CAREER POINT

JEE Main Online Exam 2024

Questions & Solution

29th January 2024 | Evening

PHYSICS

Section-A: Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Q.1 A stone of mass 900 g is tied to a string and moved in a vertical circle of radius 1 m making 10 rpm. The tension in the string, when the stone is at the lowest point is (if $\pi^2 = 9.8$ and $g = 9.8\ m/s^2$)

(1) 9.8 N (2) 8.82 N

(3) 97 N (4) 17.8 N

Ans. **[1]**

Sol. $\omega = 10\text{rpm} = 10 \times \frac{\pi}{30} = \frac{\pi}{3}$ rad/s

$T - mg = m\omega^2 r$ $\quad$ m = 0.9 kg, r = 1 m

$T - 0.9 \times 9.8 = 0.9 \times \frac{\pi^2}{9} \times 1$

$T = 0.9 \times 9.8 \left\{1 + \frac{1}{9}\right\}$

$= 0.9 \times 9.8 \times \frac{10}{9}$

= 9.8 N

Q.2 The temperature of a gas having 2.0×10^{25} molecules per cubic meter at 1.38 atm (Given, $k = 1.38 \times 10^{-23}\ JK^{-1}$) is

(1) 300 K (2) 200 K (3) 500 K (4) 100 K

Ans. **[3]**

Sol. PV = NkT

$\Rightarrow P = \frac{N}{V}kT = nkT$

$\Rightarrow 1.38 \times 101325 = 2 \times 10^{25} \times 1.38 \times 10^{-23}\ T$

$\Rightarrow T = \frac{101325}{200} \approx 500\ K$

Q.3 If the distance between object and its two times magnified virtual image produced by a curved mirror is 15 cm, the focal length of the mirror must be

(1) – 12 cm (2) $\frac{10}{3}$ cm (3) – 10 cm (4) 15 cm

Ans. **[3]**

Sol. Let |u| = x

$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$ $\qquad$ $m=-\frac{v}{u}$

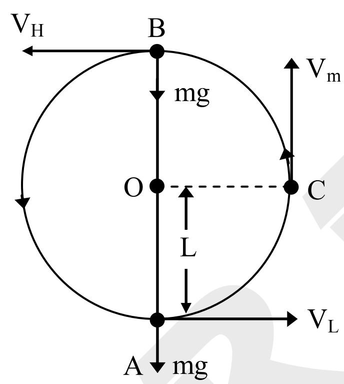
$\frac{1}{2x}-\frac{1}{x}=\frac{1}{f}$ $\qquad$ $2=\frac{-v}{-x}$

$-\frac{1}{2x}=\frac{1}{f}$ $\qquad$ v = 2x

f = –2x $\qquad$ Also 3x = 15 ⇒ x = 5 cm

f = – 10 cm

Q.4 A bob of mass 'm' is suspended by a light string of length 'L'. It is imparted a minimum horizontal velocity at the lowest point A such that it just completes half circle reaching the top most position B. The ratio of kinetic energies $\frac{(K.E)_A}{(K.E)_B}$ is



(1) 3 : 2 $\qquad$ (2) 2 : 5 $\qquad$ (3) 5 : 1 $\qquad$ (4) 1 : 5

Ans. **[3]**

Sol. $V_L=\sqrt{5gL}$ A lowest

$V_H=\sqrt{gL}$ B topmost

$$\frac{K_A}{K_B}=\frac{\frac{1}{2}m5gL}{\frac{1}{2}mgL}=\frac{5}{1}$$

Q.5 A particle is moving in a straight line. The variation of position 'x' as a function of time 't' is given as $x=(t^3-6t^2+20t+15)$ m. The velocity of the body when its acceleration becomes zero is

(1) 6 m/s $\qquad$ (2) 10 m/s $\qquad$ (3) 8 m/s $\qquad$ (4) 4 m/s

Ans. **[3]**

Sol. $x=t^3-6t^2+20t+15$

⇒ $v=3t^2-12t+20$

⇒ $a=6t-12$

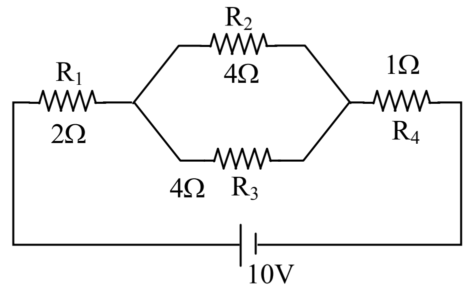
For a = 0 ⇒ t = 2

and $v_2=3\times2^2-12\times2+20$

= 12 – 24 + 20

= 8 m/s

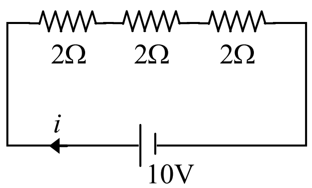
Q.6 In the given circuit, the current in resistance R_3 is :



(1) 1.5 A (2) 1 A (3) 2.5 A (4) 2 A

Ans. **[2]**

Sol.



$$i = \frac{10}{5} = 2A$$

$$\Rightarrow i_3 = \frac{i}{2} = 1A$$

Q.7 Two sources of light emit with a power of 200 W. The ratio of number of photons of visible light emitted by each source having wavelengths 300 nm and 500 nm respectively, will be :

(1) 1 : 5 (2) 3 : 5 (3) 5 : 3 (4) 1 : 3

Ans. **[2]**

Sol. $P = \frac{\Delta n}{\Delta t} E_{Ph}$ and $E_{Ph} \propto \frac{1}{\lambda}$

$$\Rightarrow \frac{\Delta n_1}{\Delta t} \times \frac{1}{300} = \frac{\Delta n_2}{\Delta t} \times \frac{1}{500}$$

$$\Rightarrow \frac{\Delta n_1}{\Delta n_2} = \frac{3}{5}$$

Q.8 A plane electromagnetic wave of frequency 35 MHz travels in free space along the X-direction. At a particular point (in space and time) $\vec{E} = 9.6\hat{j}$ V/m. The value of magnetic field at this point is :

(1) $9.6\hat{j}$ T (2) $9.6\times10^{-8}\hat{k}$ T (3) $3.2\times10^{-8}\hat{i}$ T (4) $3.2\times10^{-8}\hat{k}$ T

Ans. **[4]**

Sol. E = cB

$$\Rightarrow B = \frac{9.6}{3\times10^8} = 3.2 \times 10^{-8} \text{ T}$$

Q.9 N moles of a polyatomic gas (f = 6) must be mixed with two moles of a monoatomic gas so that the mixture behaves as a diatomic gas. The value of N is :

(1) 6 (2) 4 (3) 3 (4) 2

Ans. **[2]**

Sol. $<f> = \frac{N_1f_1 + N_2f_2}{N_1 + N_2}$

$\Rightarrow 5 = \frac{N6 + 2\times 3}{N+2}$

$\Rightarrow 5N + 10 = 6N + 6$

$\Rightarrow N = 4$

Q.10 The bob of a pendulum was released from a horizontal position. The length of the pendulum is 10 m. If it dissipates 10% of its initial energy against air resistance, the speed with which the bob arrives at the lowest point is [Use, g : 10 ms^{-2}]

(1) $6\sqrt{5}ms^{-1}$ (2) $5\sqrt{5}ms^{-1}$ (3) $2\sqrt{5}ms^{-1}$ (4) $5\sqrt{6}ms^{-1}$

Ans. **[1]**

Sol. $\frac{9}{10}mg\ell = \frac{1}{2}mv^2$

$\Rightarrow \mathbf{v} = \sqrt{1.8g\ell}$

$= \sqrt{180}$

$= 6\sqrt{5}m/s$

Q.11 An electric field is given by $(6\hat{i} + 5\hat{j} + 3\hat{k})$ N/C. The electric flux through a surface area $30\hat{i}\,m^2$ lying in YZ-plane (in SI unit) is :

(1) 180 (2) 60 (3) 150 (4) 90

Ans. **[1]**

Sol. $\phi_E = \vec{E}.\vec{A}$

$\Rightarrow \phi_E = 180$ SI unit

Q.12 A planet takes 200 days to complete one revolution around the Sun. If the distance of the planet from Sun is reduced to one fourth of the original distance, how many days will it take to complete one revolution

(1) 20 (2) 25 (3) 50 (4) 100

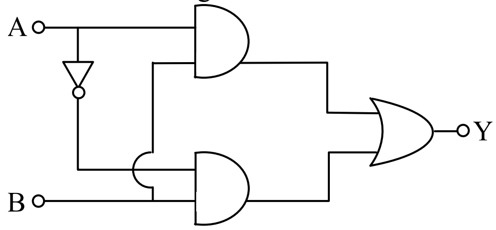
Ans. **[2]**

Sol. $T^2 \propto R^3$

$\Rightarrow \frac{T_1}{T_2} = \frac{R^{\frac{3}{2}}}{\left(\frac{R}{4}\right)^{\frac{3}{2}}}$

$\Rightarrow T_2 = \frac{200}{8} = 25$

Q.13 The truth table for this given circuit is :



(1)

A	B	Y
0	0	1
0	1	1
1	0	1
1	1	0

(2)

A	B	Y
0	0	1
0	1	0
1	0	1
1	1	0

(3)

A	B	Y
0	0	0
0	1	0
1	0	0
1	1	1

(4)

A	B	Y
0	0	0
0	1	1
1	0	0
1	1	1

Ans. **[4]**

Sol. $Y = A \cdot B + \overline{A} \cdot B$

$= (A + \overline{A})\ B = B$

Q.14 In an a.c. circuit voltage and current are given by :

$V = 100 \sin (100\ t)$ V and

$I = 100 \sin \left(100t + \dfrac{\pi}{3}\right)$ mA respectively.

The average power dissipated in one cycle is :

(1) 25 W (2) 2.5 W (3) 10 W (4) 5W

Ans. **[2]**

Sol. $P = V_{rms} I_{rms} \cos \phi$

$= \dfrac{100}{\sqrt{2}} \times \dfrac{100}{\sqrt{2}} \cos\dfrac{\pi}{3} \times 10^{-3}$

$= 2.5$ W

Q.15 A physical quantity Q is found to depend on quantities a, b, c by the relation $Q = \dfrac{a^4 b^3}{c^2}$. The percentage error in a, b and c are 3 %, 4% and 5 % respectively.

Then, the percentage error in Q is :

(1) 14 % (2) 34 % (3) 66 % (4) 43 %

Ans. **[2]**

Sol. $\dfrac{\Delta Q}{Q} = \dfrac{4\Delta a}{a} + \dfrac{3\Delta b}{b} + \dfrac{2\Delta c}{c}$

% error = 4 × 3 % + 3 × 4 % + 2 × 5 %

= (12 + 12 + 10) %

= 34 %

Q.16 A small liquid drop of radius R is divided into 27 identical liquid drops. If the surface tension is T , then the work done in the process will be :

(1) $\dfrac{1}{8}\pi R^2 T$ (2) $8\pi R^2 T$ (3) $4\pi R^2 T$ (4) $3\pi R^2 T$

Ans. **[2]**

Sol. $27\,\frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3$

$\Rightarrow R = 3r$

$\therefore\ W = \Delta V = 27 \times T \times 4\pi\left(\frac{R}{3}\right)^2 - T \times 4\pi R^2 = T4\pi R^2 \times 2$

$W = 8\pi R^2 T$

Q.17 A wire of length L and radius r is clamped at one end. If its other end is pulled by a force F, its length increases by ℓ. If the radius of the wire and the applied force both are reduced to half of their original values keeping original length constant, the increase in length will become:

(1) 3 times (2) $\frac{3}{2}$ times (3) 2 times (4) 4 times

Ans. **[3]**

Sol. $\Delta\ell = \frac{F\ell}{YA}$

$\frac{\Delta\ell_1}{\Delta\ell_2} = \frac{F_1}{A_1}\frac{A_2}{F_2}$

$= \frac{2}{4} = \frac{1}{2}$

Q.18 Two particles X and Y having equal charges are being accelerated through the same potential difference. Thereafter they enter normally in a region of uniform magnetic field and describes circular paths of radii R_1 and R_2 respectively. The mass ratio of X and Y is :

(1) $\left(\frac{R_1}{R_2}\right)^2$ (2) $\left(\frac{R_2}{R_1}\right)$ (3) $\left(\frac{R_1}{R_2}\right)$ (4) $\left(\frac{R_2}{R_1}\right)^2$

Ans. **[1]**

Sol. $qV = K$

and, $r = \frac{\sqrt{2mk}}{qB}$

$\Rightarrow \frac{r_1}{r_2} = \frac{\sqrt{m_1}}{\sqrt{m_2}} = \frac{R_1}{R_2}$

$\Rightarrow \left(\frac{R_1}{R_2}\right)^2 = \frac{m_1}{m_2}$

Q.19 In Young's double slit experiment, light from two identical sources are superimposing on a screen. The path difference between the two lights reaching at a point on the screen is $\frac{7\lambda}{4}$. The ratio of intensity of fringe at this point with respect to the maximum intensity of the fringe is :

(1) $\frac{1}{3}$ (2) $\frac{1}{4}$ (3) $\frac{3}{4}$ (4) $\frac{1}{2}$

Ans. **[4]**

Sol. $I = I_0 \cos^2\left(\frac{\Delta\phi}{2}\right)$

$\Delta\phi = \frac{2\pi}{\lambda} \times \frac{7\lambda}{4} = \frac{7}{2}\pi \equiv \frac{3}{2}\pi$

$\Rightarrow \frac{I}{I_0} = \cos^2\frac{3}{4}\pi = \frac{1}{2}$

Q.20 Given below are two statements :

Statement I : Most of the mass of the atom and all its positive charge are concentrated in a tiny nucleus and the electrons revolve around it, is Rutherford's model.

Statement II : An atom is a spherical cloud of positive charges with electrons embedded in it, is a special case of Rutherford's model.

In the light of the above statements, choose the most appropriate from the options given below

(1) Both statement I and statement II are false

(2) Both statement I and statement II are true

(3) Statement I is true but statement II is false

(4) Statement I is false but statement II is true

Ans. **[3]**

Sol. Rutherford model gave the planetary model of atom.

Statement I is correct and statement II is incorrect.

Section-B: Numerical Value Type Questions: This section contains 10 Numerical based questions. Attempt any 5 questions out of 10. The answer to each question should be rounded-off to the nearest integer.

Q.21 A charge of 4.0 μC is moving with a velocity of 4.0×10^6 ms^{-1} along the positive y-axis under a magnetic field B of strength $(2\hat{k})$ T. The force acting on the charge is $x\,\hat{i}$ N. The value of x is _______.

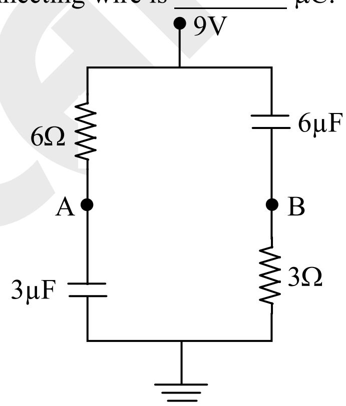
Ans. **[32]**

Sol. $F = \left|q(\vec{v} \times \vec{B})\right|$

$= 4 \times 10^{-6} \times 4 \times 10^6 \times 2$

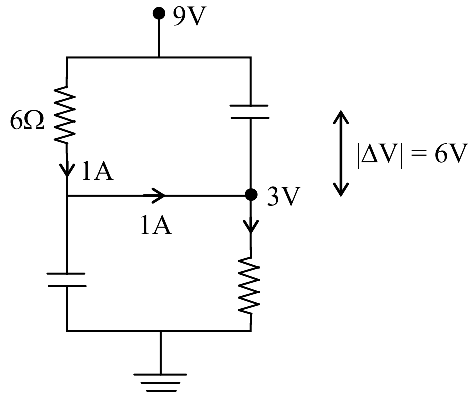
$= 32$

Q.22 In the given figure, the charge stored in 6 μF capacitor, when point A and B are joined by a connecting wire is ________ μC.



Ans. **[36]**

Sol.



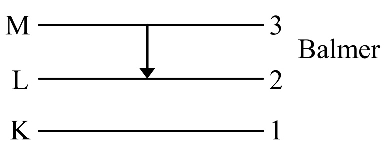
$q = 6 \times 6\ \mu C = 36\ \mu C$

Q.23 Hydrogen atom is bombarded with electrons accelerated through a potential difference of V, which causes excitation of hydrogen atoms. If the experiment is being performed at T = 0 K, the minimum potential difference needed to observe any Balmer series lines in the emission spectra will be $\frac{\alpha}{10}$ V, where

$\alpha =$ ________

Ans. **[121]**

Sol.



$|\Delta E| = 13.6\left\{1 - \frac{1}{9}\right\}$

$= 12.08 = \frac{\alpha}{10}$

$\alpha = 121$

Q.24 A horizontal straight wire 5 m long extending from east to west falling freely at right angle to horizontal component of earths magnetic field 0.60×10^{-4} Wbm^{-2}. The instantaneous value of emf induced in the wire when its velocity is 10 ms^{-1} is ________ $\times 10^{-3}$ V.

Ans. **[3]**

Sol. $e = B\ell v$

$= 0.6 \times 10^{-4} \times 5 \times 10$

$E = 3 \times 10^{-3}$ V

Q.25 Two metallic wires P and Q have same volume and are made up of same material. If their area of cross sections are in the ratio 4 : 1 and force F_1 is applied to P, an extension of $\Delta\ell$ is produced. The force which is required to produce same extension in Q is F_2.

The value of $\frac{F_1}{F_2}$ is __________.

Ans. **[16]**

Sol. $\Delta l = \frac{F\ell}{YA} = \frac{FV}{YA^2}$

$$\frac{\Delta\ell_1}{\Delta\ell_2} = \frac{F_1}{A_1^2}\frac{A_2^2}{F_2}$$

$$\frac{F_1}{16F_2} = \frac{\Delta\ell}{\Delta\ell}$$

$$\frac{F_1}{F_2} = 16$$

Q.26 A simple harmonic oscillator has an amplitude A and time period 6π second. Assuming the oscillation starts from its mean position, the time required by it to travel from x = A to x = $\frac{\sqrt{3}}{2}$A will be $\frac{\pi}{x}$ s, where x = _________.

Ans. **[2]**

Sol. $x = A\ ;\ x = \frac{\sqrt{3}}{2}A$

$\Rightarrow \theta = 30°$

$t = \frac{T}{12} = \frac{6\pi}{12} = \frac{\pi}{2} = \frac{\pi}{x} \Rightarrow x = 2.$

Q.27 A particle is moving in a circle of radius 50 cm in such a way that at any instant the normal and tangential components of it's acceleration are equal. If its speed at t = 0 is 4 m/s, the time taken to complete the first revolution will be $\frac{1}{\alpha}[1 - e^{-2\pi}]$ s, where α = ________.

Ans. **[8]**

Sol. $\frac{v^2}{r} = \left|\frac{dv}{dt}\right| \Rightarrow \frac{dt}{r} = \left|\frac{dv}{v^2}\right| \Rightarrow \frac{t}{r} = \left|\frac{1}{v_0} - \frac{1}{v}\right|$

$\frac{v^2}{r} = \left|\frac{vdv}{dx}\right| \Rightarrow \left|\frac{dv}{v}\right| = \frac{dx}{r} \Rightarrow \left|\ln\frac{v}{v_0}\right| = \frac{s}{r} = \frac{2\pi r}{r}$

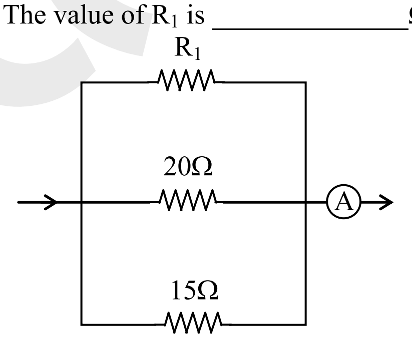
$\Rightarrow \frac{v}{v_0} = e^{\pm 2\pi}$

$v = v_0 e^{\pm 2\pi}$

$\frac{t}{r} = \left|\frac{1}{v_0} - \frac{1}{v_0 e^{\pm 2\pi}}\right|$

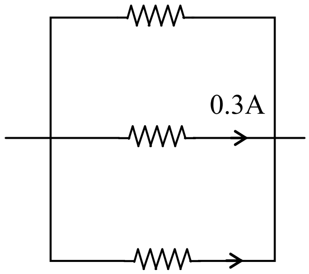
$\Rightarrow t = \frac{1}{8}[1 - e^{-2\pi}] = \frac{1}{\alpha}[1 - e^{-2\pi}] = \frac{1}{\alpha}[1 - e^{-2\pi}] \Rightarrow \alpha = 8$

Q.28 In the given circuit, the current flowing through the resistance 20 Ω is 0.3 A, while the ammeter reads 0.9 A. The value of R_1 is _____________Ω.



Ans. **[30]**

Sol. $V_{R_1} = V_{20} = V_{15} = 0.3 \times 20 = 6V$



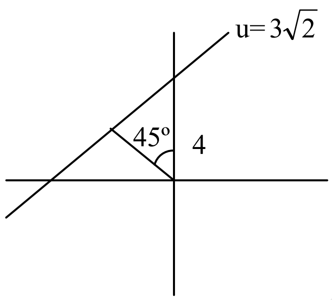
$\Rightarrow i_1 = 0.2$ A

$P_1 = \dfrac{6}{0.2} = 30\ \Omega$

Q.29 A body of mass 5 kg moving with a uniform speed $3\sqrt{2}$ ms^{-1} in X-Y plane along the line y = x + 4. The angular momentum of the particle about the origin will be ____________kg m^{2}s^{-1}.

Ans. **[60]**

Sol.



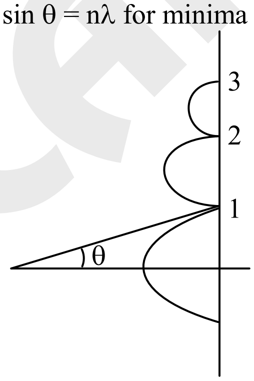
$L = mvr_1$

$= 5 \times 3\sqrt{2} \times \dfrac{4}{\sqrt{2}} = 60$ SI unit

Q.30 In a single slit diffraction pattern, a light of wavelength 6000 Å is used. The distance between the first and third minima in the diffraction pattern is found to be 3 mm when the screen in placed 50 cm away from slits. The width of the slit is _____________ $\times 10^{-4}$ m.

Ans. **[2]**

Sol. d sin θ = nλ for minima



$q_1 = \dfrac{\lambda}{d}$

$$q_3 = \frac{3\lambda}{d}$$

$$\Delta\theta = \frac{2\lambda}{d}$$

$$D\Delta\theta = \frac{2\lambda}{d}\ D = 3mm = \frac{2\times 6000\times 10^{-10}}{d}\times\frac{1}{2}$$

$$\Rightarrow\ d = \frac{6\times 10^{-7}}{3\times 10^{-3}} = 2\times 10^{-4}\ m$$

CHEMISTRY

Section-A: Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Q.31 The element having the highest first ionization enthalpy is

(1) C (2) N (3) Al (4) Si

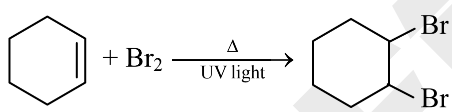
Ans. **[2]**

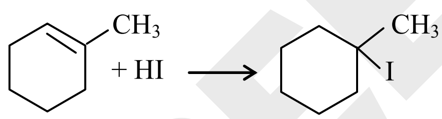
Sol. In general ionisation enthalpy increases on moving left to right in the period and decreases on moving top to bottom in group. Hence 1st ionisation enthalpy is highest for N.

Q.32 Which of the following reaction is correct ?

(1) $C_2H_5CONH_2 + Br_2 + NaOH \longrightarrow C_2H_5CH_2NH_2 + Na_2CO_3 + NaBr + H_2O$

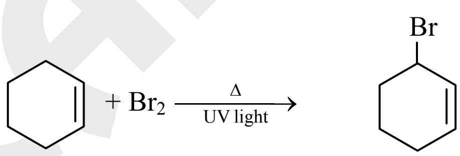
(2) $CH_3CH_2CH_2NH_2 \xrightarrow[H_2O]{HNO_2,0°C} CH_3CH_2OH + N_2 + HCl$

(3) 

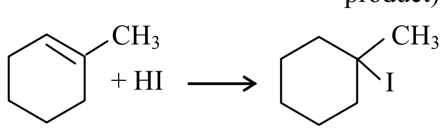
(4) 

Ans. **[4]**

Sol. The correct organic products of given reactions are as follows

1. $C_2H_5CONH_2 + Br_2 + NaOH \rightarrow C_2H_5NH_2$ (Hofmann-Bromamide reaction)
2. $CH_3CH_2CH_2NH_2 \xrightarrow[H_2O]{HNO_2,0°C} CH_3CH_2CH_2OH$
3. 

(Major monobromination product)

4. 

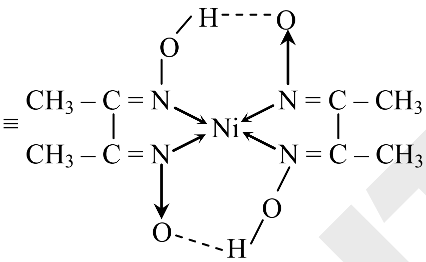
Hence option (4) is correct.

Q.33 A reagent which gives brilliant red precipitate with Nickel ions in basic medium is

(1) Dimethyl glyoxime
(2) Meta-dinitrobenzene
(3) Neutral $FeCl_3$
(4) Sodium nitroprusside

Ans. **[1]**

Sol. Ni^{2+} + dimethyl glyoxime $\xrightarrow{\text{Basic medium}}$ $\underset{\substack{\text{Cherry red} \\ \text{or} \\ \text{brilliant red}}}{[Ni(dmg)_2]}$ ≡



Q.34 The correct IUPAC name of K_2MnO_4 is

(1) Potassium tetraoxopermanganate (VI)
(2) Potassium tetraoxidomanganate (VI)
(3) Dipotassium tetraoxidomanganate (VII)
(4) Potassium tetraoxidomanganese (VI)

Ans. **[2]**

Sol. IUPAC name of K_2MnO_4 is
Potassium tetraoxidomanganate (VI)

Q.35 Given below are two statements :
Statement I : Fluorine has most negative electron gain enthalpy in its group.
Statement II : Oxygen has least negative electron gain enthalpy in its group.
In the light of the above statements, choose the most appropriate from the options given below.

(1) Statement I is true but Statement II is false
(2) Both Statement I and Statement II are true
(3) Both Statement I and Statement II are false
(4) Statement I is false but Statement II is true

Ans. **[4]**

Sol. $-\Delta egH : Cl > F > Br > I \Rightarrow 17^{th}$ group
$-\Delta egH : S > Se > Te > Po > O \Rightarrow 16^{th}$ group

Q.36 Alkyl halide is converted into alkyl isocyanide by reaction with

(1) KCN
(2) NaCN
(3) NH_4CN
(4) AgCN

Ans. **[4]**

Sol. $P_1 \rightarrow$ KCN, NaCN, NH_4CN are ionic in nature give $CN^{\ominus}$ So $R - X + CN^{\ominus} \rightarrow R–CN$
$P_2 \rightarrow$ AgCN is covalent in nature, only nitrogen donate its lone pair.
$P_3 \rightarrow RX + AgCN \longrightarrow \underset{(\text{Major})}{RNC} + AgX$

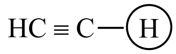
Q.37 Chromatographic technique/s based on the principle of differential adsorption is/are
A. Column chromatography
B. Thin layer chromatography
C. Paper chromatography
Choose the most appropriate answer from the options given below:

(1) A only
(2) C only
(3) B only
(4) A & B only

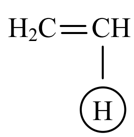
Ans. **[4]**

Sol. Column chromatography and thin layer chromatography is based on principle of differential adsorption while paper chromatography is a type of partition chromatography.

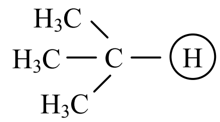
Q.38 The ascending acidity order of the following H atoms is



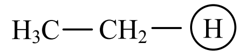
A



B



C



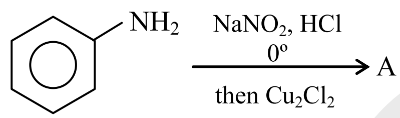
D

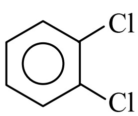
(1) D < C < B < A (2) A < B < C < D (3) C < D < B < A (4) A < B < D < C

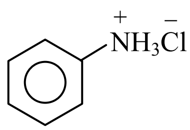
Ans. **[3]**

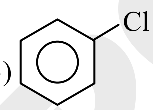
Sol. Order of acidic strength of H attached to carbon : $C_{sp} > C_{sp^2} > C_{sp^3}$. Further differentiation is based on electronic effect of substituents, hence correct order is C < D < B < A.

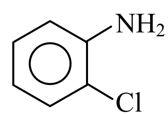
Q.39 The product A formed in the following reaction is



(1) 

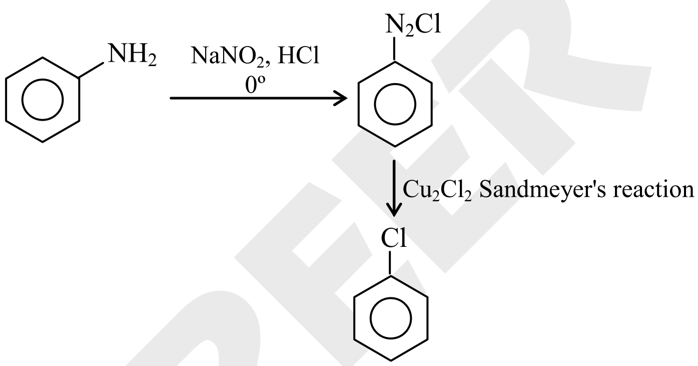
(2) 

(3) 

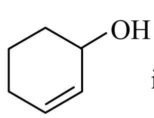
(4) 

Ans. **[3]**

Sol.



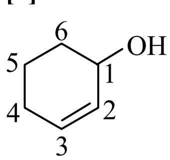
(A)

Q.40 According to IUPAC system, the compound  is named as

(1) 1-Hydroxyhex-2-ene (2) Cyclohex-1-en-2-ol

(3) Cyclohex-1-en-3-ol (4) Cyclohex-2-en-1-ol

Ans. **[4]**

Sol.  is Cyclohex-2-en-1-ol

Q.41 Match List-I with List-II.

	List-I (Bio Polymer)		List-II (Monomer)
A.	Starch	I.	Nucleotide
B.	Cellulose	II.	α-glucose
C.	Nucleic acid	III.	β-glucose
D.	Protein	IV.	α-amino acid

Choose the correct answer from the options given below:

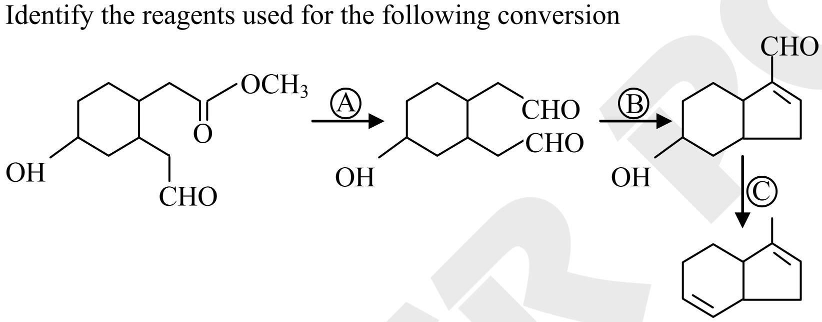
(1) A → I, B → III, C → IV, D → II
(2) A → IV, B → II, C → I, D → III
(3) A → II, B → III, C → I, D → IV
(4) A → II, B → I, C → III, D → IV

Ans. **[3]**

Sol.

Biopolymer	Monomer
Starch	α-D-glucose
Cellulose	β-D-glucose
Nucleic acid	Nucleotide
Protein	α-amino acid (Natural amino acids)

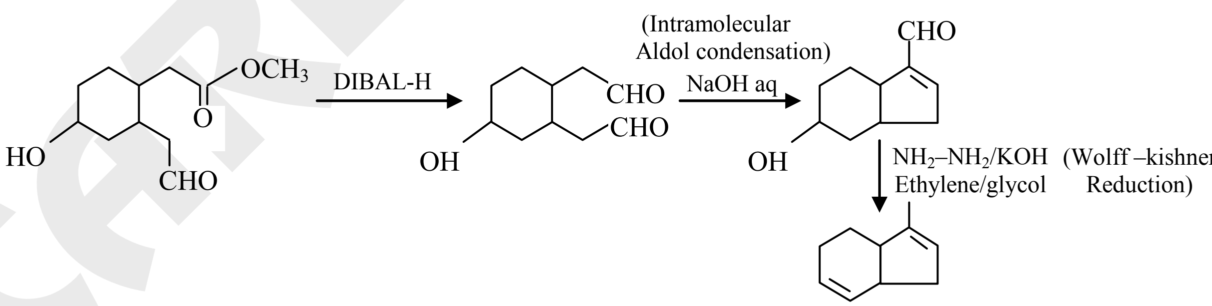
Q.42 Identify the reagents used for the following conversion



(1) A = DIBAL-H, B = $NaOH_{(alc)}$, C = Zn/HCl
(2) A = DIBAL-H, B = $NaOH_{(aq)}$, C = NH_2-NH_2/KOH, ethylene glycol
(3) A = $LiAlH_4$, B = $NaOH_{(alc)}$, C = Zn/HCl
(4) A = $LiAlH_4$, B = $NaOH_{(aq)}$, C = NH_2-NH_2/KOH, ethylene glycol

Ans. **[2]**

Sol.

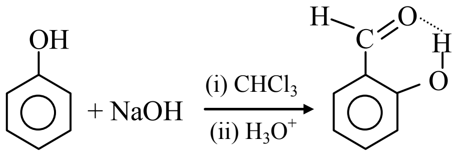


Q.43 Phenol treated with chloroform in presence of sodium hydroxide, which further hydrolyzed in presence of an acid results

(1) Benzene-1, 3-diol
(2) Salicylic acid
(3) 2-Hydroxybenzaldehyde
(4) Benzene-1, 2-diol

Ans. **[3]**

Sol.



2-Hydroxybenzaldehyde or Salicyaldehyde

is Reimer Tiemann reaction.

Q.44 Anomalous behavior of oxygen is due to its

(1) Small size and low electronegativity
(2) Large size and high electronegativity
(3) Large size and low electronegativity
(4) Small size and high electronegativity

Ans. **[4]**

Sol. Anomalous behavior of oxygen is due to small size and high electronegativity as compared to the elements of group 16.

Q.45 Match List-I with List-II.

	List-I (Compound)		**List-II (pK_a value)**
A.	Ethanol	I.	10.0
B.	Phenol	II.	15.9
C.	m-Nitrophenol	III.	7.1
D.	p-Nitrophenol	IV.	8.3

Choose the correct answer from the options given below:

(1) A $\rightarrow$ III, B $\rightarrow$ IV, C $\rightarrow$ I, D $\rightarrow$ II
(2) A $\rightarrow$ IV, B $\rightarrow$ I, C $\rightarrow$ II, D $\rightarrow$ III
(3) A $\rightarrow$ I, B $\rightarrow$ II, C $\rightarrow$ III, D $\rightarrow$ IV
(4) A $\rightarrow$ II, B $\rightarrow$ I, C $\rightarrow$ IV, D $\rightarrow$ III

Ans. **[4]**

Sol. Order of acidic strength of H attached to Oxygen depend on electronic effect of substituents attached to Oxygen.

pK_a : Ethanol > Phenol > m-Nitrophenol > p-Nitrophenol is in the order of electro withdrawing effect/strength of substituent.

Q.46 Match List I with List II.

	List I (Spectral Series for Hydrogen)		**List II (Spectral Region/Higher Energy State)**
A.	Lyman	I.	Infrared region
B.	Balmer	II.	UV region
C.	Paschen	III.	Infrared region
D.	Pfund	IV.	Visible region

Choose the correct answer from the options given below:

(1) A-I, B-III, C-II, D-IV
(2) A-II, B-III, C-I, D-IV
(3) A-I, B-II, C-III, D-IV
(4) A-II, B-IV, C-III, D-I

Ans. **[4]**

Sol. Correct match is as follows:

Spectral series for hydrogen	Spectral region/Higher Energy state
Lyman	UV region
Balmer	Visible region
Paschen	Infrared region
Pfund	Infrared region

Q.47 Which of the following acts as a strong reducing agent ? (Atomic number: Ce = 58, Eu = 63, Gd = 64, Lu = 71)

(1) Gd^{3+} (2) Eu^{2+} (3) Lu^{3+} (4) Ce^{4+}

Ans. **[2]**

Sol. Eu^{2+} is a strong reducing agent changing to the common +3 state while all other are provided at their higher oxidation states.

Q.48 Which of the following statement are correct about Zn, Cd and Hg?

A. They exhibit high enthalpy of atomization as the d-subshell is full.
B. Zn and Cd do not show variable oxidation state while Hg shows +I and +II.
C. Compounds of Zn, Cd and Hg are paramagnetic in nature.
D. Zn, Cd and Hg are called soft metals.

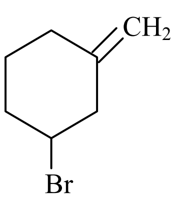
Choose the most appropriate from the options given below:

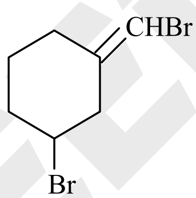
(1) B, D only (2) C, D only
(3) B, C only (4) A, D only

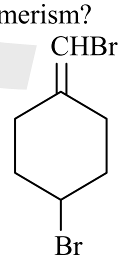
Ans. **[1]**

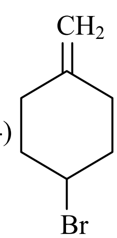
Sol. Zn, Cd and Hg are not regarded as transition elements as they have $(n-1)d^{10}ns^2$ configuration so they have low enthalpy of atomization and their compounds are diamagnetic in general.

Q.49 Which one of the following will show geometrical isomerism?

(1) 

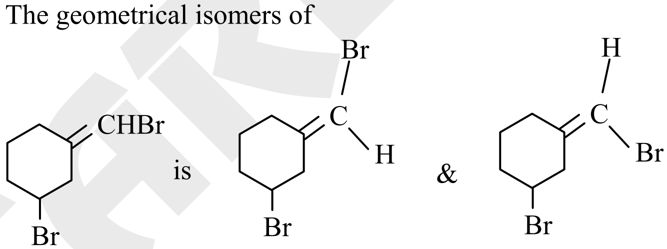
(2) 

(3) 

(4) 

Ans. **[2]**

Sol. The geometrical isomers of



While in case of other structures geometrical isomerism is not possible.

Q.50 On passing a gas, 'X', through Nessler's regent, a brown precipitate is obtained. The gas 'X' is

(1) Cl_2 (2) NH_3 (3) H_2S (4) CO_2

Ans. **[2]**

Sol. Nessler's reagent is used for the test of NH_3.

$$NH_3 + HgI_4^{2-} \xrightarrow{OH^-} \underset{\text{brown ppt}}{HgO.Hg(NH_2)I}\downarrow$$

Section-B: Numerical Value Type Questions: This section contains 10 Numerical based questions. Attempt any 5 questions out of 10. The answer to each question should be rounded-off to the nearest integer.

Q.51 Standard enthalpy of vapourisation for CCl_4 is 30.5 kJ mol^{-1}. Heat required for vapourisation of 284 g of CCl_4 at constant temperature is ____kJ. (Given molar mass in g mol^{-1} ; C = 12, Cl = 35.5)

Ans. **[56]**

Sol. Moles of $CCl_4 = \frac{284}{154} = 1.8$

Heat required = 1.8 mol × 30.5 kJ mol^{-1}

= 56.24 kJ

≃ 56 kJ

Q.52 The oxidation number of iron in the compound formed during brown ring test for NO_3^- ion is _______.

Ans. **[1]**

Sol. Brown ring complex is $[Fe(H_2O)_5NO]^{2+}$. It has Fe at +1 oxidation state that can be explained by magnetic moment data of complex. It is an example of ligand to metal charge transfer that is why Fe has +1 and NO has +1 oxidation state.

Q.53 The following concentrations were observed at 500K for the formation of NH_3 from N_2 and H_2. At equilibrium ; $[N_2] = 2 \times 10^{-2}$ M, $[H_2] = 3 \times 10^{-2}$ M and $[NH_3] = 1.5 \times 10^{-2}$ M. Equilibrium constant for the reaction is_____.

Ans. **[417]**

Sol. $N_2 + 3H_2 \rightleftharpoons 2NH_3$

$$K_C = \frac{[NH_3]^2}{[N_2][H_2]^3}$$

$$= \frac{[1.5\times10^{-2}]^2}{[2\times10^{-2}][3\times10^{-2}]^3}$$

= 416.67 ≃ 417

Q.54 A constant current was passed through a solution of $AuCl_4^-$ ion between gold electrodes. After a period of 10.0 minutes, the increase in mass of cathode was 1.314g. The total charge passed through the solution is ____ × 10^{-2} F. (Given atomic mass of Au = 197)

Ans. **[2]**

Sol. Moles of Au deposited $= \frac{1.314}{197}$

Equivalents of Au deposited = Equivalents of Charge conducted

$\frac{1.314\times3}{197}$ = Charge in F

= 0.02001

≃ 2×10^{-2}

As one equivalent charge is one faraday.

Q.55 The half-life of radioisotope bromine-82 is 36 hours. The fraction which remains after one day is _____ $\times 10^{-2}$. (Given antilog 0.2006 = 1.587)

Ans. **[63]**

Sol. $\lambda = \frac{2.303}{t} \log \frac{N_0}{N_t}$

$$\frac{2.303 \times .301}{36} = \frac{2.303}{24} \log \frac{N_0}{N_t}$$

$\log \frac{N_0}{N_t} = 0.2006$

$\frac{N_0}{N_t} = 1.587$

$\frac{N_t}{N_0} = 0.6301$

$= 63 \times 10^{-2}$

Q.56 The total number of anti bonding molecular orbitals, formed from 2s and 2p atomic orbitals in a diatomic molecules is

Ans. **[4]**

Sol. In the formation of diatomic molecule, the total number of 2s and 2p orbitals participate is 8.
So, 8 molecular orbitals will form, four of which will be antibonding, hence answer will be 4, that are
$\underbrace{\sigma^* 2s}\ \underbrace{\pi^* 2p_x}\ \underbrace{\pi^* 2p_y}\ \underbrace{\sigma^* 2p_z}$

Q.57 Molality of 0.8 M H_2SO_4 solution (density 1.06 cm^{-3}) is ________ $\times 10^{-3}$ m.

Ans. **[815]**

Sol. Molality $= \frac{\text{Molarity} \times 10^3}{(1000 \times d) - (\text{Molarity} \times \text{Molar mass})}$

$= \frac{0.8 \times 10^3}{1000 \times 1.06 - 0.8 \times 98}$

$= 0.81499$

$= 815 \times 10^{-3}$

Q.58 The total number of molecules with zero dipole moment among CH_4, BF_3, H_2O, HF, NH_3, CO_2 and SO_2 is ______.

Ans. **[3]**

Sol. CH_4, BF_3, CO_2 are nonpolar while H_2O, HF, NH_3 and SO_2 are polar that can be explained by their geometry.
CH_4 : Tetrahedral, BF_3 : Trigonal planar,
CO_2 : Linear

Q.59 If 50 mL of 0.5 M oxalic acid is required to neutralize 25 mL of NaOH solution, the amount of NaOH in 50 mL of given NaOH solution is _________g.

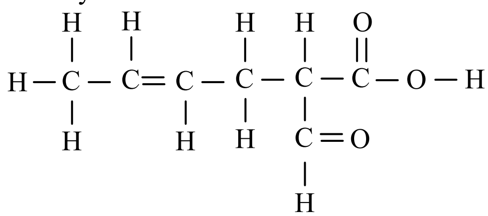
Ans. **[4]**

Sol. Let molarity of NaOH is M.
Equivalents of $H_2C_2O_4$ = Equivalents of NaOH
$0.5 \times 50 \times 10^{-3} \times 2 = M \times 25 \times 10^{-3} \times 1$
$M = 2$
Mass of NaOH in 50 mL of such solution
$= 2 \times 50 \times 10^{-3} \times 40$
$= 4$ g

Q.60 The total number of 'Sigma' and 'Pi' bonds in 2-formylhex-4-enoic acid is _______.

Ans. **[22]**

Sol. 2-formylhex-4-enoic acid is



Sigma bonds : 19

Pi bonds : 3

MATHEMATICS

Section-A: Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Q.61 If $\sin\left(\frac{y}{x}\right) = \log_e|x| + \frac{\alpha}{2}$ is the solution of the differential equation $x\cos\left(\frac{y}{x}\right)\frac{dy}{dx} = y\cos\left(\frac{y}{x}\right) + x$ and $y(1) = \frac{\pi}{3}$, then α^2 is equal to

(1) 4 (2) 9 (3) 3 (4) 12

Ans. **[3]**

Sol. $x\cos\left(\frac{y}{x}\right)\frac{dy}{dx} - y\cos\left(\frac{y}{x}\right) = x$

$\Rightarrow \cos\left(\frac{y}{x}\right)\left[x\frac{dy}{dx} - y\right] = x$

$\Rightarrow \cos\left(\frac{y}{x}\right)\left[\frac{x\frac{dy}{dx} - y}{x^2}\right] = \frac{1}{x}$

Let $\frac{y}{x} = t$

$\frac{x\frac{dy}{dx} - y}{x^2} = \frac{dt}{dx}$

$\Rightarrow \cos t\frac{dt}{dx} = \frac{1}{x}$

$\Rightarrow \int \cos t \, dt = \int \frac{1}{x}dx$

$\Rightarrow \sin t = \ln|x| + C$

$y(1) = \frac{\pi}{3}$

$\Rightarrow \sin\frac{\pi}{3} = 0 + C \Rightarrow C = \frac{\sqrt{3}}{2}$

$\therefore$ Solution is $\sin\frac{y}{x} = \ln|x| + \frac{\sqrt{3}}{2}$

$\therefore \alpha^2 = 3$

Q.62 The function $f(x) = \frac{x}{x^2 - 6x - 16}$, $x \in R - \{-2, 8\}$

(1) Decreases in $(-2, 8)$ and increases in $(-\infty, -2) \cup (8, \infty)$
(2) Decreases $(-\infty, -2) \cup (-2, 8) \cup (8, \infty)$
(3) Increase in $(-\infty, -2) \cup (-2, 8) \cup (8, \infty)$
(4) Decrease $(-\infty, -2)$ and increases in $(8, \infty)$

Ans. **[2]**

Sol. $f(x) = \frac{x}{x^2 - 6x - 16}$

$f'(x) = \frac{(x^2 - 6x - 16) - x(2x - 6)}{(x^2 - 6x - 16)^2}$

$= \frac{x^2 - 6x - 16 - 2x^2 + 6x}{(x^2 - 6x - 16)^2}$

$= \frac{-x^2 - 16}{(x^2 - 6x - 16)^2} < 0 \ \ \forall\ x \in D_f$

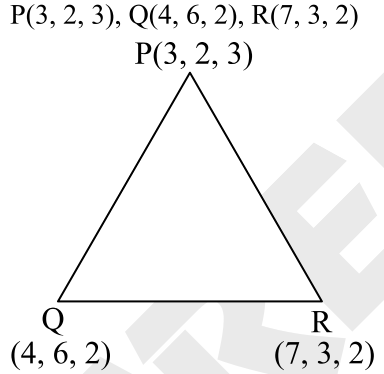
$\therefore$ f(x) is decreasing in $(-\infty, -2) \cup (-2, 8) \cup (8, \infty)$

Q.63 Let P(3, 2, 3), Q(4, 6, 2) and R(7, 3, 2) be the vertices of ΔPQR. Then, the angle ∠QPR is

(1) $\frac{\pi}{6}$ (2) $\cos^{-1}\left(\frac{7}{18}\right)$ (3) $\frac{\pi}{3}$ (4) $\cos^{-1}\left(\frac{1}{18}\right)$

Ans. **[3]**

Sol. P(3, 2, 3), Q(4, 6, 2), R(7, 3, 2)



Direction ratio of PR = 4, 1, –1
Direction ratio of PQ = 1, 4, –1

$\cos(\angle QPR) = \left|\frac{1 \times 4 + 4 \times 1 + 1 \times 1}{\sqrt{16+1+1}\sqrt{1+16+1}}\right|$

$\cos(\angle QPR) = \frac{1}{2}$

$\Rightarrow \angle QPR = \cos^{-1}\left(\frac{1}{2}\right)$

$\Rightarrow \angle QPR = \frac{\pi}{3}$

Q.64 Let $x = \frac{m}{n}$ (m, n are co-prime natural numbers) be a solution of the equation $\cos(2\sin^{-1} x) = \frac{1}{9}$ and let α, β (α > β) be the roots of the equation $mx^2 - nx - m + n = 0$.
Then the point (α, β) lies on the line
(1) $5x + 8y = 9$ (2) $5x - 8y = -9$ (3) $3x + 2y = 2$ (4) $3x - 2y = -2$

Ans. **[1]**

Sol. $\cos(2\sin^{-1} x) = \frac{1}{9}, x > 0$

$\because x > 0$

$\sin^{-1}x = \theta \in 1^{st}$ Quadrant

$\Rightarrow \cos 2\theta = \frac{1}{9}$

$= 1 - 2\sin^2\theta = \frac{1}{9}$

$\Rightarrow \sin^2\theta = \frac{4}{9}$

$\Rightarrow x^2 = \frac{4}{9}$

$\Rightarrow x = \frac{2}{3} = \frac{m}{n}$

$\Rightarrow m = 2$ and $n = 3$

$mx^2 - nx - m + n = 0$

$\Rightarrow 2x^2 - 3x + 1 = 0$

Roots are $1, \frac{1}{2}$

$\alpha = 1, \beta = \frac{1}{2}$

So, $(\alpha, \beta) = \left(1, \frac{1}{2}\right)$

and (α, β) lies on $5x + 8y = 9$

Hence, option (1) is correct

Q.65 The sum of the solution $x \in R$ of the equation $\frac{3\cos 2x + \cos^3 2x}{\cos^6 x - \sin^6 x} = x^3 - x^2 + 6$ is

(1) 3 (2) 1 (3) –1 (4) 0

Ans. **[3]**

Sol. Given : $\frac{3\cos 2x + \cos^3 2x}{\cos^6 x - \sin^6 x} = x^3 - x^2 + 6$

$\Rightarrow \frac{\cos 2x(3 + \cos^2 2x)}{(\cos^2 x - \sin^2 x)[\sin^4 x + \cos^4 x + \sin^2 x\cos^2 x]}$

$= x^3 - x^2 + 6$

$\Rightarrow \frac{3 + \cos^2 2x}{1 - \sin^2 x\cos^2 x} = x^3 - x^2 + 6$

$\Rightarrow 4\left(\frac{3 + \cos^2 2x}{4 - \sin^2 2x}\right) = x^3 - x^2 + 6$

$\Rightarrow x^3 - x^2 + 2 = 0$

$\Rightarrow (x + 1)(x^2 - 2x + 2) = 0$

$\Rightarrow (x + 1)((x - 1)^2 + 1) = 0$

$\therefore$ Sum of solution = – 1

Q.66 Let r and θ respectively be the modulus and amplitude of the complex number $z = 2 - i\left(2\tan\frac{5\pi}{8}\right)$, then (r, θ) is equal to

(1) $\left(2\sec\frac{3\pi}{8}, \frac{5\pi}{8}\right)$ (2) $\left(2\sec\frac{5\pi}{8}, \frac{3\pi}{8}\right)$ (3) $\left(2\sec\frac{11\pi}{8}, \frac{11\pi}{8}\right)$ (4) $\left(2\sec\frac{3\pi}{8}, \frac{3\pi}{8}\right)$

Ans. **[4]**

Sol. $z = 2 - 2i\,\dfrac{\sin\frac{5\pi}{8}}{\cos\frac{5\pi}{8}}$

$= \dfrac{2}{\cos\frac{5\pi}{8}}\left(\cos\frac{5\pi}{8} - i\sin\frac{5\pi}{8}\right)$

$= 2\sec\frac{5\pi}{8}e^{i\left(\frac{-5\pi}{8}\right)}$

$= \left(2\sec\frac{3\pi}{8}\right)e^{i\pi}e^{i\left(\frac{-5\pi}{8}\right)}$

$= 2\sec\frac{3\pi}{8}e^{i\frac{3\pi}{8}}$

$\therefore r = 2\sec\frac{3\pi}{8}, \theta = \frac{3\pi}{8}$

Q.67 Let A be the point of intersection of the lines 3x + 2y = 14, 5x – y = 6 and B be the point of intersection of the lines 4x + 3y = 8, 6x + y = 5. The distance of the point P(5, –2) from the line AB is

(1) $\frac{5}{2}$ (2) 8 (3) 6 (4) $\frac{13}{2}$

Ans. **[3]**

Sol. A is point of intersection of line 3x + 2y = 14

and 5x – y = 6

On solving,

$$\begin{array}{r} 3x + 2y = 14 \\ \underline{10x - 2y = 12} \\ 13x = 26 \end{array}$$

x = 2 and y = 4

Point A (2, 4)

Similarly, B is

$$\begin{array}{r} 4x + 3y = 8 \\ \underline{18x + 3y = 15} \\ 14x = 7 \end{array}$$

$x = \frac{1}{2}$ and y = 2

Point B $\left(\frac{1}{2}, 2\right)$

$\therefore$ Line AB is $\quad y - 4 = \dfrac{-2}{-\frac{3}{2}}(x - 2)$

$y - 4 = \dfrac{4}{3}(x - 2)$

$3y - 12 = 4x - 8$

$4x - 3y + 4 = 0$

$\therefore$ Distance of P (5, – 2) from AB is

$= \dfrac{|4(5) - 3(-2) + 4|}{\sqrt{(4)^2 + (-3)^2}}$

$= \dfrac{|20 + 6 + 4|}{5}$

$= 6$

Q.68 An integer is chosen at random from the integers 1, 2, 3, …, 50. The probability that the chosen integer is a multiple of atleast one of 4, 6 and 7 is

(1) $\dfrac{14}{25}$ (2) $\dfrac{8}{25}$ (3) $\dfrac{21}{50}$ (4) $\dfrac{9}{50}$

Ans. **[3]**

Sol. Take P(A) = Probability that number is multiple of 4

P(B) = Probability that number is multiple of 6

P(C) = Probability that number is multiple of 7

$P(A) = \dfrac{12}{50}$, $P(B) = \dfrac{8}{50}$ and $P(C) = \dfrac{7}{50}$

$P(A \cap B) = \dfrac{4}{50}$ (Multiple of 12)

$P(B \cap C) = \dfrac{1}{50}$ (Multiple of 42)

$P(A \cap C) = \dfrac{1}{50}$ (Multiple of 28)

$P(A \cap B \cap C) = 0$ (Multiple of 84)

$P(A \cup B \cap C) = P(A) + P(B) + P(C)$

$-P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$

$= \dfrac{12}{50} + \dfrac{8}{50} + \dfrac{7}{50} - \dfrac{4}{50} - \dfrac{1}{50} - \dfrac{1}{50} + 0 = \dfrac{21}{50}$

Q.69 Let a unit vector $\hat{u} = x\hat{i} + y\hat{j} + z\hat{k}$ make angles $\dfrac{\pi}{2}, \dfrac{\pi}{3}$ and $\dfrac{2\pi}{3}$ with the vectors $\dfrac{1}{\sqrt{2}}\hat{i} + \dfrac{1}{\sqrt{2}}\hat{k}$, $\dfrac{1}{\sqrt{2}}\hat{j} + \dfrac{1}{\sqrt{2}}\hat{k}$ and $\dfrac{1}{\sqrt{2}}\hat{i} + \dfrac{1}{\sqrt{2}}\hat{j}$ respectively, if $\vec{v} = \dfrac{1}{\sqrt{2}}\hat{i} + \dfrac{1}{\sqrt{2}}\hat{j} + \dfrac{1}{\sqrt{2}}\hat{k}$, then $|\hat{u} - \vec{v}|^2$ is equal to

(1) 9 (2) 7 (3) $\dfrac{11}{2}$ (4) $\dfrac{5}{2}$

Ans. **[4]**

Sol. $\dfrac{x}{\sqrt{2}} + \dfrac{z}{\sqrt{2}} = 0 \quad \ldots(1)$

$\frac{y}{\sqrt{2}} + \frac{z}{\sqrt{2}} = \frac{1}{2}$ …(2)

$\frac{x}{\sqrt{2}} + \frac{y}{\sqrt{2}} = \frac{-1}{2}$ …(3)

From (1) and (3)

$\frac{y}{\sqrt{2}} - \frac{z}{\sqrt{2}} = -\frac{1}{2}$ ….(4)

From (2) and (4)

$\sqrt{2}y = 0$

$y = 0$

and $z = \frac{1}{\sqrt{2}}$, $x = -\frac{1}{\sqrt{2}}$

$\therefore\ \vec{v} - \vec{u} = \sqrt{2}\,\hat{i} + \frac{1}{\sqrt{2}}\hat{j}$

$|\vec{v} - \vec{u}| = \sqrt{2 + \frac{1}{2}}$

$|\vec{v} - \vec{u}| = \sqrt{\frac{5}{2}}$

$\therefore\ |\vec{v} - \vec{u}|^2 = \frac{5}{2}$

Q.70 If each term of a geometric progression $a_1, a_2, a_3, \ldots$ with $a_1 = \frac{1}{8}$ and $a_2 \neq a_1$, is arithmetic mean of the next two terms and $S_n = a_1 + a_2 + \ldots + a_n$, then $S_{20} - S_{18}$ is equal to

(1) -2^{18} (2) 2^{18} (3) -2^{15} (4) 2^{15}

Ans. **[3]**

Sol. Given $a_1 = \frac{1}{8}$

$a, ar, ar^2, ar^3, \ldots$

Also, a_2 is arithmetic mean of next two terms

$\therefore\ 2ar = ar^2 + ar^3$

$2ar = r(ar + ar^2)$

$2a = ar + ar^2$

$2 = r + r^2$

$r^2 + r - 2 = 0$

$(r + 2)(r - 1) = 0$

$r \neq 1$

$\Rightarrow r = -2$

$\therefore\ S_{20} - S_{18} = \frac{a(1 - r^{20})}{1 - r} - \frac{a(1 - r^{18})}{1 - r}$

$= \frac{1}{8}\left(\frac{1}{3}(1 - r^{20} - 1 + r^{18})\right)$

$= \frac{1}{24} \cdot 2^{18}\ (1 - 4)$

$= \frac{-2^{18}}{8} \Rightarrow -2^{15}$

Q.71 Let $A = \begin{bmatrix} 2 & 1 & 2 \\ 6 & 2 & 11 \\ 3 & 3 & 2 \end{bmatrix}$ and $P = \begin{bmatrix} 1 & 2 & 0 \\ 5 & 0 & 2 \\ 7 & 1 & 5 \end{bmatrix}$. The sum of the prime factors of $|P^{-1}AP - 2I|$ is equal to

(1) 26 (2) 27 (3) 66 (4) 23

Ans. **[1]**

Sol. $|P^{-1}AP - 2I|$

$= |P^{-1}AP - 2(P^{-1}P)|$

$= |P^{-1}I|\,|AP - 2P|$

$= |P^{-1}I|\,|A - 2I|\,|P|$

$= |A - 2I|$

$A = \begin{bmatrix} 2 & 1 & 2 \\ 6 & 2 & 11 \\ 3 & 3 & 2 \end{bmatrix}$

$A - 2I = \begin{bmatrix} 0 & 1 & 2 \\ 6 & 0 & 11 \\ 3 & 3 & 0 \end{bmatrix}$

$|A - 2I| = -1(-33) + 2(18)$

$= 69$

Prime factors of 69 = 3, 23

Sum = 26

Q.72 Number of ways of arranging 8 identical books into 4 identical shelves where any number of shelves may remain empty is equal to

(1) 15 (2) 18 (3) 12 (4) 16

Ans. **[1]**

Sol. Given that 8 identical books have to be arranged in 4 identical shelves.

I	II	III	IV
8	0	0	0
7	1	0	0
6	2	0	0
6	1	1	0
5	3	0	0
5	2	1	0
5	1	1	1
4	4	0	0
4	3	1	0
4	2	2	0
4	2	1	1
3	3	2	0
3	3	1	1
3	2	2	1
2	2	2	2

Total 15 cases are possible.

Q.73 Let $\overrightarrow{OA} = \vec{a}$, $\overrightarrow{OB} = 12\vec{a} + 4\vec{b}$, and $\overrightarrow{OC} = \vec{b}$ where O is the origin, If S is the parallelogram with adjacent sides OA and OC, then $\frac{\text{area of the quadrilateral OABC}}{\text{area of S}}$ is equal to _______

(1) 6 (2) 7 (3) 10 (4) 8

Ans. **[4]**

Sol. $S = |\overline{OA} \times \overline{OC}| = |\bar{a} \times \bar{b}|$

Area of quadrilateral OABC

$= \frac{1}{2}\left|\overline{CA} \times \overline{OB}\right|$

$= \frac{1}{2}\left|(\bar{b} - \bar{a}) \times (12\bar{a} + 4\bar{b})\right|$

$= \frac{1}{2} \cdot 16\,|\bar{a} \times \bar{b}|$

Required ratio $= 8$

Q.74 If the mean and variance of five observations are $\frac{24}{5}$ and $\frac{194}{25}$ respectively and the mean of the first four observations is $\frac{7}{2}$, then the variance of the first four observation in equal to

(1) $\frac{77}{12}$ (2) $\frac{105}{4}$ (3) $\frac{4}{5}$ (4) $\frac{5}{4}$

Ans. **[4]**

Sol. $n = 5$

Mean $\bar{x} = \frac{24}{5}$

Take data as x_1, x_2, x_3, x_4, x_5

So, $x_1 + x_2 + x_3 + x_4 + x_5 = 24$

Mean of first 4 observation is $\frac{7}{2}$

$x_1 + x_2 + x_3 + x_4 = 14$

$\Rightarrow x_5 = 10$

Variance of first 5 observations.

$$\frac{\sum_{i=1}^{5} x_i^2}{n} - (\bar{x})^2 = \frac{194}{25}$$

$$\frac{\sum_{i=1}^{5} x_i^2}{5} - \frac{576}{25} = \frac{194}{25}$$

$$\Rightarrow 5\sum_{i=1}^{5} x_i^2 \quad - 576 = 194$$

$$\Rightarrow 5\sum_{i=1}^{5} x_i^2 = 770$$

$\Rightarrow x_1^2 + x_2^2 + x_3^2 + x_4^2 + 100 = 154$

$\Rightarrow \sum_{i=1}^{4} x_i^2 = 54$

Variance of first 4 terms :

$$\frac{\sum_{i=1}^{4} x_i^2}{4} - \frac{49}{4} = \frac{54-49}{4} = \frac{5}{4}$$

Q.75 The distance of the point (2, 3) from the line 2x – 3y + 28 = 0, measured parallel to the line $\sqrt{3}x - y + 1 = 0$, is equal to

(1) $4\sqrt{2}$ (2) $6\sqrt{3}$ (3) $3+4\sqrt{2}$ (4) $4+6\sqrt{3}$

Ans. **[4]**

Sol. Given line $y = \sqrt{3}x + 1$

Inclination $= \frac{\pi}{3}$

Any point on line though (2, 3) and parallel to given line.

$$P\left(2 + r\cos\frac{\pi}{3}, 3 + r\sin\frac{\pi}{3}\right) \equiv \left(2 + \frac{r}{2}, 3 + \frac{r\sqrt{3}}{2}\right)$$

If P lies on 2x – 3y + 28 = 0

$$2\left(2 + \frac{r}{2}\right) - 3\left(3 + \frac{r\sqrt{3}}{2}\right) + 28 = 0$$

$\Rightarrow |r| = 4 + 6\sqrt{3}$ = Required distance

Q.76 The function $f(x) = 2x + 3(x)^{\frac{2}{3}}, x \in R$, has

(1) Exactly one point of local minima and no point of local maxima

(2) Exactly one point of local maxima and no point of local minima

(3) Exactly two points of local maxima and exactly one point of local minima

(4) Exactly one point of local maxima and exactly one point of local minima

Ans. **[4]**

Sol. $f(x) = 2x + 3x^{\frac{2}{3}}, x \in R$

$$f'(x) = 2 + 3\left(\frac{2}{3}\right)x^{\frac{-1}{3}}$$

$$= 2\left(1 + \frac{1}{x^{\frac{1}{3}}}\right) = \frac{2\left(x^{\frac{1}{3}} + 1\right)}{x^{\frac{1}{3}}}$$

+ – +

–1 0

f ′(x) changes from +ve to –ve at x = –1

⇒ point of local maxima

F ′(x) changes from –ve to +ve at x = 0

⇒ point of local minima

Q.77 Let $y = \log_e\left(\frac{1-x^2}{1+x^2}\right)$, $-1 < x < 1$. Then at $x = \frac{1}{2}$, the value of 225 (y' – y") is equal to

(1) 742 (2) 746 (3) 732 (4) 736

Ans. **[4]**

Sol. $y = \ln\left(\frac{1-x^2}{1+x^2}\right), x \in (-1, 1)$

$$\frac{dy}{dx} = \frac{d}{dx}\left(\ln(1-x^2)\right) - \frac{d}{dx}\left(\ln(1+x^2)\right)$$

$$= \frac{1}{(1-x^2)}(-2x) - \frac{(2x)}{(1+x^2)}$$

$$= -2x\left[\frac{1}{1-x^2} + \frac{1}{1+x^2}\right] = \frac{-4x}{1-x^4}$$

$$y' = \frac{-4x}{1-x^4}$$

$$y'' = \frac{d}{dx}\left(\frac{-4x}{1-x^4}\right) = \frac{(1-x^4)[-4]-[-4x][-4x^3]}{(1-x^4)^2}$$

$$= (-4)\frac{[1-x^4+4x^4]}{(1-x^4)^2}$$

$$= \frac{(-4)(1+3x^4)}{(1-x^4)^2}$$

At $x = \frac{1}{2}$

$$y' = \frac{-2}{1-\frac{1}{16}} = \frac{-32}{15},$$

$$y'' = \frac{(-4)\left[1+\frac{3}{16}\right]}{\left(1-\frac{1}{16}\right)^2} = \frac{(16)(-4)(19)}{225}$$

$$225(y'-y'') = 225\left[\frac{-32}{15} + \frac{16\times4\times19}{225}\right]$$

$= 16 \times 4 \times 19 - 32 \times 15$

$= 32[38 - 15] = 32 \times 23$

$= 736$

Q.78 If $\int \frac{\sin^{\frac{3}{2}} x + \cos^{\frac{3}{2}} x}{\sqrt{\sin^3 x \cos^3 x \sin(x-\theta)}} dx =$

$A\sqrt{\cos\theta\tan x - \sin\theta} + B\sqrt{\cos\theta - \sin\theta\cot x} + C$,

Where C is the integration constant, then AB is equal to

(1) 2 sec θ (2) 4cosec (2θ) (3) 4 sec θ (4) 8 cosec (2θ)

Ans. **[4]**

Sol. $I = \int \frac{\left(\sin^{\frac{3}{2}} x + \cos^{\frac{3}{2}} x\right)dx}{\sqrt{\sin^3 x \cos^3 x \sin(x-\theta)}}$

$I = \int \frac{\sec^{\frac{3}{2}} x dx}{\sqrt{\sin(x-\theta)}} + \int \frac{\operatorname{cosec}^{\frac{3}{2}} x\, dx}{\sqrt{\sin(x-\theta)}}$

$I = \int \frac{\sec^2 x dx}{\sqrt{\sec x \sin(x-\theta)}} + \int \frac{\operatorname{cosec}^2 x dx}{\sqrt{\operatorname{cosec} x \sin(x-\theta)}}$

$I = \int \frac{\sec^2 x dx}{\sqrt{\tan x \cos\theta - \sin\theta}} + \int \frac{\operatorname{cosec}^2 x dx}{\sqrt{\cos\theta - \cot x \sin\theta}}$

$= I_1 + I_2$

$I_1 = \int \frac{\sec^2 x dx}{\sqrt{\tan x \cos\theta - \sin\theta}}$

Let tanx cosθ – sinθ = t^2

$\Rightarrow \sec^2 x dx = \frac{2tdt}{\cos\theta}$

$I_1 = \int \frac{\frac{2tdt}{\cos\theta}}{t} = \frac{2t}{\cos\theta} + C_1 = \frac{2}{\cos\theta} \sqrt{\tan x \cot\theta - \sin\theta} + C_1$

$I_2 = \frac{2}{\sin\theta} \sqrt{\cos\theta - \sin\theta \cot x} + C_2$

$\Rightarrow I_1 + I_2 \Rightarrow B = \frac{2}{\sin\theta}, A = \frac{2}{\cos\theta}$

$\Rightarrow AB = \frac{4}{\sin\theta\cos\theta} = 8 \operatorname{cosec} 2\theta$

Q.79 If R is the smallest equivalence relation on the set {1, 2, 3, 4} such that {(1, 2), (1, 3)} ⊂ R, then the number of elements in R is ______

(1) 8 (2) 10 (3) 12 (4) 15

Ans. **[2]**

Sol. R is smallest equivalence relation

⇒ R must have (1, 1), (2, 2), (3, 3), (4, 4)
(For reflexive)

⇒ Now R is {(1, 1), (1, 2), (2, 2), (1, 3), (3, 3), (4, 4)}

⇒ For symmetric relation
(1, 2) must have (2, 1)
(1, 3) must have (3, 1)

⇒ R now is
{(1, 1), (1, 2), (2, 1), (2, 2), (1, 3), (3, 1), (3, 3), (4, 4)}
Since (2, 1) and (1, 3) is in R

⇒ (2, 3) must be there

⇒ (3, 2) must be there

⇒ {(1, 1), (1, 2), (2, 1), (2, 2), (3, 3), (4, 4), (3, 2), (2, 3), (1, 3), (3, 1)}

⇒ R must have minimum 10 elements.

Q.80 If $\log_e a$, $\log_e b$, $\log_e c$ are in an A.P. and $\log_e a - \log_e 2b$, $\log_e 2b - \log_e 3c$, $\log_e 3c - \log_e a$ are also in an A.P., then a : b : c is equal to

(1) 25 : 10 : 4 (2) 9 : 6 : 4 (3) 16 : 4 : 1 (4) 6 : 3 : 2

Ans. **[2]**

Sol. $\ln a, \ln b, \ln c \rightarrow$ A.P.

$\Rightarrow 2\ln b = \ln a + \ln c$

$\Rightarrow b^2 = ac$

$\ln a - \ln 2b, \ln 2b - \ln 3c, \ln 3c - \ln a \rightarrow$ A.P.

$$\Rightarrow 2\ln\left(\frac{2b}{3c}\right) = \ln\left(\frac{a}{2b}\right) + \ln\left(\frac{3c}{a}\right) = \ln\left[\left(\frac{a}{2b}\right)\left(\frac{3c}{a}\right)\right]$$

$$\Rightarrow \frac{4b^2}{9c^2} = \left(\frac{a}{2b}\right)\left(\frac{3c}{a}\right) = \frac{3c}{2b}$$

$\Rightarrow 8b^3 = 27c^5$

$\Rightarrow 2b = 3c$

$$\frac{9c^2}{4} = ac \Rightarrow c = \frac{4}{9}a$$

$$c = \frac{2}{3}b$$

$$a : b : c = \frac{9}{4}c : \frac{3}{2}c : c$$

$$a : b : c \Rightarrow = \frac{9}{4} : \frac{3}{2} : 1 \Rightarrow a : b : c \Rightarrow 9 : 6 : 4$$

Section-B: Numerical Value Type Questions: This section contains 10 Numerical based questions. Attempt any 5 questions out of 10. The answer to each question should be rounded-off to the nearest integer.

Q.81 Let the set $C = \{(x, y) \mid x^2 - 2^y = 2023, x, y \in N\}$ Then $\sum_{(x,y)\in C}(x + y)$ is equal to _______.

Ans. **[46]**

Sol. $C = \{(x, y) \mid x^2 - 2^y = 2023, x, y \in N\}$

$x^2 - 2023 = 2^y$

$x^2 - 2025 = 2^y - 2$

$\Rightarrow x^2 - 45^2 = 2^y - 2$

$= (x - 45)(x + 45) = 2(2^{y-1} - 1)$

For $x \in$ even, no solution

$x \in$ odd, $\Rightarrow x = 2m + 1$

$(2m - 44)(2m + 46) = 2(2^{y-1} - 1)$

$\Rightarrow 4(m - 22)(m + 23) = 2\underbrace{(2^{y-1} - 1}_{\text{odd Number}}$

$\Rightarrow$ No solution form

$\Rightarrow x = 45$ or $x = -45$

$\Rightarrow 2^{y-1} = 1 \Rightarrow y = 1$

(45, 1), (– 45, 1) solution

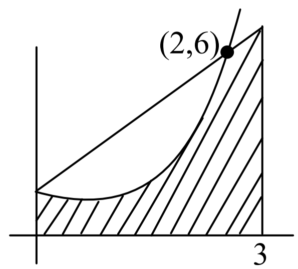
$\because$ $x, y \in N \Rightarrow x + y = 45 + 1 = 46$

Q.82 Let the area of the region $\{(x, y) : 0 \le x \le 3, 0 \le y \le \min\{x^2 + 2, 2x + 2\}\}$ be A. Then 12A is equal to ______.

Ans. **[164]**

Sol. $\{(x, y): 0 \le x \le 3, 0 \le y \le \min\{x^2 + 2, 2x + 2\}\}$

$$\min(x^2 + 2, 2x + 2) = \begin{cases} x^2 + 2 & 0 \le x \le 2 \\ 2x + 2 & 2 \le x \le 3 \end{cases}$$



$$A = \int_0^2 (x^2 + 2)dx + \frac{1}{2}[6 + 8] \times 1 = 164$$

Q.83 Let O be the origin, and M and N be the points on the lines $\frac{x-5}{4} = \frac{y-4}{1} = \frac{z-5}{3}$ and $\frac{x+8}{12} = \frac{y+2}{5} = \frac{z+11}{9}$ respectively such that MN is the shortest distance between the given lines. Then $\overrightarrow{OM} \cdot \overrightarrow{ON}$ is equal to _______.

Ans. **[9]**

Sol. $L_1 : \frac{x-5}{4} = \frac{y-4}{1} = \frac{z-5}{3} = \lambda$

Any point of L_1 : $(4\lambda + 5, \lambda + 4, 3\lambda + 5)$: M

$L_2 : \frac{x+8}{12} = \frac{y+2}{5} = \frac{z+11}{9} = \mu$

Any point on L_2 : $(12\mu - 8, 5\mu - 2, 9\mu - 11)$: N

DR of MN $< 4\lambda - 12\mu + 13, \lambda - 5\mu + 6, 3\lambda - 9\mu + 16 >$

Now

$4(4\lambda - 12\mu + 13) + (\lambda - 5\mu + 6) + 3(3\lambda - 9\mu + 16)$

$= 0$ and

$12(4\lambda - 12\mu + 13) + 5(\lambda - 5\mu + 6) + 9(3\lambda - 9\mu + 16) = 0$

$\Rightarrow \lambda = -1$ and $\mu = 1$

$\therefore$ M(1, 3, 2) and N(4, 3, –2)

$OM \cdot ON = 1 \cdot (4) + 3 \cdot (3) + 2(-2)$

$= 4 + 9 - 4 = 9$

Q.84 If $\int_{\pi/6}^{\pi/3} \sqrt{1 - \sin 2x}\, dx = \alpha + \beta\sqrt{2} + \gamma\sqrt{3}$, where α, β and γ are rational numbers, then $3\alpha + 4\beta - \gamma$ is equal to_______.

Ans. **[6]**

Sol. $= \int_{\pi/6}^{\pi/3} \sqrt{1-\sin 2x}\, dx = \int_{\pi/6}^{\pi/3} |\sin x - \cos x|\, dx$

$= \int_{\pi/6}^{\pi/4} (\cos x - \sin x)dx + \int_{\pi/4}^{\pi/3} (\sin x - \cos x)dx$

$2\sqrt{2} - \sqrt{3} - 1$

$\Rightarrow \alpha = -1 \quad \beta = 2, \qquad \gamma = -1$

$\therefore 3\alpha + 4\beta - \gamma = 3(-1) + 4(2) - (-1)$

$= 6$

Q.85 Remainder when $64^{32^{32}}$ is divided by 9 is equal to __________.

Ans. **[1]**

Sol. Remainder $64^{32^{32}}$ when divided by 9.

$64 \equiv 1 \pmod 9$

$64^{32^{32}} = 1^{32^{32}} \pmod 9$

$\Rightarrow$ Remainder = 1

Q.86 Let the slope of the line $45x + 5y + 3 = 0$ be $27r_1 + \frac{9r_2}{2}$ for some $r_1, r_2 \in R$. Then $\lim_{x\to 3}\left(\int_3^x \frac{8t^2}{\frac{3r_2 x}{2} - r_2x^2 - r_1x^3 - 3x} dt\right)$ is equal to _________.

Ans. **[12]**

Sol. $27r_1 + \frac{9r_2}{2} = -9$

$54r_1 + 9r_2 + 18 = 0$

$\ell = \lim_{x\to 3}\left(\int_3^x \frac{8t^2 dt}{\frac{3r_2 x}{2} - r_2x^2 - r_1x^3 - 3x}\right)$

$= \lim_{x\to 3} \frac{\int_3^x 8t^2 dt}{\frac{3r_2 x}{2} - r_2x^2 - r_1x^3 - 3x}$ form : $\frac{0}{0}$

Using L-H rule

$\lim_{x\to 3} \frac{8x^2}{\frac{3r_2}{2} - 2r_2x - 3r_1x^2 - 3} = \frac{8\times 9\times 2}{-9r_2 - 54r_1 - 6} = 12$

Q.87 Let $f(x) = \sqrt{\lim_{r\to x}\left\{\frac{2r^2[(f(r))^2 - f(x)f(r)]}{r^2 - x^2} - r^3 e^{\frac{f(r)}{r}}\right\}}$ be differentiable in $(-\infty, 0) \cup (0, \infty)$ and f(1) = 1. Then the value of ea, such that f(a) = 0, is equal to ________.

Ans. [2]

Sol. $f(x) = \sqrt{\lim_{r\to x}\left(\frac{2r^2 f(r)[f(r) - f(x)]}{(r+x)(r-x)} - r^3 e^{\frac{f(r)}{r}}\right)}$

$f(x) = \sqrt{\frac{2x^2 f(x) f'(x)}{2x} - x^3 e^{\frac{f(x)}{x}}}$

$\Rightarrow y^2 = xy\frac{dy}{dx} - x^3 e^{\frac{y}{x}}$

$\Rightarrow \frac{y^2}{x^2} = \frac{y}{x}\frac{dy}{dx} - xe^{\frac{y}{x}}$

Put y = tx

$\Rightarrow \frac{dy}{dx} = t + x\frac{dt}{dx}$

$t^2 = t^2 + tx\frac{dt}{dx} - xe^t$

$t\frac{dt}{dx} = e^t$

$dx = te^{-t}dt$

$\Rightarrow x = -e^{-t}(t+1) + c$

$x = -e^{-\frac{y}{x}}\left(\frac{y}{x}+1\right) + c \quad \because f(1) = 1 \Rightarrow c = 1 + \frac{2}{e}$

f(a) = 0)

$a = -1(0+1) + 1 + \frac{2}{e}$

$\Rightarrow ae = 2$

Q.88 Let P (α, β) be a point on the parabola $y^2 = 4x$. If P also lies on the chord of the parabola $x^2 = 8y$ whose mid-point is $\left(1, \frac{5}{4}\right)$, then (α – 28) (β – 8) is equal to _______.

Ans. [192]

Sol. P (α, β) lies on $y^2 = 4x \Rightarrow \beta^2 = 4\alpha$ ....(i)

Equation of chord of $x^2 = 8y$ whose mid-point is $\left(1, \frac{5}{4}\right)$ is

$T = S_1$

$x \times 1 - 4\left(y + \frac{5}{4}\right) = 1^2 - 8 \cdot \frac{5}{4}$

$x - 4y - 5 = -9$

$x - 4y + 4 = 0$

$\because$ (α, β) lies on $x - 4y + 4 = 0$

$\Rightarrow \alpha - 4\beta + 4 = 0$ …(ii)

$\frac{\beta^2}{4} - 4\beta + 4 = 0$

$\beta^2 - 16\beta + 16 = 0$

$(\beta - 8)^2 = 48 \Rightarrow \beta = 8 \pm 4\sqrt{3}$

$\because$ Intersection point of $x^2 = 8y$ and $x - 4y + 4 = 0$

are $\left(-2, \frac{1}{2}\right)$ and $(4, 2)$

$\Rightarrow \alpha \in (-2, 4),\ \beta \in \left(\frac{1}{2}, 2\right)$

$\Rightarrow \beta = 8 - 4\sqrt{3} \Rightarrow (\beta - 8) = -4\sqrt{3}$

$\Rightarrow \alpha = 32 - 16\sqrt{3} - 4$

$\alpha = 28 - 16\sqrt{3} \Rightarrow (\alpha - 28) = -16\sqrt{3}$

$\Rightarrow (\alpha - 28)(\beta - 8) = 192$

Q.89 Let α, β be the roots of the equation $x^2 - \sqrt{6}x + 3 = 0$ such that $\text{Im}(\alpha) > \text{Im}(\beta)$. Let a, b be integers not divisible by 3 and n be a natural number such that $\frac{\alpha^{99}}{\beta} + \alpha^{98} = 3^n(a + ib)$, $i = \sqrt{-1}$. Then n + a + b is equal to ________.

Ans. **[49]**

Sol. $x^2 - \sqrt{6}x + 3 = 0$

$x = \frac{\sqrt{6} \pm \sqrt{6 - 12}}{2}$, $\alpha + \beta = \sqrt{6}$, $\alpha\beta = 3$

$x = \frac{\sqrt{6}}{2} \pm \frac{\sqrt{6}}{2} i$

$\alpha = \frac{\sqrt{6}}{2}(1 + i),\ \beta = \frac{\sqrt{6}}{2}(1 - i)$

$S = \frac{\alpha^{99}}{\beta} + \alpha^{98}$

$\Rightarrow = \alpha^{99}\left(\frac{1}{\beta} + \frac{1}{\alpha}\right)$

$= \alpha^{99}\left(\frac{\alpha + \beta}{\alpha\beta}\right)$

$= \alpha^{99}\left(\frac{\sqrt{6}}{3}\right)$

$\frac{(\sqrt{6})^{100}}{2^{99} \cdot 3} \times (1 + i)^{99}$

$\because\ (1 + i) = \sqrt{2}e^{i\frac{\pi}{4}}$

$\Rightarrow (1+i)^{99} = (\sqrt{2})^{99}\ e^{i\frac{99\pi}{4}}$

$= (\sqrt{2})^{99}\left(-\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}i\right)$

$\Rightarrow S = \frac{(\sqrt{6})^{100}}{3.2^{99}} \times (\sqrt{2})^{98}\ (-1+i)$

$= \frac{(\sqrt{3})^{100}}{3}\ (-1+i) = 3^{49}(-1+i)$

$\Rightarrow\ \ n = 49, a = -1, b = 1$

$\Rightarrow\ \ n + a + b = 49$

Q.90 Let for any three distinct consecutive terms a, b, c of an A.P., the lines ax + by + c = 0 be concurrent at the point P and Q(α, β) be a point such that the system of equations.

$x + y + z = 6,$

$2x + 5y + \alpha z = \beta$ and

$x + 2y + 3z = 4$, has infinitely many solutions. Then

$(PQ)^2$ is equal to ________.

Ans. **[113]**

Sol. $b - a = c - b$

$a - 2b + c = 0$

$\Rightarrow$ ax + by + c = 0 are concurrent at P(1, – 2)

$\because\ \ x + y + z = 6$

$2x + 5y + \alpha z = \beta,$

$x + 2y + 3z = 4$

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 5 & \alpha \\ 1 & 2 & 3 \end{vmatrix} = 0$$

$\Rightarrow 8 - \alpha = 0 \Rightarrow\ \alpha = 8$

$$D_1 = \begin{vmatrix} 6 & 1 & 1 \\ \beta & 5 & \alpha \\ 4 & 2 & 3 \end{vmatrix} = 0$$

$\Rightarrow -8\alpha - \beta + 70 = 0$

$\beta = 6$

$\Rightarrow$ Q(8, 6) P(1, –2)

$\Rightarrow (PQ)^2 = 113$