JEE Main Online Exam 2024

Questions & Solution

30th January 2024 | Morning

PHYSICS

Section-A: This section contains 20 multiple choice questions. Each question has 4 choices(1), (2), (3) and (4), out of which **ONLY ONE** is correct..

Q.1 The electrostatic potential due to an electric dipole at a distance 'r' varies as :

(1) $\frac{1}{r^3}$ (2) $\frac{1}{r^2}$ (3) r (4) $\frac{1}{r}$

Ans. **[2]**

Sol. For dipole

$V = \frac{kp}{r^2}$

$\Rightarrow V \propto \frac{1}{r^2}$

Q.2 The work function of a substance is 3.0 eV. The longest wavelength of light that can cause the emission of photoelectrons from this substance is approximately;

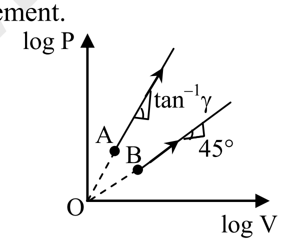
(1) 414 nm (2) 400 nm (3) 215 nm (4) 200 nm

Ans. **[1]**

Sol. $\lambda_{th} = \frac{h_c}{\phi_0} = \frac{1240}{3}\text{nm}$

= 414 nm

Q.3 Two thermodynamical processes are shown in the figure. The molar heat capacity for process A and B are C_A and C_B. The molar heat capacity at constant pressure and constant volume are represented by C_P and C_V, respectively. Choose the correct statement.



(1) $C_A > C_P > C_V$ (2) $C_B = \infty, C_A = 0$

(3) $C_A = 0$ and $C_B = \infty$ (4) $C_P > C_V > C_A = C_B$

Ans. **[None]**

Sol. $PV^{-x} = 0$

where x is slope

For A, x = 1, $C_A = C_V + \frac{R}{2}$

For B, x = λ, $C_B = C_V + \frac{R}{\gamma + 1}$

None of the options are matching.

Q.4 The diffraction pattern of a light of wavelength 400 nm diffracting from a salt of width 0.2 mm is focused on the focal plane of a convex lens of focal length 100 cm. The width of the 1st secondary

(1) 0.2 mm (2) 2 mm (3) 0.02 mm (4) 2 cm

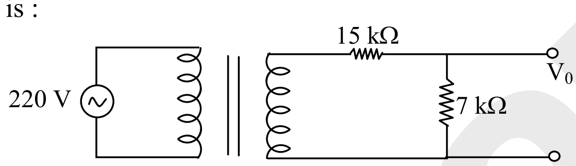
Ans. **[2]**

Sol. Angular width = $\frac{\lambda}{a}$

$\therefore$ Width = $\frac{\lambda f}{a}$

$= \frac{400\times10^{-9}\times1}{0.2\times10^{-3}} = 2$ mm

Q.5 Primary coil of a transformer is connected to 220 V ac. Primary and secondary turns of the transforms are 100 and 10 respectively. Secondary coil of transformer is connected to two series resistances shown in figure. The output voltage (V_0) is :



(1) 15 V (2) 22 V (3) 44 V (4) 7 V

Ans. **[4]**

Sol. $\frac{E_2}{E_1} = \frac{N_2}{N_1}$

$\therefore E_2 = 22$ V

$\therefore V_0 = 22 - \left(\frac{15}{22}\right) 22 = 7$ V

Q.6 The ratio of the magnitude of the kinetic energy to the potential energy of an electron in the 5th excited state of a hydrogen atom is :

(1) 4 (2) 1 (3) $\frac{1}{4}$ (4) $\frac{1}{2}$

Ans. **[4]**

Sol. $\therefore KE = \frac{1}{2}(-PE)$

$\frac{|KE|}{|PE|} = \frac{1}{2}$

Q.7 A particle of mass m is projected with a velocity 'u' making an angle of 30° with the horizontal. The magnitude of angular momentum of the projectile about the point of projection when the particle is at its maximum height h is :

(1) Zero (2) $\frac{mu^3}{\sqrt{2}g}$ (3) $\frac{\sqrt{3}}{16}\frac{mu^3}{g}$ (4) $\frac{\sqrt{3}}{2}\frac{mu^2}{g}$

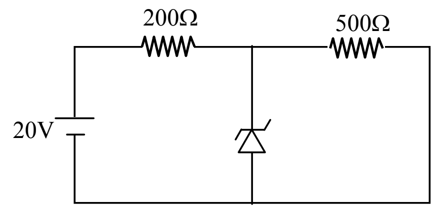
Ans. **[3]**

Sol. $L = (mu\cos\theta) H$

$H = \frac{u^2\sin^2\theta}{2g}$

$L = \frac{mu^3}{2g}\sin^2\theta\cos\theta = \frac{\sqrt{3}}{16}\frac{mu^3}{g}$

Q.8 A Zener diode of breakdown voltage 10 V is used as a voltage regulator as shown in the figure. The current through the Zener diode is:



(1) 30 mA (2) 20 mA (3) 50 mA (4) 0

Ans. **[1]**

Sol. $i_{500} = \frac{10}{500} = 20$ mA

$i_{200} = \frac{10}{200} = 50$ mA

$\therefore i_z = 30$ mA

Q.9 Match List-I with List-II.

	List-I		List-II
(A)	Coefficient of viscosity	(I)	$[ML^2T^{-2}]$
(B)	Surface tension	(II)	$[ML^2T^{-1}]$
(C)	Angular momentum	(III)	$[ML^{-1}T^{-1}]$
(D)	Rotational kinetic energy	(IV)	$[ML^0T^{-2}]$

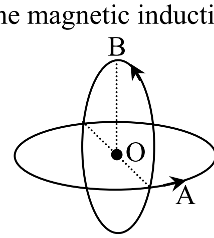
Choose the correct answer from the options given below:

(1) (A)-(I), (B)-(II), (C)-(III), (D)-(IV)
(2) (A)-(III), (B)-(IV), (C)-(II), (D)-(I)
(3) (A)-(IV), (B)-(III), (C)-(II), (D)-(I)
(4) (A)-(II), (B)-(I), (C)-(IV), (D)-(III)

Ans. **[2]**

Sol. $[\eta] = [ML^{-1}T^{-1}]$
$[S] = [MT^{-2}]$
$[L] = [ML^2T^{-1}]$
$[KE] = [ML^2T^{-2}]$

Q.10 Two insulated circular loop A and B of radius 'a' carrying a current of 'I' in the anti clockwise direction as shown in the figure. The magnitude of the magnetic induction at the centre will be:



(1) $\frac{\sqrt{2}\mu_0 I}{a}$ (2) $\frac{\mu_0 I}{2a}$ (3) $\frac{\mu_0 I}{\sqrt{2}a}$ (4) $\frac{2\mu_0 I}{a}$

Ans. **[3]**

Sol. B_1 & B_2 are perpendicular

$\therefore B_{eq} = \frac{\sqrt{2}\mu_0 I}{2a} = \frac{\mu_0 I}{\sqrt{2}a}$

Q.11 A series L.R circuit connected with an ac source E = (25 sin 1000 t) V has a power factor of $\frac{1}{\sqrt{2}}$. If the source of emf is changed to E = (20 sin 2000 t) V, the new power factor of the circuit will be:

(1) $\frac{1}{\sqrt{7}}$ (2) $\frac{1}{\sqrt{2}}$ (3) $\frac{1}{\sqrt{5}}$ (4) $\frac{1}{\sqrt{3}}$

Ans. **[3]**

Sol. For LR circuit

$$\cos\phi = \frac{R}{\sqrt{(x_L)^2 + R^2}} = \frac{1}{\sqrt{2}}$$

$R = x_L$

When $\omega' = 2\omega$

$x'_L = 2x_L$

$$\therefore \cos\phi' = \frac{R}{\sqrt{4x_L^2 + R^2}} = \frac{1}{\sqrt{5}}$$

Q.12 A spherical body of mass 100 g is dropped from a height of 10 m from the ground. After hitting the ground, the body rebounds to a height of 5 m. The impulse of force imparted by the ground to the body is given by: (given, g = 9.8 m/s^2)

(1) 43.2 kg ms^{-1} (2) 23.9 kg ms^{-1} (3) 2.39 kg ms^{-1} (4) 4.32 kg ms^{-1}

Ans. **[3]**

Sol. $V_1 = \sqrt{2gh_1}$

$V_2 = \sqrt{2gh_2}$

$\Delta V = (V_1 + V_2)$

$\therefore I = m\Delta V$

$= 0.1 \times \sqrt{2g}\left(\sqrt{h_1} + \sqrt{h_2}\right)$

= 2.39 kg m/s

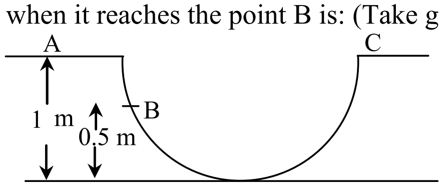
Q.13 Young's modulus of material of a wire length 'L' and cross-sectional area A is Y. If the length of the wire is doubled and cross-sectional area is halved then Young's modulus will be:

(1) Y (2) 2Y (3) $\frac{Y}{4}$ (4) 4Y

Ans. **[1]**

Sol. Young's modulus is a property of material, it is independent of dimension.

Q.14 A particle is placed at the point A of a frictionless track ABC as shown in figure. It is gently pushed towards right. The speed of the particle when it reaches the point B is: (Take g = 10 m/s^2)



(1) $2\sqrt{10}$ m/s (2) 10 m/s (3) $\sqrt{10}$ m/s (4) 20 m/s

Ans. **[3]**

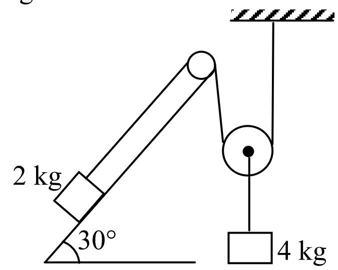
Sol. By conservation of mechanical energy

$\frac{1}{2}mv^2 = mg(\Delta h)$

$V = \sqrt{20 \times 0.5}$

$= \sqrt{10}$ m/s

Q.15 All surfaces shown in figure are assumed to be frictionless and all the pulleys and the string are light. The acceleration of the block of mass 2 kg is:



(1) g (2) $\frac{g}{2}$ (3) $\frac{g}{4}$ (4) $\frac{g}{3}$

Ans. **[4]**

Sol. $4g - 2T = 4\left(\frac{a}{2}\right)$

$\Rightarrow T = (2g - a)$

Now,

$T - 2g\sin 30° = 2a$

$\Rightarrow a = \frac{g}{3}$

Q.16 At which temperature the r.m.s. velocity of a hydrogen molecule equal to that of an oxygen molecule at 47° C?

(1) 80 K (2) 20 K (3) 4 K (4) –73 K

Ans. **[2]**

Sol. $V_{rms} = \sqrt{\frac{3RT}{M}}$

$\therefore \frac{T}{2} = \frac{(273+47)}{32}$

$\Rightarrow T = 20$ K

Q.17 The gravitational potential at a point above the surface of earth is -5.12×10^7 J/kg and the acceleration due to gravity at that point is 6.4 m/s^2 . Assume that the mean radius of earth to be 6400 km. The height of this point above the earth's surface is:

(1) 540 km (2) 1200 km (3) 1600 km (4) 1000 km

Ans. **[3]**

Sol. $V = \frac{-GM}{r}$ and $g = \frac{GM}{r^2}$

$\Rightarrow r = \frac{-V}{g} = \frac{5.12\times10^7}{6.4} = 8000\,\text{km}$

$\Rightarrow h = r - 6400 = 1600$ km

Q.18 The electric field of an electromagnetic wave in free space is represented at $\vec{E} = E_0 \cos(\omega t - kz)\,\hat{i}$ .The corresponding magnetic induction vector will be:

(1) $\vec{B} = E_0 C \cos(\omega t + kZ)\,\hat{j}$

(2) $\vec{B} = E_0 C \cos(\omega t - kZ)\,\hat{j}$

(3) $\vec{B} = \frac{E_0}{C}\cos(\omega t - kz)\,\hat{j}$

(4) $\vec{B} = \frac{E_0}{C}\cos(\omega t + kz)\hat{j}$

Ans. [3]

Sol. $\vec{E}\times\vec{B}$ is along + z axis

and $B = \frac{E_0}{C}$

$\therefore\ \vec{B} = \frac{E_0}{C}\cos(\omega t - kz)\ \hat{j}$

Q.19 An electric toaster has resistance of 60 Ω at room temperature (27°C). The toaster is connected to a 220 V supply. If the current flowing through it reaches 2.75 A, the temperature attained by toaster is around: (if $\alpha = 2\times10^{-4}$/°C)

(1) 1667 °C (2) 694 °C (3) 1235 °C (4) 1694 °C

Ans. [4]

Sol. $R = \frac{V}{I} = \frac{220}{2.75} = 80\Omega$

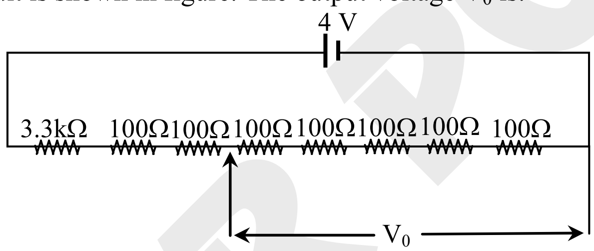
and, $R = R_0(1 + \propto \Delta T)$

$\Rightarrow 80 = 60(1 + 2\times10^{-4}\Delta T)$

$\Rightarrow \Delta T = 1667°C$

$\therefore\ T = 27 + \Delta T = 1694°C$

Q.20 A potential divider circuit is shown in figure. The output voltage V_0 is:



(1) 2 mV (2) 0.5 V (3) 4 V (4) 12 mV

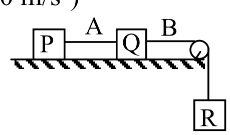
Ans. [2]

Sol. $V_{out} = = \frac{5\times100}{4000}\times4$

= 0.5 V

Section-B: Numerical Value Type Questions: This section contains 10 Numerical based questions. Attempt any 5 questions out of 10. The answer to each question should be rounded-off to the nearest integer.

Q.21 Each of three blocks P, Q and R shown in figure has a mass of 3 kg. Each of the wires A and B has cross-sectional area 0.005 cm^2 and Young's modulus 2×10^{11} N m^{-2}. Neglecting friction, the longitudinal strain on wire B is _________ $\times 10^{-4}$ (Take g = 10 m/s^2)



Ans. [2]

Sol. $a = \frac{3g}{9} = \frac{g}{3} m/s^2$

$T_A = ma = g$

$T_B - T_A = ma$

$T_B = 2g = 20$ N

$\left(\frac{\Delta L}{L}\right) = \frac{(T_B)}{AY} = \frac{20}{0.005\times10^{-4}\times2\times10^{11}} = 2\times10^{-4}$

Q.22 The distance between object and its two times magnified real image as produced by a convex lens is 45 cm. The focal length of the lens used is _______ cm.

Ans. **[10]**

Sol. $m = -2$

$v = -2u$

$v - u = 45$

$u = -15,\ v = 30$

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{f} = \frac{1}{30} - \frac{1}{-15}$$

$f = 10$ cm

Q.23 A capacitor of capacitance C and potential V has energy E. It is connected to another capacitor of capacitance 2C and potential 2V. Then the loss of energy is $\frac{x}{3}$ E, where x is __________.

Ans. **[2]**

Sol. $$V_{eq} = \frac{Q_1 + Q_2}{3C} = \frac{5V}{3}$$

$$E = \frac{1}{2}CV^2$$

$$E' = \frac{1}{2} \times 3C \times \frac{25}{9}V^2 = \frac{25}{6}CV^2$$

$$E_{Loss} = \frac{1}{2}CV^2 + 4CV^2 - \frac{25}{6}CV^2$$

$$= \frac{1}{3}CV^2$$

$$\therefore E_{Loss} = \frac{2E}{3}$$

Q.24 In a closed organ pipe, the frequency of fundamental note is 30 Hz. A certain amount of water is now poured in the organ pipe so that the fundamental frequency is increased to 110 Hz. If the organ pipe has a cross-sectional area of 2 cm^2, the amount of water poured in the organ tube is ________ g. (Take speed of sound in air is 330 m/s)

Ans. **[400]**

Sol. $$f = \frac{V}{4\ell}$$

$$\ell_1 = \frac{V}{4f_1} = \frac{11}{4}\text{m}$$

$$\ell_2 = \frac{V}{4f_2} = \frac{3}{4}\text{m}$$

$\ell_1 - \ell_2 = 2$ m

$m = V\rho = 2 \times 2 \times 10^{-4} \times 1000$

$= 0.4$ kg

Q.25 A electron of hydrogen atom on an excited state is having energy En = –0.85 eV. The maximum number of allowed transitions to lower energy level is ______.

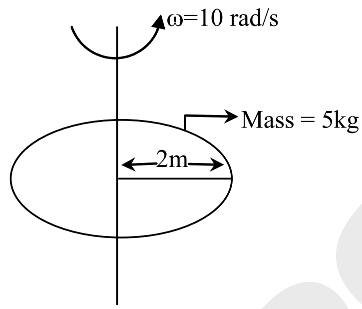
Ans. **[6]**

Sol. $E_n = \frac{-13.6}{n^2} = -0.85$

n = 4

$\therefore$ No. of transitions = $\frac{4 \times 3}{2} = 6$

Q.26



Consider a Disc of mass 5 kg, radius 2 m, rotating with angular velocity of 10 rad/s about an axis perpendicular to the plane of rotation. An identical disc is kept gently over the rotating disc along the same axis. The energy dissipated so that both the discs continue to rotate together without slipping is _______ J.

Ans. **[250]**

Sol. By conservation of angular momentum

$I\omega = 2I\omega'$

$\omega' = \frac{\omega}{2}$

$E_{loss} = \frac{1}{2}I\omega^2 - \frac{1}{2} \times 2I \times \frac{\omega^2}{4} = \frac{I\omega^2}{4}$

$I = \frac{MR^2}{2} = 10\,kg\,m^2$

$\therefore E_{loss} = \frac{10}{4} \times 100 = 250$ J

Q.27 The horizontal component of earth's magnetic field at a place is 3.5×10^{-5} T. A very long straight conductor carrying current of $\sqrt{2}$ A in the direction from South east to North west is placed. The force per unit length experienced by the conductor is _______ $\times 10^{-6}$ N/m.

Ans. **[35]**

Sol. $F = BI\ell \sin 45°$

$= 3.5 \times 10^{-5} \times \sqrt{2} \times 1 \times \frac{1}{\sqrt{2}}$

$= 35 \times 10^{-6}$ N

Q.28 The displacement and the increase in the velocity of a moving particle in the time interval of t to (t + 1) s are 125 m and 50 m/s, respectively. The distance travelled by the particle in $(t + 2)^{th}$ s is _______m.

Ans. **[175]**

Sol. $S_n = u + a\left(n - \frac{1}{2}\right)$

$S_{t+1} = u + a\left(t + 1 - \frac{1}{2}\right)$

$a = 50\ m/s^2$

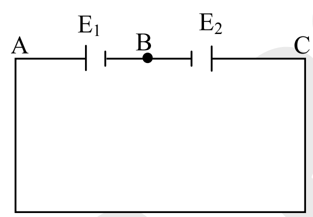
$125 = u + 50\left(t + \frac{1}{2}\right)$

$S_{t+2} = u + a\left(t + 2 - \frac{1}{2}\right)$

$= u + 50\left(t + \frac{3}{2}\right)$

$= 100 + 75 = 175$ m

Q.29 Two cells are connected in opposition as shown. Cell E_1 is of 8 V emf and 2 Ω internal resistance ; the cell E_2 is 2 V emf and 4 Ω internal resistance. The terminal potential difference of cell E_2 is __________V.



Ans. **[6]**

Sol. $i = \frac{E_1 - E_2}{r_1 + r_2} = 1A$

$\therefore V_2 = E_2 + ir_2 = 2 + 4 = 6$ V

Q.30 A ceiling fan having 3 blades of length 80 cm each is rotating with an angular velocity of 1200 rpm. The magnetic field of earth in that region is 0.5 G and angle of dip is 30°. The emf induced across the blades is N $\pi \times 10^{-5}$ V. The value of N is ______________.

Ans. **[32]**

Sol. $B_v = B\sin 30° = \frac{0.5}{2} \times 10^{-4}$ T

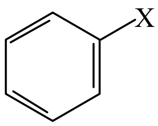
$E = \frac{(B_v)\omega\ell^2}{2}$

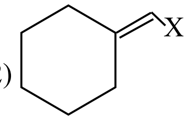
$= \frac{0.5}{4} \times 10^{-4} \times 40\pi \times 0.64 = 32\pi \times 10^{-5}$ V

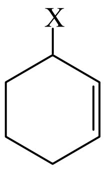
CHEMISTRY

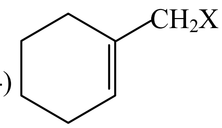
Section-A: This section contains 20 multiple choice questions. Each question has 4 choices(1), (2), (3) and (4), out of which **ONLY ONE** is correct..

Q.31 Example of vinylic halide is

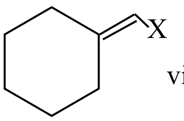
(1) 

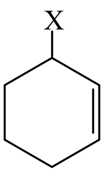
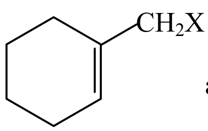
(2) 

(3) 

(4) 

Ans. **[2]**

Sol.  vinylic halide

 and  are allylic halide.

Q.32 Given below are two statements.

Statement (I) : The gas liberated on warming a salt with dil H_2SO_4, turns a piece of paper dipped in lead acetate into black, it is a confirmatory test for sulphide ion.

Statement (II) : In statement-I the colour of paper turns black because of formation of lead sulphite. In the light of the above statements, choose the **most appropriate** answer from the options given below

(1) Both **Statement I** and **Statement II** are false

(2) **Statement I** is true but **Statement II** is false

(3) Both **Statement I** and **Statement II** are true

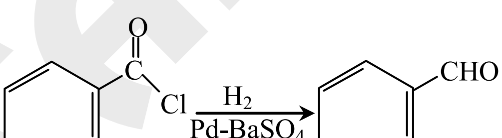
(4) **Statement I** is false but **Statement II** is true

Ans. **[2]**

Sol. H_2S turns pb $(CH_3COO)_2$ black.

$$Pb\,(CH_3\,COO)_2 + H_2S \longrightarrow \underset{\text{Black}}{PbS} + 2CH_3COOH$$

Statement I is true but **Statement II** is false.

Q.33 

This reduction reaction is known as

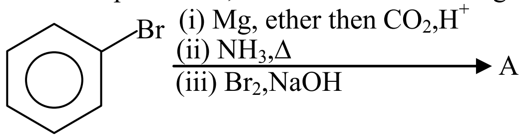
(1) Etard reduction
(2) Wolff-Kishner reduction
(3) Stephen reduction
(4) Rosenmund reduction

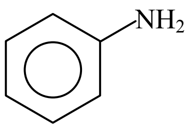
Ans. **[4]**

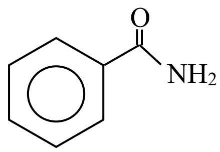
Sol. $C_6H_5COCl \xrightarrow{H_2\,/\,Pd\,/\,BaSO_4} C_6H_5CHO$

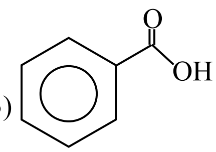
Rosenmund's reduction convert acid halides to aldehydes.

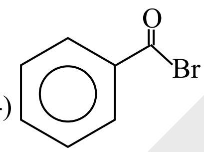
Q.34 The final product A, formed in the following multistep reaction sequence is



(1) 

(2) 

(3) 

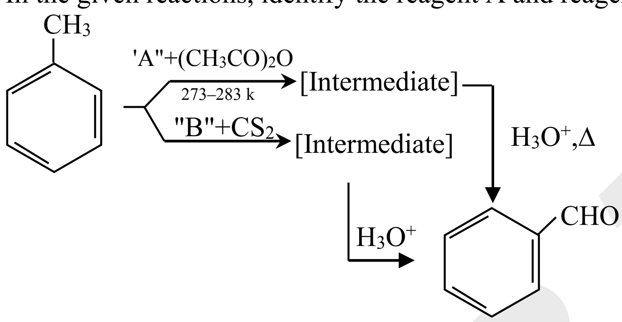
(4) 

Ans. **[1]**

Sol. $C_6H_5Br \xrightarrow[CO_2,\ H^+]{Mg,\ ether} C_6H_5COOH \xrightarrow{NH_3,\Delta} C_6H_5CONH_2$

$C_6H_5CONH_2 \xrightarrow{Br_2, NaOH} \underset{\text{Aniline}}{C_6H_5NH_2}$

Q.35 In the given reactions, identify the reagent A and reagent B.

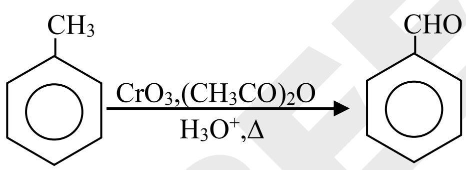


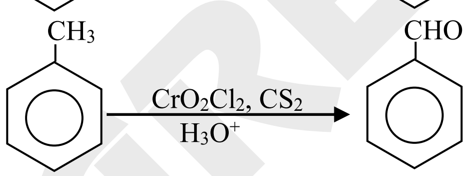
(1) A-CrO_2Cl_2, B-CrO_3 (2) A-CrO_3, B-CrO_2Cl_2

(3) A-CrO_2Cl_2, B-CrO_2Cl_2 (4) A-CrO_3, B-CrO_3

Ans. **[2]**

Sol.





A = CrO_3

B = CrO_2Cl_2

Q.36 Match **List-I** with **List-II**.

	List-I Species		**List-II Electronic distribution**
(A)	Cr^{+2}	(I)	$3d^8$
(B)	Mn^+	(II)	$3d^3 4s^1$
(C)	Ni^{+2}	(III)	$3d^4$
(D)	V^+	(IV)	$3d^5 4s^1$

Choose the **correct** answer from the options given below

(1) (A)-(II), (B)-(I), (C)-(IV), (D)-(III) (2) (A)-(III), (B)-(IV), (C)-(I), (D)-(II)

(3) (A)-(IV), (B)-(III), (C)-(I), (D)-(II) (4) (A)-(I), (B)-(II), (C)-(III), (D)-(IV)

Ans. **[2]**

Sol. $Cr^{+2} = 3d^4$

$Mn^{+} = 3d^5 4s^1$

$Ni^{+2} = 3d^8$

$V^{+} = 3d^3\ 4s^1$

Q.37 Diamagnetic Lanthanoid ions are

(1) Nd^{3+} & Ce^{4+} (2) La^{3+} & Ce^{4+} (3) Nd^{3+} & Eu^{3+} (4) Lu^{3+} & Eu^{3+}

Ans. **[2]**

Sol. $La^{+3} = 4f^{0}$

$Ce^{+4} = 4f^{0}$

Diamagnetic

Q.38 Sugar which does not give reddish brown precipitate with Fehling's reagent, is

(1) Maltose (2) Glucose (3) Lactose (4) Sucrose

Ans. **[4]**

Sol. Aldehyde (carbonyl) group is not free in sucrose.

Q.39 Match **List-I** with **List-II**.

	List-I Molecule		**List-II Shape**
(A)	BrF_5	(I)	T-shape
(B)	H_2O	(II)	See saw
(C)	ClF_3	(III)	Bent
(D)	SF_4	(IV)	Square pyramidal

Choose the **correct** answer from the options given below

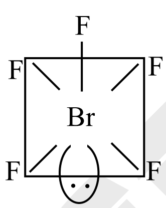
(1) (A)-(I), (B)-(II), (C)-(IV), (D)-(III) (2) (A)-(II), (B)-(I), (C)-(III), (D)-(IV)

(3) (A)-(III), (B)-(IV), (C)-(I), (D)-(II) (4) (A)-(IV), (B)-(III), (C)-(I), (D)-(II)

Ans. **[4]**

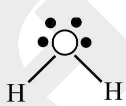
Sol. BrF_5

Electron pair = $\frac{7+5}{2} = 6$



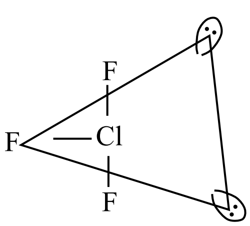
square pyramidal

H_2O



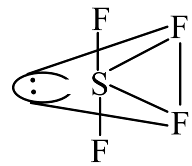
Bent

ClF_3 e^{-}pair = $\frac{7+3}{2} = 5$



T-shape

SF_4 e^-pair $= \frac{6+4}{2} = 5$



See saw

Q.40 Structure of 4-Methylpent-2-enal is

(1) $CH_3-CH_2-\underset{\underset{CH_3}{|}}{C}=CH-\overset{\overset{O}{||}}{C}-H$ (2) $CH_3-\underset{\underset{CH_3}{|}}{CH}-CH=CH-\overset{\overset{O}{||}}{C}-H$

(3) $H_2C=\underset{\underset{H}{|}}{C}-\overset{\overset{CH_3}{|}}{\underset{\underset{H}{|}}{CH}}-CH_2-\overset{\overset{O}{||}}{C}-H$ (4) $CH_3-CH_2-CH=\underset{\underset{CH_3}{|}}{C}-\overset{\overset{O}{||}}{C}-H$

Ans. **[2]**

Sol. $\overset{5}{C}H_3-\underset{\underset{CH_3}{|}}{\overset{4}{C}H}-\overset{3}{C}H=\overset{2}{C}H-\overset{\overset{O}{1||}}{C}-H$

4-Methylpent-2-enal

Q.41 Compound A formed in the following reaction reacts with B gives the product C. Find out A and B.

$CH_3-C\equiv CH + Na \longrightarrow A \xrightarrow{B} \underset{(C)}{CH_3-C\equiv C-CH_2}-\underset{\underset{CH_3}{|}}{CH_2} + NaBr$

(1) $A = CH_3-C\equiv \overset{-}{C}\overset{+}{Na}$, $B = CH_3-CH_2-CH_2-Br$

(2) $A = CH_3-C\equiv \overset{-}{C}\overset{+}{Na}$, $B = CH_3-CH_2-CH_3$

(3) $A = CH_3-CH_2-CH_3$, $B = CH_3-C\equiv CH$

(4) $A = CH_3-CH=CH_2$, $B = CH_3-CH_2-CH_2-Br$

Ans. **[1]**

Sol. $CH_3-C\equiv CH \xrightarrow{Na} \underset{(A)}{CH_3-C\equiv C^-Na^+}$

$CH_3-C\equiv C^-Na^+ + \underset{(B)}{CH_3CH_2CH_2Br} \longrightarrow CH_3-C\equiv C-CH_2CH_2CH_3$

Q.42 Choose the correct statements from the following :

(A) Ethane-1, 2-diamine is a chelating ligand.

(B) Metallic aluminium is produced by electrolysis of aluminium oxide in presence of cryolite.

(C) Cyanide ion is used as ligand for leaching of silver

(D) Phosphine act as a ligand in Wilkinson catalyst.

(E) The stability constants of Ca^{2+} and Mg^{2+} are similar with EDTA complexes.

Choose the **correct** answer from the options given below :

(1) (C), (D), (E) only (2) (A), (B), (C) only

(3) (A), (D), (E) only (4) (B), (C), (E) only

Ans. **[2]**

Sol. (A) $H_2N-CH_2-CH_2-NH_2$ is a chelating ligand.

(B) Aluminium is extracted by electrolysis of Al_2O_3 in the presence of cryolite.

(C) Cyanide ion form complex with silver $Ag_2S + CN^- \longrightarrow [Ag(CN)_2]^-$

(A), (B) and (C) are true.

Q.43 Given below are two statements
Statement (I) : The orbitals having same energy are called as degenerate orbitals.
Statement (II) : In hydrogen atom, 3p and 3d orbitals are not degenerate orbitals.
In the light of the above statements, choose the **most appropriate** answer from the options given below :
(1) Both **Statement I** and **Statement II** are true (2) Both **Statement I** and **Statement II** are false
(3) **Statement I** is true but **Statement II** is false (4) **Statement I** is false but **Statement II** is true

Ans. **[3]**

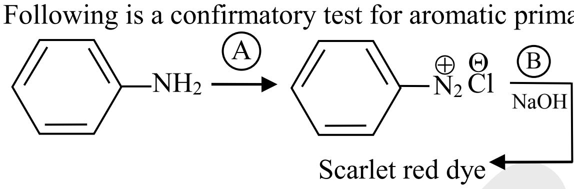
Sol. In hydrogen 3s, 3p and 3d orbitals have same energy.

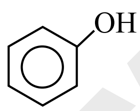
Q.44 Given below are two statements : one is labelled as **Assertion (A)** and the other is labelled as **Reason (R)**.
Assertion (A) : $CH_2 = CH - CH_2 - Cl$ is an example of allyl halide.
Reason (R) : Allyl halides are the compounds in which the halogen atom is attached to sp^2 hybridised carbon atom.
In the light of the above statements, choose the **most appropriate** answer from the options given below :
(1) Both **(A)** and **(R)** are true and **(R)** is the correct explanation of **(A)**
(2) **(A)** is false but **(R)** is true
(3) **(A)** is true but **(R)** is false
(4) Both **(A)** and **(R)** are true but **(R)** is not the correct explanation of **(A)**

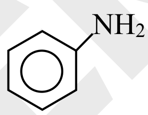
Ans. **[3]**

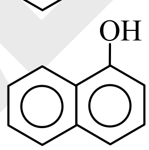
Sol. $CH_2 = CH - \overset{sp^3}{CH_2} - Cl$ is allyl halide.
Halogen is attached to sp^3 hybridised carbon in allyl halide.

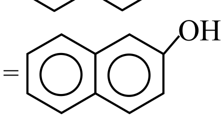
Q.45 Following is a confirmatory test for aromatic primary amines. Identify reagent (A) and (B).



(1) $A = HNO_3/H_2SO_4$; B = 

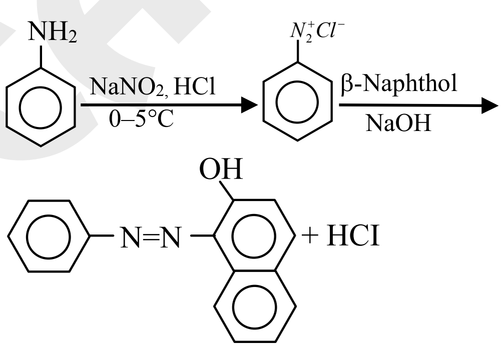
(2) $A = NaNO_2 + HCl$, 0 – 5°C; B = 

(3) $A = NaNO_2 + HCl$, 0 – 5°C; B = 

(4) $A = NaNO_2 + HCl$, 0 – 5° C; B = 

Ans. **[4]**

Sol.



Scarlet red dye

Q.46 The Lassaigne's extract is boiled with dil. HNO_3 before testing for halogens because,

(1) Silver halides are soluble in HNO_3
(2) Ag_2S is soluble in HNO_3
(3) Na_2S and NaCN are decomposed by HNO_3
(4) AgCN is soluble in HNO_3

Ans. **[3]**

Sol. The Lassaigne's extract is boiled with dil. HNO_3 because it decomposes Na_2S and NaCN, if formed

$NaCN + HNO_3 \longrightarrow NaNO_3 + HCN\uparrow$

$Na_2S + 2HNO_3 \longrightarrow 2NaNO_3 + H_2S$

Q.47 Given below are two statements : one is labelled as **Assertion (A)** and the other is labelled as **Reason (R)**.

Assertion (A) : There is a considerable increase in covalent radius from N to P. However from As to Bi only a small increase in covalent radius is observed.

Reason (R) : Covalent and ionic radii in a particular oxidation state increases down the group.

In the light of the above statements, choose the **most appropriate** answer from the options given below :

(1) Both **(A)** and **(R)** are true and **(R)** is not the correct explanation of **(A)**
(2) **(A)** is true but **(R)** is false
(3) Both **(A)** and **(R)** are true but **(R)** is the correct explanation of **(A)**
(4) **(A)** is false but **(R)** is true

Ans. **[1]**

Sol. There is considerable increase in covalent radius from N to P. However from As to Bi increment in covalent radii is very small due to presence of *d* and *f* orbital electron results in increase in effective nuclear charge.

Covalent and ionic radius at particular oxidation state generally increases down the group, due to increase in shell number.

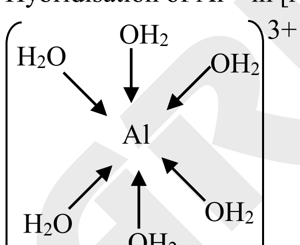
Q.48 Aluminium chloride in acidified aqueous solution forms an ion having geometry

(1) Octahedral
(2) Tetrahedral
(3) Square planar
(4) Trigonal bipyramidal

Ans. **[1]**

Sol. $AlCl_3 \xrightarrow[\text{Aq.solution}]{\text{Acidified}} [Al(H_2O)_6]^{3+}$

Hybridisation of Al^{+3} in $[Al(H_2O)_6]^{3+}$ is sp^3d^2 and geometry is octahedral.



Octahedral

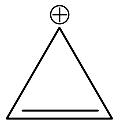
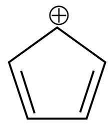
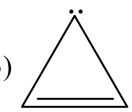
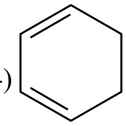
Q.49 What happens to freezing point of benzene when small quantity of naphthalene is added to benzene?

(1) First decreases and then increases
(2) Increases
(3) Decreases
(4) Remains unchanged

Ans. **[3]**

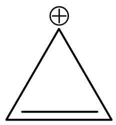
Sol. When small quantity of naphthalene is added to benzene freezing point decreases due to decrease in vapour pressure of solution.

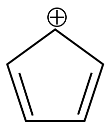
Q.50 Which of the following molecule/species is most stable?

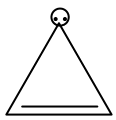
(1)  (2)  (3)  (4) 

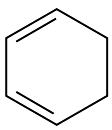
Ans. **[1]**

Sol. The compound cyclopropenium ion is most stable, because it is an aromatic compound

 ⇒ Aromatic

 ⇒ Anti aromatic

 ⇒Anti aromatic

 ⇒ Non aromatic

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Section-B: Numerical Value Type Questions: This section contains 10 Numerical based questions. Attempt any 5 questions out of 10. The answer to each question should be rounded-off to the nearest integer.

Q.51 On a thin layer chromatographic plate, an organic compound moved by 3.5 cm, while the solvent moved by 5 cm. The retardation factor of the organic compound is _______ $\times 10^{-1}$.

Ans. **[7]**

Sol. Retardation factor = $\frac{\text{Distance travelled by solute}}{\text{Distance travelled by solvent}}$

$R_f = \frac{3.5}{5} = 0.7$

$= 7 \times 10^{-1}$

Ans = 7

Q.52 $2MnO_4^- + bI^- + cH_2O \rightarrow xI_2 + yMnO_2 + z\overline{O}H$

If the above equation is balanced with integer coefficients, the value of z is _________.

Ans. **[8]**

Sol. $2MnO_4^- + bI^- + cH_2O \rightarrow xI_2 + yMnO_2 + z\overline{O}H$

Balanced chemical reaction is

$2MnO_4^{\ominus} + 6I^{\ominus} + 4H_2O \longrightarrow 3I_2 + 2MnO_2 + 8OH^{\ominus}$

z = 8

Q.53 0.05 cm thick coating of silver is deposited on a plate of 0.05 m^2 area. The number of silver atoms deposited on plate are _______ $\times 10^{23}$. (At mass Ag = 108, d = 7.9 g cm^{-3})

Ans. **[11]**

Sol. Given density of silver coating = 7.9 g/cm^3

Volume of silver coating = 0.05×10^4 cm^2 × 0.05 cm = 25 cm^3

$$d = \frac{\text{Mass of silver coating}}{\text{Volume of silver coating}}$$

$$7.9 = \frac{\text{Mass of silver coating}}{25\,\text{cm}^3}$$

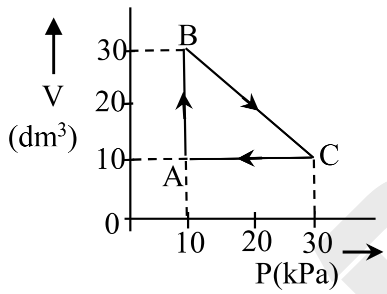
Mass of silver coating = 7.9 × 25 g = 197.5 g

Moles of silver coating = $\frac{197.5}{108}$ = 1.83 mol

Atoms of silver = $1.83 \times 6.022 \times 10^{23} = 11.02 \times 10^{23}$

Ans = 11

Q.54



An ideal gas undergoes a cyclic transformation starting from the point A and coming back to the same point by tracing the path A → B → C → A as shown in the diagram above. The total work done in the process is ______ J.

Ans. **[200]**

Sol. Work done = Area under curve

Work done $\frac{1}{2}$ × AC × AB

1 dm^3 = 0.001 m^3

AB = 20 dm^3 = 0.02 m^3

1 kPa = 1000 Nm^{-2}

20 kPa = 20000 Nm^{-2}

Work done = $\frac{1}{2}$ × 0.02 × 20000 J =200 J

Q.55 The mass of sodium acetate (CH_3COONa) required to prepare 250 mL of 0.35 M aqueous solution is _____ g. (Molar mass of CH_3COONa is 82.02 g mol^{-1})

Ans. **[7]**

Sol. Given volume of solution = 250 mL

Molarity of solution of CH_3COONa = 0.35 M

$$\text{Molarity} = \frac{\text{Moles of } CH_3COONa}{\text{Volume of solution (in mL)}} \times 1000$$

$$0.35 = \frac{\text{Moles of } CH_3COONa}{250} \times 1000$$

Moles of CH_3COONa = $\frac{0.35 \times 250}{1000}$ = 0.0875 mol

Mass of CH_3COONa = 0.0875 × 82.02 g = 7.18 g

Q.56 If IUPAC name of an element is "Unununnium" then the element belongs to n^{th} group of Periodic table. The value of n is _____.

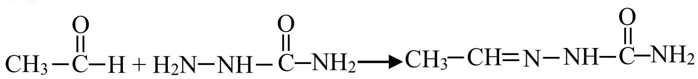
Ans. **[11]**

Sol. IUPAC name given is Unununnium
un = 1
un = 1
un = 1
Atomic number is 111 belongs to group 11^{th}.

Q.57 The compound formed by the reaction of ethanal with semicarbazide contains ______ number of nitrogen atoms.

Ans. **[3]**

Sol.



Ethanol Semicarbazide Semicarbazone

When ethanal reacts with semicarbazide, semi carbazone is formed via condensation, number of nitrogen atoms present in product is 3.

Q.58 The total number of molecular orbitals from 2s and 2p atomic orbitals of a diatomic molecule is _____.

Ans. **[8]**

Sol. 2s – 2s combine to form two molecular orbitals $\sigma 2_s$ and σ^*_{2s}
2p – 2p combine to form 6 molecular orbital

$\sigma_{2p_z}, \sigma^*_{2p_z}, \pi_{2p_x}, \pi_{2p_y}, \pi^*_{2p_x}, \pi^*_{2p_y}$

Total molecular orbitals formed are 6 + 2 = 8

Q.59 The rate of First order reaction is 0.04 mol L^{-1} s^{-1} at 10 minutes and 0.03 mol L^{-1} s^{-1} at 20 minutes after initiation. Half life of the reaction is ______ minutes. (Given log2 = 0.3010, log3 = 0.4771)

Ans. **[24]**

Sol. Rate of first order reaction = 0.04 mol $L^{-1}s^{-1}$ at time = 10 min
Rate of first order reaction = 0.03 mol L^{-1} s^{-1} at time = 20 min
$r_1 = 0.04$ mol $L^{-1}s^{-1}$
$r_2 = 0.03$ mol $L^{-1}s^{-1}$
$t_1 = 10$ min
$t_2 = 20$ min

$$k = \frac{2.303}{t_2 - t_1} \log \frac{r_1}{r_2}$$

$$k = \frac{2.303}{10} \log \frac{0.04}{0.03}$$

$k = 0.02876\ \text{min}^{-1}$

$$t_{\frac{1}{2}} = \frac{0.693}{k} = \frac{0.0693}{0.02876} = 24.09$$

≈ 24 min

Q.60 The pH at which $Mg(OH)_2$ [$K_{sp} = 1 \times 10^{-11}$] begins to precipitate from a solution containing 0. 10 M Mg^{2+} ions is ______ .

Ans. **[9]**

Sol. $Mg(OH)_2(s) \rightleftharpoons Mg^{+2}(aq) + 2OH^{\ominus}(aq)$ At limiting condition, for precipitation to be start K_{sp} $= Q_{sp}$
$K_{sp} = [Mg^{+2}]\,[OH^-]^2$
$1 \times 10^{-11} = [0.1]\,[OH^-]^2$

$$[OH^-] = \frac{10^{-11}}{0.1}$$

$10^{-10} = [OH^-]^2$
$[OH^-] = 10^{-5}$ M
pOH = –log $[OH^-]$ = 5
pH = 14 – 5 = 9

MATHEMATICS

Section-A: This section contains 20 multiple choice questions. Each question has 4 choices(1), (2), (3) and (4), out of which **ONLY ONE** is correct..

Q.61 If $f(x) = \begin{vmatrix} 2\cos^4 x & 2\sin^4 x & 3+\sin^2 2x \\ 3+2\cos^4 x & 2\sin^4 x & \sin^2 2x \\ 2\cos^4 x & 3+2\sin^4 x & \sin^2 2x \end{vmatrix}$, then $\frac{1}{5}f'(0)$ is equal to :

(1) 1 (2) 2 (3) 6 (4) 0

Ans. **[4]**

Sol. $f(x) = \begin{vmatrix} 2\cos^4 x & 2\sin^4 x & 3+\sin^2 2x \\ 3+2\cos^4 x & 2\sin^4 x & \sin^2 2x \\ 2\cos^4 x & 3+2\sin^4 x & \sin^2 2x \end{vmatrix}$

$$f'(x) = \begin{vmatrix} 8\cos^3 x(-\sin x) & 2\sin^4 x & 3+\sin^2 2x \\ 8\cos^3 x(-\sin x) & 2\sin^4 x & \sin^2 2x \\ 8\cos^3 x(-\sin x) & 3+2\sin^4 x & \sin^2 2x \end{vmatrix} + \begin{vmatrix} 2\cos^4 x & 8\sin^3 x\cos x & 3+\sin^2 2x \\ 3+2\cos^4 x & 8\sin^3 x\cos x & \sin^2 2x \\ 2\cos^4 x & 8\sin^3 x\cos x & \sin^2 2x \end{vmatrix}$$

$$+ \begin{vmatrix} 2\cos^4 x & 2\sin^4 x & 4\sin 2x\cos 2x \\ 3+2\cos^4 x & 2\sin^4 x & 4\sin 2x\cos 2x \\ 2\cos^4 x & 3+2\sin^4 x & 4\sin 2x\cos 2x \end{vmatrix}$$

$$f'(0) = \begin{vmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{vmatrix} + \begin{vmatrix} 2 & 0 & 0 \\ 5 & 0 & 0 \\ 2 & 0 & 0 \end{vmatrix} + \begin{vmatrix} 2 & 0 & 0 \\ 5 & 0 & 0 \\ 2 & 0 & 0 \end{vmatrix}$$

$f'(0) = 0$

$\therefore \frac{1}{5}f'(0) = 0$

Q.62 Let $f : \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \to \mathbf{R}$ be a differentiable function such that $f(0) = \frac{1}{2}$. If the $\lim\limits_{x\to 0} \frac{x\int_0^x f(t)dt}{e^{x^2}-1} = \alpha$, then $8\alpha^2$ is equal to :

(1) 1 (2) 16 (3) 4 (4) 2

Ans. **[4]**

Sol. $\lim\limits_{x\to 0} \frac{x\int_0^x f(t)dt}{e^{x^2}-1} = \alpha$ and $f(0) = \frac{1}{2}$.

$$\lim_{x\to 0} \frac{\int_0^x f(t)dt + x f(x)}{e^{x^2}(2x)} = \lim_{x\to 0} \frac{\int_0^x f(t)dt}{e^{x^2}(2x)} + \lim_{x\to 0} \frac{f(x)}{e^{x^2}(2)}$$

$$= \lim_{x\to 0} \frac{f(x)}{2(e^{x^2} + xe^{x^2}(2x))} + \frac{1}{4} = \frac{1}{2}$$

$\therefore \alpha = \frac{1}{2}$

$\therefore 8\alpha^2 = 8 \times \frac{1}{4} = 2$

Q.63 If $2\sin^3x + \sin2x\cos x + 4\sin x - 4 = 0$ has exactly 3 solutions in the interval $\left[0, \frac{n\pi}{2}\right]$, $n \in \mathbf{N}$, then the roots of the equation $x^2 + nx + (n - 3) = 0$ belong to:

(1) $(-\infty, 0)$ (2) $\mathbf{Z}$ (3) $\left(-\frac{\sqrt{17}}{2}, \frac{\sqrt{17}}{2}\right)$ (4) $(0, \infty)$

Ans. **[1]**

Sol. $2\sin^3x + \sin2x\cos x + 4\sin x - 4 = 0$

$2\sin^3x + 2\sin x\cos^2x + 4\sin x - 4 = 0$

$\Rightarrow \sin^3x + \sin x\,(1 - \sin^2x) + 2\sin x - 2 = 0$

$\Rightarrow \sin^3x + \sin x - \sin^3x + 2\sin x - 2 = 0$

$\Rightarrow 3\sin x - 2 = 0$

$\Rightarrow \sin x = \frac{2}{3}$

$\because$ It has exactly three solution in the interval $\left[0, \frac{n\pi}{2}\right]$, $n \in \mathbf{N}$

$\Rightarrow n = 5$

$\Rightarrow x^2 + 5x + 2 = 0$

$\Rightarrow x = \frac{-5 \pm \sqrt{25-8}}{2}$

$\Rightarrow x = \frac{-5 \pm \sqrt{17}}{2} \in (-\infty, 0)$

$\Rightarrow$ Roots belongs in the interval $(-\infty, 0)$

Q.64 If the domain of the function $f(x) = \cos^{-1}\left(\frac{2-|x|}{4}\right) + \left(\log_e\{3-x\}\right)^{-1}$ is $[-\alpha, \beta) - \{\gamma\}$, then $\alpha + \beta + \gamma$ is equal to

(1) 8 (2) 12 (3) 9 (4) 11

Ans. **[4]**

Sol. $f(x) = \cos^{-1}\left(\frac{2-|x|}{4}\right) + \{\log_e(3 - x)\}^{-1}$

$-1 \le \frac{2-|x|}{4} \le 1$

$\Rightarrow -4 \le 2 - |x| \le 4$

$\Rightarrow -4 \le |x| - 2 \le 4$

$\Rightarrow -2 \le |x| \le 6$

$|x| \le 6$

$x \in [-6, 6]$ …(1)

also, $3 - x \ne 1$

$x \ne 2$ …(2)

and $3 - x > 0$

$\Rightarrow x < 3$ …(3)

From (1), (2) and (3)

$\Rightarrow x \in [-6, 3) - \{2\}$

$\Rightarrow \alpha = 6, \beta = 3, \gamma = 2$

$\alpha + \beta + \gamma = 6 + 3 + 2 = 11$

So, option (4) is correct

Q.65 Let M denote the median of the following frequency distribution

Class	0-4	4-8	8-12	12-16	16-20
Frequency	3	9	10	8	6

Then 20M is equal to:

(1) 52 (2) 104 (3) 208 (4) 416

Ans. **[3]**

Sol.

x_i	f_i	C.f
0-4	3	3
4-8	9	12
8-12	10	22
12-16	8	30
16-20	6	36

$N = \sum f_i = 36$

$\left(\frac{N}{2}\right) = \frac{36}{2} = 18$

So, we have median lies in the class 8-12

$\therefore\ l_1 = 8,\ f = 10,\ h = 4,\ C.f = 12$

Here, we apply formula

$$M = l_1 + \frac{\frac{N}{2} - C.f}{f} \times h$$

$$= 8 + \frac{18 - 12}{10} \times 4$$

$$= 8 + \frac{12}{5} = \frac{52}{5}$$

$\therefore\ 20M = 4 \times 52$

$= 208$

Q.66 If the circles $(x + 1)^2 + (y + 2)^2 = r^2$ and $x^2 + y^2 - 4x - 4y + 4 = 0$ intersect at exactly two distinct points, then

(1) $\frac{1}{2} < r < 7$ (2) $3 < r < 7$ (3) $5 < r < 9$ (4) $0 < r < 7$

Ans. **[2]**

Sol. $\because$ Circles intersect at two distinct points,

$\Rightarrow |r_1 - r_2| < c_1c_2 < r_1 + r_2$

$\Rightarrow |r - 2| < \sqrt{9+16} < r + 2$

$\Rightarrow |r - 2| < 5$ and $r + 2 > 5$

$\therefore\ -5 < r - 2 < 5$ and $r > 3$

$\Rightarrow -3 < r < 7$ and $r > 3$

$\therefore\ 3 < r < 7$

Q.67 Two integers x and y are chosen with replacement from the set {0, 1, 2, 3, …, 10}. Then the probability that $|x - y| > 5$, is:

(1) $\frac{62}{121}$ (2) $\frac{60}{121}$ (3) $\frac{30}{121}$ (4) $\frac{31}{121}$

Ans. **[3]**

Sol. If x = 0, y = 6, 7, 8, 9, 10
If x = 1, y = 7, 8, 9, 10
If x = 2, y = 8, 9, 10
If x = 3, y = 9, 10
If x = 4, y = 10
If x = 5, y = no possible value
Total possible ways = (5 + 4 + 3 + 2 + 1) × 2 = 30

Required probability = $\frac{30}{11 \times 11} = \frac{30}{121}$

Q.68 If the length of the minor axis of an ellipse is equal to half of the distance between the foci, then the eccentricity of the ellipse is:

(1) $\frac{1}{\sqrt{3}}$ (2) $\frac{2}{\sqrt{5}}$ (3) $\frac{\sqrt{3}}{2}$ (4) $\frac{\sqrt{5}}{3}$

Ans. **[2]**

Sol. $\because$ ae = 2b

$\therefore \frac{4b^2}{a^2} = e^2$

or $4(1 - e^2) = e^2$

$\therefore 4 = 5e^2$

$\Rightarrow e = \frac{2}{\sqrt{5}}$

Q.69 Let A(2, 3, 5) and C(–3, 4, –2) be opposite vertices of a parallelogram ABCD. If the diagonal $\overrightarrow{BD} = \hat{i} + 2\hat{j} + 3\hat{k}$, then the area of the parallelogram is equal to:

(1) $\frac{1}{2}\sqrt{306}$ (2) $\frac{1}{2}\sqrt{474}$ (3) $\frac{1}{2}\sqrt{586}$ (4) $\frac{1}{2}\sqrt{410}$

Ans. **[2]**

Sol. Given opposite vertices of parallelogram.
A(2, 3, 5) and C(–3, 4, –2)

$\overrightarrow{CA}(d_1) = 5\hat{i} - \hat{j} + 7\hat{k}$

Given $\overrightarrow{BD}(d_2) = \hat{i} + 2\hat{j} + 3\hat{k}$

$\therefore$ As area of parallelogram

$= \frac{1}{2}|\vec{d}_1 \times \vec{d}_2|$

Now, $\vec{d}_1 \times \vec{d}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 5 & -1 & 7 \\ 1 & 2 & 3 \end{vmatrix}$

$= \hat{i}(-3 - 14) - \hat{j}(15 - 7) + \hat{k}(10 + 1)$

$= -17\hat{i} - 8\hat{j} + 11\hat{k}$

$|\vec{d}_1 \times \vec{d}_2| = \sqrt{(-17)^2 + (-8)^2 + (11)^2}$

$= \sqrt{289 + 64 + 121} = \sqrt{474}$

$\therefore \frac{1}{2}|\vec{d}_1 \times \vec{d}_2| = \frac{1}{2}\sqrt{474}$

Q.70 The value of $\lim_{n\to\infty}\sum_{k=1}^{n}\frac{n^3}{(n^2+k^2)(n^2+3k^2)}$ is:

(1) $\frac{\pi}{8(2\sqrt{3}+3)}$ (2) $\frac{(2\sqrt{3}+3)\pi}{24}$ (3) $\frac{13\pi}{8(4\sqrt{3}+3)}$ (4) $\frac{13(2\sqrt{3}-3)\pi}{8}$

Ans. **[3]**

Sol. $$\lim_{n\to\infty}\sum_{k=1}^{n}\frac{n^3}{n^4\left(1+\frac{k^2}{n^2}\right)\left(1+\frac{3k^2}{n^2}\right)}$$

$$\lim_{n\to\infty}\frac{1}{n}\sum_{k=1}^{n}\frac{1}{\left(1+\frac{k^2}{n^2}\right)\left(1+\frac{3k^2}{n^2}\right)}$$

$$=\int_0^1\frac{dx}{3(1+x^2)\left(\frac{1}{3}+x^2\right)}$$

$$=\frac{1}{3}\times\frac{3}{2}\int_0^1\frac{(x^2-1)-\left(x^2+\frac{1}{3}\right)}{(1+x^2)\left(\frac{1}{3}+x^2\right)}$$

$$=\frac{1}{2}\int_0^1\left[\frac{1}{x^2+\left(\frac{1}{\sqrt{3}}\right)^2}-\frac{1}{1+x^2}\right]dx$$

$$=\frac{1}{2}\left[\sqrt{3}\tan^{-1}(\sqrt{3}x)\right]_0^1-\frac{1}{2}(\tan^{-1}x)_0^1$$

$$=\frac{\sqrt{3}}{2}\left(\frac{\pi}{3}\right)-\frac{1}{2}\left(\frac{\pi}{4}\right)$$

$$=\frac{\pi}{2\sqrt{3}}-\frac{\pi}{8}$$

$$=\frac{\pi}{2}\left(\frac{1}{\sqrt{3}}-\frac{1}{4}\right)$$

$$=\frac{\pi}{2}\left(\frac{4-\sqrt{3}}{4\sqrt{3}}\right)\left(\frac{4+\sqrt{3}}{4+\sqrt{3}}\right)$$

$$=\frac{\pi}{2}\left(\frac{16-3}{4\sqrt{3}(4+\sqrt{3})}\right)$$

$$=\frac{13\pi}{8(4\sqrt{3}+3)}$$

Option (3) is correct

Q.71 Let S_n denote the sum of first n terms of an arithmetic progression. If $S_{20} = 790$ and $S_{10} = 145$, then $S_{15} - S_5$ is:

(1) 410 (2) 390 (3) 405 (4) 395

Ans. **[4]**

Sol. Given, $S_{20} = 790$ and $S_{10} = 145$

$S_{20} = 790 = 10[2a + (19)d]$

$\Rightarrow 79 = 2a + 19d$ …(1)

and $S_{10} = 145 = 5[2a + 9d]$

$\Rightarrow 29 = 2a + 9d$ …(2)

Subtract equation (2) from equation (1)

$\Rightarrow 50 = 10d$

$d = 5$

Put it in equation (2)

$29 = 2a + 45$

$= -16 = 2a$

$\Rightarrow \quad a = -8$

$S_{15} = \frac{15}{2}[2(-8) + (14)5]$

$= \frac{15}{2}[-16 + 70]$

$= 27\cdot 15$

$= 405$

$S_5 = \frac{5}{2}[2(-8) + 4 \times 5]$

$= \frac{5}{2}[-16 + 20] = 10$

$S_{15} - S_5 = 395$

Q.72 Consider the system of linear equation $x + y + z = 4\mu$, $x + 2y + 2\lambda z = 10\mu$, $x + 3y + 4\lambda^2 z = \mu^2 + 15$, where $\lambda, \mu \in \mathbf{R}$. Which one of the following statements is **NOT** correct?

(1) The system has infinite number of solutions if $\lambda = \frac{1}{2}$ and $\mu = 15$

(2) The system is consistent if $\lambda \neq \frac{1}{2}$

(3) The system is inconsistent if $\lambda = \frac{1}{2}$ and $\mu \neq 1$

(4) The system has unique solution if $\lambda \neq \frac{1}{2}$ and $\mu \neq 1, 15$

Ans. **[3]**

Sol. Given system of equation is

$x + y + z = 4\mu$

$x + 2y + 2\lambda z = 10\mu$

$x + 3y + 4\lambda^2 z = \mu^2 + 15$

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 2\lambda \\ 1 & 3 & 4\lambda^2 \end{vmatrix} = (2\lambda - 1)^2$$

If $\lambda \neq \frac{1}{2}$ the system is consistent and has a unique solution.

If $\lambda = \frac{1}{2}$ the system reduces to

$x + y + z = 4\mu$ ...(i)

$x + 2y + z = 10\mu$ ...(ii)

$x + 3y + z = \mu^2 + 15$ ...(iii)

From (i) and (ii) $y = 6\mu$ and $x + z = -2\mu$.

Putting in (iii) gives $-2\mu + 18\mu = \mu^2 + 15$.

$\Rightarrow (\mu - 1)(\mu - 15) = 0$.

Hence for $\lambda = \frac{1}{2}$ and u = 1 or 15 we have consistent system with infinite number of solution.

Q.73 Let g : **R** → **R** be a non constant twice differentiable function such that $g'\left(\frac{1}{2}\right) = g'\left(\frac{3}{2}\right)$. If a real valued function f is defined as $f(x) = \frac{1}{2}[g(x) + g(2 - x)]$, then

(1) $f''(x) = 0$ for exactly one x in (0, 1)

(2) $f'\left(\frac{3}{2}\right) + f'\left(\frac{1}{2}\right) = 1$

(3) $f'(x) = 0$ for atleast two x in (0, 2)

(4) $f'(x) = 0$ for no x in (0, 1)

Ans. **[3]**

Sol. $f(x) = \frac{1}{2}[g(x) + g(2 - x)]$

$f'(x) = \frac{1}{2}[g'(x) - g'(2 - x)]$

Put $x = \frac{1}{2}$

$f'\left(\frac{1}{2}\right) = \frac{1}{2}[g'(1/2) - g'(3/2)]$

$= 0$ [Given]

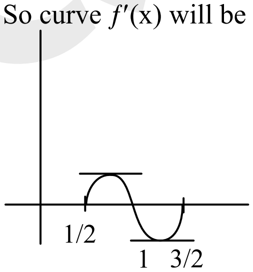
$f'(3/2) = 1/2[g'(3/2) - g'(1/2)]$

$= 0$

$f'(1) = 1/2[g'(1) - g'(1)]$

$= 0$

So curve $f'(x)$ will be like



So $f''(x) = 0$, atleast for 2 values of x in (0, 2)

Q.74 Let y = y(x) be the solution of the differential equation secx dy + {2(1 – x)tanx + x(2 – x)}dx = 0 such that y(0) = 2. Then y(2) is equal to :

(1) 2{sin(2) + 1} (2) 1 (3) 2{1 – sin(2)} (4) 2

Ans. [4]

Sol. Given differential equation can be written as

$dy = (2(x-1)\sin x + x(x-2)\cos x)dx.$

Integrating both sides.

$\Rightarrow \int dy = \int (2(x-1)\sin x + (x^2 - 2x)\cdot\cos x)dx$

$y(x) = \int 2(x-1)\sin x + (x^2 - 2x)\sin x - \int 2(x-1)\sin x + c$

$y(x) = (x^2 - 2x)\sin x + c$

$y(0) = 2 \Rightarrow c = 2$

Hence, $y(x) = (x^2 - 2x)\sin x + 2$

$y(2) = 2$

Q.75 A line passing through the point A(9, 0) makes angle of 30° with the positive direction of x-axis. If this line is rotated about A through an angle of 15° in the clockwise direction, then its equation in the new position is :

(1) $\frac{y}{\sqrt{3}-2} + x = 9$ (2) $\frac{y}{\sqrt{3}-2} + y = 9$ (3) $\frac{y}{\sqrt{3}+2} + x = 9$ (4) $\frac{y}{\sqrt{3}+2} + y = 9$

Ans. [1]

Sol. Inclination of line in new position = 15°.

$\Rightarrow$ Slope = tan15° = $2 - \sqrt{3}$

Required equation

$y - 0 = (2 - \sqrt{3})(x - 9)$

$\Rightarrow x + \frac{y}{\sqrt{3}-2} = 9$

Q.76 If z = x + iy, xy ≠ 0, satisfies the equation $z^2 + i\bar{z} = 0$, then $|z^2|$ is equal to :

(1) 4 (2) 9 (3) 1 (4) $\frac{1}{4}$

Ans. [3]

Sol. $z = x + iy, x \neq 0, y \neq 0 \Rightarrow |z| \neq 0$

$z^2 + i\bar{z} = 0$

$\Rightarrow z^2 = -i\bar{z}$

$\Rightarrow |z^2| = |-i\bar{z}| = |-i||\bar{z}|$

$\Rightarrow |z|^2 = |z| \Rightarrow |z|(|z| - 1) = 0$

$\Rightarrow |z| = 1$

Q.77 Let $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ be two vectors such that $|\vec{a}| = 1$ $\vec{a}\cdot\vec{b} = 2$ and $|\vec{b}| = 4$. If $\vec{c} = 2(\vec{a}\times\vec{b}) - 3\vec{b}$, then the angle between $\vec{b}$ and $\vec{c}$ is equal to :

(1) $\cos^{-1}\left(\frac{2}{3}\right)$ (2) $\cos^{-1}\left(\frac{2}{\sqrt{3}}\right)$ (3) $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$ (4) $\cos^{-1}\left(-\frac{1}{\sqrt{3}}\right)$

Ans. [3]

Sol. $|a|^2 |b|^2 = |a.b|^2 + |a \times \vec{b}|^2$

$\Rightarrow |\vec{a} \times \vec{b}|^2 = 16 - 4 = 12$

$\vec{c} = 2\vec{a} \times \vec{b} - 3\vec{b}$

$\vec{b}.\vec{c} = 2[a\ b\ b] - 3|b|^2$

$= 0 - 3(16) = -48$

$\vec{c}.\vec{c} = (2\vec{a} \times \vec{b} - 3\vec{b}).(2\vec{a} \times \vec{b} - 3\vec{b})$

$= (4(\vec{a} \times \vec{b})(\vec{a} \times \vec{b}) + 9|\vec{b}|^2 - 6[a\ b\ b])$

$|\vec{c}|^2 = (4|\vec{a} \times \vec{b}|^2 + 9 \times 16)$

$= (48 + 144) = 192$

Let angle between $\vec{b}$ and $\vec{c}$ is θ.

$\Rightarrow \vec{b}.\vec{c} = |\vec{b}||\vec{c}|\cos\theta = -48$

$\Rightarrow (4)\sqrt{192}\cos\theta = -48$

$$\Rightarrow \cos\theta = \frac{-48}{4 \times \sqrt{192}} = -\sqrt{\frac{48 \times 48}{16 \times 192}}$$

$$= -\sqrt{\frac{3}{4}} = \frac{-\sqrt{3}}{2}$$

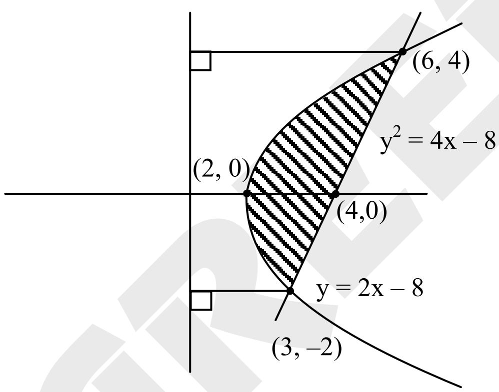
$$\theta = \cos^{-1}\left(\frac{-\sqrt{3}}{2}\right)$$

Q.78 The area (in square units) of the region bounded by the parabola $y^2 = 4(x - 2)$ and the line $y = 2x - 8$, is:

(1) 7 (2) 9 (3) 6 (4) 8

Ans. **[2]**

Sol.



Area along y-axis

$$\int_{-2}^{4}\left[\left(\frac{y+8}{2}\right) - \left(\frac{y^2+8}{4}\right)\right]dy$$

$$= \frac{y^2}{4} + 4y - \frac{y^3}{12} - 2y\Big|_{-2}^{4}$$

$$\Rightarrow \left(4 + 16 - \frac{16}{3} - 8\right) - \left(1 - 8 + \frac{8}{12} + 4\right)$$

$$\Rightarrow \left(12 - \frac{16}{3}\right) - \left(\frac{8}{12} - 3\right) = 15 - \left(\frac{64+8}{12}\right)$$

$= 15 - 6 = 9$ sq. unit

Q.79 The maximum area of a triangle whose one vertex is at (0, 0) and the other two vertices lie on the curve $y = -2x^2 + 54$ at points (x, y) and (–x, y) where y > 0, is :

(1) 122 (2) 92 (3) 88 (4) 108

Ans. **[4]**

Sol. Area of triangle

$$= \frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ x & y & 1 \\ -x & y & 1 \end{vmatrix}$$

$$= \frac{1}{2}|2xy| = |xy| \; y > 0$$

$\Rightarrow -2x^2 + 54 > 0$

$\Rightarrow x^2 - 27 < 0$

$\Rightarrow x \in (-3\sqrt{3}, 3\sqrt{3})$

Area $= |x(-2x^2 + 54)|$, $x \in (-3\sqrt{3}, 3\sqrt{3})$

$= |-2x^3 + 54x|$

$$\frac{dA}{dx} = -6x^2 + 54$$

$\Rightarrow \frac{dA}{dx}$ is zero at x = 3 or – 3

$$\Rightarrow \frac{d^2A}{dx^2} = -12x$$

at $x = 3 \; \frac{d^2A}{dx^2} < 0$

$\Rightarrow$ Maxima at x = 3

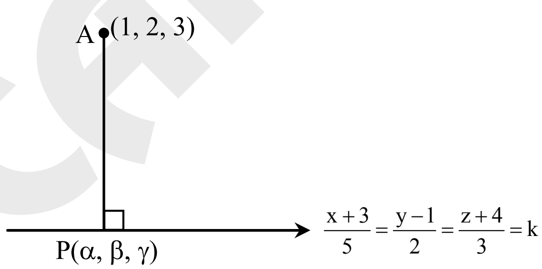
$\Rightarrow$ Area $= |3(-2 \times 9 + 54)| = |36 \times 3| = 108$

Q.80 Let (α, β, γ) be the foot of perpendicular from the point (1, 2, 3) on the line $\frac{x+3}{5} = \frac{y-1}{2} = \frac{z+4}{3}$. Then $19(\alpha + \beta + \gamma)$ is equal to :

(1) 101 (2) 100 (3) 102 (4) 99

Ans. **[1]**

Sol.



$\Rightarrow \alpha = 5k - 3$

$\beta = 2k + 1$

$\gamma = 3k - 4$

$\alpha + \beta + \gamma = 10k - 6$

AP $\perp$ to the line

$\Rightarrow (5\hat{i}+2\hat{j}+3\hat{k}).((\alpha-1)\hat{i}+(\beta-2)\hat{j}+(\gamma-3)\hat{k})=0$

$\Rightarrow 5(\alpha-1)+2(\beta-2)+3(\gamma-3)=0$

$\Rightarrow 5\alpha+2\beta+3\gamma-5-4-9=0$

$5(5k-3)+2(2k+1)+3(3k-4)=18$

$\Rightarrow k(25+4+9)-15+2-12-18=0$

$\Rightarrow 38k=43$

$$\alpha+\beta+\gamma=\left(10\times\frac{43}{38}-6\right)=\frac{5\times 43}{19}-6$$

$19(\alpha+\beta+\gamma)=5\times 43-19\times 6=215-114=101$

Section-B: Numerical Value Type Questions: This section contains 10 Numerical based questions. Attempt any 5 questions out of 10. The answer to each question should be rounded-off to the nearest integer.

Q.81 A group of 40 students appeared in an examination of 3 subjects – Mathematics, Physics and Chemistry. It was found that all students passed in atleast one of the subjects, 20 students passed in Mathematics, 25 students passed in Physics, 16 students passed in Chemistry, atmost 11 students passed in both Mathematics and Physics, atmost 15 students passed in both Physics and Chemistry, atmost 15 students passed in both Mathematics and Chemistry. The maximum number of students passed in all the three subjects is __________.

Ans. **[10]**

Sol. $n(M)=20$

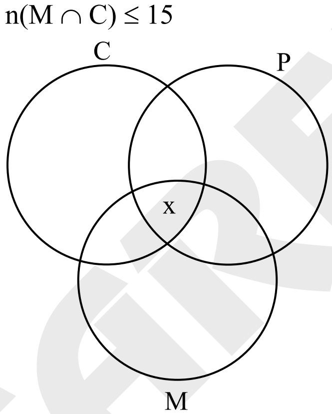
$n(P)=25$

$n(C)=16$

$n(M\cap P)\le 11$

$n(P\cap C)\le 15$

$n(M\cap C)\le 15$

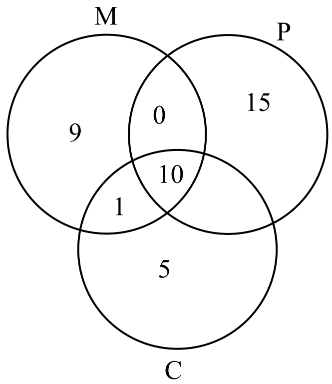


$11-x\ge 0$

$x\le 11$

x = 11 does not satisfied the data

x = 10



Q.82 The value of $9\int_0^9 \left[\sqrt{\frac{10x}{x+1}}\right] dx$, where [t] denotes the greatest integer less than or equal to t, is_______.

Ans. **[155]**

Sol. $9\int_0^9 \left[\sqrt{\frac{10x}{x+1}}\right] dx$

Let $I = \int_0^9 \left[\sqrt{\frac{10x}{x+1}}\right] dx$

$\frac{10x}{x+1} = 1 \Rightarrow x = \frac{1}{9}$

$\frac{10x}{x+1} = 4$

$\Rightarrow x = \frac{2}{3}$

$\frac{10x}{x+1} = 9$

$\Rightarrow x = 9$

$I = \int_0^{1/9} 0\,dx + \int_{1/9}^{2/3} 1\,dx + \int_{2/3}^{9} 2\,dx$

$= 0 + [x]_{1/9}^{2/3} + [2x]_{2/3}^{9}$

$I = \frac{2}{3} - \frac{1}{9} + 18 - \frac{4}{3}$

$\Rightarrow I = \frac{155}{9}$

$\Rightarrow 9I = 155$

Q.83 Number of integral terms in the expansion of $\left\{7^{\left(\frac{1}{2}\right)} + 11^{\left(\frac{1}{6}\right)}\right\}^{824}$ is equal to _______.

Ans. **[138]**

Sol. $\left\{7^{\frac{1}{2}} + 11^{\frac{1}{6}}\right\}^{824}$

Number of integral term

$$T_{r+1} = {}^{824}C_r\left(7^{\frac{1}{2}}\right)^{824-r}\left(11^{\frac{1}{6}}\right)^{r}$$

$\Rightarrow$ r must be multiple of 6

$\Rightarrow$ r = 0, 6, 12, ….822

$\Rightarrow$ 138 term

Q.84 Let A = {1, 2, 3, …., 7} and let P(A) denote the power set of A. If the number of functions f : A → P(A) such that $a \in f(a)$, $\forall$ a $\in$ A is m^n, m and n $\in$ **N** and m is least, then m + n is equal to ___________.

Ans. **[44]**

Sol. $n(P(A)) = 2^7$

$a \in f(a)$

$\Rightarrow f(a)$ will have (2^6) different subsets having a in them as choice

$\Rightarrow (2^6)^7 = 2^{42}$ function

$m^n = 2^{42}$

$\Rightarrow$ m + n = 44

Q.85 Let $\alpha = 1^2 + 4^2 + 8^2 + 13^2 + 19^2 + 26^2 + \ldots..$ up to 10 terms and $\beta = \sum_{n=1}^{10} n^4$. If $4\alpha - \beta = 55k + 40$, them k is equal to ______.

Ans. **[353]**

Sol. $\alpha = 1^2 + 4^2 + 8^2 + 13^2 + 19^2 + 26^2 + \ldots..$

$$T_n = \left(\frac{n^2+3n-2}{2}\right)^2$$

$$\sum T_n = \alpha = \sum_{n=1}^{10}\left(\frac{n^2+3n-2}{2}\right)^2$$

$$4\alpha = \sum_{n=1}^{10}(n^2+3n-2)^2$$

$$\beta = \sum_{n=1}^{10} n^4$$

$$4\alpha - \beta = \sum_{n=1}^{10}(9n^2+4+6n^3-12n-4n^2)$$

$$= \sum_{n=1}^{10}(6n^3+5n^2-12n+4)$$

$$= 6\sum_{n=1}^{10} n^3 + 5\sum_{n=1}^{10} n^2 - 12\sum_{n=1}^{10} n + 4\sum_{n=1}^{10} 1$$

$$= 6\times\left(\frac{10\times11}{2}\right)^2 + 5\left(\frac{10\times11\times21}{6}\right) - 12\left(\frac{10\times11}{2}\right) + 4\times10$$

$= 19455 = 55k + 40$

$\Rightarrow k = 353$

Q.86 If d_1 is the shortest distance between the lines x + 1 = 2y = –12z, x = y + 2 = 6z – 6 and d_2 is the shortest distance between the lines

$\frac{x-1}{2}=\frac{y+8}{-7}=\frac{z-4}{5}, \frac{x-1}{2}=\frac{y-2}{1}=\frac{z-6}{-3}$, then the value of $\frac{32\sqrt{3}d_1}{d_2}$ is :

Ans. **[16]**

Sol. $\frac{x+1}{-12}=\frac{y}{-6}=\frac{z}{1}, \frac{x}{6}=\frac{y+2}{6}=\frac{z-1}{1}$

$A_1(-1, 0, 0), A_2(0, -2, 1), \overrightarrow{A_1A_2}=\hat{i}-2\hat{j}+\hat{k}$

$\vec{n}_1=(-12\hat{i}-6\hat{j}+\hat{k}), \vec{n}_2=6\hat{i}+6\hat{j}+\hat{k}$

$$\vec{n}_1\times\vec{n}_2=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ -12 & -6 & 1\\ 6 & 6 & 1\end{vmatrix}=6(-2\hat{i}+3\hat{j}-6\hat{k})$$

$\Rightarrow |\vec{n}_1\times\vec{n}_2|=42$

$$d_1=\left|\frac{\overrightarrow{A_1A_2}.(\vec{n}_1\times\vec{n}_2)}{|\vec{n}_1\times\vec{n}_2|}\right|=\left|\frac{6(-2-6-6)}{7\times 6}\right|=2$$

$\because \frac{x-1}{2}=\frac{y+8}{-7}=\frac{z-4}{5}, \frac{x-1}{2}=\frac{y-2}{1}=\frac{z-6}{-3}$

$A_3=(1, -8, 4), A_4=(1, 2, 6), \overrightarrow{A_3A_4}=10\hat{j}+2\hat{k}$

$$\vec{n}_3\times\vec{n}_4=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & -7 & 5\\ 2 & 1 & -3\end{vmatrix}=16\hat{i}+16\hat{j}+16\hat{k}$$

$\Rightarrow |\vec{n}_3\times\vec{n}_4|=16\sqrt{3}$

$$d_2=\left|\frac{\overrightarrow{A_3A_4}.(\vec{n}_3\times\vec{n}_4)}{|\vec{n}_3\times\vec{n}_4|}\right|=\frac{16(10+2)}{16\sqrt{3}}=\frac{12}{\sqrt{3}}$$

$$\Rightarrow \frac{32\sqrt{3}\,d_1}{d_2}=\frac{32\sqrt{3}\times 2\times\sqrt{3}}{12}=16$$

Q.87 If the function $f(x)=\begin{cases}\frac{1}{|x|} & ,|x|\ge 2\\ ax^2+2b & ,|x|<2\end{cases}$ is differentiable on R, then 48(a + b) is equal to _________ .

Ans. **[15]**

Sol. $f(x)=\begin{cases}\frac{1}{|x|} & ,|x|\ge 2\\ ax^2+2b & ,|x|<2\end{cases}$

$$f(x)=\begin{cases}-\frac{1}{x}, & x\le -2\\ ax^2+2b, & -2<x<2\\ \frac{1}{x}, & x\ge 2\end{cases}$$

$$\Rightarrow f'(x) = \begin{cases} \frac{1}{x^2}, & x \le -2 \\ 2ax, & -2 < x < 2 \\ -\frac{1}{x^2}, & x \ge 2 \end{cases}$$

f(x) is continuous at x = –2 $\Rightarrow \frac{1}{2} = 4a + 2b$

f(x) is continuous at x = 2 $\Rightarrow \frac{1}{2} = 4a + 2b$

f(x) is differentiable at x = –2 $\Rightarrow \frac{1}{4} = -4a$

f(x) is differentiable at x = 2 $\Rightarrow 4a = -\frac{1}{4}$

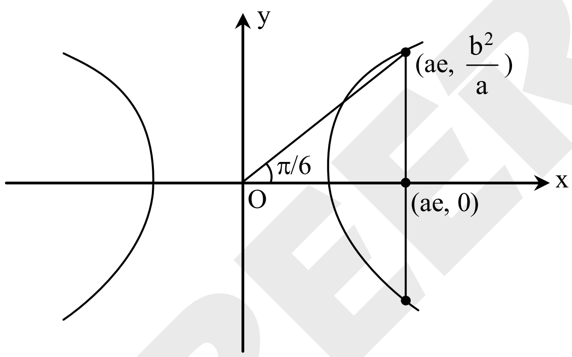
$\Rightarrow a = -\frac{1}{16}, b = \frac{3}{8}$

$\Rightarrow 48(a + b) = -3 + 18 = 15$

Q.88 Let the latus rectum of the hyperbola $\frac{x^2}{9} - \frac{y^2}{b^2} = 1$ subtend an angle of $\frac{\pi}{3}$ at the centre of the hyperbola. If b^2 is equal to $\frac{\ell}{m}(1 + \sqrt{n})$, where ℓ and m are co-prime numbers, then $\ell^2 + m^2 + n^2$ is equal to ___________.

Ans. **[182]**

Sol.



$\Rightarrow \tan\frac{\pi}{6} = \frac{b^2}{a^2e} \Rightarrow \frac{b^2}{9e} = \frac{1}{\sqrt{3}} \Rightarrow e = \frac{\sqrt{3}b^2}{9}$

$e^2 = 1 + \frac{b^2}{a^2}$

$e^2 = 1 + \frac{b^2}{9} \Rightarrow \frac{3b^4}{81} = 1 + \frac{b^2}{9}$

$\Rightarrow b^4 - 3b^2 - 27 = 0$

$b^2 = \frac{3 \pm \sqrt{9 + 108}}{2}$

$b^2 = \frac{3 + 3\sqrt{13}}{2} = \frac{3}{2}\left(1 + \sqrt{13}\right)$

$\Rightarrow \ell = 3, m = 2, n = 13$

$\Rightarrow \ell^2 + m^2 + n^2 = 182$

Q.89 Let y = y(x) be the solution of the differential equation $(1 - x^2)dy = \left[xy + (x^3+2)\sqrt{3(1-x^2)}\right]dx$, $-1 < x < 1$, y(0) = 0. If $y\left(\frac{1}{2}\right) = \frac{m}{n}$, m and n are co-prime numbers, then m + n is equal to ________.

Ans. **[97]**

Sol. $\frac{dy}{dx} + \left(\frac{x}{x^2-1}\right)y = \frac{(x^3+2)\sqrt{3}}{\sqrt{1-x^2}}$

I.F. = $e^{\int \frac{x}{x^2-1}} = e^{\frac{1}{2}\ln|x^2-1|} = \sqrt{1-x^2}$

Solution of D.E.

$y.\sqrt{1-x^2} = \int \sqrt{3}(x^3+2)dx + C$

$y.\sqrt{1-x^2} = \sqrt{3}\left(\frac{x^4}{4} + 2x\right) + C$

$\because$ y(0) = 0 $\Rightarrow$ C = 0

$y\sqrt{1-x^2} = \sqrt{3}\left(\frac{x^4}{4} + 2x\right)$

$\Rightarrow y\left(\frac{1}{2}\right) = \frac{\sqrt{3}\left(\frac{1}{64} + 1\right)}{\frac{\sqrt{3}}{2}} = \frac{65}{32}$

$\Rightarrow$ m + n = 97

Q.90 Let α, β ∈ N be roots of the equation $x^2 - 70x + \lambda = 0$. Where $\frac{\lambda}{2}, \frac{\lambda}{3} \notin$ **N**. If λ assumes the minimum possible value, then $\frac{(\sqrt{\alpha-1} + \sqrt{\beta-1})(\lambda+35)}{|\alpha-\beta|}$ is equal to :

Ans. **[60]**

Sol. $x^2 - 70x + \lambda = 0$

$\alpha + \beta = 70, \alpha\beta = \lambda$

$\because \frac{\lambda}{2}, \frac{\lambda}{3} \notin N, \alpha, \beta \in N$

$\Rightarrow$ λ is not divisible by 2 or 3

$\Rightarrow$ α, β not divisible by 2 or 3

$\lambda = \alpha(70-\alpha) \Rightarrow \frac{d\lambda}{d\alpha} = 70 - 2\alpha$

λ is increasing when α ≤ 35

$\Rightarrow$ λ is minimum when α = 5 or α = 65

When α = 1, β = 69 (divisible by 3) not possible

α ≠ 2, α ≠ 3, α ≠ 4

$\Rightarrow$ α = 5, β = 65

$\Rightarrow \frac{(\sqrt{\alpha-1} + \sqrt{\beta-1})(\lambda+35)}{|\alpha-\beta|} = \frac{(2+8).(5\times 65+35)}{60} = 60$