JEE Main Online Exam 2024

Questions & Solution

31st January 2024 | Morning

PHYSICS

Section-A: This section contains 20 multiple choice questions. Each question has 4 choices(1), (2), (3) and (4), out of which **ONLY ONE** is correct..

Q.1 Four identical particles of mass m are kept at the four corners of a square. If the gravitational force exerted on one of the masses by the other masses is $\left(\frac{2\sqrt{2}+1}{32}\right)\frac{Gm^2}{L^2}$, the length of the sides of the square is

(1) 2L (2) 3L (3) 4L (4) $\frac{L}{2}$

Ans. **[3]**

Sol. Let a side length

Force on a mass = $\frac{Gm^2}{a^2}\left(\sqrt{2}+\frac{1}{2}\right)$

$\therefore \left(\frac{2\sqrt{2}+1}{32}\right)\frac{Gm^2}{L^2} = \frac{Gm^2}{a^2}\left(\sqrt{2}+\frac{1}{2}\right)$

$\Rightarrow$ a= 4L

Q.2 Two charges q and 3q are separated by a distance 'r' in air. At a distance x from charge q, the resultant electric field is zero. The value of x is :

(1) $\frac{r}{3(1+\sqrt{3})}$ (2) $\frac{(1+\sqrt{3})}{r}$ (3) $\frac{r}{(1+\sqrt{3})}$ (4) $r(1+\sqrt{3})$

Ans. **[3]**

Sol. $\frac{kq}{x^2} = k\frac{(3q)}{(r-x)^2}$

$\Rightarrow x = \frac{r}{(\sqrt{3}+1)}$

Q.3 In a plane EM wave, the electric field oscillates sinusoidally at a frequency of 5×10^{10} Hz and an amplitude of 50 Vm^{-1}. The total average energy density of the electromagnetic field of the wave is :
[Use $\varepsilon_0 = 8.85 \times 10^{-12}\ C^2/Nm^2$]

(1) $4.425 \times 10^{-8}\ Jm^{-3}$ (2) $2.212 \times 10^{-8}\ Jm^{-3}$
(3) $1.106 \times 10^{-8}\ Jm^{-3}$ (4) $2.212 \times 10^{-10}\ Jm^{-3}$

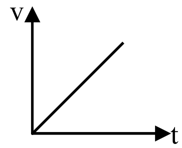
Ans. **[3]**

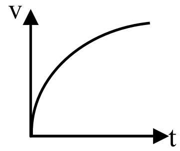
Sol. $u = \frac{1}{2}\varepsilon_0 E^2$

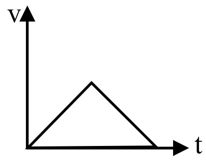
$= \frac{1}{2} \times (8.85 \times 10^{-12}) \times (50)^2$

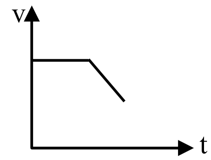
$= 1.106 \times 10^{-8}\ Jm^{-3}$

Q.4 A small steel ball is dropped into a long cylinder containing glycerine. Which one of the following is the correct representation of the velocity time graph for the transit of the ball?

(1) 

(2) 

(3) 

(4) 

Ans. **[2]**

Sol. $\mathbf{a} = \left(\frac{mg - v\rho g}{m}\right) - \left(\frac{6\pi nr}{m}\right)v$

$\mathbf{a = C_1 - C_2\, v}$

$\therefore \int_0^v \frac{dv}{C_1 - C_2} v = \int_0^t dt$

$\Rightarrow v = \frac{C_1}{C_2}\left(1 - e^{-C_2 t}\right)$

$\therefore$ Exponentially growing graph is possible $\therefore$

Q.5 When a metal surface is illuminated by light of wavelength λ, the stopping potential is 8V. When the same surface is illuminated by light of wavelength 3λ, stopping potential is 2V. The threshold wavelength for this surface is:

(1) 3λ (2) 4.5λ (3) 9λ (4) 5λ

Ans. **[3]**

Sol. $8\text{ eV} = \frac{hc}{\lambda} - \phi_0$ ....(i)

$2\text{ eV} = \frac{hc}{3\lambda} - \phi_0$ ...(ii)

From (i) & (ii)

$\phi = \frac{1}{9}\frac{hcv}{\lambda}$

$\therefore \lambda_0 = 9\lambda$

Q.6 The relation between time 't' and distance 'x' is $t = \alpha x^2 + \beta x$, where α and β are constants. The relation between acceleration (a) and velocity (v) is:

(1) $a = -3\alpha v^2$ (2) $a = -5\alpha v^5$ (3) $a = -2\alpha v^3$ (4) $a = -4\alpha v^4$

Ans. **[3]**

Sol. $\frac{1}{v} = \frac{dt}{dx} = 2\alpha x + \beta$

Now, $-\frac{1}{v^2}\frac{dv}{dt} = 2\alpha\frac{dx}{dt}$

$\Rightarrow -\frac{1}{v^2}a = 2\alpha v$

$\Rightarrow a = -2\alpha v^3$

Q.7 A force is represented by $F = ax^2 + bt^{\frac{1}{2}}$

Where x = distance and t = time. The dimensions of b^2/a are :

(1) $[ML^3T^{-3}]$ (2) $[ML^2T^{-3}]$ (3) $[ML^{-1}T^{-1}]$ (4) $[MLT^{-2}]$

Ans. **[1]**

Sol. $[a] = \left[\frac{F}{x^2}\right], [b] = \left[\frac{F}{\sqrt{t}}\right]$

$\therefore \left[\frac{b^2}{a}\right] = \left[\frac{Fx^2}{t}\right] = [ML^3T^{-3}]$

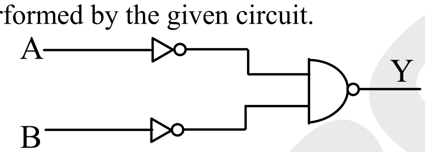
Q.8 The parameter that remains the same for molecules of all gases at a given temperature is :

(1) momentum (2) speed (3) kinetic energy (4) mass

Ans. **[3]**

Sol. K.E. is independent of the mass of the molecules.

Q.9 Identify the logic operation performed by the given circuit.



(1) NOR (2) OR (3) AND (4) NAND

Ans. **[2]**

Sol. Truth table is

A	**B**	**Y**
0	0	0
1	1	1
0	1	1
1	0	1

$\therefore$ OR gate

Q.10 The refractive index of a prism with apex angle A is $\cot\frac{A}{2}$. The angle of minimum deviation is :

(1) $\delta_m = 180° - A$ (2) $\delta_m = 180° - 2A$ (3) $\delta_m = 180° - 4A$ (4) $\delta_m = 180° - 3A$

Ans. **[2]**

Sol. $\therefore \mu = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}$

and $\mu = \cot\left(\frac{A}{2}\right)$

$\therefore \delta_m = 180 - 2A$

Q.11 The fundamental frequency of a closed organ pipe is equal to the first overtone frequency of an open organ pipe. If length of the open pipe is 60 cm, the length of the closed pipe will be:

(1) 30 cm (2) 45 cm (3) 15 cm (4) 60 cm

Ans. **[3]**

Sol. $\frac{v}{4\ell_c} = 2\left(\frac{v}{2\ell_0}\right)$ and $\ell_0 = 60$ cm

$\ell c = 15$ cm

Q.12 A coin is placed on a disc. The coefficient of friction between the coin and the disc is μ. If the distance of the coin from the centre of the disc is r, the maximum angular velocity which can be given to the disc, so that the coin does not slip away, is:

(1) $\sqrt{\frac{\mu g}{r}}$ (2) $\frac{\mu}{\sqrt{rg}}$ (3) $\frac{\mu g}{r}$ (4) $\sqrt{\frac{r}{\mu g}}$

Ans. [1]

Sol. $F_{\text{centripetal}} \le f_{max}$

$m\omega^2 r \le \mu mg$

$\omega \le \sqrt{\frac{\mu g}{r}}$

Q.13 An artillery piece of mass M_1 fires a shell of mass M_2 horizontally. Instantaneously after the firing, the ratio of kinetic energy of the artillery and that of the shell is:

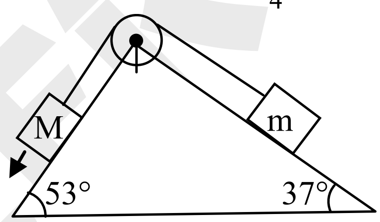
(1) $\frac{M_2}{M_1}$ (2) $\frac{M_2}{(M_1+M_2)}$ (3) $\frac{M_1}{M_2}$ (4) $\frac{M_1}{(M_1+M_2)}$

Ans. [1]

Sol. $\therefore P_1 = P_2$ and $K.E = \frac{p^2}{2M}$

$\therefore \frac{K.E_1}{K.E_2} = \frac{M_2}{M_1}$

Q.14 In the given arrangement of a doubly inclined plane two blocks M and m are placed. The blocks are connected by a light string passing over an ideal pulley as shown. The coefficient of friction between the surface of the plane and the blocks is 0.25. The value of m, for which M = 10 kg will move down with an acceleration of 2 m/s^2, is: (take g = 10 m/s^2 and $\tan 37° = \frac{3}{4}$)



(1) 6.5 kg (2) 9 kg (3) 4.5 kg (4) 2.25 kg

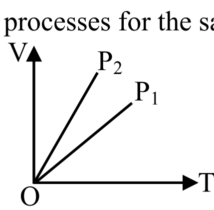
Ans. [3]

Sol. $a = \frac{\text{Net pulling force}}{\text{Total mass}}$

$\mathbf{2} = \frac{Mg\sin 53° - \mu Mg\cos 53° - mg\sin 37° - \mu mg\cos 37°}{M+m}$

$= 20 + 2m = 65 - 6m - 2m$ $\therefore m = 4.5$ kg

Q.15 The given figure represents two isobaric processes for the same mass of an ideal gas, then



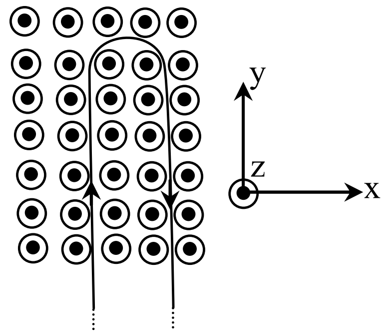
(1) $P_1 > P_2$ (2) $P_1 = P_2$ (3) $P_2 \ge P_1$ (4) $P_2 > P_1$

Ans. [1]

Sol. at same temperature

$V_2 > V_1$ $\therefore P_2 < P_1$

Q.16 A rigid wire consists of a semicircular portion of radius R and two straight sections. The wire is partially immerged in a perpendicular magnetic field $B = B_0\ \hat{k}$ as shown in figure. The magnetic force on the wire if it has a current i is:



(1) $2iBR\,\hat{j}$ (2) $-2iBR\,\hat{j}$ (3) $iBR\,\hat{j}$ (4) $-iBR\,\hat{j}$

Ans. **[2]**

Sol. $F = i\left(\int \vec{dl} \times \vec{B}\right)$

$\therefore\ \vec{F} = i(2R)B_0\,(-\hat{j})$

Q.17 A coil is places perpendicular to a magnetic field of 5000 T. When the field is changed to 3000 T in 2 s, an induced emf of 22 V is produced in the coil. If the diameter of the coil is 0.02 m, then the number of turns in the coil is

(1) 35 (2) 70 (3) 7 (4) 140

Ans. **[2]**

Sol. $\varepsilon = \frac{\Delta\phi}{\Delta t} = \frac{N\Delta(BA)}{\Delta t}$

$22 = \frac{N\left(2000\left(\frac{22}{7}.\frac{0.02^2}{4}\right)\right)}{2}$

$\Rightarrow N = 70$

Q.18 If the wavelength of the first member of Lyman series of hydrogen is λ. The wavelength of the second member will be

(1) $\frac{5}{27}\lambda$ (2) $\frac{27}{5}\lambda$ (3) $\frac{32}{27}\lambda$ (4) $\frac{27}{32}\lambda$

Ans. **[4]**

Sol. $\frac{1}{\lambda} = R\left(\frac{1}{1^2} - \frac{1}{2^2}\right)$ .....(i)

Also $\frac{1}{\lambda'} = R\left(\frac{1}{1^2} - \frac{1}{3^2}\right)$ ....(ii)

from (i) and (ii)

$\lambda' = \frac{27}{32}\lambda$

Q.19 Two conductors have the same resistances at 0°C but their temperature coefficients of resistance are α_1 and α_2. The respective temperature coefficients for their series and parallel combinations are:

(1) $\alpha_1 + \alpha_2, \frac{\alpha_1 + \alpha_2}{2}$ (2) $\frac{\alpha_1 + \alpha_2}{2}, \alpha_1 + \alpha_2$ (3) $\frac{\alpha_1 + \alpha_2}{2}, \frac{\alpha_1 + \alpha_2}{2}$ (4) $\alpha_1 + \alpha_2, \frac{\alpha_1 a_2}{\alpha_1 + \alpha_2}$

Ans. **[3]**

Sol. For series

$$R_{series} = 2R_0\left[1+\left(\frac{\alpha_1+\alpha_2}{2}\right)\Delta t\right]$$

$$\therefore\ \alpha_{series} = \frac{\alpha_1+\alpha_2}{2}$$

For parallel

$$-\frac{dR}{R_p^2} = \frac{-dR}{R_1^2} - \frac{dR_2}{R_2^2}$$

$$\frac{\alpha_p R_p dT}{R_p^2} = \frac{\alpha_1 R_0 dT}{R_0^2} + \frac{\alpha_2 R_0 dT}{R_0^2}$$

$$\Rightarrow \alpha_p = \frac{\alpha_1+\alpha_2}{2}$$

Q.20 If the percentage errors in measuring the length and the diameter of a wire are 0.1% each. The percentage error in measuring its resistance will be:

(1) 0.2% (2) 0.144% (3) 0.3% (4) 0.1%

Ans. **[3]**

Sol. $\frac{\Delta R}{R} = \frac{\Delta \ell}{\ell} + \frac{2\Delta D}{D}\ \left(\because R = \frac{4\rho I}{\pi D^2}\right)$

$\Rightarrow \frac{\Delta R}{R} \times 100 = 0.1 + 2\,(0.1)$

= 0.3%

Section-B: Numerical Value Type Questions: This section contains 10 Numerical based questions. Attempt any 5 questions out of 10. The answer to each question should be rounded-off to the nearest integer.

Q.21 An electron moves through a uniform magnetic field $\vec{B} = B_0\hat{i} + 2B_0\hat{j}$T. At a particular instant of time, the velocity of electron is $\vec{u} = 3\hat{i} + 5\hat{j}$ m/s. If the magnetic force acting on electron is $\vec{F} = 5e\hat{k}$ N, where e is the charge of electron then the value of B_0 is ____________T.

Ans. **[5]**

Sol. $\therefore\ \vec{F} = e(\vec{V}\times\vec{B})$

$= \mathbf{e}\ (3\hat{i}+5\hat{j})\times(B_0\hat{i}+3B_0\hat{j})$

$= eB_0\hat{k}$

$\therefore\ B_0 = 5T$

Q.22 The mass defect in a particular reaction is 0.4 g. The amount of energy liberated is n × 10^7 kWh. Where n = _______.

(Speed of light = 3 × 10^8 m/s)

Ans. **[1]**

Sol. $\Delta E = \Delta mc^2$

$n \times 10^7 \times 3.6 \times 10^6 = 0.4 \times 10^{-3} \times 9 \times 10^{16}$

$\Rightarrow n = 1$

Q.23 The depth below the surface of sea to which a rubber ball be taken so as to decrease its volume by 0.02% is ________ m.

(Take density of sea water = 10^3 kgm^{-3}, Bulk modulus of rubber = 9 × 10^8 Nm^{-2}, and g = 10 ms^{-2}

Ans. **[18]**

Sol. $\therefore B = \left|\frac{\Delta P}{\Delta V / V}\right|$

$\therefore \Delta P = \rho g h = \frac{B\Delta V}{V}$

$h = \frac{B}{\rho g}\left(\frac{\Delta V}{V}\right)$

$= \frac{9\times 10^{8}}{10^{3}\times 10}\left(0.02\times 10^{-2}\right)$

= 18 m

Q.24 A body starts falling freely from height H hits an inclined plane in its path at height h. As a result of this perfectly elastic impact, the direction of the velocity of the body becomes horizontal. The value of $\frac{H}{h}$ for which the body will take the maximum time to reach the ground is _______.

Ans. **[2]**

Sol. $\sqrt{\frac{2(H-h)}{g}} + \sqrt{\frac{2h}{g}} = T$

for T $\rightarrow$ maximum

$\frac{dT}{dh} = 0$

$\Rightarrow \Rightarrow \frac{1}{2\sqrt{(H-h)}}(-1) + \frac{1}{2\sqrt{h}}(1) = 0$

$\Rightarrow h = \frac{H}{2}$

$\Rightarrow \frac{H}{h} = 2$

Q.25 A particle performs simple harmonic motion with amplitude A. Its speed is increased to three times at an instant when its displacement is $\frac{2A}{3}$. The new amplitude of motion is $\frac{nA}{3}$. The value of n is _______.

Ans. **[7]**

Sol. $3\omega\sqrt{A^2 - \left(\frac{2A}{3}\right)^2} = \omega\sqrt{(A')^2 - \left(\frac{2A}{3}\right)^2}$

$\Rightarrow A' = \frac{7}{3}A$

Q.26 A small square loop of wire of side ℓ is placed inside a large square loop of wire of side L ($L = \ell^2$). The loops are coplanar and their centres coincide. The value of the mutual inductance of the system is $\sqrt{x} \times 10^{-7}$ H, where x = __________ .

Ans. **[128]**

Sol. $M = \frac{\phi}{i}$

$= \frac{4\left[\frac{\mu_0 i}{4\pi(L/2)}\sqrt{2}\right]\ell^2}{i}$

$= 8\sqrt{2}\times 10^{-7}$ H

$= \sqrt{128}\times 10^{-7}$ H

Q.27 A solid circular disc of mass 50 kg rolls along a horizontal floor so that its center of mass has speed of 0.4 m/s. The absolute value of work done on the disc to stop it is _______ J.

Ans. **[6]**

Sol. $W = \frac{1}{2}\left(\frac{mR^2}{2}\right)\left(\frac{V^2}{R^2}\right) + \frac{1}{2}mv^2$

$= \frac{3}{4}mv^2$

$= \frac{3}{4}(50)\,(0.4)^2 = 6\text{ J}$

Q.28 A parallel plate capacitor with plate separation 5 mm is charged up by a battery. It is found that on introducing a dielectric sheet of thickness 2 mm, while keeping the battery connections intact, the capacitor draws 25% more charge from the battery than before. The dielectric constant of the sheet is _______.

Ans. **[2]**

Sol. Let $C = \frac{\varepsilon_0 A}{(5mm)}$

$\Rightarrow C' = \frac{\varepsilon_0 A}{3 + \frac{2}{k}}$

Now

$C'V = \frac{5}{4}(CV)$

$\Rightarrow \frac{\varepsilon_0 A}{3 + \frac{2}{k}} = \frac{\varepsilon_0 A}{5}\left(\frac{5}{4}\right) \Rightarrow k = 2$

Q.29 Two waves of intensity ratio 1 : 9 cross each other at a point. The resultant intensities at that point, when (a) Waves are incoherent is I_1 (b) Waves are coherent is I_2 and differ in phase by 60°. If $\frac{I_1}{I_2} = \frac{10}{x}$ then x = ________.

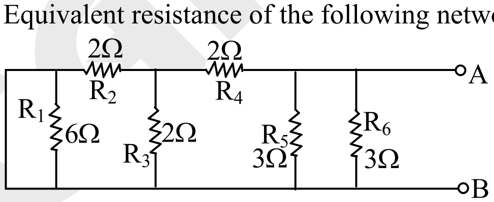
Ans. **[13]**

Sol. $I_1 = 10C$

$I_2 = C + 9C + 6C\cos 60° = 13C$

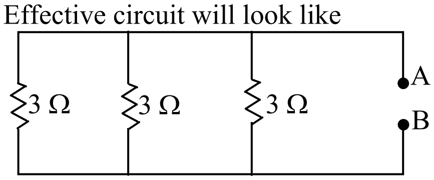
$\therefore \frac{I_1}{I_2} = \frac{10}{13} \Rightarrow x = 13$

Q.30 Equivalent resistance of the following network is ___________Ω



Ans. **[1]**

Sol. Effective circuit will look like



$R_{AB} = 1\ \Omega$

CHEMISTRY

Section-A: This section contains 20 multiple choice questions. Each question has 4 choices(1), (2), (3) and (4), out of which **ONLY ONE** is correct..

Q.31 'Adsorption' principle is used for which of the following purification method?

(1) Sublimation (2) Chromatography (3) Extraction (4) Distillation

Ans. [2]

Sol. Adsorption chromatography is based upon the differential adsorption of the various components of a mixture on a suitable adsorbent such as silica gel or alumina.

Q.32 The correct sequence of electron gain enthalpy of the elements listed below is

A. Ar B. Br

C. F D. S

Choose the most appropriate from the options given below

(1) A > D > C > B (2) C > B > D > A (3) A > D > B > C (4) D > C > B > A

Ans. [3]

Sol. Electron gain enthalpy of

$D \Rightarrow S = -200\ kJ\ mol^{-1}$

$C \Rightarrow F = -333\ kJ\ mol^{-1}$

$B \Rightarrow Br = -325\ kJ\ mol^{-1}$

$A \Rightarrow \ Ar = 96\ kJ\ mol^{-1}$

A > D > B > C (source NCERT)

Q.33 The compound that is white in colour is

(1) ammonium arsinomolybdate (2) lead sulphate

(3) ammonium sulphide (4) lead iodide

Ans. [2]

Sol. Lead sulphate is a white solid, which appears white in microcrystalline form.

Q.34 The linear combination of atomic orbitals to form molecular orbitals takes place only when the combining atomic orbitals

A. have the same energy

B. have the minimum overlap

C. have same symmetry about the molecular axis

D. have different symmetry about the molecular axis

Choose the **most appropriate** from the options given below.

(1) B, C, D only (2) B and D only (3) A, B, C only (4) A and C only

Ans. [4]

Sol. Linear combination of atomic orbitals to form molecular orbitals takes place when

• They have same energy

• Maximum overlap

• Have same symmetry about the molecular axis

Hence, only A and C are correct.

Q.35 Consider the oxides of group 14 elements

SiO_2, GeO_2 SnO_2, PbO_2, CO and GeO. The amphoteric oxides are

(1) SnO_2, CO (2) SiO_2, GeO_2 (3) GeO, GeO_2 (4) SnO_2, PbO_2

Ans. [4]

Sol. SnO_2 and $PbO_2 \rightarrow$ Amphoteric oxides

$CO \rightarrow$ Neutral oxide

GeO, SiO_2, $GeO_2 \rightarrow$ Acidic oxides

Q.36 The metals that are employed in the battery industries are

A. Fe
B. Mn
C. Ni
D. Cr
E. Cd

Choose the correct answer from the options given below:

(1) A, B, C and D only (2) A, B, C, D and E (3) B, C and E only (4) B, D and E only

Ans. **[3]**

Sol. Mn, Ni and Cd are metals that are employed in the battery industries.

Commercial dry cell has → Zinc anode and MnO_2 + carbon black + NH_4Cl paste cathode.

Ni – Cd cell is a rechargeable cell.

$Cd(s) + Ni(OH)_3(s) \rightarrow CdO(s) + 2Ni(OH)_2(s) + H_2O(l)$

Q.37 Match List I with List II

LIST I		LIST II	
A.	Glucose/$NaHCO_3$/Δ	I.	Gluconic acid
B.	Glucose/HNO_3	II.	No reaction
C.	Glucose/HI/Δ	III.	n-hexane
D	Glucose/Bromine water	IV.	Saccharic acid

Choose the correct answer from the options given below:

(1) A-I, B-IV, C-III, D-II (2) A-III, B-II, C-I, D-IV
(3) A-IV, B-I, C-III, D-II (4) A-II, B-IV, C-III, D-I

Ans. **[4]**

Sol.

$$\text{Glucose} \xrightarrow[\Delta]{NaHCO_3} \text{No reaction}$$

$$\text{Glucose} \xrightarrow{HNO_3} \begin{array}{c} COOH \\ | \\ (CHOH)_4 \\ | \\ CO_2OH \end{array} \text{(saccharic acid)}$$

$$\text{Glucose} \xrightarrow[\Delta]{HI} \text{N –Hexane}$$

$$\text{Glucose} \xrightarrow[H_2O]{Br_2} \begin{array}{c} COOH \\ | \\ (CHOH)_4 \\ | \\ CH_2OH \end{array} \text{(Gluconic acid)}$$

Q.38 The product (C) in the below mentioned reaction is:

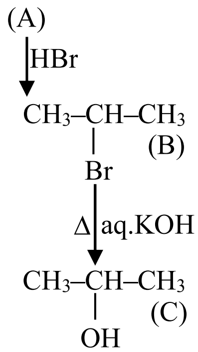
$$CH_3 - CH_2 - CH_2 - Br \xrightarrow[\Delta]{KOH_{(alc)}} A \xrightarrow{HBr} B \xrightarrow[KOH_{(aq)}]{\Delta} C$$

(1) Propene (2) Propan-2- ol (3) Propan-1- ol (4) Propyne

Ans. **[2]**

Sol. Propan-2 – ol

$$CH_3\,CH_2\,CH_2\,Br \xrightarrow[\Delta]{alc.KOH} CH_3 - CH = CH_2$$



Q.39 Identify correct statements from below:
A. The chromate ion is square planar.
B. Dichromates are generally prepared from chromates.
C. The green manganate ion is diamagnetic.
D. Dark green coloured K_2MnO_4 disproportionates in a neutral or acidic medium to give permanganate.
E. With increasing oxidation number of transition metal, ionic character of the oxides decreases.
Choose the correct answer from the options given below:
(1) B, D, E only (2) A, D, E only (3) B, C, D only (4) A, B, C only

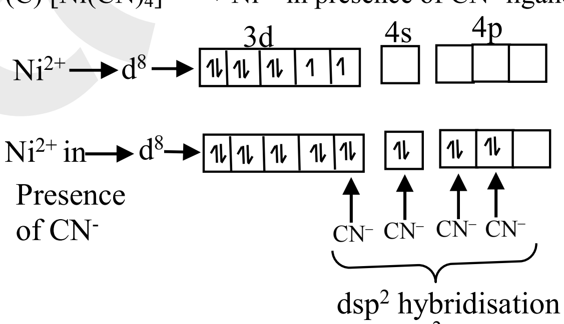
Ans. [1]

Sol. (A) Chromate ions are tetrahedral and not square planar.
(B) $FeCr_2O_4 + Na_2CO_3 + O_2 \rightarrow Na_2CrO_4 + Fe_2O_3 + CO_2$
$Na_2CrO_4 + H^+ \rightarrow Na_2Cr_2O_7 + Na^+$
(C) Manganate $\rightarrow MnO_4^{2-} \rightarrow$ 1 unpaired e^- in $Mn^{+6} \rightarrow$ Paramagnetic
(D) $K_2\overset{+6}{M}nO_4 \xrightarrow{H^+} K\overset{+7}{M}nO_4 + \overset{+4}{M}nO_2 + H_2O$ (Disproportionation)
(E) $Mn_2O_7 \rightarrow$ Covalent; CrO_3, V_2O_5 has low melting point. Oxidation state increases then ionic character decreases.

Q.40 The correct statements from following are:
A. The strength of anionic ligands can be explained by crystal field theory.
B. Valence bond theory does not give a quantitative interpretation of kinetic stability of coordination compounds.
C. The hybridisation involved in formation of $[Ni(CN)_4]^{2-}$ complex is dsp^2
D. The number of possible isomer(s) of cis- $[PtCl_2(en)_2]^{2+}$ is one
Choose the correct answer from the options given below :
(1) A, C only (2) A, D only (3) B, D only (4) B, C only

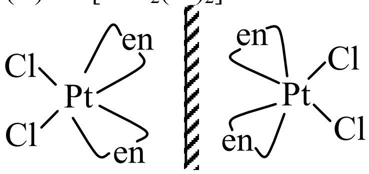
Ans. [4]

Sol. (A) Strength of anionic ligands cannot be explained by CFT instead LFT i.e., ligand field theory explains the strength of ligands.
(B) VBT does not give a quantitative interpretation of kinetic stability of coordination compounds.
(C) $[Ni(CN)_4]^{2-} \rightarrow Ni^{2+}$ in presence of CN^- ligand.



Square planar geometry since $\rightarrow 4dsp^2$ hybridised orbitals created.

(D) cis-$[PtCl_2(en)_2]$ ⇒ It has two possible isomers



Q.41 For the given reaction, choose the correct expression of K_C from the following.

Fe^{3+} (aq.) SCN^- (aq.) $\rightleftharpoons$ $(FeSCN)^{2+}$ (aq.)

(1) $K_c = \dfrac{[Fe^{3+}][SCN^-]}{[FeSCN^{2+}]}$

(2) $K_c = \dfrac{[FeSCN^{2+}]^2}{[Fe^{3+}][SCN^-]}$

(3) $K_c = \dfrac{[FeSCN^{2+}]}{[Fe^{3+}]^2[SCN^-]^2}$

(4) $K_c = \dfrac{[FeSCN^{2+}]}{[Fe^{3+}][SCN^-]}$

Ans. **[4]**

Sol. Given reaction :

Fe^{3+} (aq.) SCN^- (aq.) $\rightleftharpoons$ $(FeSCN)^{2+}$ (aq.)

$$K_c = \frac{[\text{Product ions}]}{[\text{Reactant ions}]} = \frac{[FeSCN^{2+}]}{[Fe^{3+}][SCN^-]}$$

Correct answer is (4)

Q.42 Integrated rate law equation for a first order gas phase reaction is given by (where P_i is initial pressure and P_t is total pressure at time t)

(1) $K = \dfrac{2.303}{t} \times \log \dfrac{2P_i}{(2P_i - P_t)}$

(2) $K = \dfrac{2.303}{t} \times \log \dfrac{P_i}{(2P_i - P_t)}$

(3) $K = \dfrac{2.303}{t} \times \dfrac{P_i}{(2P_i - P_t)}$

(4) $K = \dfrac{2.303}{t} \times \log \dfrac{(2P_i - P_t)}{P_i}$

Ans. **[2]**

Sol.

$$A(g) \longrightarrow 2B(g)$$

Initial : P_i

Pressure at time t: $P_i - x$ $\quad 2x$

$P_t = P_i - x + 2x$

$P_t = P_i + x$

$x = P_t - P_i$

$$K = \frac{2.303}{t} \log \frac{\text{Initial pressure of A}}{\text{Pressure of A at time t}}$$

$$K = \frac{2.303}{t} \log \frac{P_i}{P_i - x} \Rightarrow \frac{2.303}{t} \log \frac{P_i}{P_i - P_t + p_i}$$

$$K = \frac{2.303}{t} \log \frac{P_i}{2P_i - P_t}$$

Q.43 Match List I with List II

LIST I (Technique)		LIST II (Application)	
A.	Distillation	I.	Separation of glycerol from spent-lye
B.	Fractional distillation	II.	Aniline-Water mixture
C.	Steam distillation	III.	Separation of crude oil fractions
D	Distillation under reduced pressure	IV.	Chloroform-Aniline

Choose the correct answer from the options given below.

(1) A-IV, B-III, C-II, D-I

(2) A-IV, B-I, C-II, D-III

(3) A-I, B-II, C-IV, D-III

(4) A-II, B-III, C-I, D-IV

Ans. **[1]**

Sol. A-IV: Chloroform and aniline can be separated by distillation.
B-III: Separation of crude oil fractions in petroleum industry is done by fractional distillation.
C-II: Aniline water mixture can be separated by steam distillation due to difference in boiling point.
D-I: Separation of glycerol from spent-lye done by distillation under reduced pressure (vacuum distillation).

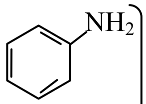
Q.44 Identify the mixture that shows positive deviations from Raoult's law.

(1) $(CH_3)_2CO + C_6H_5NH_2$ (2) $CHCl_3 + C_6H_6$

(3) $CHCl_3 + (CH_3)_2 CO$ (4) $(CH_3)_2CO + CS_2$

Ans. **[4]**

Sol.

(1) $CH_3-\underset{\underset{O}{\|}}{C}-CH_3$ + 

acetone Aniline

H-Bonding between acetone and aniline is more stronger then acetone-acetone and aniline-aniline

(2) $CHCl_3$ + C_6H_6

Chloroform Benzene

H-Bonding between π-cloud of Benzene and H-atom of choloroform (Negative deviation)

(3) $CHCl_3$ + $CH_3-\underset{\underset{O}{\|}}{C}-CH_3$

Chloroform Acetone

Shows negative deviation due to H-Bonding

(4) $CH_3-\underset{\underset{O}{\|}}{C}-CH_3$ + $S=C=S$

Acetone Carbondisulphide

Shows positive deviation because solute solvent interaction is weaker then solute-solute and solvent-solvent.

Q.45 Given below are two statements: One is labelled as **Assertion A** and the other is labelled as **Reason R.**

Assertion A: Alcohols react both as nucleophiles and electrophiles.

Reason R: Alcohol is react with active metals such as sodium, potassium and aluminium to yield corresponding alkoxides and liberate hydrogen.

In the light of the above statements, choose the correct answer from the options given below.

(1) Both **A** and **R** are true but **R** is NOT the correct explanation of **A**

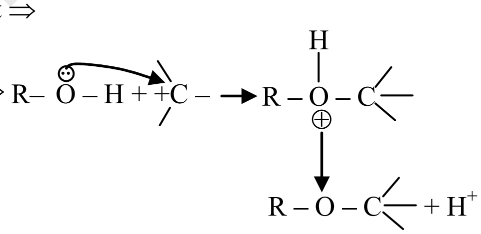
(2) **A** is true but **R** is false

(3) Both **A** and **R** are true and **R** is the correct explanation of **A**

(4) **A** is false but **R** is true

Ans. **[1]**

Sol. Assertion is correct ⇒

Alcohols as $Nu^{\ominus}$ ⇒ 

Bond between R — H ┼ H is broken when alcohol act as nucleophile

Alcohols as electrophile ⇒ Bond between

CH_3 ┼ OH is broken where alcohol act as electrophile.

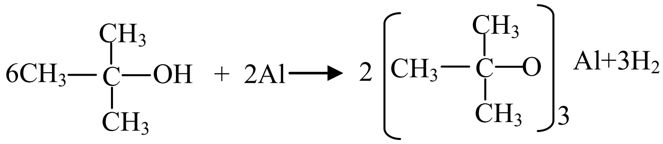
$R - CH_2 - OH + H^+ \longrightarrow R - CH_2 - \overset{\oplus}{O}H_2$

$Br^- + R - CH_2 - \overset{+}{O}H_2 \longrightarrow Br - CH_2 - R + H_2O$

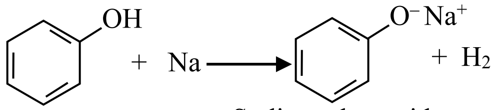
Reason is also correct :

$2R - OH + 2Na \longrightarrow 2R - O - Na + H_2$

Sodium alkoxide



t-Butylalcohol   Aluminium tert-butoxide



Phenol   Sodium phenoxide

Q.46 Given below are two statements: One is labelled as **Assertion A** and the other is labelled as **Reason R.**

Assertion A: pK_a value of phenol is 10.0 while that of ethanol is 15.9.

Reason R: Ethanol is stronger acid than phenol.

In the light of the above statements, choose the correct answer from the options given below.

(1) Both **A** and **R** are true but **R** is NOT the correct explanation of **A**

(2) **A** is true but **R** is false

(3) Both **A** and **R** are true and **R** is the correct explanation of **A**

(4) **A** is false but **R** is true

Ans. **[2]**

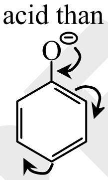
Sol. pK_a of phenol = 10.0

pK_a of ethanol = 15.9

Acid which has more pK_a is weaker acid.

Ethanol is weaker acid than phenol

$CH_3CH_2O^{\ominus}$



Negative charge is destabilised by + I effect of ethyl group.

Negative charge is stabilised by – R effect of benzene ring.

Hence, removal of H^+ is easier in case of phenol as compared to ethanol.

Q.47 A species having carbon with sextet of electrons and can act as electrophile is called

(1) Pentavalent carbon (2) Carbon free radical (3) Carbocation (4) Carbanion

Ans. **[3]**

Sol. Sextet of electron means 6 electrons in outermost shell.

A species having six electron in outermost shell is electron deficient and hence can act as electrophile (electron loving). Carbocation is a species which have $6e^-$s in outermost shell and can act as E^+ .

Q.48 Given below are two statements:

Statement I: IUPAC name of $HO - CH_2 - (CH_2)_3 - CH_2 - COCH_3$ is 7-hydroxyheptan-2-one.

Statement II: 2-oxoheptan-7-ol is the correct IUPAC name for above compound.

In the light of the above statements, choose the most appropriate answer from the options given below.

(1) Both **statement I** and **statement II** are incorrect
(2) **Statement I** is incorrect but **statement II** is correct
(3) **Statement I** is correct but **statement II** is incorrect
(4) Both **statement I** and **statement II** are correct

Ans. **[3]**

Sol. $HO-\overset{7}{C}H_2-\overset{6}{C}H_2-\overset{5}{C}H_2-\overset{4}{C}H_2-\overset{3}{C}H_2-\overset{2}{\underset{\underset{O}{\|}}{C}}-\overset{1}{C}H_3$

Correct IUPAC name is
7-Hydroxyheptan – 2 – one
Hence, **statement I** is correct and **statement II** is incorrect.

Q.49 Given below are two statements:
Statement I: Noble gases have very high boiling points.
Statement II: Noble gases are monoatomic gases. They are held together by strong dispersion forces. Because of this they are liquefied at very low temperature. Hence, they have very high boiling points.
In the light of the above statements, choose the correct answer from the options given below.
(1) Both **statement I** and **statement II** are true
(2) Both **statement I** and **statement II** are false
(3) **Statement I** is false but **statement II** is true
(4) **Statement I** is true but **statement II** is false

Ans. **[2]**

Sol. Noble gases have very low melting and boiling points because noble gases are held together by weak dispersion forces.
Even helium has lowest boiling point (4.2 K) of any known substance.
Therefore, both **statement I** and **II** are incorrect

Q.50 Identify the factor from the following that does not affect electrolytic conductance of a solution.
(1) The nature of solvent used
(2) Concentration of the electrolyte
(3) The nature of the electrolyte added
(4) The nature of the electrode used

Ans. **[4]**

Sol. Conductivity of electrolytic solution depends upon:
(i) Nature of electrolyte added.
(ii) Size of ions produced and their solvation
(iii) Nature of the solvent and viscosity.
(iv) Concentration of the electrolyte.
(v) Temperature (Increases with increase of temp.).
It does not depend on nature of electrode used.

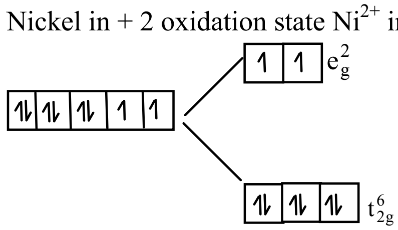
Section-B: Numerical Value Type Questions: This section contains 10 Numerical based questions. Attempt any 5 questions out of 10. The answer to each question should be rounded-off to the nearest integer.

Q.51 The 'spin only' magnetic moment for $[Ni(NH_3)_6]^{2+}$ is
_________ $\times 10^{-1}$ BM.
(given = atomic number of Ni : 28)

Ans. **[28]**

Sol. $[Ni(NH_3)_6]^{2+} \Rightarrow$ Nickel in + 2 oxidation state Ni^{2+} in presence of 6 NH_3 ligand (strong field).

$Ni^{2+} \Rightarrow$



Ni^{2+} have two unpaired electrons, and forms sp^3d^2 hybridisation in presence of 6 NH_3 ligand

$\mu = \sqrt{n(n+2)} = \sqrt{2(2+2)} = \sqrt{2\times 4}$

$\mu = \sqrt{8} = 2.82 \approx 2.8$ BM

$\approx 28 \times 10^{-1}$ BM

Hence, answer is $\Rightarrow$ 28

Q.52 Number of moles of methane required to produce 22 g $CO_{2(g)}$ after combustion is x $\times 10^{-2}$ moles. The value of x is ______.

Ans. **[50]**

Sol. Reaction of combustion of methane:

$CH_4 + 2O_2 \longrightarrow CO_2 + 2H_2O$

1 mol methane produce 1 mol CO_2.

$\therefore$ 1 mol methane produce 44 g CO_2.

Hence, $\frac{1}{2}$ mol CH_4 will produce 22 g CO_2.

$\therefore$ Moles of methane required to produce 22 g CO_2.

$\Rightarrow$ 0.5 mol

$\Rightarrow 50 \times 10^{-2}$ mol

Hence, the value of x = 50

Q.53 Number of alkanes obtained on electrolysis of a mixture of CH_3COONa and C_2H_5COONa is ______.

Ans. **[3]**

Sol.

$2CH_3-\overset{\overset{O}{||}}{C}-O^- \xrightarrow{-2e^-} 2CH_3 + \overset{\overset{O}{||}}{C}-\dot{O} \longrightarrow 2\dot{C}H_3 + 2CO_2$

$2C_2H_5COO^- \xrightarrow{-2e^-} 2C_2H_5\overset{\overset{O}{||}}{C}O \longrightarrow 2\dot{C}_2H_5 + 2CO_2$

$\dot{C}_2H_5$ and $\dot{C}H_3$ created.

$\dot{C}H_3 + \dot{C}H_3 \longrightarrow CH_3-CH_3$ (Ethane)

$\dot{C}H_3 + \dot{C}_2H_5 \longrightarrow CH_3CH_2CH_3$ (Propane)

$\dot{C}_2H_5 + \dot{C}_2H_5 \longrightarrow CH_3CH_2CH_2CH_3$ (Butane)

3 alkanes were obtained on electrolysis of

$CH_3COO^-Na^+$ and $C_2H_5COO^-Na^+$ mixture.

Q.54 Molar mass of the salt from NaBr, $NaNO_3$, KI and CaF_2 which does not evolve coloured vapours on heating with concentrated H_2SO_4 is ______ g mol^{-1}.

(Molar mass in g mol^{-1} : Na : 23, N : 14, K : 39,

O : 16, Br : 80, I : 127, F : 19, Ca : 40)

Ans. **[78]**

Sol. (1) $NaBr + H_2SO_4 \xrightarrow{\Delta} Br_2 + SO_2 + H_2O + Na_2SO_4$

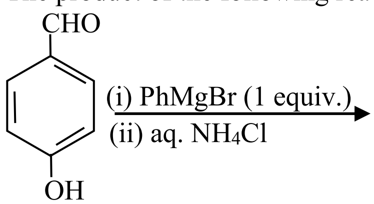
(2) $NaNO_3 + H_2SO_4 \xrightarrow{\Delta} NO_2 + O_2 + H_2O + Na_2SO_4$

(3) $KI + H_2SO_4 \xrightarrow{\Delta} I_2 + H_2S + H_2O + K_2SO_4$

(4) $CaF_2 + H_2SO_4 \xrightarrow{\Delta} CaSO_4 + HF$

CaF_2 will not give coloured vapours on heating with conc. H_2SO_4.
Other will give coloured vapours due to formation of Br_2, NO_2, I_2.
$Br_2 \rightarrow$ Red-brown colour
$I_2 \rightarrow$ Violet colour
$NO_2 \rightarrow$ Deep red orange gas.
Molar mass of $CaF_2 \Rightarrow 40 + 19 + 19 = 78$ g/mol

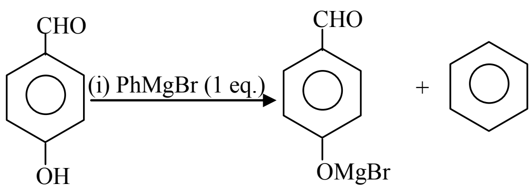
Q.55 The product of the following reaction is P.



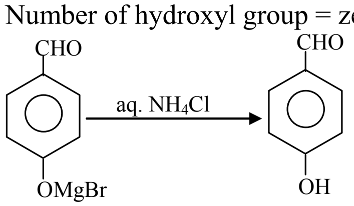
The number of hydroxyl groups present in the product P is ______.

Ans. **[1]**

Sol.



Number of hydroxyl group = zero.



Phenolic (– OH) group = 1

Q.56 The ionization energy of sodium in kJ mol^{-1} , if electromagnetic radiation of wavelength 242 nm is just sufficient to ionize sodium atom is ______.

Ans. **[494]**

Sol. $\lambda = 242 \text{ nm} = 242 \times 10^{-9}$ m

Energy to just remove 1 electron from 1 mol sodium atom =
Energy for removal of $1e^-$ from 1 atom

$$\Rightarrow E = \frac{hc}{\lambda} = \frac{6.62 \times 10^{-34} \times 3 \times 10^8}{242 \times 10^{-9}}$$

Energy for removal of $1e^-$ from 1 mol atoms

$$\Rightarrow E = \frac{hc}{\lambda} \times N_A$$

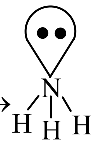
$$E = \frac{6.62 \times 10^{-34} \times 3 \times 10^8 \times 6.023 \times 10^{23}}{242 \times 10^{-9}}$$

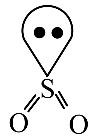
E = 494. 28 kJ/mol
≈ 494

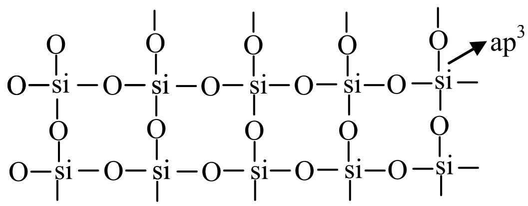
Q.57 The number of species from the following in which the central atom uses sp^3 hybrid orbitals in its bonding is ______.

NH_3, SO_2, SiO_2, $BeCl_2$, CO_2, H_2O, CH_4, BF_3

Ans. **[4]**

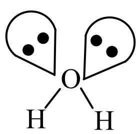
Sol. $NH_3 \rightarrow$  $\rightarrow sp^3$

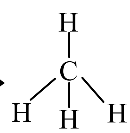
$SO_2 \rightarrow$  $\rightarrow Sp^2$

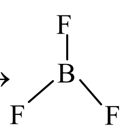
$SiO_2 \rightarrow$ 

$BeCl_2 \rightarrow Cl — Be — Cl \rightarrow sp$

$CO_2 \rightarrow C = C = O \rightarrow sp$

$H_2O \rightarrow$  $\rightarrow sp^3$

$CH_4 \rightarrow$  $\rightarrow sp^3$

$BF_3 \rightarrow$  $\rightarrow sp^2$

Q.58 One Faraday of electricity liberates $x \times 10^{-1}$ gram atom of copper from copper sulphate. x is ______.

Ans. **[5]**

Sol. $Cu^{2+} + 2e^- \longrightarrow Cu$

2 Faraday will liberate = 1 mol Cu

= 1 gram atom Cu

1 Faraday will liberate = 0.5 mol Cu

= 0.5 gram atom Cu

$0.5 = 5 \times 10^{-1}$

$\therefore$ Ans $\Rightarrow x = 5$

Q.59 Consider the following reaction at 298 K. $\frac{3}{2} O_{2(g)} \rightleftharpoons O_{3(g)}$. $Kp = 2.47 \times 10^{-29}$.

$\Delta_r G^\ominus$ for the reaction is ___________ KJ. (Given $R = 8.314\ JK^{-1}\ mol^{-1}$)

Ans. **[163]**

Sol. $\Delta°G = -8.314 \times 298 \times 2.303 \times \log 2.47 \times 10^{-29}$

$\Delta G° = +163199.52$ J

$\Delta G° = 163.2$ J

$\Delta G° \approx 163$ KJ

Q.60 $CH_3CH_2Br + NaOH \begin{cases} \xrightarrow{C_2H_5OH} \text{Product A} \\ \xrightarrow{H_2O} \text{Product B} \end{cases}$

The total number of hydrogen atoms in product A and product B is ______.

Ans. **[10]**

Sol. $CH_3CH_2Br + NaOH \begin{cases} \xrightarrow{C_2H_5OH_2} CH_2 = CH_2\ (A) \\ \xrightarrow{H_2O} CH_3\ CH_2 - OH\ (B) \end{cases}$

In product A = 4 Hydrogen atoms are present In product B = 6 Hydrogen atoms are present Total H-atom = 4 + 6 = 10

MATHEMATICS

Section-A: This section contains 20 multiple choice questions. Each question has 4 choices(1), (2), (3) and (4), out of which **ONLY ONE** is correct..

Q.61 The solution curve of the differential equation $y\frac{dx}{dy} = x\ (\log_e x - \log_e y + 1)$, $x > 0, y > 0$ passing through the point (e, 1) is

(1) $\left|\log_e \frac{x}{y}\right| = y$ (2) $\left|\log_e \frac{y}{x}\right| = x$ (3) $\left|\log_e \frac{y}{x}\right| = y^2$ (4) $2\left|\log_e \frac{x}{y}\right| = y+1$

Ans. **[1]**

Sol. $y\frac{dx}{dy} = x\ (\log_e x - \log_e y + 1)$

$\frac{dx}{dy} = \frac{x}{y}\left(\log_e \frac{x}{y} + 1\right)$

Let x = vy

$\frac{dx}{dy} = v + y\ \frac{dv}{dy}$

$\Rightarrow v + y\ \frac{dv}{dy} = v\ \log_e v + v$

$\Rightarrow y\ \frac{dv}{dy} = v\ \log_e v$

$\Rightarrow \int \frac{dv}{v \log_e v} = \int \frac{dy}{y}$

$\Rightarrow \log_e (\log_e v) = \log_e y + \log_e c$

$\Rightarrow |\log_e v| = yc$

$\Rightarrow \left|\log_e\left(\frac{x}{y}\right)\right| = yc$

It passes through (e, 1)

$\Rightarrow \log_e e = c \Rightarrow c = 1$

$\Rightarrow y = \left|\log_e\left(\frac{x}{y}\right)\right|$

Q.62 For $0 < c < b < a$, let $(a + b - 2c)x^2 + (b + c - 2a)x + (c + a - 2b) = 0$ and $\alpha \neq 1$ be one of its root. Then, among the two statements

(I) If $\alpha \in (-1, 0)$, then b cannot be the geometric mean of a and c

(II) If $\alpha \in (0, 1)$ then b may be the geometric mean of a and c

(1) Only (I) is true (2) Only (II) is true

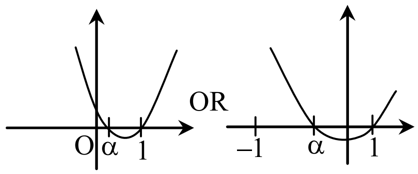
(3) Neither (I) nor (II) is true (4) Both (I) and (II) are ture

Ans. **[4]**

Sol. $f(x) = (a + b - 2c)x^2 + (b + c - 2a)x + (c + a - 2b)$

Clearly $f(1) = 0 \Rightarrow x = 1$ in a root.

Coefficient of $x^2 = a + b - 2c = (a - c) + (b - c) > 0$.



For I, if $\alpha \in (-1, 0) \Rightarrow f(-1)\, f(0) < 0$

$\Rightarrow 2(2a - b - c)(c + a - 2b) < 0$

$\Rightarrow c + a - 2b < 0$ as $2a - b - c > 0$

$\Rightarrow \frac{c+a}{2} < b \Rightarrow$ A. M. of a and c is less than b.

Hence b cannot be G.M of a and c

For II, if $\alpha \in (0, 1) \Rightarrow f(0)\, f(1^-) < 0$

$\Rightarrow c + a - 2b > 0$ as $f(1^-) < 0$

$\Rightarrow \frac{c+a}{2} > b \Rightarrow$ b can be G.M of a and c.

Q.63 If $f(x) = \begin{vmatrix} x^3 & 2x^2+1 & 1+3x \\ 3x^2+2 & 2x & x^3+6 \\ x^3-x & 4 & x^2-2 \end{vmatrix}$ for all $x \in R$, then $2f(0) + f'(0)$ is equal to

(1) 24 (2) 42 (3) 48 (4) 18

Ans. **[2]**

Sol. $f(x) = \begin{vmatrix} x^3 & 2x^2+1 & 1+3x \\ 3x^2+2 & 2x & x^3+6 \\ x^3-x & 4 & x^2-2 \end{vmatrix}$

$$f'(x) = \begin{vmatrix} 3x^2 & 2x^2+1 & 1+3x \\ 6x & 2x & x^3+6 \\ 3x^2-1 & 4 & x^2-2 \end{vmatrix} + \begin{vmatrix} x^3 & 4x & 1+3x \\ 3x^2+2 & 2 & x^3+6 \\ x^3-x & 0 & x^2-2 \end{vmatrix} + \begin{vmatrix} x^3 & 2x^2+1 & 3 \\ 3x^2+2 & 2x & 3x^2 \\ x^3-x & 4 & 2x \end{vmatrix}$$

$$f'(0) = \begin{vmatrix} 0 & 1 & 1 \\ 0 & 0 & 6 \\ -1 & 4 & -2 \end{vmatrix} + \begin{vmatrix} 0 & 0 & 1 \\ 2 & 2 & 6 \\ 0 & 0 & -2 \end{vmatrix} + \begin{vmatrix} 0 & 1 & 3 \\ 2 & 0 & 0 \\ 0 & 4 & 0 \end{vmatrix}$$

$f'(0) = 6 + 0 + 12$

$f'(0) = 18$

$f(0) = \begin{vmatrix} 0 & 1 & 1 \\ 2 & 0 & 6 \\ 0 & 4 & -2 \end{vmatrix}$

$f(0) = 12$

$2f(0) + f'(0) = 24 + 18 = 42$

Q.64 $\lim_{x\to 0}\frac{e^{2|\sin x|}-2|\sin x|-1}{x^2}$

(1) Is equal to –1 (2) Is equal to 2 (3) Is equal to 1 (4) Does not exist

Ans. **[2]**

Sol. $\lim_{x\to 0}\frac{e^{2|\sin x|}-2|\sin x|-1}{x^2}$

Since, it is an even function

$\Rightarrow$ LHL = RHL

RHL = $\lim_{x\to 0^+}\frac{e^{2\sin x}-2\sin x-1}{x^2}$

$= \lim_{x\to 0^+}\frac{e^{2\sin x}(2\cos x)-2\cos x}{2x}$

$= \lim_{x\to 0^+}\left(\frac{e^{2\sin x}-1}{2\sin x}\right)\left(\frac{2\sin x}{x}\right)\cos x$

$= 1 \times 2 \times 1$

$= 2$

Q.65 If $f(x) = \frac{4x+3}{6x-4}, x \neq \frac{2}{3}$ and (fof) (x) = g(x), where $g : R - \left\{\frac{2}{3}\right\} \to R - \left\{\frac{2}{3}\right\}$, then (gogog) (4) is equal to

(1) $\frac{19}{20}$ (2) 4 (3) $-\frac{19}{20}$ (4) –4

Ans. **[2]**

Sol. Given $f(x) = \frac{4x+3}{6x-4}, x \neq \frac{2}{3}$ and g(x) = (fof) (x)

$= \frac{4\left(\frac{4x+3}{6x-4}\right)+3}{6\left(\frac{4x+3}{6x-4}\right)-4}$

$= \frac{16x+12+18x-12}{24x+18-24x+16}$

$= \frac{34x}{34} = x$

$\therefore$ (gogog) (x) = x

$\Rightarrow$ (gogog) (4) = 4

Q.66 For $\alpha,\beta,\gamma \neq 0$, if $\sin^{-1}\alpha + \sin^{-1}\beta + \sin^{-1}\gamma = \pi$ and $(\alpha + \beta + \gamma)(\alpha - \gamma + \beta) = 3\alpha\beta$, then γ equals

(1) $\sqrt{3}$ (2) $\frac{\sqrt{3}}{2}$ (3) $\frac{\sqrt{3}-1}{2\sqrt{2}}$ (4) $\frac{1}{\sqrt{2}}$

Ans. **[2]**

Sol. Let $\sin^{-1}\alpha = A$, $\sin^{-1}\beta = B$ and $\sin^{-1}\gamma = C$

$\therefore A + B + C = \pi$

Also, $(\alpha + \beta + \gamma)(\alpha - \gamma + \beta) = 3\alpha\beta$

$\Rightarrow (\alpha + \beta)^2 - \gamma^2 = 3\alpha\beta$

$\Rightarrow \alpha^2 + \beta^2 - \gamma^2 = \alpha\beta$

$\Rightarrow \frac{\alpha^2+\beta^2-\gamma^2}{2\alpha\beta} = \frac{1}{2}$

$\Rightarrow \cos C = \frac{1}{2} \Rightarrow C = 60°$

$\therefore \sin C = \frac{\sqrt{3}}{2} = \gamma$

Q.67 The area of the region

$\left\{(x, y) : y^2 \le 4x, x < 4, \frac{xy(x-1)(x-2)}{(x-3)(x-4)} > 0. x \ne 3\right\}$ is

(1) $\frac{8}{3}$ (2) $\frac{32}{3}$ (3) $\frac{16}{3}$ (4) $\frac{64}{3}$

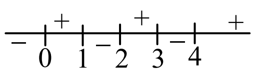
Ans. **[2]**

Sol. **Case I:**

When $y < 0$

Then $\frac{xy(x-1)(x-2)}{(x-3)(x-4)} > 0$

$\frac{x(x-1)(x-2)}{(x-3)(x-4)} < 0$



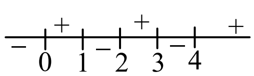
$x \in (1, 2) \cup (3, 4)$

Case II:

$y > 0$

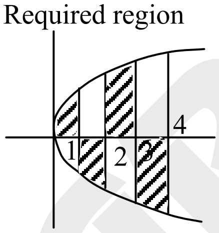
$\frac{xy(x-1)(x-2)}{(x-3)(x-4)} > 0$

$\frac{x(x-1)(x-2)}{(x-3)(x-4)} > 0$



$x \in (0, 1) \cup (2, 3)$

Required region



So, required area

$= \int_0^4 \sqrt{4x}\, dx = \left(2\left(x^{3/2}\right)\frac{2}{3}\right)_0^4$

$= \frac{4}{3}(4 \times 2 - 0) = \frac{32}{3}$

Q.68 Let α, β, γ, δ ∈ Z and let A (α, β), B (1, 0), C (γ, δ) and D(1, 2) be the vertices of a parallelogram ABCD. If AB = $\sqrt{10}$ and the points A and C lie on the line 3y = 2x + 1, then 2(α + β + γ + δ) is equal to

(1) 10 (2) 8 (3) 12 (4) 5

Ans. **[2]**

Sol. ABCD is a parallelogram so mid-point of AC = mid-point of BD

$\Rightarrow \left(\frac{\alpha+\gamma}{2}, \frac{\beta+\delta}{2}\right) = \left(\frac{1+1}{2}, \frac{0+2}{2}\right)$

$\Rightarrow \left(\frac{\alpha+\gamma}{2}, \frac{\beta+\delta}{2}\right) = (1,1)$

$\Rightarrow \alpha + \gamma = 2$ and $\beta + \delta = 2$

So, $2(\alpha + \beta + \gamma + \delta) = 2(2 + 2) = 8$

Q.69 Let y = y(x) be the solution of the differential equation $\frac{dy}{dx} = \frac{(\tan x)+y}{\sin x(\sec x - \sin x \tan x)}, x \in \left(0, \frac{\pi}{2}\right)$ satisfying the condition $y\left(\frac{\pi}{4}\right) = 2$. Then $y\left(\frac{\pi}{3}\right)$ is

(1) $\sqrt{3}(2+\log_e 3)$ (2) $\sqrt{3}(1+2\log_e 3)$ (3) $\frac{\sqrt{3}}{2}(2+\log_e 3)$ (4) $\sqrt{3}(2+\log_e \sqrt{3})$

Ans. **[4]**

Sol.

$\frac{dy}{dx} = \frac{\tan x}{\sin x(\sec x - \sin x \tan x)} + \frac{y}{\sin x(\sec x - \sin x \tan x)}$

$\frac{dy}{dx} = \frac{1}{\cos x(\sec x - \sin x \tan x)} + \frac{y}{\sin x(\sec x - \sin^2 x \sec x)}$

$\frac{dy}{dx} = \frac{1}{1-\sin^2 x} + \frac{y}{\sin x \sec x(1-\sin^2 x)}$

$\frac{dy}{dx} = \frac{1}{\cos^2 x} + \frac{y}{\sin x \cos x}$

$\frac{dy}{dx} - y(\sec x \operatorname{cosec} x) = \sec^2 x$

I.F = $e^{-\int \frac{1}{\sin x \cos x} dx}$

$= e^{-2\int \operatorname{cosec} 2x dx}$

$= e^{-\ln \tan x}$

$= \frac{1}{\tan x} = \cot x$

$\therefore$ y. cot x = $\int \frac{1}{\cos^2 x}.\cot x dx$

y cotx = $\int \frac{1}{\sin x \cos x} dx$

y cot x = $2\int \operatorname{cosec} 2x\, dx$

y cot x = ln tan x + c` …(i)

$y\left(\frac{\pi}{4}\right) = 2$

$\therefore 2\cot\frac{\pi}{4} = \ln \tan\frac{\pi}{4} + c$

2 = 0 + c

2 = c

$\therefore$ ycotx = ln tanx + 2 (from i)

Put x = $\frac{\pi}{3}$

$y \cot\left(\frac{\pi}{3}\right) = \ln \tan\frac{\pi}{3} + 2$

$y\ \frac{1}{\sqrt{3}} = \ln.\sqrt{3} + 2$

$y = \sqrt{3}\left[\ln\left(\sqrt{3}\right) + 2\right]$

Option (4) is correct

Q.70 Let g(x) be a linear function and $f(x) = \begin{cases} g(x), & x \le 0 \\ \left(\frac{1+x}{2+x}\right)^{\frac{1}{x}}, & x > 0 \end{cases}$, is continuous at x = 0. If f'(1) = f(–1), then the value g(3) is

(1) $\log_e\left(\frac{4}{9}\right) - 1$ (2) $\log_e\left(\frac{4}{9e^{1/3}}\right)$ (3) $\frac{1}{3}\log_e\left(\frac{4}{9}\right) + 1$ (4) $\frac{1}{3}\log_e\left(\frac{4}{9e^{1/3}}\right)$

Ans. **[2]**

Sol. Let g(x) = ax + b f(x) is continuous at x = 0

$\therefore \underset{x\to 0^+}{\text{Lt}}\left(\frac{1+x}{2+x}\right)^{\frac{1}{x}} = \left(\frac{1}{2}\right)^{\infty} = 0$

$\underset{x\to 0^-}{\text{Lt}}\ g(x) = \underset{x\to 0^-}{\text{Lt}}\ ax + b = b$

So, b = 0

Now, $f(x) = \left(\frac{1+x}{2+x}\right)^{\frac{1}{x}}$

$\ln f(x) = \frac{1}{x}$ [ln (1 + x) – ln (2 + x)]

$$\frac{1}{f(x)}.f'(x) = \frac{x\left[\frac{1}{1+x} - \frac{1}{2+x}\right] - \ln\left(\frac{1+x}{2+x}\right)}{x^2}$$

$$f'(1) = f(1)\left[\frac{\left(\frac{1}{2}-\frac{1}{3}\right) - \ln\left(\frac{2}{3}\right)}{1}\right]$$

$$f'(1) = \frac{2}{3}\left[\frac{1}{6} - \ln\frac{2}{3}\right] \qquad ...(1)$$

Also f(– 1) = a (– 1) + b

f(–1) = – a ....(2)

$\therefore$ from (1) and (2)

$-a = \frac{1}{9} - \frac{2}{3}\ln\frac{2}{3}$

$a = \frac{2}{3}\ln\left(\frac{2}{3}\right) - \frac{1}{9}$

$\therefore$ g(x) = ax

$g(3) = 3\left(\frac{2}{3}\ln\left(\frac{2}{3}\right) - \frac{1}{9}\right)$

$g(3) = 2\ln\left(\frac{2}{3}\right) - \frac{1}{3}$

$g(3) = \ln\frac{4}{9} - \frac{1}{3}$

$g(3) = \ln\frac{4}{9} - \ln e^{1/3}$

$g(3) = \ln\left(\frac{4}{9.e^{1/3}}\right)$

$g(3) = \log_e\left(\frac{4}{9e^{1/3}}\right)$

Option (2) is correct

Q.71 The distance of the point Q(0, 2, –2) form the line passing through the point P(5, –4, 3) and perpendicular to the lines $\vec{r} = (-3\hat{i} + 2\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 5\hat{k}), \lambda \in R$ and $\vec{r} = (\hat{i} - 2\hat{j} + \hat{k}) + \mu(-\hat{i} + 3\hat{j} + 2\hat{k}), \mu \in R$ is:

(1) $\sqrt{20}$ (2) $\sqrt{54}$ (3) $\sqrt{86}$ (4) $\sqrt{74}$

Ans. **[4]**

Sol. $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 5 \\ -1 & 3 & 2 \end{vmatrix}$

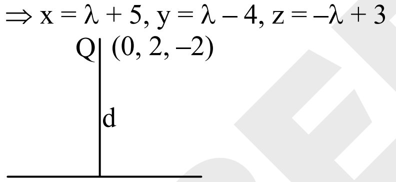
$\hat{i}(6 - 15) - \hat{j}(4 + 5) + \hat{k}(6 + 3)$

$-9\hat{i} - 9\hat{j} + 9\hat{k}$

Required line

$\frac{x-5}{1} = \frac{y+4}{1} = \frac{Z-3}{-1} = \lambda$

$\Rightarrow x = \lambda + 5, y = \lambda - 4, z = -\lambda + 3$



P(5, –4, 3) R(λ + 5, λ – 4, – λ + 3)

$\vec{QR}.\vec{l} = (\lambda + 5)1 + (\lambda - 6)1 + (3 - \lambda + 2)(-1) = 0$

$\Rightarrow \lambda = 2$

$R = (7, -2, 1)$

$QR = \sqrt{49 + 16 + 9}$

$= \sqrt{74}$

Q.72 If the foci of a hyperbola are same as that of the ellipse $\frac{x^2}{9} + \frac{y^2}{25} = 1$ and the eccentricity of the hyperbola is $\frac{15}{8}$ times the eccentricity of the ellipse, then the smaller focal distance of the point $\left(\sqrt{2}, \frac{14}{3}\sqrt{\frac{2}{5}}\right)$ on the hyperbola, is equal to

(1) $7\sqrt{\frac{2}{5}} + \frac{8}{3}$ (2) $14\sqrt{\frac{2}{5}} - \frac{4}{3}$ (3) $7\sqrt{\frac{2}{5}} - \frac{8}{3}$ (4) $14\sqrt{\frac{2}{5}} - \frac{16}{3}$

Ans. **[3]**

Sol. Foci of hyperbola is same as that of ellipse $\frac{x^2}{9}+\frac{y^2}{25}=1$

$a_1 = 3$
$b_1 = 5$

foci = $(0, \pm b_1e_1)$ $\quad e_1 = \sqrt{1-\frac{a_1^2}{b_1^2}}$

foci = $\left(0, \pm 5.\frac{4}{5}\right)$ $\quad e_1 = \sqrt{1-\frac{9}{25}} = \frac{4}{5}$

foci = $(0, \pm 4)$

Take eccentricity of hyperbola = e_2

$e_2 = \frac{15}{8} \times e_1 = \frac{15}{8}\times\frac{4}{5} = \frac{3}{2}$

foci of hyperbola

$(0, \pm b_2e_2) = (0, \pm 4)$

$b_2.\ \frac{3}{2} = 4$

$b_2 = \frac{8}{3}$

$e_2 = \sqrt{1+\frac{a_2^2}{b_2^2}} = \sqrt{1+\frac{9a_2^2}{64}}$

$e_2 = \sqrt{\frac{64+9a_2^2}{64}}$

$\frac{9}{4} = \frac{64+9a_2^2}{64}$

$144 - 64 = 9a_2^2$

$\frac{80}{9} = a_2^2$

Now, smaller focal distance

$\left(\sqrt{2}, \frac{14}{3}\sqrt{\frac{2}{5}}\right)$ and $(0, 4)$

$\sqrt{2+\left(\frac{14}{3}\sqrt{\frac{2}{5}}-4\right)^2} = \sqrt{\frac{1202}{45}-\frac{28}{3}\sqrt{\frac{2}{5}}}$

Which is $7\sqrt{\frac{2}{5}}-\frac{8}{3}$

Q.73 Let a be the sum of all coefficients in the expansion of $(1-2x+2x^2)^{2023}(3-4x^2+2x^3)^{2024}$ and $b = \lim_{x\to 0}\left(\frac{\int_0^x \frac{\log(1+t)}{t^{2024}+1}dt}{x^2}\right)$. If the equations $cx^2 + dx + e = 0$ and $2bx^2 + ax + 4 = 0$ have a common root, where c, d, e $\in$ R, then d: c: e equal

(1) 1 : 1 : 4 (2) 4 : 1 : 4 (3) 2 : 1 : 4 (4) 1 : 2 : 4

Ans. **[1]**

Sol. Sum of all coefficients in the expansion of $(1 - 2x + 2x^2)^{2023} (3 - 4x^2 + 2x^3)^{2024}$

Put x = 1

$1^{2023.}\ 1^{2024} = 1$

a = 1

$$b = \lim_{x\to 0}\left[\frac{\int_0^x \frac{\log(1+t)}{t^{2024}+1}dt}{x^2}\right]$$

Apply L-Hospital here

$$\lim_{x\to 0}\frac{\frac{\log(1+x)}{x^{2024}+1}}{2x} = \lim_{x\to 0}\frac{\log(1+x)}{2x(x^{2024}+1)}$$

Again apply L. Hospital here :

$$\lim_{x\to 0}\frac{\frac{1}{1+x}}{2\left(x^{2024}+1\right)+2x\left(2024x^{2023}+1\right)}$$

$$b = \frac{1}{2}$$

$2bx^2 + ax + 4 = x^2 + x + 4 = 0$

D < 0

So roots are imaginary. So they occur in pair. So both roots have to be common. It implies Ratio of both equation are same as c, d, e $\in$ R $cx^2 + dx + e = 0$ is same as

d : c : e = 1 : 1 : 4

Q.74 Let $\vec{a} = 3\hat{i} + \hat{j} - 2\hat{k}, \vec{b} = 4\hat{i} + \hat{j} + 7\hat{k}$ and $\vec{c} = \hat{i} - 3\hat{j} + 4\hat{k}$ be three vectors. If a vectors $\vec{p}$ satisfies $\vec{p}\times\vec{b} = \vec{c}\times\vec{b}$ and $\vec{p}.\vec{a} = 0$, then $\vec{p}.\left(\hat{i} - \hat{j} - \hat{k}\right)$ is equal to

(1) 36 (2) 24 (3) 28 (4) 32

Ans. **[4]**

Sol. $\vec{a} = (3\hat{i} + \hat{j} - 2\hat{k})$

$\vec{b} = (4\hat{i} + \hat{j} + 7\hat{k})$

$\vec{c} = (\hat{i} - 3\hat{j} + 4\hat{k})$

$\vec{p}\times\vec{b} = \vec{c}\times\vec{b}$

$\Rightarrow \left(\vec{p} - \vec{c}\right)\times\vec{b} = 0$

$\Rightarrow \vec{p} = \vec{c} + \lambda\vec{b}$

$\therefore\ \vec{p}.\vec{a} = 0 \Rightarrow \vec{c}.\vec{a} + \lambda\vec{b}.\vec{a} = 0$

$$\Rightarrow \lambda = -\left(\frac{\vec{c}.\vec{a}}{\vec{b}.\vec{a}}\right) = -\frac{\left(3\times 1 + 1\times(-3) + (-2)\times 4\right)}{\left(3\times 4 + 1\times 1 + (-2)\times 7\right)} = -8$$

Here $\vec{p} = -31\hat{i} - 11\hat{j} - 52\hat{k}$

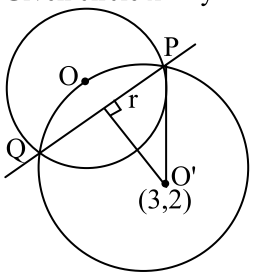
$\vec{p}(\hat{i} - \hat{j} - \hat{k}) = 32$

Q.75 If one of the diameters of the circle $x^2 + y^2 - 10x + 4y + 13 = 0$ is a chord of another circle C, whose center is the point of intersection of the lines 2x + 3y = 12 and 3x – 2y = 5, then the radius of the circle C is :

(1) $3\sqrt{2}$ (2) $\sqrt{20}$ (3) 4 (4) 6

Ans. [4]

Sol. Given circle $x^2 + y^2 - 10x + 4y + 13 = 0$



Centre (O) ≡ (5, –2)

Radius (r) = $\sqrt{25+4-13}$

= 4

Point of intersection of $2x + 3y = 12$ and $3x - 2y = 5$ is (3, 2)

⇒ Centre of C (O') = (3, 2)

$= d = OO' = \sqrt{(5-3)^2 + (-2-2)^2} = \sqrt{20}$

Radius of C = $\sqrt{r^2 + d^2}$

$= \sqrt{16+20}$

= 6

Q.76 If the system of linear equations

$x - 2y + z = -4$

$2x + \alpha y + 3z = 5$

$3x - y + \beta z = 3$

has infinitely many solutions, then $12\alpha + 13\beta$ is equal to

(1) 64 (2) 58 (3) 60 (4) 54

Ans. [2]

Sol. $\Delta_2 = \begin{vmatrix} 1 & -4 & 1 \\ 2 & 5 & 3 \\ 3 & 3 & \beta \end{vmatrix} = 0$

$(5\beta - 9) + 4(2\beta - 9) - 9 = 0$

$\Rightarrow \beta = \frac{54}{13}$

$\Delta_3 = \begin{vmatrix} 1 & -2 & -4 \\ 2 & \alpha & 5 \\ 3 & -1 & 3 \end{vmatrix} = 0$

$\Rightarrow (3\alpha + 5) + 2(-9) - 4(-2 - 3\alpha) = 0$

$\Rightarrow \alpha = \frac{1}{3}$

$\Rightarrow 12\alpha + 13\beta = 4 + 54 = 58$

Q.77 The sum of the series $\frac{1}{1-3.1^2+1^4} + \frac{2}{1-3.2^2+2^4} + \frac{3}{1-3.3^2+3^4} + ...$up to 10-terms is

(1) $-\frac{55}{109}$ (2) $-\frac{45}{109}$ (3) $\frac{55}{109}$ (4) $\frac{45}{109}$

Ans. [1]

Sol. $T_r = \dfrac{r}{1-3.r^2+r^4} = \dfrac{r}{(r^4-2r^2+1)-r^2}$

$= \dfrac{1}{2}\left[\dfrac{1}{r^2-r-1} - \dfrac{1}{r^2+r-1}\right]$

Sum of 10 terms is

$$\sum_{r-1}^{10} T_r = \frac{1}{2}[-1-1] + \frac{1}{2}\left[1-\frac{1}{5}\right] + \frac{1}{2}\left[\frac{1}{5}-\frac{1}{11}\right] + ... + \frac{1}{2}\left[\frac{1}{89}-\frac{1}{109}\right]$$

$= \dfrac{1}{2}\left[-1-\dfrac{1}{109}\right] = \dfrac{-55}{109}$

Q.78 Two marbles are drawn in succession from a box containing 10 red, 30 white, 20 blue and 15 orange marbles, with replacement being made after each drawing. Then the probability, that first drawn marble is red and second drawn marble is white, is

(1) $\dfrac{4}{75}$ (2) $\dfrac{2}{3}$ (3) $\dfrac{2}{25}$ (4) $\dfrac{4}{25}$

Ans. **[1]**

Sol. Total marbles = 10 + 30 + 20 + 15 = 75

R : drawing a red marble in 1st

W = drawing a white marble in 2nd

$\Rightarrow P(R \cap W) = \left(\dfrac{10}{75}\right)\left(\dfrac{30}{75}\right)$

$= \left(\dfrac{300}{75}\right) \times \dfrac{1}{75} = \dfrac{4}{75}$

Q.79 Let S be the set of positive integral values of a for which $\dfrac{ax^2+2(a+1)x+9a+4}{x^2-8x+32} < 0$, $\forall\ x \in R$. Then, the number of elements is S is :

(1) 0 (2) ∞ (3) 3 (4) 1

Ans. **[1]**

Sol. $x^2 - 8x + 32 > 0\ \forall\ x \in R$ as discriminat of this quadratic is $64 - 4 \times 32 < 0$

$\Rightarrow ax^2 + 2(a+1)x + 9a + 4 < 0\ \forall\ x \in R$

$\Rightarrow$ Only possible when $a < 0$ and $D < 0$

$\Rightarrow$ Since S is set of positive

values of a $\Rightarrow$ S is a null set

$\Rightarrow n(S) = 0$

Q.80 Three rotten apples are accidently mixed with fifteen good apples. Assuming the random variable x to be the number of rotten apples in a draw of two apples, the variance of x is

(1) $\dfrac{57}{153}$ (2) $\dfrac{37}{153}$ (3) $\dfrac{47}{153}$ (4) $\dfrac{40}{153}$

Ans. **[4]**

Sol.

X	0	1	2
P(x)	$\dfrac{35}{51}$	$\dfrac{15}{51}$	$\dfrac{1}{51}$

$P(X=0) = \dfrac{{}^{15}C_2}{{}^{18}C_2} = \dfrac{35}{51}$

$$P(X=1)=\frac{^{15}C_1\times{}^{3}C_1}{^{18}C_2}=\frac{15}{51}$$

$$P(X=2)=\frac{^{3}C_2}{^{18}C_2}=\frac{1}{51}$$

$$\mu_x=\sum XP(X)=0\times\frac{35}{51}+\frac{1\times15}{51}+\frac{2\times1}{51}=\frac{17}{51}$$

$$\sigma_x^2=\text{ Variance }=\left(\sum X^2P(X)\right)-\mu_x^2$$

$$=\left(0\times\frac{35}{51}+\frac{1\times15}{51}+\frac{4\times1}{51}\right)-\left(\frac{17}{51}\right)^2$$

$$=\frac{19}{51}-\left(\frac{17}{51}\right)^2=\frac{19}{51}-\frac{1}{9}=\frac{40}{153}$$

Section-B: Numerical Value Type Questions: This section contains 10 Numerical based questions. Attempt any 5 questions out of 10. The answer to each question should be rounded-off to the nearest integer.

Q.81 Let S = (–1, ∞) and $f : S \to R$ be define as $f(x)=\int_{-1}^{x}(e^t-1)^{11}(2t-1)^5(t-2)^7(t-3)^{12}(2t-10)^{61}dt.$

Let p = sum of squares of the values of x, where f(x) attains local maxima on S, and q = sum of the values of x, where f(x) attains local minima on S. Then, the value of $p^2 + 2q$ is ___________

Ans. **[27]**

Sol. $f'(x)=(e^x-1)(2x-1)^5(x-2)^7(x-3)^{12}(2x-10)^{61}=0$

Point of extremum = 0, $\frac{1}{2}$,2,3,5

+ | – | + | – | – | +
0 1/2 2 3 5

maxima = {0, 2}

minima = $\left\{\frac{1}{2},5\right\}$

$p = 0 + (2)^2 = 4$

$q=\frac{1}{2}+5=\frac{11}{2}$

$P^2+2q=16+\frac{11}{2}\times2=27$

Q.82 If α denotes the number of solutions of $|1-i|^x=2^x$ and $\beta=\left(\frac{|z|}{\arg(z)}\right)$, where

$z=\frac{\pi}{4}(1+i)^4\left[\frac{1-\sqrt{\pi}i}{\sqrt{\pi}+i}+\frac{\sqrt{\pi}-i}{1+\sqrt{\pi}i}\right]$, $i=\sqrt{-1}$, then the distance of the point (α, β) from the line 4x – 3y = 7 is ___________

Ans. **[3]**

Sol. $z=\frac{\pi}{4}(1+i)^4\left[\frac{\sqrt{\pi}-\pi i-i-\sqrt{\pi}}{\pi+1}+\frac{\sqrt{\pi}-i-\pi i-\sqrt{\pi}}{1+\pi}\right]$

$=\frac{-\pi}{2}i(1+i)^4$

$|z| = \left|\frac{-\pi}{2}\right||i||1+i|^4 = 2\pi$

$z = \frac{-\pi}{2}(-4i) = 2\pi i \Rightarrow \arg(z) = \frac{\pi}{2}$

$\Rightarrow \beta = \frac{2\pi}{\frac{\pi}{2}} = 4$

Also $|1 - i|^x = 2^x$

$(\sqrt{2})^x = 2^x$

$\Rightarrow x = 0$

$\Rightarrow$ 1 solution $\Rightarrow \alpha = 1$

Now perpendicular distance of $4x - 3y = 7$ from (1, 4)

$\left|\frac{4(1)-3(4)-7}{5}\right| = 3$

Q.83 The total number of words (with or without meaning) that can be formed out of the letters of the word 'DISTRIBUTION' taken four at a time, is equal

Ans. **[3734]**

Sol. $I \to 3, T \to 2, D, S, R, B, U, O, N \to 1.$

All distinct = ${}^9C_4 \times 4! = 3024$

2 Alike + 2 different = ${}^2C_1 \times {}^8C_2 \times \frac{4!}{2!} = 672$

2 Alike + 2 other Alike = ${}^2C_2 \times \frac{4!}{2!2!} = 6$

3 Alike + 1 different = ${}^1C_1 \times {}^8C_1 \times \frac{4!}{3!} = 32$

Total words = 3734

Q.84 Let the foci and length of the latus rectum of an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1,\ a > b$ be $(\pm 5, 0)$ and $\sqrt{50}$, respectively.

Then, the square of the eccentricity of the hyperbola $\frac{x^2}{b^2} - \frac{y^2}{a^2b^2} = 1$ equals

Ans. **[51]**

Sol. $ae = 5$

$\frac{2b^2}{a} = \sqrt{50}$

$b^2 = \frac{5\sqrt{2}}{2}a$

$b^2 = a^2(1 - e^2)$

$\frac{5\sqrt{2}}{2} = a(1 - e^2)$

$\Rightarrow \frac{5}{e}(1 - e^2) = \frac{5}{\sqrt{2}}$

$\sqrt{2}(5 - 5e^2) = 5e$

$\Rightarrow 5\sqrt{2e^2} + 5e - 5\sqrt{2} = 0$

$\sqrt{2}e^2 + e - \sqrt{2} = 0$

$\Rightarrow e = -\sqrt{2}$ (rejected), $e = \dfrac{1}{\sqrt{2}}$

from $ae = 5 \Rightarrow a = 5\sqrt{2}$ and $b^2 = 25$

Now eccentricity of $\dfrac{x^2}{b^2} - \dfrac{y^2}{a^2b^2} = 1$

$e^2 = 1 + \dfrac{a^2b^2}{b^2} = 1 + a^2 = 51$

Q.85 Let A = {1, 2, 3, 4} and R = {(1, 2), (2, 3), (1, 4)} be a relation on A. Let S be the equivalence relation on A such that $R \subset S$ and the number of elements in S is n. Then, the minimum value of n is ______________

Ans. **[16]**

Sol. A = {1, 2, 3, 4}

R = {(1, 2), (2, 3), (1, 4)}

S is equivalence for R < S and reflexive {(1, 1), (2, 2), (3, 3), (4, 4)}

for symmetric

{(2, 1), (4, 1), (3, 2)}

for transitive

{(1, 3), (3, 1), (4, 2), (2, 4)}

Now set

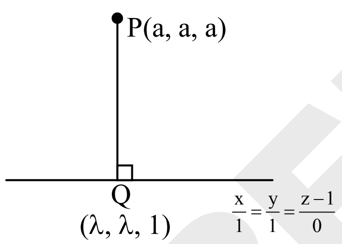
S = {(1, 1), (2, 2), (3, 3), (4, 4), (1, 2), (2, 3), (1, 4), (4, 3), (3, 4), (2, 1), (4, 1), (3, 2), (1, 3), (3, 1), (4, 2), (2, 4)}

n(S) = 16

Q.86 Let Q and R be the feet of perpendiculars from the point P(a, a, a) on the lines x = y, z = 1 and x = –y, z = –1 respectively. If $\angle QPR$ is a right angle, then $12a^2$ is equal to _____

Ans. **[12]**

Sol.

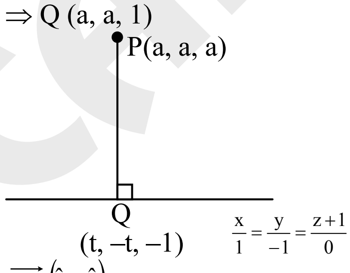


$\overrightarrow{PQ}.(\hat{i} + \hat{j}) = 0$

$\Rightarrow (\lambda - a) + (\lambda - a) = 0$

$\Rightarrow \lambda = a$

$\Rightarrow Q\ (a, a, 1)$



$\overrightarrow{PR}.(\hat{i} - \hat{j}) = 0$

$\Rightarrow (t - a) - (-t - a) = 0$

$\Rightarrow t = 0$

$\Rightarrow R(0, 0, -1)$

$\therefore\ \overrightarrow{PQ}.\overrightarrow{PR}. = 0$

$\left((1-a)\hat{k}\right)(-a\hat{i}-a\hat{j}-(a+1)\hat{k} = 0$

$\Rightarrow a^2 = 1$

$12a^2 = 12$

Q.87 In the expansion of $(1 + x)(1 - x^2)$ $\left(1+\frac{3}{x}+\frac{3}{x^2}+\frac{1}{x^3}\right)^5$, $x \neq 0$, the sum of the coefficients of x^3 and x^{-13} is equal to ______

Ans. **[118]**

Sol. $E(x) = \frac{1}{x^{15}}(1+x)(1-x^2)(1+3x^2+x^3)^5$

$= \frac{1}{x^{15}}(1+x)(1-x)(1+x)(1+x)^{15}$

$\frac{1}{x^{15}}(1-x)(1+x)^{17}$

$\frac{(1+x)^{17}}{x^{15}} - \frac{(1+x)^{17}}{x^{14}}$

$\Rightarrow$ Coefficient of x^3 in E(x) = 0 – 1 = –1

Coefficient of x^{-13} in E(x) = $^{17}C_2 - {}^{17}C_1$

= 136 – 17 = 119 $\Rightarrow$ Sum = 118

Q.88 If the integral $525\int_0^{\frac{\pi}{2}} \sin 2x \cos^{\frac{11}{2}} x \left(1+\cos^{\frac{5}{2}} x\right)^{\frac{1}{2}} dx$ is equal to $(n\sqrt{2}-64)$, then n is equal to _______

Ans. **[176]**

Sol. $I = 525\int_0^{\frac{\pi}{2}} \sin 2x \cos^{\frac{11}{2}} x \left(1+\cos^{\frac{5}{2}} x\right)^{\frac{1}{2}} dx$

$= 525\int_0^{\frac{\pi}{2}} 2\sin x \cos x \cos x^{\frac{11}{2}} x \left(1+\cos^{\frac{5}{2}} x\right)^{\frac{1}{2}} dx$

$= 1050\int_0^{\frac{\pi}{2}} \sin x \cos x^{\frac{13}{2}} x \left(1+\cos^{\frac{5}{2}} x\right)^{\frac{1}{2}} dx$

Put $\left(1+\cos^{\frac{5}{2}} x\right) = t \Rightarrow \frac{5}{2}\cos^{\frac{3}{2}} x(-\sin x)dx = dt$

$I = 1050\int_2^1 -\frac{2}{5}\cos^5 x.\sqrt{t}dt = -420\int_2^1 (t-1)^2\sqrt{t}dt$

$= 420\int_1^2 \left(t^{\frac{5}{2}} - 2t^{\frac{3}{2}} + t^{\frac{1}{2}}\right)dt$

$= 420\left[\frac{2}{7}t^{\frac{7}{2}} - 2\times\frac{2}{5}t^{\frac{5}{2}} + \frac{2}{3}t^{\frac{3}{2}}\right]_1^2$

$= 420\left(\sqrt{2}\left(\frac{16}{7}-\frac{16}{5}+\frac{4}{3}\right)-\left(\frac{2}{7}-\frac{4}{5}+\frac{2}{3}\right)\right)$

$= 176\sqrt{2} - 64$

$\Rightarrow n = 176$

Q.89 Let f: R → R be a function defined by $f(x) = \frac{4^x}{4^x+2}$ and $M = \int_{f(a)}^{f(1-a)} x\sin^4(x(1-x))dx$,

$N = \int_{f(a)}^{f(1-a)} \sin^4(x(1-x))dx; a \neq \frac{1}{2}$. If $\alpha M = \beta N$,

$\alpha, \beta, \in N$, then the least value of $\alpha^2 + \beta^2$ is equal to ________

Ans. **[5]**

Sol. $f(x) + f(1-x) = \frac{4^x}{4^x+2} + \frac{4^{1-x}}{4^{1-x}+2}$

$= \frac{4^x}{4^x+2} + \frac{4}{4+2.4^x}$

$= \frac{4^x+2}{4^x+2} = 1$

$\Rightarrow M = \int_{f(a)}^{f(1-a)} x\sin^4(x(1-x))dx$

$x \to f(a) + f(1-a) - x = 1 - x$

$M = \int_{f(a)}^{f(1-a)} (1-x)\sin^4((1-x)x)dx$

$M = N - M \Rightarrow \frac{M}{N} = \frac{1}{2} = \frac{\beta}{\alpha}$

$\Rightarrow (\alpha^2 + \beta^2)_{least} = 1 + 4 = 5$

Q.90 Let $\vec{a}$ and $\vec{b}$ be two vectors such that $|\vec{a}| = 1$, $|\vec{b}| = 4$, and $\vec{a}\,.\,\vec{b} = 2$. If $\vec{c} = (2\vec{a}\times\vec{b}) - 3\vec{b}$ and the angle between $\vec{b}$ and $\vec{c}$ is α, then 192 $\sin^2\alpha$ is equal to

Ans. **[48]**

Sol. $\vec{c} = (2\vec{a}\times\vec{b}) - 3\vec{b}$

$|\vec{c}|^2 = |(2\vec{a}\times\vec{b}) - 3\vec{b}|^2 = \left(2|\vec{a}\times\vec{b}|\right)^2 + |3\vec{b}|^2 - 12(\vec{a}\times\vec{b})^2.\vec{b} = 4\left(|\vec{a}|^2|\vec{b}|^2 - (\vec{a}.\vec{b})^2\right) + 9|\vec{b}|^2$

$= 4(16 - 4) + 144 = 192$

$\vec{b}.\vec{c} = -3\vec{b}.\vec{b} = -48$

$|\vec{b}|.|\vec{c}|\cos\alpha = -48$

$\cos\alpha = \frac{-48}{4\times\sqrt{192}} = -\frac{12}{\sqrt{192}}$

$\sin^2\alpha = 1 - \frac{144}{192} \Rightarrow 192\sin^2\alpha = 48$