CAREER POINT

JEE Main Online Exam 2024

Questions & Solution

29th January 2024 | Morning

PHYSICS

Section-A: This section contains 20 multiple choice questions. Each question has 4 choices(1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Q.1 The deflection in moving coil galvanometer falls from 25 divisions to 5 division when a shunt of 24 Ω is applied. The resistance of galvanometer coil will be :

(1) 100 Ω (2) 12 Ω (3) 96 Ω (4) 48 Ω

Ans. **[3]**

Sol. i ∝ number of divisions

$\therefore\ 5(R_G) = (25 - 5)(24)$

$R_G = 96\ \Omega$

Q.2 Match List-I with List-II

	List-I		List-II
a.	$\oint \vec{B}.\overrightarrow{d\ell} = \mu_0 i_c + \mu_0 \varepsilon_0 \frac{d\phi_E}{dt}$	(i)	Gauss law for electricity
b.	$\oint \vec{E}.\overrightarrow{d\ell} = \frac{d\phi_B}{dt}$	(ii)	Gauss's law for magnetism
c.	$\oint \vec{E}.\overrightarrow{dA} = \frac{Q}{\varepsilon_0}$	(iii)	Faraday law
d.	$\oint \vec{B}.\overrightarrow{dA} = 0$	(iv)	Ampere-Maxwell law

Choose the correct answer from the options given below :

(1) a(iv), b(iii), c(i), d (ii) (2) a(ii), b(iii), c(i), d(iv)

(3) a(iv), b(i), c(iii), d(ii) (4) a(i), b(ii), c(iii), d(iv)

Ans. **[1]**

Sol. $\oint \vec{E}.\overrightarrow{dx} = \frac{Q}{\varepsilon_0}$ (Gauss'a law)

$\oint \vec{E}.\overrightarrow{d\ell} = \frac{d\phi_B}{dt}$ (Faraday law)

a(iv), b(iii), c(i), d(ii)

Q.3 At what distance above and below the surface of the earth a body will have same weight. (take radius of earth as R.)

(1) $\frac{\sqrt{5}R - R}{2}$ (2) $\sqrt{5}R - R$ (3) $\frac{R}{2}$ (4) $\frac{\sqrt{3}R - R}{2}$

Ans. **[1]**

Sol. $g\frac{R^2}{(R+h)^2} = g\left(1 - \frac{h}{R}\right)$

$h = \left(\frac{\sqrt{5}-1}{2}\right)R$

Q.4 A biconvex lens of refractive index 1.5 has a focal length of 20 cm in air. Its focal length when immersed in a liquid of refractive index 1.6 will be :

(1) +16 cm (2) +160 cm (3) –160 cm (4) –16 cm

Ans. **[3]**

Sol. $\frac{1}{f_{air}} = (1.5-1)\left(\frac{2}{R}\right) \Rightarrow R = 20\text{ cm}$

$\frac{1}{f_{liq}} = \left(\frac{1.5}{1.6}-1\right)\left(\frac{2}{20}\right)$

$f_{liq} = -160\text{ cm}$

Q.5 Two vessels A and B are of the same size and are at same temperature. A contains 1 g of hydrogen and B contains 1 g of oxygen. P_A and P_B are the pressure of the gases in A and B respectively, then $\frac{P_A}{P_B}$ is-

(1) 4 (2) 32 (3) 8 (4) 16

Ans. **[4]**

Sol. $\because$ PV = nRT

$n_A = \frac{1}{2}, n_B = \frac{1}{32}$

$\frac{P_A}{P_B} = \frac{32}{2} = 16$

Q.6 The explosive in a hydrogen bomb is a mixture of ${}_1H^2$, ${}_1H^3$ and ${}_3Li^6$ in some condensed form. The chain reaction is given by

${}_3Li^6 + {}_0n^1 \rightarrow {}_2He^4 + {}_1H^3$

${}_1H^2 + {}_1H^3 \rightarrow {}_2He^4 + {}_0n^1$

During the explosion the energy released is approximately

[Given : M(Li) = 6.01690 amu, $M({}_1H^2)$ = 2.01471 amu, $M({}_2He^4)$ = 4.00388 amu, and 1 amu = 931.5 MeV]

(1) 22.22 MeV (2) 16.48 MeV (3) 28.12 MeV (4) 12.64 MeV

Ans. **[1]**

Sol. Q value = energy released

$= c^2[m({}_3Li^6) + m({}_0n^1) - m({}_2He^4) - m({}_1H^3) + m({}_1H^2) + m({}_1H^3) - m({}_2He^4) - m({}_0n^1)]$

= 22.22 MeV

Q.7 If the radius of curvature of the path of two particles of same mass are in the ratio 3 : 4, then in order to have constant centripetal force, their velocities will be in the ratio of :

(1) $\sqrt{3}$: 1 (2) 2 : $\sqrt{3}$ (3) $\sqrt{3}$: 2 (4) 1 : $\sqrt{3}$

Ans. **[3]**

Sol. $\frac{mv_1^2}{R_1} = \frac{mv_2^2}{R_2}$

$\Rightarrow \frac{v_1}{v_2} = \sqrt{\frac{R_1}{R_2}} = \frac{\sqrt{3}}{2}$

Q.8 A block of mass 100 kg slides over a distance of 10 m on a horizontal surface. If the co-efficient of friction between the surface is 0.4, then the work done against friction (in J) is :

(1) 4200 (2) 4000 (3) 4500 (4) 3900

Ans. **[2]**

Sol. $W_f = |\mu mg\, x|$

$= 0.4 \times 100 \times 10 \times 10$

= 4000 J

Q.9 The de-Broglie wavelength of an electron is the same as that of a photon. If velocity of electron is 25% of the velocity of light, then the ratio of K.E. of electron and K.E. of photon will be

(1) $\frac{1}{8}$ (2) $\frac{1}{1}$ (3) $\frac{1}{4}$ (4) $\frac{8}{1}$

Ans. **[1]**

Sol. $\frac{v_e}{v_p} = \frac{1}{4}$ and $p_e = p_p$

$$\therefore \frac{KE_e}{KE_p} = \frac{\frac{1}{2}(p_e)v_e}{(p_p)v_p} = \frac{1}{8}$$

Q.10 A galvanometer having coil resistance 10 Ω shows a full scale deflection for a current of 3mA. For it to measure a current of 8A, the value of the shunt should be :

(1) $3 \times 10^{-3}\Omega$ (2) $4.85 \times 10^{-3}\Omega$ (3) $3.75 \times 10^{-3}\Omega$ (4) $2.75 \times 10^{-3}\Omega$

Ans. **[3]**

Sol. $i_G(G) = (i - i_G)\, s$

$\therefore s = 3.75 \times 10^{-3}\ \Omega$

Q.11 The resistance $R = \frac{V}{I}$ where V = (200 ± 5) V and I = (20 ± 0.2) A, the percentage error in the measurement of R is-

(1) 3.5 % (2) 3% (3) 5.5% (4) 7%

Ans. **[1]**

Sol. $\frac{\Delta R}{R} = \frac{\Delta V}{V} + \frac{\Delta I}{I}$

$\therefore \frac{\Delta R}{R} \times 100 = 3.5\ \%$

Q.12 Two charges of 5Q and –2Q are situated at the points (3a, 0) and (–5a, 0) respectively. The electric flux through a sphere of radius '4a' having center at origin is-

(1) $\frac{2Q}{\varepsilon_0}$ (2) $\frac{3Q}{\varepsilon_0}$ (3) $\frac{7Q}{\varepsilon_0}$ (4) $\frac{5Q}{\varepsilon_0}$

Ans. **[4]**

Sol. $\phi = \frac{q_{enclosed}}{\varepsilon_0}$

$= \frac{5Q}{\varepsilon_0}$

Q.13 Given below are two statements:

Statement I: If a capillary tube is immersed first in cold water and then in hot water, the height of capillary rise will be smaller in hot water.

Statement II: If a capillary tube is immersed first in cold water and then in hot water, the height of capillary rise will be smaller in cold water.

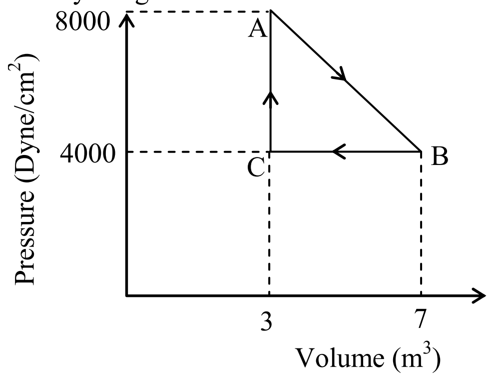
In the light of the above statements, choose the most appropriate from the options given below

(1) Statement I is false but Statement II is true
(2) Both Statement I and Statement II are true
(3) Both Statement I and Statement II are false
(4) Statement I is true but Statement II is false

Ans. **[4]**

Sol. As temperature increases, surface tension decreases. And [$h \propto$ surface tension]

Q.14 A thermodynamic system is taken from an original state A to an intermediate state B by a linear process as shown in the figure. It's volume is then reduced to the original value from B to C by an isobaric process. The total work done by the gas from A to B and B to C would be:



(1) 1200 J (2) 2200 J (3) 600 J (4) 33800 J

Ans. **[None]**

Sol. Work done = $\frac{1}{2}(4)(4000) \times 10^{-1}$ J

$= 800$ J

No option is matching

Q.15 A convex mirror of radius of curvature 30 cm forms an image that is half the size of the object. The object distance is-

(1) 15 cm (2) 45 cm (3) –45 cm (4) –15 cm

Ans. **[4]**

Sol. $m = \frac{1}{2} = \frac{-v}{u}$

$\Rightarrow v = -\frac{u}{2}$

Now

$\frac{1}{v} + \frac{1}{u} = \frac{1}{15}$

$\frac{-2}{u} + \frac{1}{u} = \frac{1}{15}$

$\boxed{u = -15 \text{ cm}}$

Q.16 A body starts moving from rest with constant acceleration covers displacement S_1 in first (p – 1) seconds and S_2 in first p seconds. The displacement $S_1 + S_2$ will be made in time:

(1) $(2p^2 - 2p + 1)$s (2) $(2p + 1)$ (3) $\sqrt{(2p^2 - 2p + 1)s}$ (4) $(2p - 1)$s

Ans. **[3]**

Sol. $S_1 = \frac{1}{2}a(p-1)^2$

$S_2 = \frac{1}{2}a(p)^2$

$\therefore S_1 + S_2 = \frac{1}{2}a(p^2 + 1 - 2p + p^2)$

$= \frac{1}{2}at^2$

$\therefore t = \sqrt{2p^2 - 2p + 1}$

Q.17 A capacitor of capacitance 100 μF is charged to a potential of 12 V and connected to a 6.4 mH inductor to produce oscillations. The maximum current in the circuit would be:

(1) 2.0 A (2) 1.2 A (3) 1.5 A (4) 3.2 A

Ans. [3]

Sol. $i = Q\omega$

$$= CV\sqrt{\frac{1}{LC}}$$

$$= \sqrt{\frac{C}{L}}V = \sqrt{\frac{10^{-2}}{64\times10^{-4}}}(12) = 1.5\text{ A}$$

Q.18 The potential energy function (in J) of a particle in a region of space is given as $U = (2x^2 + 3y^2 + 2z)$. Here x, y and z are in meter. The magnitude of x component of force (in N) acting on the particle at point P(1, 2, 3)m is:

(1) 6 (2) 8 (3) 2 (4) 4

Ans. [4]

Sol. $F_x = \frac{-\partial u}{-\partial x} = -4x$

$\therefore$ magnitude = 4 N

Q.19 The electric current through a wire varies with time as $I = I_0 + \beta t$, where $I_0 = 20$ A and $\beta = 3$ A/s. The amount of electric charge crossed through a section of the wire in 20 s is:

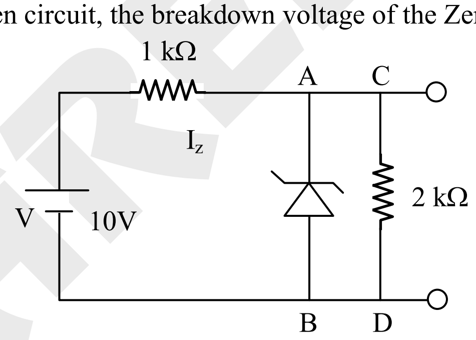
(1) 800 C (2) 1600 C (3) 80 C (4) 1000 C

Ans. [4]

Sol. $\Delta q = \int i\,dt$

$$= \int_0^{20}(20+3t)\,dt = \left[20t + \frac{3t^2}{2}\right]_0^{20} = 1000\text{ C}$$

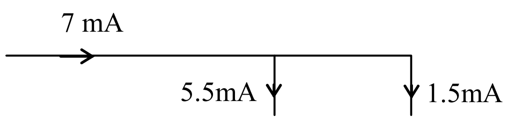
Q.20 In the given circuit, the breakdown voltage of the Zener diode is 3.0 V. What is the value of I_z?



(1) 7 mA (2) 5.5 mA (3) 3.3 mA (4) 10 mA

Ans. [2]

Sol.



$$i_Z = \left(\frac{7V}{1\,k\Omega} - \frac{3V}{2\,k\Omega}\right) = 5.5\text{ mA}$$

Section-B: Numerical Value Type Questions: This section contains 10 Numerical based questions. Attempt any 5 questions out of 10. The answer to each question should be rounded-off to the nearest integer.

Q.21 In a test experiment on a model aeroplane in wind tunnel, the flow speeds on the upper and lower surfaces of the wings are 70 ms^{-1} and 65 ms^{-1} respectively. If the area is 2 m^2, the lift of the wing is ________N. (Given density of air = 1.2 kg m^{-3})

Ans. **[810]**

Sol. $F = \Delta P(A)$

$= \frac{1}{2}\rho(v_u^2 - v_L^2)(A)$

$= \frac{1}{2}1.2(70^2 - 65^2)2$

$= 810$ N

Q.22 A cylinder is rolling down on an inclined plane of inclination 60°. It's acceleration during rolling down will be $\frac{x}{\sqrt{3}}$ m/s^2, where x = __________ (use g = 10 m/s^2)

Ans. **[10]**

Sol. $a = \frac{g\sin\theta}{1+\frac{I}{mR^2}}$ $\left(\because I = \frac{mR^2}{2}\right)$

$= \frac{2g\sin 60^\circ}{3}$

$= \frac{10}{\sqrt{3}}$ m/s^2

Q.23 A ball rolls off the top of a stairway with horizontal velocity u. The steps are 0.1 m high and 0.1 m wide. The minimum velocity u with which that ball just hits the step 5 of the stairway will be $\sqrt{x}$ ms^{-1} where x = _______ [use g = 10 m/s^2]

Ans. **[2]**

Sol. To hit 5^{th} step it should just cross the 4th step.

$\therefore$ Range = 0.4 m, Height = 0.4 m.

$\therefore R = u\sqrt{\frac{2H}{g}}$

$0.4 = u\sqrt{\frac{2(0.4)}{10}}$

$u = \sqrt{2}$ m/s

Q.24 The magnetic potential due to a magnetic dipole at a point on its axis situated at a distance of 20 cm from its center is 1.5×10^{-5} T m. The magnetic moment of the dipole is ______ A m^2. (Given : $\frac{\mu_0}{4\pi} = 10^{-7}$ T mA^{-1})

Ans. **[6]**

Sol. $V_{axial} = \frac{\mu_0}{4\pi}\frac{M}{r^2}$

$\therefore 1.5 \times 10^{-5} = 10^{-7}\frac{M}{(20\times10^{-2})^2}$

$\Rightarrow M = 6$ Am^2

Q.25 A electron is moving under the influence of the electric field of a uniformly charged infinite plane sheet S having surface charge density +σ. The electron at t = 0 is at a distance of 1 m from S and has a speed of 1 m/s. The maximum value of σ if the electron strikes S at t = 1 s is $\alpha\left[\frac{m\in_0}{e}\right]\frac{C}{m^2}$, the value of α is ________.

Ans. **[8]**

Sol. e^- should be thrown away from sheet.

$\because s = ut + \frac{1}{2}at^2$

$-1 = 1(1) + \frac{1}{2}\left(\frac{-\sigma e}{2\in_0 m}\right)(1)^2$

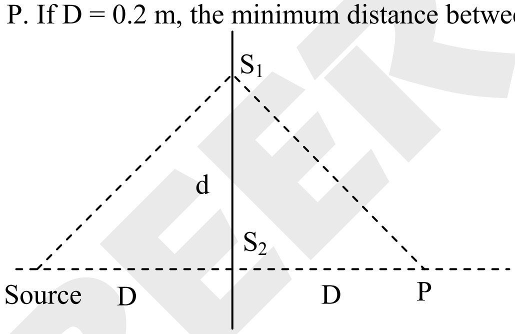
$\therefore \sigma = \frac{8\in_0 m}{e}$

Q.26 When the displacement of a simple harmonic oscillator is one third of its amplitude, the ratio of total energy to the kinetic energy is $\frac{x}{8}$, where x = ________.

Ans. **[9]**

Sol. $\frac{T.E}{K.E} = \frac{\frac{1}{2}KA^2}{\frac{1}{2}K(A^2 - x^2)} = \frac{9}{8}$

Q.27 In a double slit experiment shown in figure, when light of wavelength 400 nm is used, dark fringe is observed at P. If D = 0.2 m, the minimum distance between the slits S_1 and S_2 is __________ mm.



Ans. **[0.2]**

Sol. $\Delta x = \frac{\lambda}{2}$ (for min. d)

$2\left[\sqrt{D^2 + d^2} - D\right] = 200 \times 10^{-9}$

$D\left(1 + \frac{d^2}{2D^2}\right) - D = 10^{-7}$

$\frac{d^2}{20} = 10^{-7}$

$\Rightarrow d^2 = 2 \times (0.2) \times 10^{-7} = 4 \times 10^{-8}$ m

$d = 2 \times 10^{-4}$ m = 0.2 mm

Q.28 A 16 Ω wire is bend to form a square loop. A 9 V battery with internal resistance 1 Ω is connected across one of its sides. If a 4 μF capacitor is connected across one of its diagonals, the energy stored by the capacitor will be $\frac{x}{2}$ μ J. Where x = __________.

Ans. **[81]**

Sol. $i_{battery} = \frac{9}{3+1} = \frac{9}{4}A$

$(\Delta V)_{cap.} = \frac{9}{4}\left(\frac{1}{4}\right)(8) = \frac{9}{2}V$

$\Delta U = \frac{1}{2}CV^2$

$= \frac{1}{2}(4)\left(\frac{9}{2}\right)^2 \mu J = \frac{81}{2} \mu J$

x = 81

Q.29 When a hydrogen atom going from n = 2 to n = 1 emits a photon, its recoil speed is $\frac{x}{5}$ m/s. Where x = _________ . (Use mass of hydrogen atom = 1.6×10^{-27} kg)

Ans. **[17]**

Sol. $\Delta E = 13.6\left(1-\frac{1}{4}\right) = \frac{3(13.6)}{4}$ eV

now, $v = \frac{p}{m} = \frac{E}{mC}$

$= \frac{\frac{3}{4}(13.6)\times 1.6\times 10^{-19}}{3\times 10^8 \times 1.6\times 10^{-27}} = \frac{17}{5}$ m/s

Q.30 A square loop of side 10 cm and resistance 0.7 Ω is placed vertically in east-west plane. A uniform magnetic field of 0.20 T is set up across the plane in north east direction. The magnetic field is decreased to zero in 1 s at a steady rate. Then magnitude of induced emf is $\sqrt{x}\times 10^{-3}$ V. The value of x is _________ .

Ans. **[2]**

Sol. $\varepsilon_{ind} = \frac{\Delta\phi}{\Delta t} = \frac{BA\cos 45^o - 0}{\Delta t}$

$= \frac{0.2(10^{-2})\left(\frac{1}{\sqrt{2}}\right)}{1}$

$\boxed{x = 2}$

CHEMISTRY

Section-A: This section contains 20 multiple choice questions. Each question has 4 choices(1), (2), (3) and (4), out of which **ONLY ONE** is correct..

Q.31 Match List I with List II.

	List I (Substances)		List II (Element Present)
A.	Ziegler catalyst	I.	Rhodium
B.	Blood pigment	II.	Cobalt
C.	Wilkinson catalyst	III.	Iron
D.	Vitamin B_{12}	IV.	Titanium

Choose the correct answer from the options given below.

(1) A-II, B-IV, C-I, D-III
(2) A-III, B-II, C-IV, D-I
(3) A-II, B-III, C-IV, D-I
(4) A-IV, B-III, C-I, D-II

Ans. **[4]**

Sol. Ziegler catalyst – $TiCl_4 \cdot Al(C_2H_5)_3$
Blood pigment – Compound containing iron
Wilkinson catalyst – $[RhCl(PPh_3)_3]$
Vitamin B_{12} – Compound containing cobalt
∴ Correct match is A-IV, B-III, C-I, D-II.

Q.32 Given below are two statements :
Statement I : The electronegativity of group 14- elements from Si to Pb, gradually decreases.
Statement II : Group 14 contains non-metallic, metallic, as well as metalloid elements.
In the light of the above statements, choose the most appropriate from the options given below.

(1) Both Statement I and Statement II are true
(2) Both Statement I and Statement II are false
(3) Statement I is true but Statement II is false
(4) Statement I is false but Statement II is true

Ans. **[4]**

Sol. Statement-I is false because electronegativity of Group-14 elements from Si to Pb is almost constant, i.e., 1.8.
Statement-II is correct because Group-14 contains non-metals, metalloids as well as metals.
Carbon – Non-metal
Silicon – Metalloid
Germanium – Metalloid
Tin – Metal
Lead – Metal

Q.33 In chromyl chloride test for confirmation of Cl^- ion, a yellow solution is obtained. Acidification of the solution and addition of amyl alcohol and 10% H_2O_2 turns organic layer blue indicating formation of chromium pentoxide. The oxidation state of chromium in that is

(1) +6 (2) +10 (3) +5 (4) +3

Ans. **[1]**

Sol. The reactions involved in chromyl chloride test and subsequent formation of chromium pentaoxide are as follows :

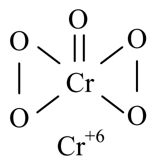
$$Cr_2O_7^{2-} + 6H^+ + 4Cl^- \longrightarrow \underset{\text{(Red)}}{2CrO_2Cl_2(g)} + 3H_2O$$

$$CrO_2Cl_2\,(g) + 4NaOH\,(aq) \longrightarrow \underset{\text{(Yellow)}}{Na_2CrO_4(aq)} + 2NaCl + 2H_2O$$

$$\underset{\text{(Yellow)}}{2\,Na_2CrO_4(aq)} + 2H^+ \longrightarrow \underset{\text{(Orange)}}{Na_2Cr_2O_7(aq)} + 2Na^+ + H_2O$$

$Na_2Cr_2O_7 + 4H_2O_2 + 2H^+ \longrightarrow \underset{\text{(Blue in amyl alcohol)}}{2CrO_5} + 5H_2O + 2Na^+$

Oxidation state of chromium in CrO_5 is +6.



Q.34 Identify the incorrect pair from the following.

(1) Cryolite – Na_3AlF_6
(2) Fluoroapatite – $3Ca_3(PO_4)_2 \cdot CaF_2$
(3) Fluorspar – BF_3
(4) Carnallite – $KCl \cdot MgCl_2 \cdot 6H_2O$

Ans. **[3]**

Sol. The correct pairs are

1. Cryolite – Na_3AlF_6
2. Fluorapatite – $3Ca_3(PO_4)_2 \cdot CaF_2$
3. Fluorspar – CaF_2
4. Carnallite – $KCl \cdot MgC_{l2} \cdot 6H_2O$

Q.35 Given below are two statements : one is labelled as **Assertion A** and the other is labelled as **Reason R.**
Assertion A : Aryl halides cannot be prepared by replacement of hydroxyl group of phenol by halogen atom.
Reason R : Phenols react with halogen acids violently.
In the light of the above statements, choose the **most appropriate** from the options given below.

(1) **A** is false but **R** is true
(2) Both **A** and **R** are true but **R** is NOT the correct explanation of **A**
(3) **A** is true but **R** is false
(4) Both **A** and **R** are true and **R** is the correct explanation of **A**

Ans. **[3]**

Sol. Assertion is true because aryl halides cannot be prepared by replacement of OH group of phenol by halogen atom. Because double bond character present between carbon and oxygen
Reason is false because phenols do not react with halogen acids.

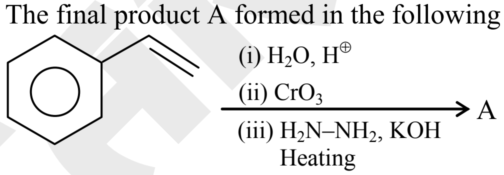
Q.36 Type of amino acids obtained by hydrolysis of proteins is :

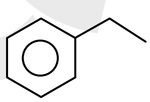
(1) β (2) γ (3) δ (4) α

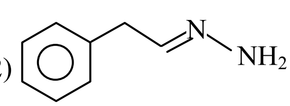
Ans. **[4]**

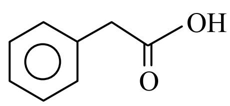
Sol. Hydrolysis of proteins results in the formation of α-amino acids only.

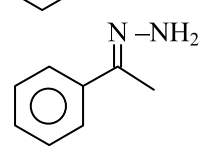
Q.37 The final product A formed in the following multistep reaction sequence is



(1) 

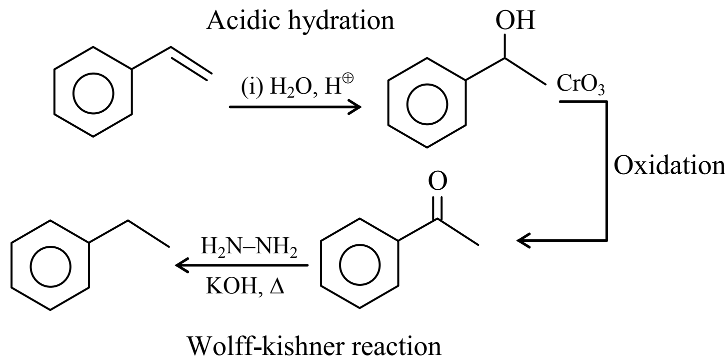
(2) 

(3) 

(4) 

Ans. **[1]**

Sol.



Q.38 Appearance of blood red colour, on treatment of the sodium fusion extract of an organic compound with $FeSO_4$ in presence of concentrated H_2SO_4 indicates the presence of element/s

(1) N and S (2) N (3) Br (4) S

Ans. **[1]**

Sol. Appearance of blood red colour on treatment of sodium fusion extract of an organic compound with $FeSO_4$ in presence of conc. H_2SO_4 is due to the formation of $[Fe(SCN)]^{2+}$

$Na + C + N + S \rightarrow NaSCN$

$Fe^{3+} + SCN^- \rightarrow [Fe(SCN)]^{2+}$ (Blood red)

This indicates the presence of both N and S in the organic compound.

Q.39 The difference in energy between the actual structure and the lowest energy resonance structure for the given compound is

(1) hyperconjugation energy (2) electromeric energy

(3) resonance energy (4) ionization energy

Ans. **[3]**

Sol. The difference in energy between the actual structure and the lowest energy resonance structure of the given compound is resonance energy.

Q.40 In alkaline medium, MnO_4^-, oxidises I^- to

(1) IO^- (2) I_2 (3) IO_3^- (4) IO_4^-

Ans. **[3]**

Sol. In alkaline medium, MnO_4^- oxidises I^- to IO_3^-

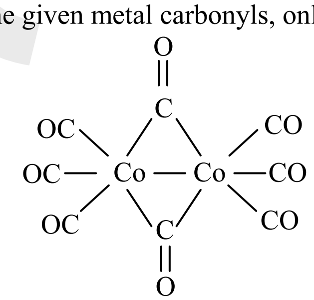
$$2MnO_4^- + I^- + H_2O \longrightarrow 2MnO_2 + IO_3^- + 2OH^-$$

Q.41 In which one of the following metal carbonyls, CO forms a bridge between metal atoms?

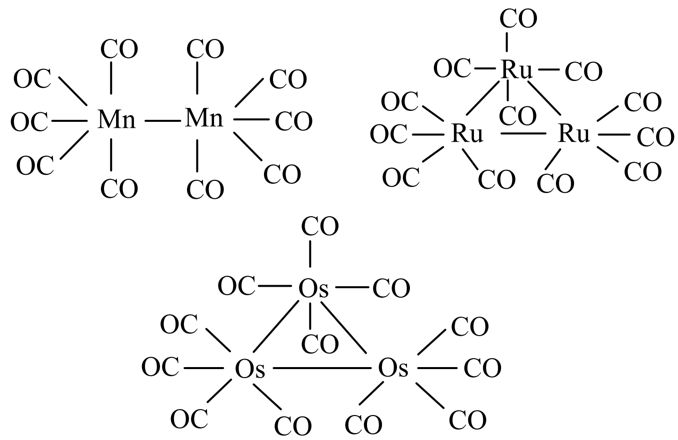
(1) $[Mn_2(CO)_{10}]$ (2) $[Co_2(CO)_8]$ (3) $[Ru_3(CO)_{12}]$ (4) $[Os_3(CO)_{12}]$

Ans. **[2]**

Sol. Among the given metal carbonyls, only $[Co_2(CO)_8]$ shows bridging CO ligands between two metal atoms.



Structures of other metal carbonyls are



Q.42 Chlorine undergoes disproportionation in alkaline medium as shown below :

$aCl_{2(g)} + bOH^{-}_{(aq)} \rightarrow cClO^{-}_{(aq)} + dCl^{-}_{(aq)} + eH_2(O)_{(l)}$

The values of a, b, c and d in a balanced redox reaction are respectively :

(1) 2, 4, 1 and 3 (2) 1, 2, 1 and 1 (3) 2, 2, 1 and 3 (4) 3, 4, 4 and 2

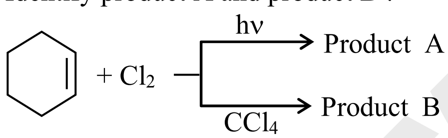
Ans. **[2]**

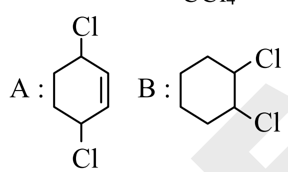
Sol. The balanced disproportionation reaction of C_{l2} in alkaline medium is

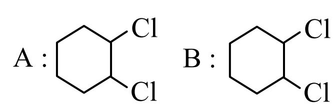
$Cl_2 + 2OH^{-} \rightarrow ClO^{-} + Cl^{-} + H_2O$

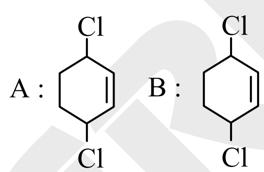
$\therefore$ a = 1, b = 2, c = 1, d = 1

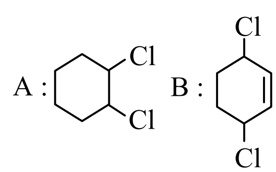
Q.43 Identify product A and product B :



(1) 

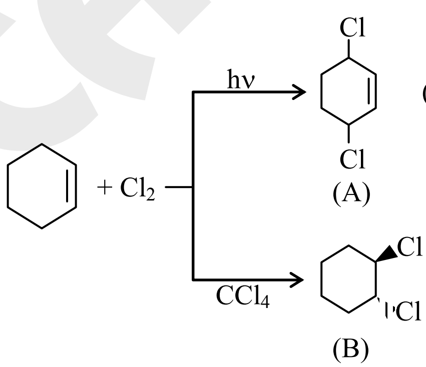
(2) 

(3) 

(4) 

Ans. **[1]**

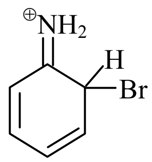
Sol.

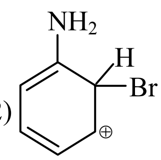


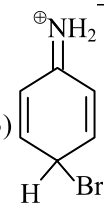
(Formed by free radical mechanism)

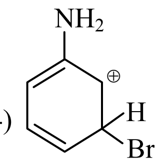
(Formed by electrophilic addition reaction on alkene)

Q.44 The arenium ion which is not involved in the bromination of Aniline is _______.

(1) 

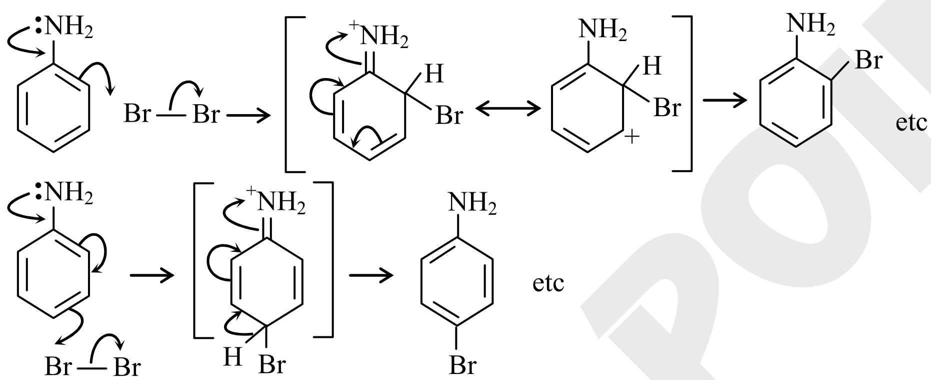
(2) 

(3) 

(4) 

Ans. **[4]**

Sol. Bromination of aniline results in the formation of 2, 4, 6-tribromoaniline because NH_2 group strongly activates the benzene ring particularly at the ortho and para positions. So, arenium ion will be formed when Br_2 attacks the ortho and para positions but not the meta position. Therefore arenium ion having Br atom at the meta position will not be involved in the given reaction.



Q.45 Given below are two statements : one is labelled as **Assertion A** and the other is labelled as **Reason R**

Assertion A: The first ionisation enthalpy decreases across a period.

Reason R: The increasing nuclear charge outweighs the shielding across the period.

In the light of the above statements, choose the **most appropriate** from the options given below :

(1) **A** is true but **R** is false

(2) **A** is false but **R** is true

(3) Both **A** and **R** are true but **R** is NOT the correct explanation of **A**

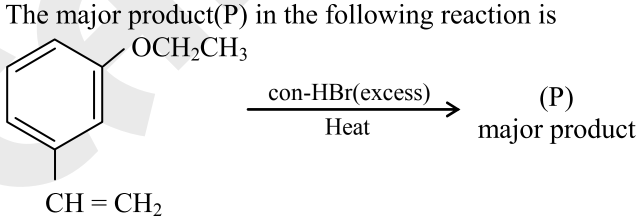
(4) Both **A** and **R** are true and **R** is the correct explanation of **A**

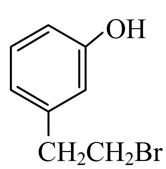
Ans. **[2]**

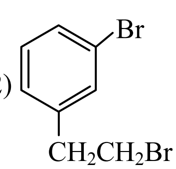
Sol. Assertion is false because the first ionisation enthalpy generally increases across a period barring few exceptions.

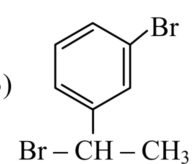
Reason is true because the increase in first ionisation enthalpy across a period is due to the increasing nuclear charge outweighs the shielding effect.

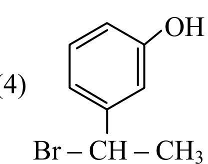
Q.46 The major product(P) in the following reaction is



(1) 

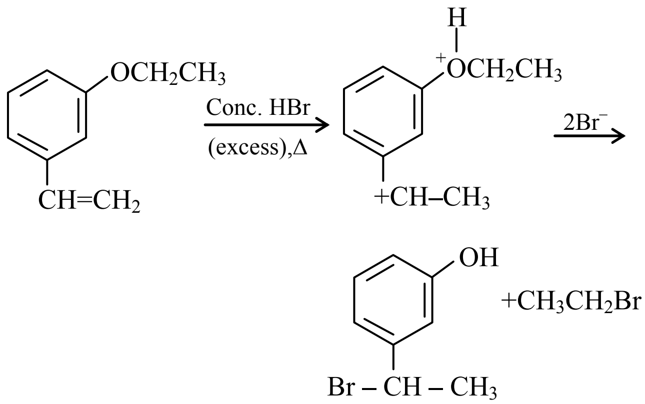
(2) 

(3) 

(4) 

Ans. **[4]**

Sol.



Q.47 Which of the following is not correct?

(1) ΔG is negative for a spontaneous reaction

(2) ΔG is positive for a non-spontaneous reaction

(3) ΔG is zero for a reversible reaction

(4) ΔG is positive for a spontaneous reaction

Ans. **[4]**

Sol. ΔG is negative for a spontaneous process, zero for a reversible process and positive for a non- spontaneous process.

Q.48 The correct set of four quantum numbers for the valence electron of rubidium atom (Z = 37) is:

(1) $5, 0, 0, +\frac{1}{2}$ (2) $5, 0, 1, +\frac{1}{2}$ (3) $5, 1, 0, +\frac{1}{2}$ (4) $5, 1, 1, +\frac{1}{2}$

Ans. **[1]**

Sol. The electronic configuration of Rb-atom (Z = 37) is

$1s^2 2s^2 2p^6 3s^2 3p^6 3d^{10} 4s^2 4p^6 5s^1$

The values of four quantum numbers of the valence electron of Rb-atom are

$n = 5, l = 0, m = 0, s = +\frac{1}{2}$

Q.49 $KMnO_4$ decomposes on heating at 513K to form O_2 along with

(1) K_2MnO_4 & MnO_2 (2) K_2MnO_4 & Mn (3) MnO_2 & K_2O_2 (4) Mn & KO_2

Ans. **[1]**

Sol. $KMnO_4$ decomposes on heating at 513 K as per the following reaction

$2KMnO_4 \xrightarrow{513\text{ K}} K_2MnO_4 + MnO_2 + O_2$

Therefore, K_2MnO_4 and MnO_2 are formed alongwith O_2.

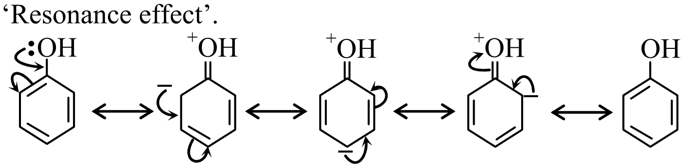
Q.50 The interaction between π bond and lone pair of electrons present on an adjacent atom is responsible for

(1) Hyperconjugation (2) Inductive effect

(3) Resonance effect (4) Electromeric effect

Ans. **[3]**

Sol. The interaction between π bond and lone pair of electrons present on adjacent atom is responsible for 'Resonance effect'.



Section-B: Numerical Value Type Questions: This section contains 10 Numerical based questions. Attempt any 5 questions out of 10. The answer to each question should be rounded-off to the nearest integer.

Q.51 The osmotic pressure of a dilute solution is 7×10^5 Pa at 273K. Osmotic pressure of the same solution at 283K is ______ $\times 10^4$ Nm^{-2}.

Ans. **[73]**

Sol. P_1, osmotic pressure at 273K = 7×10^5 Pa

P_2, osmotic pressure at 283K is given by

$$\frac{P_2}{P_1} = \frac{T_2}{T_1} \qquad (P \propto T)$$

$$P_2 = 7 \times 10^5 \times \frac{283}{273} = 72.56 \times 10^4 \text{ Pa} \simeq 73 \times 10^4 \text{ Nm}^{-2}$$

Q.52 The mass of zinc produced by the electrolysis of zinc sulphate solution with a steady current of 0.015 A for 15 minutes is ______ $\times 10^{-4}$ g. (Atomic mass of zinc = 65.4 amu)

Ans. **[46]**

Sol. I = 0.015 amp

t = 15 × 60 = 900 sec

Q = I × t

= 0.015 × 900 C = 13.5 C

$= \frac{13.5}{96500}$

$Zn^{2+} + 2e \rightarrow Zn$

Number of moles of Zn produced = $\frac{13.5}{2 \times 96500}$

Mass of Zn produced = $\frac{13.5 \times 65.4}{2 \times 96500}$ g

$= 45.75 \times 10^{-4}$ g

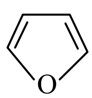
$\simeq 46 \times 10^{-4}$ g

Q.53 Number of compounds among the following which contain sulphur as heteroatom is ______.

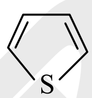
Furan, Thiophene, Pyridine, Pyrrole, Cysteine, Tyrosine

Ans. **[2]**

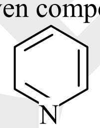
Sol. The structures of the given compounds are



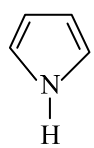
Furan



Thiophene



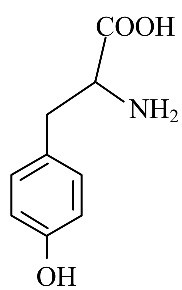
Pyridine



Pyrrole

$HS - CH_2 - CH(NH_2) - COOH$

Cysteine



Tryosine

Number of compounds containing S-atom = 2

Q.54 From the compounds given below, number of compounds which give positive Fehling's test is ______.
Benzaldehyde, Acetaldehyde, Acetone, Acetophenone, Methanal, 4-nitrobenzaldehyde, cyclohexane carbaldehyde.

Ans. **[3]**

Sol. Fehling's test is given by aliphatic aldehydes only.
The following compounds give positive Fehling's test.
Acetaldehyde, Methanal, Cyclohexane carbaldehyde

Q.55 For a reaction taking place in three steps at same temperature, overall rate constant $K = \frac{K_1K_2}{K_3}$. If Ea_1, Ea_2 and Ea_3 are 40, 50 and 60 kJ/mol respectively, the overall Ea is __________ kJ/mol.

Ans. **[30]**

Sol. The rate constant of a three steps reaction is

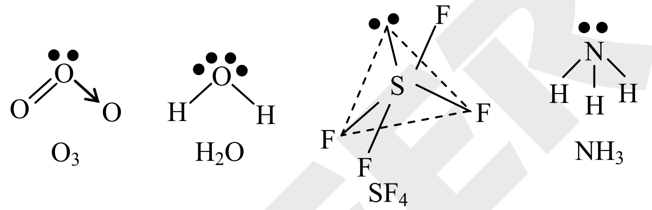
$$K = \frac{K_1K_2}{K_3}$$

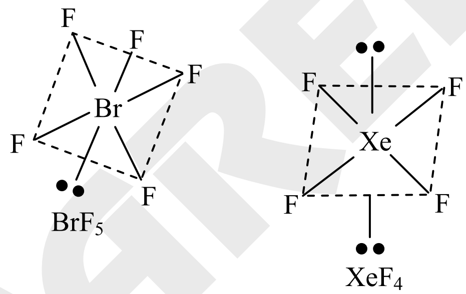
If Ea_1, Ea_2 and Ea_3 are activation energies of three steps respectively then
$Ea_1 = 40\ kJ\ mol^{-1}$, $Ea_2 = 50\ kJ\ mol^{-1}$, $Ea_3 = 60\ kJ\ mol^{-1}$
Using Arrhenius equation,
$K = Ae^{-Ea/RT}$
$Ea = Ea_1 + Ea_2 - Ea_3$
$= 40 + 50 - 60 = 30\ kJ\ mol^{-1}$

Q.56 Number of compounds with one lone pair of electrons on central atom amongst following is ________
O_3, H_2O, SF_4, ClF_3, NH_3, BrF_5, XeF_4

Ans. **[4]**

Sol. The structures of the given compounds are





$\therefore$ No. of compounds with one lone pair of electrons on central atom = 4

Q.57 For the reaction $N_2O_{4(g)} \rightleftharpoons 2NO_{2(g)}$, $K_p = 0.492$ atm at 300 K. K_C for the reaction at same temperature is __________ $\times 10^{-2}$. (Given : $R = 0.082\ L\ atm\ mol^{-1}\ K^{-1}$)

Ans. **[2]**

Sol. $N_2O_4(g) \rightleftharpoons 2NO_2(g)$ $K_p = 0.492$ atm at 300 K
$K_p = K_c(RT)^{\Delta n_g}$
$0.492 = K_c(0.082 \times 300)$ $(\because \Delta n_g = 1)$

$$K_c = \frac{0.492}{24.6} = 2 \times 10^{-2}$$

Q.58 A solution of H_2SO_4 is 31.4% H_2SO_4 by mass and has a density of 1.25g/mL.
The molarity of the H_2SO_4 solution is ______M (nearest integer)
[Given molar mass of H_2SO_4 = 98g mol^{-1}]

Ans. **[4]**

Sol. Density of H_2SO_4 solution = 1.25 $gmol^{-1}$
Mass percentage of H_2SO_4 = 31.4 %
Mass of 1 L H_2SO_4 solution = 1250 g
Mass of H_2SO_4 in 1L solution = $\frac{1250 \times 31.4}{100}$ g
Molarity of H_2SO_4 solution = $\frac{1250 \times 31.4}{98 \times 100}$ = 4.00 M

Q.59 The number of species from the following which are paramagnetic and with bond order equal to one is ______.
H_2, He_2^+, O_2^+, N_2^{2-}, O_2^{2-}, F_2, Ne_2^+, B_2

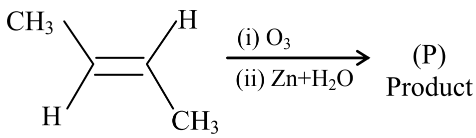
Ans. **[1]**

Sol. According to M.O. theory electronic configurations of the given species are

			Bond order	
H_2	:	σ_{1s}^2	1	Diamagnetic
He_2^+	:	$\sigma_{1s}^2\sigma_{1s}^{*1}$	0.5	Paramagnetic
O_2^+	:	$\sigma_{1s}^2\sigma_{1s}^{*2}\sigma_{2s}^2$ σ_{2s}^{*2} $\sigma_{2p_z}^2$ $\pi_{2p_x}^2$ $\pi_{2p_y}^2$ $\pi_{2p_x}^{*1}$	2.5	Paramagnetic
N_2^{2-}	:	$\sigma_{1s}^2\sigma_{1s}^{*2}\sigma_{2s}^2$ σ_{2s}^{*2} $\sigma_{2p_z}^2$ $\pi_{2p_x}^2$ $\pi_{2p_y}^2$ $\pi_{2p_x}^{*1}$ $\pi_{2p_y}^{*1}$	2.0	Paramagnetic
F_2, O_2^{2-}	:	$\sigma_{1s}^2\sigma_{1s}^{*2}\sigma_{2s}^2$ σ_{2s}^{*2} $\sigma_{2p_z}^2$ $\pi_{2p_x}^2$ $\pi_{2p_y}^2$ $\pi_{2p_x}^{*2}$ $\pi_{2p_y}^{*2}$	1.0	Diamagnetic
Ne_2^+	:	$\sigma_{1s}^2\sigma_{1s}^{*2}\sigma_{2s}^2$ σ_{2s}^{*2} $\sigma_{2p_z}^2$ $\pi_{2p_x}^2$ $\pi_{2p_y}^2$ $\pi_{2p_x}^{*2}$ $\pi_{2p_y}^{*2}$ $\sigma_{2p_z}^{*1}$	0.5	Paramagnetic
B_2	:	$\sigma_{1s}^2\sigma_{1s}^{*2}\sigma_{2s}^2$ σ_{2s}^{*2} $\pi_{2p_x}^1$ $\pi_{2p_y}^1$	1.0	Paramagnetic

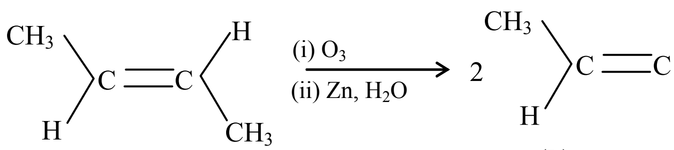
No. of compounds which are paramagnetic and with bond order equal to one is 1 i.e. only Br_2.

Q.60 Consider the given reaction. The total number of oxygen atom/s present per molecule of the product (P) is _______.



Ans. **[1]**

Sol.



(P)

No. of O-atom present per molecule of (P) = 1

MATHEMATICS

Section-A: This section contains 20 multiple choice questions. Each question has 4 choices(1), (2), (3) and (4), out of which **ONLY ONE** is correct..

Q.61 If the value of the integral $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\left(\frac{x^2\cos x}{1+\pi^x}+\frac{1+\sin^2 x}{1+e^{\sin x^{2003}}}\right)dx = \frac{\pi}{4}(\pi+a)-2$, then the value of *a* is-

(1) $\frac{3}{2}$ (2) 3 (3) 2 (4) $-\frac{3}{2}$

Ans. **[2]**

Sol.
$$I=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\left(\frac{x^2\cos x}{1+\pi^x}+\frac{1+\sin^2 x}{1+e^{\sin x^{2003}}}\right)dx$$

$$I=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\left(\frac{x^2\cos x}{1+\pi^{-x}}+\frac{1+\sin^2 x}{1+e^{-\sin x^{2003}}}\right)dx$$

$$2I=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}(x^2\cos x+\sin^2 x+1)\,dx$$

$$2I=2\int_{0}^{\frac{\pi}{2}}(x^2\cos x+\sin^2 x+1)\,dx$$

$$I=\int_{0}^{\frac{\pi}{2}}\left(x^2\cos x+\frac{1-\cos 2x}{2}+1\right)dx$$

$$=\int_{0}^{\frac{\pi}{2}}\left(x^2\cos x+\frac{3}{2}-\frac{\cos 2x}{2}\right)dx$$

$= x^2\sin x - \int 2x\sin x dx + \frac{3}{2}x - \frac{\sin 2x}{4}$

$= x^2\sin x - \left[2x(-\cos x) - \int 2(-\cos x)dx + \frac{3x}{2} - \frac{\sin 2x}{4}\right]$

$= \left[x^2\sin x + 2x\cos x - 2\sin x + \frac{3x}{2} - \frac{\sin 2x}{4}\right]_0^{\frac{\pi}{2}}$

$= \frac{\pi^2}{4} + \frac{3\pi}{4} - 2$

$\Rightarrow a = 3$

Q.62 For $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, if $y(x) = \int \frac{\operatorname{cosec} x + \sin x}{\operatorname{cosec} x \sec x + \tan x \sin^2 x} dx$, and $\lim_{x \to \left(\frac{\pi}{2}\right)^-} y(x) = 0$ then $y\left(\frac{\pi}{4}\right)$ is equal to-

(1) $\frac{1}{2}\tan^{-1}\left(\frac{1}{\sqrt{2}}\right)$

(2) $-\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{1}{\sqrt{2}}\right)$

(3) $\tan^{-1}\left(\frac{1}{\sqrt{2}}\right)$

(4) $\frac{1}{\sqrt{2}}\tan^{-1}\left(-\frac{1}{2}\right)$

Ans. **[4]**

Sol. $y(x) = \int \frac{\cos ex + \sin x}{\cos ecx \sec x + \tan x \sin^2 x} dx$

$= \int \frac{\frac{1}{\sin x} + \sin x}{\frac{1}{\sin x \cos x} + \frac{\sin^3 x}{\cos x}} dx$

$= \int \frac{\frac{1 + \sin^2 x}{\sin x}}{\frac{1 + \sin^4 x}{\sin x \cos x}} dx = \int \frac{(1 + \sin^2 x)\cos x}{1 + \sin^4 x} dx$

Put sinx = t

$\therefore$ cosx dx = dt

$\int \frac{1 + t^2}{1 + t^4} dx = \int \frac{1 + \frac{1}{t^2}}{t^2 + \frac{1}{t^2}} dx$

Let $t - \frac{1}{t} = u$

$\left(1 + \frac{1}{t^2}\right) dt = du$

and $\left(t - \frac{1}{t}\right)^2 = u^2 \Rightarrow t^2 + \frac{1}{t^2} = u^2 + 2$

$\therefore \int \frac{du}{u^2 + 2} = \frac{1}{\sqrt{2}}\tan^{-1}\frac{u}{\sqrt{2}} + c$

$$y(x) = \frac{1}{\sqrt{2}}\tan^{-1}\frac{\left(\sin x - \frac{1}{\sin x}\right)}{\sqrt{2}} + c$$

$$\lim_{x\to\left(\frac{\pi}{2}\right)^-} \frac{1}{\sqrt{2}}\tan^{-1}\frac{\left(\sin x - \frac{1}{\sin x}\right)}{\sqrt{2}} + c = 0$$

$$\lim_{x\to\left(\frac{\pi}{2}\right)^-} \frac{1}{\sqrt{2}}\tan^{-1}\frac{\sin^2 x - 1}{\sqrt{2}\sin x} + c = 0$$

$\Rightarrow c = 0$

$$\therefore\ y(x) = \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{\sin^2 x - 1}{\sqrt{2}\sin x}\right)$$

$$y\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{\frac{1}{2} - 1}{\sqrt{2}\times\frac{1}{\sqrt{2}}}\right)$$

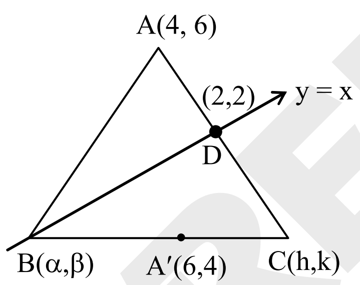
$$= \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{-1}{2}\right)$$

Q.63 In a ΔABC, suppose $y = x$ is the equation of the bisector of the angle B and the equation of the side AC is $2x - y = 2$. If $2AB = BC$ and the points A and B are respectively $(4, 6)$ and (α, β), then $\alpha + 2\beta$ is equal to

(1) 39 (2) 42 (3) 48 (4) 45

Ans. **[2]**

Sol.



$\because$ B lies on $y = x$

$\therefore\ \alpha = \beta$

Let coordinate of C be (h, k)

Here AB : CB = 1 : 2 = AD : CD

Equation of line AC : $2x - y = 2$ ...(1)

and BD : $x = y$ . ...(2)

From (1) and (2) coordinate of D = (2, 2)

$$\because \left(\frac{h+8}{3}, \frac{k+12}{3}\right) = (2, 2)$$

$\therefore$ (h, k) = (–2, –6)

Equation of line BA'C is : $y - 4 = \frac{5}{4}(x - 6)$

$\because x = y = \alpha \Rightarrow \alpha = 14$

$\therefore\ \alpha + 2\beta = 3 \times 14 = 42$

Q.64 $\lim_{x\to\frac{\pi}{2}}\left(\frac{1}{\left(x-\frac{\pi}{2}\right)^2}\int_{x^3}^{\left(\frac{\pi}{2}\right)^3}\cos(t^{1/3})dt\right)$ is equal to-

(1) $\frac{3\pi}{4}$ (2) $\frac{3\pi}{8}$ (3) $\frac{3\pi^2}{4}$ (4) $\frac{3\pi^2}{8}$

Ans. **[4]**

Sol. $\lim_{x\to\frac{\pi}{2}}\left(\frac{1}{\left(x-\frac{\pi}{2}\right)^2}\int_{x^3}^{\left(\frac{\pi}{2}\right)^3}\cos(t^{1/3})dt\right)$

Applying L-H rule

$\lim_{x\to\frac{\pi}{2}}\frac{\cos\left(\frac{\pi}{2}\right)\times 0-\cos x(3x^2)}{2\left(x-\frac{\pi}{2}\right)(1)}$ By Leibnitz theorem

$\lim_{x\to\frac{\pi}{2}}\left(-\frac{\sin\left(\frac{\pi}{2}-x\right)}{\left(x-\frac{\pi}{2}\right)}\right)\left(\frac{3}{2}x^2\right)$

$=\frac{3}{2}\left(\frac{\pi}{2}\right)^2=\frac{3\pi^2}{8}$

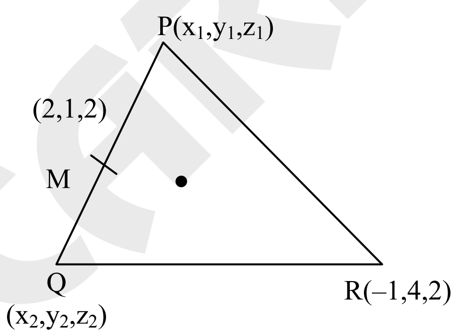
So, option (4) is correct.

Q.65 Let PQR be a triangle with R(–1, 4, 2). Suppose M (2, 1, 2) is the mid-point of PQ. The distance of the centroid of ΔPQR from the point of intersection of the lines $\frac{x-2}{0}=\frac{y}{2}=\frac{z+3}{-1}$ and $\frac{x-1}{1}=\frac{y+3}{-3}=\frac{z+1}{1}$ is-

(1) $\sqrt{69}$ (2) 69 (3) $\sqrt{99}$ (4) 9

Ans. **[1]**

Sol.



Given two lines are

$\frac{x-2}{0}=\frac{y}{2}=\frac{z+3}{-1}=\lambda(\text{let})$

$\frac{x-1}{1}=\frac{y+3}{-3}=\frac{z+1}{1}=\mu(\text{let})$

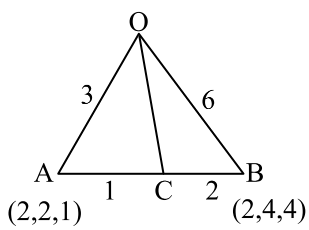
for point of intersection
$x = 2, y = 2\lambda, z = -\lambda - 3$ (General point of line 1)
$x = \mu + 1, y = -3\mu - 3, z = \mu - 1$ (General point of line 2)
$2 = \mu + 1 \Rightarrow \mu = 1$
$2\lambda = -3\mu - 3 \Rightarrow \lambda = -3$
Point of intersection of two lines is (2, –6, 0)
$\because$ M is the mid point of PQ,
$\therefore x_1 + x_2 = 4, y_1 + y_2 = 2, z_1 + z_2 = 4$
Centroid of ΔPQR
$= \left(\frac{x_1 + x_2 - 1}{3}, \frac{y_1 + y_2 + 4}{3}, \frac{z_1 + z_2 + 2}{3}\right) = (1, 2, 2)$
$\therefore$ distance between (1, 2, 2) and (2, –6, 0)
$= \sqrt{1^2 + 8^2 + 2^2} = \sqrt{69}$

Q.66 Let O be the origin and the position vectors of A and B be $2\hat{i} + 2\hat{j} + \hat{k}$ and $2\hat{i} + 4\hat{j} + 4\hat{k}$ respectively. If the internal bisector of $\angle AOB$ meets the line AB at C, then the length of OC is-

(1) $\frac{2}{3}\sqrt{34}$ (2) $\frac{3}{2}\sqrt{34}$ (3) $\frac{3}{2}\sqrt{31}$ (4) $\frac{2}{3}\sqrt{31}$

Ans. **[1]**

Sol.



Position vector of C = $\frac{6\hat{i} + 8\hat{j} + 6\hat{k}}{3}$

$\therefore |\overrightarrow{OC}| = \frac{1}{3}\sqrt{36 + 64 + 36}$

$= \frac{1}{3}\sqrt{136}$

$= \frac{2}{3}\sqrt{34}$

Q.67 A function y = f(x) satisfies $f(x)\sin 2x + \sin x - (1 + \cos^2 x) f'(x) = 0$ with condition f(0) = 0. Then, $f\left(\frac{\pi}{2}\right)$ is equal to-

(1) –1 (2) 1 (3) 2 (4) 0

Ans. **[2]**

Sol. $y \cdot \sin 2x + \sin x - (1 + \cos^2 x)\frac{dy}{dx} = 0$

$(1 + \cos^2 x)\frac{dy}{dx} - (\sin 2x)y = \sin x$

$\frac{dy}{dx} - \left(\frac{\sin 2x}{1 + \cos^2 x}\right)y = \frac{\sin x}{1 + \cos^2 x}$

It is of the form : $\frac{dy}{dx} + P(x)y = Q(x)$

$$\text{I.F.} = e^{\int P(x)\,dx} = e^{-\int \frac{\sin 2x}{1+\cos^2 x}dx}$$

$$\text{Let } I = \int \frac{\sin 2x}{1+\cos^2 x}dx = 2\int \frac{\sin x \cos x}{1+\cos^2 x}dx$$

$$= -\int \frac{2tdt}{1+t^2} \qquad \begin{bmatrix} \cos x = t \\ -\sin x\ dx = dt \end{bmatrix}$$

$$= -\int \frac{du}{u} \qquad \begin{bmatrix} 1+t^2 = u \\ 2tdt = du \end{bmatrix}$$

$= -\ln u$

$= -\ln |1 + t^2|$

$I = -\ln |1 + \cos^2 x|$

$\therefore \text{I.F.} = e^{\ln|1+\cos^2 x|} = 1 + \cos^2 x$

$\Rightarrow y \cdot (\text{I.F}) = \int Q(x)(\text{I.F})\,dx$

$$y(1 + \cos^2 x) = \int \frac{\sin x}{(1+\cos^2 x)}(1+\cos^2 x)dx$$

$y(1 + \cos^2 x) = -\cos x + c$

$\because f(0) = 0\ ;\ c = 1$

$\Rightarrow y(1 + \cos^2 x) = -\cos x + 1$

Now, $f\left(\frac{\pi}{2}\right)$ is

$$\Rightarrow y\left(1+\cos^2\frac{\pi}{2}\right) = -\cos\frac{\pi}{2}+1$$

$\Rightarrow y = 1$

Q.68 If $z = \frac{1}{2} - 2i$ is such that $|z + 1| = \alpha z + \beta(1 + i)$, $i = \sqrt{-1}$ and $\alpha, \beta \in \mathbb{R}$, then $\alpha + \beta$ is equal to-

(1) –4 (2) 2 (3) –1 (4) 3

Ans. **[4]**

Sol.

$$\left|\frac{1}{2} - 2i + 1\right| = \alpha\left(\frac{1}{2} - 2i\right) + \beta(1+i)$$

$$\sqrt{\frac{9}{4}+4} = \alpha\left(\frac{1}{2} - 2i\right) + \beta(1+i)$$

$$\frac{5}{2} = \alpha\left(\frac{1}{2}\right) + \beta + i(-2\alpha+\beta)$$

$$\frac{\alpha}{2} + \beta = \frac{5}{2} \qquad \text{...(i)}$$

$$-2\alpha + \beta = 0 \qquad \text{...(ii)}$$

Solving (i) and (ii)

$$\frac{\alpha}{2} + 2\alpha = \frac{5}{2}$$

$$\frac{5}{2}\alpha = \frac{5}{2}$$

$\alpha = 1$ and $\beta = 2$

$\therefore \alpha + \beta = 3$

Q.69 Consider the function $f : \left[\frac{1}{2}, 1\right] \to \mathbb{R}$ defined by $f(x) = 4\sqrt{2}\,x^3 - 3\sqrt{2}\,x - 1$. Consider the statements

(I) The curve y = f(x) intersects the x-axis exactly at one point.

(II) The curve y = f(x) intersects the x-axis at $x = \cos\frac{\pi}{12}$

Then

(1) Both (I) and (II) are incorrect.
(2) Only (II) is correct.
(3) Only (I) is correct.
(4) Both (I) and (II) are correct.

Ans. **[4]**

Sol. $f'(x) = 12\sqrt{2}\,x^2 - 3\sqrt{2}$

Which is positive for $x \in \left[\frac{1}{2}, 1\right]$ and $f\left(\frac{1}{2}\right) < 0,\ f(1) > 0$

Hence, only one solution in $\left[\frac{1}{2}, 1\right]$

$\therefore$ Statement I is correct.

Let $x = \cos\theta$

$f(x) = \sqrt{2}\,(4\cos^3\theta - 3\cos\theta) - 1 = 0$

$= \sqrt{2}\,\cos 3\theta - 1 = 0$

$= \cos 3\theta = \cos\frac{\pi}{4}$

$\theta = \frac{\pi}{12}$

Statement II is also correct.

Q.70 In an A.P., the sixth term $a_6 = 2$. If the product $a_1a_4a_5$ is the greatest, then the common difference of the A.P. is equal to

(1) $\frac{5}{8}$ (2) $\frac{2}{3}$ (3) $\frac{3}{2}$ (4) $\frac{8}{5}$

Ans. **[4]**

Sol. Let first term = a

Common difference = d

Given, a + 5d = 2 …(1)

Product (P) = $a_1a_4a_5 = a(a + 3d)(a + 4d)$

From (1)

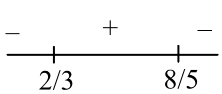
$P = (2 - 5d)(2 - 2d)(2 - d)$

$\frac{dP}{dd} = (2 - 5d)(2 - d)(-2) + (2 - 5d)(2 - 2d)(-1) + (-5)(2 - d)(2 - 2d)$

$= -2[(d - 2)(5d - 2) + (d - 1)(5d - 2) + 5(d - 1)(d - 2)]$

$= -2[5d^2 + 4 - 12d + 5d^2 + 2 - 7d + 5d^2 + 10 - 15d]$

$= -2[15d^2 - 34d + 16] \Rightarrow d = \frac{8}{5}$ or $\frac{2}{3}$ (Critical points)



at $\left(\frac{8}{5}\right)$, Product attains maxima

Answer is $\frac{8}{5}$

Q.71 If α, $-\frac{\pi}{2} < \alpha < \frac{\pi}{2}$ is the solution of $4\cos\theta + 5\sin\theta = 1$, then the value of $\tan\alpha$ is-

(1) $\frac{10-\sqrt{10}}{6}$ (2) $\frac{\sqrt{10}-10}{12}$ (3) $\frac{\sqrt{10}-10}{6}$ (4) $\frac{10-\sqrt{10}}{12}$

Ans. **[2]**

Sol. α, $-\frac{\pi}{2} < \alpha < \frac{\pi}{2}$ is solution of $4\cos\theta + 5\sin\theta = 1$

So, $4\cos\alpha + 5\sin\alpha = 1$

Divide both sides by $\cos\alpha$

$$\left(\text{as } \alpha \neq \frac{\pi}{2}, -\frac{\pi}{2}, \text{so we can divide by } \cos\alpha\right)$$

$\Rightarrow 4 + 5\tan\alpha = \sec\alpha$

$\Rightarrow 4 + 5\tan\alpha = \sqrt{1+\tan^2\alpha}$

Square both sides,

$\Rightarrow 16 + 25\tan^2\alpha + 40\tan\alpha = 1 + \tan^2\alpha$

$\Rightarrow 24\tan^2\alpha + 40\tan\alpha + 15 = 0$

$\Rightarrow \tan\alpha = \frac{-40 \pm \sqrt{1600 - 4\times 24\times 15}}{48}$

$\Rightarrow \tan\alpha = \frac{-10 \pm \sqrt{10}}{12}$

So $\tan\alpha = \frac{-10 + \sqrt{10}}{12}$

Q.72 Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \alpha & \beta \\ 0 & \beta & \alpha \end{bmatrix}$ and $|2A|^3 = 2^{21}$ where $\alpha, \beta \in Z$, then a value of α is-

(1) 5 (2) 9 (3) 17 (4) 3

Ans. **[1]**

Sol. $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \alpha & \beta \\ 0 & \beta & \alpha \end{bmatrix}$

$2A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2\alpha & 2\beta \\ 0 & 2\beta & 2\alpha \end{bmatrix}$

$|2A| = 2(4\alpha^2 - 4\beta^2)$

$|2A|^3 = 8(4\alpha^2 - 4\beta^2)^3 = 2^{21}$ (given)

$\Rightarrow (4\alpha^2 - 4\beta^2)^3 = 2^{18} = (2^6)^3$

$\Rightarrow 4(\alpha^2 - \beta^2) = 2^6$

$\Rightarrow \alpha^2 - \beta^2 = 2^4$

$\Rightarrow \alpha^2 - \beta^2 = 16$

Now as $\alpha, \beta \in Z$

$(\alpha, \beta) = (5, 3)$ is a possible pair.

So $\alpha = 5$

Q.73 Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three non-zero vectors such that $\vec{b}$ and $\vec{c}$ are non-collinear. If $\vec{a}+5\vec{b}$ is collinear with $\vec{c}$, $\vec{b}+6\vec{c}$ is collinear with $\vec{a}$ and $\vec{a}+\alpha\vec{b}+\beta\vec{c}=\vec{0}$, then α + β is equal to-

(1) 35 (2) –30 (3) –25 (4) 30

Ans. **[1]**

Sol. $\vec{a}+5\vec{b}$ is collinear with $\vec{c}$

$\vec{a}+5\vec{b}=\lambda\vec{c}$

$\vec{a}=\lambda\vec{c}-5\vec{b}$ ...(i)

$\vec{b}+6\vec{c}$ is collinear with $\vec{a}$

$\vec{b}+6\vec{c}=\mu\vec{a}$

$\vec{b}+6\vec{c}=\mu(\lambda\vec{c}-5\vec{b})$

$\vec{b}+6\vec{c}=\mu\lambda\vec{c}-5\mu\vec{b}$

$\vec{b}(1+5\mu)=\vec{c}(\lambda\mu-6)$

$\Rightarrow 1+5\mu=0$

$\Rightarrow \mu=-\frac{1}{5}$ and $\lambda\mu-6=0$

$\lambda\left(-\frac{1}{5}\right)=6$

$\lambda=-30$

By equation (i), we get :

$\vec{a}+5\vec{b}+30\vec{c}=0$

Compare it with

$\vec{a}+\alpha\vec{b}+\beta\vec{c}=0$

$\alpha=5,\ \beta=30$

$\Rightarrow \alpha+\beta=35$

Q.74 If $f(x)=\begin{cases}2+2x, & -1\le x<0\\ 1-\frac{x}{3}, & 0\le x\le 3\end{cases}$; $g(x)=\begin{cases}-x, & -3\le x\le 0\\ x, & 0<x\le 1\end{cases}$, then range of (fog)(x) is-

(1) [0, 1] (2) (0, 1] (3) [0, 1) (4) [0, 3)

Ans. **[1]**

Sol. $f(x)=\begin{cases}2+2x, & -1\le x<0\\ 1-\frac{x}{3}, & 0\le x\le 3\end{cases}$

$g(x)=\begin{cases}-x, & -3\le x\le 0\\ x, & 0<x\le 1\end{cases}$ $\Rightarrow$ g(x) is nothing but a part of $|x|$

$\Rightarrow g(x)=|x|,\ x\in[-3,1]\Rightarrow g(x)\ge 0\ (\because |x|\ge 0)$

$\Rightarrow f(g(x))=\begin{cases}2+2|x|, & -1<|x|<0\\ 1-\frac{|x|}{3}, & 0\le|x|\le 3\end{cases}$

$\Rightarrow f(g(x))=1-\frac{|x|}{3},\ |x|\in[0,3]$

Range of f(g(x))

$|x| \in [0, 3] \Rightarrow -\frac{|x|}{3} \in [-1, 0]$

$1 - \frac{[x]}{3} \in [0, 1]$

Range of f(g(x)) = [0, 1]

Q.75 If in a G.P. of 64 terms, the sum of all the terms is 7 times the sum of the odd terms of the G.P. then the common ratio of the G.P. is equal to

(1) 6 (2) 7 (3) 4 (4) 5

Ans. **[1]**

Sol. Let the G.P. is a, ar, ar^2, …..

As given

$$\frac{a(r^{64}-1)}{r-1} = 7.\frac{a(r^2)^{32}-1)}{r^2-1}$$

$\Rightarrow r^2 - 1 = 7r - 7$

$\Rightarrow r^2 - 7r + 6 = 0$

$\Rightarrow r = 1$ (Rejected) or $r = 6$.

Q.76 Suppose $f(x) = \frac{(2^x + 2^{-x})\tan x\sqrt{\tan^{-1}(x^2 - x + 1)}}{(7x^2 + 3x + 1)^3}$. Then the vale of $f'(0)$ is equal to-

(1) $\frac{\pi}{2}$ (2) $\sqrt{\pi}$ (3) 0 (4) π

Ans. **[2]**

Sol. Let f(x) = tanx g(x)

where $g(x) = \frac{(2^x + 2^{-x})\sqrt{\tan^{-1}(x^2 - x + 1)}}{(7x^2 + 3x + 1)^3}$

$\Rightarrow f'(x) = \sec^2 x g(x) + \tan x g'(x)$

$f'(0) = \sec^2(0)g(0) + \tan 0 g'(0)$

$= g(0)$

$$g(0) = \frac{(2^0 + 2^{-0})\sqrt{\tan^{-1}(0^2 - 0 + 1)}}{(7(0)^2 + 3(0) + 1)^3}$$

$$= 2\sqrt{\tan^{-1} 1} = 2\sqrt{\frac{\pi}{4}} = \sqrt{\pi}$$

Q.77 Let A be a square matrix such that $AA^T = I$. Then $\frac{1}{2}A[(A + A^T)^2 + (A - A^T)^2]$ is equal to-

(1) $A^2 + A^T$ (2) $A^2 + I$ (3) $A^3 + I$ (4) $A^3 + A^T$

Ans. **[4]**

Sol. Let $B = \frac{1}{2}A[(A + A^T)^2 + (A - A^T)^2]$

$$B = \frac{1}{2}A(A^2 + (A^T)^2 + AA^T + A^TA + A^2 + (A^T)^2 - AA^T - A^TA)$$

$$B = \frac{1}{2}A(2A^2 + 2(A^T)^2) = A(A^2 + A^T.A^T)$$

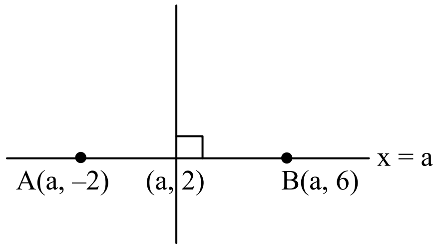
$$= A^3 + AA^TA^T = A^3 + A^T$$

Q.78 Let $\left(5, \frac{a}{4}\right)$ be the circumcentre of a triangle with vertices A(a, –2), B(a, 6) and $C\left(\frac{a}{4}, -2\right)$. Let α denote the circumradius, β denote the area and γ denote the perimeter of the triangle. Then $\alpha + \beta + \gamma$ is-

(1) 62 (2) 60 (3) 30 (4) 53

Ans. **[4]**

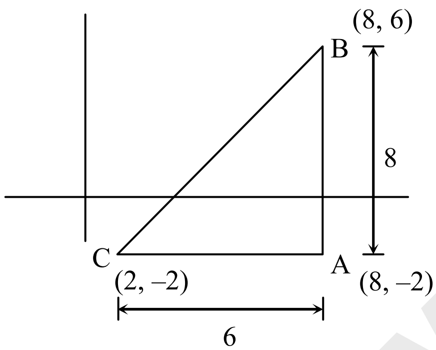
Sol.



Circumcentre lie on perpendicular bisector of

AB $\Rightarrow y = 2$

$\Rightarrow \frac{a}{4} = 2 \Rightarrow a = 8$



$\Rightarrow$ Perimeter = 6 + 8 + 10 = 24

Area = $\frac{6 \times 8}{2} = 24$

Circumradius = half of hypotenuse

$= \frac{10}{2} = 5$

$\alpha + \beta + \gamma \Rightarrow 24 + 24 + 5 = 53$

Q.79 Let R be a relation on Z × Z defined by (a, b) R (c, d) if and only if ad – bc is divisible by 5. Then R is

(1) Reflexive and symmetric but not transitive

(2) Reflexive but neither symmetric nor transitive

(3) Reflexive and transitive but not symmetric

(4) Reflexive, symmetric and transitive

Ans. **[1]**

Sol. (a, b) R (c, d) $\Rightarrow$ ad – bc is divisible by 5

= cb – da is also divisible by 5

= (c, d) R (a, b)

$\Rightarrow$ R is symmetric relation

If (a, b) R (a, b) $\Rightarrow$ ab – ba = 0 is divisible by 5

$\Rightarrow$ R is reflexive

$\Rightarrow$ Not transitive

For example take : (a, b) ≡ (6, 1), (c, d) = (0, 0), (e, f)

= (1, 7)

Q.80 A fair die is thrown until 2 appears. Then the probability, that 2 appears in even number of throws, is-

(1) $\frac{6}{11}$ (2) $\frac{5}{6}$ (3) $\frac{5}{11}$ (4) $\frac{1}{6}$

Ans. **[3]**

Sol. A : 2 appears in even number of throws

B : 2 appears on die

$\Rightarrow P(B) = \frac{1}{6}, P(\overline{B}) = \frac{5}{6}$

$P(A) = P(\overline{B})P(B) + P(\overline{B})P(\overline{B})P(\overline{B})P(B) + P(\overline{B})P(\overline{B})P(\overline{B})P(\overline{B})P(\overline{B})P(B) + ...$

$\Rightarrow P(A) = \frac{5}{6}.\frac{1}{6} + \left(\frac{5}{6}\right)^3.\frac{1}{6} + \left(\frac{5}{6}\right)^5\frac{1}{6} + ...$

$$\frac{\frac{5}{36}}{1-\frac{5^2}{6^2}} = \frac{\frac{5}{36}}{\frac{36-25}{36}} = \frac{5}{11}$$

Section-B: Numerical Value Type Questions: Numerical Value Type Questions: This section contains 10 Numerical based questions. Attempt any 5 questions out of 10. The answer to each question should be rounded-off to the nearest integer..

Q.81 If $\frac{^{11}C_1}{2} + \frac{^{11}C_2}{3} + ... + \frac{^{11}C_9}{10} = \frac{n}{m}$ with gcd(n, m) = 1, then n + m is equal to _______.

Ans. **[2041]**

Sol. $(1 + x)^{11} = {}^{11}C_0 + {}^{11}C_1x + {}^{11}C_2x^2 + ... + {}^{11}C_{11}x^{11}$

$$\int_0^1 (1+x)^{11}\,dx = {}^{11}C_0x + \left[\frac{^{11}C_1x^2}{2} + \frac{^{11}C_2x^3}{3} + ... + \frac{^{11}C_9x^{10}}{10} + \frac{^{11}C_{10}x^{11}}{11} + \frac{^{11}C_{11}x^{12}}{12}\right]_0^1$$

$$\therefore \frac{^{11}C_1}{2} + \frac{^{11}C_2}{3} + ... + \frac{^{11}C_9}{10} = \frac{2^{12}-1}{12} - 1 - 1 - \frac{1}{12}$$

$$= \frac{2035}{6} = \frac{n}{m}$$

n + m = 2041

Q.82 Let α, β be the roots of the equation $x^2 - x + 2 = 0$ with Im (α) > Im (β). Then $\alpha^6 + \alpha^4 + \beta^4 - 5\alpha^2$ is equal to ______.

Ans. **[13]**

Sol. $x^2 + 2 = x$

$\Rightarrow x^4 + 4 + 4x^2 = x^2$

So,

$x^4 = -4 - 3x^2$

$\Rightarrow x^6 = -4x^2 - 3x^4$

$\Rightarrow x^6 = 12 + 5x^2$

Now,

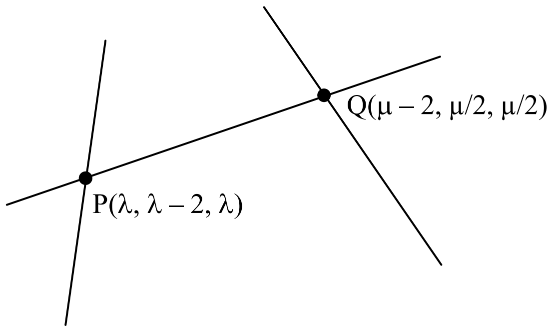
$\alpha^6 - 5\alpha^2 = 12$

So $\alpha^4 + \beta^4 + 12$
$= 4 - 3(\alpha^2 + \beta^2)$
$= 4 - 3[1 - 4]$
$= 13$

Q.83 A line with direction ratios 2, 1, 2 meets the lines x = y + 2 = z and x + 2 = 2y = 2z respectively at the points P and Q. If the length of the perpendicular from the point (1, 2, 12) to the line PQ is l, then l^2 is _______.

Ans. **[65]**

Sol.



$x = y + 2 = z = \lambda$
$x + 2 = 2y = 2z = \mu$
Now PQ is proportional to <2, 1, 2>

$$\therefore \frac{\lambda - \mu + 2}{2} = \frac{\lambda - 2 - \frac{\mu}{2}}{1} = \frac{\lambda - \frac{\mu}{2}}{2}$$

$\therefore \mu = 4, \lambda = 6$
$\therefore$ P(6, 4, 6) and Q(2, 2, 2)

$$L_{PQ} = \frac{x-2}{2} = \frac{y-2}{1} = \frac{z-2}{2} = K$$

Point of PQ : (2K + 2, K + 2, 2K + 2)
< 2K + 1, K, 2K – 10 >
$2(2K + 1) + K + 2(2K - 10) = 0$
$9K - 18 = 0 \Rightarrow \boxed{K = 2}$
$\therefore$ Point on PQ ≡ (6, 4, 6)
$l^2 = (6 - 1)^2 + (4 - 2)^2 + (12 - 6)^2$
$= 25 + 4 + 36 = 65$

Q.84 All the letters of the word "GTWENTY" are written in all possible ways with or without meaning and these words are written as in a dictionary. The serial number of word "GTWENTY" is ______.

Ans. **[553]**

Sol. GTWENTY

(1) E : $\frac{6!}{2!} = 360$

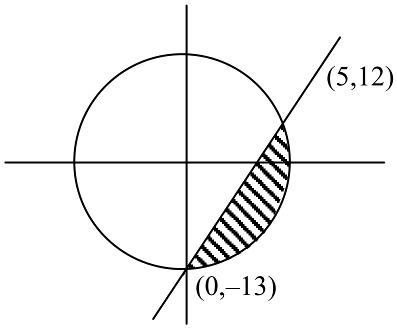
(2) $\overline{GE} : \frac{5!}{2!}, \overline{GN} : \frac{5!}{2!}$

(3) GTE : 4!, GTN : 4!, GTT : 4!
(4) GTWENTY = 1
$\Rightarrow 360 + 60 + 60 + 24 + 24 + 24 + 1 = 553$

Q.85 The area (in sq. units) of the part of the circle $x^2 + y^2 = 169$ which is below the line $5x - y = 13$ is $\frac{\pi\alpha}{2\beta} - \frac{65}{2} + \frac{\alpha}{\beta}\sin^{-1}\left(\frac{12}{13}\right)$, where α, β are coprime numbers. Then α + β is equal to ________.

Ans. **[171]**

Sol.



$$\text{Area} = \frac{\pi(13)^2}{2} - \left[\frac{1}{2}\times 25\times 5 + \int_{12}^{13}\sqrt{169 - y^2}\ dy\right]$$

$$\frac{169\pi}{4} - \frac{65}{2} + \frac{169}{2}\sin^{-1}\frac{12}{13}$$

$\Rightarrow \alpha = 169$

$\beta = 2$

$\therefore\ \alpha + \beta = 171$

Q.86 If the mean and variance of the data 65, 68, 58, 44, 48, 45, 60, α, β, 60 where α > β are 56 and 66.2 respectively, then $\alpha^2 + \beta^2$ is equal to _____.

Ans. **[6344]**

Sol. $56 = \frac{65 + 68 + 58 + 44 + 48 + 45 + 60 + 60 + \alpha + \beta}{10}$

$\Rightarrow \alpha + \beta = 112$

$66.2 \times 10 = (65 - 56)^2 + (68 - 56)^2 + (58 - 56)^2 + (44 - 56)^2 + (48 - 56)^2 + (45 - 56)^2 + (60 - 56)^2 + (60 - 56)^2 + (\alpha - 56)^2 + (\beta - 56)^2$

$662 = 81 + 144 + 4 + 144 + 64 + 121 + 16 + 16 + 2 \times (56)^2 + \alpha^2 + \beta^2 - 112(\alpha + \beta)$

$\Rightarrow \alpha^2 + \beta^2 = 6344$

Q.87 If the solution curve y = y(x) of the differential equation $(1 + y^2)(1 + \log_e x)\,dx + x\,dy = 0$, x > 0 passes through the point (1, 1) and $y(e) = \frac{\alpha - \tan\left(\frac{3}{2}\right)}{\beta + \tan\left(\frac{3}{2}\right)}$ then α + 2β is.

Ans. **[3]**

Sol. $(1 + y^2)(1 + \ln x)\,dx + x\,dy = 0$

$$\int\left(\frac{1 + \ln x}{x}\right)dx + \int\frac{dy}{1 + y^2} = 0$$

Put $1 + \ln x = t \Rightarrow \frac{1}{x}dx = dt$

$\int t\,dt + \tan^{-1}y + C = 0$

$\frac{(1+\ln x)^2}{2} + \tan^{-1}y + C = 0$

$\Rightarrow \frac{(1+\ln x)^2}{2} = -\tan^{-1}y + C$ passes through (1, 1)

$\Rightarrow C = \frac{1}{2}+\frac{\pi}{4}$

$\Rightarrow (1 + \ln x)^2 = -2\tan^{-1}y + 1 + \frac{\pi}{2}$

Put x = e

$\Rightarrow 4 = -2\tan^{-1}(y(e)) + 1 + \frac{\pi}{2}$

$\tan^{-1}(y(e)) = \frac{\pi}{4}-\frac{3}{2}$

$y(e) = \frac{1-\tan\left(\frac{3}{2}\right)}{1+\tan\left(\frac{3}{2}\right)}$

$\Rightarrow \alpha + 2\beta = 3$

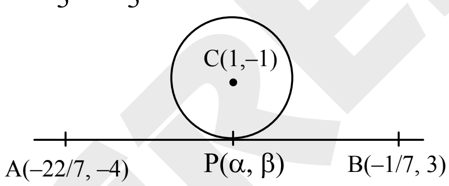
Q.88 Equations of two diameters of a circle are $2x - 3y = 5$ and $3x - 4y = 7$. The line joining the points $\left(-\frac{22}{7},-4\right)$ and $\left(-\frac{1}{7},3\right)$ intersects the circle at only one point $P(\alpha, \beta)$. Then $17\beta - \alpha$ is equal to _____.

Ans. **[2]**

Sol. Intersection point of diameters $2x - 3y = 5$ and $3x - 4y = 7$ is centre of circle $C(1, -1)$

Equation of line joining $A\left(-\frac{22}{7},-4\right)$ and $B\left(-\frac{1}{7},3\right)$ is

$y = \frac{7}{3}x + \frac{10}{3} \Rightarrow 7x - 3y + 10 = 0$ ....(i)



Equation of CP is

$3x + 7y + 4 = 0$ ...(ii)

Solving (i) and (ii)

$\alpha = -\frac{41}{29},\ \beta = \frac{1}{29}$

$17\beta - \alpha = \frac{17}{29}+\frac{41}{29} = 2$

Q.89 If the points of intersection of two distinct conics $x^2 + y^2 = 4b$ and $\frac{x^2}{16}+\frac{y^2}{b^2} = 1$ lie on the curve $y^2 = 3x^2$ then $3\sqrt{3}$ times the area of the rectangle formed by the intersection points is ________.

Ans. **[432]**

Sol. $x^2 + y^2 = 4b$ ...(i)

$\frac{x^2}{16} + \frac{y^2}{b^2} = 1$ ...(ii)

$y^2 = 3x^2$ ...(iii)

From (i) and (iii)

$x^2 = b,\ y^2 = 3b$

From (ii)

$\frac{b}{16} + \frac{3b}{b^2} = 1$

$\frac{b}{16} + \frac{3}{b} = 1$

$b^2 - 16b + 48 = 0$

$b = 12,\ b = 4$

(i) & (ii) are distinct $\Rightarrow b \neq 4,\ b = 12$

$\Rightarrow x = \pm 2\sqrt{3}$, $y = \pm 6 \Rightarrow 3\sqrt{3}$ A $= 3\sqrt{3} \times 4\sqrt{3} \times 12 = 432$

Q.90 Let $f(x) = 2^x - x^2$, $x \in R$. If m and n are respectively the number of points at which the curves $y = f(x)$ and $y = f'(x)$ intersect the x-axis then the value of m + n is ______

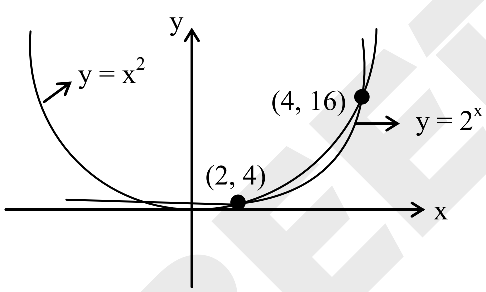
Ans. **[5]**

Sol. $f(x) = 2^x - x^2$

$f'(x) = 2^x \ln 2 - 2x$

When $f(x) = 0$

$2^x = x^2$

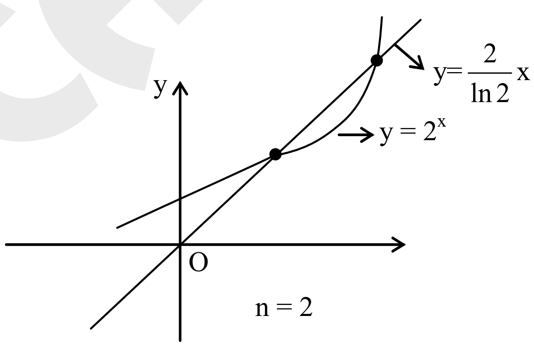


$\Rightarrow m = 3$

When $f'(x) = 0$

$2^x \ln 2 - 2x = 0$

$2^x = \frac{2}{\ln 2}x$



n = 2

$\Rightarrow m + n = 5$