JEE Main Online Exam 2024

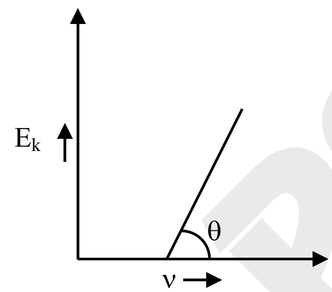
Questions & Solution

30th January 2024 | Evening

PHYSICS

Section-A: This section contains 20 multiple choice questions. Each question has 4 choices(1), (2), (3) and (4), out of which **ONLY ONE** is correct..

Q.1 For the photoelectric effect, the maximum kinetic energy (E_k) of the photoelectrons is plotted against the frequency (ν) of the incident photons as shown in figure. The slope of the graph gives



(1) Charge of electron
(2) Work function of the metal
(3) Ratio of Planck's constant to electric charge
(4) Planck's constant

Ans. **[4]**

Sol. For photoelectric effect

$h\nu = k + \phi_0$

$k = h\nu - \phi_0$

Compare with $y = mx \pm c$

Slope (m) = h

Q.2 An alternating voltage $V(t) = 220 \sin 100 \pi t$ volt is applied to a purely resistive load of 50 Ω. The time taken for the current to rise from half of the peak value to the peak value is:

(1) 3.3 ms (2) 2.2 ms (3) 5 ms (4) 7.2 ms

Ans. **[1]**

Sol. $i = I_0 \sin(100\pi t)$

Time taken to reach $i = \frac{I_0}{2}$ from i = 0

$\frac{I_0}{2} = I_0 \sin(100\pi t_1)$

$\Rightarrow t_1 = \frac{1}{600}$ second

Time taken to reach I_0 from i = 0

$t_2 = \frac{T}{4} = \frac{1}{200}$ second

So, $\Delta T = 3.3 \times 10^{-3}$

Q.3 Projectiles A and B are thrown at angles of 45° and 60° with vertical respectively from top of a 400 m high tower. If their ranges and times of flight are same, the ratio of their speeds of projection $v_A : v_B$ is :
[Take $g = 10\ m/s^2$]

(1) $1 : \sqrt{3}$ (2) $1 : \sqrt{2}$ (3) $\sqrt{2} : 1$ (4) 1 : 2

Ans. **[2]**

Sol. For same time of flight, vertical component of velocity of projection must be same.
$v_A \sin 45 = v_B \sin 30$

$\Rightarrow \frac{v_A}{v_B} = 1 : \sqrt{2}$

Q.4 If mass is written as $m = kc^P G^{-1/2} h^{1/2}$ then the value of P will be : (Constant have their usual meaning with k a dimensionless constant)

(1) –1/3 (2) 1/2 (3) 2 (4) 1/3

Ans. **[2]**

Sol. $m = kC^p G^{-1/2} h^{1/2}$
$[M] = [LT^{-1}]^P [M^{-1}L^3T^{-2}]^{-1/2} [ML^2T^{-1}]^{1/2}$

On solving $P = \frac{1}{2}$

Q.5 A particle of charge '–q' and mass 'm' moves in a circle of radius 'r' around an infinitely long line charge of linear charge density '+λ'. Then time period will be given as :
(Consider k as Coulomb's constant)

(1) $T = \frac{1}{2\pi r}\sqrt{\frac{m}{2k\lambda q}}$ (2) $T^2 = \frac{4\pi^2 m}{2k\lambda q} r^3$ (3) $T = 2\pi r\sqrt{\frac{m}{2k\lambda q}}$ (4) $T = \frac{1}{2\pi}\sqrt{\frac{2k\lambda q}{m}}$

Ans. **[3]**

Sol. Centrifugal force = Centripetal force

$\frac{mv^2}{r} = \frac{\lambda}{2\pi\varepsilon_0 r} q$

$\Rightarrow v = \sqrt{\frac{\lambda q}{2m\varepsilon_0}}$

$\therefore T = \frac{2\pi r}{v} = 2\pi r\sqrt{\frac{m}{2k\lambda q}}$

Q.6 Match List I with List II

	List I		*List II*
A.	Gauss's law of magnetostatics	I.	$\oint \vec{E}\cdot d\vec{a} = \frac{1}{\varepsilon_0}\int \rho dv$
B.	Faraday's law of electro magnetic induction	II.	$\oint \vec{B}\cdot d\vec{a} = 0$
C.	Ampere's law	III.	$\oint \vec{E}\cdot d\vec{\ell} = \frac{-d}{dt}\int \vec{B}\cdot d\vec{a}$
	Gauss's law of electrostatics	*IV.*	$\oint \vec{B}\cdot d\vec{\ell} = \mu_0 I$

Chosse the correct answer from the options given below.

(1) A-I, B-III, C-IV, D-II (2) A-III, B-IV, C-I, D-II
(3) A-IV, B-II, C-III, D-I (4) A-II, B-III, C-IV, D-I

Ans. **[4]**

Sol. Gausss' s law of magnetostatics $\Rightarrow \oint \vec{B}\cdot\vec{da} = 0$

Faraday's law of electromagnetic induction

$\Rightarrow \oint \vec{E}\cdot\vec{d\ell} = \frac{-d}{dt}\int \vec{B}\cdot\vec{da}$

Ampere's law $\Rightarrow \oint \vec{B}\cdot\vec{d\ell} = \mu_0 I$

Gauss's law of electrostatics $\Rightarrow \oint \vec{E}\cdot\vec{da} = \frac{1}{\varepsilon_0}\int \rho dv$

Q.7 A beam of unpolarised light of intensity I_0 is passed through a Polaroid A and then through another Polaroid B which is oriented so that its principal plane makes an angle of 45^o relative to that of A. The intensity of emergent light is:

(1) $I_0/8$ (2) $I_0/4$ (3) $I_0/2$ (4) I_0

Ans. **[2]**

Sol. $I_1 = \frac{I_0}{2}$

$I_1 = \frac{I_0}{2}\cos^2(45^o)$

$= \frac{I_0}{4}$

Q.8 An electron revolving in n^{th} Bohr orbit has magnetic moment μ_n. If $\mu_n \alpha n^x$, the |value of x is

(1) 1 (2) 3 (3) 0 (4) 2

Ans. **[1]**

Sol. Magnetic moment of electron = $\frac{ehn}{4\pi m_e}$

From given value x = 1

Q.9 If the total energy transferred to a surface in time t is 6.48×10^5 J, then the magnitude of the total momentum delivered to this surface for complete absorption will be :

(1) 2.46×10^{-3} kg m/s (2) 4.32×10^{-3} kg m/s

(3) 1.58×10^{-3} kg m/s (4) 2.16×10^{-3} kg m/s

Ans. **[4]**

Sol. E = PC

$\Rightarrow P = \frac{E}{C}$

$= \frac{6.48\times10^5}{3\times10^8}$

$= 2.16 \times 10^{-3}$ kg m/s

Q.10 In a nuclear fission reaction of an isotope of mass M, three similar daughter nuclei of same mass are formed. The speed of a daughter nuclei in terms of mass defect ΔM will be :

(1) $c\sqrt{\frac{3\Delta M}{M}}$ (2) $\sqrt{\frac{2c\Delta M}{M}}$ (3) $\frac{\Delta Mc^2}{3}$ (4) $c\sqrt{\frac{2\Delta M}{M}}$

Ans. **[4]**

Sol. $(M - 3m)c^2 = 3\left(\frac{1}{2}mV^2\right)$

$V = \sqrt{\frac{2\Delta mc^2}{3m}}$

Again $\Delta m = M - 3m$

$m = \frac{M - \Delta m}{3}$

$V = \sqrt{\frac{2\Delta mc^2}{M - \Delta m}}$

$= c\sqrt{\frac{2\Delta M}{M}}$

Q.11 Escape velocity of a body from earth is 11.2 km/s. If the radius of a planet be one third the radius of earth and mass be one-sixth that of earth, the escape velocity from the planet is :

(1) 4.2 km/s (2) 8.4 km/s (3) 7.9 km/s (4) 11.2 km/s

Ans. **[3]**

Sol. Escape velocity (V) = $\sqrt{\frac{2GM}{R}}$

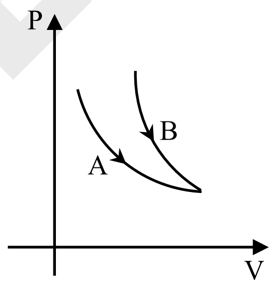
$V_e = \sqrt{\frac{2GM_e}{R_e}}$

$V_p = \sqrt{\frac{2GM_e \times 3}{6R_e}}$

$V_p = \frac{1}{\sqrt{2}} V_e$

= 7.9 km/s

Q.12 Choose the correct statement for processes A and B shown in figure.



(1) PV = k for process B and A

(2) $\frac{P^{\gamma-1}}{T^{\gamma}}$ = k for process B and T = k for process A

(3) $\frac{T^{\gamma}}{P^{\gamma-1}}$ = k for process A and PV = k for process B

(4) PV^{γ} = k for process B and PV = k for process A

Ans. **[2, 4]**

Sol. Slope of curve B > slope of curve A so A is isothermal process while B is adiabatic process.

Curve (B) is adiabatic

Adiabatic $\Rightarrow pv^{\gamma}$ = constant or $p\left(\frac{T}{P}\right)^{\gamma}$ = constant

$\frac{T^{\gamma}}{P^{\gamma-1}}$ = constant

Curve (A) is isothermal

T = constant

PV = constant

Q.13 A block of mass m is placed on a surface having vertical cross-section given by $y = \frac{x^2}{4}$. If coefficient of friction is 0.5, the maximum height above the ground at which block can be placed without slipping is:

(1) $\frac{1}{2}$ m (2) $\frac{1}{6}$ m (3) $\frac{1}{3}$ m (4) $\frac{1}{4}$ m

Ans. **[4]**

Sol. at equilibrium, $\tan\theta = \mu$

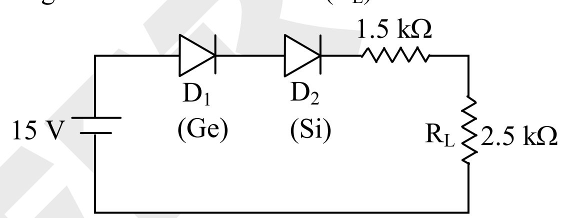
$\Rightarrow \frac{dy}{dx} = \frac{x}{2}$

$\Rightarrow \frac{1}{2} = \frac{x}{2}$

$\Rightarrow x = 1$

$\Rightarrow y = \frac{1}{4}$ m

Q.14 In the given circuit, the voltage across load resistance (R_L) is



(1) 14.00 V (2) 9.00 V (3) 8.50 V (4) 8.75 V

Ans. **[4]**

Sol. From KVL

15 – (0.3 + 0.7) = (1.5 + 2.5) K × I

$I = \frac{7}{2}$ mA

$V_1 = \left(\frac{7}{2}m\right)\left(\frac{5}{2}k\right)$

= 8.75 V

Q.15 If three moles of monoatomic gas $\left(\gamma = \frac{5}{3}\right)$ is mixed with two moles of a diatomic gas $\left(\gamma = \frac{7}{5}\right)$, the value of adiabatic exponent γ for the mixture is

(1) 1.35 (2) 1.52 (3) 1.75 (4) 1.40

Ans. **[2]**

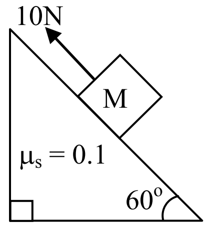
Sol. $F_{mix} = \frac{3\times3+2\times5}{5} = \frac{19}{5}$

$\therefore \gamma = 1 + \frac{2}{\left(\frac{19}{5}\right)} = 1 + \frac{10}{19}$

$\Rightarrow \gamma = \frac{29}{19}$

$\Rightarrow \gamma = 1.52$

Q.16 A block of mass 1 kg is pushed up a surface inclined to horizontal at an angle of 60° by a force of 10 N parallel to the inclined surface as shown in figure. When the block is pushed up by 10 m along inclined surface, the work done against frictional force is [g = 10 m/s^2]



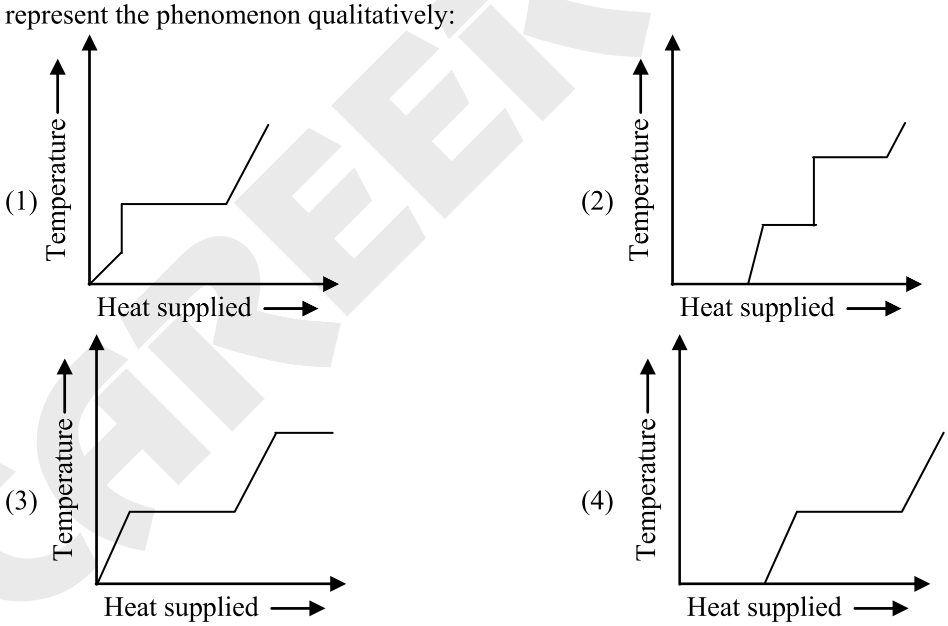
(1) 5 J (2) $5\sqrt{3}$ J (3) 5×10^3J (4) 10 J

Ans. **[1]**

Sol. Maximum friction force = μmgcosθ = $\frac{1}{2}$

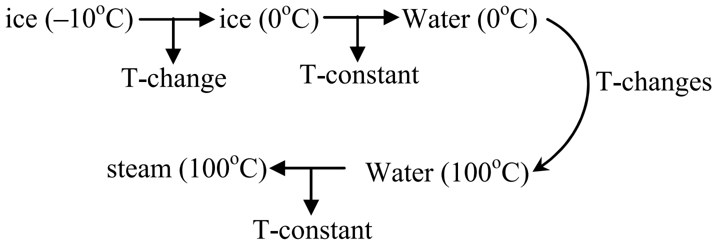
Work done against friction = $\left(\frac{1}{2}\right) \times 10 = 5$ J

Q.17 A block of ice at –10°C is slowly heated and converted to steam at 100°C. Which of the following curves represent the phenomenon qualitatively:

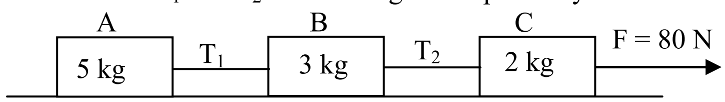


Ans. **[3]**

Sol.



Q.18 Three blocks A, B and C are pulled on a horizontal smooth surface by a force of 80 N as shown in figure The tensions T_1 and T_2 in the string are respectively:



(1) 80 N, 100 N (2) 88 N, 96 N (3) 40 N, 64 N (4) 60 N, 80 N

Ans. **[3]**

Sol. Acceleration of system (a) = $\frac{80}{10}$ = 8 m/s^2

$\Rightarrow T_1 = (5 \times 8)$ N

and, $T_2 - T_1 = (3 \times 8)$ N

$T_2 = (8 \times 8)$ N

Q.19 If 50 Vernier divisions are equal to 49 main scale divisions of a traveling microscope and one smallest reading of main scale is 0.5 mm, the Vernier constant of traveling microscope is

(1) 0.01 mm (2) 0.1 cm (3) 0.01 cm (4) 0.1 mm

Ans. **[1]**

Sol. MSD = 0.5 mm

LC = MSD – VSD

$= \frac{MSD}{50} = 0.01$ mm

Q.20 When a potential difference V is applied across a wire of resistance R, it dissipates energy at a rate W. If the wire is cut into two halves and these halves are connected mutually parallel across the same supply, the energy dissipation rate will become:

(1) $\frac{1}{2}$W (2) 4 W (3) $\frac{1}{4}$W (4) 2 W

Ans. **[2]**

Sol. In 1st case : $W = \frac{V^2}{R}$

In 2nd case : $W' = \frac{V^2}{\left(\frac{R}{4}\right)}$

$\Rightarrow W' = 4W$

Section-B: Numerical Value Type Questions: This section contains 10 Numerical based questions. Attempt any 5 questions out of 10. The answer to each question should be rounded-off to the nearest integer..

Q.21 Two identical charged spheres are suspended by strings of equal lengths. The strings make an angle of 37° with each other. When suspended in a liquid of density 0.7 g/cm^3, the angle remains same. If density of material of the sphere is 1.4 g/cm^3, the dielectric constant of the liquid is _______.

$\left(\tan 37^\circ = \frac{3}{4}\right)$

Ans. **[2]**

Sol. $\frac{Fe}{mg} = \frac{\left(\frac{Fe}{k}\right)}{mg - \sigma vg}$

$k = \frac{\rho}{\rho - \sigma}$

$= \frac{1.4}{1.4 - 0.7}$

$k = \frac{1.7}{0.7}$

$k = 2$

Q.22 A point source is emitting sound waves of intensity 16×10^{-8} Wm^{-2} at the origin. The difference in intensity (magnitude only) at two points located at a distance of 2 m and 4 m from the origin respectively will be _____ $\times 10^{-8}$ Wm^{-2}.

Ans. **[3]**

Sol. Question is not correctly represented lossely, it can be solved as below

$I \propto \frac{1}{r^2}$

$\therefore I_1 = \frac{16 \times 10^{-8}}{(2)^2} = 4 \times 10^{-8}$

$I_2 = \frac{16 \times 10^{-8}}{(4)^2} = 1 \times 10^{-8}$

$\therefore \Delta I = I_1 - I_2$

$= 3 \times 10^{-8}$ W/m^2

Q.23 A power transmission line feeds input power at 2.3 kV to a step-down transformer with its primary winding having 3000 turns. The output power is delivered at 230 V by the transformer. The current in the primary of the transformer is 5 A and its efficiency is 90%. The winding of transformer is made of copper. The output current of transformer is _______ A.

Ans. **[45]**

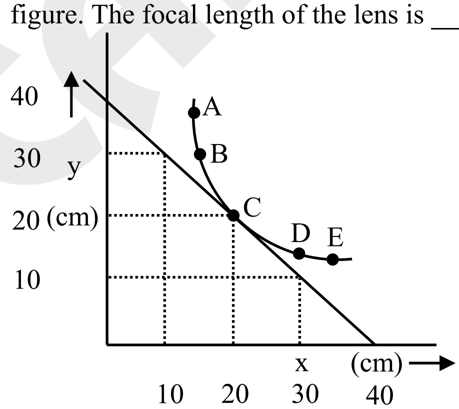
Sol. $V_1 = 2.3$ kV $\quad P_0 = 10.35$ kW

$N_i = 3000 \quad I_i = 5$ $AV_0 = 230$ V

$P_0 = I_0 V_0$

$I_0 = \frac{P_0}{V_0} = 45$ A

Q.24 In an experiment to measure the focal length (f) of a convex lens, the magnitude of object distance (x) and the image distance (y) are measured with reference to the focal point of the lens. The y-x plot is shown in figure. The focal length of the lens is _______ cm.



Ans. **[20]**

Sol. **Method I**

$x_1 x_2 = f^2$

$\Rightarrow f = 20$ cm

Method II

$$\frac{1}{f+20} - \frac{1}{-(f+20)} = \frac{1}{f}$$

$$\frac{2}{f+20} = \frac{1}{f}$$

$\Rightarrow f = 20$ cm

Q.25 The current of 5 A flows in a square loop of sides 1 m is placed in air. The magnetic field at the centre of the loop is $X\sqrt{2} \times 10^{-7}$ T. The value of X is ______.

Ans. **[40]**

Sol. Magnetic field at centre $= 4\left[\frac{\mu_0 I}{4\pi d}(\sin 45 + \sin 45)\right]$

$d = \frac{1}{2}$ m

$B_c = 4 \times 10^{-7} \times 5 \times 2 \times \frac{2}{\sqrt{2}}$

$= 40\sqrt{2} \times 10^{-7}$ T

Q.26 Two discs of moment of inertia $I_1 = 4$ kg m^2 and $I_2 = 2$ kg m^2, about their central axes and normal to their planes, rotating with angular speeds 10 rad/s and 4 rad/s respectively are brought into contact face to face with their axes of rotation coincident. The loss in kinetic energy of the system in the process is _______ J.

Ans. **[24]**

Sol. From angular momentum conservation

$40 + 8 = 6\,\omega$

$\omega = 8$ rad/s (common velocity)

Loss in K.E. $= \frac{1}{2}[4 \times 100 + 2 \times 16 - 6 \times 64]$

$= 24$ J

Q.27 Two resistance of 100 Ω and 200 Ω are connected in series with a battery of 4 V and negligible internal resistance. A voltmeter is used to measure voltage across 100 Ω resistance, which gives reading as 1 V. The resistance of voltmeter must be ______ Ω.

Ans. **[200]**

Sol. Voltage across 200 Ω = 3 V

Current in loop $= \frac{3}{200}$

For voltmeter $= \left(\frac{100R}{100+R}\right) \times \frac{3}{200} = 1$

$R = 200\ \Omega$

Q.28 A vector has magnitude same as that of $\vec{A} = 3\hat{i} + 4\hat{j}$ and is parallel to $\vec{B} = 4\hat{i} + 3\hat{j}$. The x and y components of this vector in first quadrant are x and 3 respectively where x = _______.

Ans. **[4]**

Sol. The vector $(\vec{C}) = 5\left(\frac{4\hat{i}+3\hat{j}}{5}\right)$

$= 4\hat{i}+3\hat{j}$

x-component = 4

Q.29 A simple pendulum is placed at a place where its distance from the earth's surface is equal to the radius of the earth. If the length of the string is 4m, then the time period of small oscillations will be _____s. (Take $g = \pi^2 ms^{-2}$)

Ans. **[8]**

Sol. $g' = \frac{g}{4}$

$T' = 2\pi\sqrt{\frac{4\ell}{g}}$

$T' = 2\pi\sqrt{\frac{4\times4}{\pi^2}} = \frac{2\pi\times4}{\pi}$

= 8 sec

Q.30 A big drop is formed by coalescing 1000 small identical drops of water. If E_1 be the total surface energy of 1000 small drops of water and E_2 be the surface energy of single big drop of water, then $E_1 : E_2$ is x : 1 where x = ______.

Ans. **[10]**

Sol. Radius of small drop = r

Radius of big drop = R

R = 10 r

$E_1 = 4\pi r^2 \times 1000$

$E_2 = 4\pi(10r)^2$

$\frac{E_1}{E_2} = 10$

CHEMISTRY

Section-A: This section contains 20 multiple choice questions. Each question has 4 choices(1), (2), (3) and (4), out of which **ONLY ONE** is correct..

Q.31 Choose the correct statements about the hydrides of group 15 elements.

A. The stability of the hydrides decreases in the order $NH_3 > PH_3 > AsH_3 > SbH_3 > BiH_3$

B. The reducing ability of the hydride increases in the order $NH_3 < PH_3 < AsH_3 < SbH_3 < BiH_3$

C. Among the hydrides, NH_3 is strong reducing agent while BiH_3 is mild reducing agent.

D. The basicity of the hydrides increases in the order $NH_3 < PH_3 < AsH_3 < SbH_3 < BiH_3$

Choose the **most appropriate** from the option given below

(1) B and C only
(2) A and B only
(3) A and D only
(4) C and D only

Ans. **[2]**

Sol. BiH_3 is a stronger reducing agent than NH_3. Basicity of hydrides decreases down the group.

Q.32 Given below are two statements :

Statement-I : Since Fluorine is more electronegative than nitrogen, the net dipole moment of NF_3 is greater than NH_3.

Statement-II : In NH_3, the orbital dipole due to lone pair and the dipole moment of NH bonds are in opposite direction, but in NF_3 the orbital dipole due to lone pair and dipole moments of N-F bonds are in same direction.

In the light of the above statements, choose the **most appropriate** from the options given below :

(1) Both **Statement I** and **Statement II** are true

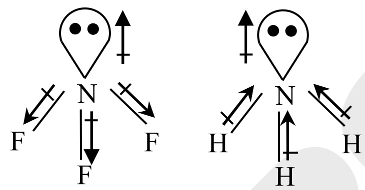
(2) Both **Statement I** and **Statement II** are false

(3) **Statement I** is false but **Statement II** is true

(4) **Statement I** is true but **Statement II** is false

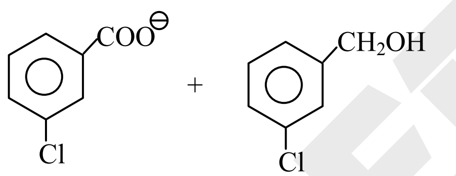
Ans. **[3]**

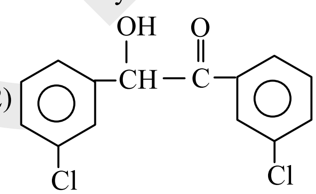
Sol. In NF_3 the orbital dipole due to lone pair and dipole moments of N – F bonds are in opposite direction. In NH_3, they are the same direction

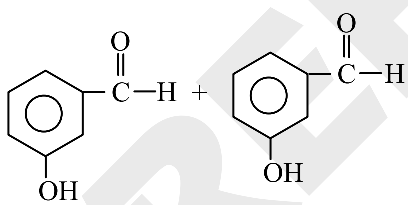


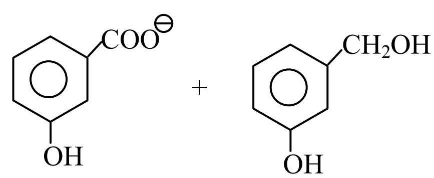
Thus, in NH_3 the dipole moments add up which leads to higher dipole moment than NF_3.

Q.33 m-chlorobenzaldehyde, on treatement with 50% KOH solution yields

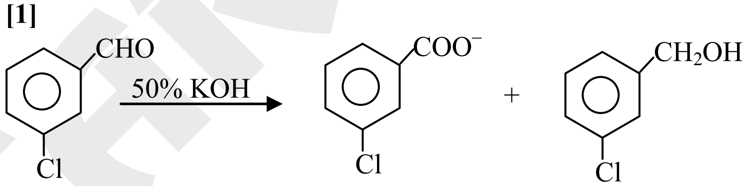
(1) 

(2) 

(3) 

(4) 

Ans. **[1]**

Sol. 

Cannizzaro reaction.

Q.34 Reduction potential of ions are given below:

ClO_4^-	IO_4^-	BrO_4^-
$E^o = 1.19V$	$E^o = 1.65V$	$E^o = 1.74V$

The correction order of their oxidising power is

(1) $IO_4^- > BrO_4^- > ClO_4^-$

(2) $BrO_4^- > IO_4^- > ClO_4^-$

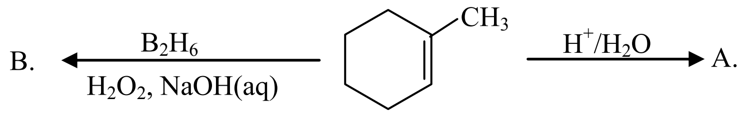
(3) $ClO_4^- > IO_4^- > BrO_4^-$

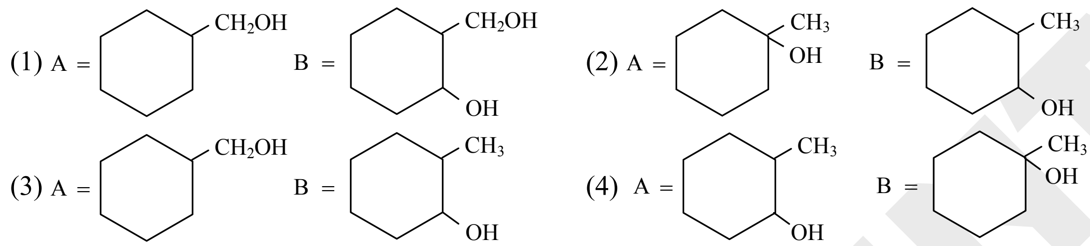
(4) $BrO_4^- > ClO_4^- > IO_4^-$

Ans. [2]

Sol. Higher the reduction potential higher is the oxidizing power.

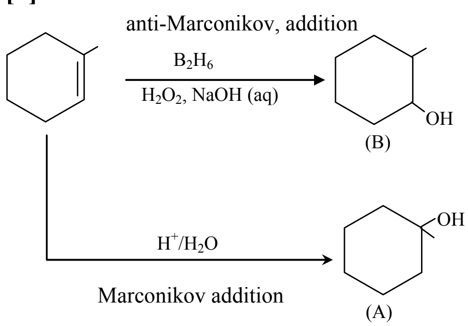
Q.35 Product A and B formed in the following set of reaction are





Ans. [2]

Sol.



Q.36 Given below are two statements:

Statement I : High concentration of strong nucleophilic reagent with secondary alkyl halides which do not have bulky substituents will follow $S_N{}^2$mechanism.

Statement II : A secondary alkyl halide when treated with a larger excess of ethanol follows $S_N{}^1$ mechanism.

In the light of the above statements, choose the **most appropriate** from the options given below :

(1) **Statement I** true but **Statement II** is false

(2) Both **Statement I** and **Statement II** are true

(3) **Statement I** is false but **Statement II** is false

(4) **Statement I** is false but **Statement II** is true

Ans. [2]

Sol. High concentration of strong nucleophile and lack of bulky substituents favors $S_N{}^2$ mechanism.
Ethanol is a weak nucleophile so $S_N{}^1$ mechanism should be preferred

Q.37 The correct stability order of carbocations is

(1) $\overset{+}{C}H_3 > CH_3-\overset{+}{C}H_2 > CH_3-\underset{\underset{CH_3}{|}}{\overset{+}{C}H} > (CH_3)_3\overset{+}{C}$

(2) $(CH_3)_3C^+ > CH_3-\overset{+}{C}H_2 > (CH_3)_2\overset{+}{C}H > \overset{+}{C}H_3$

(3) $\overset{+}{C}H_3 > (CH_3)_2\overset{+}{C}H > CH_3-\overset{+}{C}H_2 > (CH_3)_3\overset{+}{C}$

(4) $(CH_3)_3\overset{+}{C} > (CH_3)_2\overset{+}{C}H > CH_3-\overset{+}{C}H_2 > \overset{+}{C}H_3$

Ans. [4]

Sol. Stability order of carbocations
tertiary > secondary > primary > methyl
More is alkyl substituents in a carbocation more is hyperconjugation and inductive effect, thus more is the stability.

Q.38 The solution from the following with highest depression in freezing point/lowest freezing point is
(1) 180 g of acetic acid dissolved in benzene
(2) 180 g of acetic acid dissolved in water
(3) 180 g of benzoic acid dissolved in benzene
(4) 180 g of glucose dissolved in water

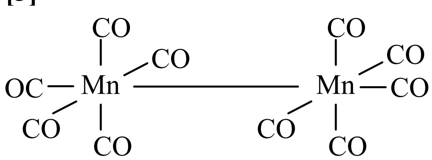
Ans. **[2]**

Sol. Assuming same quantity of solvent, 180 g of acetic acid will have highest moles among given solutes. Further, in water acetic acid will dissociate leading to increase in number of solute particles.

Q.39 The coordination geometry around the manganese in decacarbonyldimanganese(0) is
(1) Square pyramidal
(2) Square planar
(3) Octahedral
(4) Trigonal bipyramidal

Ans. **[3]**

Sol.



Each Mn has 6 atoms surrounding it in an octahedral geometry

Q.40 Given below are two statements:
Statement-I : Along the period, the chemical reactivity of the elements gradually increases from group 1 to group 18.
Statement-II : The nature of oxides formed by group 1 elements is basic while that of group 17 elements is acidic.
In the light of the above statements, choose the **most appropriate** from the options given below :
(1) Both **statement I** and **Statement II** are true
(2) Both **statement I** and **Statement II** are false
(3) **Statement I** is false but **statement II** is true
(4) **Statement I** is true but **statement II** is false

Ans. **[3]**

Sol. Chemical reactivity decreases in a period from left to right. Group-1 elements being metals form basic oxides, while group 17 elements being non-metals form acidic oxides.

Q.41 A and B formed in the following reaction are :
$CrO_2Cl_2 + 4NaOH \rightarrow A + 2NaCl + 2H_2O$,
$A + 2HCl + 2H_2O_2 \rightarrow B + 3H_2O$
(1) A = $Na_2Cr_2O_7$, B = CrO_3
(2) A = $Na_2Cr_2O_7$, B = CrO_5
(3) A = $Na_2Cr_2O_4$, B = CrO_4
(4) A = Na_2CrO_4, B = CrO_5

Ans. **[4]**

Sol. A is Na_2CrO_4
B is CrO_5

Q.42 If a substance 'A' dissolves in solution of a mixture of 'B' and 'C' with their respective number of moles in n_A, n_B and n_c, mole fraction C in the solution is

(1) $\frac{n_B}{n_A + n_B}$ (2) $\frac{n_C}{n_A - n_B - n_C}$ (3) $\frac{n_C}{n_A + n_B + n_C}$ (4) $\frac{n_C}{n_A \times n_B \times n_C}$

Ans. **[3]**

Sol. Moles fraction = $\frac{\text{Moles of a component}}{\text{total moles of all components}}$

$$= \frac{n_C}{n_A + n_B + n_C}$$

Q.43 Alkaline oxidative fusion of MnO_2 gives "A" which on electrolytic oxidation in alkaline solution produces B. A and B respectively are

(1) MnO_4^{2-} and Mn_2O_7 (2) Mn_2O_7 and MnO_4^- (3) Mn_2O_3 and MnO_4^{2-} (4) MnO_4^{2-} and MnO_4^-

Ans. **[4]**

Sol. $2MnO_2 + 4\,KOH + O_2 \longrightarrow \underset{A}{2K_2MnO_4} + 2H_2O$

$$\underset{A}{MnO_4^{-2}} \xrightarrow[\text{Oxidation}]{\text{Electrolytic}} \underset{B}{MnO_4^{-1}}$$

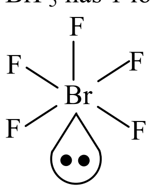
A is MnO_4^{2-} ; B is MnO_4^-

Q.44 The molecular/ion with square pyramidal shape is

(1) PCl_5 (2) PF_5 (3) $[Ni(CN)_4]^{2-}$ (4) BrF_5

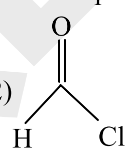
Ans. **[4]**

Sol. BrF_5 has 1 lone pair and 5 bond pair



It has square pyramidal shape.

Q.45 Salicyladehyde is synthesized from phenol, when reacted with

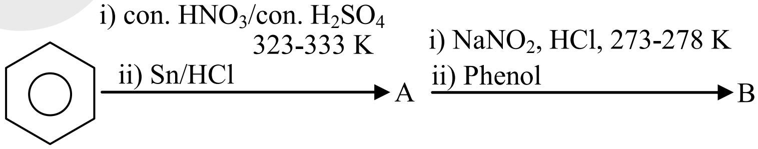
(1) CO_2, NaOH (2) 

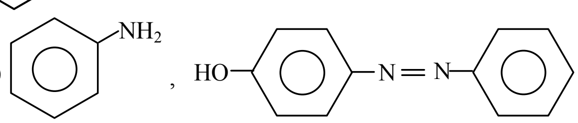
, NaOH (3) CCl_4, NaOH (4) $HCCl_3$, NaOH

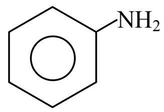
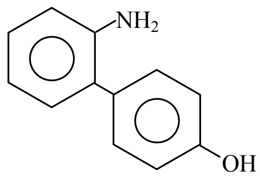
Ans. **[4]**

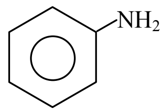
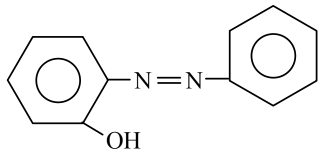
Sol. Phenol reacts with $CHCl_3$, NaOH to give salicyladehyde. This is Reimer-Tiemann reaction.

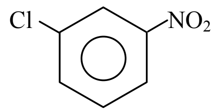
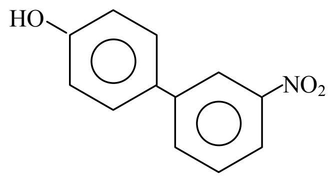
Q.46 The products A and B formed in the following reaction scheme are respectively



(1) 

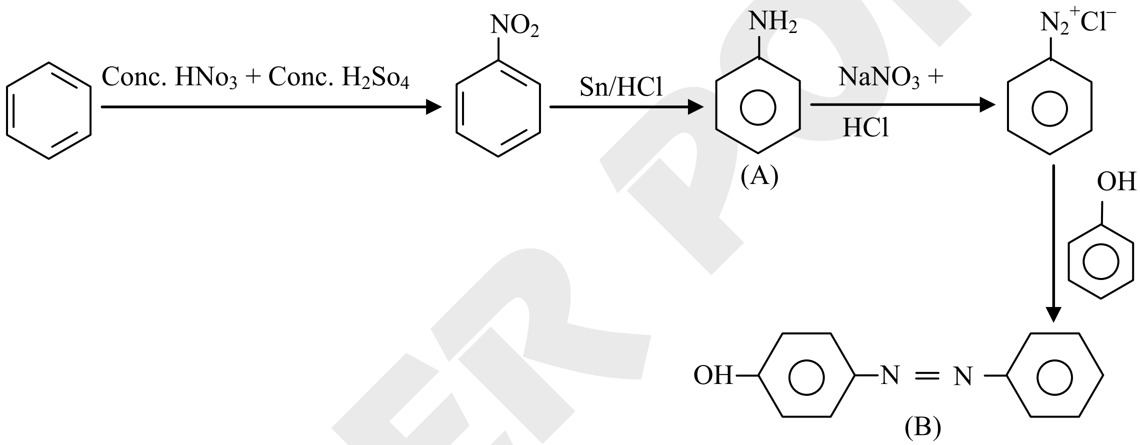
(2)  , 

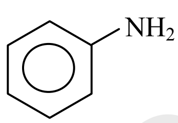
(3)  , 

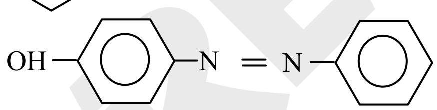
(4)  , 

Ans. **[1]**

Sol.



A is 

B is 

Q.47 IUPAC name of following compound is :

$$CH_3 - \underset{\underset{NH_2}{|}}{CH} - CH_2 - CN$$

(1) 2-Aminopentanenitrile
(2) 2-Aminobutanenitrile
(3) 3-Aminopropanenitrile
(4) 3-Aminobutanenitrile

Ans. **[4]**

Sol.

$$\overset{4}{CH_3} - \underset{\underset{NH_2}{|}}{\overset{3}{CH}} - \overset{2}{CH_2} - \overset{1}{CN}$$

3-Aminobutanenitrile

Q.48 The orange colour of $K_2Cr_2O_7$ and purple colour of $KMnO_4$ is due to
(1) d → d transitions in $K_2Cr_2O_7$ and charge transfer transitions in KMnO4
(2) d → d transitions in KMnO4 and charge transfer transitions in $K_2Cr_2O_7$
(3) Charge transfer transition in both
(4) d → d transitions in both

Ans. **[3]**

Sol. Colour in $K_2Cr_2O_7$ and $KMnO_4$, both d° complexes is due to charge transfer transitions.

Q.49 Which among the following purification methods is based on the principle of "Solubility" in two different solvents?
(1) Distillation
(2) Differential Extraction
(3) Column Chromatography
(4) Sublimation

Ans. **[2]**

Sol. Differential extraction is based on the principle of solubility in two different solvents.

Q.50 Given below are two statements : One is labelled as **Assertion A** and the other is labelled as **Reason R**.
Assertion A : H_2Te is more acidic than H_2S.
Reason R : Bond dissociation enthalpy of H_2Te is lower than H_2S.
In the light of the above statements, choose the **most appropriate** from the options given below :
(1) **A** is false but **R** is true
(2) Both **A** and **R** are true but **R** is NOT the correct explanation of **A**
(3) **A** is true but **R** is false
(4) Both **A** and **R** are true and **R** is the correct explanation of **A**

Ans. **[4]**

Sol. Bond enthalpy of H_2Te is lower than H_2S, thus H_2Te releases H^+ more easily. So, it is a stronger acid.

Section-B: Numerical Value Type Questions: This section contains 10 Numerical based questions. Attempt any 5 questions out of 10. The answer to each question should be rounded-off to the nearest integer.

Q.51 The total number of correct statements, regarding the nucleic acid is ______.
A. RNA is regarded as the reserve of genetic information.
B. DNA molecule self-duplicates during cell division.
C. DNA synthesizes proteins in the cell.
D. The message for the synthesis of particular proteins is present in DNA.
E. Identical DNA strands are transferred to daughter cells.

Ans. **[3]**

Sol. DNA is regarded as reserve of genetic information. Proteins are synthesised by various RNA molecules in the cell. Statements B, D, E are correct

Q.52 Total number of species from the following which can undergo disproportionation reaction is ____.
H_2O_2, ClO_3^-, P_4, Cl_2, Ag, Cu^{+1}, F_2, NO_2, K^+

Ans. **[6]**

Sol. H_2O_2, ClO_3^-, P_4, Cl_2, Cu^{+1}, NO_2, have atoms in intermediate oxidation states.

Q.53 Number of complexes which show optical isomerism among the following is ____.
cis-$[Cr(ox)_2Cl_2]^{3-}$, $[Co(en)_3]^{3+}$, cis-$[Pt(en)_2Cl_2]^{2+}$, cis-$[Co(en)_2Cl_2]^+$, trans-$[Pt(en)_2Cl_2]^{2+}$, trans-$[Cr(ox)_2Cl_2]^{3-}$

Ans. **[4]**

Sol. cis-$[Cr(ox)_2Cl_2]^{3-}$, $[Co(en)_3]^{3+}$, cis-$[Pt(en)_2Cl_2]^{2+}$, cis-$[Co(en)_2Cl_2]^+$ show optical isomerism

Q.54 NO_2 required for a reaction is produced by decomposition of N_2O_5 in CCl_4 as by equation

$2N_2O_5(g) \rightarrow 4NO_2(g) + O_2(g)$

The initial concentration of N_2O_5 is 3 mol L^{-1} and it is 2.75 mol L^{-1} after 30 minutes.

The rate of formation of NO_2 is x × 10^{-3} mol L^{-1} min^{-1}, value of x is _______. (nearest integer)

Ans. **[17]**

Sol. $$\frac{-\Delta[N_2O_5]}{\Delta t} = \frac{3-2.75}{30} = \frac{0.25}{30}$$

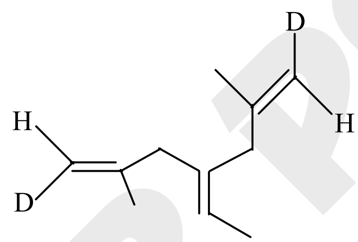
$$\frac{\Delta[NO_2]}{\Delta t} = 2\times\left[\frac{-\Delta[N_2O_5]}{\Delta t}\right]$$

$$= 2 \times \frac{0.25}{30}$$

$= 0.017$ mol L^{-1} min^{-1}

$= 17 \times 10^{-3}$ Mol L^{-1} min^{-1}

Q.55 Number of geometrical isomers possible for the given structute is/are _____.



Ans. **[4]**

Sol. Possible geometries

Left DB	Middle DB	Right DB
E	–	E
Z	–	Z
E	Z	Z
Z	E	Z

Total 4 possibillities

Q.56 Number of spectral lines obtained in He^+ spectra, when an electron makes transition from fifth excited state to first excited state will be

Ans. **[10]**

Sol. Number of spectral lines = $\frac{(n_2 - n_1)(n_2 - n_1 + 1)}{2}$

Where, $n_2 = 6$, $n_1 = 2$

$\Rightarrow$ Number of spectral lines = $\frac{(6-2)(6-2+1)}{2}$

$$= \frac{4\times 5}{2} = 10$$

Q.57 Number of metal ions characterized by flame test among the following is _____.
Sr^{2+}, Ba^{2+}, Ca^{2+}, Cu^{2+}, Zn^{2+}, Co^{2+}, Fe^{2+}

Ans. **[4]**

Sol. Ca^{2+} – Brick red
Sr^{2+} – Crimson red
Ba^{2+} – Grassy green
Cu^{2+} – Blue green

Q.58 Two reaction are given below :

$$2Fe_{(s)} + \frac{3}{2}O_{2(g)} \rightarrow Fe_2O_{3(s)},\ \Delta H^o = -822\text{ kJ/mol}$$

$$C_{(s)} + \frac{1}{2}O_{2(g)} \rightarrow CO_{(g)},\ \Delta H^o = -110\text{ kJ/mol}$$

Then enthalpy change for following reaction
$3C_{(s)} + Fe_2O_{3(s)}, \rightarrow 2Fe_{(s)}, + 3CO_{(g)}$ is ____kJ/mol.

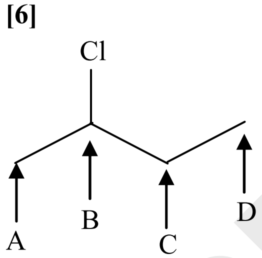
Ans. **[492]**

Sol. Net enthalpy change
= –(Enthalpy change of first reaction)
+3 (Enthalpy change of second reaction)
= 822 – 330
= 492

Q.59 2-chlorobutane + $Cl_2 \rightarrow C_4H_8Cl_2$ (isomers)
Total number of optically active isomers shown by $C_4H_8Cl_2$, obtained in the above reaction is ____.

Ans. **[6]**

Sol.



When Cl attaches to
Carbon A → 2 optically active isomers are formed
Carbon B → No optically active isomer
Carbon C → 2 optically active isomers
Carbon D → 2 optically active isomers

Q.60 The pH of an aqueous solution containing 1 M benzoic acid (pK_a = 4.20) and 1 M sodium benzoate is 4.5. The volume of benzoic acid solution in 300 mL of this buffer solution s _____ mL. (given : log2 = 0.3)

Ans. **[100]**

Sol. $$pH = pK_a + \log\left[\frac{(300 - V)}{V}\right]$$

$$\Rightarrow 4.5 = 4.2 + \log\frac{(300 - V)}{V}$$

$$\Rightarrow \frac{300 - V}{V} = 2 \Rightarrow V = 100\text{ mL}$$

MATHEMATICS

Section-A: This section contains 20 multiple choice questions. Each question has 4 choices(1), (2), (3) and (4), out of which **ONLY ONE** is correct..

Q.61 Let $f : R - \{0\} \to R$ be a function satisfying $f\left(\frac{x}{y}\right) = \frac{f(x)}{f(y)}$ for all, x, y, $f(y) \neq 0$. If $f'(1) = 2024$, then

(1) $xf'(x) - 2024\, f(x) = 0$
(2) $xf'(x) - 2023\, f(x) = 0$
(3) $xf'(x) + 2024\, f(x) = 0$
(4) $xf'(x) + f(x) = 2024$

Ans. **[1]**

Sol. $f\left(\frac{x}{y}\right) = \frac{f(x)}{f(y)}$

Put x = y = 1

f(1) = 1

Now put x = 1

$\Rightarrow f\left(\frac{1}{y}\right) = \frac{1}{f(y)}$

$f(y) = \pm y^n$

but $f(1) = 1 \Rightarrow f(y) = y^n$

$f'(y) = ny^{n-1}$

$f'(1) = n = 2024$

$\therefore f(x) = x^{2024}$

$f'(x) = 2024x^{2023}$

$\Rightarrow xf'(x) = 2024f(x)$

$\Rightarrow xf'(x) - 2024f(x) = 0$

Q.62 Let $\vec{a}$ and $\vec{b}$ be two vectors such that $|\vec{b}| = 1$ and $|\vec{b}\times\vec{a}| = 2$. Then $|(\vec{b}\times\vec{a}) - \vec{b}|^2$ is equal to

(1) 5
(2) 1
(3) 3
(4) 4

Ans. **[1]**

Sol. $|\vec{b}| = 1$ and $|\vec{b}\times\vec{a}| = 2$

Now, $|(\vec{b}\times\vec{a}) - \vec{b}|^2 = |\vec{b}\times\vec{a}|^2 + |\vec{b}|^2 - 2\,\vec{b}\cdot(\vec{b}\times\vec{a})$

$= (2)^2 + (1)^2 - 0 = 5$

Q.63 Let A(α, 0) and B(0, β) be the points on the line 5x + 7y = 50. Let the point P divide the line segment AB internally in the ratio 7 : 3. Let 3x – 25 = 0 be a directrix of the ellipse E : $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and the corresponding focus be S. If from S, the perpendicular on the x-axis passes through P, then the length of the latus rectum of E is equal to

(1) $\frac{32}{5}$
(2) $\frac{25}{3}$
(3) $\frac{32}{9}$
(4) $\frac{25}{9}$

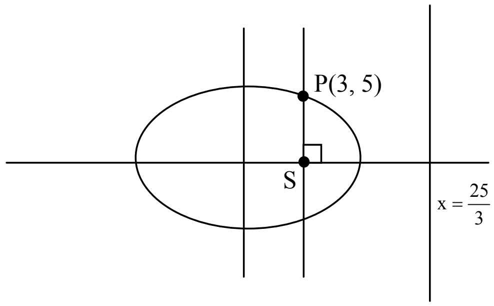
Ans. **[1]**

Sol. 5x + 7y = 50

When x = 0, y = $\frac{50}{7}$ $\Rightarrow B\left(0, \frac{50}{7}\right)$

When y = 0, x = 10 ⇒ A(10, 0)

$$P = \left(\frac{0+3\times10}{10}, \frac{7\times\frac{50}{7}+0}{10}\right) = (3, 5)$$



ae = 3

$\frac{a}{e} = \frac{25}{3}$

$\Rightarrow$ a = 5, b = 4

Length of latus rectum = $\frac{2b^2}{a} = \frac{32}{5}$

Q.64 Let y = f(x) be a thrice differentiable function in (–5, 5). Let the tangents to the curve y = f(x) at (1, f(1)) and (3, f(3)) makes angle $\frac{\pi}{6}$ and $\frac{\pi}{4}$, respectively with positive x-axis. If $27\int_1^3 \left((f'(t))^2+1\right) f''(t)dt = \alpha + \beta\sqrt{3}$, where α, β are integers, then the value of α + β equal

(1) –14 (2) –16 (3) 36 (4) 26

Ans. **[4]**

Sol. $f'(1) = \tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$

$f'(3) = \tan\frac{\pi}{4} = 1$

$I = 27\int_1^3 \left((f'(t))^2+1\right) f''(t)\, dt$

Let f '(t) = u

f''(t)dt = du

$I = 27\int_{\frac{1}{\sqrt{3}}}^{1} \left(u^2+1\right)du$

$= 27\left[\frac{u^3}{3}+u\right]_{1/\sqrt{3}}^{1}$

$= 27\left[\frac{1}{3}+1-\frac{1}{9\sqrt{3}}-\frac{1}{\sqrt{3}}\right]$

$= 27\left[\frac{4}{3}-\frac{10}{9\sqrt{3}}\right]$

$= \left[36-\frac{30}{\sqrt{3}}\right] = 36 - 10\sqrt{3}$

α = 36, β = –10

$\Rightarrow$ α + β = 36 – 10 = 26

Q.65 Let a and b be real constants such that the function f defined by $f(x) = \begin{cases} x^2 + 3x + a & , \ x \le 1 \\ bx + 2 & , \ x > 1 \end{cases}$ be differentiable on R. Then, the value of $\int_{-2}^{2} f(x)dx$ equals

(1) $\frac{15}{6}$ (2) 17 (3) 21 (4) $\frac{19}{6}$

Ans. **[2]**

Sol. Given function

$f(x) = \begin{cases} x^2 + 3x + a & , \ x \le 1 \\ bx + 2 & , \ x > 1 \end{cases}$ is differentiable on R

$\therefore$ f(x) is continuous at x = 1,

$\Rightarrow f(1) = \lim_{x \to 1^+} f(x)$

$\Rightarrow a + 4 = b + 2$

$\Rightarrow -a + b = 2$ ….(1)

also, $f'(x) = \begin{cases} 2x + 3 & , \ x \le 1 \\ b & , \ x > 1 \end{cases}$

$\because$ f is differentiable at x = 1

$\Rightarrow b = 5$

$\therefore$ from equation (1), a = 3

$\therefore \int_{-2}^{2} f(x)dx = \int_{-2}^{1} (x^2 + 3x + 3)dx + \int_{1}^{2} (5x + 2)dx$

$= \left[\frac{x^3}{3} + \frac{3x^2}{2} + 3x\right]_{-2}^{1} + \left[\frac{5x^2}{2} + 2x\right]_{1}^{2}$

$= 6 + \frac{3}{2} + 12 - \frac{5}{2}$

$= 18 - 1 = 17$

Q.66 If the domain of the function $f(x) = \log_e\left(\frac{2x+3}{4x^2+x-3}\right) + \cos^{-1}\left(\frac{2x-1}{x+2}\right)$ is (α, β], then the value of 5β – 4α is equal to

(1) 10 (2) 11 (3) 12 (4) 9

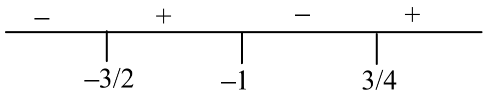
Ans. **[3]**

Sol. Given function,

$f(x) = \log_e\left(\frac{2x+3}{4x^2+x-3}\right) + \cos^{-1}\left(\frac{2x-1}{x+2}\right)$

Clearly, $\frac{2x+3}{4x^2+x-3} > 0$

$\Rightarrow \frac{2x+3}{(4x-3)(x+1)} > 0$

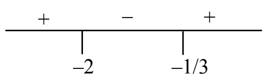


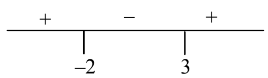
$\therefore\ x \in \left(\frac{-3}{2}, -1\right) \cup \left(\frac{3}{4}, \infty\right)$

Also, $-1 \le \frac{2x-1}{x+2} \le 1$

$\Rightarrow \frac{2x-1}{x+2} + 1 \ge 0$ | $\frac{2x-1}{x+2} - 1 \le 0$

$\Rightarrow \frac{3x+1}{x+2} \ge 0$ | $\frac{x-3}{x+2} \le 0$





$\therefore\ x \in (-\infty, -2) \cup [-\frac{1}{3}, \infty)$ and $x \in (-2, 3]$

$\therefore\ x \in (\frac{3}{4}, 3] \equiv (\alpha, \beta]$

$\therefore\ 5\beta - 4\alpha \equiv 15 - 3 = 12$

Q.67 Let $\vec{a} = \hat{i} + \alpha\hat{j} + \beta\hat{k}$, $\alpha, \beta, \in R$. Let a vector $\vec{b}$ be such that the angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{4}$ and $|\vec{b}|^2 = 6$. If $\vec{a}\cdot\vec{b} = 3\sqrt{2}$, then the value of $(\alpha^2 + \beta^2)\,|\vec{a}\times\vec{b}|^2$ is

(1) 75 (2) 85 (3) 90 (4) 95

Ans. **[3]**

Sol. $\vec{a}\cdot\vec{b} = 3\sqrt{2}$

$|\vec{a}||\vec{b}| \cos\frac{\pi}{4} = 3\sqrt{2}$

$|\vec{a}|\ \sqrt{6}\cdot\frac{1}{\sqrt{2}} = 3\sqrt{2}$

$|\vec{a}| = \sqrt{6} = \sqrt{1+\alpha^2+\beta^2}$

$\alpha^2 + \beta^2 = 5$

$|\vec{a}\times\vec{b}|^2 = |\vec{a}|^2|\vec{b}|^2 \sin^2\frac{\pi}{4}$

$= 6 \times 6 \times \frac{1}{2} = 18$

So, $(\alpha^2+\beta^2)|\vec{a}\times\vec{b}|^2 = 5 \times 18 = 90$

Q.68 Let $f : R \to R$ be a function defined by $f(x) = \frac{x}{(1+x^4)^{1/4}}$, and $g(x) = f(f(f(f(x))))$. Then $18\int_0^{\sqrt{2\sqrt{5}}} x^2\, g(x)dx$ is equal to

(1) 33 (2) 39 (3) 36 (4) 42

Ans. **[2]**

Sol. $f(x) = \frac{x}{(1+x^4)^{1/4}}$

$f(f(x)) = \dfrac{x}{(1+2x^4)^{\frac{1}{4}}}$

$f(f(f(x))) = \dfrac{x}{(1+3x^4)^{\frac{1}{4}}}$

$\therefore\ g(x) = f(f(f(f(x)))) = \dfrac{x}{(1+4x^4)^{\frac{1}{4}}}$

$\therefore\ I = 18\int_0^{\sqrt{2\sqrt{5}}} x^2\, g(x)dx = 18\int_0^{\sqrt{2\sqrt{5}}} \dfrac{x^3}{(1+4x^4)^{\frac{1}{4}}}$

Put $1 + 4x^4 = t^4$

$\Rightarrow 16x^3dx = 4t^3dt \Rightarrow 4x^3dx = t^3dt$

$\therefore\ I = 18\int_1^3 \dfrac{t^2}{4}dt$

$= 18\left(\dfrac{1}{12}(3^3 - 1^3)\right) = \dfrac{13}{6} \times 18 = 39$

Q.69 If z is a complex number, then the number of common roots of the equations $z^{1985} + z^{100} + 1 = 0$ and $z^3 + 2z^2 + 2z + 1 = 0$, is equal to

(1) 2 (2) 1 (3) 0 (4) 3

Ans. **[1]**

Sol. Given $z^{1985} + z^{100} + 1 = 0$ and

$z^3 + 2z^2 + 2z + 1 = 0$

$(z + 1)(z^2 + z + 1) = 0$

$z = -1, \omega, \omega^2$

but '–1' not satisfies

$z^{1985} + z^{100} + 1 = 0$

So the roots of $z^{1985} + z^{100} + 1 = 0$

be ω & ω^2.

$\therefore$ ω and ω^2 are common solutions

(where ω is cube root of unity)

$\therefore$ 2 solutions

Option (1) is correct.

Q.70 Let $R = \begin{pmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{pmatrix}$ be a non-zero 3 × 3 matrix, where $x\sin\theta = y\sin\left(\theta + \dfrac{2\pi}{3}\right) = z\sin\left(\theta + \dfrac{4\pi}{3}\right) \neq 0$, $\theta \in (0, 2\pi)$. For a square matrix M, let trace (M) denote the sum of all the diagonals entries of M. Then, among the statements:

(I) Trace (R) = 0

(II) If trace (adj(adj(R)) = 0, then R has exactly one non-zero entry.

(1) Only (II) is true (2) Only (I) is true

(3) Both (I) and (II) are true (4) Neither (I) nor (II) is true

Ans. [4]

Sol. $x\sin\theta = y\sin\theta\left(\theta+\frac{2\pi}{3}\right) = z\sin\left(\theta+\frac{4\pi}{3}\right) \neq 0,$

$\therefore\ y = \frac{x\sin\theta}{\sin\left(\theta+\frac{2\pi}{3}\right)},\ z = \frac{x\sin\theta}{\sin\left(\theta+\frac{4\pi}{3}\right)}$

$\therefore\ x+y+z = \frac{-3x}{4\sin\left(\theta+\frac{2\pi}{3}\right)\sin\left(\theta+\frac{4\pi}{3}\right)} \neq 0$

$\Rightarrow$ statement 1 is wrong.

$$R = \begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix}$$

$$\text{adj}R = \begin{bmatrix} yz & 0 & 0 \\ 0 & xz & 0 \\ 0 & 0 & xy \end{bmatrix}$$

$$\text{adj}(\text{adj}R) = \begin{bmatrix} x^2yz & 0 & 0 \\ 0 & y^2xz & 0 \\ 0 & 0 & z^2xy \end{bmatrix}$$

$\therefore$ Trace (adj(adjR) = xyz (x + y + z) $\neq 0$

Statement (2) is wrong

$\therefore$ Option (4) is correct.

Q.71 Bag A contains 3 white, 7 red balls and Bag B contains 3 white, 2 red balls. One bag is selected at random and a ball is drawn from it. The probability of drawing the ball from the bag A, if the ball drawn is white, is

(1) $\frac{1}{3}$ (2) $\frac{1}{4}$ (3) $\frac{1}{9}$ (4) $\frac{3}{10}$

Ans. [1]

Sol. Bag A contains 3 white, 7 red balls

Bag B contains 3 white, 2 red balls

Given, drawn bal lis white

So, probability that ball has drawn from bag A

$$= \frac{\frac{1}{2}\times\frac{3}{10}}{\frac{1}{2}\times\frac{3}{10}+\frac{1}{2}\times\frac{3}{5}}$$

$$= \frac{\frac{3}{20}}{\frac{3}{20}+\frac{3}{10}} = \frac{\frac{3}{20}}{\frac{3+6}{20}} = \frac{3}{9} = \frac{1}{3}$$

Q.72 Let $L_1 : \vec{r} = (\hat{i} - \hat{j} + 2\hat{k}) + \lambda\ (\hat{i} - \hat{j} + 2\hat{k})$, $\lambda \in R$,

$L_2 : \vec{r} = (\hat{j} - \hat{k}) + \mu\,(3\hat{i} + \hat{j} + p\hat{k})$, $\mu \in R$, and

$L_3 : \vec{r} = \delta(\ell\hat{i} + m\hat{j} + n\hat{k})$, $\delta \in R$ be three lines such that L_1 is perpendicular to L_2 and L_3 is perpendicular to both L_1 and L_2. Then the point which lies on L_3 is

(1) (–1, 7, 4) (2) (1, –7, 4) (3) (1, 7, –4) (4) (–1, –7, 4)

Ans. **[1]**

Sol. $L_1 : \vec{r} = (\hat{i} - \hat{j} + 2\hat{k}) + \lambda\ (\hat{i} - \hat{j} + 2\hat{k})$

$L_2 : \vec{r} = (\hat{j} - \hat{k}) + \mu\,(3\hat{i} + \hat{j} + p\hat{k})$

$L_3 : \vec{r} = \delta(\ell\hat{i} + m\hat{j} + n\hat{k})$

$L_1 \perp L_2$

$3 - 1 + 2P = 0$

$\Rightarrow P = -1$

$L_3 \perp L_1 \Rightarrow \ell - m + 2n = 0$ …..(1)

$L_2 \perp L_3 \Rightarrow 3\ell + m + nP = 0$

$3\ell + m - n = 0$ …..(2)

So, we need to check point which satisfy both equations

(–1, 7, 4) satisfies the line L_3

Q.73 Consider the system of linear equations

$x + y + z = 5$, $x + 2y + \lambda^2 z = 9$, $x + 3y + \lambda z = \mu$, where $\lambda, \mu \in R$. Then, which of the following statement is **NOT** correct?

(1) System has unique solution if $\lambda \neq 1$ and $\mu \neq 13$

(2) System is inconsistent if $\lambda = 1$ and $\mu \neq 13$

(3) System has infinite number of solutions if $\lambda = 1$ and $\mu = 13$

(4) System is consistent if $\lambda \neq 1$ and $\mu = 13$

Ans. **[1]**

Sol. Given equation: $x + y + z = 5$

$x + 2y + \lambda^2 z = 9$

$x + 3y + \lambda z = \mu$

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & \lambda^2 \\ 1 & 3 & \lambda \end{vmatrix}$$

$= -(2\lambda + 1)(\lambda + 1)$

For $\lambda = \frac{-1}{2}$ the equation reduces to

$x + y + z = 5$ …(i)

$4x + 8y + z = 36$ …(ii)

$2x + 6y - z = 2\mu$ …(iii)

Taking z = k in (i) and (ii) and solving for x and y

gives $x = 1 - \frac{7k}{4}$ and $y = 4 + \frac{3k}{4}$

Putting in (iii) gives $\mu = 13$

Hence, for $\lambda = \frac{-1}{2}$ $(\lambda \neq 1)$ & $\mu \neq 13$

The system of equation has no solution.

Hence, statement gives in option (1) is not correct.

Q.74 Let $f(x) = (x + 3)^2 (x - 2)^3$, $x \in [-4, 4]$. If M and m are the maximum and minimum values of f, respectively in $[-4, 4]$, then the value of M – m is

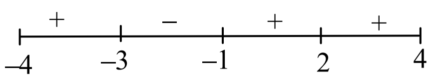
(1) 108 (2) 608 (3) 600 (4) 392

Ans. **[2]**

Sol. $f(x) = (x + 3)^2 (x - 2)^3 \ \forall\ x \in [4, 4]$

$f'(x) = 5(x - 2)^2 (x + 1)(x + 3)$

Sign variation of $f'(x)$ on $x \in [-4, 4]$



By first derivative test

Points of local minimum are

$x = -4$ and $x = -1$

$f(-4) = -216$ and $f(-1) = -108$

Points of local maximum are

$x = -3$ and $x = 4$

$f(-3) = 0$ and $f(4) = 392$

Hence, M = 392 and m = –216

M – m = 608

Q.75 If $x^2 - y^2 + 2hxy + 2gx + 2fy + c = 0$ is the locus of a point, which moves such that it is always equidistant from the lines $x + 2y + 7 = 0$ and $2x - y + 8 = 0$, then the value of $g + c + h - f$ equals

(1) 29 (2) 8

(3) 14 (4) 6

Ans. **[3]**

Sol. Locus of the point which moves equidistant from given lines $x + 2y + 7 = 0$ and $2x - y + 8 = 0$

$$\left(\frac{x+2y+7}{\sqrt{1^2+2^2}}\right)^2 = \left(\frac{2x-y+8}{\sqrt{2^2+1^2}}\right)^2$$

On simplifying

$$x^2 - y^2 + \left(-\frac{8}{3}\right)xy + \left(\frac{18}{3}\right)x + \left(\frac{-44}{3}\right)y + 5 = 0$$

Given, $x^2 - y^2 + 2hxy + 2gx + 2fy + c = 0$

Hence, $h = -\frac{4}{3}$, $g = \frac{9}{3}$, $f = \frac{-22}{3}$, $c = 5$

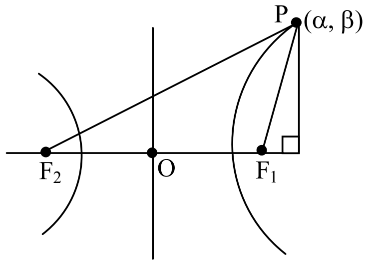
$\therefore\ g + c + h - f = 14$

Q.76 Let P be a point on the hyperbola $H : \frac{x^2}{9} - \frac{y^2}{4} = 1$, in the first quadrant such that the area of triangle formed by P and the two foci of H is $2\sqrt{13}$. Then, the square of the distance of P from the origin is

(1) 22 (2) 20 (3) 26 (4) 18

Ans. **[1]**

Sol.



$e = \sqrt{1+\frac{4}{9}} = \frac{\sqrt{13}}{3}$

$F_1 = (\sqrt{13}, 0)$

$F_2 = (-\sqrt{13}, 0)$

Area of $\Delta PF_1F_2 = 2\sqrt{13}$

$\Rightarrow \frac{1}{2} \cdot \beta(2\sqrt{13}) = 2\sqrt{13}$

$\Rightarrow \beta = 2$

(α, β) lie on hyperbola

$\Rightarrow \frac{\alpha^2}{9} - 1 = 1 \Rightarrow \alpha = \sqrt{18} = 3\sqrt{2}$

$\Rightarrow (\alpha, \beta) = (3\sqrt{2}, 2)$

(Distance of P from origin)$^2 = \left(\sqrt{18+4}\right)^2 = 22$

Q.77 Let a and b be two distinct real numbers. Let 11^{th} term of a GP, whose first term is a and third term is b, is equal to p^{th} term of another GP. Whose first term is a and fifth term is b. Then p is equal to

(1) 25 (2) 24 (3) 21 (4) 20

Ans. **[3]**

Sol. G.P first: $T_1 = a$

$T_3 = b$

$\Rightarrow T_3 = ar^2 = b$

$\Rightarrow r = \pm\sqrt{\frac{b}{a}}$

G. P Second: $T_1' = a$

$T_5' = b$

$\Rightarrow T_5' = ar_1^4 = b$

$\Rightarrow r_1 = \pm\left(\frac{b}{a}\right)^{\frac{1}{4}}$

$\Rightarrow T_{11} = ar^{10}$

$T_p' = ar_1^{p-1}$

$\Rightarrow T_{11} = T_p' \Rightarrow r^{10} = r_1^{p-1}$

$\Rightarrow \frac{b^5}{a^5} = \left(\frac{b}{a}\right)^{\frac{p-1}{4}} \Rightarrow \frac{p-1}{4} = 5 \Rightarrow p = 21$

Q.78 Suppose $2 - p, p, 2 - \alpha, \alpha$ are the coefficients of four consecutive terms in the expansion of $(1 + x)^n$. Then the value of $p^2 - \alpha^2 + 6\alpha + 2p$ equals

(1) 8 (2) 4 (3) 10 (4) 6

Ans. **[Bonus]**

Sol. Let 4 consecutive terms

$\Rightarrow {}^nC_r, {}^nC_{r+1}, {}^nC_{r+2}, {}^nC_{r+3}$

$\left.\begin{matrix} 2-p = {}^nC_r \\ p = {}^nC_{r+1} \end{matrix}\right\}$ and $2 - \alpha = {}^nC_{r+2}$

$\alpha = {}^nC_{r+3}$

$\Rightarrow {}^nC_r + {}^nC_{r+1} = 2$

Similarly, ${}^nC_{r+2} + {}^nC_{r+3} = 2$

$\Rightarrow {}^{n+1}C_{r+1} = 2$

$\Rightarrow {}^{n+1}C_{r+3} = 2$

$\Rightarrow {}^{n+1}C_{r+1} = {}^{n+1}C_{r+3} \Rightarrow r + 1 = r + 3$

$\Rightarrow$ absurd

$\Rightarrow (r + 1) + (r + 3) = n + 1$

$\Rightarrow n = 2r + 3$

$\Rightarrow {}^{2r+4}C_{r+1} = 2 \Rightarrow$ no such r exists

Data inconsistent

Q.79 For $\alpha, \beta \in (0, \pi/2)$, let $3\sin(\alpha + \beta) = 2\sin(\alpha - \beta)$ and a real number k be such that $\tan\alpha = k\tan\beta$. Then, the value of k is equal to

(1) 2/3 (2) – 5

(3) 5 (4) –2/3

Ans. **[Bonus]**

Sol. $3\sin(\alpha + \beta) = 2\sin(\alpha - \beta)$

$$\frac{\sin(\alpha+\beta)}{\sin(\alpha-\beta)} = \frac{2}{3}$$

$$\frac{\sin(\alpha+\beta)+\sin(\alpha-\beta)}{\sin(\alpha+\beta)-\sin(\alpha-\beta)} = \frac{2+3}{2-3}$$

$$\frac{2\sin\alpha\cos\beta}{2\sin\beta\cos\alpha} = -5$$

$\Rightarrow \tan\alpha = -5\tan\beta$

Not possible because $\alpha, \beta \in (0, \frac{\pi}{2}) \Rightarrow \tan\alpha, \tan\beta > 0$

$\Rightarrow$ Data inconsistent

Q.80 Let $f : R \to R$ be defined as $f(x) = ae^{2x} + be^x + cx$. If $f(0) = -1$, $f'(\log_e 2) = 21$ and $\int_0^{\log_e 4} (f(x) - cx)dx = \frac{39}{2}$, then the value of $|a + b + c|$ equals

(1) 12 (2) 16 (3) 10 (4) 8

Ans. **[4]**

Sol. $f(x) = ae^{2x} + be^x + cx$

$f(0) = a + b = -1$ …(1)

$f'(x) = 2ae^{2x} + be^{x} + c$

$f'(\ln 2) = 2ae^{\ln 4} + be^{\ln 2} + c$

$= 8a + 2b + c = 21$ …(2)

$$\Rightarrow \int_0^{\ln 4}(f(x) - cx)dx = \int_0^{\ln 4}(ae^{2x} + be^{x})dx$$

$$= \frac{ae^{2x}}{2} + be^{x}\Big|_0^{\ln 4} = \left(\frac{a}{2}(16) + b(4)\right) - \left(\frac{a}{2} + b\right)$$

$$= 8a + 4b - \frac{a}{2} - b = \frac{15a}{2} + 3b = \frac{39}{2}$$

$\Rightarrow 15a + 6b = 39$ …(3)

Using (1), (2) and (3)

$a = 5, b = -6, c = -7 \Rightarrow |a + b + c| = 8$

Section-B: Numerical Value Type Questions: This section contains 10 Numerical based questions. Attempt any 5 questions out of 10. The answer to each question should be rounded-off to the nearest integer.

Q.81 In an examination of Mathematics paper, there are 20 questions of equal marks and the questions paper is divided into three sections: A, B and C. A student is required to attempt total 15 questions taking at least 4 questions from each section. If section A has 8 questions, section B has 6 questions and section C has 6 questions then the total number of ways a student can select 15 questions is ________.

Ans. **[11376]**

Sol.

A	B	C	$\Rightarrow$	No. of question	*Total marks*
4	5	6	$\to$	${}^8C_4\,{}^6C_5\,{}^6C_5$	$6 \times {}^8C_4$
4	6	5	$\to$	${}^8C_4\,{}^6C_6\,{}^6C_5$	$6 \times {}^8C_4$
7	4	4	$\to$	${}^8C_7\,{}^6C_4\,{}^6C_4$	$8 \times (15)^2$
6	5	4	$\to$	${}^8C_6\,{}^6C_5\,{}^6C_4$	$28 \times 6 \times 15$
6	4	5	$\to$	${}^8C_5\,{}^6C_4\,{}^6C_5$	$28 \times 15 \times 6$
5	5	5	$\to$	${}^8C_5\,{}^6C_5\,{}^6C_5$	${}^8C_5 \times 36$
5	6	4	$\to$	${}^8C_5\,{}^6C_6\,{}^6C_4$	${}^8C_5 \times 15$
5	4	6	$\to$	${}^8C_5\,{}^6C_4\,{}^6C_6$	${}^8C_5 \times 15$

Total ways of select = 11376

Q.82 The number of real solution of the equation $x(x^2 + 3|x| + 5|x - 1| + 6|x - 2| = 0$ is _____.

Ans. **[1]**

Sol. $x = 0$ and $x^2 + 3|x| + 5|x - 1| + 6|x - 2| = 0$

(all the terms are positive so no real values)

only solution $x = 0$

No. of solution = 1

Q.83 Let S_n be the sum to n-terms of an arithmetic progression 3, 7, 11, …… . If $40 < \left(\frac{6}{n(n+1)}\sum_{k=1}^{n} S_k\right) < 42$, then n equals _____.

Ans. **[9]**

Sol. $S_k = 3 + 7 + 11 + \ldots$ up to k term

$= \frac{k}{2}[6 + (k - 1)4] = k(2k + 1)$

$\sum_{k=1}^{n}(2k^2 + k) = \frac{2n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}$

$40 < 2(2n + 1) + 3 < 42$

$35 < 4n < 37$

$8.75 < n < 9.25$

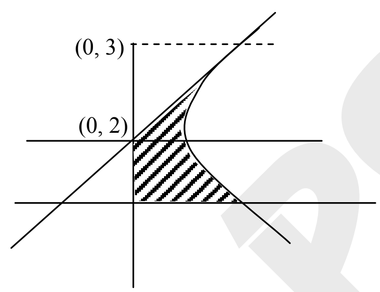
$\Rightarrow n = 9$

Q.84 The area of the region enclosed by the parabola $(y - 2)^2 = x - 1$, the line $x - 2y + 4 = 0$ and the positive coordinate axes is ______.

Ans. **[5]**

Sol. Solving $y - 2 = \frac{x}{2}$ and $(y - 2)^2 = x - 1$

We will get $x = 2$, $y = 3$



$\text{Area} = \int_0^3 (x_P - x_L)\, dy$

$= \int_0^3 ((y-2)^2 + 1 - 2y + 4)\, dy$

$\int_0^3 (y-3)^2 dy = \left.\frac{(y-3)^3}{3}\right]_0^3 = 9$

$\therefore$ Required area $= 9 - \frac{1}{2} \times 4 \times 2 = 5$

Q.85 The number of symmetric relations defined on the set {1, 2, 3, 4} which are not reflexive is ______.

Ans. **[960]**

Sol. A {1, 2, 3, 4,}

$n(A) = 4$

Number of symmetric relations $= 2^{\frac{n^2+n}{2}} = 2^{10}$ ($\because$ n = 4)

Number of reflexive and symmetric relations $= 2^{\frac{n^2-n}{2}} = 2^6$ ($\because$ n = 4)

$\therefore$ Number of relations which is symmetric but not reflexive $= 2^{10} - 2^6 = 960$

Q.86 Let $\alpha = \sum_{k=0}^{n}\left(\frac{({}^nC_k)^2}{k+1}\right)$ and $\beta = \sum_{k=0}^{n-1}\left(\frac{{}^nC_k\,{}^nC_{k+1}}{k+2}\right)$. If $5\alpha = 6\beta$, then n equals _____.

Ans. **[10]**

Sol. $\alpha = \sum_{k=0}^{n}\frac{{}^nC_k\cdot{}^nC_k}{k+1}\times\frac{n+1}{n+1}$

$= \frac{1}{n+1}\sum_{k=0}^{n}{}^{n+1}C_{k+1}\cdot{}^nC_{n-k}$ $\qquad \because \left(\frac{n+1}{k+1}\cdot{}^nC_k = {}^{n+1}C_{k+1}\right)$

$\alpha = \frac{1}{n+1}\,{}^{2n+1}C_{n+1}$

$\beta = \sum_{k=0}^{n-1}\frac{{}^nC_k\cdot{}^nC_{k+1}}{k+2}\times\frac{n+1}{n+1}$

$= \frac{1}{n+1}\sum_{k=0}^{n-1}{}^nC_{n-k}\cdot{}^{n+1}C_{k+2}$

$= \frac{1}{n+1}\,{}^{2n+1}C_{n+2}$

$\Rightarrow \frac{\beta}{\alpha} = \frac{{}^{2n+1}C_{n+2}}{{}^{2n+1}C_{n+1}} = \frac{2n+1-(n+2)+1}{n+2}$

$\frac{\beta}{\alpha} = \frac{n}{n+2} = \frac{5}{6}$

$\Rightarrow n = 10$

Q.87 Let a line passing through the point (–1, 2, 3) intersect the lines $L_1 : \frac{x-1}{3} = \frac{y-2}{2} = \frac{z+1}{-2}$ at M(α, β, γ) and $L_2 : \frac{x+2}{-3} = \frac{y-2}{-2} = \frac{z-1}{4}$ at N(a, b, c). Then the value of $\frac{(\alpha+\beta+\gamma)^2}{(a+b+c)^2}$ equals _____.

Ans. **[196]**

Sol. $M \equiv (\alpha, \beta, \gamma) \equiv (3\lambda + 1, 2\lambda + 2, -2\lambda - 1)$

$N \equiv (a, b, c) \equiv (-3t - 2, -2t + 2, 4t + 1)$

A(–1, 2, 3)

Direction ratios of line passing through A (–1, 2, 3) are $3\lambda + 2, 2\lambda, -2\lambda - 4$ or $-3t - 1, -2t, 4t - 2$

$\Rightarrow \frac{3\lambda+2}{-3t-1} = \frac{2\lambda}{-2t} = \frac{-2\lambda-4}{4t-2}$

$\Rightarrow \lambda = 4, t = 2$

$\Rightarrow$ M(13, 10, –9), N(–8, –2 ,9)

$\Rightarrow \left(\frac{\alpha+\beta+\gamma}{a+b+c}\right)^2 = 196$

Q.88 The variance σ^2 of the data

x_i	0	1	5	6	10	12	17
f_i	3	2	3	2	6	3	3

is ______.

Ans. **[29]**

Sol. $\bar{x} = \dfrac{3\times0+2\times1+3\times5+2\times6+6\times10+3\times12+3\times17}{3+2+3+2+6+3+3}$

$= 8$

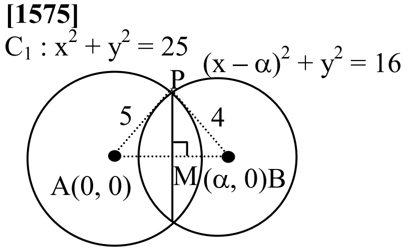
$\sigma^2 = \dfrac{1}{22}(3\times(8-0)^2+2\times(8-1)^2+3\times(8-5)^2+2\times(8-6)^2+6\times(8-10)^2+3\times(8-12)^2+3\times(8-17)^2)$

$= \dfrac{1}{22}(640) = 29.09$

Q.89 Consider two circles $C_1 : x^2 + y^2 = 25$ and $C_2 : (x - \alpha)^2 + y^2 = 16$, where $\alpha \in (5, 9)$. Let the angle between the two radii (one to each circle) drawn from one of the intersection points C_1 and C_2 be $\sin^{-1}\left(\dfrac{\sqrt{63}}{8}\right)$. If the length of common chord of C_1 and C_2 is β, then the value of $(\alpha\beta)^2$ equals ______.

Ans. **[1575]**

Sol.



$\because \sin\theta = \dfrac{\sqrt{63}}{8}$

area of $\Delta ABP = \dfrac{1}{2}\times\alpha\times\dfrac{\beta}{2} = \dfrac{1}{2}\times5\times4\times\sin\theta$

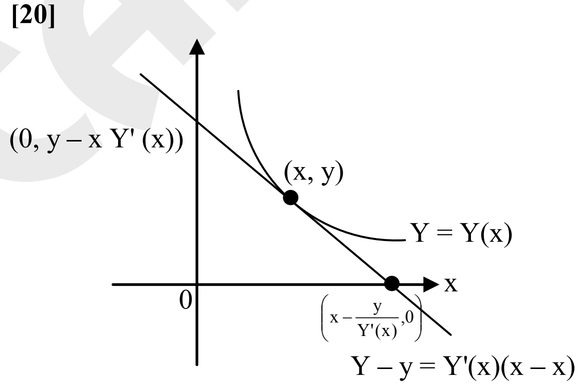
$\alpha\beta = 5\sqrt{63} \Rightarrow (\alpha\beta)^2 = 25\times63 = 1575$

Q.90 Let $Y = Y(X)$ be a curve lying in the first quadrant such that the area enclosed by the line $Y - y = Y'(x)(X - x)$ and the co-ordinate axes, where (x, y) is any point on the curve, is always $\dfrac{-y^2}{2Y'(x)} + 1$, $Y'(x) \neq 0$.

If $Y(1) = 1$, then $12\,Y(2)$ equals _____.

Ans. **[20]**

Sol.



$$\Rightarrow \frac{1}{2}(y - xY'(x))\left(x - \frac{y}{Y'(x)}\right) = -\frac{y^2}{2Y'(x)} + 1$$

$$(y - x\,Y'(x))\,(x\,Y'(x) - y) = -y^2 + 2Y'(x)$$

$$yxY'(x) - y^2 - x^2(Y'(x)) + xyY'(x) = -y^2 + 2Y'(x)$$

$$\left(y - x\frac{dy}{dx}\right)\left(x\frac{dy}{dx} - y\right) = -y^2 + 2\frac{dy}{dx}$$

$$-y^2 - x^2\left(\frac{dy}{dx}\right)^2 + 2xy\left(\frac{dy}{dx}\right) = -y^2 + 2\frac{dy}{dx}$$

$$-x^2\frac{dy}{dx} + 2xy = 2$$

$$\Rightarrow \frac{dy}{dx} + \left(-\frac{2}{x}\right)y = \left(-\frac{2}{x^2}\right)$$

$$\text{I.F.} = \frac{1}{x^2}$$

$\Rightarrow$ Solution is $\frac{y}{x^2} = -\frac{-2\cdot x^{-3}}{-3} + c$ $\qquad$ $\because y(1) = 1$

$$\Rightarrow c = \frac{1}{3}$$

$\frac{y}{x^2} = \frac{2}{3x^3} + \frac{1}{3}$, when $x = 2$, $y = 4\left(\frac{1}{12} + \frac{1}{3}\right) = \frac{5}{3}$

$$\Rightarrow 12y(2) = 20$$