CAREER POINT

JEE Main Online Exam 2024

Questions & Solution

31st January 2024 | Evening

PHYSICS

Section-A: This section contains 20 multiple choice questions. Each question has 4 choices(1), (2), (3) and (4), out of which **ONLY ONE** is correct..

Q.1 The mass of the moon is $\frac{1}{144}$ times the mass of a planet and its diameter is $\frac{1}{16}$ times the diameter of a planet. If the escape velocity on the planet is v, the escape velocity on the moon will be

(1) $\frac{v}{12}$ (2) $\frac{v}{6}$ (3) $\frac{v}{4}$ (4) $\frac{v}{3}$

Ans. **[4]**

Sol. $v_e = \sqrt{\frac{2GM}{r}}$

$\Rightarrow \frac{v_1}{v_2} = \sqrt{\frac{M_1R_2}{M_2R_1}}$

$\Rightarrow \frac{v}{v_m} = \sqrt{\frac{M_1 \frac{1}{16}R_1}{\frac{1}{144}M_1R_1}} = \sqrt{9} = 3$

Q.2 A uniform magnetic field of 2×10^{-3} T acts along positive Y-direction. A rectangular loop of sides 20 cm and 10 cm with current of 5 A is in Y–Z plane. The current is in anticlockwise sense with reference to negative X axis. Magnitude and direction of the torque is

(1) 2×10^{-4} N-m along positive Z-direction
(2) 2×10^{-4} N-m along negative Z-direction
(3) 2×10^{-4} N-m along positive Y-direction
(4) 2×10^{-4} N-m along positive X-direction

Ans. **[2]**

Sol. m = iA

$\tau = iAB$

$\tau = 5 \times 200 \times 10^{-4} \times 2 \times 10^{-3}$

$= 2 \times 10^{-4}$ Nm

$\hat{M} = -\hat{i}\ ;\ \hat{B} = \hat{j}$

$\hat{M} \times \hat{B} \parallel -\hat{i} \times \hat{j} \parallel -\hat{k}$

Q.3 Given below are two statements:

Statement I : Electromagnetic waves carry energy as they travel through space and this energy is equally shared by the electric and magnetic fields.

Statement II: When electromagnetic waves strike a surface, a pressure is exerted on the surface.

In the light of the above statements, choose the **most appropriate** answer from the options given below:

(1) Both **Statement I** and **Statement II** are correct
(2) **Statement I** is correct but **Statement II** is incorrect
(3) **Statement I** is incorrect but **Statement II** is correct
(4) Both **Statement I** and **Statement II** are incorrect

Ans. [1]

Sol. $\Rightarrow$ EMW carries energy shared equally in magnetic and electric field.

$\Rightarrow$ EMW carrying momentum exerts force.

Q.4 A gas mixture consists of 8 moles of argon and 6 moles of oxygen at temperature T. Neglecting all vibrational modes, the total internal energy of the system is

(1) 27 RT (2) 20 RT (3) 21 RT (4) 29 RT

Ans. [1]

Sol. $U = \sum u_i f_i RT_i$

$= 8 \times \frac{3}{2} R \times T + 6 \times \frac{5}{2} RT$

$= 27\ RT$

Q.5 In a photoelectric effect experiment a light of frequency 1.5 times the threshold frequency is made to fall on the surface of photosensitive material. Now if the frequency is halved and intensity is doubled, the number of photo electrons emitted will be

(1) halved (2) doubled (3) quadrupled (4) Zero

Ans. [4]

Sol. $f_i = 1.5\ f_o$

$f_{ii} = 0.75\ f_o < f_o$

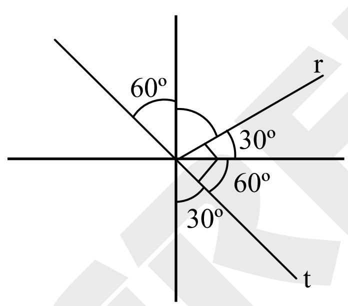
$\Rightarrow$ No electrons emitted

Q.6 When unpolarized light is incident at an angle of 60° on a transparent medium from air, the reflected ray is completely polarized. The angle of refraction in the medium is

(1) 30º (2) 45º (3) 60º (4) 90º

Ans. [1]

Sol.



$\Rightarrow r = 30°$

Q.7 The mass number of nucleus having radius equal to half of the radius of nucleus with mass number 192 is

(1) 40 (2) 20 (3) 32 (4) 24

Ans. [4]

Sol. $r_2 = \frac{r_1}{2}$, $N_2 = ?$, $N_1 = 192$

$\left(\frac{r_1}{r_2}\right)^3 = \frac{N_1}{N_2}$

$8 = \frac{192}{N_2} \Rightarrow N_2 = 24$

Q.8 Consider two physical quantities A and B related to each other as $E = \frac{B - x^2}{At}$ where E, x and t have dimensions of energy, length and time respectively. The dimension of AB is

(1) $L^{-2}M^1T^0$ (2) $L^{-2}M^{-1}T^1$ (3) $L^0M^{-1}T^1$ (4) $L^2M^{-1}T^1$

Ans. **[4]**

Sol. $E = \frac{B - x^2}{At} \Rightarrow B \equiv L^2$

and, $A \equiv \frac{L^2}{[Et]}$

$= \frac{L^2}{ML^2T^{-2}}.T$

$A \equiv M^{-1}T$

$\therefore AB = M^{-1}L^2T$

Q.9 The measured value of the length of a simple pendulum is 20 cm with 2 mm accuracy. The time for 50 oscillations was measured to be 40 seconds with 1 second resolution. From these measurements, the accuracy in the measurement of acceleration due to gravity is N%. The value of N is

(1) 5 (2) 8 (3) 4 (4) 6

Ans. **[4]**

Sol. $T = 2\pi\sqrt{\frac{L}{g}} \Rightarrow g = \frac{4\pi^2 L}{T^2}$

$\frac{\Delta g}{g} = \frac{\Delta L}{L} + \frac{2T}{T}$

$\frac{\Delta g}{g} \times 100 = \left(\frac{2}{200} + \frac{2}{40}\right) \times 100$

$= 1 + 5 = 6\%$

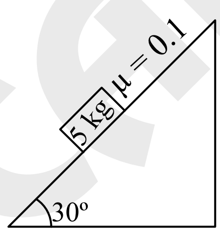
Q.10 The speed of sound in oxygen at S.T.P. will be approximately
(given, $R = 8.3\ JK^{-1}$, $\gamma = 1.4$)

(1) 310 m/s (2) 333 m/s (3) 325 m/s (4) 341 m/s

Ans. **[1]**

Sol. $\therefore v = \sqrt{\frac{\gamma RT}{M_0}} \Rightarrow v = \sqrt{\frac{1.4 \times 8.3 \times 273.15}{32 \times 10^{-3}}} = 314$ m/s

Q.11



A block of mass 5 kg is placed on a rough inclined surface as shown in the figure. If $\vec{F}_1$ is the force required to just move the block up the inclined plane and $\vec{F}_2$ is the force required to just prevent the block from sliding down, then the value of $|\vec{F}_1| - |\vec{F}_2|$ is [Use $g = 10$ m/s^2]

(1) 10 N (2) $\frac{5\sqrt{3}}{2}$ N (3) $50\sqrt{3}$ N (4) $25\sqrt{3}$ N

Ans. **[No option is correct]**

Sol. $F_1 = mg\sin\theta + \mu mg\cos\theta$

$F_2 = mg\sin\theta - \mu mg\cos\theta$

$F_1 - F_2 = 2\mu mg\cos\theta$

$= 2 \times 0.1 \times 5 \times 10 \times \dfrac{\sqrt{3}}{2}$

$= 5\sqrt{3}$ N

No option is correct.

Q.12 An AC voltage V = 20 sin 200πt is applied to a series LCR circuit which drives a current $I = 10\sin\left(200\pi t + \dfrac{\pi}{3}\right)$. The average power dissipated in

(1) 50 W (2) 200 W (3) 173.2 W (4) 21.6 W

Ans. **[1]**

Sol. $P = I_{rms} V_{rms} \cos\theta$

$= \dfrac{20}{\sqrt{2}} \times \dfrac{10}{\sqrt{2}} \times \cos\dfrac{\pi}{3}$

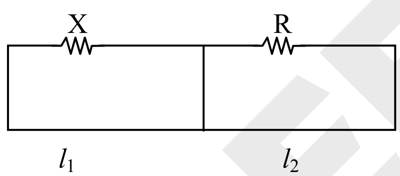
$\Rightarrow P = 50$ W

Q.13 The resistance per centimeter of a meter bridge wire is r, with XΩ resistance in left gap. Balancing length from left end is at 40 cm with 25 Ω resistance in right gap. Now the wire is replaced by another wire of 2r resistance per centimeter. The new balancing length for same settings will be at

(1) 10 cm (2) 20 cm (3) 40 cm (4) 80 cm

Ans. **[3]**

Sol.



$\dfrac{X}{R} = \dfrac{l_1}{l_2}$ is independent of area

Q.14 A small spherical ball of radius r, falling through a viscos medium of negligible density has terminal velocity 'v'. Another ball of the same mass but of radius 2r, falling through the same viscous medium will have terminal velocity

(1) 4v (2) $\dfrac{v}{2}$ (3) 2v (4) $\dfrac{v}{4}$

Ans. **[2]**

Sol. $mg = 6\pi\eta rv$

$\Rightarrow \dfrac{v_1}{v_2} = \dfrac{r_2}{r_1} \Rightarrow \dfrac{v}{v_2} = \dfrac{2r}{r}$

Q.15 Force between two point charges q_1 and q_2 placed in vacuum at 'r' cm apart is F. Force between them when placed in a medium having dielectric constant K = 5 at 'r/5' cm apart will be

(1) F/25 (2) 25F (3) 5F (4) F/5

Ans. **[3]**

Sol. $F = \dfrac{1}{4\pi\varepsilon_0 v}\dfrac{q_1q_2}{kr^2}$

$\Rightarrow \dfrac{F_1}{F_2} = \dfrac{1}{r^2 \dfrac{1}{k\left(\dfrac{r}{5}\right)^2}} = 5\times\dfrac{1}{25}$

$\Rightarrow F_2 = 5F_1$

Q.16 By what percentage will the illumination of the lamp decrease if the current drops by 20%?

(1) 26% (2) 56% (3) 36% (4) 46%

Ans. **[3]**

Sol. Brightness $\propto$ Power

$\Rightarrow P = i^2R \rightarrow \dfrac{4i}{5}$

$\dfrac{P}{P'} = \dfrac{i^2R}{\left(\dfrac{4}{5}i\right)^2 R} = \dfrac{25}{16}$

$\Rightarrow \dfrac{16}{25}P = P'$

$\Rightarrow \left|\dfrac{P'-P}{P}\times 100\right| = \dfrac{9}{25}\times 100 = 36\%$

Q.17 A body of mass 2 kg begins to move under the action of a time dependent force given by $\vec{F} = (6t\,\hat{i} + 6t^2\,\hat{j})$N. The power developed by the force at the time t is given by

(1) $(9t^3 + 6t^5)$W (2) $(3t^3 + 6t^5)$W (3) $(6t^4 + 9t^5)$W (4) $(9t^5 + 6t^3)$W

Ans. **[1]**

Sol. $a = 3t\,\hat{i} + 3t^2\,\hat{j}$

$V = \dfrac{3t^2}{2}\,\hat{i} + t^3\,\hat{j}$

$F = 6t\,\hat{i} + 6t^2\,\hat{j}$

$P = 9t^3 + 6t^5$

Q.18 If two vectors $\vec{A}$ and $\vec{B}$ having equal magnitude R are inclined at an angle θ, then

(1) $|\vec{A}+\vec{B}| = 2R\sin\left(\dfrac{\theta}{2}\right)$ (2) $|\vec{A}+\vec{B}| = 2R\cos\left(\dfrac{\theta}{2}\right)$

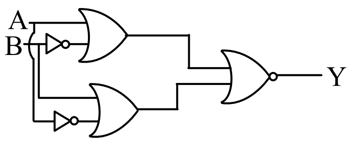
(3) $|\vec{A}-\vec{B}| = \sqrt{2}R\sin\left(\dfrac{\theta}{2}\right)$ (4) $|\vec{A}-\vec{B}| = 2R\cos\left(\dfrac{\theta}{2}\right)$

Ans. **[2]**

Sol. $|\vec{A}+\vec{B}|$ for A = B = R

$|\vec{A}+\vec{B}| = 2R\cos\dfrac{\theta}{2}$

Q.19



The output of the given circuit diagram is

(1)

A	B	Y
0	0	0
1	0	0
0	1	1
1	1	0

(2)

A	B	Y
0	0	0
1	0	0
0	1	0
1	1	1

(3)

A	B	Y
0	0	0
1	0	1
0	1	1
1	1	0

(4)

A	B	Y
0	0	0
1	0	0
0	1	0
1	1	0

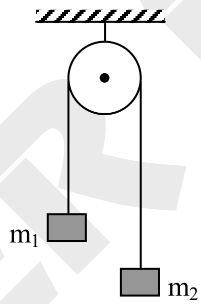
Ans. **[4]**

Sol. $Y = \overline{(A+\overline{B})+(\overline{A}+B)}$

$= \overline{(A+\overline{B})}.\overline{(\overline{A}+B)}$

$= (A.\overline{B}).(\overline{A}.B) = 0$

Q.20 A light string passing over a smooth light fixed pulley connects two blocks of masses m_1 and m_2. If the acceleration of the system is g/8, then the ratio of masses is



(1) $\frac{9}{7}$ (2) $\frac{8}{1}$ (3) $\frac{5}{3}$ (4) $\frac{4}{3}$

Ans. **[1]**

Sol. $\frac{(m_2 - m_1)}{m_2 + m_1} g = \frac{g}{8}$

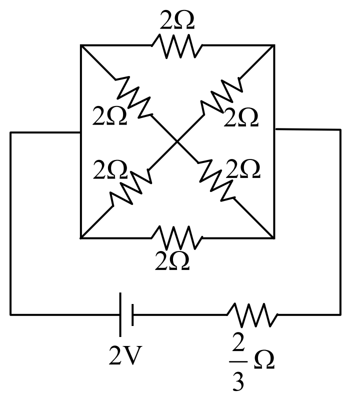
$8m_2 - 8m_1 = m_2 + m_1$

$7m_2 = 9m_1$

$\frac{m_1}{m_2} = \frac{7}{9}$

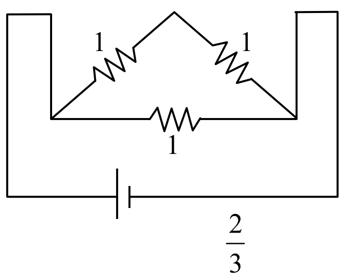
Section-B: Numerical Value Type Questions: This section contains 10 Numerical based questions. Attempt any 5 questions out of 10. The answer to each question should be rounded-off to the nearest integer.

Q.21 In the following circuit, the battery has an emf of 2 V and an internal resistance of $\frac{2}{3}\Omega$. The power consumption in the entire circuit is ______ W.



Ans. [3]

Sol.

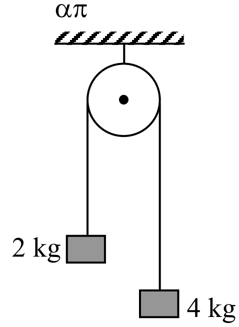


$R_4 = \frac{4}{3}$

$P = \frac{V^2}{R_4}$

$P = \frac{2 \times 2 \times 3}{4} = 3W$

Q.22 Two blocks of mass 2 kg and 4 kg are connected by a metal wire going over a smooth pulley as shown in figure. The radius of wire is 4.0×10^{-5} m and Young's modulus of the metal is 2.0×10^{11} N/m^2. The longitudinal strain developed in the wire is $\frac{1}{\alpha\pi}$. The value of α is _______. [Use $g = 10$ m/s^2]

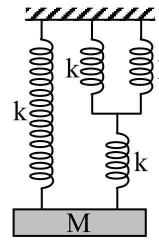


Ans. [12]

Sol. $T = \frac{2 \times 2 \times 4 \times 10}{6} = \frac{80}{3}$ N

$\text{Strain} = \frac{F}{AY} = \frac{80}{3 \times 16\pi \times 10^{-10} \times 2 \times 10^{11}} = \frac{1}{12\pi} \Rightarrow \alpha = 12$

Q.23 The time period of simple harmonic motion of mass M in the given figure is $\pi\sqrt{\frac{\alpha M}{5k}}$, where the value of α is _______.



Ans. **[12]**

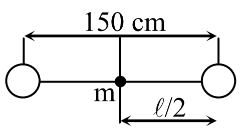
Sol. $k_{eq} = \frac{2k}{3} + k = \frac{5k}{3}$

$\therefore T = 2\pi\sqrt{\frac{3M}{5k}} = \pi\sqrt{\frac{12M}{5k}}$

Q.24 Two identical spheres each of mass 2 kg and radius 50 cm are fixed at the ends of a light rod so that the separation between the centers is 150 cm. Then, moment of inertia of the system about an axis perpendicular to the rod and passing through its middle point is $\frac{x}{20}$ kg m^2, where the value of x is _______.

Ans. **[53]**

Sol.



$$\ell = \left\{\frac{2}{5}mr^2 + m\left(\frac{\ell}{2}\right)^2\right\} \times 2$$

$$= \frac{4}{5} \times 2 \times \frac{1}{4} + 2 \times 2 \times \frac{3}{4} \times \frac{3}{4}$$

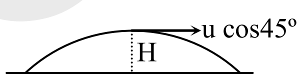
$$= \frac{2}{5} + \frac{9}{4}$$

$$= \frac{8+45}{20} = \frac{53}{20}$$

Q.25 A body of mass 'm' is projected with a speed 'u' making an angle of 45° with the ground. The angular momentum of the body about the point of projection, at the highest point is expressed as $\frac{\sqrt{2}mu^3}{Xg}$. The value of 'X' is _______.

Ans. **[8]**

Sol.

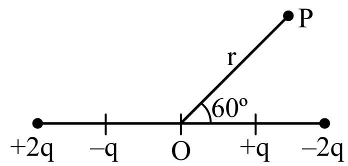


$$H = \frac{u^2 \sin^2 45^\circ}{2g}$$

$$L = mvr = mu\cos45^\circ \times u^2 \frac{1}{4g} = \frac{mu^3}{4\sqrt{2}g}$$

Q.26 The distance between charges +q and –q is 2ℓ and between +2q and –2q is 4ℓ. The electrostatic potential at point P at a distance r from center O is $-\alpha\left[\frac{q\ell}{r^2}\right] \times 10^9$V, where the value of α is _______.

$\left(\text{Use} \frac{1}{4\pi\varepsilon_0} = 9\times10^9\,\text{Nm}^2\text{C}^{-2}\right)$



Ans. **[27]**

Sol. Assuming r >> l

$P = 2q \times 2\ell - q\ell = 3q\ell \times 2$

$V = \frac{KP\cos\theta}{r^3}$

$V = \frac{9\times10^9 \times 3q\ell \times \cos 120^\circ}{r^3} \times 2$

$= -13.5\frac{q\ell}{r^3} \times 10^9 \times 2$

$\alpha = 13.5 \times 2 = 27$

Q.27 The magnetic flux ϕ (in weber) linked with a closed circuit of resistance 8Ω varies with time (in seconds) as $\phi = 5t^2 - 36t + 1$. The induced current in the circuit at t = 2 s is _______ A.

Ans. **[2]**

Sol. $|\varepsilon| = |10t - 36|$

$= 16$ V

$\therefore\ i = \frac{16}{8} = 2A$

Q.28 Two circular coils P and Q of 100 turns each have same radius of π cm. The currents in P and R are 1 A and 2 A respectively. P and Q are placed with their planes mutually perpendicular with their centers coincide. The resultant magnetic field induction at the center of the coils is $\sqrt{x}$ mT, where x = _______.

[Use $\mu_0 = 4\pi \times 10^{-7}$ TmA^{-1}]

Ans. **[20]**

Sol. $B_1 = \frac{4\pi\times10^{-7}\times1}{2\times\pi\times10^{-2}}\times100$

$B_1 = 2$mT

$B_2 = \frac{4\pi\times10^{-7}\times2}{2\times\pi\times10^{-2}}\times100 = 4$mT

$B = \sqrt{B_1^2 + B_2^2} = \sqrt{20}$ mT

Q.29 A nucleus has mass number A_1 and volume V_1. Another nucleus has mass number A_2 and Volume V_2. If relation between mass number is $A_2 = 4A_1$, then $\frac{V_2}{V_1} =$ _______ .

Ans. **[4]**

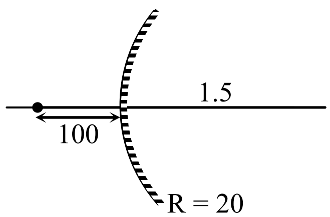
Sol. $V \propto A$

$$\frac{V_1}{V_2} = \frac{A_1}{A_2} = \frac{1}{4}$$

Q.30 Light from a point source in air falls on a convex curved surface of radius 20 cm and refractive index 1.5. If source is located at 100 cm from the convex surface, the image will be formed at _______ cm from the object.

Ans. **[200]**

Sol.



$$\frac{1.5}{v} - \frac{1}{-100} = \frac{1.5-1}{20}$$

$$\frac{1.5}{v} = \frac{1}{40} - \frac{1}{100}$$

$$\frac{1.5}{v} = \frac{5-2}{200} = \frac{3}{200}$$

$v = 100$

$\therefore$ 100 + 100 = 200 cm

CHEMISTRY

Section-A: Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Q.31 Select the option with correct property-

(1) $[Ni(CO)_4]$ and $[NiCl_4]^{2-}$ both Paramagnetic
(2) $[Ni(CO)_4]$ and $[NiCl_4]^{2-}$ both Diamagnetic
(3) $[NiCl_4]^{2-}$ Diamagnetic, $[Ni(CO)_4]$ Paramagnetic
(4) $[Ni(CO)_4]$ Diamagnetic, $[NiCl_4]^{2-}$ Paramagnetic

Ans. **[4]**

Sol. $[Ni(CO)_4]$ is diamagnetic due to d^{10} configuration. Ni^{2+} with no pairing will be paramagnetic due to presence of 2 unpaired electrons.

Q.32 The four quantum numbers for the electron in the outer most orbital of potassium (atomic no. 19) are

(1) $n = 4, \ell = 0, m = 0, s = +\frac{1}{2}$

(2) $n = 4, \ell = 2, m = -1, s = +\frac{1}{2}$

(3) $n = 3, \ell = 0, m = 1, s = +\frac{1}{2}$

(4) $n = 2, \ell = 0, m = 0, s = +\frac{1}{2}$

Ans. **[1]**

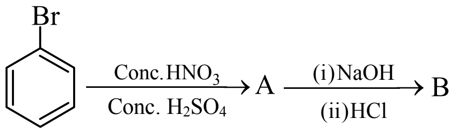
Sol. $K_{19} : 1s^2 2s^2 2p^6 3s^2 3p^6 4s^1$
Outermost electron is $4s^1$
$n = 4$

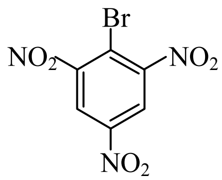
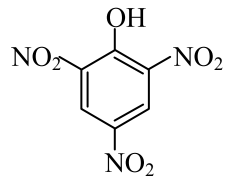
$\ell = 0$

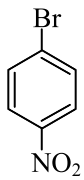
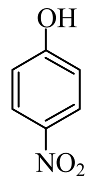
$m = 0$

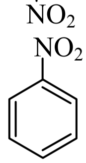
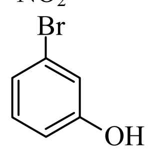
$s = +\frac{1}{2}$

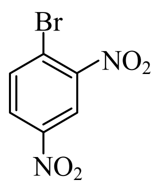
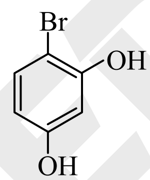
Q.33 Identify A and B in the following reaction sequence.



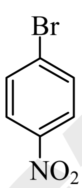
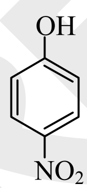
(1) A=  B= 

(2) A=  B= 

(3) A=  B= 

(4) A=  B= 

Ans. **[2]**

Sol. A=  B= 

Q.34 Given below are two statements:

Statement I: S_8 solid undergoes disproportionation reaction under alkaline conditions to from S^{2-} and $S_2O_3^{2-}$.

Statement II: ClO_4^- can undergo disproportionation reaction under acidic condition.

In the light of the above statements, choose the **most appropriate answer** from the options given below:

(1) Both **statement I** and **statement II** are correct

(2) **Statement I** is incorrect but **statement II** is correct

(3) **Statement I** is correct but **statement II** is incorrect

(4) Both **statement I** and **statement II** are incorrect

Ans. **[3]**

Sol. Cl is present in highest oxidation state (+7) and hence cannot undergo disproportionation reaction.

Statement I is correct but **statement II** is incorrect

Q.35 Consider the following elements.

Group ↓ A' B' → Period
C' D'

Which of the following is/are true about A', B', C' and D' ?

A. Order of atomic radii: B' < A' < D' < C'

B. Order of metallic character: B' < A' < D' < C'

C. Size of the element: D' < C' < B' < A'

D. Order of ionic radii : $B'^{+} < A'^{+} < D'^{+} < C'^{+}$

Choose the correct answer from the option given below:

(1) A only (2) A and B only

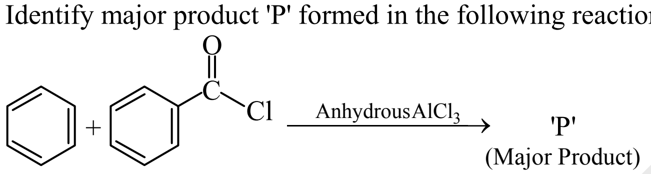
(3) B, C and D only (4) A, B and D only

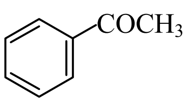
Ans. **[4]**

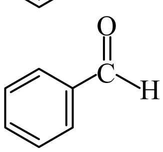
Sol. Size of element : B' < A' < D' < C'

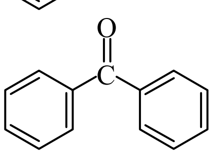
Statement A, B and D are correct.

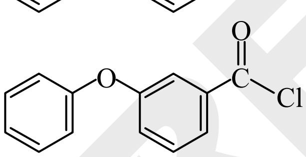
Q.36 Identify major product 'P' formed in the following reaction.



(1) 

(2) 

(3) 

(4) 

Ans. **[3]**

Sol. Friedel craft acylation reaction to produce benzophenone

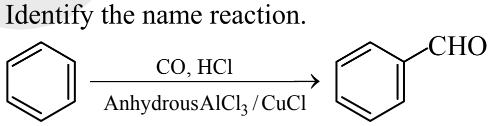
Q.37 Which of the following is least ionic?

(1) $BaCl_2$ (2) KCl (3) $CoCl_2$ (4) AgCl

Ans. **[4]**

Sol. AgCl is least ionic due to pseudo inert gas configuration

Q.38 Identify the name reaction.



(1) Gatterman-Koch Reaction (2) Rosenmund Reduction

(3) Stephen Reaction (4) Etard Reaction

Ans. **[1]**

Sol. Named reaction Gattermann – Koch reaction is used to convert benzene directly into benzenecarbaldehyde.

Q.39 The fragrance of flowers is due to the presence of some steam volatile organic compounds called essential oils. These are generally insoluble in water at room temperature but are miscible with water vapour in vapour phase. A suitable method for the extraction these oils from the flower is-

(1) distillation
(2) crystallisation
(3) distillation under reduced pressure
(4) steam distillation

Ans. **[4]**

Sol. Steam distillation is used for those substances which are miscible with water vapour.

Q.40 Given below are two statements:

Statement I: Aniline reacts with con. H_2SO_4, followed by heating at 453-473 K gives p-aminobenzene sulphonic acid, which gives blood red colour in the 'Lassaigne's test'.

Statement II: In Friedel-Craft's alkylation and acylation reactions, aniline forms salt with the $AlCl_3$ catalyst. Due to this, nitrogen of aniline acquires a positive charge and acts as deactivating group.

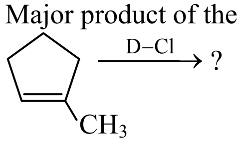
In the light of the above statements, choose the **correct answer** from the options given below:

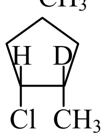
(1) **Statement I** is true but **statement II** is false
(2) **Statement I** is false but **statement II** is true
(3) Both **statement I** and **statement II** are true
(4) Both **statement I** and **statement II** are false

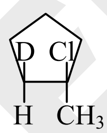
Ans. **[3]**

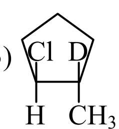
Sol. Due to presence of sulphur, carbon and nitrogen compound will give blood red colour in "Lassaigne" test.
Aniline forms salt which makes it deactivating group.
Both S(I) and S(II) are correct.

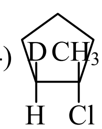
Q.41 Major product of the following reaction is –



(1) 

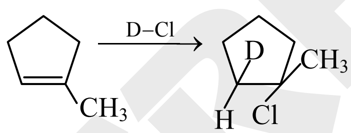
(2) 

(3) 

(4) 

Ans. **[4]**

Sol.



Q.42 Choose the correct statements from the following

A. Mn_2O_7 is an oil at room temperature
B. V_2O_4 reacts with acid to give VO_2^{2+}
C. CrO is a basic oxide
D. V_2O_5 does not react with acid

Choose the correct answer from the options given below:

(1) A, B and D only
(2) A, B and C only
(3) B and C only
(4) A and C only

Ans. **[4]**

Sol. V_2O_5 is Amphoteric and hence can react with acid as well as base.
V_2O_4 reacts with acid to give VO^{+2}
Statements A and C are correct.

Q.43 Choose the correct statements from the following

A. All group 16 elements form oxides of general formula EO_2 an EO_3, where E = S, Se, Te and Po. Both the types of oxides are acidic in nature.

B. TeO_2 is an oxidising agent while SO_2 is reducing in nature.

C. The reducing property decreases from H_2S to H_2Te down the group.

D. The ozone molecule contains five lone pairs of electrons.

Choose the correct answer from the options given below:

(1) C and D only (2) B and C only (3) A and D only (4) A and B only

Ans. **[4]**

Sol. (C) Reducing property increases from H_2S to H_2Te

(D) Ozone molecule have 6 lone pair of electrons.

$\Rightarrow$ Statements A and B are correct.

Q.44 $A_{(g)} \rightleftharpoons B_{(g)} + \frac{C}{2}(g)$. The correct relationship between Kp, α and equilibrium pressure P is

(1) $K_P = \dfrac{\alpha^{1/2}P^{1/2}}{(2+\alpha)^{1/2}}$

(2) $K_P = \dfrac{\alpha^{1/2}P^{3/2}}{(2+\alpha)^{3/2}}$

(3) $K_P = \dfrac{\alpha^{3/2}P^{1/2}}{(2+\alpha)^{1/2}(1-\alpha)}$

(4) $K_P = \dfrac{\alpha^{1/2}P^{1/2}}{(2+\alpha)^{3/2}}$

Ans. **[3]**

Sol. $A_{(g)} \rightleftharpoons B_{(g)} + \frac{C}{2}(g)$.

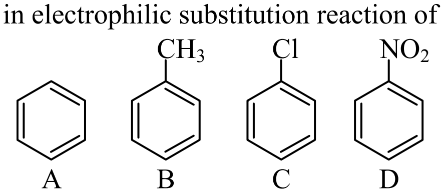
$(1-\alpha) \quad \alpha \quad \frac{\alpha}{2}$

$K_p = K_x (P_{total})^{1/2}$

$K_P = \dfrac{\alpha^{3/2}}{(2+\alpha)^{1/2}(1-\alpha)}(P)^{1/2}$

Option (3) is correct.

Q.45 The correct order of reactivity in electrophilic substitution reaction of the following compound is:



(1) A > B > C > D (2) B > A > C > D (3) D > C > B > A (4) B > C > A > D

Ans. **[2]**

Sol. Rate $\propto$ electron density on benzene ring.

Correct order is B > A > C > D

Option (2) is correct.

Q.46 A sample of $CaCO_3$ and $MgCO_3$ weighed 2.21 g is ignited to constant weight of 1.152 g. The composition of mixture is:

(Given molar mass in g mol^{-1} $CaCO_3$: 100, $MgCO_3$: 84)

(1) 1.187 g $CaCO_3$ + 1.023 g $MgCO_3$
(2) 1.187 g $CaCO_3$ + 1.187 g $MgCO_3$
(3) 1.023 g $CaCO_3$ + 1.187 g $MgCO_3$
(4) 1.023 g $CaCO_3$ + 1.023 g $MgCO_3$

Ans. **[1]**

Sol. Assume mass of $CaCO_3$ = x gm

Mass of $MgCO_3$ = (2.21 – x) gm

$CaCO_3 \xrightarrow{\Delta} CaO + CO_2\uparrow$

$MgCO_3 \xrightarrow{\Delta} MgO + CO_2\uparrow$

(Mass of CaO) + (Mass of MgO) = 1.152 gm

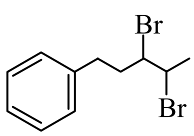
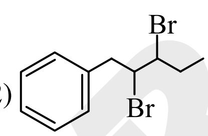
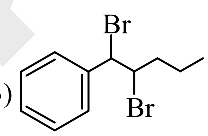
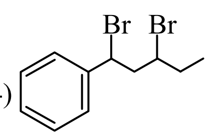
$$\left(\frac{x}{100}\times 56\right)+\left(\frac{2.21-x}{84}\right)\times 40 = 1.152 \text{ gm}$$

x = 1.187 gm

$\Rightarrow CaCO_3$ = 1.187 gm

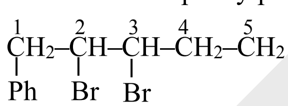
$\Rightarrow MgCO_3$ = 1.023 gm

Q.47 Identify structure of 2,3-dibromo-1-phenylpentane.

(1) 
(2) 
(3) 
(4) 

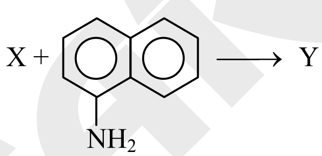
Ans. **[2]**

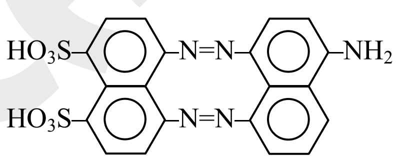
Sol. 2-3–dibromo-1-phenylpentane

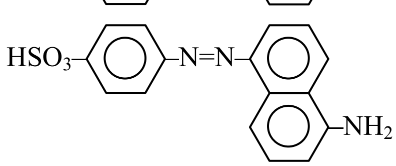


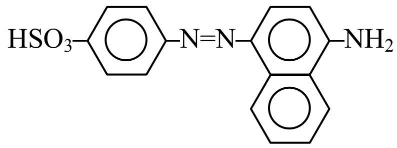
Q.48 The azo-dye (Y) formed in the following reaction is

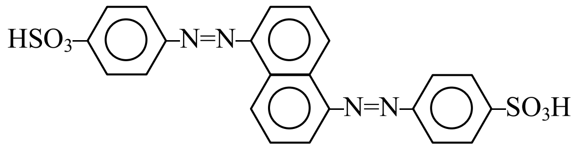
Sulphanilic acid + $NaNO_2$ + $CH_3COOH \rightarrow$ X.



(1) 

(2) 

(3) 

(4) 

Ans. **[3]**

Sol. Coupling reaction at para-position of NH_2 will take place.
Option (3) is correct.

Q.49 Match List I with List II

	List I (Complex ion)		***List II (Electronic Configuration)***
A.	$[Cr(H_2O_6)]^{3+}$	I.	$t_{2g}^2e_g^0$
B.	$[Fe(H_2O)_6]^{3+}$	II.	$t_{2g}^3e_g^0$
C.	$[Ni(H_2O)_6]^{2+}$	III.	$t_{2g}^3e_g^2$
D.	$[V(H_2O)_6]^{3+}$	*IV.*	$t_{2g}^6e_g^2$

Choose the correct answer from the option given below:

(1) A-IV, B-I, C-II, D-III
(2) A-III, B-II, C-IV, D-I
(3) A-II, B-III, C-IV, D-I
(4) A-IV, B-III, C-I, D-II

Ans. **[3]**

Sol. A : Cr^{3+} : $t_{2g}^3e_g^0$

B : Fe^{3+} : $t_{2g}^3e_g^2$

C : Ni^{2+} : $t_{2g}^6e_g^2$

D : V^{3+} : $t_{2g}^2e_g^0$

Q.50 Given below are two statements:

Statement I: Group 13 trivalent halides get easily hydrolyzed by water due to their covalent nature.

Statement II: $AlCl_3$ upon hydrolysis in acidified aqueous solution forms octahedral $[Al(H_2O_6)^{3+}]$ ion.

In the light of the above statements, choose the **correct answer** from the options given below:

(1) Both **statement I** and **statement II** are true
(2) **Statement I** is true but **statement II** is false
(3) **Statement I** is false but **statement II** is true
(4) Both **statement I** and **statement II** are false

Ans. **[1]**

Sol. Al can expand its octet due to presence of vacant d-orbital.
Both **statement I** and **statement II** are true.

Section-B: Numerical Value Type Questions: This section contains 10 Numerical based questions. Attempt any 5 questions out of 10. The answer to each question should be rounded-off to the nearest integer.

Q.51 The molarity of 1 L orthophosphoric acid (H_3PO_4) having 70% purity by weight (specific gravity 1.54 g cm^{-3}) is _______ M.
(Molar mass of H_3PO_4 = 98 g mol^{-1})

Ans. **[11]**

Sol. Mass of solution = 1540 gm

Mass of $H_3PO_4 = 1540 \times \frac{70}{100} = 1078$ gm

Moles of solute $= \frac{1078}{98} = 11$

Q.52 A compound (x) with molar mass 108 g mol^{-1} undergoes acetylation to give product with molar mass 192 g mol^{-1}. The number of amino groups in the compound (x) is ______.

Ans. **[2]**

Sol. Number of amino groups $= \left(\frac{192-108}{42}\right) = 2$

Q.53 r = k[A] for a reaction, 50% of A is decomposed in 120 minutes. The time taken for 90% decomposition of A is _______ minutes.

Ans. **[399]**

Sol. $t_{90} = \frac{2.303}{K}\log 10 = \frac{2.303}{K}$

$t_{50} = \frac{2.303}{K}\log 2 = 120$

$\frac{2.303}{K} = \frac{120}{\log 2}$

$t_{90} = \frac{120}{0.3010} = 398.67$

Nearest integer = 399

Q.54 A diatomic molecule has a dipole moment of 1.2 D. If the bond distance is 1 Å, then fractional charge on each atom is ______ $\times 10^{-1}$ esu. (Given 1 D = 10–18 esu cm)

Ans. **[1.2×10^{-9}]**

Sol. Dipole moment : 1.2 D = 1.2×10^{-18} esu cm

$\mu = q.d$

$1.2 \times 10^{-18} = (q) \times 10^{-8}$

$q = 1.2 \times 10^{-10}$ esu

$q = 1.2 \times 10^{-9} \times 10^{-1}$ esu

Q.55 The values of conductivity of some materials at 298.15 K in Sm^{-1} are 2.1×10^3, 1.0×10^{-16}, 1.2×10, 3.91, 1.5×10^{-2}, 1×10^{-7}, 1.0×10^3. The number of conductors among the materials is _____.

Ans. **[4]**

Sol. Conductors with conductivity

2.1×10^3 : Sodium

1.2×10 : Graphite

3.91 : 0.1 M HCl

1.0×10^3 : Iron

[Ref. - NCERT]

Q.56 If 5 moles of an ideal gas expands from 10 L to a volume of 100 L at 300 K under isothermal and reversible condition then work, w, is –x J. The value of x is _____.
(Given R = 8.314 J K^{-1} mol^{-1})

Ans. **[28721]**

Sol. W = –2.303(5)(8.314)(300) log10
= –28720.713 J
x = 28720.713
Nearest integer = 28721

Q.57 In the reaction of potassium dichromate, potassium chloride and sulfuric acid (conc.), the oxidation state of the chromium in the product is (+) _____.

Ans. **[6]**

Sol. $K_2Cr_2O_7 + KCl + H_2SO_4 \longrightarrow KHSO_4 + CrO_2Cl_2 + H_2O$
Product is $Cr_2O_2Cl_2$
Oxidation state of Cr is +6.

Q.58 Number of moles of H^+ ions required by 1 mole of MnO_4^- to oxidise oxalate ion to CO_2 is _______.

Ans. **[8]**

Sol. Balanced reaction:

$$16H^+ + 2MnO_4^- + 5C_2O_4^{2-} \longrightarrow 10CO_2 + 2Mn^{2+} + 8H_2O$$

Moles of H^+ required = 8

Q.59 Number of isomeric products formed by monochlorination of 2-methylbutane in presence of sunlight is ______.

Ans. **[6]**

Sol.

$$\underset{\displaystyle CH_3}{CH_2\text{–}\overset{}{C}H}\text{–}CH_2\text{–}CH_3 \xrightarrow{Cl_2/h\nu} 6 \text{ Products}$$

4 structural isomers are possible out of which 2 are optically active.
⇒ Total 6 isomers are possible

Q.60 From the vitamins A, B_1, B_6, B_{12}, C, D, E and K, the number of vitamins that can be stored in our body is _____.

Ans. **[5]**

Sol. Following vitamins can be stored in human body Vitamin A, D, E, K, B_{12}

MATHEMATICS

Section-A: This section contains 20 multiple choice questions. Each question has 4 choices(1), (2), (3) and (4), out of which **ONLY ONE** is correct..

Q.61 Let f : R → (0, ∞) be strictly increasing function such that $\lim_{x\to\infty}\frac{f(7x)}{f(x)} = 1$. Then, the value of

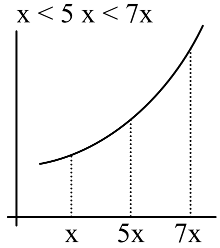
$$\lim_{x\to\infty}\left[\frac{f(5x)}{f(x)} - 1\right] \text{ is equal to}$$

(1) 0 (2) 7/5 (3) 1 (4) 4

Ans. **[1]**

Sol. $f(x) \le f(5x) \le f(7x),\ \forall x > 0$

$x < 5x < 7x$



$$1 \le \frac{f(5x)}{f(x)} \le \frac{f(7x)}{f(x)}$$

$$1 \le \lim_{x\to\infty} \frac{f(5x)}{f(x)} \le \lim_{x\to\infty} \frac{f(7x)}{f(x)} = 1$$

As $x \to \infty$, $\frac{f(5x)}{f(x)} \to 1^+$

$$\lim_{x\to\infty}\left[\frac{f(5x)}{f(x)} - 1\right] = 0$$

Q.62 If for some m, n: ${}^6C_m + 2({}^6C_{m+1}) + {}^6C_{m+2} > {}^8C_3$ and ${}^{n-1}P_3 : {}^nP_4 = 1 : 8$, then ${}^nP_{m+1} + {}^{n+1}C_m$ is equal to

(1) 376 (2) 372 (3) 384 (4) 380

Ans. **[2]**

Sol. ${}^6C_m + 2({}^6C_{m+1}) + {}^6C_{m+2} > {}^8C_3$

${}^6C_m + {}^6C_{m+1} + {}^6C_{m+1} + {}^6C_{m+2} > {}^8C_3$

${}^7C_{m+1} + {}^7C_{m+2} > {}^8C_3$

${}^8C_{m+2} > {}^8C_3$

$\Rightarrow m = 2$

Now, $\frac{{}^{n-1}P_3}{{}^nP_4} = \frac{1}{8} \Rightarrow \frac{1}{n} = \frac{1}{8} \Rightarrow n = 8$

Now, ${}^nP_{m+1} + {}^{n+1}C_m$

$= {}^8P_3 + {}^9C_2$

$= \frac{8!}{5!} + \frac{9!}{2!7!} = 336 + 36 = 372$

Q.63 If the function $f : (-\infty, -1] \to (a, b]$ defined by $f(x) = e^{x^3-3x+1}$ is one-one and onto, then the distance of the point $P(2b + 4, a + 2)$ from the line $x + e^{-3}y = 4$ is

(1) $\sqrt{1+e^6}$ (2) $3\sqrt{1+e^6}$ (3) $2\sqrt{1+e^6}$ (4) $4\sqrt{1+e^6}$

Ans. **[3]**

Sol. $f(x) = e^{x^3-3x+1}$

$f'(x) = e^{x^3-3x+1}(3x^2 - 3)$

$= 3\,e^{x^3-3x+1}(x - 1)(x + 1)$

For $x \in (-\infty, -1]$, $f'(x) \ge 0$

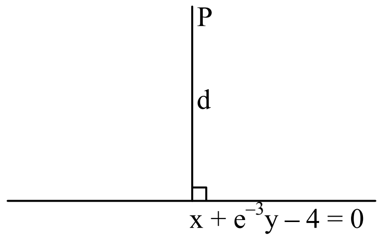
$\therefore$ f(x) is increasing function

$\therefore a = e^{-\infty} = 0 = f(-\infty)$

$b = e^{-1+3+1} = e^3 = f(-1)$

$\therefore P(2e^3 + 4, 2)$

$$d = \frac{2e^3 + 4 + 2e^{-3} - 4}{\sqrt{1+e^{-6}}}$$



$$d = \frac{2\left(\frac{e^6+1}{e^3}\right)}{\frac{\sqrt{e^6+1}}{e^3}} \Rightarrow d = 2\sqrt{1+e^6}$$

Q.64 Let 2^{nd}, 8^{th} and 44^{th} terms of a non-constant A.P. be respectively the 1^{st}, 2^{nd} and 3^{rd} terms of a G.P. If the first term of the A.P. is 1, then the sum of its first 20 terms is equal to

(1) 990 (2) 960 (3) 980 (4) 970

Ans. **[4]**

Sol. a + d, a + 7d, a + 43d are 1^{st}, 2^{nd}, 3^{rd} terms of a G.P.

$$\frac{a+7d}{a+d} = \frac{a+43d}{a+7d}$$

$\Rightarrow (a + 7d)^2 = (a + 43d)(a + d)$

$\Rightarrow a^2 + 49d^2 + 14ad = a^2 + 44ad + 43d^2$

$\Rightarrow 6d^2 = 30ad$

$\Rightarrow d = 0, 5$

But $d \neq 0$

$\Rightarrow d = 5, a = 1$

$S_{20}, = \frac{20}{2}[2 + 19 \times 5]$

$= 10(95 + 2) = 970$

Q.65 Let A be a 3 × 3 real matrix such that

$$A\begin{pmatrix}1\\0\\1\end{pmatrix} = 2\begin{pmatrix}1\\0\\1\end{pmatrix},\ A\begin{pmatrix}-1\\0\\1\end{pmatrix} = 4\begin{pmatrix}-1\\0\\1\end{pmatrix},\ A\begin{pmatrix}0\\1\\0\end{pmatrix} = 2\begin{pmatrix}0\\1\\0\end{pmatrix}$$

Then, the system $(A - 3I)\begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}1\\2\\3\end{pmatrix}$ has

(1) No solution

(2) Exactly two solutions

(3) Infinitely many solutions

(4) Unique solution

Ans. **[4]**

Sol. Let $A = \begin{pmatrix}x_1 & y_1 & z_1\\x_2 & y_2 & z_2\\x_3 & y_3 & z_3\end{pmatrix}$

Given $A\begin{pmatrix}1\\0\\1\end{pmatrix}=\begin{pmatrix}2\\0\\2\end{pmatrix}$ ……(1)

$$\Rightarrow \begin{pmatrix}x_1 & + & z_1\\ x_2 & + & z_2\\ x_3 & + & z_3\end{pmatrix}=\begin{pmatrix}2\\0\\2\end{pmatrix}$$

$\therefore\ x_1 + z_1 = 2$ ...(2)

$x_2 + z_2 = 0$ ...(3)

$x_3 + z_3 = 0$ ...(4)

Similarly, from $A\begin{pmatrix}-1\\0\\1\end{pmatrix}=\begin{pmatrix}-4\\0\\4\end{pmatrix}$

$\Rightarrow -x_1 + z_1 = -4$ ...(5)

$-x_2 + z_2 = 0$ ...(6)

$-x_3 + z_3 = 4$ ...(7)

Similarly, from $A\begin{pmatrix}0\\1\\0\end{pmatrix}=\begin{pmatrix}0\\2\\0\end{pmatrix}$

$\Rightarrow y_1 = 0,\ y_2 = 2,\ y_3 = 0$

From (2), (3), (4), (5), (6) and (7)

$x_1 = 3,\ x_2 = 0,\ x_3 = -2$

$y_1 = 0,\ y_2 = 2,\ y_3 = 0$

$z_1 = -1,\ z_2 = 0,\ z_3 = 2$

$$\therefore A = \begin{pmatrix}3 & 0 & -2\\ 0 & 2 & 0\\ -1 & 0 & 2\end{pmatrix}$$

Now, $(A - 3I)\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}1\\2\\3\end{pmatrix}$

$$\Rightarrow \begin{pmatrix}0 & 0 & -2\\ 0 & -1 & 0\\ -1 & 0 & -1\end{pmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}1\\2\\3\end{pmatrix}$$

$\Rightarrow z = -3/2,\ y = -2$ and $x = -3/2$

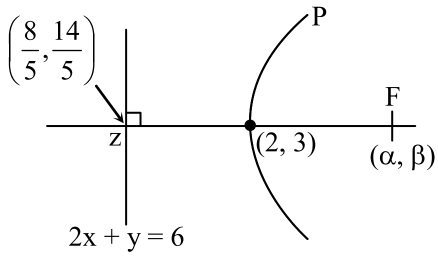
(Unique solution)

Q.66 Let P be a parabola with vertex (2, 3) and directrix $2x + y = 6$. Let an ellipse E : $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, $a > b$, of eccentricity $\frac{1}{\sqrt{2}}$ pass through the focus of the parabola P. Then, the square of the length of the latus rectum of E, is

(1) $\frac{347}{8}$ (2) $\frac{512}{25}$ (3) $\frac{385}{8}$ (4) $\frac{656}{25}$

Ans. **[4]**

Sol.



Let z be the foot of perpendicular from vertex (2, 3) to the directrix.

$\therefore \ \frac{x-2}{2} = \frac{y-3}{1} = -\frac{(4+3-6)}{5}$

$\therefore \ z = \left(\frac{8}{5}, \frac{14}{5}\right)$

$\because$ (2, 3) is the mid-point of z and focus F.

$\therefore \ (\alpha, \beta) \equiv \left(\frac{12}{5}, \frac{16}{5}\right)$

$\because$ Eccentricity of ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Where a > b is $\frac{1}{\sqrt{2}}$

$\therefore \quad b^2 = \frac{a^2}{2}$

$\therefore \quad \frac{144}{25a^2} + \frac{256}{25 \times \frac{a^2}{2}} = 1$

$\Rightarrow \quad a^2 = \frac{656}{25} \Rightarrow a = \frac{\sqrt{656}}{5}$

Square of length of latus rectum of E

$= 4\left(\frac{b^2}{a}\right)^2$

$= 4.\frac{a^2}{4} = \frac{656}{25}$

Q.67 The shortest distance between lines L_1 and L_2, where $L_1 : \frac{x-1}{2} = \frac{y+1}{-3} = \frac{z+4}{2}$ and L_2 is the line, passing through the points A(–4, 4, 3), B(–1, 6, 3) and perpendicular to the line $\frac{x-3}{-2} = \frac{y}{3} = \frac{z-1}{1}$, is

(1) $\frac{141}{\sqrt{221}}$ (2) $\frac{24}{\sqrt{117}}$ (3) $\frac{121}{\sqrt{221}}$ (4) $\frac{42}{\sqrt{117}}$

Ans. **[1]**

Sol. $L_1 : \frac{x-1}{2} = \frac{y+1}{-3} = \frac{z+4}{2}$

L_2 is line passing through A(–4, 4, 3) & B(–1, 6, 3)

$\therefore$ Equation of line L_2 : $\dfrac{x+4}{3}=\dfrac{y-4}{2}=\dfrac{z-3}{0}$

$\therefore$ Which is perpendicular to $\dfrac{x-3}{-2}=\dfrac{y}{3}=\dfrac{z-1}{1}$

$\therefore$ Distance between L_1 and L_2 is :

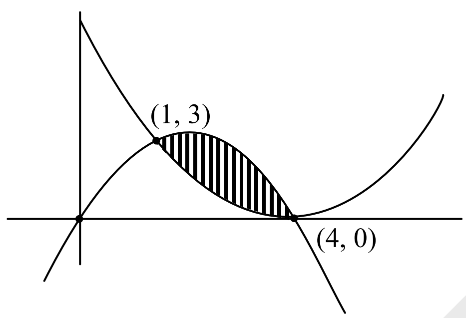
$$\frac{\begin{vmatrix} -5 & 5 & 7 \\ 2 & -3 & 2 \\ 3 & 2 & 0 \end{vmatrix}}{\sqrt{(0-4)^2+(0-6)^2+(4+9)^2}}=\frac{20+30+91}{\sqrt{16+36+169}}=\frac{141}{\sqrt{221}}$$

Q.68 The area of the region enclosed by the parabolas $y = 4x - x^2$ and $3y = (x-4)^2$ is equal to

(1) $\dfrac{14}{3}$ (2) $\dfrac{32}{9}$ (3) 4 (4) 6

Ans. **[4]**

Sol.



$$\text{Area} = \left|\int_1^4\left[(4x-x^2)-\frac{(x-4)^2}{3}\right]\right|dx$$

$$\text{Area} = \left|\frac{4x^2}{2}-\frac{x^3}{3}-\frac{(x-4)^3}{9}\right|_1^4$$

$$= \left|\left(32-\frac{64}{3}\right)-\left(2-\frac{1}{3}+\frac{27}{9}\right)\right|$$

$$= \left|32-\frac{64}{3}-5+\frac{1}{3}\right|$$

$$= |27-21| = 6$$

Q.69 The number of ways in which 21 identical apples can be distributed among three children such that each child gets at least 2 apples, is

(1) 406 (2) 130 (3) 136 (4) 142

Ans. **[3]**

Sol. After giving 2 apples to each child.

15 apples left now 15 apples can be distributed in

${}^{15+3-1}C_2 = {}^{17}C_2$

$= \dfrac{17\times16}{2} = 136$

$\therefore$ Option (3) is correct.

Q.70 Let a variable line passing through the centre of the circle $x^2 + y^2 - 16x - 4y = 0$, meet the positive coordinate axes at the points A and B. Then the minimum value of OA + OB, where O is the origin, is equal to

(1) 12 (2) 18 (3) 20 (4) 24

Ans. **[2]**

Sol. Given $x^2 + y^2 - 16x - 4y = 0$

$\Rightarrow (x-4)^2 + (y-2)^2 = 20$

As variable line passes through center of circle

$\therefore (y-2) = m(x-8)$

$\therefore$ X intercept $= \left(\frac{-2}{m} + 8\right)$

Y intercept $= (-8m + 2)$

$\therefore$ OA + OB $= \frac{-2}{m} + 8 - 8m + 2$

$f'(m) = \frac{2}{m^2} - 8 = 0$

$\Rightarrow m^2 = \frac{1}{4}$

$\Rightarrow m = \frac{-1}{2}$

$\therefore f\left(\frac{-1}{2}\right) = 18$

Option (2) is correct

Q.71 Let the mean and the variance of 6 observations a, b, 68, 44, 48, 60 be 55 and 194, respectively. If a > b, then a + 3b is

(1) 180 (2) 210 (3) 190 (4) 200

Ans. **[1]**

Sol. Mean = 55 = $\overline{X}$

$\frac{a + b + 68 + 44 + 48 + 60}{6} = 55$

a + b + 220 = 330

a + b = 110

a > b

Variance is 194

$\frac{\sum x_i^2}{n} - (\overline{X})^2 = 194$

$\frac{\sum x_i^2}{6} = 3219$

$\sum x_i^2 = 19314$

$a^2 + b^2 = 6850$

a > b

a = 75, b = 35

a + 3b = 75 + 105 = 180

Q.72 If $a = \sin^{-1}(\sin(5))$ and $b = \cos^{-1}(\cos(5))$, then $a^2 + b^2$ is equal to

(1) $4\pi^2 - 20\pi + 50$ (2) 25 (3) $4\pi^2 + 25$ (4) $8\pi^2 - 40\pi + 50$

Ans. **[4]**

Sol. $a = \sin^{-1}(\sin(5))$

$b = \cos^{-1}(\cos(5))$

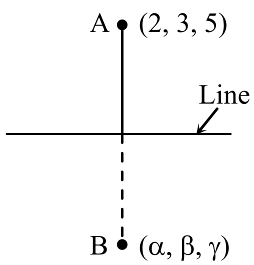
$a = \sin^{-1}(\sin(5))$
$= 5 - 2\pi$
$b = \cos^{-1}(\cos(5))$
$= 2\pi - 5$
$a^2 + b^2 = (5 - 2\pi)^2 + (2\pi - 5)^2$
$= 25 + 4\pi^2 - 20\pi + 4\pi^2 + 25 - 20\pi$
$= 8\pi^2 - 40\pi + 50$

Q.73 Let (α, β, γ) be the mirror image of the point (2, 3, 5) in the line $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$. Then $2\alpha + 3\beta + 4\gamma$ is equal to

(1) 34 (2) 31 (3) 32 (4) 33

Ans. **[4]**

Sol.



Take

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} = \lambda$$

$x = 2\lambda + 1,\ y = 3\lambda + 2,\ z = 4\lambda + 3$

$\overrightarrow{AB} = (\alpha - 2)\hat{i} + (\beta - 3)\hat{j} + (\gamma - 5)\hat{k}$

Now, $(\alpha - 2) \cdot 2 + (\beta - 3) \cdot 3 + (\gamma - 5) \cdot 4 = 0$

$\Rightarrow 2\alpha - 4 + 3\beta - 9 + 4\gamma - 20 = 0$

$\Rightarrow 2\alpha + 3\beta + 4\gamma = 33$

Q.74 A coin is biased so that a head is twice as likely to occur as a tail. If the coin is tossed 3 times then the probability of getting two tails and one head is

(1) $\frac{2}{9}$ (2) $\frac{1}{9}$ (3) $\frac{1}{27}$ (4) $\frac{2}{27}$

Ans. **[1]**

Sol. **H:** Getting a head on tossing the given coin

T: Getting a tail on tossing the given coin

$PH = \frac{2}{3}$

P(Getting two tails and a head on tossing the coin three times)

$$= 3\left(\frac{2}{3}\right)\left(\frac{1}{3}\right)^2 = \frac{2}{9}$$

Q.75 Let f, g: $(0, \infty) \rightarrow \mathbf{R}$ be two functions defined by $f(x) = \int_{-x}^{x} (|t| - t^2)e^{-t^2}dt$ and $g(x) \int_0^{x^2} t^{1/2}e^{-t}dt$. Then, the value of $9\left(f\left(\sqrt{\log_e 9}\right) + g\left(\sqrt{\log_e 9}\right)\right)$ is equal to

(1) 10 (2) 9 (3) 6 (4) 8

Ans. **[4]**

Sol. $f(x) = 2\int_0^x (t - t^2)e^{-t^2}dt$

Put $t^2 = z \Rightarrow 2tdt = dz$

$f(x) = \int_0^{x^2} (1-\sqrt{z})e^{-z}dz$

Switching back to t

$f(x) = \int_0^{x^2} (1-\sqrt{t})e^{-t}dt$

And $g(x) = \int_0^{x^2} \sqrt{t}e^{-t}dt$

$h(x) = f(x) + g(x) = \int_0^{x^2} e^{-t}dt = 1 - e^{-x^2}$

$h\left(\sqrt{\log_e g}\right) = 1 - e^{-\log_e g} = 1 - \frac{1}{9} = \frac{8}{9}$

Required value $= 9 \times \frac{8}{9} = 8$

Q.76 Let z_1 and z_2 be two complex numbers such that $z_1 + z_2 = 5$ and $z_1^3 + z_2^3 = 20 + 15i$. Then $\left|z_1^4 + z_2^4\right|$ equals

(1) 75 (2) $25\sqrt{3}$ (3) $15\sqrt{15}$ (4) $30\sqrt{3}$

Ans. **[1]**

Sol. $z_1 + z_2 = 5$

$z_1^3 + z_2^3 = 20 + 15i$

$z_1^3 + z_2^3 = (z_1 + z_2)^3 - 3z_1z_2(z_1 + z_2)$

$= 125 - 15z_1z_2 = 20 + 15i$

$\Rightarrow z_1z_2 = 7 - i$

$(z_1 + z_2)^2 = 25$

$\Rightarrow z_1^2 + z_2^2 = 25 - 2(7 - i) = 11 + 2i$

$\left(z_1^2 + z_2^2\right)^2 = z_1^4 + z_2^4 + 2z_1^2z_2^2 = 117 + 44i$

$\Rightarrow \left(z_1^4 + z_2^4\right) = 21 + 72i$

$\Rightarrow \left|z_1^4 + z_2^4\right| = 75$

Q.77 The number of solutions, of the equation $e^{\sin x} - 2e^{-\sin x} = 2$, is

(1) 0 (2) 1 (3) 2 (4) More than 2

Ans. **[1]**

Sol. $e^{\sin x} - 2e^{-\sin x} = 2$

Let $y = e^{\sin x}$, $e^{\sin x} > 0 \; \forall x \Rightarrow y > 0$

$\Rightarrow y - \frac{2}{y} = 2 \quad \Rightarrow y^2 - 2y - 2 = 0$

$\Rightarrow (y - 1)^2 = 3$

$\Rightarrow y = \sqrt{3} + 1$

$\Rightarrow e^{\sin x} = \sqrt{3} + 1$

$\sin x = \ln(\sqrt{3} + 1) > 1$

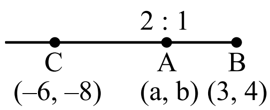
$\Rightarrow$ No solution $\Rightarrow$ Number of solution is zero

Q.78 Let A(a, b), B(3, 4) and C(–6, –8) respectively denote the centroid, circumcentre and orthocenter of a triangle. Then, the distance of the point P(2a + 3, 7b + 5) from the line 2x + 3y – 4 = 0 measured parallel to the line x – 2y – 1 = 0 is

(1) $\frac{17\sqrt{5}}{7}$ (2) $\frac{15\sqrt{5}}{7}$ (3) $\frac{\sqrt{5}}{17}$ (4) $\frac{17\sqrt{5}}{6}$

Ans. **[1]**

Sol. Centroid divides orthocentre and circumcentre in 2 : 1.

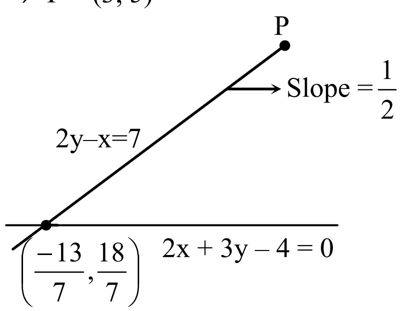


$a \equiv \frac{2(3)-6}{3} = 0$

$b \equiv \frac{2(4)-8}{3} = 0$

$\Rightarrow (a, b) \equiv (0, 0)$

$\Rightarrow P \equiv (3, 5)$



$\Rightarrow (y - 5) = \frac{1}{2}(x - 3)$

$\Rightarrow 2y - 10 = x - 3$

$\Rightarrow 2y - x = 7$

$\Rightarrow \sqrt{\left(3+\frac{13}{7}\right)^2 + \left(5-\frac{18}{7}\right)^2} = \frac{\sqrt{34^2+17^2}}{7} = \frac{17\sqrt{5}}{7}$

Q.79 The temperature T(t) of a body at time t = 0 is 160°F and it decreases continuously as per the differential equation $\frac{dT}{dt} = -K(T - 80)$, where K is a positive constant. If T(15) = 120°F, then T(45) is equal to

(1) 95°F (2) 90°F (3) 80°F (4) 85°F

Ans. **[2]**

Sol. $\frac{dT}{dt} = -K(T - 80)$

$\Rightarrow \frac{dT}{T-80} = -K\,dt$

$\Rightarrow$ Integrating,

$\ln|T - 80| = -Kt + C$

at t = 0, T = 160°F

$\ln|(80)| = C$

$\Rightarrow \ln|T - 80| = -Kt + \ln 80$

$\Rightarrow \ln\left|\frac{T-80}{80}\right| = -Kt$

$$\Rightarrow \left|\frac{T-80}{80}\right| = e^{-Kt}$$

at T(15) = 120°F

$$\Rightarrow \frac{40}{80} = e^{-K(15)} \Rightarrow \frac{1}{2} = e^{-15K}$$

$$\Rightarrow \frac{|T-80|}{80} = e^{-45K} = (e^{-15K})^3 = \left(\frac{1}{2}\right)^3 = \frac{1}{8}$$

$\Rightarrow |T - 80| = 10 \Rightarrow T = 90°F$

Q.80 Consider the function $f : (0, \infty) \to \mathbf{R}$ defined by $f(x) = e^{-|\log_e x|}$. If m and n be respectively the number of points at which f is **not** continuous and f is **not** differentiable, then m + n is

(1) 1 (2) 3 (3) 2 (4) 0

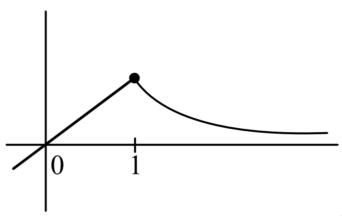
Ans. **[1]**

Sol. $f(x) = e^{-|\log_e x|}$, $x \in (0, \infty)$

$$f(x) = \begin{cases} e^{\ell nx}, & x \in (0,1) \\ e^{-\ell nx}, & x \in [1,\infty) \end{cases}$$

$$\Rightarrow f(x) = \begin{cases} x, & x \in (0,1) \\ 1/x, & x \in [1,\infty) \end{cases}$$

$\Rightarrow$



f(x) is continuous for $\forall x \in R \Rightarrow m = 0$ and f(x) is not differentiable at x = 1

$n \Rightarrow 1$

m + n = 1

Section-B: Numerical Value Type Questions: This section contains 10 Numerical based questions. Attempt any 5 questions out of 10. The answer to each question should be rounded-off to the nearest integer.

Q.81 Let A be a 3 × 3 matrix and det(A) = 2. If

$n = \det(\underbrace{\text{adj}(\text{adj}(.....(\text{adj}A))))}_{2024-\text{times}}$, then the remainder

when n is divided by 9 is equal to ________.

Ans. **[7]**

Sol. |A|=2

$n = |\underbrace{\text{adj}(\text{adj}.....(\text{adj}A))}_{2024-\text{times}}|$

$|\text{adj}A| = |A|^2$

$|\text{adj}(\text{adj}.....(\text{adj}A))| = |A|^{2^{2024}} = 2^{2^{2024}}$

$\because 2^{2024} = 4^{1012} = (3 + 1)^{1012} = 3k + 1$

where k is odd

$|\text{adj}(\text{adj}.......\text{adj}(A)| = 2^{3k+1} = 2.8^k$

$2(9 - 1)^k = 9m - 2$

= 9P + 7

$\Rightarrow$ Remainder = 7

Q.82 Let the coefficient of x^r in the expansion of $(x + 3)^{n-1} + (x + 3)^{n-2}(x + 2) + (x + 3)^{n-3}(x + 2)^2 + \ldots\ldots (x + 2)^{n-1}$ be α_r. If $\sum_{r=0}^{n}\alpha_r = \beta^n - \gamma^n$, $\beta, \gamma \in \mathbf{N}$, then the value of $\beta^2 + \gamma^2$ equals ________.

Ans. **[25]**

Sol. $(x+3)^{n-1}\dfrac{\left[1-\left(\dfrac{x+2}{x+3}\right)^n\right]}{\dfrac{1}{(x+3)}}$

$= (x+3)^n\left(1-\left(\dfrac{x+2}{x+3}\right)^n\right)$

$= (x + 3)^n - (x + 2)^n$

Coefficient of $x^r = {}^nC_r3^{n-r} - {}^nC_r2^{n-r} = \alpha_r$

$\sum_{r=0}^{n}({}^nC_r3^{n-r} - {}^nC_r2^{n-r})$

$= ({}^nC_0 3^n + {}^nC_1 3^{n-1} + \ldots. + {}^nC_n) - ({}^nC_0 2^n + {}^nC_1 2^{n-1} + \ldots.. + {}^nC_n)$

$= (3 + 1)^n - (2 + 1)^n$

$= 4^n - 3^n \equiv \beta^n - \gamma^n$

$\beta = 4 \quad \gamma = 3$

$\beta^2 + \gamma^2 = 4^2 + 3^2 = 25$

Q.83 $\left|\dfrac{120}{\pi^3}\int_0^{\pi}\dfrac{x^2\sin x\cos x}{\sin^4 x+\cos^4 x}dx\right|$ is equal to ________.

Ans. **[15]**

Sol. $I = \int_0^{\pi}\dfrac{x^2\sin x\cos x}{\sin^4 x+\cos^4 x}dx$

$2I = \int_0^{\pi}\dfrac{\sin x\cos x}{\sin^4 x+\cos^4 x}(x^2-(\pi-x)^2)dx$

$2I = \int_0^{\pi}\dfrac{\pi(2x-\pi)\sin x\cos x}{\sin^4 x+\cos^4 x}dx$

$\Rightarrow\ I = \pi\int_0^{\pi/2}\dfrac{(2x-\pi)\sin x\cos x}{\sin^4 x+\cos^4 x}dx$

$\Rightarrow\ I = \pi\int_0^{\pi/2}\dfrac{-2x\sin x\cos x}{\sin^4 x+\cos^4 x}dx$

$2I = -\pi^2\int_0^{\pi/2}\dfrac{\sin x\cos x}{\sin^4 x+\cos^4 x}dx$

$I = \dfrac{-\pi^2}{2\times 2}\int_0^{\pi/2}\dfrac{\sin 2x}{\sin^4 x+\cos^4 x}dx$

$$I = \frac{-\pi^2}{4}\int_0^{\pi/2}\frac{2\tan x\sec^2 x}{\tan^4 x+1}dx$$

$$I = \frac{-\pi^2}{4}\tan^{-1}(\tan^2 x)_0^{\pi/2}$$

$$\Rightarrow\ I = \frac{-\pi^3}{8}$$

$$\therefore\ \left|\frac{120}{\pi^3}I\right| = 15$$

Q.84 Let y = y(x) be the solution of the differential equation $\sec^2 x\,dx + (e^{2y}\tan^2 x + \tan x)dy = 0$, $0 < x < \frac{\pi}{2}$, $y\left(\frac{\pi}{4}\right) = 0$. If $y\left(\frac{\pi}{6}\right) = \alpha$, then $e^{8\alpha}$ is equal to _______.

Ans. **[9]**

Sol. $\sec^2 x\frac{dx}{dy} + e^{2y}\tan^2 x + \tan x = 0$

Put $\tan x = t$

$\Rightarrow \sec^2 x\ \frac{dx}{dy} = \frac{dt}{dy}$

$\frac{dt}{dy} + e^{2y}\times t^2 + t = 0$

$\frac{1}{t^2}\frac{dt}{dy} + \frac{1}{t} = -e^{2y}$

Put $\frac{1}{t} = u$

$\frac{-1}{t^2}\frac{dt}{dy} = \frac{du}{dy}$

$\frac{-du}{dy} + u = e^{2y}$

$\Rightarrow \frac{du}{dy} - u = e^{2y}$

$IF = e^{-y}$

$ue^{-y} = \int e^{-y}e^{2y}$

$\frac{1}{\tan x}\times e^{-y} = e^y + C$

$x = \frac{\pi}{4}\ \ y = 0\ \ \ c = 0$

$x = \frac{\pi}{6}\ \ y = \alpha$

$\sqrt{3}e^{-\alpha} = e^{\alpha} + 0$

$\Rightarrow\ e^{2\alpha} = \sqrt{3}$

$e^{8\alpha} = 9$

Q.85 Let A = {1, 2, 3, …., 100}. Let R be a relation on A defined by $(x, y) \in R$ if and only if $2x = 3y$. Let R_1 be a symmetric relation on A such that $R \subset R_1$ and the number of elements in R_1 is n. Then, the minimum value of n is _____

Ans. **[66]**

Sol. $R_1 = \{(3, 2), (6, 4), (9, 6), (12, 8)\ldots(99.66)\}$ and symmetric elements

$\Rightarrow 2 \times 33 = 66$

$\Rightarrow n(R_1) = 66$

Q.86 If $\lim\limits_{x\to 0} \dfrac{ax^2e^x - b\log_e(1+x) + cxe^{-x}}{x^2 \sin x} = 1$, then $16(a^2 + b^2 + c^2)$ is equal to ______

Ans. **[81]**

Sol.

$$\lim_{x\to 0} \frac{ax^2e^x - b\log_e(1+x) + cxe^{-x}}{x^2 \sin x} = 1$$

$$= \lim_{x\to 0} \frac{ax^2e^x - b\log_e(1+x) + cxe^{-x}}{x^3 \times \dfrac{\sin x}{x}}$$

$$= \lim_{x\to 0} \frac{ax^2e^x - b\log_e(1+x) + cxe^{-x}}{x^3 \times 1}$$

$$= \lim_{x\to 0} \frac{ax^2\left(1 + x + \dfrac{x^2}{2!} + \dfrac{x^3}{3!} + ...\right) - b\left(x - \dfrac{x^2}{2} + \dfrac{x^3}{3} - ....\right) + cx\left(1 - x + \dfrac{x^2}{2!} - \dfrac{x^3}{3!} + ...\right)}{x^3}$$

$$= \lim_{x\to 0} \frac{(c-b)x + \left(\dfrac{b}{2} - c + a\right)x^2 + \left(a - \dfrac{b}{3} + \dfrac{c}{2}\right)x^3 + ...}{x^3} = 1$$

$\Rightarrow c - b = 0,\ \dfrac{b}{2} - c + a = 0$ and $a - \dfrac{b}{3} + \dfrac{c}{2} = 1$

$\Rightarrow a = \dfrac{3}{4},\ b = c = \dfrac{3}{2}$

$a^2 + b^2 + c^2 = \dfrac{9}{16} + \dfrac{9}{4} + \dfrac{9}{4} = \dfrac{81}{16}$

Q.87 A line passes through A(4, –6, –2) and B(16, –2, 4). The point P(a, b, c), where a, b, c are non-negative integers, on the line AB lies at a distance of 21 units, from the point A. The distance between the points P(a, b, c) and Q(4, –12, 3) is equal to _____

Ans. **[22]**

Sol.

$$\frac{x-4}{12} = \frac{y+6}{4} = \frac{z+2}{6}$$

$$\frac{x-4}{\frac{6}{7}} = \frac{y+6}{\frac{2}{7}} = \frac{z+2}{\frac{3}{7}} = 21$$

$$\left(21 \times \frac{6}{7} + 4,\ \frac{2}{7} \times 21 - 6,\ \frac{3}{7} \times 21 - 2\right)$$

$\equiv (22, 0, 7) \equiv (a, b, c) : P$

$PQ = \sqrt{324 + 144 + 16} = 22$

Q.88 Let a, b, c be the lengths of three sides of a triangle satisfying the condition $(a^2 + b^2)x^2 - 2b(a + c)x + (b^2 + c^2) = 0$. If the set of all possible values of x is the interval (α, β), then $12(\alpha^2 + \beta^2)$ is equal to _____

Ans. **[36]**

Sol. $(a^2 + b^2)x^2 - 2b(a + c)x + (b^2 + c^2)=0$

$\Rightarrow (a^2x^2 - 2ab + b^2) + (b^2x^2 - 2bcx + c^2)=0$

$\Rightarrow (ax - b)^2 + (bx - c)^2 = 0$

$\Rightarrow ax - b = 0,\ bx - c = 0$

$\Rightarrow \dfrac{b}{a} = \dfrac{c}{b} \Rightarrow b^2 = ac$

Using triangular inequality

$a + b > c$

$\Rightarrow a + ax > bx$

$\Rightarrow a + ax > ax^2 > 0$

$\Rightarrow x^2 - x - 1 < 0$

$b + c > a$

$ax + bx > a$

$ax + ax^2 > a$

$x^2 + x - 1 > 0$

$c + a > b$

$bx + a > ax$

$ax^2 + a > ax$

$x^2 - x + 1 > 0$ ($D < 0 \Rightarrow$ always true)

$\Rightarrow x^2 - x - 1 < 0$ and $x^2 + x - 1 > 0$

$$\Rightarrow x \in \left(\frac{1-\sqrt{5}}{2}, \frac{1+\sqrt{5}}{2}\right) \text{ and } x \in \left(-\infty, \frac{-1-\sqrt{5}}{2}\right) \cup \left(\frac{-1+\sqrt{5}}{2}, \infty\right)$$

$$\Rightarrow x \in \left(\frac{\sqrt{5}-1}{2}, \frac{\sqrt{5}+1}{2}\right)$$

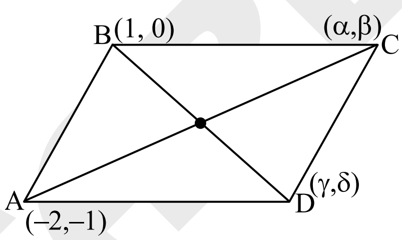
$$\Rightarrow \alpha^2 + \beta^2 = \left(\frac{\sqrt{5}-1}{2}\right)^2 + \left(\frac{\sqrt{5}+1}{2}\right)^2 = 3$$

$$\Rightarrow 12(\alpha^2 + \beta^2) = 36$$

Q.89 Let A(–2, –1), B(1, 0), C(α, β) and D(γ, δ) be the vertices of a parallelogram ABCD. If the point C lies on $2x - y = 5$ and the point D lies on $3x - 2y = 6$, then the value of $|\alpha + \beta + \gamma + \delta|$ is equal to _____

Ans. **[32]**

Sol.



(α, β) lie on $2x - y = 5$ …(i)

(γ, δ) lie on $3x - 2y = 6$ …(ii)

Also, mid-point of diagonal coincide

$\Rightarrow \dfrac{1+\gamma}{2} = \dfrac{\alpha - 2}{2}$

$\Rightarrow \gamma = \alpha - 3$

$\Rightarrow \dfrac{0+\delta}{2} = \dfrac{\beta - 1}{2}$

$\Rightarrow \delta = \beta - 1$

(γ, δ) lie on (ii)

$\Rightarrow 3(\alpha - 3) - 2(\beta - 1) = 6$

$\Rightarrow 3\alpha - 2\beta = 13$

$2\alpha - \beta = 5$

$\Rightarrow \alpha = -3, \beta = -11$

$\Rightarrow \gamma = -6, \delta = -12$

$\Rightarrow |\alpha + \beta + \gamma + \delta| = 32$

Q.90 Let $\vec{a} = 3\hat{i} + 2\hat{j} + \hat{k}$, $\vec{b} = 2\hat{i} - \hat{j} + 3\hat{k}$ and $\vec{c}$ be a vector such that $(\vec{a} + \vec{b}) \times \vec{c} = 2(\vec{a} \times \vec{b}) + 24\hat{j} - 6\hat{k}$ and $(\vec{a} - \vec{b} + \hat{i}).\vec{c} = -3$. then $|\vec{c}|^2$ is equal to

Ans. **[38]**

Sol. $(\vec{a} + \vec{b}) \times \vec{c} = 2(\vec{a} \times \vec{b}) + 24\hat{j} - 6\hat{k}$

$\Rightarrow (5\hat{i} + \hat{j} + 4\hat{k}) \times \vec{c} = 2(7\hat{i} - 7\hat{j} - 7\hat{k}) + 24\hat{j} - 6\hat{k}$

$\Rightarrow$ Let $\vec{c} = x\hat{i} + y\hat{j} + z\hat{k}$

$$\Rightarrow \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 5 & 1 & 4 \\ x & y & z \end{vmatrix} = 14\hat{i} + 10\hat{j} - 20\hat{k}$$

$\Rightarrow \hat{i}(z - 4y) - \hat{j}(5z - 4x) + \hat{k}(5y - x) = 14\hat{i} + 10\hat{j} - 20\hat{k}$

$\Rightarrow z - 4y = 14$

$4x - 5z = 10$

$5y - x = -20$

Also, $(a - b + i) . \vec{c} = -3$

$\Rightarrow (2\hat{i} + 3\hat{j} - 2\hat{k}) . \vec{c} = -3$

$\Rightarrow 2x + 3y - 2z = -3$

$\Rightarrow x = 5, y = -3, z = 2$

$|\vec{c}| = \sqrt{5^2 + (-3)^2 + (2)^2} = \sqrt{38}$

$|\vec{c}|^2 = 38$