# JEE Main Online Exam 2024

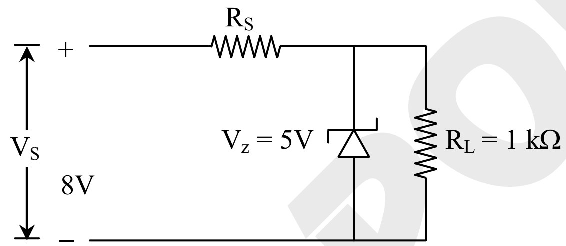
## Questions & Solution

### 01st February 2024 | Morning

## PHYSICS

**Section-A:** This section contains 20 multiple choice questions. Each question has 4 choices(1), (2), (3) and (4), out of which **ONLY ONE** is correct..

**Q.1** In the given circuit if the power rating of Zener diode is 10 mW, the value of series resistance $R_S$ to regulate the input unregulated supply is :




(1) 5 kΩ (2) 10 Ω (3) 10 kΩ (4) 1 kΩ

**Ans.** **[4]**

**Sol.** $$\frac{3}{R_S} + \frac{10mW}{5V} = \frac{5}{1k\Omega}$$

$$\frac{3}{R_S} = 3mA$$

$$\Rightarrow \quad R_S = 1\ k\Omega$$

**Q.2** A particle moving in a circle of radius R with uniform speed takes time T to complete one revolution. If this particle is projected with the same speed at an angle θ to the horizontal, the maximum height attained by it is equal to 4R. The angle of projection θ is then given by:

(1) $\cos^{-1}\left[\frac{2gT^2}{\pi^2 R}\right]^{\frac{1}{2}}$ (2) $\sin^{-1}\left[\frac{2gT^2}{\pi^2 R}\right]^{\frac{1}{2}}$ (3) $\sin^{-1}\left[\frac{\pi^2 R}{2gT^2}\right]^{\frac{1}{2}}$ (4) $\cos^{-1}\left[\frac{\pi R}{2gT^2}\right]^{\frac{1}{2}}$

**Ans.** **[2]**

**Sol.** $$T = \frac{2\pi R}{V}$$

$$H_{max} = \frac{v^2 \sin^2\theta}{2g} \Rightarrow 4R = \frac{v^2 \sin^2\theta}{2g}$$

$$\sin^2\theta = \frac{8gR}{v^2} = \frac{2gT^2}{\pi^2 R}$$

$$\theta = \sin^{-1}\left(\frac{2gT^2}{\pi^2 R}\right)^{1/2}$$

**Q.3** Two identical capacitors have same capacitance C. One of them is charged to the potential V and other to the potential 2V. The negative ends of both are connected together. When the positive ends are also joined together, the decrease in energy of the combined system is :

(1) $\frac{1}{4}CV^2$ (2) $\frac{3}{4}CV^2$ (3) $2CV^2$ (4) $\frac{1}{2}CV^2$

**Ans.** **[1]**

**Sol.** $E_i = \frac{1}{2}CV^2 + \frac{1}{2}C(2V)^2 = \frac{5}{2}CV^2$

$E_f = 2 \times \left(\frac{Q^2}{2C}\right) = \frac{9C^2V^2}{4C} = \frac{9}{4}CV^2$

Decrease in energy

$= E_i - E_f = \left(\frac{5}{2} - \frac{9}{4}\right)CV^2 = \frac{1}{4}CV^2$

**Q.4** A galvanometer has a resistance of 50 Ω and it allows maximum current of 5 mA. It can be converted into voltmeter to measure upto 100 V by connecting in series a resistor of resistance:

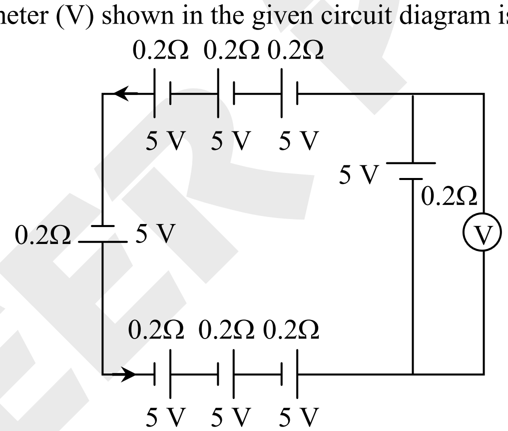
(1) 20050 Ω (2) 5975 Ω (3) 19950 Ω (4) 19500 Ω

**Ans.** **[3]**

**Sol.** $V = i_g(r_g + s) \Rightarrow 100 = 5 \times 10^{-3}(50 + s)$

$2 \times 10^4 = 50 + s \Rightarrow s = 20000 - 50 = 19950\ \Omega$

**Q.5** The reading in the ideal voltmeter (V) shown in the given circuit diagram is :




(1) 0 V (2) 5 V (3) 10 V (4) 3 V

**Ans.** **[1]**

**Sol.** Total 8 cells : $40 - i \times (8 \times 0.2) = 0 \Rightarrow i = 25$ A

$V_B - (25 \times 0.2) + 5 = V_A$

$V_B + 0 = V_A \Rightarrow V_A - V_B = 0$ (Reading)

**Q.6** A ball of mass 0.5 kg is a string of length 50 cm. The ball is rotated on a horizontal circular path about its vertical axis. The maximum tension that the string can bear is 400 N. The maximum possible value of angular velocity of the ball in rad/s is, :

(1) 1000 (2) 40 (3) 20 (4) 1500

**Ans.** **[2]**

**Sol.** $T\cos\theta = mg,\ T\sin\theta = m\omega^2\ell\sin\theta$

$\Rightarrow T = m\omega^2\ell$

$\Rightarrow 400 = 0.5 \times \omega^2 \times 0.5$

$\Rightarrow \omega^2 = 1600$

$\Rightarrow \omega = 40$ rad/s

**Q.7** The minimum energy required by a hydrogen atom in ground state to emit radiation in Balmer series is nearly :

(1) 13.6 eV (2) 1.5 eV (3) 12.1 eV (4) 1.9 eV

**Ans.** **[3]**

**Sol.** Balmer emission: transition to n = 2

To transition to n = 2, minimum state to transition from is n = 3

$\Delta E = E_{ground} - E_{n=3} = 12.1$ eV

**Q.8** The dimensional formula of angular impulse is :

(1) $[ML^2T^{-2}]$ (2) $[ML^{-2}T^{-1}]$ (3) $[ML^2T^{-1}]$ (4) $[MLT^{-1}]$

**Ans.** **[3]**

**Sol.** Angular impulse = Impulse × distance from axis

[Angular impulse] = [Force] [Time] [Length] = $[ML^2T^{-1}]$

**Q.9** The pressure and volume of an ideal gas are related as $PV^{3/2} = K$ (Constant). The work done when the gas is taken from state $A(P_1, V_1, T_1)$ to state $B(P_2, V_2, T_2)$ is :

(1) $2(P_2\sqrt{V_2} - P_1\sqrt{V_1})$ (2) $2(P_1V_1 - P_2V_2)$

(3) $2(P_2V_2 - P_1V_1)$ (4) $2(\sqrt{P_1}V_1 - \sqrt{P_2}V_2)$

**Ans.** **[2]**

**Sol.** $W = \frac{P_F V_f - P_i V_i}{1-\gamma} = \frac{P_2V_2 - P_1V_1}{1-\frac{3}{2}} = = -2(P_2V_2 - P_1V_1)$

$W = 2(P_1V_1 - P_2V_2)$

**Q.10** With rise in temperature, the Young's modulus of elasticity:

(1) Decreases (2) Remains unchanged

(3) Increases (4) Changes erratically

**Ans.** **[1]**

**Sol.** With rise in temperature, Young's modulus of elasticity decreases due to the increase in atomic vibrations which leads to decrease in atomic forces.

**Q.11** Two moles a monoatomic gas is mixed with six moles of a diatomic gas. The molar specific heat of the mixture at constant volume is:

(1) $\frac{3}{2}R$ (2) $\frac{9}{4}R$ (3) $\frac{7}{4}R$ (4) $\frac{5}{2}R$

**Ans.** **[2]**

**Sol.** $C_{v_{mix}} = \frac{n_1C_{v_1} + n_2C_{v_2}}{n_1 + n_2} = \frac{\frac{2\times 3R}{2} + \frac{6\times 5R}{2}}{2+6}$

$C_{v_{mix}} = \frac{9R}{4}$

**Q.12** A simple pendulum of length 1 m has a wooden bob of mass 1 kg. It is struck by a bullet of mass $10^{-2}$ kg moving with a speed of $2 \times 10^2$ $ms^{-1}$. The bullet gets embedded into the bob. The height to which the bob rises before swinging back is (use g = 10 m/s$^2$)

(1) 0.20 m (2) 0.35 m (3) 0.30 m (4) 0.40 m

**Ans.** **[1]**

**Sol.** Linear momentum conservation:

$10^{-2} \times 2 \times 10^{2} = (1 + 0.01)v$

v = 2 m/s

Energy conservation:

$\frac{1}{2} \times 1 \times 2^2 = 1 \times 10 \times h$

h = 0.20 m

**Q.13** A parallel plate capacitor has a capacitance C = 200 pF. It is connected to 230 V ac supply with an angular frequency 300 rad/s. The rms value of conduction current in the circuit and displacement current in the capacitor respectively are:

(1) 14.3 μA and 143 μA
(2) 13.8 μA and 138 μA
(3) 13.8 μA and 13.8 μA
(4) 1.38 μA and 1.38 μA

**Ans.** **[3]**

**Sol.** $i_{displacement} = \frac{C.dv}{dt}$ ; $(i_{displacement})_{rms} = C \cdot \omega E_{rms}$

$= 230 \times 200 \times 10^{-12} \times 300$

$= 13.8 \times 10^{-6} A = 13.8\ \mu A$

$(i_{conduction})_{rms} = E_{rms} \cdot \omega C = 230 \times 200 \times 10^{-12} \times 300$

$= 13.8 \times 10^{-6} A = 13.8\ \mu A$

**Q.14** The de Broglie wavelengths of a proton and an α particle are λ and 2λ respectively. The ratio of the velocities of proton and α particle will be :

(1) 4 : 1
(2) 1 : 8
(3) 8 : 1
(4) 1 : 2

**Ans.** **[3]**

**Sol.** $\therefore \quad \lambda = \frac{h}{mv} \Rightarrow v = \frac{h}{m\lambda} : v \propto \frac{1}{m\lambda}$

$\frac{V_p}{V_\alpha} = \frac{m_\alpha \lambda_\alpha}{m_P \lambda_P} \frac{4m_P . 2\lambda}{m_P . \lambda} = \frac{8}{1}$

**Q.15** In series LCR circuit, the capacitance is changed from C to 4C. To keep the resonance frequency unchanged, the new inductance should be:

(1) Increased to 4L
(2) Increased by 2L
(3) Reduced by $\frac{1}{4}L$
(4) Reduced by $\frac{3}{4}L$

**Ans.** **[4]**

**Sol.** $\omega_1 = \frac{1}{\sqrt{LC}}$ ; $\omega_f = \frac{1}{\sqrt{4C \cdot L'}}$

For $\omega_i = \omega_f = L' = \frac{L}{4}$ $\left(\text{reduced by } \frac{3L}{4}\right)$

**Q.16** If R is the radius of the earth and the acceleration due to gravity on the surface of earth is $g = \pi^2\ m/s^2$, then the length of the second's pendulum at a height h = 2R from the surface of earth will be :

(1) $\frac{4}{9}$m
(2) $\frac{8}{9}$m
(3) $\frac{1}{9}$m
(4) $\frac{2}{9}$m

**Ans.** **[3]**

**Sol.** $g' = \frac{GM}{(R+2R)^2} = \frac{GM}{9R^2} = \frac{g}{9}$

$2 = 2\pi\sqrt{\frac{\ell \times 9}{g}} \Rightarrow \ell = \frac{1}{9}$ m

**Q.17** A monochromatic light of wavelength 6000Å is incident on the single slit of width 0.01 mm. If the diffraction pattern is formed at the focus of the convex lens of focal length 20 cm, the linear width of the central maximum is :

(1) 12 mm (2) 120 mm (3) 60 mm (4) 24 mm

**Ans.** **[4]**

**Sol.** Linear width = $\frac{2\lambda_0 f}{d} = \frac{2(6000\times10^{-10})(20\times10^{-2})}{0.01\times10^{-3}}$ = $24 \times 10^{-3}$ m = 24 mm

**Q.18** 10 divisions on the main scale of a Vernier caliper coincide with 11 divisions on the Vernier scale. If each division on the main scale is of 5 units, the least count of the instrument is :

(1) $\frac{10}{11}$ (2) $\frac{1}{2}$ (3) $\frac{5}{11}$ (4) $\frac{50}{11}$

**Ans.** **[3]**

**Sol.** $10M = 11v \Rightarrow 1v = \frac{10}{11}$; $v' = \frac{10}{11}\times 5 = \frac{50}{11}$

Least count = $M - v' = 5 - \frac{50}{11} = \frac{5}{11}$

**Q.19** The radius (r), length ($\ell$) and resistance (R) of a metal wire was measured in the laboratory as

$r = (0.35 \pm 0.05)$ cm

$R = (100 \pm 10)$ ohm

$\ell = (15 \pm 0.2)$ cm

The percentage error in resistivity of the material of the wire is:

(1) 35.6% (2) 39.9% (3) 25.6% (4) 37.3%

**Ans.** **[2]**

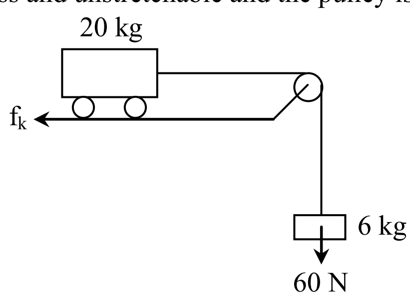
**Sol.** $\rho = \frac{RA}{\ell} = \frac{R.\pi r^2}{\ell}$

$\therefore \frac{d\rho}{\rho}\times 100 = \frac{2dr}{r}\times 100 + \frac{dR}{R}\times 100 + \frac{d\ell}{\ell}\times 100$

$= 2\times\frac{0.05}{0.35}\times 100 + \frac{10}{100}\times 100 + \frac{0.2}{15}\times 100 = 39.9\%$

**Q.20** Consider a block and trolley system as shown in figure. If the coefficient of kinetic friction between the trolley and the surface is 0.04, the acceleration of the system in $ms^{-2}$ is:

(Consider that the string is massless and unstretchable and the pulley is also massless and frictionless):




(1) 4 (2) 3 (3) 2 (4) 1.2

**Ans.** **[3]**

**Sol.** $60 - f_k = (20 + 6)a$

$60 - 200 \times 0.04 = 26a \Rightarrow a = 2\ m/s^2$

**Section-B: Numerical Value Type Questions:** This section contains 10 Numerical based questions. Attempt any 5 questions out of 10. The answer to each question should be rounded-off to the nearest integer.

**Q.21** A rectangular loop of sides 12 cm and 5 cm, with its sides parallel to the x-axis and y-axis respectively, moves with a velocity of 5 cm/s in the positive x-axis direction, in a space containing a variable magnetic field in the positive z direction. The field has a gradient of $10^{-3}$ T/cm along the negative x direction and it is decreasing with time at the rate of $10^{-3}$ T/s. If the resistance of the loop is 6 mΩ, the power dissipated by the loop as heat is _______ $\times 10^{-9}$ W.

**Ans.** **[216]**

**Sol.**

$$I_1 = \frac{-dB}{dx} \times \frac{dx}{dt} \times A$$

$$= 30 \times 10^{-6}\ \text{V}$$

$$I_2 = \frac{-dB}{dt} A$$

$$= 6 \times 10^{-6}\text{V}$$

$$P = \frac{(l_1 + l_2)^2}{R} = 216 \times 10^{-9}\ \text{W}$$

**Q.22** A plane is in level fight at constant speed and each of its two wings has an area of 40 $m^2$. If the speed of the air is 180 km/h over the lower wing surface and 252 km/h over the upper wing surface, the mass of the plane is _______ kg. (Take air density to be 1 kg $m^{-3}$ and g = 10 $ms^{-2}$)

**Ans.** **[9600]**

**Sol.**

$$P_1 - P_2 = \frac{1}{2}\rho(V_2^2 - V_1^2)$$

$$\Rightarrow \quad P_1 - P_2 = \frac{1}{2} \times 1 \times (70^2 - 50^2)$$

$$P_1 - P_2 = 1200\ \text{N/m}^2$$

$$\therefore \quad F = mg = (P_1 - P_2)A$$

$$\Rightarrow \quad m = \frac{(P_1 - P_2)A}{g} = \frac{1200 \times (2 \times 40)}{10} = 9600\ \text{kg}$$

**Q.23** Two identical charged spheres are suspended by strings of equal lengths. The strings make an angle θ with each other. When suspended in water the angle remains the same. If density of the material of the sphere is 1.5 g/cc, the dielectric constant of water will be __________ (Take density of water = 1 g/cc)

**Ans.** **[3]**

**Sol.**

$$\therefore \quad \tan\theta = \frac{qE}{mg} = \frac{qE}{mg - F_B} \Rightarrow \frac{E}{g} = \frac{E/K}{mg - 1.vg}$$

$$\frac{E}{mg} = \frac{E/K}{mg/3} \Rightarrow 1 = \frac{3}{K} \Rightarrow K = 3$$

**Q.24** A particle is moving in one dimension (along x-axis) under the action of a variable force. It's initial position was 16 m right of origin. The variation of its position (x) with time (t) is given as $x = -3t^3 + 18t^2 + 16t$, where x is in m and t is in s. The velocity of the particle when its acceleration becomes zero is _______ m/s.

**Ans.** **[52]**

**Sol.**

$$x = -3t^3 + 18t^2 + 16t \Rightarrow v = -9t^2 + 36t + 16$$

$$a = -18t + 36 = 0 \Rightarrow t = 2\ \text{sec}$$

$$v_{(t=2)} = -9(2)^2 + 36(2) + 16 = 52\ \text{m/s}$$

**Q.25** The current in a conductor is expressed as $I = 3t^2 + 4t^3$, where I is in Ampere and t is in second. The amount of electric charge that flows through a section of the conductor during t = 1 s to t = 2 s is _______ C.

**Ans.** **[22]**

**Sol.** $\int dq = \int i \, . \, dt \Rightarrow q = \int_1^2 (3t^2 + 4t^3) . dt$

$q = \left[t^3 + t^4\right]_1^2 = [8 + 16 - 2] = 22C.$

**Q.26** The distance between object and its 3 times magnified virtual image as produced by a convex lens is 20 cm. The focal length of the lens used is _______ cm.

**Ans.** **[15]**

**Sol.** $|m| = \frac{v}{u} \Rightarrow v = 3u; |v| - |u| = 20$ cm

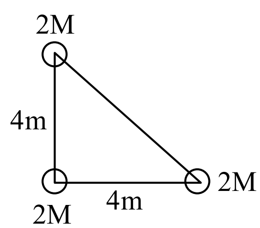
$u = -10$ cm, $v = 30$ cm

$\therefore \frac{1}{f} = \frac{1}{v} - \frac{1}{u} \Rightarrow \frac{1}{f} = \frac{-1}{30} + \frac{1}{10} \Rightarrow f = 15$ cm

**Q.27** The identical spheres each of mass 2M are placed at the corners of a right angled triangle with mutually perpendicular sides equal to 4 m each. Taking point of intersection of these two sides as origin, the magnitude of position vector of the centre of mass of the system is $\frac{4\sqrt{2}}{x}$ where the value of x is _______ .

**Ans.** **[3]**

**Sol.**




$\vec{r}_{com} = \frac{2M(0) + 2M4\hat{i} + 2M4\hat{j}}{6M}$

$\vec{r} = \frac{4}{3}\hat{i} + \frac{4}{3}\hat{j} \Rightarrow |\vec{r}| = \frac{4\sqrt{2}}{3}$

$x = 3$

**Q.28** A tuning fork resonates with a sonometer wire of length 1 m stretched with a tension of 6 N. When the tension in the wire is changed to 54 N, the same tuning fork produces 12 beats per second with it. The frequency of the tuning fork is _______ Hz.

**Ans.** **[6]**

**Sol.** Let $f_0$ = tuning fork frequency.

$\therefore \quad f_0 = \frac{1}{2\ell}\sqrt{\frac{6}{\mu}}$

And $\quad \frac{1}{2\ell}\sqrt{\frac{54}{\mu}} - \frac{1}{2\ell}\sqrt{\frac{6}{\mu}} = 12$

$\therefore \quad \frac{1}{2\ell}\sqrt{\frac{6}{\mu}} = 6$

$\therefore \quad \boxed{f_0 = 6}$

**Q.29** The radius of a nucleus of mass number 64 is 4.8 fermi. Then the mass number of another nucleus having radius of 4 fermi is $\frac{1000}{x}$, where x is _______.

**Ans.** **[27]**

**Sol.** $R = R_0(A)^{1/3} \Rightarrow \frac{R_1}{R_2} = \left(\frac{A_1}{A_2}\right)^{\frac{1}{3}} \Rightarrow \frac{4.8}{4} = \left(\frac{64}{A_2}\right)^{\frac{1}{3}}$

$1.2 = \frac{4}{(A_2)^{\frac{1}{3}}} \Rightarrow A_2^{1/3} = \frac{10}{3} \Rightarrow A_2 = \frac{1000}{27}$

$\Rightarrow \boxed{x = 27}$

**Q.30** A regular polygon of 6 sides is formed by bending a wire of length 4 π meter. If an electric current of $4\pi\sqrt{3}$ A is flowing through the sides of the polygon, the magnetic field at the centre of the polygon would be $x \times 10^{-7}$ T. The value of x is _______.

**Ans.** **[72]**

**Sol.** $B = 6 \times \left(\frac{\mu_0 (4\pi\sqrt{3})}{4\pi \times \left(\frac{\pi}{\sqrt{3}}\right)}(\sin 30 + \sin 30)\right)$

$B = 6 \times \frac{\mu_0 \times 3}{\pi} = \frac{6 \times 4\pi \times 10^{-7} \times 3}{\pi} = 72 \times 10^{-7}$ T

## CHEMISTRY

**Section-A:** This section contains 20 multiple choice questions. Each question has 4 choices(1), (2), (3) and (4), out of which **ONLY ONE** is correct..

**Q.31** Given below are two statements : one is labeled as **Assertion (A)** and the other is labelled as **Reason (R)**.
**Assertion (A) :** $PH_3$ has lower boiling point than $NH_3$.
**Reason (R) :** In liquid state $NH_3$ molecules are associated through van der Waal's forces, but $PH_3$ molecules are associated through hydrogen bonding.
In the light of above statements, choose the **most appropriate** answer from the options given below.
(1) Both **(A)** and **(R)** are correct and **(R)** is the correct explanation of **(A).**
(2) Both **(A)** and **(R)** are correct but **(R)** is not the correct explanation of **(A).**
(3) **(A)** is correct but **(R)** is not correct.
(4) **(A)** is not correct but **(R)** is correct.

**Ans.** **[3]**

**Sol.** $PH_3$ has lower boiling point than $NH_3$ due to lower electronegativity of larger $PH_3$ molecules. They are unable to form hydrogen bonds among themselves.

**Q.32** Which of the following complex is homoleptic?
(1) $[Co(NH_3)_4Cl_2]^+$ (2) $[Ni(NH_3)_2Cl_2]$ (3) $[Fe(NH_3)_4Cl_2]^+$ (4) $[Ni(CN)_4]^{2-}$

**Ans.** **[4]**

**Sol.** Homoleptic complexes are compounds in which all the ligands attached to metal centre are the same.
e.g., $[Ni(CN)_4]^{2-}$

**Q.33** Match List - I with List - II.

| | List - I (Reactions) | | List - II (Reagents) |
|---|---|---|---|
| (A) | $CH_3(CH_2)_5-\underset{\underset{O}{\Vert}}{C}-OC_2H_5 \rightarrow CH_3(CH_2)_5CHO$ | (i) | $CH_3MgBr, H_2O$ |
| (B) | $C_6H_5COC_6H_5 \rightarrow C_6H_5CH_2C_6H_5$ | (ii) | Zn(Hg) and conc. HCl |
| (C) | $C_6H_5CHO \rightarrow C_6H_5CH(OH)CH_3$ | (iii) | $NaBH_4, H^+$ |
| (D) | $CH_3COCH_2COOC_2H_5 \rightarrow CH_3\underset{\underset{H}{\vert}}{C}(OH)CH_2COOC_2H_5$ | (iv) | DIBAL-H, $H_2O$ |

Choose the correct answer from the options given below.

(1) (A)-(IV), (B)-(II), (C)-(III), (D)-(I)
(2) (A)-(III), (B)-(IV), (C)-(II), (D)-(I)
(3) (A)-(IV), (B)-(II), (C)-(I), (D)-(III)
(4) (A)-(III), (B)-(IV), (C)-(I), (D)-(II)

**Ans.** [3]

**Sol.**

$$CH_3(CH_2)_5-\overset{\overset{O}{\Vert}}{C}-OC_2H_5 \xrightarrow{\text{DIBAL-H}} CH_3(CH_2)_5CHO + C_2H_5OH$$

$$C_6H_5COC_6H_5 \xrightarrow[\text{Conc. HCl}]{\text{Zn(Hg) and}} C_6H_5CH_2C_6H_5$$

$$C_6H_5CHO \xrightarrow{CH_3MgBr,\ H_2O} C_6H_5CH(OH)CH_3$$

$$CH_3COCH_2COOC_2H_5 \xrightarrow{NaBH_4/H^+} CH_3\underset{\underset{H}{\vert}}{C}(OH)CH_2COOC_2H_5$$

**Q.34** We have three aqueous solutions of NaCl labeled as ‘A’, ‘B’ and ‘C’ with concentration 0.1 M, 0.01 M and 0.001 M, respectively. The value of van’t Hoff factor (i) for these solutions will be in the order :

(1) $i_A > i_B > i_C$
(2) $i_A < i_B < i_C$
(3) $i_A = i_B = i_C$
(4) $i_A < i_C < i_B$

**Ans.** [3]

**Sol.** For strong electrolyte i does not depends on concentration.

**Q.35** Which of the following reactions are disproportionation reactions?

(A) $Cu^+ \rightarrow Cu^{2+} + Cu$

(B) $3MnO_4^{2-} + 4H^+ \rightarrow 2MnO_4^- + MnO_2 + 2H_2O$

(C) $3KMnO_4 \rightarrow K_2MnO_4 + MnO_2 + O_2$

(D) $2MnO_4^- + 3Mn^{2+} + 2H_2O \rightarrow 5MnO_2 + 4H^+$

Choose the correct answer from the options given below.

(1) (B), (C), (D)
(2) (A), (D)
(3) (A), (B)
(4) (A), (B), (C)

**Ans.** [3]

**Sol.** Disproportionation reaction is a reaction in which, the same element is simultaneously oxidised and reduced.

Oxidation

(A) $Cu^{+} \longrightarrow Cu^{2+} + Cu^{0}$

Reduction

Oxidation

(B) $3\overset{+6}{Mn}O_4^{2-} + 4H^{+} \longrightarrow K_2\overset{+7}{Mn}O_4 + \overset{+4}{Mn}O_2 + O_2$

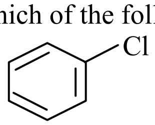
Reduction

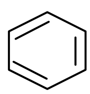
(C) $2K\overset{+7}{Mn}O_4 \longrightarrow K_2\overset{+6}{Mn}O_4 + \overset{+4}{Mn}O_2 + O_2$

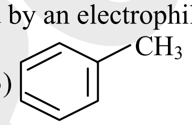
Reduced only

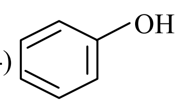
(D) $2\overset{+7}{Mn}O_4^{-} + 3Mn^{2+} + 2H_2O \rightarrow 5\overset{+4}{Mn}O_2 + 4H^{+}$

**Q.36** Which of the following compound will most easily be attacked by an electrophile?

(1) 

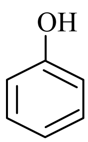

(2) 

(3) 

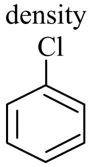

(4) 


**Ans.** **[4]**

**Sol.**




Phenol is most easily attacked by an electrophile because of presence of –OH group which increases electron density at ortho and para position mainly.




Here, chlorine atom shows +R effect o-, p-directive.
But deactivate benzene ring.
–OH and $–CH_3$ are activating and ortho para directing groups.
–OH activates more than $–CH_3$.

**Q.37** Given below are two statements :

**Statement (I) :** The $NH_2$ group in Aniline is ortho and para directing and a powerful activating group.

**Statement (II) :** Aniline does not undergo Friedel-Craft's reaction (alkylation and acylation). In the light of the above statements, choose the **most appropriate** answer from the options given below.

(1) Both **Statement I** and **Statement II** are correct
(2) Both **Statement I** and **Statement II** are incorrect
(3) **Statement I** is incorrect but **Statement II** is correct
(4) **Statement I** is correct but **Statement II** is incorrect

**Ans.** **[1]**

**Sol.**
- $NH_2$ group in aniline is ortho and para directing and a powerful activating group due to its strong electron donating nature.
- Aniline does not undergo Friedel-Craft's reaction (alkylation and acylation) due to salt formation with aluminium chloride which is used as a Lewis acid catalyst.

**Q.38** Given below are two statements :

**Statement (I) :** A solution of $[Ni(H_2O)_6]^{2+}$ is green in colour.

**Statement (II) :** A solution of $[Ni(CN)_4]^{2-}$ is colourless.

In the light of the above statements, choose the **most appropriate** answer from options given below.

(1) Both **Statement I** and **Statement II** are incorrect

(2) **Statement I** is correct but **Statement II** is incorrect

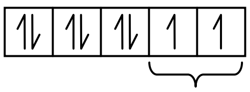
(3) **Statement I** is incorrect but **Statement II** is correct

(4) Both **Statement I** and **Statement II** are correct

**Ans.** **[4]**

**Sol.** $[Ni(H_2O)_6]^{2+}$ is green in colour.

$Ni^{2+}$ has $3d^8$ configuration.



$H_2O$ is weak ligand; no-pairing of unpaired electrons, d-d transition absorbs light and emits green light.

$[Ni(CN)_4]^{2-}$ is colourless.

As $CN^-$ is strong ligand, causes pairing of electrons.

Therefore, d-d transition is not possible.

**Q.39** In case of isoelectronic species the size of $F^-$, Ne and $Na^+$ is affected by :

(1) Electron-electron interaction in the outer orbitals.

(2) Principal quantum number (n)

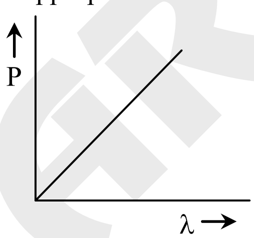
(3) Nuclear charge (z)

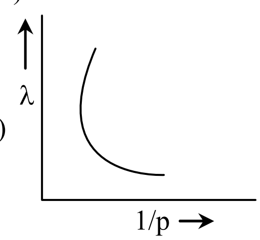
(4) None of the factors because their size is the same

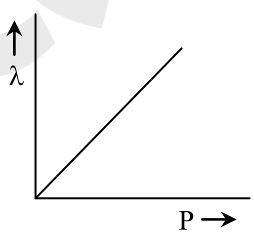
**Ans.** **[3]**

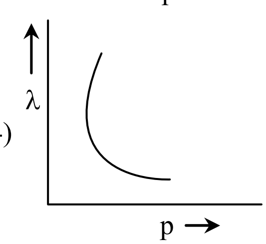
**Sol.** For isoelectronic species, size depends on nuclear charge. More nuclear charge, lesser will be the size of species, this is because the valence electron will experience greater attractive force due to increase in nuclear charge.

**Q.40** According to the wave-particle duality of matter by de-Broglie, which of the following graph plot presents most appropriate relationship between wavelength of electron ($\lambda$) and momentum of electron (p)?

(1)


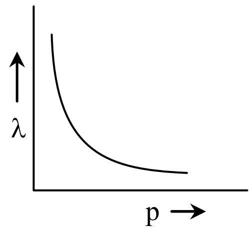

(2)



(3)



(4)



**Ans.** **[4]**

**Sol.**




$\lambda = \frac{h}{p}$

$\lambda \propto \frac{1}{p}$

So, graph between λ and p is rectangular hyperbola.

**Q.41** Arrange the bonds in order of increasing ionic character in the molecules. LiF, $K_2O$, $N_2$, $SO_2$ and $ClF_3$.

(1) $LiF < K_2O < ClF_3 < SO_2 < N_2$
(2) $N_2 < SO_2 < ClF_3 < K_2O < LiF$
(3) $ClF_3 < N_2 < SO_2 < K_2O < LiF$
(4) $N_2 < ClF_3 < SO_2 < K_2O < LiF$

**Ans.** **[2]**

**Sol.** The ionic character of molecule depends on electronegativity difference between atoms of molecule. Greater the difference, greater will be ionic character.
On this basis, the order of increasing ionic character in the given molecule is –
$N_2 < SO_2 < ClF_3 < K_2O < LiF$

**Q.42** Given below are two statements : one is labelled as **Assertion (A)** and the other is labelled as **Reason (R)**.
**Assertion (A) :** Haloalkanes react with KCN to form alkyl cyanides as a main product while with AgCN form isocyanide as the main product.
**Reason (R) :** KCN and AgCN both are highly ionic compounds.
In the light of the above statements, choose the **most appropriate** answer from the options given below.
(1) **(A)** is not correct but **(R)** is correct.
(2) Both **(A)** and **(R)** are correct and **(R)** is the correct explanation of **(A)**.
(3) Both **(A)** and **(R)** are correct but **(R)** is not the correct explanation of **(A)**.
(4) **(A)** is correct but **(R)** is not correct.

**Ans.** **[4]**

**Sol.** $R - X + KCN \rightarrow R - CN + KX$
$R - X + AgCN \rightarrow R - NC + AgX$
KCN is ionic so it provides cyanide ions in solution and attacks from carbon side on alkyl halide.
But AgCN is covalent it cannot form cyanide ion so it attacks from nitrogen side and isocyanide is formed predominantly.
A is correct R is not correct.

**Q.43** In acidic medium, $K_2Cr_2O_7$ shows oxidising action as represented in the half reaction :
$Cr_2O_7^{2-} + XH^+ + Ye^{\ominus} \rightarrow 2A + ZH_2O$
X, Y, Z and A are respectively:

(1) 8, 4, 6 and $Cr_2O_3$
(2) 14, 6, 7 and $Cr^{3+}$
(3) 14, 7, 6 and $Cr^{3+}$
(4) 8, 6, 4 and $Cr_2O_3$

**Ans.** **[2]**

**Sol.** In acidic medium
$K_2Cr_2O_7 + 14H^+ + 6e^- \rightarrow 2Cr^{+3} + 7H_2O$
X = 14
Y = 6
Z = 7
$A = Cr^{+3}$

**Q.44** Given below are two statements :

**Statement (I):** Aminobenzene and aniline are same organic compounds.

**Statement (II):** Aminobenzene and aniline are different organic compounds.

In the light of the above statements, choose the **most appropriate** answer from the options given below.

(1) **Statement I** is correct but **Statement II** is incorrect.

(2) Both **Statement I** and **Statement II** are incorrect.

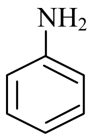
(3) **Statement I** is incorrect but **Statement II** is correct.

(4) Both **Statement I** and **Statement II** are correct.

**Ans.** **[1]**

**Sol.** Aminobenzene and aniline are same organic compound.

Aniline is also known as aminobenzene or phenylamine.



because aniline has amino group in its structure.

**Q.45** In Kjeldahl's method for estimation of nitrogen,

$CuSO_4$ acts as :

(1) Oxidising agent (2) Catalytic agent (3) Reducing agent (4) Hydrolysis agent

**Ans.** **[2]**

**Sol.** In Kjeldahl's Method $CuSO_4$ acts as a catalytic agent.

Organic compound + $H_2SO_4 \xrightarrow{CuSO_4} (NH_4)_2SO_4$

**Q.46** Given below are two statements :

**Statement (I) :** Potassium hydrogen phthalate is a primary standard for standardisation of sodium hydroxide solution.

**Statement (II) :** In this titration phenolphthalein can be used as indicator.

In the light of the above statements, choose the **most appropriate** answer from the options given below.

(1) Both **Statement I** and **Statement II** are incorrect.

(2) Both **Statement I** and **Statement II** are correct.

(3) **Statement I** is incorrect but **Statement II** is correct.

(4) **Statement I** is correct but **Statement II** is incorrect.

**Ans.** **[2]**

**Sol.** Potassium hydrogen phthalate is a primary standard, generally used to standardize a solution of NaOH.

Indicator used for titration of weak acids is phenolphthalein as it goes from colourless at acidic pH to pink at basic pH.

**Q.47** Choose the correct option for free expansion of an ideal gas under adiabatic condition from the following.

(1) $q = 0, \Delta T < 0, w \neq 0$ (2) $q \neq 0, \Delta T = 0, w = 0$

(3) $q = 0, \Delta T \neq 0, w = 0$ (4) $q = 0, \Delta T = 0, w = 0$

**Ans.** **[4]**

**Sol.** In adiabatic free expansion, there is no external pressure for gas to expand.

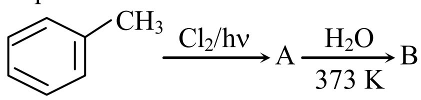
So, work done is zero.

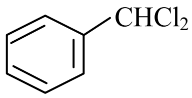
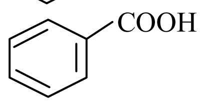
$w = 0, \Delta T = 0.$

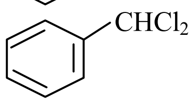
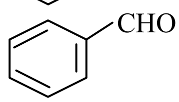
For adiabatic process, no heat is exchanged.

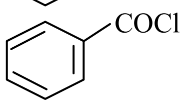
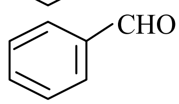
$q = 0$

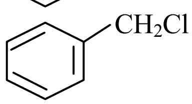
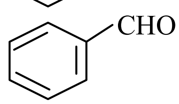
**Q.48** Identify A and B in the following sequence of reaction.



(1) A=  B = 

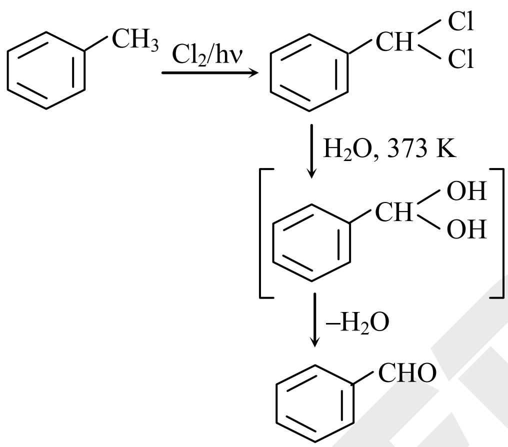
(2) A=  B = 

(3) A=  B = 

(4) A=  B = 

**Ans.** **[2]**

**Sol.**



**Q.49** If one strand of a DNA has the sequence ATGCTTCA, sequence of the bases in complementary strand is

(1) TACGAAGT (2) CATTAGCT (3) GTACTTAC (4) ATGCGACT

**Ans.** **[1]**

**Sol.** Adenine forms hydrogen bonds with thymine whereas cytosine forms hydrogen bonds with guanine.

Given sequence → [A T G C T T C A]

↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓

Complementary strand → [T A C G A A G T]

**Q.50** Ionic reactions with organic compounds proceed through

(A) Homolytic bond cleavage

(B) Heterolytic bond cleavage

(C) Free radical formation

(D) Primary free radical

(E) Secondary free radical

Choose the correct answer from the options given below.

(1) (C) only (2) (D) and (E) only (3) (A) only (4) (B) only

**Ans.** **[4]**

**Sol.** Ionic reactions occur when covalent bond between two atoms undergoes heterolytic cleavage by transferring electrons and in process forms positively and negatively charged ions.

Heterolytic cleavage

$CH_3$—Br ⟶ $H_3C^+$ + $Br^-$

Ionic compound Carbocation Carbonium ion

**Section-B: Numerical Value Type Questions:** This section contains 10 Numerical based questions. Attempt any 5 questions out of 10. The answer to each question should be rounded-off to the nearest integer.

**Q.51** Ka for $CH_3COOH$ is $1.8 \times 10^{-5}$ and $K_b$ for $NH_4OH$ is $1.8 \times 10^{-5}$. The pH of ammonium acetate solution will be ______.

**Ans.** **[7]**

**Sol.** $K_a$ for $CH_3COOH = 1.8 \times 10^{-5}$

$K_b$ for $NH_4OH = 1.8 \times 10^{-5}$

$pK_a = 4.74$; $pK_b = 4.74$

pH of $CH_3COONH_4 = \frac{1}{2}(pK_w + pK_a - pK_b)$

$pH = \frac{1}{2}(14 + 4.74 - 4.74) = 7$

**Q.52** Among the following oxides of p-block elements, number of oxides having amphoteric nature is ________.

$Cl_2O_7$, $CO$, $PbO_2$, $N_2O$, $NO$, $Al_2O_3$, $SiO_2$, $N_2O_5$, $SnO_2$

**Ans.** **[3]**

**Sol.** Amphoteric oxides are those which can react with both acids and bases.

Amphoteric oxides are: $PbO_2$, $Al_2O_3$, $SnO_2$

CO, NO and $N_2O$ are neutral.

$Cl_2O_7$, $SiO_2$ and $N_2O_5$ are acidic

**Q.53** The lowest oxidation number of an atom in a compound $A_2B$ is –2. The number of electrons in its valence shell is _______.

**Ans.** **[6]**

**Sol.** In compound $A_2B$

Lowest oxidation state of B is –2

It means it has $6e^-$ in its valence shell.

**Q.54** The ratio of $\frac{^{14}C}{^{12}C}$ in a piece of wood is $\frac{1}{8}$ part that of atmosphere. If half life of $^{14}C$ is 5730 years, the age of wood sample is ________ years.

**Ans.** **[17190]**

**Sol.** $N = N_0e^{-\lambda t}$

$\frac{1}{8} = 1e^{-\lambda t}$

$e^{\lambda t} = 8$

$\lambda t = \ln 8$

$\frac{0.693}{t_{1/2}} t = \ln 8$

$t = \frac{\ln 8}{\ln 2} \times t_{1/2}$

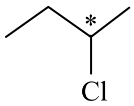
$t = 3 \times 5730$ years

$= 17190$ years

**Q.55** Number of optical isomers possible for 2- chlorobutane _______.

**Ans.** **[2]**

**Sol.** 2-chlorobutane




2-chlorobutane contains only one chiral centre. So,
it can show two optical isomers

**Q.56** Consider the following reaction:

$3PbCl_2 + 2(NH_4)_3PO_4 \rightarrow Pb_3(PO_4)_2 + 6NH_4Cl$

If 72 mmol of $PbCl_2$ is mixed with 50 mmol of $(NH_4)_3PO_4$, then the amount of $Pb_3(PO_4)_2$ formed is _______ mmol (nearest integer).

**Ans.** **[24]**

**Sol.** $\underset{72\,\text{mmol}}{3PbCl_2} + \underset{50\,\text{mmol}}{2(NH_4)_3PO_4} \rightarrow \underset{-}{Pb_3(PO_4)_2} + \underset{-}{6NH_4Cl}$

$PbCl_2$ is limiting reagent

3 mol $PbCl_2$ produces 1 mol of $Pb_3(PO_4)_2$

72 mmol of $PbCl_2$ will produce

$\frac{1}{3} \times 72$ mmol of $Pb_3(PO_4)_2$

= 24 mmol

**Q.57** The number of white coloured salts, among the following is _______.

(a) $SrSO_4$ (b) $Mg(NH_4)PO_4$

(c) $BaCrO_4$ (d) $Mn(OH)_2$

(e) $PbSO_4$ (f) $PbCrO_4$

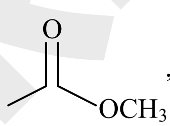
(g) AgBr (h) $PbI_2$

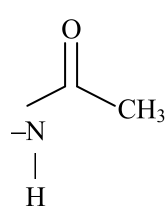
(i) $CaC_2O_4$ (j) $[Fe(OH)_2(CH_3COO)]$

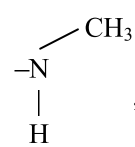
**Ans.** **[5]**

**Sol.** $SrSO_4$, $PbSO_4$, $Mg(NH_4)PO_4$, $Mn(OH)_2$ and $CaC_2O_4$ are white coloured salts.

**Q.58** Total number of deactivating groups in aromatic electrophilic substitution reaction among the following is _______.



,


,


, –C≡N, $-OCH_3$

**Ans.** **[2]**

**Sol.**

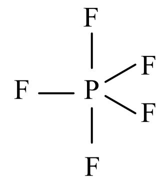
$-\overset{\overset{\displaystyle O}{\|}}{C}-OCH_3$, –C≡N are deactivating groups in aromatic electrophilic substitution reaction, because they are –R group which pulls electron density towards themselves.

**Q.59** The number of molecules/ions having trigonal bipyramidal shape is _______.

$PF_5$, $BrF_5$, $PCl_5$, $[Pt\,Cl_4]^{2-}$, $BF_3$, $Fe(CO)_5$

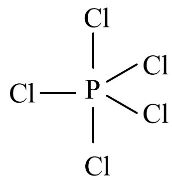
**Ans.** [3]

**Sol.**



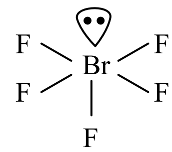
$sp^3d$

(trigonal bipyramidal)



$sp^3d$

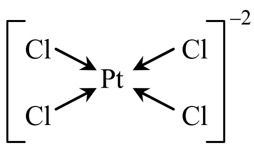
(trigonal bipyramidal)



$sp^3d^2$

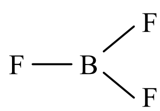
(square pyramidal)

$[PtCl_4]^{-2}$



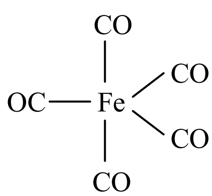
$dsp^2$

square planar



$sp^2$

triangular planar



$sp^3d$

(trigonal bipyramidal)

**Q.60** The potential for the given half cell at 298 K is (–) ________ $\times 10^{-2}$ V.

$2H^+_{(aq)} + 2e^- \rightarrow H_2(g)$

$[H^+] = 1M,\ P_{H_2} = 2atm$

(Given : 2.303RT/F = 0.06 V, log2 = 0.3)

**Ans.** [1]

**Sol.** $E^o_{H^+/H_2} = 0V$

$2H^+_{(aq)} + 2e^- \rightarrow H_2(g)$

$$E_{Half\ Cell} = E^o_{Half\ Cell} - \frac{0.06}{n}\log\frac{p_{H_2}}{[H^+]^2}$$

$$E_{half\ Cell} = 0 - \frac{0.06}{2}\log\frac{2}{[1]^2}$$

$E_{Half\ Cell} = -\ 0.03 \times 0.30$

$= -0.009$

$= 0.9 \times 10^{-2}$

$0.9 \approx 1$

## MATHEMATICS

**Section-A:** This section contains 20 multiple choice questions. Each question has 4 choices(1), (2), (3) and (4), out of which **ONLY ONE** is correct..

**Q.61** Let $S = \{z \in C : |z - 1| = 1 \text{ and } (\sqrt{2} - 1)(z + \bar{z}) - i(z - \bar{z}) = 2\sqrt{2}\}$. Let $z_1, z_2 \in S$ be such that $|z_1| = \max_{z \in S} |z|$ and $|z_2| = \min_{z \in S} |z|$. Then $|\sqrt{2}z_1 - z_2|^2$ equals :

(1) 2 (2) 4 (3) 3 (4) 1

**Ans.** **[1]**

**Sol.** Let $z = x + iy$

$|z - 1| = 1 \Rightarrow |x + iy - 1| = 1$

$(x - 1)^2 + y^2 = 1$ …(1)

$(\sqrt{2} - 1)(z + \bar{z}) - i(z - \bar{z}) = 2\sqrt{2}$ (Given)

$(\sqrt{2} - 1)(2x) - i(2iy) = 2\sqrt{2}$

$(\sqrt{2} - 1)x + y = \sqrt{2}$ …(2)

Solving (1) and (2), we get

$(x-1)^2 + (\sqrt{2} - (\sqrt{2} - 1)x)^2 = 1$

$(x^2 - 2x + 1) + 2 + (\sqrt{2} - 1)^2 x^2 - 2\sqrt{2}(\sqrt{2} - 1)x = 0$

$x^2(1 + (\sqrt{2} - 1)^2) + x(-2 - 2\sqrt{2}(\sqrt{2} - 1)) + 2 = 0$

$x^2(4 - 2\sqrt{2}) + x(2\sqrt{2} - 6) + 2 = 0$

$x^2(2 - \sqrt{2}) + x(\sqrt{2} - 3) + 1 = 0$

$\Rightarrow x = 1 \text{ and } x = \dfrac{1}{2 - \sqrt{2}}$ …(3)

When $x = 1$, $y = 1 \Rightarrow z_2 = 1 + i$

When $x = \dfrac{1}{2 - \sqrt{2}}$, $y = \sqrt{2} - \dfrac{1}{\sqrt{2}}$

$\Rightarrow z_1 = \left(1 + \dfrac{1}{\sqrt{2}}\right) + \dfrac{i}{\sqrt{2}}$

Now,

$$|\sqrt{2}z_1 - z_2|^2 = \left|\left(\frac{1}{\sqrt{2}} + 1\right)\sqrt{2} + i - (1 + i)\right|^2 = (\sqrt{2})^2 = 2$$

**Q.62** Let $S = \{x \in R : (\sqrt{3} + \sqrt{2})^x + (\sqrt{3} - \sqrt{2})^x = 10\}$.

Then the number of elements in S is :

(1) 2 (2) 1 (3) 4 (4) 0

**Ans.** **[1]**

**Sol.** $(\sqrt{3} + \sqrt{2})^x = t$

$t + \dfrac{1}{t} = 10$

$\Rightarrow \quad t^2 - 10t + 1 = 0$

$\Rightarrow \quad t = \dfrac{10 \pm \sqrt{96}}{2}$

$\Rightarrow \quad t = 5 \pm 2\sqrt{6}$

$\Rightarrow \quad (\sqrt{3} + \sqrt{2})^x = 5 \pm 2\sqrt{6}$

$\Rightarrow \quad x = \pm 2$

**Q.63** Let $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, a > b be an ellipse, whose eccentricity is $\frac{1}{\sqrt{2}}$ and the length of the latusrectum is $\sqrt{14}$.

Then the square of the eccentricity of $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is :

(1) $\frac{3}{2}$ (2) $\frac{5}{2}$ (3) 3 (4) $\frac{7}{2}$

**Ans.** **[1]**

**Sol.** $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \quad a>b$

$e=\frac{1}{\sqrt{2}}$

$\Rightarrow 1-\frac{b^2}{a^2}=\frac{1}{2}$

$\Rightarrow \frac{b^2}{a^2}=\frac{1}{2}$

Also, $\frac{2b^2}{a}=\sqrt{14}$

$\Rightarrow \frac{2b^2}{a^2}\times a=\sqrt{14}$

$\Rightarrow 2\times\frac{1}{2}\times a=\sqrt{14}$

$\Rightarrow a=\sqrt{14}$

$\Rightarrow b^2=7$

Now, $\frac{x^2}{14}-\frac{y^2}{7}=1$

$e_1^2=1+\frac{b^2}{a^2}$

$e_1^2=1+\frac{7}{14}$

$\boxed{e_1^2=\frac{3}{2}}$

**Q.64** For $0<\theta<\pi/2$, if the eccentricity of the hyperbola $x^2-y^2\text{cosec}^2\theta=5$ is $\sqrt{7}$ times eccentricity of the ellipse $x^2\text{cosec}^2\theta+y^2=5$, then the value of θ is :

(1) $\frac{\pi}{4}$ (2) $\frac{5\pi}{12}$ (3) $\frac{\pi}{3}$ (4) $\frac{\pi}{6}$

**Ans.** **[3]**

**Sol.** $x^2-y^2\text{cosec}^2\theta=5$

$\Rightarrow \frac{x^2}{5}-\frac{y^2}{5\sin^2\theta}=1$

$e_H=\sqrt{1+\sin^2\theta}$

and $x^2\text{cosec}^2\theta+y^2=5$

$\Rightarrow \frac{x^2}{5\sin^2\theta}+\frac{y^2}{5}=1$

$e_E = \sqrt{1-\sin^2\theta}$

$e_H = \sqrt{7}\, e_E$

$\sqrt{1+\sin^2\theta} = \sqrt{7}\sqrt{1-\sin^2\theta}$

$\Rightarrow 1 + \sin^2\theta = 7 - 7\sin^2\theta$

$\Rightarrow 8\sin^2\theta = 6$

$\Rightarrow \sin\theta = \frac{\sqrt{3}}{2}$

$\Rightarrow \theta = \frac{\pi}{3} \quad \left[\because 0 < \theta < \frac{\pi}{2}\right]$

So, option (3) is correct

**Q.65** Let f : **R** → **R** and g : **R** → **R** be defined as $f(x) = \begin{cases} \log_e x, & x > 0 \\ e^{-x}, & x \le 0 \end{cases}$ and $g(x) = \begin{cases} x, & x \ge 0 \\ e^{x}, & x < 0 \end{cases}$. Then, gof : R → R is :

(1) Neither one-one nor onto
(2) Onto but not one-one
(3) Both one-one and onto
(4) One-one but not onto

**Ans.** **[1]**

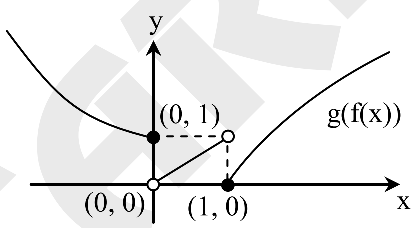
**Sol.** Given $f(x) = \begin{cases} e^{-x} & ; \quad x \le 0 \\ \ln x & ; \quad x > 0 \end{cases}$

and $g(x) = \begin{cases} x & ; \quad x \ge 0 \\ e^{x} & ; \quad x < 0 \end{cases}$

then gof(x)

$= g(f(x)) = \begin{cases} f(x) & ; \quad f(x) \ge 0 \\ e^{f(x)} & ; \quad f(x) < 0 \end{cases}$

$= \begin{cases} e^{-x} & ; \quad x \le 0 \\ x & ; \quad 0 < x < 1 \\ \ln x & ; \quad x \ge 1 \end{cases}$




$\therefore$ f(x) is neither one-one nor onto.

**Q.66** Let the median and the mean deviation about the median of 7 observation 170, 125, 230, 190, 210, a, b be 170 and $\frac{205}{7}$ respectively. Then the mean deviation about the mean of these 7 observations is :

(1) 31
(2) 32
(3) 28
(4) 30

**Ans.** **[4]**

**Sol.** Mean deviation about median = $\frac{0+45+60+20+40+170-a+170-b}{7} = \frac{205}{7}$

$\Rightarrow$ a + b = 300

Mean = $\frac{170+125+230+190+210+a+b}{7} = 175$

Mean deviation about mean = $\frac{50+175-a+175-b+5+15+35+55}{7} = 30$

**Q.67** If $\tan A = \frac{1}{\sqrt{x(x^2+x+1)}}$, $\tan B = \frac{\sqrt{x}}{\sqrt{x^2+x+1}}$ and $\tan C = (x^{-3}+x^{-2}+x^{-1})^{1/2}$, $0 < A, B, C < \frac{\pi}{2}$, then A + B is equal to :

(1) $2\pi - C$ (2) C (3) $\frac{\pi}{2} - C$ (4) $\pi - C$

**Ans.** **[2]**

**Sol.** $\tan A = \frac{1}{\sqrt{x(x^2+x+1)}}$, $\tan B = \frac{\sqrt{x}}{\sqrt{x^2+x+1}}$ and $\tan C = (x^{-3}+x^{-2}+x^{-1})^{1\backslash 2}$

$$A = \tan^{-1}\left(\frac{1}{\sqrt{x(x^2+x+1)}}\right) \text{ and } B = \tan^{-1}\left(\frac{\sqrt{x}}{\sqrt{x^2+x+1}}\right)$$

$$A + B = \tan^{-1}\left(\frac{\frac{1+x}{\sqrt{x(x^2+x+1)}}}{\left(1-\frac{1}{x^2+x+1}\right)}\right) = \tan^{-1}\left(\frac{\frac{1+x}{\sqrt{x(x^2+x+1)}}}{\left(\frac{x^2+x}{x^2+x+1}\right)}\right)$$

$$= \tan^{-1}\left(\frac{1}{\sqrt{x(x^2+x+1)}} \times \frac{x^2+x+1}{x}\right) = \tan^{-1}\left(\frac{\sqrt{x^2+x+1}}{x^{3/2}}\right) = \tan^{-1}C$$

$\therefore$ A + B = C

**Q.68** If n is the number of ways five different employees can sit into four indistinguishable offices where any office may have any number of persons including zero, then n is equal to :

(1) 43 (2) 53 (3) 47 (4) 51

**Ans.** **[4]**

**Sol.** Since rooms are identical so we can distribute in following way

| | (1) | (2) | (3) | (4) |
|---|---|---|---|---|
| 1 way = 1 | 0 | 0 | 0 | 5 |
| $\frac{5!}{4!1!}$ ways = 5 | 0 | 0 | 1 | 4 |
| $\frac{5!}{2!3!}$ ways = 10 | 0 | 0 | 2 | 3 |
| $\frac{5!}{3!1!1!} \times \frac{1}{2!} = 10$ | 0 | 1 | 1 | 3 |
| $\frac{5!}{1!2!2!2!} = 15$ | 0 | 1 | 2 | 2 |
| $\frac{5!}{1!1!2!} \times \frac{1}{3!} = 10$ | 1 | 1 | 1 | 2 |

$\therefore$ Total ways = 1 + 5 + 10 + 10 + 15 + 10 = 51

**Q.69** If the system of equations

$2x + 3y - z = 5$

$x + \alpha y + 3z = -4$

$3x - y + \beta z = 7$

has infinitely many solutions, then $13\alpha\beta$ is equal to ________ .

(1) 1220 (2) 1110 (3) 1120 (4) 1210

**Ans.** [3]

**Sol.** Given $2x + 3y - z = 5$

$x + \alpha y + 3z = -4$

$3x - y + \beta z = 7$

$$\Delta_2 = \begin{vmatrix} 2 & -1 & 5 \\ 1 & 3 & -4 \\ 3 & \beta & 7 \end{vmatrix}$$

$\Delta_2 = 2(21 + 4\beta) + 1(7 + 12) + 5(\beta - 9)$

$\Delta_2 = 42 + 8\beta + 19 + 5\beta - 45$

$\Delta_2 = 13\beta + 16$

$\Delta_2 = 0$

$\therefore \boxed{\beta = -\frac{16}{13}}$

$$\Delta_3 = \begin{vmatrix} 2 & 3 & 5 \\ 1 & \alpha & -4 \\ 3 & -1 & 7 \end{vmatrix}$$

$\Delta_3 = 2(7\alpha - 4) - 3(7 + 12) + 5(-1 - 3\alpha)$

$\Delta_3 = 14\alpha - 8 - 57 - 5 - 15\alpha$

$\Delta_3 = -\alpha - 70$

$\Delta_3 = 0$

$\boxed{\alpha = -70}$

$$13\alpha\beta = (13)(-70)\left(\frac{-16}{13}\right) = +1120$$

**Q.70** A bag contains 8 balls, whose colours are either white or black. 4 balls are drawn at random without replacement and it was found that 2 balls are white and other 2 balls are black. The probability that the bag contains equal number of white and black balls is :

(1) $\frac{2}{5}$ (2) $\frac{1}{7}$ (3) $\frac{1}{5}$ (4) $\frac{2}{7}$

**Ans.** [4]

**Sol.** P(2W and 2B) = P(2B, 6W) × P(2W and 2B) + P(3B, 5W) × P(2W and 2B) + P(4B, 4W) × P(2W and 2B) + P(5B, 3W) × P(2W and 2B) + P(6B, 2W) × P(2W and 2B)

$$\frac{1}{9}\left(0 + 0 + \frac{{}^2C_2 \times {}^6C_2}{{}^8C_4} + \frac{{}^3C_2 \times {}^5C_2}{{}^8C_4} + \frac{{}^4C_2 \times {}^4C_2}{{}^8C_2} + \frac{{}^5C_2 \times {}^3C_2}{{}^8C_4} + \frac{{}^8C_2 \times {}^2C_2}{{}^8C_4} + 0 + 0\right)$$

$$= \frac{1}{9} \times \frac{1}{{}^8C_4}(15 + 30 + 36 + 30 + 15) = \frac{1}{9} \times \frac{1}{{}^8C_4} \times 126$$

$$P\left(\frac{4B \text{ and } 4W}{2W \text{ and } 2B}\right) = \frac{\frac{1}{9} \times \frac{{}^4C_2 \times {}^4C_2}{{}^8C_4}}{\frac{1}{9} \times \frac{1}{{}^8C_4 \times 126}} = \frac{36}{126} = \frac{6}{21} = \frac{2}{7}$$

**Q.71** The value of the integral $\int_0^{\pi/4} \frac{x\,dx}{\sin^4(2x)+\cos^4(2x)}$ equals :

(1) $\frac{\sqrt{2}\pi^2}{64}$ (2) $\frac{\sqrt{2}\pi^2}{8}$ (3) $\frac{\sqrt{2}\pi^2}{32}$ (4) $\frac{\sqrt{2}\pi^2}{16}$

**Ans.** **[3]**

**Sol.** $\int_0^{\pi/4} \frac{x\,dx}{\sin^4(2x)+\cos^4(2x)}$

Take $I = \int_0^{\pi/4} \frac{x\,dx}{\sin^4(2x)+\cos^4(2x)}$

Let $2x = t$

$2dx = dt$

$dx = \frac{dt}{2}$

$$I = \int_0^{\pi/2} \frac{t/2 \cdot \frac{dt}{2}}{\sin^4 t + \cos^4 t}$$

$$I = \frac{1}{4}\int_0^{\pi/2} \frac{t\,dt}{\sin^4 t + \cos^4 t} = \frac{1}{4}\int_0^{\pi/2} \frac{\left(\frac{\pi}{2}-t\right)dt}{\sin^4(\pi/2-t)+\cos^4(\pi/2-t)} = \frac{1}{4}\int_0^{\pi/2} \frac{\left(\frac{\pi}{2}-t\right)}{\sin^4 t + \cos^4 t}$$

$$2I = \frac{1}{4}\int_0^{\pi/2} \frac{\frac{\pi}{2}}{\sin^4 t + \cos^4 t}dt$$

$$2I = \frac{\pi}{8}\int_0^{\pi/2} \frac{dt}{\sin^4 t + \cos^4 t}$$

$$2I = \frac{\pi}{8}\int_0^{\pi/2} \frac{\sec^4 t}{1+\tan^4 t}dt$$

Put $\tan t = y$

$\sec^2 t\,dt = dy$

$$2I = \frac{\pi}{8}\int_0^{\infty} \frac{(1+y^2)\,dy}{1+y^4} = \frac{\pi}{8}\int_0^{\infty} \frac{1+\frac{1}{y^2}}{y^2+\frac{1}{y^2}-2+2}dy$$

$$\frac{\pi}{8}\int_0^{\infty} \frac{\left(1+\frac{1}{y^2}\right)dy}{2+\left(y-\frac{1}{y}\right)^2}$$

Put, $y - \frac{1}{y} = 4$

$$2I = \frac{\pi}{8}\int_{-\infty}^{\infty} \frac{du}{2+u^2} = \frac{\pi}{8\sqrt{2}}\left[\tan^{-1}\frac{y}{\sqrt{2}}\right]_{-\infty}^{\infty} = \frac{\sqrt{2}\pi^2}{32}$$

**Q.72** Let 3, a, b, c be in A.P. and 3, a – 1, b + 1, c + 9 be in G.P. Then, the arithmetic mean of a, b and c is :

(1) –4 (2) –1 (3) 11 (4) 13

**Ans.** **[3]**

**Sol.** 3, a, b, c be in A.P.

$a - 3 = b - a = c - b$

3, a – 1, b + 1, c + 9 in G.P.

$\frac{a-1}{3} = \frac{b+1}{a-1} = \frac{c+9}{b+1}$ $(a \neq 1, b = -1)$

$(a - 1)^2 = 3(b + 1)$

$a^2 + 1 - 2a = 3b + 3$

$a^2 - 8a + 7 = 0$

$(a - 1)(a - 7) = 0$

$a = 7$ as $a \neq 1$

$b = 2a - 3 = 14 - 3 = 11$

$c = 2b - a = 22 - 7 = 15$

Mean of a, b, c

$\frac{a+b+c}{3} = \frac{7+11+15}{3} = \frac{33}{3} = 11$

**Q.73** If $5f(x) + 4f\left(\frac{1}{x}\right) = x^2 - 2, \forall\ x \neq 0$ and $y = 9x^2 f(x)$, then y is strictly increasing in :

(1) $\left(-\infty, \frac{1}{\sqrt{5}}\right) \cup \left(0, \frac{1}{\sqrt{5}}\right)$ (2) $\left(-\frac{1}{\sqrt{5}}, 0\right) \cup \left(0, \frac{1}{\sqrt{5}}\right)$

(3) $\left(0, \frac{1}{\sqrt{5}}\right) \cup \left(\frac{1}{\sqrt{5}}, \infty\right)$ (4) $\left(-\frac{1}{\sqrt{5}}, 0\right) \cup \left(\frac{1}{\sqrt{5}}, \infty\right)$

**Ans.** **[4]**

**Sol.** $5f(x) + 4f(1/x) = x^2 - 2$ …(1)

Replace x by 1/x

$5f(1/x) + 4f(x) = \frac{1}{x^2} - 2$ …(2)

Multiply equation (1) by 5 and multiply equation (2) by 4 and then subtract equation (2) from (1)

$25f(x) - 16f(x) = 5x^2 - 10 - \frac{4}{x^2} + 8$

$9f(x) = 5x^2 - \frac{4}{x^2} - 2$

$9f(x) = \frac{5x^4 - 4 - 2x^2}{x^2}$

$y = 9x^2 f(x)$

$y = 5x^4 - 2x^2 - 4$

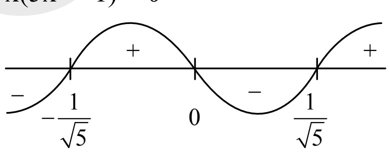
$y' = 20x^3 - 4x$

Put $y' > 0$

$20x^3 - 4x > 0$

$5x^3 - x > 0$

$x(5x^2 - 1) > 0$




$x \in \left(-\frac{1}{\sqrt{5}}, 0\right) \cup \left(\frac{1}{\sqrt{5}}, \infty\right)$

**Q.74** Let $\vec{a} = -5\hat{i} + \hat{j} - 3\hat{k}$, $\vec{b} = \hat{i} + 2\hat{j} - 4\hat{k}$ and $\vec{c} = (((\vec{a} \times \vec{b}) \times \hat{i}) \times \hat{i}) \times \hat{i}$. Then $\vec{c}\,.\,(-\hat{i} + \hat{j} + \hat{k})$ is equal to :

(1) –10 (2) –15 (3) –12 (4) –13

**Ans.** **[3]**

**Sol.** $\vec{a} = -5\cdot\hat{i} + \hat{j} - 3\hat{k}$, $\vec{b} = \hat{i} + 2\hat{j} - 4\hat{k}$

$\vec{c} = (((\vec{a} \times \vec{b}) \times \hat{i}) \times \hat{i}) \times \hat{i}$

$= (((\vec{a}\cdot\hat{i})\vec{b} - (\vec{b}\cdot\hat{i})\vec{a}) \times \hat{i}) \times \hat{i}$

$= ((-5\vec{b} - \vec{a}) \times \hat{i}) \times \hat{i}$

$= (\vec{u} \times \hat{i}) \times \hat{i}$

$\{\vec{u} = -(5\vec{b} + \vec{a}) = 0\cdot\hat{i} - 11\hat{j} + 23\hat{k}\}$

$= (\vec{u}\cdot\hat{i})\hat{i} - (\hat{i}\cdot\hat{i})\cdot\vec{u}$

$= 0 - \vec{u} = 0\cdot\hat{i} + 11\hat{j} - 23\hat{k}$

$\vec{c}\cdot(-\hat{i} + \hat{j} + \hat{k}\} = 0 + 11 - 23 = -12$

**Q.75** If $A = \begin{bmatrix} \sqrt{2} & 1 \\ -1 & \sqrt{2} \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$, $C = ABA^T$ and $X = A^TC^2A$, then det X is equal to :

(1) 243 (2) 891 (3) 729 (4) 27

**Ans.** **[3]**

**Sol.** $C = A\cdot B\cdot A^T \Rightarrow |C| = |A|\cdot|B|\cdot|A^T|$

$= |A|^2\cdot|B|$ ...(i)

$X = A^T\cdot C^2\cdot A \Rightarrow |X| = |A^T|\cdot|C|^2\cdot|A|$

$= (|A|\cdot|C|)^2$ ...(ii)

From (i) and (ii)

$|X| = (|A|\cdot|A|^2\cdot|B|)^2$

$= (|A|^3\cdot|B|)^2$ ...(iii)

$|A| = \begin{vmatrix} \sqrt{2} & 1 \\ -1 & \sqrt{2} \end{vmatrix} = 2 + 1 = 3$ ...(iv)

$|B| = \begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix} = 1$ ...(v)

From (iii), (iv) and (v)

$|X| = 729$

**Q.76** The area enclosed by the curves $xy + 4y = 16$ and $x + y = 6$ is equal to :

(1) $32 - 30\log_e 2$ (2) $30 - 32\log_e 2$ (3) $30 - 28\log_e 2$ (4) $28 - 30\log_e 2$

**Ans.** **[2]**

**Sol.** $xy + 4y = 16$ ; $x + y = 6$

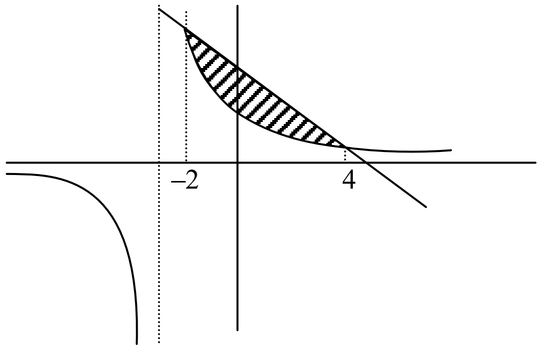
$y(x + 4) = 16$ ; $x + y = 6$

$\Rightarrow (6 - x)(x + 4) = 16$ $y = 6 - x$

$\Rightarrow -x^2 + 2x + 24 = 16$

$\Rightarrow$ $x^2 - 2x - 8 = 0$

$\Rightarrow$ $x = 4, x = -2$




$$\text{Area} = \int_{-2}^{4}\left[(6-x)-\left(\frac{16}{x+4}\right)\right]dx$$

$= 30 - 32\ln 2$

**Q.77** If the shortest distance between the lines $\frac{x-\lambda}{-2} = \frac{y-2}{1} = \frac{z-1}{1}$ and $\frac{x-\sqrt{3}}{1} = \frac{y-1}{-2} = \frac{z-2}{1}$ is 1, then the sum of all possible values of $\lambda$ is :

(1) 0 (2) $2\sqrt{3}$ (3) $3\sqrt{3}$ (4) $-2\sqrt{3}$

**Ans.** **[2]**

**Sol.** Shortest distance

$$= \left|\frac{\vec{V}_1\times\vec{V}_2}{|\vec{V}_1\times\vec{V}_2|}\cdot\overrightarrow{AB}\right| = 1$$

where $V_1 = -2\hat{i} + \hat{j} + \hat{k}$, $\overrightarrow{AB} = (\lambda - \sqrt{3}, 2-1, 1-2)$

$V_2 = \hat{i} - 2\hat{j} + \hat{k}$

$$\vec{V}_1\times\vec{V}_2 = \begin{vmatrix} i & j & k \\ -2 & 1 & 1 \\ 1 & -2 & 1 \end{vmatrix} = 3\hat{i} + 3\hat{j} + 3\hat{k}$$

$\Rightarrow$ $\vec{V}_1\times\vec{V}_2 = 3(\hat{i} + \hat{j} + \hat{k})$

$\Rightarrow$ $|\vec{V}_1\times\vec{V}_2| = 3\sqrt{3}$

$$\Rightarrow \left|\frac{(\hat{i}+\hat{j}+\hat{k}).((\lambda-\sqrt{3})\hat{i}+\hat{j}-\hat{k}}{\sqrt{3}}\right| = 1$$

$|(\lambda - \sqrt{3}) + 1 - 1| = \sqrt{3}$

$\Rightarrow$ $|\lambda - \sqrt{3}| = \sqrt{3}$

$\Rightarrow$ $\lambda - \sqrt{3} = \sqrt{3}$ or $\lambda - \sqrt{3} = -\sqrt{3}$

$\Rightarrow$ $\lambda = 2\sqrt{3}$ or 0

$\Rightarrow$ Sum of $\lambda = 2\sqrt{3}$

**Q.78** Let f : R → R be define as :

$$f(x) = \begin{cases} \dfrac{a - b\cos 2x}{x^2} & ; \quad x < 0 \\ x^2 + cx + 2 & ; \quad 0 \le x \le 1 \\ 2x + 1 & ; \quad x > 1 \end{cases}$$

If f is continuous every where in R and m is the number of points where f is NOT differential then m + a + b + c equal :

(1) 4 (2) 2 (3) 1 (4) 3

**Ans.** **[2]**

**Sol.** f(x) is continuous $\forall\, x \in R$

$\Rightarrow f(0) = f(0^-)$

$\Rightarrow 2 = \lim\limits_{x \to 0^-} \dfrac{a - b\cos 2x}{x^2}$

$\Rightarrow a - b = 0 \Rightarrow a = b$

$\lim\limits_{x \to 0^-} \dfrac{2b\sin 2x}{2x} = 2b = 2 \Rightarrow b = 1 = a$

Also $f(1) = f(1^+)$

$\Rightarrow 1 + c + 2 = 3$

$\Rightarrow c = 0$

$$\Rightarrow f(x) = \begin{cases} \dfrac{1 - \cos 2x}{x^2} & , \quad x < 0 \\ x^2 + 2 & , \quad x \in [0, 1] \\ 2x + 1 & , \quad x > 1 \end{cases}$$

$$f'(x) = \begin{cases} \dfrac{d}{dx}\left[2\left(\dfrac{\sin x}{x}\right)^2\right] & , \quad x < 0 \\ 2x & , \quad x \in [0, 1] \\ 2 & , \quad x > 1 \end{cases}$$

Differentiability at x = 1 holds

$\Rightarrow m = 0$

$m + a + b + c = 0 + 1 + 1 + 0 = 2$

**Q.79** Let y = y(x) be the solution of the differential equation $\dfrac{dy}{dx} = 2x(x + y)^3 - x(x + y) - 1$, y(0) = 1.

Then $\left(\dfrac{1}{\sqrt{2}} + y\left(\dfrac{1}{\sqrt{2}}\right)\right)^2$ equals :

(1) $\dfrac{2}{1 + \sqrt{e}}$ (2) $\dfrac{3}{3 - \sqrt{e}}$ (3) $\dfrac{1}{2 - \sqrt{e}}$ (4) $\dfrac{4}{4 + \sqrt{e}}$

**Ans.** **[3]**

**Sol.** $\frac{dy}{dx} = 2x(x+y)^3 - x(x+y) - 1$

Put $x + y = t$

$\Rightarrow \frac{dy}{dx} = \frac{dt}{dx} - 1$

$\frac{dt}{dx} - 1 = 2x(t)^3 - xt - 1$

$\Rightarrow \frac{dt}{2t^3 - t} = xdx$

$\int \frac{1}{2t^3 - t} dt = \int xdx$

$\Rightarrow \int \frac{t}{2t^4 - t^2} dt = \int xdx$

$t^2 = z$

$2tdt = dz$

$\frac{1}{2}\int \frac{dz}{2z^2 - z} = \int xdx$

$\ln \left| \frac{z - \frac{1}{2}}{z} \right| = x^2 + c$

$\ln \left| \frac{(x+y)^2 - \frac{1}{2}}{(x+y)^2} \right| = x^2 + c$

$y(0) = 1 \Rightarrow c = \ln\left(\frac{1}{2}\right)$

$\Rightarrow \frac{(x+y)^2 - \frac{1}{2}}{(x+y)^2} = e^{x^2} \times \frac{1}{2}$

$\frac{(x+y)^2 - \frac{1}{2}}{(x+y)^2} = \sqrt{e} \times \frac{1}{2}$

$\Rightarrow (x+y)^2 = \frac{1}{2 - \sqrt{e}}$

**Q.80** Let $C : x^2 + y^2 = 4$ and $C' : x^2 + y^2 - 4\lambda x + 9 = 0$ be two circles. If the set of all values of $\lambda$ so that the circles C and C′ intersect at two distinct points, is $R - [a, b]$, then the point $(8a + 12, 16b - 20)$ lies on the curve :

(1) $5x^2 - y = -11$ (2) $x^2 + 2y^2 - 5x + 6y = 3$ (3) $x^2 - 4y^2 = 7$ (4) $6x^2 + y^2 = 42$

**Ans.** **[4]**

**Sol.** $C : x^2 + y^2 = 4 \Rightarrow C(0, 0), r_1 = 2$

$C' : x^2 + y^2 - 4\lambda x + 9 = 0 \Rightarrow C'(2\lambda, 0), r_2 = \sqrt{4\lambda^2 - 9}$

$|r_1 - r_2| < CC' < |r_1 + r_2|$

$|2 - \sqrt{4\lambda^2 - 9}| < |2\lambda| < 2 + \sqrt{4\lambda^2 - 9}$

$|2\lambda| > |2 - \sqrt{4\lambda^2 - 9}|$

$\Rightarrow\ 4\lambda^2 > 4 + 4\lambda^2 - 9 - 4\sqrt{4\lambda^2 - 9}$

$4\sqrt{4\lambda^2 - 9} + 5 > 0 \Rightarrow \lambda \in R$

$|2\lambda| < 2 + \sqrt{4\lambda^2 - 9}$

$\Rightarrow\ 4\lambda^2 < 4 + 4\lambda^2 - 9 + 4\sqrt{(4\lambda^2) - 9}$

$5 < 4\sqrt{4\lambda^2 - 9} \Rightarrow \lambda^2 \geq \frac{9}{4} \rightarrow$ Domain

$\frac{25}{16} < 4\lambda^2 - 9$

$\Rightarrow\ \lambda^2 > \frac{169}{64}$

$\lambda \in \left(-\infty, \frac{-13}{8}\right) \cup \left(\frac{13}{8}, \infty\right)$

$\lambda \in R - \left[\frac{-13}{8}, \frac{13}{8}\right]$

$a = \frac{-13}{8}, b = \frac{13}{8}$

$\Rightarrow$ (8a + 12, 16b – 20) = (–1, 6)

$\Rightarrow$ $6(-1)^2 + (6)^2 = 42$

**Section-B: Numerical Value Type Questions:** This section contains 10 Numerical based questions. Attempt any 5 questions out of 10. The answer to each question should be rounded-off to the nearest integer.

**Q.81** If x = x(t) is the solution of the differential equation $(t + 1)dx = (2x + (t + 1)^4)dt$, x(0) = 2, then, x(1) equals_____.

**Ans.** **[14]**

**Sol.** $(t + 1)dx = (2x + (t + 1)^4)dt.$

$\frac{dx}{dt} - \frac{2x}{(t+1)} = (t+1)^3$

$IF = e^{-\int \frac{2}{t+1}dt} = \frac{1}{(t+1)^2}$

$\frac{x}{(t+1)^2} = \int (t+1)\,dt$

$\frac{x}{(t+1)^2} = \frac{t^2}{2} + t + C$

Now x(0) = 2

$\Rightarrow$ C = 2

$\therefore\ x = \left(\frac{t^2}{2} + t + 2\right)(t+1)^2$

$x(1) = \left(\frac{1}{2} + 1 + 2\right)(1+1)^2$

$= \frac{7}{2} \times 4 = 14$

**Q.82** Let $P = \{z \in C: |z + 2 - 3i| \le 1\}$ and $Q = \{z \in C: z(1 + i) + \bar{z}(1 - i) \le -8\}$. Let in $P \cap Q$, $|z - 3 + 2i|$ be maximum and minimum at $z_1$ and $z_2$ respectively. If $|z_1|^2 + 2|z_2|^2 = \alpha + \beta\sqrt{2}$, where $\alpha, \beta$ are integers, then $\alpha + \beta$ equals ________.

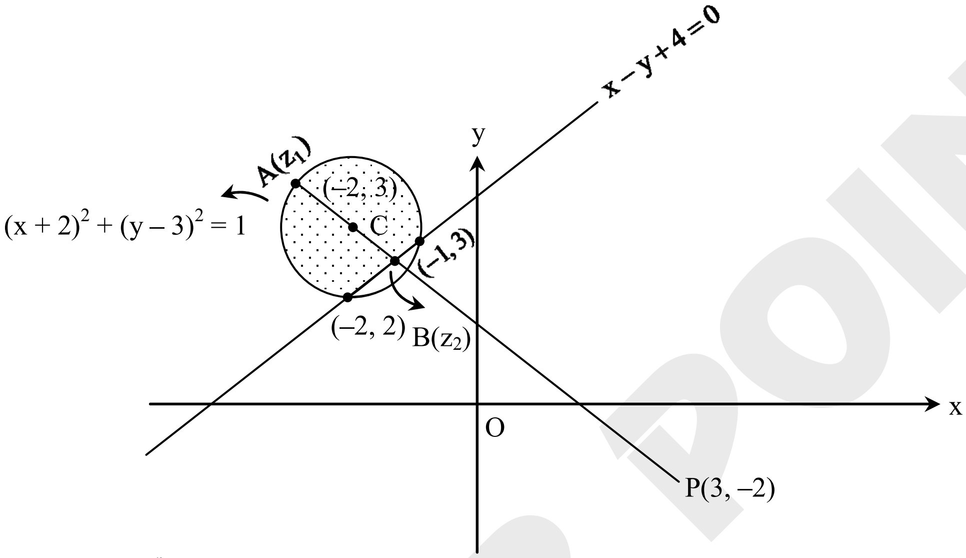
**Ans.** **[36]**

**Sol.** $P : (x + 2)^2 + (y - 3)^2 \le 1$

$\because$ $(x + iy)(1 + i) + (x - iy)(1 - i) \le -8$

$2(x - y) \le -8$

$\Rightarrow$ $Q : (x - y) \le -4$




$m_{CP} = -1$, eq$^n$ of line AP is $y = -x + 1$

$\Rightarrow$ $|z - 3 + 2i|$ be maximum, when z is at point A.

$|z - 3 + 2i|$ be minimum when z is at point B.

A is intersection point of $(x + 2)^2 + (y - 3)^2 = 1$ and $y = -x + 1$

$\Rightarrow$ $A\left(-2 - \frac{1}{\sqrt{2}}, 3 + \frac{1}{\sqrt{2}}\right)$

$\Rightarrow$ $z_1 = \left(-2 - \frac{1}{\sqrt{2}}\right) + i\left(3 + \frac{1}{\sqrt{2}}\right)$

B is intersection point of $x - y + 4 = 0$ and $y = -x + 1$

$\Rightarrow$ $B\left(-\frac{3}{2}, \frac{5}{2}\right) \Rightarrow z_2 = -\frac{3}{2} + \frac{5}{2}i$

$\Rightarrow$ $|z_1|^2 + 2|z_2|^2 = 31 + 5\sqrt{2} \Rightarrow \alpha + \beta = 36$

**Q.83** The number of elements in the set $S = \{(x, y, z) : x, y, z \in\ , x + 2y + 3z = 42, x, y, z \ge 0\}$ equal ______.

**Ans.** **[169]**

**Sol.** $x + 2y + 3z = 42$

$x, y, z \ge 0$

as

$z = 0$ $x + 2y = 42 \Rightarrow 22$ cases

$z = 1$ $x + 2y = 39 \Rightarrow 20$ cases

$z = 2$ $x + 2y = 36 \Rightarrow 19$ cases

$z = 3$ $x + 2y = 33 \Rightarrow 17$ cases

$z = 4$ $x + 2y = 30 \Rightarrow 16$ cases

$z = 5$ $x + 2y = 27 \Rightarrow 14$ cases

$z = 6 \quad x + 2y = 24 \Rightarrow 13$ cases
$z = 7 \quad x + 2y = 21 \Rightarrow 11$ cases
$z = 8 \quad x + 2y = 18 \Rightarrow 10$ cases
$z = 9 \quad x + 2y = 15 \Rightarrow 8$ cases
$z = 10 \quad x + 2y = 12 \Rightarrow 7$ cases
$z = 11 \quad x + 2y = 9 \Rightarrow 5$ cases
$z = 12 \quad x + 2y = 6 \Rightarrow 4$ cases
$z = 13 \quad x + 2y = 3 \Rightarrow 2$ cases
$z = 14 \quad x + 2y = 0 \Rightarrow 1$ case
169 cases.

**Q.84** Let 3, 7, 11, 15, …, 403 and 2, 5, 8, 11, …, 404 be two arithmetic progressions. Then the sum of the common terms in them, is equal to _________.

**Ans.** **[6699]**

**Sol.** 3, 7, 11, 15….403
2, 5, 8, 11….404.
So common term AP
11, 23, 35…., 395
$\Rightarrow \quad 395 = 11 + (n - 1)12$
$\Rightarrow \quad n = 33$

$$\text{Sum} = \frac{33}{2}[2 \times 11 + (32)\,12]$$
$$= \frac{33}{2}[22 + 384]$$
$$= 6699$$

**Q.85** Let A = {1, 2, 3, …, 20}. Let $R_1$ and $R_2$ two relation on A such that
$R_1 = \{(a, b) : b \text{ is divisible by } a\}$
$R_2 = \{(a, b) : a \text{ is an integral multiple of } b\}$.
Then, number of elements in $R_1 - R_2$ is equal to ____.

**Ans.** **[46]**

**Sol.** S = {1, 2, 3…., 20}

$$R_1 : \begin{Bmatrix} (1,1), (1,2)....,(1,20), \\ (2,2),(2,4)....,(2,20), \\ (3,3),(3,6)....,(3,18), \\ (4,4),(4,8)....(4,20), \\ (5,5),(5,10)....,(5,20), \\ (6,6),(6,12),(6,18),(7,7),(7,14), \\ (8,8),(8,16),(9,9),(9,18),(10,10), \\ (10,20),(11,11),(12,12)...,(20,20) \end{Bmatrix}$$

$n(R_1) = 66$
$R_2$ : {a is integral multiple of b.}
$R_1 \cap R_2 : \{(1, 1), (2, 2)…,(20, 20)\}$
$n(R_1 \cap R_2) = 20$
$n(R_1 - R_2) = 66 - 20 = 46$

**Q.86** If $\displaystyle\int_{-\pi/2}^{\pi/2} \frac{8\sqrt{2}\cos x\,dx}{(1 + e^{\sin x})(1 + \sin^4 x)} = \alpha\pi + \beta \log_e(3 + 2\sqrt{2})$
where $\alpha$, $\beta$ are integers, then $\alpha^2 + \beta^2$ equals_____.

**Ans.** **[8]**

**Sol.** $$= \int_0^{\frac{\pi}{2}} \left\{ \frac{8\sqrt{2}\cos x}{(1+e^{\sin x})(1+\sin^4 x)} + \frac{8\sqrt{2}\cos x}{(1+e^{-\sin x})(1+\sin^4 x)} \right\} dx$$

$$= 8\sqrt{2} \int_0^{\frac{\pi}{2}} \frac{\cos x}{1+\sin^4 x} dx$$

Let $\sin x = t$

$$I = 8\sqrt{2} \int_0^1 \frac{dt}{1+t^4}$$

$$= 4\sqrt{2} \int_0^1 \frac{\left(1+\frac{1}{t^2}\right) - \left(1-\frac{1}{t^2}\right)}{t^2 + \frac{1}{t^2}} dt$$

$$= 4\sqrt{2} \int_0^1 \frac{\left(1+\frac{1}{t^2}\right)dt}{\left(t-\frac{1}{t}\right)^2 + 2} - 4\sqrt{2} \int_0^1 \frac{\left(1-\frac{1}{t^2}\right)dt}{\left(t+\frac{1}{t}\right)^2 - 2}$$

$$= 4\sqrt{2} \cdot \frac{1}{\sqrt{2}} \left( \tan^{-1} \frac{t-\frac{1}{t}}{\sqrt{2}} \right)_0^1 - 4\sqrt{2} \cdot \frac{1}{2\sqrt{2}} \left[ \log \left| \frac{t+\frac{1}{t}-\sqrt{2}}{t+\frac{1}{t}+\sqrt{2}} \right| \right]_0^1$$

$$= 2\pi - 2\log \left| \frac{2-\sqrt{2}}{2+\sqrt{2}} \right|$$

$= 2\pi + 2\log(3 + 2\sqrt{2}) = \alpha\pi + \beta\log_e(3 + 2\sqrt{2})$

$\Rightarrow \alpha = 2, \beta = 2$

$\Rightarrow \alpha^2 + \beta^2 = 8$

**Q.87** Let $\{x\}$ denote the fractional part of x and $f(x) = \frac{\cos^{-1}(1-\{x\}^2)\sin^{-1}(1-\{x\})}{\{x\}-\{x^3\}}$, $x \neq 0$. If L and R respectively denotes the left hand limit and the right hand limit of f(x) at x = 0, then $\frac{32}{\pi^2}(L^2 + R^2)$ is equal to _______.

**Ans.** **[18]**

**Sol.** RHL $\Rightarrow \lim_{x\to 0^+} \frac{\cos^{-1}(1-x^2)\sin^{-1}(1-x)}{x-x^3}$

$$\Rightarrow \lim_{x\to 0^+} \frac{\pi}{2} \cdot \frac{\cos^{-1}(1-x^2)}{x}$$

$$= \frac{\pi}{2} \lim_{x\to 0^+} \frac{-1}{\sqrt{(1-(1-x^2)^2}}(-2x)$$

$$= \frac{\pi}{2} \lim_{x\to 0^+} \frac{2x}{\sqrt{2x^2-x^4}} = \pi \lim_{x\to 0^+} \frac{x}{x\sqrt{2-x^2}} = \frac{\pi}{\sqrt{2}} = R$$

LHL $\Rightarrow \lim_{x\to 0^-} \dfrac{\cos^{-1}(1-(1+x)^2)\sin^{-1}(1-(1+x))}{(1+x)\,(1-(1+x)^2)} \Rightarrow \lim_{x\to 0^-} \dfrac{\cos^{-1}(-x^2-2x)\cdot\sin^{-1}(-x)}{-x^2-2x}$

$= \dfrac{\pi}{2} \lim_{x\to 0^-} \dfrac{-\sin^{-1}x}{-x(x+2)} = \dfrac{\pi}{2} \times \dfrac{1}{2} = \dfrac{\pi}{4} = L.$

Requried value $= \dfrac{32}{\pi^2}(L^2 + R^2)$

$= \dfrac{32}{\pi^2}\left(\dfrac{\pi^2}{2} + \dfrac{\pi^2}{16}\right) = 18$

**Q.88** If the coefficient of $x^{30}$ in the expansion of $\left(1+\dfrac{1}{x}\right)^6 (1 + x^2)^7 (1 - x^3)^8$ ; $x \neq 0$ is $\alpha$, then $|\alpha|$ equals_______.

**Ans.** **[678]**

**Sol.** Coefficient of $x^{30}$ in $\dfrac{(1+x)^6(1+x^2)^7(1-x^3)^8}{x^6}$

$\Rightarrow$ Coefficient of $x^{36}$ in $(1 + x)^6 (1 + x^2)^7 (1 - x^3)^8$

$\Rightarrow$ General term = ${}^6C_{r_1}\ {}^7C_{r_2}\ {}^8C_{r_3}\ (-1)^{r_3}\ x^{r_1+2r_2+3r_3}$

$\Rightarrow r_1 + 2r_2 + 3r_3 = 36$

Then possible value of $r_1$, $r_2$ and $r_3$ are

| $r_1$ | $r_2$ | $r_3$ | |
|---|---|---|---|
| 0 | 6 | 8 | ${}^8C_8 \times \left[{}^6C_0 \times {}^7C_6 + {}^6C_2 \times {}^7C_5 + {}^6C_4 \times {}^7C_4 + {}^6C_6 \times {}^7C_3\right]$ |
| 2 | 5 | 8 | |
| 4 | 4 | 8 | |
| 6 | 3 | 8 | |
| 1 | 7 | 7 | $-{}^8C_7 \times \left[{}^6C_1 \times {}^7C_3 + {}^6C_3 \times {}^7C_6 + {}^6C_5 \times {}^7C_5\right]$ |
| 3 | 6 | 7 | |
| 5 | 5 | 7 | |
| 4 | 7 | 6 | ${}^8C_6 \times [{}^6C_4 \times {}^7C_7 + {}^6C_6 \times {}^7C_6]$ |
| 6 | 6 | 6 | |

$\therefore\ \alpha = 882 - 8 \times 272 + 28 \times 22$

$\alpha = -678$

$|\alpha| = 678$

**Q.89** Let the line L : $\sqrt{2}\,x + y = \alpha$ pass through the point of the intersection P (in the first quadrant) of the circle $x^2 + y^2 = 3$ and the parabola $x^2 = 2y$. Let the line L touch two circles $C_1$ and $C_2$ of equal radius $2\sqrt{3}$ . If the centres $Q_1$ and $Q_2$ of the circles $C_1$ and $C_2$ lie on the y-axis, then the square of the area of the triangle $PQ_1Q_2$ is equal to_________.

**Ans.** **[72]**

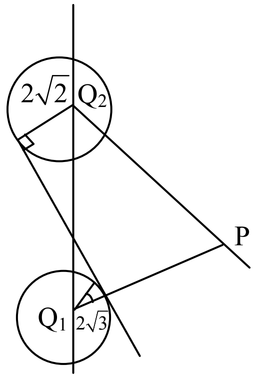
**Sol.** Solving $x^2 + y^2 = 3$ and $x^2 = 2y$

$\Rightarrow$ y = 1 or y = –3 (Rejected)

Then P : $(\sqrt{2}, 1) \Rightarrow \alpha = 3$

Let centre of $C_1$ or $C_2$ be $(0, \beta)$

If touches $\sqrt{2}\,x + y = 3$




$$\Rightarrow \left|\frac{\beta - 3}{\sqrt{2+1}}\right| = 2\sqrt{3}$$

$\beta = 9$ or $\beta = -3$

Area of $\Delta PQ_1Q_2 = \frac{1}{2}\sqrt{2}\,|\beta_1 - \beta_2| = 6\sqrt{2}$

Required $= (6\sqrt{2})^2 = 72$

**Q.90** Let the line of the shortest distance between the lines

$L_1 : \vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(\hat{i} - \hat{j} + \hat{k})$ and

$L_2 : \vec{r} = (4\hat{i} + 5\hat{j} + 6\hat{k}) + \mu(\hat{i} + \hat{j} - \hat{k})$

Intersect $L_1$ and $L_2$ at P and Q respectively.

If $(\alpha, \beta, \gamma)$ is the mid point of the line segment PQ,

then $2(\alpha + \beta + \gamma)$ is equal to________.

**Ans.** **[21]**

**Sol.** $L_1 \equiv \vec{r} = (1, 2, 3) + \lambda(1, -1, 1) \quad (\vec{r} = \vec{a_1} + \lambda\vec{b_1})$

$L_2 \equiv \vec{r} = (4, 5, 6) + \mu(1, -1, 1) \quad (\vec{r} = \vec{a_2} + \lambda\vec{b_2})$

$P \equiv (\lambda + 1, -\lambda + 2, \lambda + 3)$

$Q \equiv (\mu + 4, \mu + 5, -\mu + 6)$

$\overline{PQ} = (\mu - \lambda + 3, \mu + \lambda + 3, -\mu - \lambda + 3)$

$\overline{PQ} \cdot \vec{b_1} = 0 \Rightarrow 3\lambda + \mu = 3$ ...(i)

$\overline{PQ} \cdot \vec{b_2} = 0 \Rightarrow 3\mu + \lambda = 3$ ...(ii)

From (i) and (ii),1

$P \equiv \left(\frac{5}{2}, \frac{1}{2}, \frac{9}{2}\right)$ & $Q \equiv \left(\frac{5}{2}, \frac{7}{2}, \frac{15}{2}\right)$

$\alpha = \frac{5}{2}, \beta = \frac{4}{2}, \gamma = \frac{12}{2}$

$2(\alpha + \beta + \gamma) = 21$