2a + 19d]$

Putting values of a and d in above equation

$S_{15} - S_5 = 5[2(3) + 19(8)]$

$S_{15} - S_5 = 790$

Q.65 Let α and β be the roots the equation $px^2 + qx - r = 0$, where $p \neq 0$. If p, q and r be the consecutive terms of a non-constant G.P. and $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{3}{4}$, then the value of $(\alpha - \beta)^2$ is :

(1) $\frac{80}{9}$ (2) 9 (3) 8 (4) $\frac{20}{3}$

Ans. **[1]**

Sol. Given: $px^2 + qx - r = 0$

Let $p = \frac{a}{r_1}$, $q = a$, $r = ar_1$

and $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{3}{4}$

$\Rightarrow \frac{\alpha + \beta}{\alpha\beta} = \frac{3}{4}$

$\Rightarrow \frac{-\frac{q}{p}}{-\frac{r}{p}} = \frac{3}{4}$

$\Rightarrow \frac{q}{r} = \frac{3}{4}$

$\Rightarrow \frac{1}{r_1} = \frac{3}{4}$

$\Rightarrow r_1 = \frac{4}{3}$

$(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$

$= \left(\frac{-q}{p}\right)^2 - 4\left(\frac{-r}{p}\right)$

$= \frac{q^2}{p^2} + \frac{4r}{p}$

$= r_1^2 + 4r_1^2 = 5r_1^2$

$= 5\left(\frac{4}{3}\right)^2 = \frac{80}{9}$

Q.66 Let the locus of the midpoints of the chords of the circle $x^2 + (y - 1)^2 = 1$ drawn from the origin intersect the line $x + y = 1$ at P and Q. Then, the length of PQ is :

(1) $\sqrt{2}$ (2) 1 (3) $\frac{1}{\sqrt{2}}$ (4) $\frac{1}{2}$

Ans. [3]

Sol. Let mid-point is (x, y)

$x^2 + y^2 - 2y = 0$

$xx_1 + yy_1 - (y + y_1) = x_1^2 + y_1^2 - 2y_1$

It is passing through origin

So, $0 + 0 - (0 + y_1) = x_1^2 + y_1^2 - 2y_1$

$\Rightarrow -y_1 = x_1^2 + y_1^2 - 2y_1$

$\Rightarrow x_1^2 + y_1^2 - y_1 = 0$

$x^2 + y^2 - y = 0$ ...(1)

$\because$ It intersects the line $x + y = 1$

So put $x = 1 - y$ in equation (1)

$(1 - y)^2 + y^2 - y = 0$

$2y^2 - 3y + 1 = 0$

$\Rightarrow (y - 1)(2y - 1) = 0$

$\Rightarrow y = 1, \frac{1}{2}$

$\therefore$ P(0, 1) and Q $\left(\frac{1}{2}, \frac{1}{2}\right)$

So, $PQ = \sqrt{\left(\frac{1}{2} - 0\right)^2 + \left(\frac{1}{2} - 1\right)^2} = \frac{1}{\sqrt{2}}$

Q.67 If the mirror image of the point P(3, 4, 9) in the line $\frac{x-1}{3} = \frac{y+1}{2} = \frac{z-2}{1}$ is (α, β, γ), then $14(\alpha + \beta + \gamma)$ is :

(1) 138 (2) 102 (3) 132 (4) 108

Ans. [4]

Sol. P (3, 4, 9)

$\frac{x-1}{3} = \frac{y+1}{2} = \frac{z-2}{1} = \lambda$

Any point on line

Q $(3\lambda + 1, 2\lambda - 1, \lambda + 2)$

$PQ < 3, 2, 1 > = 0$

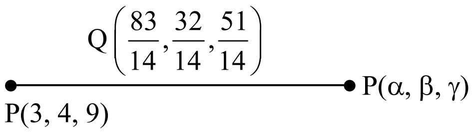
$< 3\lambda - 2, 2\lambda - 5, \lambda - 7 > < 3, 2, 1 > = 0$

$9\lambda - 6 + 4\lambda - 10 + \lambda - 7 = 0$

$14\lambda - 23 = 0$

$\lambda = \frac{23}{14}$

$\therefore Q\left(\frac{83}{14}, \frac{32}{14}, \frac{51}{14}\right)$



$\frac{3+\alpha}{2} = \frac{83}{14} \Rightarrow \alpha = \frac{62}{7}$

$\frac{4+\beta}{2} = \frac{32}{14} \Rightarrow \beta = \frac{4}{7}$

$\frac{9+\gamma}{2} = \frac{51}{14} \Rightarrow \gamma = -\frac{12}{7}$

Now, $14(\alpha + \beta + \gamma) = 14\left(\frac{62 + 4 - 12}{7}\right) = 108$

Q.68 Let m and n be the coefficients of seventh and thirteenth terms respectively in the expansion of $\left(\frac{1}{3}x^{\frac{1}{3}} + \frac{1}{2x^{\frac{2}{3}}}\right)^{18}$. Then $\left(\frac{n}{m}\right)^{\frac{1}{3}}$ is :

(1) $\frac{4}{9}$ (2) $\frac{1}{4}$ (3) $\frac{9}{4}$ (4) $\frac{1}{9}$

Ans. **[3]**

Sol. $T_7 = m = {}^{18}C_6 \left(\frac{1}{3}\right)^{12} \left(\frac{1}{2}\right)^{6}$

$T_{13} = n = {}^{18}C_{12} \left(\frac{1}{3}\right)^{6} \left(\frac{1}{2}\right)^{12}$

$$\left(\frac{m}{n}\right)^{\frac{1}{2}} = \left(\frac{{}^{18}C_6 \left(\frac{1}{3}\right)^{12}\left(\frac{1}{2}\right)^{6}}{{}^{18}C_{12}\left(\frac{1}{3}\right)^{6}\left(\frac{1}{2}\right)^{12}}\right)^{\frac{1}{3}} = \left(\frac{\left(\frac{1}{3}\right)^{6}}{\left(\frac{1}{2}\right)^{6}}\right)^{\frac{1}{3}} = \left(\left(\frac{2}{3}\right)^{6}\right)^{\frac{1}{3}} = \frac{4}{9}$$

$\therefore \left(\frac{n}{m}\right)^{\frac{1}{3}} = \frac{9}{4}$

Q.69 If z is a complex number such that $|z| \geq 1$, then the minimum value of $\left|z + \frac{1}{2}(3+4i)\right|$ is :

(1) 3 (2) 2 (3) $\frac{3}{2}$ (4) $\frac{5}{2}$

Ans. **[3]**

Sol. $|z| \geq 1$

$$\left|z + \frac{1}{2}(3+4i)\right| \geq \left||z| - \left|\frac{3}{2} + 2i\right|\right|$$

$$\geq \left|1 - \sqrt{\frac{9}{4} + 4}\right| = \left|1 - \sqrt{\frac{9+16}{4}}\right|$$

$$\geq \left|1 - \frac{5}{2}\right| = \left|\frac{2-5}{2}\right|$$

$$\geq \frac{3}{2}$$

$\therefore$ Option (3) is corect.

Q.70 Let the system of equations x + 2y + 3z = 5, 2x + 3y + z = 9, 4x + 3y + λz = μ have infinite number of solutions. Then λ + 2μ is equal to :

(1) 15 (2) 28 (3) 22 (4) 17

Ans. **[4]**

Sol. Given $x + 2y + 3z = 5$

$2x + 3y + z = 9$

$4x + 3y + \lambda z = \mu$

$$\Delta = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 4 & 3 & \lambda \end{vmatrix}$$

$\Delta = 1(3\lambda - 3) - 2(2\lambda - 4) + 3(6 - 12)$

$\Delta = 3\lambda - 3 - 4\lambda + 8 + 18 - 36$

$\Delta = -\lambda - 13$

$\Delta = 0 \Rightarrow \lambda = -13$

$$\Delta_3 = \begin{vmatrix} 1 & 2 & 5 \\ 2 & 3 & 9 \\ 4 & 3 & \mu \end{vmatrix}$$

$= 1(3\mu - 27) - 2(2\mu - 36) + 5(6 - 12)$

$= 3\mu - 27 - 4\mu + 72 + 30 - 60$

$= -\mu + 15 \quad \Rightarrow \quad \mu = 15$

$\therefore \quad \lambda + 2\mu = -13 + 30 = 17$

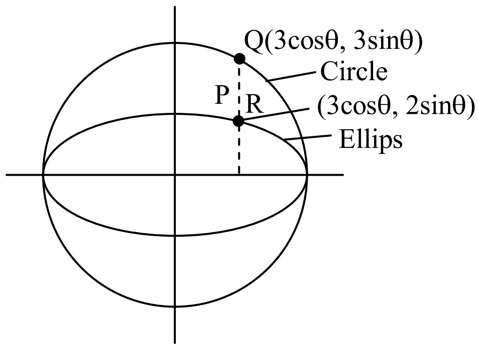
Option (4) is correct.

Q.71 Let P be a point on the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$. Let the line passing through P and parallel to y-axis meet the circle $x^2 + y^2 = 9$ at point Q such that P and Q are on the same side of the x-axis. Then, the eccentricity of the locus of the point R on PQ such that PR : RQ = 4 : 3 as P moves on the ellipse, is :

(1) $\frac{\sqrt{139}}{23}$ (2) $\frac{\sqrt{13}}{7}$ (3) $\frac{11}{19}$ (4) $\frac{13}{21}$

Ans. **[2]**

Sol.



P 4 R 3 Q

(3cosθ, 2sinθ) (3cosθ, 3sinθ)

$$h = \frac{12\cos\theta + 9\cos\theta}{7}$$

$$\Rightarrow \cos\theta = \frac{7h}{21}$$

$$k = \frac{12\sin\theta + 6\sin\theta}{7}$$

$$\sin\theta = \frac{7k}{18}$$

$$\left(\frac{7h}{21}\right)^2 + \left(\frac{7k}{18}\right)^2 = 1$$

$$e^2 = 1 - \frac{18^2}{21^2}$$

$$e^2 = \frac{21^2 - 18^2}{21^2}$$

$$e^2 = \frac{117}{21^2}$$

$$e^2 = \frac{\sqrt{117}}{21} = \frac{\sqrt{13}}{7}$$

Q.72 Consider the relations R_1 and R_2 defined as $aR_1b \Leftrightarrow a^2 + b^2 = 1$ for all a, b $\in$ R and (a, b) R_2 (c, d) $\Leftrightarrow a + d = b + c$ for all (a, b), (c, d) $\in N \times N$. Then :

(1) Neither R_1 nor R_2 is an equivalence relation
(2) Only R_2 is an equivalence relation
(3) R_1 and R_2 both are equivalence relations
(4) Only R_1 is an equivalence relation

Ans. **[2]**

Sol. $aRb \Leftrightarrow a^2 + b^2 = 1$

For reflexive (a R a) $\forall a \in IR$

If a R a

$\Rightarrow a^2 + a^2 = 1$

$\Rightarrow 2a^2 = 1$

$\Rightarrow a \Rightarrow \pm \frac{1}{\sqrt{2}}$

So, R_1 is not reflexive as (a, a) $\notin R \ \forall a \in IR$

(a, b) R_2 (c, d) $\Rightarrow a + d = b + c$

For reflexive
(a, b) R_2 (a, b) $\forall$ (a, b) $\in$ IN × IN
a + b = b + a
[(a, b), (a, b)] $\in IR_2 \forall$ (a, b) $\in$ IN × IN
So R_2 is reflexive relation
For symmetric.
If (a, b) R_2 (c, d)
Then (c, d) R_2 (a, b)
So (a, b) R_2 (c, d) $\Rightarrow$ a + d = b + c
For (c, d) R_2 (a, b) $\Rightarrow$ c + b = a + d
So R_2 is symmetric
For transitive:
If (a, b) R_2 (c, d) and (c, d) R_2 (e, f)
Then (a, b) R_2 (e, f)
(a, b) R_2 (c, d) $\Rightarrow$ a + d = b + c … (i)
(c, d) R_2 (e, f) $\Rightarrow$ c + f = e + d … (ii)
Add equation (i) and (ii)
a + d + c + f = b + c + e + d
$\Rightarrow$ a + f = b + e
$\Rightarrow$ (a, b) R_2 (e, f)
So R_2 is transitive
Only R_2 is equivalence relation.

Q.73 If $\int_0^{\frac{\pi}{3}} \cos^4 x\, dx = a\pi + b\sqrt{3}$, where a and b are rational numbers, then 9a + 8b is equal to :

(1) 3 (2) 1 (3) 2 (4) $\frac{3}{2}$

Ans. **[3]**

Sol.

$$\int_0^{\frac{\pi}{3}} \cos^4 x dx = \frac{1}{4}\int_0^{\frac{\pi}{3}} (1+\cos 2x)^2 dx = \frac{1}{4}\int_0^{\frac{\pi}{3}} (1+\cos^2 2x + 2\cos 2x)\, dx$$

$$= \frac{1}{4}\left[\left(\frac{\pi}{3}\right) + \int_0^{\frac{\pi}{3}}\left(\frac{1+\cos 4x}{2}\right)dx + \frac{2\sin 2x}{2}\Bigg|_0^{\frac{\pi}{3}}\right]$$

$$= \frac{1}{4}\left[\left(\frac{\pi}{3}\right) + \frac{1}{2}\left(x + \frac{\sin 4x}{4}\right)\Bigg|_0^{\frac{\pi}{3}} + \left(\sin\frac{2\pi}{3}\right)\right]$$

$$= \frac{1}{4}\left[\frac{\pi}{3} + \frac{1}{2}\left(\frac{\pi}{3} + \frac{1}{4}\left(\sin\frac{4\pi}{3}\right)\right) + \frac{\sqrt{3}}{2}\right]$$

$$= \frac{1}{4}\left[\frac{\pi}{3} + \frac{\pi}{6} + \frac{1}{8}\left(\frac{-\sqrt{3}}{2}\right) + \frac{\sqrt{3}}{2}\right]$$

$$= \frac{\pi}{8} + \frac{7\sqrt{3}}{64} = a\pi + b\sqrt{3}$$

$$\Rightarrow a = \frac{1}{8}, b = \frac{7}{64}$$

9a + 8b = 2

Q.74 Let $f(x) = \begin{cases} x-1 & , \quad x \text{ is even} \\ 2x & , \quad x \text{ is odd} \end{cases}$ $x \in N$. If for some $a \in N$, $f(f(f(a))) = 21$, then $\lim\limits_{x\to a^-} \left\{\frac{|x|^3}{a} - \left[\frac{x}{a}\right]\right\}$, where [t] denotes the greatest integer less than or equal to t, is equal to :

(1) 225 (2) 144 (3) 169 (4) 121

Ans. **[2]**

Sol. $f(x) = \begin{cases} x-1 & x \text{ is even} \\ 2x & x \text{ is odd} \end{cases}$ $x \in N$

Let a is odd

$\Rightarrow$ f(a) = 2a

$\Rightarrow$ f(f(a)) = 2a – 1

$\Rightarrow$ f(f(f(a))) = 2(2a –1)

2(2a –1) = 21 Not possible for any $a \in N$

Let a is even

$\Rightarrow$ f(a) = a – 1

$\Rightarrow$ f(f(a)) = 2(a – 1)

$\Rightarrow$ f(f(f(a))) = 2(a –1) – 1 = 2a – 3

2a – 3 = 21 $\Rightarrow$ a =12

Now, $\lim\limits_{x\to 12^-} \left(\frac{|x|^3}{12} - \left[\frac{x}{12}\right]\right) = 144$

Q.75 Let P and Q be the points on the line $\frac{x+3}{8} = \frac{y-4}{2} = \frac{Z+1}{2}$ which are at a distance of 6 units from the point R(1, 2, 3). If the centroid of the triangle PQR is (α, β, γ), then $\alpha^2 + \beta^2 + \gamma^2$ is :

(1) 26 (2) 18 (3) 24 (4) 36

Ans. **[2]**

Sol. Any point on line $\frac{x+3}{8} = \frac{y-4}{2} = \frac{z+1}{2}$

can be taken as (8λ – 3, 2λ + 4, 2λ – 1)

If at a distance of 6 units from R(1, 2, 3)

$\Rightarrow$ $(8\lambda - 3 - 1)^2 + (2\lambda + 4 - 2)^2 + (2\lambda - 1 - 3)^2 = 36$

$\Rightarrow$ $\lambda^2 - \lambda = 0$ {on simplification}

$\Rightarrow$ λ = 0, λ = 1

Here P & Q are (–3, 4, –1) and (5, 6, 1)

Centroid of ΔPQR

$(\alpha, \beta, \gamma) \equiv \left(\frac{5-3+1}{3}, \frac{6+4+2}{3}, \frac{1-1+3}{3}\right)$

$\Rightarrow$ α = 1, β = 4, γ = 1

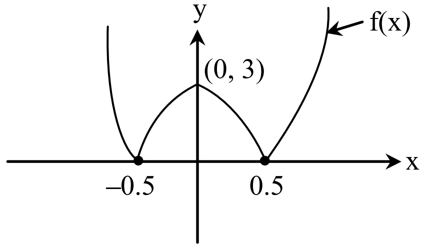
$\Rightarrow$ $\alpha^2 + \beta^2 + \gamma^2 = 18$

Q.76 Let $f(x) = |2x^2 + 5|x| - 3|$, $x \in R$. If m and n denote the number of points where f is not continuous and not differentiable respectively, then m + n is equal to :

(1) 3 (2) 5 (3) 0 (4) 2

Ans. **[1]**

Sol. $f(x) = |2x^2 + 5|x| - 3|$



Now f(x) is continuous $\forall\ x \in R$

but non- differentiable at $x = \frac{-1}{2}, \frac{1}{2}, 0$

$\therefore$ m = 0

n = 3

m + n = 3

Q.77 Consider 10 observations $x_1, x_2, \ldots, x_{10}$ such that $\sum_{i=1}^{10}(x_i - \alpha) = 2$ and $\sum_{i=1}^{10}(x_i - \beta)^2 = 40$, where α, β are positive integers. Let the mean and the variance of the observations be $\frac{6}{5}$ and $\frac{84}{25}$ respectively. Then $\frac{\beta}{\alpha}$ is equal to :

(1) 1 (2) $\frac{3}{2}$ (3) 2 (4) $\frac{5}{2}$

Ans. **[3]**

Sol. We have given $\bar{x}$ (mean) = $\frac{6}{5}$

Variance = $\frac{84}{25}$

$$\sum_{i=1}^{10}(x_i - \alpha) = 2$$

$\Rightarrow x_1 + x_2 + \ldots + x_{10} - 10\alpha = 2$

$\Rightarrow \frac{x_1 + x_2 + \ldots + x_{10}}{10} - \alpha = \frac{2}{10}$

$\Rightarrow \frac{6}{5} - \alpha = \frac{2}{10}$

$\Rightarrow \alpha = 1$

and $\sum_{i=1}^{10}(x_i - \beta)^2 = 40$

$(x_1 - \beta)^2 + (x_2 - \beta)^2 + \ldots + (x_{10} - \beta)^2 = 40$

$x_1^2 + x_2^2 + \ldots + x_{10}^2 + 10\beta^2 - 2\beta(x_1 + x_2 + \ldots + x_{10}) = 40$

$\Rightarrow \frac{x_1^2 + x_2^2 + \ldots + x_{10}^2}{10} + \beta^2 - \frac{2\beta(x_1 + x_2 + \ldots + x_{10})}{10} = 4$

$$\Rightarrow \frac{x_1^2 + x_2^2 + ... + x_{10}^2}{10} - \frac{36}{25} + \frac{36}{25} + \beta^2 - 2\beta \times \frac{6}{5} = 4$$

$$\left[\text{Variance} = \frac{\sum_{i=1}^{n} x_i^2}{n} - (\bar{x})^2 \right]$$

$$\Rightarrow \frac{84}{25} + \frac{36}{25} + \beta^2 - \frac{12\beta}{5} - 4 = 0$$

$$\Rightarrow \frac{120}{25} + \beta^2 - \frac{12\beta}{5} - 4 = 0$$

$$\Rightarrow 25\beta^2 - 60\beta + 20 = 0 \Rightarrow 5\beta^2 - 12\beta + 4 = 0$$

$$\Rightarrow \beta = 2, \frac{2}{5}$$

Take $\beta = 2$

$$\frac{\beta}{\alpha} = \frac{2}{1} = 2$$

Q.78 The number of solutions of the equation $4\sin^2 x - 4\cos^3 x + 9 - 4\cos x = 0$; $x \in [-2\pi, 2\pi]$ is :

(1) 2 (2) 0 (3) 3 (4) 1

Ans. **[2]**

Sol. $4 - 4\cos^2 x - 4\cos^3 x - 4\cos x + 9 = 0$

$4\cos^3 x + 4\cos^2 x + 4\cos x - 13 = 0$

$\Rightarrow 4\cos^3 x + 4\cos^2 x + 4\cos x = 13$

LHS. ≥ 12 can't be equal to 13.

$\therefore$ Number of solutions = 0

Q.79 The value of $\int_0^1 (2x^3 - 3x^2 - x + 1)^{\frac{1}{3}} dx$ is equal to :

(1) 1 (2) 0 (3) 2 (4) –1

Ans. **[2]**

Sol.

$$I = \int_0^1 (2x^3 - 3x^2 - x + 1)^{1/3}\, dx$$

$$I = \int_0^1 ((2x - 1)(x^2 - x - 1)^{1/3} dx$$

$$I = \int_0^1 [(2(1 - x) - 1)((1 - x)^2 - (1 - x) - 1)]^{1/3} dx$$

$$I = \int_0^1 ((1 - 2x)(x^2 - x - 1))^{1/3} dx$$

$$I = -\int_0^1 ((2x - 1)(x^2 - x - 1))^{1/3} dx$$

$I = -1$

$2I = 0$

$I = 0$

Q.80 Let α be non-zero real number. Suppose $f : R \to R$ is a differentiable function such that $f(0) = 2$ and $\lim_{x\to-\infty} f(x) = 1$. If $f'(x) = \alpha f(x) + 3$, for all $x \in R$, then $f(-\log_e 2)$ is equal to ______.

(1) 5 (2) 3 (3) 7 (4) 9

Ans. **[Bonus]**

Sol. $f'(x) = \alpha f(x) + 3$

$f'(x) + (-\alpha) f(x) = 3$

I.F. $= e^{\int -\alpha dx} = e^{-\alpha x}$

$\Rightarrow f(x)\cdot e^{-\alpha x} = \int 3e^{-\alpha x} dx + c$

$\Rightarrow f(x)e^{-\alpha x} = -\frac{3}{\alpha} e^{-\alpha x} + c$

$f(x) = -\frac{3}{\alpha} + ce^{\alpha x}$, $\because$ $f(0) = 2 \Rightarrow c = 2 + \frac{3}{\alpha}$

$\Rightarrow f(x) = -\frac{3}{\alpha} + \left(2 + \frac{3}{\alpha}\right)e^{\alpha x}$

$\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty}\left(-\frac{3}{\alpha} + ce^{\alpha x}\right) = 1$

$-\frac{3}{\alpha} + c\lim_{x\to-\infty} e^{\alpha x} = 1$

$\Rightarrow \alpha = -3$ (not possible as $\alpha > 0$)

If $\alpha < 0$,

$\because f(x) = -\frac{3}{\alpha} + \left(2 + \frac{3}{\alpha}\right)e^{\alpha x}$

$\lim_{x\to-\infty} e^{-\alpha x} f(x) = \lim_{x\to-\infty}\left(-\frac{3}{\alpha}e^{-\alpha x} + \left(2 + \frac{3}{\alpha}\right)\right)$

$2 + \frac{3}{\alpha} = 0 \Rightarrow \alpha = -\frac{3}{2} \Rightarrow f(x) = 2\ \forall\ x \in R$

$\Rightarrow \lim_{x\to-\infty} f(x) = 2$ (but it given that $\lim_{x\to-\infty} f(x) = 1$)

not possible

Section-B: Numerical Value Type Questions: This section contains 10 Numerical based questions. Attempt any 5 questions out of 10. The answer to each question should be rounded-off to the nearest integer.

Q.81 Let $A = I_2 - 2MM^T$, where M is a real matrix of order 2 × 1 such that the relation $M^TM = I_1$ holds. If λ is a real number such that the relation $AX = \lambda X$ holds for some non-zero real matrix X of order 2 × 1, then the sum of squares of all possible values of λ is equal to ________.

Ans. **[2]**

Sol. $A = I_2 - 2MM^T$

$A^2 = (I_2 - 2MM^T)(I_2 - 2MM^T) = I_2 - 2MM^T - 2MM^T + 4MM^TMM^T = I_2 - 4MM^T + 4MM^T = I_2$

$AX = \lambda X$

$A^2X = \lambda AX$

$X = \lambda(\lambda X)$

$X = \lambda^2 X$

$X(\lambda^2 - 1) = 0$

$\lambda^2 = 1$

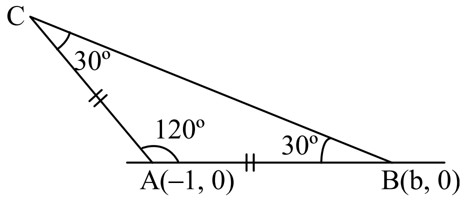
$\lambda = \pm 1$

Sum of square of all values = 2

Q.82 Let ABC be an isosceles triangle in which A is at (–1, 0), $\angle A = \frac{2\pi}{3}$, AB = AC and B is on the positive x-axis. If BC = $4\sqrt{3}$ and the line BC intersects the line y = x + 3 at (α, β), then $\frac{\beta^4}{\alpha^2}$ is ______.

Ans. **[36]**

Sol.



$$\frac{c}{\sin 30} = \frac{4\sqrt{3}}{\sin 120}$$

$\Rightarrow$ c = 4

AB = |(b + 1)| = 4

b = 3 $m_{AB} = 0$

$m_{BC} = \frac{-1}{\sqrt{3}}$

BC : $y = -\frac{1}{\sqrt{3}}(x - 3)$

: $\sqrt{3}\,y + x = 3$

Point of intersection of y = x + 3, $\sqrt{3}\,y + x = 3$

$y = \frac{6}{\sqrt{3}+1}$ and $x = \frac{-6}{(1+\sqrt{3})^2}$

$\frac{\beta^4}{\alpha^2} = 36$

Q.83 If three successive terms of a G.P. with common ratio r(r > 1) are the lengths of the sides of a triangle and [r] denotes the greatest integer less than or equal to r, then 3[r] + [–r] is equal to ________.

Ans. **[1]**

Sol. Let three terms of G.P. are $\frac{a}{r}$, a , ar(r > 1)

Sum of two sides > third side

$\frac{a}{r} + a > ar \Rightarrow 1 + r > r^2$

$\Rightarrow r^2 - r - 1 < 0$

$\Rightarrow \frac{1-\sqrt{5}}{2} < r < \frac{1+\sqrt{5}}{2}$

but r > 1

$\Rightarrow r \in \left(1, \frac{1+\sqrt{5}}{2}\right)$

$\Rightarrow$ [r] = 1 and [–r] = –2

3[r] + [–r] = 3 – 2 = 1

Q.84 The lines $L_1, L_2, \ldots, L_{20}$ are distinct. For n = 1, 2, 3, …, 10 all the lines L_{2n-1} are parallel to each other and all the lines L_{2n} pass through a given point P. the maximum number of points of intersection of pairs of lines from the set $[L_1, L_2, \ldots, L_{20}]$ is equal to ______.

Ans. **[101]**

Sol. 10 lines are concurrent, 10 lines are parallel.

Odd lines $\in \{l_1, l_3, ..., l_{19}\}$

Even lines $\in \{l_2, l_4, ..., l_{20}\}$

For maximum intersection

${}^{\text{Even lines}}C_2$ × (Zero point of intersection) + (One line from odd) × (One line from even lines) + 1 point of intersection of concurrent lines

$= {}^{10}C_2(0) + {}^{10}C_1\ {}^{10}C_1 + 1 = 101$

Q.85 The sum of squares of all possible values of k, for which area of the region bounded by the parabolas $2y^2 = kx$ and $ky^2 = 2(y - x)$ is maximum, is equal to _______.

Ans. **[08]**

Sol. Given $ky^2 = 2(y - x)$ …(i)

$2y^2 = kx$ …(ii)

Point of intersection of (i) and (ii)

$$ky^2 = 2\left(y - \frac{2y^2}{k}\right)$$

$$\Rightarrow\ y = 0,\ ky = 2\left(1 - \frac{2y}{k}\right)$$

$$ky + \frac{4y}{k} = 2$$

$$y = \frac{2}{k + \frac{4}{k}} = \frac{2k}{k^2 + 4}$$

$$A = \int_6^{\frac{2k}{k^2+4}} \left(\left(y - \frac{ky^2}{2}\right) - \frac{2y^2}{k}\right) dy$$

$$A = \left[\frac{y^2}{2} - \left(\frac{k}{2} + \frac{2}{k}\right)\frac{y^3}{3}\right]_0^{\frac{2k}{k^2+4}} = \left(\frac{2k}{k^2+4}\right)^2 \left[\frac{1}{2} - \frac{k^2+4}{2k}\left(\frac{1}{3}\right)\left(\frac{2k}{k^2+4}\right)\right] = \frac{1}{6} \times 4 \times \left(\frac{1}{k + \frac{4}{k}}\right)^2$$

A.M. ≥ G.M.

$$\frac{\left(k + \frac{4}{k}\right)}{2} \geq 2$$

$$k + \frac{4}{k} \geq 4$$

∴ Area is maximum when $k = \frac{4}{k}$

∴ $k^2 = 4$

$k = \pm 2$

$k_1 = 2,\ k_2 = -2$

∴ $k_1^2 + k_2^2 = (+2)^2 + (-2)^2 = 4 + 4 = 08$

Q.86 If $y = \dfrac{(\sqrt{x}+1)(x^2-\sqrt{x})}{x\sqrt{x}+x+\sqrt{x}} + \dfrac{1}{15}(3\cos^2 x - 5)\cos^3 x$, then $96\, y'\left(\dfrac{\pi}{6}\right)$ is equal to ______.

Ans. **[105]**

Sol.
$$y = \frac{(\sqrt{x}+1)(x^2-\sqrt{x})}{x\sqrt{x}+x+\sqrt{x}} + \frac{1}{15}(3\cos^2 x - 5)\cos^3 x$$

$$\Rightarrow y = \frac{(\sqrt{x}+1)\sqrt{x}(\sqrt{x}-1)(x+\sqrt{x}+1)}{\sqrt{x}(x+\sqrt{x}+1)} + \frac{1}{5}\cos^5 x - \frac{1}{3}\cos^3 x$$

$$\Rightarrow y = x - 1 + \frac{1}{5}\cos^5 x - \frac{1}{3}\cos^3 x$$

$$\Rightarrow \frac{dy}{dx} = 1 + \cos^4 x(-\sin x) + \cos^2 x \cdot \sin x$$

$$\Rightarrow \left.\frac{dy}{dx}\right|_{x=\frac{\pi}{6}} = 1 + \left(\frac{\sqrt{3}}{2}\right)^4\left(-\frac{1}{2}\right) + \left(\frac{\sqrt{3}}{2}\right)^2\left(\frac{1}{2}\right) = \frac{35}{32}$$

Required value $= 96 \times \dfrac{35}{32} = 105$

Q.87 Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = -\hat{i} - 8\hat{j} + 2\hat{k}$ and $\vec{c} = 4\hat{i} + c_2\hat{j} + c_3\hat{k}$, be three vectors such that $\vec{b} \times \vec{a} = \vec{c} \times \vec{a}$. If the angle between the vector $\vec{c}$ and the vector $3\hat{i} + 4\hat{j} + \hat{k}$ is θ, then the greatest integer less than or equal to $\tan^2\theta$ is _______.

Ans. **[38]**

Sol. $\vec{b} \times \vec{a} = \vec{c} \times \vec{a}$

$\Rightarrow (\vec{c} - \vec{b}) \times \vec{a} = 0$

$\Rightarrow \vec{c} = \vec{b} + \lambda\vec{a}$

$\Rightarrow \vec{c} = (-1+\lambda)\hat{i} + (-8+\lambda)\hat{j} + (2+\lambda)\hat{k}$

But given $\vec{c} = 4\hat{i} + c_2\hat{j} + c_3\hat{k}$

$\Rightarrow \lambda = 5, c_2 = -3$ and $c_3 = 7$

Hence $\vec{c} = 4\hat{i} - 3\hat{j} + 7\hat{k}$

Let $\vec{x} = 3\hat{i} + 4\hat{j} + \hat{k}$

If angle between $\vec{c}$ and $\vec{x}$ is θ then

$$\cos\theta = \frac{12-12+7}{\sqrt{16+9+49}\sqrt{9+16+1}}$$

$$\tan^2\theta = \frac{1}{\cos^2\theta} - 1 = \frac{74\times 26}{49} - 1 = 38.26$$

Required value = 38

Q.88 If $\dfrac{dx}{dy} = \dfrac{1+x-y^2}{y}$, $x(1) = 1$, then $5x(2)$ is equal to ______.

Ans. **[5]**

Sol. $\frac{dx}{dy} = \frac{1+x-y^2}{y}$

$\Rightarrow \frac{dx}{dy} + \left(-\frac{1}{y}\right) x = \frac{1-y^2}{y}$

Integrating factor (I.F.) = $e^{-\int \frac{1}{y}dy} = e^{\ln\left(\frac{1}{y}\right)} = \frac{1}{y}$

$\Rightarrow x \cdot \frac{1}{y} = \int\left(\frac{1-y^2}{y^2}\right) dy$

$\Rightarrow x = -1 - y^2 + cy$

$x(1) = 1 \Rightarrow c = 3$

$\Rightarrow x(y) = 3y - 1 - y^2$

$5 \cdot x(2) = 5\,(6 - 1 - 4) = 5$

Q.89 Let $f : (0, \infty) \to R$ and $F(x) = \int_0^x t\,f(t)dt$. It $F(x^2) = x^4 + x^5$, then $\sum_{r=1}^{12} f(r^2)$ is equal to ___________ .

Ans. [219]

Sol. $\because F(x) = \int_0^x tf(t)\,dt$ ∫

$\Rightarrow F'(x) = x \cdot f(x)$ ....(i)

Also $F(x^2) = x^4 + x^5$

$\Rightarrow F(x) = x^2 + x^{\frac{5}{2}}$

$F'(x) = 2x + \frac{5}{2} \cdot x^{\frac{3}{2}}$ ...(ii)

From (i) and (ii)

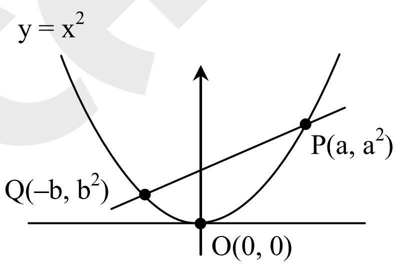
$f(x) = 2 + \frac{5}{2}\sqrt{x}$

$\sum_{r=1}^{12} f(r^2) = \sum_{r=1}^{12}\left(2 + \frac{5}{2}r\right) = 2 \times 12 + \frac{5}{2}\left(\frac{12 \times 13}{2}\right) = 219$

Q.90 Three points O(0, 0), P(a, a^2), Q(–b, b^2), a > 0, b > 0, are on the parabola $y = x^2$. Let S_1 be the area of the region bounded by the line PQ and the parabola, and S_2 be the area of the triangle OPQ. If the minimum value of $\frac{S_1}{S_2}$ is $\frac{m}{n}$, gcd(m, n) = 1, then m + n is equal to _____

Ans. [7]

Sol.



Equation of PQ

$y - b^2 = \frac{a^2 - b^2}{a + b}(x + b)$

$\Rightarrow$ $y = (a - b)x + ab$

$$S_1 = \int_{-b}^{a} ((a-b)x + ab - x^2)\,dx$$

$$= \left(\frac{(a-b)}{2}x^2 + abx - \frac{x^3}{3}\right)_{-b}^{a}$$

$$= \frac{1}{6}(a+b)^3$$

$$S_2 = \frac{1}{2}\begin{vmatrix} -b & b^2 & 1 \\ 0 & 0 & 1 \\ a & a^2 & 1 \end{vmatrix} = \frac{1}{2}\,ab\,(a+b)$$

$$\frac{S_1}{S_2} = \frac{\frac{1}{6}(a+b)^3}{\frac{1}{2}\cdot ab(a+b)}$$

$$= \frac{1}{3}\,\frac{(a+b)^2}{ab} = \frac{1}{3}\left(\frac{a}{b} + \frac{b}{a} + 2\right)$$

$\because$ $\frac{b}{a} + \frac{a}{b} \geq 2$

$\Rightarrow$ $\left(\frac{S_1}{S_2}\right)_{min} = \frac{4}{3} = \frac{m}{n}$

$\Rightarrow$ $m + n = 7$