



JEE Main Online Exam 2024

Questions & Solution
27th January 2024 | Evening

PHYSICS

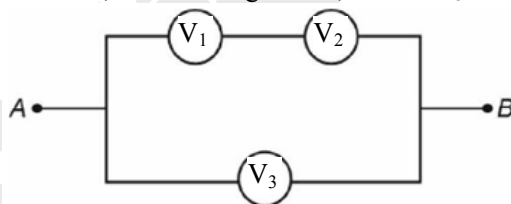
Section-A: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct..

- Q.1** Given below are two statements: one is labelled as **Assertion (A)** and the other is labelled as **Reason (R)**.
Assertion (A) : The property of body, by virtue of which it tends to regain its original shape when the external force is removed, is Elasticity.
Reason (R) : The restoring force depends upon the bonded inter atomic and inter molecular force of solid.
In the light of the above statements, choose the **correct** answer form the options given below.
(1) Both (A) and (R) are true and (R) is the correct explanation of (A)
(2) (A) is false but (R) is true
(3) (A) is true but (R) is false
(4) Both (A) and (R) are true but (R) is not the correct explanation of (A)

Ans. [1]

Sol. The elasticity is responsible for shape regain of material and it cause due to inter atomic bond and intermolecular bond.

- Q.2** Three voltmeters, all having different internal resistance are joined as shown in figure when some potential difference is applied across A and B, their reading are V_1 , V_2 and V_3 . choose the correct option.



- (1) $V_1 + V_2 > V_3$ (2) $V_1 = V_2$ (3) $V_1 \neq V_3 - V_2$ (4) $V_1 + V_2 = V_3$

Ans. [4]

Sol. V_1 and V_2 are in series and they are in parallel with V_3 .
 $\therefore V_1 + V_2 = V_3$

- Q.3** Given below are two statements : one is labelled as **Assertion (A)** and the other is labelled as **Reason (R)**.
Assertion (A) : Work done by electric field on moving a positive charge on an equipotential surface is always zero.
Reason (R) : Electric lines of forces are always perpendicular to equipotential surfaces.
In the light of the above statements, choose the **most appropriate** answer from the options given below.
(1) (A) is correct but (R) not correct
(2) (A) is not correct but (R) is correct
(3) Both (A) and (R) are true and (R) is the correct explanation of (A)
(4) Both (A) and (R) are true and (R) is not the correct explanation of (A)

Ans. [3]

Sol. The potential equipotential surface remain same everywhere, so no work is needed to move any charged particle and \vec{E} is perpendicular to equipotential surface.

- Q.4** Primary side of a transformer is connected to 230 V, 50 Hz supply. Turns ratio of primary to secondary winding is 10 : 1. Load resistance connected to secondary side is 46 Ω . The power consumed in it is
(1) 10.0 W (2) 12.0 W (3) 11.5 W (4) 12.5 W

Ans. [3]

Sol. $\therefore \frac{N_p}{N_s} = \frac{V_p}{V_s}$

$$\Rightarrow V_s = 23 \text{ V}$$

$$\Rightarrow \text{Power} = \frac{V_s^2}{R} = 11.5 \text{ W}$$

- Q.5** A current of 200 μA deflects the coil of a moving coil galvanometer through 60° . The current to cause deflection through $\frac{\pi}{10}$ radian is

- (1) 120 μA (2) 180 μA (3) 30 μA (4) 60 μA

Ans. [4]

Sol. $i \propto \theta$ (angle of deflection)

$$\text{Current (i)} = \frac{200\mu}{\left(\frac{\pi}{3}\right)} \times \frac{\pi}{10} = 60 \mu\text{A}$$

- Q.6** The threshold frequency of a metal with work function 6.63 eV is :
(1) 1.6×10^{15} Hz (2) 16×10^{12} Hz (3) 1.6×10^{12} Hz (4) 16×10^{15} Hz

Ans. [1]

Sol. Here $\phi = 6.63 \text{ eV}$, $h = 6.63 \times 10^{-34} \text{ J.s}$

$$\nu = \frac{\phi}{h} = 1.6 \times 10^{15} \text{ Hz}$$

- Q.7** Given below are two statements:

Statements (I) : The limiting force of static friction depends on the area of contact and independent of material

Statement (II) : The limiting force of kinetic friction is independent of the area of contact and depends on materials.

In the light of the above statements, choose the most appropriate answer from the options given below :

- (1) **Statement I** is correct but **Statement II** is incorrect
(2) **Statement I** is incorrect but **Statement II** is correct
(3) Both **Statement I** and **Statement II** are incorrect
(4) Both **Statement I** and **Statement II** are correct

Ans. [2]

Sol. Friction force is independent on area and it only depends on nature of material in contact.

- Q.8** A bullet is fired into a fixed target losses one third of its velocity after travelling 4 cm. It penetrates further $D \times 10^{-3}$ m before coming to rest. The value of D is
(1) 5 (2) 3 (3) 4 (4) 2

Ans. [Bonus]

Sol. No option is correct

$$v = \frac{2u}{3}$$

$$v^2 - u^2 = 2a(0.04)$$

$$-\frac{5}{9}u^2 = 2a \times (0.04)$$

$$-\frac{u^2}{2a} = \frac{9}{5} \times 0.04$$

\therefore distance travelled further

$$= \frac{9}{5} \times 0.04 - 0.04 = 32 \times 10^{-3}$$

$$D = 32$$

None of the option is correct.

Q.9 A ball suspended by a thread swings in a vertical plane so that its magnitude of acceleration in the extreme position and lowest position are equal. The angle (θ) of thread deflection in the extreme position will be :

- (1) $2 \tan^{-1}\left(\frac{1}{2}\right)$ (2) $\tan^{-1}(\sqrt{2})$ (3) $\tan^{-1}\left(\frac{1}{2}\right)$ (4) $2 \tan^{-1}\left(\frac{1}{\sqrt{5}}\right)$

Ans. [1]

Sol. At extreme position

$$a_1 = g \sin \theta$$

At mean position

$$a_2 = \frac{v^2}{\ell}$$

$$a_2 = 2g(1 - \cos \theta)$$

$$2g(1 - \cos \theta) = g \sin \theta$$

$$\tan \frac{\theta}{2} = \frac{1}{2}$$

$$\theta = 2 \tan^{-1}\left(\frac{1}{2}\right)$$

Q.10 Wheatstone bridge principal is used to measure of specific resistance (S_1) of given wire, having length L , radius r . If X is the resistance of wire, then specific resistance is ; $S_1 = X \left(\frac{\pi r^2}{L}\right)$. If the length of the wire gets doubled, then the value of specific resistance will be

- (1) S_1 (2) $\frac{S_1}{2}$ (3) $\frac{S_1}{4}$ (4) $2S_1$

Ans. [1]

Sol. $S_1 = X \left(\frac{\pi r^2}{L}\right)$

Specific resistance independent on the other factor. So, it will remain same.

- Q.11** The total kinetic energy of 1 mole of oxygen at 27°C is
[Use universal gas constant (R) = 8.31 J/mole K]
(1) 6232.5 J (2) 5670.5 J (3) 5942.0 J (4) 6845.5 J

Ans. [1]

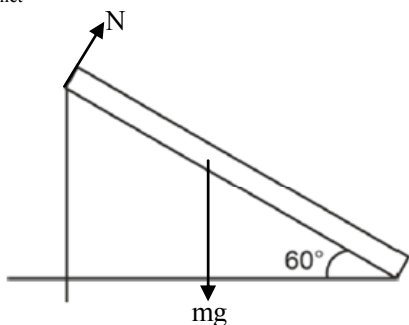
Sol. Kinetic energy $= \frac{5}{2} RT$
 $= \frac{5}{2} \times 8.31 \times 300$
 $= 6232.5 \text{ J}$

- Q.12** A heavy iron bar of weight 12 kg is having its one end on the ground and the other on the shoulder of a man. The rod makes an angle 60° with the horizontal, the weight experienced by the man is :
(1) $6\sqrt{3}$ kg (2) 12 kg (3) 3 kg (4) 6 kg

Ans. [3]

Sol. Net torque about point of contact on ground

$$\tau_{\text{net}} = 0$$



$$Nl = mg \times \frac{l}{2} \cos 60^\circ$$

- Q.13** During an adiabatic process, the pressure of a gas is found to be proportional to the cube of its absolute temperature. The ratio of $\frac{C_p}{C_v}$ for the gas is :

- (1) $\frac{7}{5}$ (2) $\frac{5}{3}$ (3) $\frac{3}{2}$ (4) $\frac{9}{7}$

Ans. [3]

Sol. Here, process is adiabatic

$$\text{and } P \propto T^3$$

$$\Rightarrow PT^{-3} = \text{const}$$

$$\text{or } pV^{\frac{3}{2}} = \text{const}$$

$$\text{so } \frac{C_p}{C_v} = \frac{3}{2}$$

- Q.14** Given below are two statements: one is labelled as **Assertion (A)** and the other is labelled as **Reason (R)**.

Assertion (A) : The angular speed of the moon in its orbit about the earth is more than the angular speed of the earth in its orbit about the sun.

Reason (R) : The moon takes less time to move around the earth than the time taken by the earth to move around the sun.

In the light of the above statements, choose the **most appropriate** answer form the options given below.

- (1) Both (A) and (R) are true and (R) is the correct explanation of (A)
- (2) (A) is false but (R) is true
- (3) (A) is true but (R) is false
- (4) Both (A) and (R) are true but (R) is not the correct explanation of (A)

Ans. [1]

Sol. Angular speed of moon (ω_m) > Angular speed of Earth (ω_e)

$$\text{Time period (T)} = \frac{2\pi}{\omega}$$

So moon take less time to move around the earth than earth around sun.

Q.15 Given below are two statements: one is labelled as **Assertion (A)** and the other is labelled as **Reason (R)**.

Assertion (A) : In Vernier caliper if positive zero error exists, then while taking measurements, the reading taken will be more than the actual reading.

Reason (R) : The zero error in Vernier Calliper might have happened due to manufacturing defect or due to rough handling.

In the light of the above statements, choose the **most appropriate** answer form the options given below.

- (1) (A) is true but (R) is false
- (2) Both (A) and (R) are true and (R) is the correct explanation of (A)
- (3) Both (A) and (R) are true but (R) is not the correct explanation of (A)
- (4) (A) is false but (R) is true

Ans. [2]

Sol. In case of positive error, measured value will be more than real and it may happen due to manufacturing defect or due to rough handling.

Q.16 The equation of state of a real gas is given by $\left(P + \frac{a}{V^2}\right)(V - b) = RT$, where P, V and T are pressure, volume and temperature respectively and R is the universal gas constant. The dimensions of $\frac{a}{b^2}$ is similar to that of :

- (1) P
- (2) RT
- (3) R
- (4) PV

Ans. [1]

Sol. $\left(P + \frac{a}{V^2}\right)(V - b) = RT$

$$[a] = [RTV]$$

$$[b] = \left[\frac{RT}{P}\right]$$

$$[ab] = [RTV^2]$$

$$[PV] = [RT]$$

$$\text{Then } \left[\frac{a}{b^2}\right] = \left[\frac{RTV}{(RT)^2} P^2\right] = [P]$$

Q.17 The atomic mass of ${}_6\text{C}^{12}$ is 12.000000 u and that of ${}_6\text{C}^{13}$ is 13.003354 u. The required energy to remove a neutron from ${}_6\text{C}^{13}$, if mass of neutron is 1.008665 u, will be :

- (1) 49.5 MeV
- (2) 62.5 MeV
- (3) 6.25 MeV
- (4) 4.95 MeV

Ans. [4]

Sol. ${}_6\text{C}^{13} \longrightarrow {}_6\text{C}^{12} + {}_0\text{n}^1$

$$\Delta E = \Delta mc^2$$

$$= (0.005311) \times 931.5 \text{ MeV}$$

$$= 4.95 \text{ MeV}$$

- Q.18** An object is placed in a medium of refractive index 3. An electromagnetic wave of intensity $6 \times 10^8 \text{ W/m}^2$ falls normally on the object and it is absorbed completely. The radiation pressure on the object would be (speed of light in free space = $3 \times 10^8 \text{ m/s}$):
- (1) 6 Nm^{-2} (2) 18 Nm^{-2} (3) 2 Nm^{-2} (4) 36 Nm^{-2}

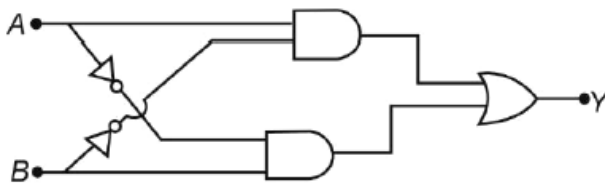
Ans. [1]

Sol. Radiation pressure (P) = $\frac{I_0}{C}$

$$= \frac{6 \times 10^8}{\left(\frac{3 \times 10^8}{3}\right)}$$

$$= 6 \text{ Nm}^{-2}$$

- Q.19** The truth table of the given circuit diagram is :



	A	B	Y		A	B	Y		A	B	Y		A	B	Y				
	0	0	0	(1)	0	1	0	(2)	0	1	0	(3)	0	1	0	(4)	0	1	1
	1	0	0		1	0	1		1	0	0		1	0	0		1	0	1
	1	1	1		1	1	0		1	1	1		1	1	1		1	1	0

Ans. [2]

Sol. $Y = A \cdot \bar{B} + B \cdot \bar{A}$

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

- Q.20** When a Polaroid sheet is rotated between two crossed polaroids then the transmitted light intensity will be maximum for a rotation of :

- (1) 30° (2) 45° (3) 60° (4) 90°

Ans. [2]

Sol. $I = I_0 \cos^2 \theta \cdot \sin^2 \theta$

$$I = I_0 \left(\frac{2 \sin \theta \cdot \cos \theta}{2}\right)^2$$

$$= \frac{I_0}{4} (\sin 2\theta)^2$$

$I \rightarrow \text{max}, \sin 2\theta = 1$

$$\theta = 45^\circ$$

Section-B: Numerical Value Type Questions: This section contains 10 Numerical based questions. Attempt any 5 questions out of 10. The answer to each question should be rounded-off to the nearest integer

Q.21 A closed organ pipe 150 cm long gives 7 beats per second with an open organ pipe of length 350 cm, both vibrating in fundamental mode. The velocity of sound is ____ m/s.

Ans. [294]

Sol. For close pipe: $n_1 = \frac{V}{4L_1} = \frac{V}{6}$

For open pipe: $n_2 = \frac{V}{2L_2} = \frac{V}{7}$

$$n_1 - n_2 = 7$$

$$V = 294.00 \text{ m/s}$$

Q.22 A body falling under gravity covers two points A and B separated by 80 m in 2 s. The distance of upper point A from the starting points is ____ m (use $g = 10 \text{ ms}^{-2}$)

Ans. [45]

Sol. Let time taken to reach point A is t,

Then
$$\begin{aligned} V_A &= gt \\ V_B &= gt + 2g \end{aligned}$$

$$80 = (gt)(2) + \frac{1}{2}9(2)^2$$

$$t = 3 \text{ sec.}$$

$$\text{Distance of point A, } S = \frac{1}{2}(g)(3)^2 = 45 \text{ m}$$

Q.23 Two charges of $-4\mu\text{C}$ and $+4\mu\text{C}$ are placed at the point A(1, 0, 4) m and B(2, -1, 5) m located in an electric field $\vec{E} = 0.20 \hat{i} \text{ V/cm}$. The magnitude of the torque acting on the dipole is $8\sqrt{\alpha} \times 10^{-5} \text{ Nm}$, where $\alpha = \underline{\hspace{1cm}}$.

Ans. [2]

Sol. Torque ($\vec{\tau}$) = $\vec{P} \times \vec{E}$

$$= q(\vec{\ell} \times \vec{E})$$

$$= 4 \times 10^{-6} (\hat{i} - \hat{j} + \hat{k}) \times (0.2\hat{i}) \frac{\text{V}}{10^{-2}}$$

$$|\vec{\tau}| = 8\sqrt{2} \times 10^{-5} \text{ N-m}$$

Q.24 The electric potential at the surface of an atomic nucleus ($z = 50$) of radius $9 \times 10^{-13} \text{ cm}$ is ____ $\times 10^6 \text{ V}$.

Ans. [8]

Sol. Potential (V) = $\frac{Kz(e)}{r}$

$$= \frac{9 \times 10^9 \times 50(1.6 \times 10^{-19})}{9 \times 10^{-15}}$$
$$= 8 \times 10^6 \text{ V}$$

Q.25 If Rydberg's constant is R, the longest wavelength of radiation in Paschen series will be $\frac{\alpha}{7R}$, where $\alpha = \underline{\hspace{1cm}}$.

Ans. [144]

Sol.
$$\frac{1}{\lambda} = RZ^2 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$$

For given data, $n_1 = 3$
 $n_2 = 4$
 $Z_0 = 1$

$$\frac{1}{\lambda} = R \left(\frac{1}{9} - \frac{1}{16} \right)$$

$$\frac{1}{\lambda} = \frac{7R}{144}$$

$$\lambda = \frac{144}{7R}$$

Q.26 A series LCR circuit with $L = \frac{100}{\pi}$ mH, $C = \frac{10^{-3}}{\pi}$ F and $R = 10 \Omega$, is connected across an ac source of 220 V, 50 Hz supply. The power factor of the circuit would be _____.

Ans. [1]

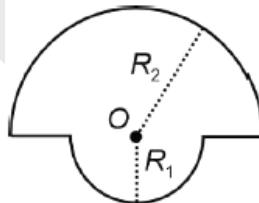
Sol. Power factor = $\frac{R}{Z}$

here, $X_c = X_L = 10\Omega$

so $Z = 10 \Omega$

power factor = $\frac{10}{10} = 1$

Q.27 The magnetic field at the centre of a wire loop formed by two semicircular wires of radii $R_1 = 2\pi$ m and $R_2 = 4\pi$ m, carrying current $I = 4$ A as per figure given below is $\alpha \times 10^{-7}$ T. The value of α is _____ .
 (Centre O is common for all segments)



Ans. [3]

Sol. Net magnetic field (B) = $B_1 + B_2$

$$= \frac{\mu_0 I}{4} \left(\frac{1}{R_1} + \frac{1}{R_2} \right)$$

$$= \frac{\mu_0 I}{4} \times \frac{3}{4\pi}$$

$$= 3 \times 10^{-7} \text{ T}$$

Q.28 The reading of pressure metre attached with a closed pipe is 4.5×10^4 N/m². On opening the valve, water starts flowing and the reading of pressure metre falls to 2.0×10^4 N/m². The velocity of water is found to be \sqrt{V} m/s. The value of V is _____ .

Ans. [50]

Sol. Change in pressure (ΔP) = $\frac{1}{2} \rho V^2$

$$2.5 \times 10^4 = \frac{1}{2} \times 10^3 V^2$$

$$V = \sqrt{50} \text{ m/s}$$

Q.29 A ring and a solid sphere roll down the same inclined plane without slipping. They start from rest. The radii of both bodies are identical and the ratio of their kinetic energies is $\frac{7}{x}$, where x is _____.

Ans. [7]

Sol. If mass of ring and solid sphere will be same, then total kinetic energy of ring will be equal to total kinetic energy of solid sphere.

$$\text{so } \frac{(\text{K.E.})_{\text{ring}}}{(\text{K.E.})_{\text{sphere}}} = 1$$

Q.30 A parallel beam of monochromatic light of wavelength 5000 Å is incident normally on a single narrow slit of width 0.001 mm. The light is focused by convex lens on screen, placed on its focal plane. The first minima will be formed for the angle of diffraction of _____ (degree).

Ans. [30]

Sol. for first minima

$$a \sin \theta = \lambda$$

$$\sin \theta = \frac{5000 \times 10^{-10}}{0.001 \times 10^{-3}}$$

$$\sin \theta = \frac{1}{2} \Rightarrow \theta = 30^\circ$$

CHEMISTRY

Section-A: Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Q.31 Identify from the following species in which d^2sp^3 hybridization is shown by central atom :

- (1) BrF_5 (2) SF_6 (3) $[\text{Pt}(\text{Cl}_4)]^{2-}$ (4) $[\text{Co}(\text{NH}_3)_6]^{3+}$

Ans. [4]

Sol. $\text{Co}^{3+} : 3d^6 4s^0$

Pairing will take place and hybridization will be d^2sp^3 .

Q.32 Which structure of protein remains intact after coagulation of egg white on boiling?

- (1) Tertiary (2) Quaternary (3) Secondary (4) Primary

Ans. [4]

Sol. Primary structure of protein remains intact (it represents the sequence of amino acids).

Q.33 Choose the correct option having all the elements with d^{10} electronic configuration from the following :

- (1) ^{27}Co , ^{28}Ni , ^{26}Fe , ^{24}Cr (2) ^{46}Pd , ^{28}Ni , ^{26}Fe , ^{24}Cr
(3) ^{29}Cu , ^{30}Zn , ^{48}Cd , ^{47}Ag (4) ^{28}Ni , ^{24}Cr , ^{26}Fe , ^{29}Cu

Ans. [3]

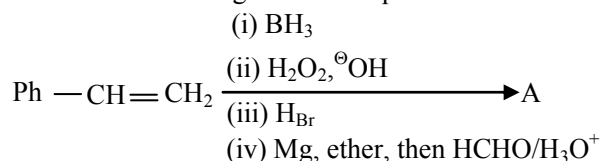
Sol. $\text{Cu} : 4s^1 3d^{10}$

$\text{Zn} : 4s^2 3d^{10}$

$\text{Cd} : 5s^2 4d^{10}$

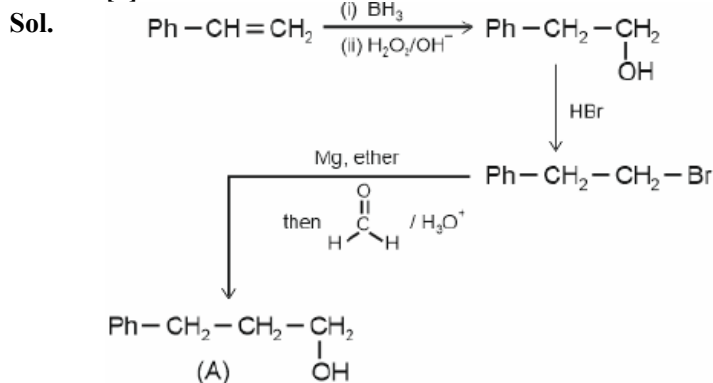
$\text{Ag} : 5s^1 4d^{10}$

Q.34 The final product A, formed in the following reaction sequence is



- (1) $\text{Ph}-\text{CH}_2-\text{CH}_2-\text{CH}_3$
- (2) $\text{Ph}-\underset{\text{CH}_3}{\text{CH}}-\text{CH}_3$
- (3) $\text{Ph}-\text{CH}_2-\text{CH}_2-\text{CH}_2-\text{OH}$
- (4) $\text{Ph}-\underset{\text{CH}_2\text{OH}}{\text{CH}}-\text{CH}_3$

Ans. [3]



Option (3) is correct.

Q.35 Given below are two statements :

Statement (I) : Oxygen being the first member of group 16 exhibits only -2 oxidation state.

Statement (II) : Down the group 16 stability of $+4$ oxidation state decreases and $+6$ oxidation state increases.

In the light of the above statements, choose the most appropriate answer from the options given below :

- (1) Statement I is incorrect but Statement II is correct
- (2) Both Statement I and Statement II are incorrect
- (3) Statement I is correct but Statement II is incorrect
- (4) Both Statement I and Statement II are correct

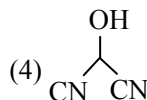
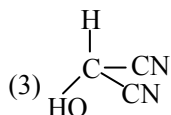
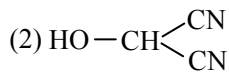
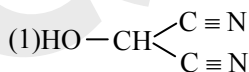
Ans. [2]

Sol. Oxygen can show other oxidation states apart from (-2) .

Stability of $+6$ oxidation state decreases down the group due to inert pair effect.

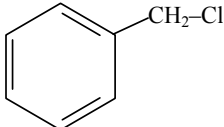
Both Statement I and Statement II are incorrect.

Q.36 Bond line formula of $\text{HOCH}(\text{CN})_2$ is :



Ans. [4]

Sol. Bond line representation does not include representation of H-atoms.

- Q.37** Which among the following halide/s will not show S_N1 reaction ?
- (A) $H_2C = CH - CH_2Cl$ (B) $CH_3 - CH = CH - Cl$
- (C)  (D) $\begin{matrix} CH_3 \\ \diagdown \\ CH - Cl \\ \diagup \\ CH_3 \end{matrix}$

Choose the **most appropriate** answer form the options given below :

- (1) (B) and (C) only (2) (B) only
 (3) (A) and (B) only (4) (A), (B) and (D) only

Ans. [2]

Sol. Vinyl halide will not show S_N1 reaction, due to very less stability of carbocation formed and partial double bond character in C - Cl bond.

Q.38 Identify the incorrect pair from the following:

- (1) Polythene preparation - $TiCl_4, Al(CH_3)_3$
 (2) Haber process - Iron
 (3) Wacker process - $PtCl_2$
 (4) Photography - AgBr

Ans. [3]

Sol. Wacker process - $PdCl_2, CuCl_2$

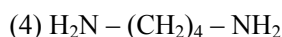
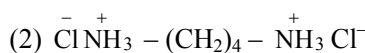
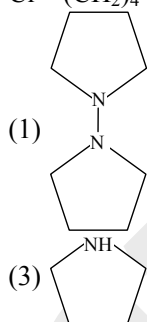
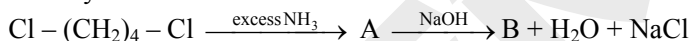
Q.39 The molecular formula of second homologue in the homologous series of mono carboxylic acids is _____.

- (1) CH_2O (2) $C_2H_4O_2$ (3) $C_3H_6O_2$ (4) $C_2H_2O_2$

Ans. [2]

Sol. Second homologue : CH_3COOH
 $= C_2H_4O_2$

Q.40 Identify B formed in the reaction



Ans. [4]

Sol. A : $Cl^- NH_3^+ - (CH_2)_4 - NH_3^+ Cl^-$

B : $H_2N - (CH_2)_4 - NH_2$

Q.41 The incorrect statement regarding conformations of ethane is:

- (1) Ethane has infinite number of conformations
 (2) Eclipsed conformation is the most stable conformation
 (3) The conformations of ethane are interconvertible to one-another
 (4) The dihedral angle in staggered conformation is 60°

Ans. [2]

Sol. Staggered conformation is most stable
 \Rightarrow Option (2) is incorrect

Q.42 The technique used for purification of steam volatile water immiscible substances is:

- (1) Steam distillation
- (2) Distillation
- (3) Fractional distillation under reduced pressure
- (4) Fractional distillation

Ans. [1]

Sol. Steam distillation is used for purification of steam volatile water immiscible substances.

Q.43 The quantity which changes with temperature is:

- (1) Molarity
- (2) Molality
- (3) Mass percentage
- (4) Mole fraction

Ans. [1]

Sol. Formula of molarity includes volume which depends upon temperature.

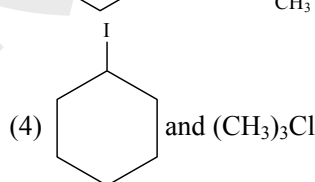
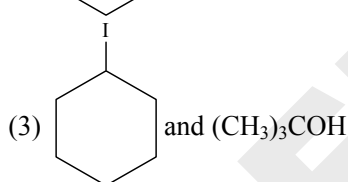
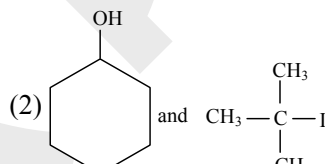
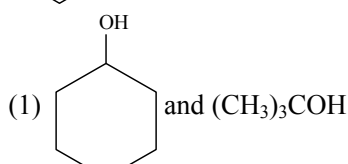
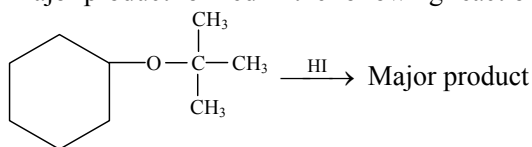
Q.44 Which of the following cannot function as an oxidising agent?

- (1) N^{3-}
- (2) SO_4^{2-}
- (3) MnO_4^-
- (4) BrO_3^-

Ans. [1]

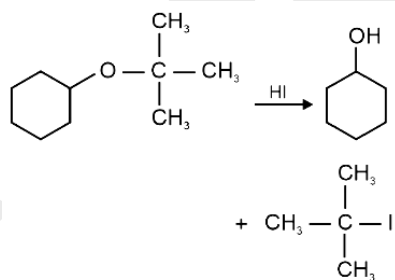
Sol. Nitrogen is present in its lowest oxidation state in N^{3-} .

Q.45 Major product formed in the following reaction is a mixture of:



Ans. [2]

Sol.



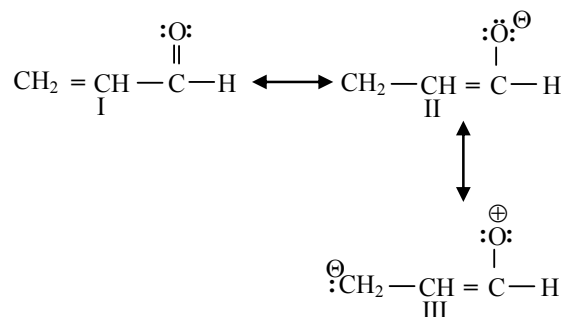
Q.46 Phenolic group can be identified by a positive:

- (1) Lucas test
- (2) Carbylamine test
- (3) Tollen's test
- (4) Phthalein dye test

Ans. [4]

Sol. Phenolic group is identified by phthalein dye test.

Q.47 The order of relative stability of the contributing structure is :



Choose the correct answer from the options given below:

- (1) III > II > I (2) I > II > III (3) II > I > III (4) I = II = III

Ans. [2]

Sol. In structure I, octet of every element is complete.

Q.48 Match List-I with List-II.

	List-I (Reaction)		List-II (Reagent (s))
(A)		(I)	Na ₂ Cr ₂ O ₇ , H ₂ SO ₄
(B)		(II)	(i) NaOH (ii) CHCl ₃
(C)		(III)	(i) NaOH, CHCl ₃ (ii) NaOH (iii) HCl
(D)		(IV)	(i) NaOH (ii) CO ₂ (iii) HCl

Choose the correct answer from the options given below:

- (1) (A)-(IV); (B)-(I); (C)-(III); (D)-(II)
 (2) (A)-(IV); (B)-(III); (C)-(I); (D)-(II)
 (3) (A)-(II); (B)-(III); (C)-(I); (D)-(IV)
 (4) (A)-(II); (B)-(I); (C)-(III); (D)-(IV)

Ans. [2]

Sol. (A) Kolbe reaction in presence of NaOH; CO₂ and HCl.
 (B) Reimer-Tiemann reaction
 (C) Oxidation of phenol
 (D) Williamson ether synthesis

Q.49 Given below are two statements:

Statement (I) : In the Lanthanoids, the formation Ce^{+4} is favoured by its noble gas configuration.

Statement (II) : Ce^{+4} is a strong oxidant reverting to the common +3 state.

In the light of the above statements, choose the most appropriate answer from the options given below:

- (1) Statement I is true but Statement II is false
- (2) Statement I is false but Statement II is true
- (3) Both Statement I and Statement II are false
- (4) Both Statement I and Statement II are true

Ans. [4]

Sol. $Ce_{58} = [Xe] 6s^2 5d^1 4f^1$
 $Ce^{4+} = [Xe] 6s^0 5d^0 4f^0$
 $E^0_{Ce^{4+}/Ce^{3+}}$ is very high
 $E^0_{Ce^{4+}/Ce^{3+}} = 1.74 \text{ V}$

Both statement I and statement II are true.

Q.50 Which of the following statements is not correct about rusting of iron?

- (1) Dissolved acidic oxides SO_2 , NO_2 in water act as catalyst in the process of rusting.
- (2) Rusting of iron is envisaged as setting up of electrochemical cell on the surface of iron object.
- (3) When pH lies above 9 or 10, rusting of iron does not take place.
- (4) Coating of iron surface by tin prevents rusting, even if the tin coating is peeling off.

Ans. [4]

Sol. If Tin coating is peeling off, then rusting of iron will take place as contact of iron with air will take place.

Section-B: Numerical Value Type Questions: This section contains 10 Numerical based questions. Attempt any 5 questions out of 10. The answer to each question should be rounded-off to the nearest integer..

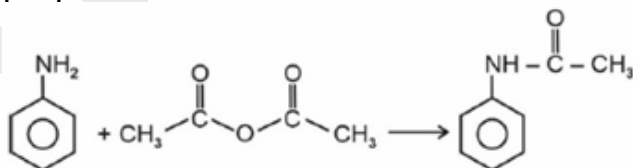
Q.51 Time required for completion of 99.9% of a First order reaction is _____ times of half life ($t_{1/2}$) of the reaction.

Ans. [10]

Sol. $t_{99.9} = \frac{2.303}{K} \log\left(\frac{100}{0.1}\right)$
 $= \frac{2.303}{K} \log 1000$
 $t_{50} = \frac{2.303}{K} \log 2$
 $\frac{t_{99.9}}{t_{50}} = \frac{3}{0.3} = 10$

Q.52 9.3 g of aniline is subjected to reaction with excess of acetic anhydride to prepare acetanilide. The mass of acetanilide produced if the reaction is 100% completed is _____ $\times 10^{-1}$ g.
 (Given molar mass in g mol^{-1} N : 14, O : 16, C : 12, H : 1)

Ans. [135]



Sol.

Moles of aniline = $\frac{9.3}{93} = 0.1$
 Mass of acetanilide = $(0.1) \times 135$
 $= 13.5 = 135 \times 10^{-1} \text{ gm}$

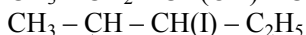
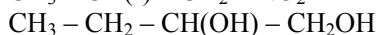
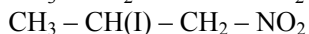
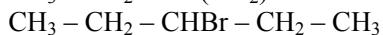
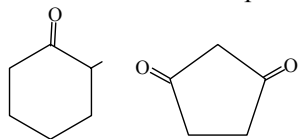
Q.53 Volume of 3 M NaOH (formula weight 40 g mol^{-1}) which can be prepared from 84 g of NaOH is $\text{_____} \times 10^{-1} \text{ dm}^3$.

Ans. [7]

Sol. Moles of NaOH = $\frac{84}{40} = 2.1$

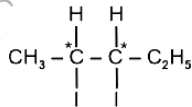
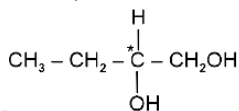
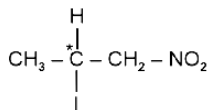
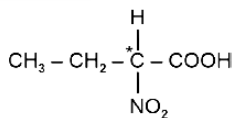
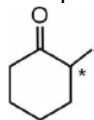
$$\text{Volume} = \frac{\text{Moles}}{\text{Molarity}} = \frac{2.1}{3} = 0.7 = 7 \times 10^{-1}$$

Q.54 Total number of compound with Chiral carbon atoms from following is _____.



Ans. [5]

Sol. Compounds with chiral carbon are



Q.55 The number of non-polar molecules from the following is _____.

HF, H₂O, SO₂, H₂, CO₂, CH₄, NH₃, HCl, CHCl₃, BF₃

Ans. [4]

Sol. H₂; CO₂; CH₄; BF₃

Q.56 For a certain thermochemical reaction $\text{M} \rightarrow \text{N}$ at $T = 400 \text{ K}$, $\Delta H^\circ = 77.2 \text{ kJ mol}^{-1}$, $\Delta S = 122 \text{ JK}^{-1}$, log equilibrium constant (log K) is - $\text{_____} \times 10^{-1}$.

Ans. [37]

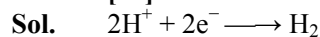
Sol. $\Delta G^\circ = \Delta H^\circ - T\Delta S^\circ$
 $= (77.2 \times 1000) - 400(122)$
 $= 77200 - 48800$
 $= 28400$

$$\Delta G^\circ = - (2.303)(8.314)(400)\log K$$

$$\begin{aligned}\log K &= -3.708 \\ &= -37.08 \times 10^{-1} \\ &= 37\end{aligned}$$

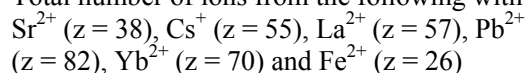
Q.57 The hydrogen electrode is dipped in a solution of pH = 3 at 25°C. The potential of the electrode will be – _____ $\times 10^{-2}$ V. $\left(\frac{2.303}{F} = 0.059\text{V}\right)$

Ans. [18]



$$\begin{aligned}E_{\text{cell}} &= 0 + \frac{.059}{2} \log(\text{H}^+)^2 \\ &= +.059(\log\text{H}^+) \\ &= -0.059 \times \text{PH} \\ &= -0.059 \times 3 \\ &= -0.177 \text{ V} \\ &= -17.7 \times 10^{-2} \text{ V} \\ \text{Nearest integer} &= 18\end{aligned}$$

Q.58 Total number of ions from the following with noble gas configuration is _____.

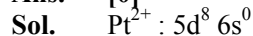


Ans. [3]

Sol. Species with noble gas configuration;
 Sr^{2+} ; Cs^+ ; Yb^{2+}

Q.59 The Spin only magnetic moment value of square planar complex $[\text{Pt}(\text{NH}_3)_2\text{Cl}(\text{NH}_2\text{CH}_3)]\text{Cl}$ is _____ B.M. (Nearest integer)
(Given atomic number for Pt = 78)

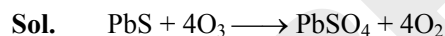
Ans. [0]



Pairing will take place and hence number of unpaired electrons = 0.

Q.60 1 mole of PbS is oxidised by “X” moles of O_3 to get “Y” moles of O_2 . $X + Y =$ _____.

Ans. [8]



$$X = 4$$

$$Y = 4$$

$$(X + Y) = 8$$

MATHEMATICS

Section-A: This section contains 20 multiple choice questions. Each question has 4 choices(1), (2), (3) and (4), out of which **ONLY ONE** is correct..

Q.61 Let e_1 be the eccentricity of the hyperbola $\frac{x^2}{16} - \frac{y^2}{9} = 1$ and e_2 be the eccentricity of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, $a > b$, which passes through the foci of the hyperbola. If $e_1 e_2 = 1$, then the length of the chord of the ellipse parallel to the x-axis and passing through (0, 2) is:

(1) $\frac{10\sqrt{5}}{3}$

(2) $3\sqrt{5}$

(3) $4\sqrt{5}$

(4) $\frac{8\sqrt{5}}{3}$

Ans. [1]

Sol. $\frac{x^2}{16} - \frac{y^2}{9} = 1$

$$e_1 = \frac{\sqrt{16+9}}{4} = \frac{5}{4}$$

foci $(\pm 5, 0)$

$$e_1 e_2 = 1$$

$$\therefore e_2 = \frac{4}{5}$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \text{ passes through } (\pm 5, 0)$$

$$\therefore \frac{25}{a^2} = 1 \Rightarrow a^2 = 25$$

$$e_2 = \frac{\sqrt{a^2 - b^2}}{a}$$

$$\frac{4}{5} = \frac{\sqrt{25 - b^2}}{5}$$

$$\Rightarrow b = 3$$

$$\frac{x^2}{25} + \frac{y^2}{9} = 1$$

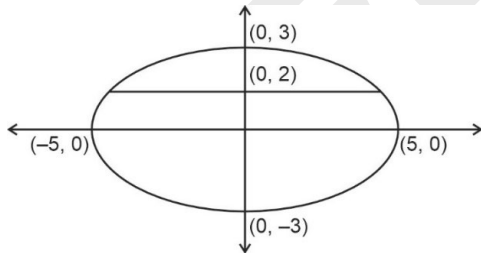
For $y = 2$

$$\frac{x^2}{25} + \frac{4}{9} = 1$$

$$\Rightarrow \frac{x^2}{25} = 1 - \frac{4}{9} = \frac{5}{9}$$

$$\Rightarrow x^2 = \frac{25 \times 5}{9} \Rightarrow x = \frac{5\sqrt{5}}{3}$$

$$\therefore \text{Length of chord} = 2 \times \frac{5\sqrt{5}}{3} = \frac{10\sqrt{5}}{3}$$



Q.62 The values of α , for which

$$\begin{vmatrix} 1 & \frac{3}{2} & \alpha + \frac{3}{2} \\ 1 & \frac{1}{3} & \alpha + \frac{1}{3} \\ 2\alpha + 3 & 3\alpha + 1 & 0 \end{vmatrix} = 0, \text{ lie in the interval}$$

(1) $(-2, 1)$

(2) $(0, 3)$

(3) $(-3, 0)$

(4) $\left(-\frac{3}{2}, \frac{3}{2}\right)$

Ans. [3]

Sol.
$$\begin{vmatrix} 1 & \frac{3}{2} & \alpha + \frac{3}{2} \\ 1 & \frac{1}{3} & \alpha + \frac{1}{3} \\ 2\alpha + 3 & 3\alpha + 1 & 0 \end{vmatrix} = 0$$

$$C_3 \rightarrow C_3 - (\alpha C_1 + C_2)$$

$$\begin{vmatrix} 1 & \frac{3}{2} & 0 \\ 1 & \frac{1}{3} & 0 \\ 2\alpha + 3 & 3\alpha + 1 & -(2\alpha^2 + 6\alpha + 1) \end{vmatrix} = 0$$

$$\Rightarrow 2\alpha^2 + 6\alpha + 1 = 0$$

$$\alpha = \frac{-3 + \sqrt{7}}{2}, \frac{-3 - \sqrt{7}}{2}$$

Q.63 Let R be the interior region between the lines $3x - y + 1$ and $x + 2y - 5 = 0$ containing the origin. The set of all values of a, for which the points $(a^2, a + 1)$ lie in R, is:

(1) $(-3, 0) \cup \left(\frac{2}{3}, 1\right)$ (2) $(-3, -1) \cup \left(\frac{1}{3}, 1\right)$ (3) $(-3, -1) \cup \left(\frac{1}{3}, 1\right)$ (4) $(-3, 0) \cup \left(\frac{1}{3}, 1\right)$

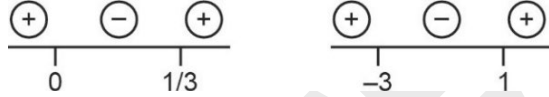
Ans. [4]

Sol. $(a^2, a + 1)$ and $(0, 0)$ lies on the same side of $3x - y + 1 = 0$ and $x + 2y - 5 = 0$

$$\Rightarrow 3a^2 - (a + 1) + 1 > 0 \text{ and } a^2 + 2a + 2 - 5 < 0$$

$$\Rightarrow a(3a - 1) > 0 \text{ and } (a^2 + 2a - 3) < 0$$

$$\Rightarrow a(3a - 1) > 0 \text{ and } (a + 3)(a - 1) < 0$$



$$\Rightarrow a \in (-\infty, 0) \cup \left(\frac{1}{3}, \infty\right) \text{ and } a \in (-3, 1)$$

$$\Rightarrow a \in (-3, 0) \cup \left(\frac{1}{3}, 1\right)$$

So, option (4) is correct.

Q.64 The integral $\int \frac{(x^8 - x^2)dx}{(x^{12} + 3x^6 + 1)\tan^{-1}\left(x^3 + \frac{1}{x^3}\right)}$ is equal to:

(1) $\log_e \left(\left| \tan^{-1}\left(x^3 + \frac{1}{x^3}\right) \right| \right) + C$

(2) $\log_e \left(\left| \tan^{-1}\left(x^3 + \frac{1}{x^3}\right) \right|^3 \right) + C$

(3) $\log_e \left(\left| \tan^{-1}\left(x^3 + \frac{1}{x^3}\right) \right|^{1/3} \right) + C$

(4) $\log_e \left(\left| \tan^{-1}\left(x^3 + \frac{1}{x^3}\right) \right|^{1/2} \right) + C$

Ans. [3]

Sol. Let $\tan^{-1}\left(x^3 + \frac{1}{x^3}\right) = t$

$$\Rightarrow \frac{3}{1 + \left(x^3 + \frac{1}{x^3}\right)^2} \left(x^2 - \frac{1}{x^4}\right) dx = dt$$

$$\Rightarrow \frac{3(x^8 - x^2)}{x^{12} + 3x^6 + 1} dx = dt$$

$$\int \frac{1}{3} \frac{dt}{t} = \frac{1}{3} \ln |t| + C$$

$$= \frac{1}{3} \ln \left| \tan^{-1} \left(x^3 + \frac{1}{x^3}\right) \right| + C$$

$$= \log_e \left(\left| \tan^{-1} \left(x^3 + \frac{1}{x^3}\right) \right| \right)^{1/3} + C$$

Hence, option (3) is correct.

Q.65 If α, β are the roots of the equation $x^2 - x - 1 = 0$ and $S_n = 2023 \alpha^n + 2024 \beta^n$, then:

(1) $S_{11} = S_{10} + S_{12}$ (2) $2S_{11} = S_{12} + S_{10}$ (3) $2S_{12} = S_{11} + S_{10}$ (4) $S_{12} = S_{11} + S_{10}$

Ans. [4]

Sol. We have $x^2 - x - 1 = 0$

By Newton's method

$$S_n - S_{n-1} - S_{n-2} = 0$$

$$\therefore S_n = S_{n-1} + S_{n-2}$$

$$\therefore S_{12} = S_{11} + S_{10}$$

Q.66 Consider the function $f : (0, 2) \rightarrow \mathbb{R}$ defined by $f(x) = \frac{x}{2} + \frac{2}{x}$ and the function $g(x)$ defined by $g(x) =$

$$\begin{cases} \min\{f(t)\}, & 0 < t \leq x \text{ and } 0 < x \leq 1 \\ \frac{3}{2} + x, & 1 < x < 2 \end{cases} \text{ . Then,}$$

- (1) g is not continuous for all $x \in (0, 2)$
- (2) g is continuous and differentiable for all $x \in (0, 2)$
- (3) g is continuous but not differentiable at $x = 1$
- (4) g is neither continuous nor differentiable at $x = 1$

Ans. [3]

Sol. $f(x) = \frac{x}{2} + \frac{2}{x}$

$$\Rightarrow f'(x) = \frac{1}{2} - \frac{2}{x^2} = \left(\frac{x^2 - 4}{2x^2}\right) \text{ for } x \in (0, 2)$$

$\Rightarrow f'(x) < 0 \Rightarrow f(x)$ is decreasing

$$g(x) = \begin{cases} \min\{f(t)\}, & 0 < t \leq x \text{ and } 0 < x \leq 1 \\ \frac{3}{2} + x, & 1 < x < 2 \end{cases}$$

Since $f(x)$ is decreasing, $f(t)$ is minimum for x in $t \in (0, x] \Rightarrow \min\{f(t)\} = f(x)$

$$\Rightarrow g(x) = \begin{cases} f(x), & x \in (0, 1] \\ \frac{3}{2} + x, & 1 < x < 2 \end{cases}$$

$$\Rightarrow g(1) = \frac{1}{2} + \frac{2}{1} = \frac{5}{2} \text{ also } g(1^+) = \frac{3}{2} + 1 = \frac{5}{2}$$

$\Rightarrow g(x)$ is continuous in $(0, 2)$

$$g'(x) = \begin{cases} f'(x), & x \in (0,1] \\ 1, & x \in (1,2) \end{cases}$$

$$g'(1) = f'(1) = \frac{1-4}{2} = \frac{-3}{2}$$

$\Rightarrow g'(1) \neq g'(1^+) \Rightarrow$ not differentiable at $x = 1$

Q.67 Let $f : \mathbb{R} - \left\{ \frac{-1}{2} \right\} \rightarrow \mathbb{R}$ and $g : \mathbb{R} - \left\{ \frac{-5}{2} \right\} \rightarrow \mathbb{R}$ be defined as $f(x) = \frac{2x+3}{2x+1}$ and $g(x) = \frac{|x|+1}{2x+5}$. Then, the domain of the function fog is :

- (1) \mathbb{R} (2) $\mathbb{R} - \left\{ \frac{-5}{2} \right\}$ (3) $\mathbb{R} - \left\{ \frac{-7}{4} \right\}$ (4) $\mathbb{R} - \left\{ \frac{-5}{2}, \frac{-7}{4} \right\}$

Ans. [2]

Sol. $f(g(x))$

$$\Rightarrow g(x) \neq -\frac{1}{2}$$

$$\frac{|x|+1}{2x+5} \neq -\frac{1}{2}$$

Case I : $x \geq 0$

$$\frac{x+1}{2x+5} = -\frac{1}{2}$$

$$2x+2 = -2x-5$$

$$4x = -7$$

$$x = -\frac{7}{4} \text{ (Rejected)}$$

Case II: $x < 0$

$$\frac{-x+1}{2x+5} = -\frac{1}{2}$$

$$-2x+2 = -2x-5$$

$$2 = -5 \text{ (Not possible)}$$

\Rightarrow Domain of $f(g(x)) =$ domain of $g(x)$

$$D_{\text{fog}} = \mathbb{R} - \left\{ \frac{-5}{2} \right\}$$

Q.68 For $0 < a < 1$, the value of the integral $\int_0^{\pi} \frac{dx}{1-2a \cos x + a^2}$ is:

- (1) $\frac{\pi}{\pi+a^2}$ (2) $\frac{\pi}{1+a^2}$ (3) $\frac{\pi}{1-a^2}$ (4) $\frac{\pi^2}{\pi-a^2}$

Ans. [3]

Sol. Let $I = \int_0^{\pi} \frac{dx}{1+a^2-2a \cos x}$

$$I = \frac{1}{2a} \int_0^{\pi} \frac{dx}{\frac{1+a^2}{2a} - \cos x}$$

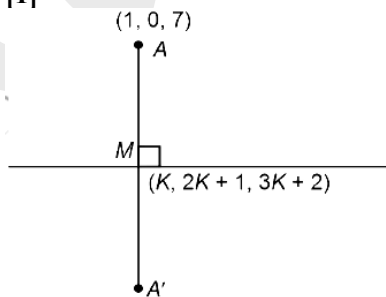
$$\text{Let } \alpha = \frac{1+a^2}{2a}$$

$$\begin{aligned}
 I &= \frac{1}{2a} \int_0^\pi \frac{dx}{\alpha - \cos x} \\
 &= \frac{1}{2a} \int_0^\pi \frac{dx}{\alpha - \left(\frac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}} \right)} \\
 &= \frac{1}{2a} \int_0^\pi \frac{\sec^2 \frac{x}{2} dx}{(\alpha - 1) + (\alpha + 1) \tan^2 \frac{x}{2}} \\
 \text{Let } \tan \frac{x}{2} &= t \Rightarrow \frac{1}{2} \sec^2 \frac{x}{2} dx = dt \\
 I &= \frac{1}{a} \int_0^\infty \frac{dt}{(\alpha - 1) + (\alpha + 1)t^2} \\
 &= \frac{1}{a(\alpha + 1)} \int_0^\infty \frac{dt}{\left(\frac{\alpha - 1}{\alpha + 1} \right) + t^2} \\
 &= \frac{1}{a(\alpha + 1)} \frac{1}{\sqrt{\frac{\alpha - 1}{\alpha + 1}}} \tan^{-1} \left(\frac{t}{\sqrt{\frac{\alpha - 1}{\alpha + 1}}} \right) \Bigg|_0^\infty \\
 &= \frac{1}{a\sqrt{\alpha^2 - 1}} \left[\tan^{-1} \infty - \tan^{-1} 0 \right] \\
 &= \frac{\pi}{2a\sqrt{\alpha^2 - 1}} = \frac{\pi}{2a\sqrt{\left(\frac{1+a^2}{2a} \right)^2 - 1}} = \frac{\pi}{\sqrt{(1-a^2)^2}} = \frac{\pi}{(1-a^2)} = \frac{\pi}{1-a^2}
 \end{aligned}$$

Q.69 Let the image of the point $(1, 0, 7)$ in the line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$ be the point (α, β, γ) . Then which one of the following points lies on the line passing through (α, β, γ) and making angles $\frac{2\pi}{3}$ and $\frac{3\pi}{4}$ with y-axis and z-axis respectively and an acute angle with x-axis?

- (1) $(3, 4, 3 - 2\sqrt{2})$ (2) $(1, 2, 1 - \sqrt{2})$ (3) $(3, -4, 3 + 2\sqrt{2})$ (4) $(1, -2, 1 + \sqrt{2})$

Ans. [1]



Sol.

$$\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3} = K$$

$$\vec{AM} = (K-1)\hat{i} + (2K+1)\hat{j} + (3K-5)\hat{k}$$

$$\vec{AM} \perp (\hat{i} + 2\hat{j} + 3\hat{k})$$

$$\Rightarrow (K-1) + 2(2K+1) + 3(3K-5) = 0$$

$$K + 4K + 9K - 1 + 2 - 15 = 0$$

$$14K = 14 \Rightarrow K = 1$$

$$A' \equiv (1, 6, 3)$$

Line passing through (1, 6, 3) and angle

$$\left(\frac{2\pi}{3}, \frac{3\pi}{4}, \theta\right), \theta \in \left(0, \frac{\pi}{2}\right)$$

$$\Rightarrow \cos^2\left(\frac{2\pi}{3}\right) + \cos^2\left(\frac{3\pi}{4}\right) + \cos^2\theta = 1$$

$$\Rightarrow \frac{1}{4} + \frac{1}{2} + \cos^2\theta = 1$$

$$\Rightarrow \cos^2\theta = \frac{1}{4} \Rightarrow \cos\theta = \frac{1}{2}$$

$$\Rightarrow \frac{x-1}{\frac{1}{2}} = \frac{y-6}{-\frac{1}{2}} = \frac{z-3}{-\frac{1}{\sqrt{2}}}$$

Q.70 The 20th term from the end of the progression $20, 19\frac{1}{4}, 18\frac{1}{2}, 17\frac{3}{4}, \dots, -129\frac{1}{4}$ is:
 (1) -118 (2) -100 (3) -115 (4) -110

Ans. [3]

Sol. Given $20, 19\frac{1}{4}, 18\frac{1}{2}, 17\frac{3}{4}, \dots, -129\frac{1}{4}$

$$-\frac{517}{4} = 20 + (n-1)\left(-\frac{3}{4}\right)$$

$$\Rightarrow -517 = 80 + (-3n + 3)$$

$$\Rightarrow -597 = -3n + 3$$

$$\Rightarrow -600 = -3n$$

$$\Rightarrow n = 200$$

n^{th} term from end is $(n-r+1)^{\text{th}}$

$$\therefore 200 - 20 + 1 = 181^{\text{st}} \text{ term}$$

$$a_{181} = 20 + (181-1)\left(-\frac{3}{4}\right)$$

$$a_{181} = 20 - 135$$

$$a_{181} = -115$$

Q.71 Let $g(x) = 3f\left(\frac{x}{3}\right) + f(3-x)$ and $f'(x) > 0$ for all $x \in (0, 3)$. If g is decreasing in $(0, \alpha)$ and increasing in $(\alpha, 3)$, then 8α is:

(1) 24 (2) 18 (3) 20 (4) 0

Ans. [2]

Sol. $g(x) = 3f\left(\frac{x}{3}\right) + f(3-x)$

$$g'(x) = f'\left(\frac{x}{3}\right) + f'(3-x)(-1)$$

Putting $g'(x) = 0$

$$f'\left(\frac{x}{3}\right) = f'(3-x)$$

$$\left(\frac{x}{3}\right) = 3-x$$

$$\Rightarrow x = 9 - 3x$$

$$\Rightarrow 4x = 9$$

$$\Rightarrow x = \frac{9}{4}$$

As g is decreasing in $(0, \alpha)$

$$f'\left(\frac{x}{3}\right) < f'(3-x) \text{ in } (0, \alpha)$$

g is increasing in $(\alpha, 3)$

$$f'\left(\frac{x}{3}\right) > f'(3-x) \text{ in } (\alpha, 3)$$

$$\Rightarrow \alpha = \frac{9}{4}$$

$$8\alpha = 8 \times \frac{9}{4}$$

$$= 18$$

Q.72 The position vectors of the vertices A, B and C of a triangle are $2\hat{i} - 3\hat{j} + 3\hat{k}$, $2\hat{i} + 2\hat{j} + 3\hat{k}$, and $-\hat{i} + \hat{j} + 3\hat{k}$, respectively. Let ℓ denotes the length of the angle bisector AD of $\angle BAC$ where D is on the line segment BC, the $2\ell^2$ equals:

(1) 42

(2) 50

(3) 45

(4) 49

Ans. [3]

Sol. Position vectors are

$$A(2, -3, 3), B(2, 2, 3), C(-1, 1, 3)$$

$$\vec{AB} = \vec{B} - \vec{A} = (2\hat{i} + 2\hat{j} + 3\hat{k}) - (2\hat{i} - 3\hat{j} + 3\hat{k})$$

$$= 5\hat{j}$$

$$\vec{AC} = \vec{C} - \vec{A} = (-\hat{i} + \hat{j} + 3\hat{k}) - (2\hat{i} - 3\hat{j} + 3\hat{k})$$

$$= 3\hat{i} + 4\hat{j}$$

$$|\vec{AB}| = 5$$

$$|\vec{AC}| = 5$$

Bisector of angle A meets \vec{BC} at D So \vec{AD} divides \vec{BC} in the ratio $\vec{AB} : \vec{AC}$

Position vector of D

$$\frac{|\vec{AC}|(2\hat{i} + 2\hat{j} + 3\hat{k}) + |\vec{AB}|(-\hat{i} + \hat{j} + 3\hat{k})}{|\vec{AB}| + |\vec{AC}|}$$
$$= \frac{5(2\hat{i} + 2\hat{j} + 3\hat{k}) + 5(-\hat{i} + \hat{j} + 3\hat{k})}{5 + 5}$$
$$= \frac{10\hat{i} + 10\hat{j} + 15\hat{k} - 5\hat{i} + 5\hat{j} + 15\hat{k}}{10}$$
$$= \frac{5\hat{i} + 15\hat{j} + 30\hat{k}}{10}$$

So,

$$\vec{AD} = \vec{OD} - \vec{OA} = \frac{5\hat{i} + 15\hat{j} + 30\hat{k}}{10} - 2\hat{i} - 3\hat{j} + 3\hat{k}$$
$$= \frac{5\hat{i} + 15\hat{j} + 30\hat{k} - 20\hat{i} + 30\hat{j} - 30\hat{k}}{10}$$
$$= \frac{-15\hat{i} + 45\hat{j}}{10}$$
$$2\ell^2 = \frac{4500}{100}$$
$$2\ell^2 = 45$$

Q.73 If $\lim_{x \rightarrow 0} \frac{3 + \alpha \sin x + \beta \cos x + \log_e(1-x)}{3 \tan^2 x} = \frac{1}{3}$, then $2\alpha - \beta$ is equal to:

(1) 7

(2) 2

(3) 1

(4) 5

Ans. [4]

Sol. $\lim_{x \rightarrow 0} \frac{3 + \alpha \sin x + \beta \cos x + \log_e(1-x)}{3 \tan^2 x} = \frac{1}{3}$

As the limit is finite when $x \rightarrow 0$, denominator $3 \tan^2 x \rightarrow 0$ so numerator has to be zero when $x \rightarrow 0$ to become $\frac{0}{0}$ form.

So, when $x \rightarrow 0$

$$3 + \alpha \sin 0 + \beta \cos 0 + \log_e(1-0) = 0$$

$$3 + \beta = 0$$

$$\beta = -3$$

As this is $\frac{0}{0}$ form, we apply L. Hopital rule here,

$$\text{So, } \lim_{x \rightarrow 0} \frac{\alpha \cos x - \beta \sin x + \frac{1(-1)}{1-x}}{6 \tan x \sec^2 x} = \frac{1}{3}$$

Again when $x \rightarrow 0$ denominator $6 \tan x \sec^2 x \rightarrow 0$

So, numerator has to be zero

So, when $x \rightarrow 0$

$$\alpha \cos 0 - \beta \sin 0 - \frac{1}{1-0} = 0$$

$$\Rightarrow \alpha - 1 = 0$$

$$\Rightarrow \alpha = 1$$

Now,

$$2\alpha - \beta = 2(1) - (-3)$$

$$= 2 + 3$$

$$= 5$$

- Q.74** Let A and B be two finite sets with m and n elements respectively. The total number of subsets of the set A is 56 more than the total number of subsets of B. Then the distance of the point P(m, n) from the point Q(-2, -3) is:
 (1) 6 (2) 4
 (3) 10 (4) 8

Ans. [3]

Sol. As given, $2^m - 2^n = 56$

$$\Rightarrow m = 6, n = 3$$

P(6, 3) and Q(-2, -3)

$$\begin{aligned} \text{Distance PQ} &= \sqrt{(6+2)^2 + (3+3)^2} \\ &= 10 \end{aligned}$$

- Q.75** Considering only the principal values of inverse trigonometric functions, the number of positive real values of x satisfying $\tan^{-1}(x) + \tan^{-1}(2x) = \frac{\pi}{4}$ is :

- (1) More than 2 (2) 1 (3) 0 (4) 2

Ans. [2]

Sol. $f(x) = \tan^{-1}x + \tan^{-1}2x$

$$f(x) = \frac{1}{1+x^2} + \frac{2}{1+4x^2} > 0 \quad \forall x \in \mathbb{R}$$

$$\lim_{x \rightarrow \infty} f(x) = \pi$$

$$\lim_{x \rightarrow -\infty} f(x) = -\pi$$

Hence, f(x) is monotonically increasing function for all $x \in \mathbb{R}$ and range is $(-\pi, \pi)$.

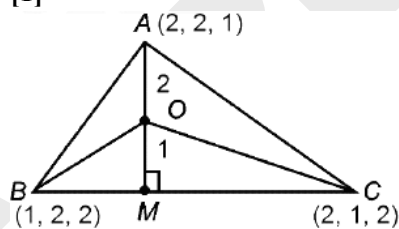
Hence, $f(x) = \frac{\pi}{4}$ has exactly one solution for $x > 0$.

- Q.76** Let the position vectors of the vertices A, B and C of a triangle be $2\hat{i} + 2\hat{j} + \hat{k}$, $\hat{i} + 2\hat{j} + 2\hat{k}$ and $2\hat{i} + \hat{j} + 2\hat{k}$ respectively. Let l_1 , l_2 and l_3 be the length of perpendicular drawn from the ortho center of the triangle on the sides AB, BC and CA respectively, then $l_1^2 + l_2^2 + l_3^2$ equals :

- (1) $\frac{1}{2}$ (2) $\frac{1}{3}$ (3) $\frac{1}{4}$ (4) $\frac{1}{5}$

Ans. [1]

Sol.



$$|AB| = \sqrt{1^2 + 0^2 + (-1)^2} = \sqrt{2} = BC = CA$$

\Rightarrow ABC is an equilateral triangle.

\Rightarrow Orthocentre = centroid \equiv circumcentre

$$\text{Centroid} = \left(\frac{5}{3}, \frac{5}{3}, \frac{5}{3} \right) \equiv O$$

$$\text{And } M = \left(\frac{3}{2}, \frac{3}{2}, 2 \right) \quad (\text{Mid-point of BC})$$

$$\begin{aligned}
 |OM| &= \sqrt{\left(\frac{5}{3}-\frac{3}{2}\right)^2 + \left(\frac{5}{3}-\frac{3}{2}\right)^2 + \left(\frac{5}{3}-2\right)^2} \\
 &= \sqrt{\frac{1}{36} + \frac{1}{36} + \frac{1}{9}} = \sqrt{\frac{6}{36}} = \frac{1}{\sqrt{6}} = \ell \\
 I_1 &= I_2 = I_3 = I \\
 \Rightarrow \ell_1^2 + \ell_2^2 + \ell_3^2 &= 3\ell^2 = 3 \times \frac{1}{6} = \frac{1}{2}
 \end{aligned}$$

Q.77 If $y = y(x)$ is the solution curve of the differential equation $(x^2 - 4)dy - (y^2 - 3y)dx = 0$, $x > 2$, $y(4) = \frac{3}{2}$ and the slope of the curve is never zero, then the value of $y(10)$ equals :

- (1) $\frac{3}{1+(8)^{\frac{1}{4}}}$ (2) $\frac{3}{1+2\sqrt{2}}$ (3) $\frac{3}{1-(8)^{\frac{1}{4}}}$ (4) $\frac{3}{1-2\sqrt{2}}$

Ans. [1]

Sol. $(x^2 - 4)dy - (y^2 - 3y)dx = 0$, $x > 2$, $y(4) = \frac{3}{2}$

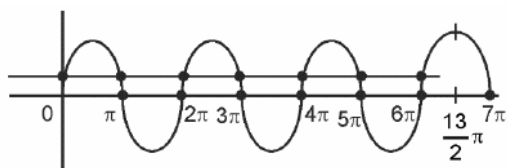
$$\begin{aligned}
 \frac{dy}{y^2 - 3y} &= \frac{dx}{x^2 - 4} \\
 \Rightarrow \frac{\ln|y-3|}{3} - \frac{\ln|y|}{3} &= \frac{\ln|x-2|}{4} - \frac{\ln|x+2|}{4} + C \\
 \Rightarrow \frac{1}{3} \ln \left| \frac{y-3}{y} \right| &= \frac{1}{4} \ln \left| \frac{x-2}{x+2} \right| + \ln C, C > 0 \\
 \Rightarrow \left(\frac{y-3}{y} \right)^{\frac{1}{3}} \left(\frac{x+2}{x-2} \right)^{\frac{1}{4}} &= K \\
 y(4) = \frac{3}{2} \Rightarrow \left(\frac{\frac{3}{2}-3}{\frac{3}{2}} \right)^{\frac{1}{3}} \left(\frac{6}{4} \right)^{\frac{1}{4}} &= K \Rightarrow K = 3^{\frac{1}{4}} \\
 \Rightarrow \left(\frac{y-3}{y} \right)^{\frac{1}{3}} \left(\frac{x+2}{x-2} \right)^{\frac{1}{4}} &= 3^{\frac{1}{4}} \\
 \text{At } x = 10, \left(\frac{y-3}{y} \right)^{\frac{1}{3}} \left(\frac{12}{8} \right)^{\frac{1}{4}} &= 3^{\frac{1}{4}} \\
 \Rightarrow \left(\frac{y-3}{y} \right)^{\frac{1}{3}} &= 2^{\frac{1}{4}} \\
 \Rightarrow \left| \frac{y-3}{y} \right| &= 2^{\frac{3}{4}} = 8^{\frac{1}{4}} \Rightarrow \frac{3-y}{y} = 8^{\frac{1}{4}} \\
 \Rightarrow y(10) &= \frac{3}{1+8^{\frac{1}{4}}}
 \end{aligned}$$

Q.78 If $2\tan^2\theta - 5\sec\theta = 1$ has exactly 7 solutions in the interval $\left[0, \frac{n\pi}{2}\right]$, for the least value of $n \in \mathbb{N}$, then $\sum_{k=1}^n \frac{k}{2^k}$ is equal to:

- (1) $1 - \frac{15}{2^{13}}$ (2) $\frac{1}{2^{13}}(2^{14} - 15)$ (3) $\frac{1}{2^{14}}(2^{15} - 15)$ (4) $\frac{1}{2^{15}}(2^{14} - 14)$

Ans. [2]

Sol. $2\tan^2\theta - 5\sec\theta - 1 = 0$
 $\Rightarrow 2(\sec^2\theta - 1) - 5\sec\theta - 1 = 0, \sec\theta \in (-\infty, -1] \cup [1, \infty)$
 $\Rightarrow 2\sec^2\theta - 5\sec\theta - 3 = 0$
 $2\sec^2\theta - 6\sec\theta + \sec\theta - 3 = 0$
 $2\sec\theta(\sec\theta - 3) + 1(\sec\theta - 3) = 0$
 $\Rightarrow \sec\theta = \frac{-1}{2}$ or $\sec\theta = 3$
 But $\sec\theta \in (-\infty, -1] \cup [1, \infty)$
 $\Rightarrow \sec\theta = 3$
 $\Rightarrow \cos\theta = \frac{1}{3}$



\Rightarrow Least value of $n = 13$

$$\Rightarrow \sum_{k=1}^{13} \frac{k}{2^k}$$

$$\Rightarrow \text{A. G. P.} \Rightarrow S = \frac{1}{2} + \frac{2}{2^2} + \frac{3}{2^3} + \dots + \frac{13}{2^{13}}$$

$$\Rightarrow \frac{S}{2} = \frac{1}{2^2} + \frac{1}{2^3} + \dots + \frac{12}{2^{13}} + \frac{13}{2^{14}}$$

$$\Rightarrow S - \frac{S}{2} = \frac{1}{2} + \frac{1}{2^2} + \dots + \frac{1}{2^{13}} - \frac{13}{2^{14}}$$

$$\Rightarrow \frac{S}{2} = \frac{\frac{1}{2} \left(1 - \left(\frac{1}{2}\right)^{13} \right)}{\left(1 - \frac{1}{2} \right)} - \frac{13}{2^{14}} = 1 - \frac{1}{2^{13}} - \frac{13}{2^{14}}$$

$$\Rightarrow S = 2 - \frac{1}{2^{12}} - \frac{13}{2^{13}} = \frac{2^{14} - 2 - 13}{2^{13}} = \frac{2^{14} - 15}{2^{13}}$$

Q.79 Let $\alpha = \frac{(4!)!}{(4!)^{3!}}$ and $\beta = \frac{(5!)!}{(5!)^{4!}}$. Then:

- (1) $\alpha \in \mathbb{N}$ and $\beta \in \mathbb{N}$ (2) $\alpha \notin \mathbb{N}$ and $\beta \in \mathbb{N}$
 (3) $\alpha \in \mathbb{N}$ and $\beta \notin \mathbb{N}$ (4) $\alpha \notin \mathbb{N}$ and $\beta \notin \mathbb{N}$

Ans. [1]

Sol. $\alpha = \frac{(4!)!}{(4!)^{3!}}$ and $\beta = \frac{(5!)!}{(5!)^{4!}}$.

$$4! = 24$$

⇒ Divide 24 different objects into 6 persons of each

⇒ Numbers of ways of grouping

$$= \frac{24!}{4!4!4!...4!} = \frac{24!}{(4!)^6} = \frac{(4!)!}{(4!)^6}$$

$$\Rightarrow \alpha \in \mathbb{N}$$

$$5! = 120$$

⇒ Divide 120 different into 24 persons 5 each

⇒ Number of ways of grouping

$$= \frac{(5!)!}{(5!)^{24}} \Rightarrow \beta \in \mathbb{N}$$

⇒ α, β both are natural number as they represent number of ways to group objects.

Q.80 An urn contains 6 white and 9 black balls. Two successive draws of 4 balls are made without replacement. The probability, that the first draw gives all white balls and the second draw gives all black balls, is:

(1) $\frac{5}{256}$

(2) $\frac{5}{715}$

(3) $\frac{3}{715}$

(4) $\frac{3}{256}$

Ans. [3]

Sol. 6 white, 9 black balls

First draw of 4 balls

⇒ Number of ways to get only white = 6C_4

Second draw of 4 balls

⇒ Number of ways to get only black = 9C_4

Total cases ⇒ First 4 balls out of 15 and then 4 balls out of remaining 11 balls

$$\Rightarrow {}^{15}C_4 \times {}^{11}C_4$$

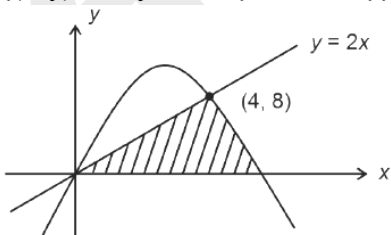
$$\Rightarrow \text{Probability} = \frac{{}^6C_4 \times {}^9C_4}{{}^{11}C_4 \cdot {}^{15}C_4} = \frac{3}{715}$$

Section-B: Numerical Value Type Questions: This section contains 10 Numerical based questions. Attempt any 5 questions out of 10. The answer to each question should be rounded-off to the nearest integer.

Q.81 If the area of the region $\{(x, y) : 0 \leq y \leq \min \{2x, 6x - x^2\}\}$ is A, then 12 A is equal to

Ans. [304]

Sol. $\{(x, y) : 0 \leq y \leq \min \{2x, 6x - x^2\}\}$



Shaded region is required area.

Solving $y = 2x$ and $y = 6x - x^2$

We will get (0, 0) or (4, 8)

$$\begin{aligned}\therefore A &= \frac{1}{2} \times 4 \times 8 + \int_4^6 (6x - x^2) dx \\ &= 16 + \left(3x^2 - \frac{x^3}{3} \right)_4^6 \\ &= 16 + \frac{28}{3} \\ \therefore 12A &= 304\end{aligned}$$

Q.82 Let $f(x) = \int_0^x g(t) \log_e \left(\frac{1-t}{1+t} \right) dt$, where g is a continuous odd function.

If $\int_{-\pi/2}^{\pi/2} \left(f(x) + \frac{x^2 \cos x}{1+e^x} \right) dx = \left(\frac{\pi}{\alpha} \right)^2 - \alpha$, then α is equal to _____.

Ans. [2]

Sol. $f(x) = \int_0^x g(t) \log_e \left(\frac{1-t}{1+t} \right) dt$ is odd.

$$\begin{aligned}\text{Now, } \int_{-\pi/2}^{\pi/2} \left(f(x) + \frac{x^2 \cos x}{1+e^x} \right) dx \\ &= \int_0^{\pi/2} \frac{(x^2 \cos x)(1+e^x)}{(1+e^x)} dx \\ &= \int_0^{\pi/2} x^2 \cos x dx = [x^2 \sin x]_0^{\pi/2} - 2 \int_0^{\pi/2} x \sin x dx \\ &= \frac{\pi^2}{4} - 2 \times 1 = \frac{\pi^2}{4} - 2 \\ \Rightarrow \alpha &= 2\end{aligned}$$

Q.83 Let the complex number α and $\frac{1}{\alpha}$ lie on the circles $|z - z_0|^2 = 4$ and $|z - z_0|^2 = 16$ respectively, where $z_0 = 1 + i$.

Then, the value of $100 |\alpha|^2$ is _____.

Ans. [20]

Sol. $|z - z_0|^2 = 4$ and $|z_0|^2 = 2$

$$\Rightarrow (z - z_0)(\bar{z} - \bar{z}_0) = 4$$

$$\Rightarrow z\bar{z} - z\bar{z}_0 - z_0\bar{z} + z_0\bar{z}_0 = 4$$

$$\Rightarrow |z|^2 - z\bar{z}_0 - z_0\bar{z} + 2 = 4$$

α lies on it

$$\Rightarrow |\alpha|^2 - \alpha\bar{z}_0 - z_0\bar{\alpha} - 2 = 0 \quad \dots(1)$$

Similarly

$$|z|^2 - z\bar{z}_0 - z_0\bar{z} - 14 = 0$$

$$\left[\because \left(\frac{1}{\alpha} \right) = \frac{\alpha}{|\alpha|^2} \right]$$

$\frac{1}{\alpha}$ lies on it

$$\Rightarrow \frac{1}{|\alpha|^2} - \frac{\alpha}{|\alpha|^2} \cdot \bar{z}_0 - z_0 \frac{\alpha}{|\alpha|^2} - 14 = 0$$

$$\Rightarrow 1 - \alpha \bar{z}_0 - z_0 \alpha - 14 |\alpha|^2 = 0 \quad \dots(2)$$

Using (1) and (2)

$$\Rightarrow |\alpha|^2 - 2 = 1 - 14 |\alpha|^2$$

$$\Rightarrow 15 |\alpha|^2 = 3 \Rightarrow |\alpha|^2 = \frac{1}{5}$$

$$\Rightarrow 100|\alpha|^2 = 20$$

* **Note** : There is no such complex number α satisfy both circles.

Q.84 If the sum of squares of all real values of α , for which the lines $2x - y + 3 = 0$, $6x + 3y + 1 = 0$ and $\alpha x + 2y - 2 = 0$ do not form a triangle is p . then the greatest integer less than or equal to p is _____.

Ans. [32]

Sol. $2x - y + 3 = 0 \quad \dots(1)$

$6x + 3y + 1 = 0 \quad \dots(2)$

$\alpha x + 2y - 2 = 0 \quad \dots(3)$

Triangle will not form if either at least two lines are parallel or lines are concurrent.

Case I

if two lines are parallel

$$\frac{2}{\alpha} = \frac{-1}{2}$$

$$\Rightarrow \alpha = -4$$

or

$$\frac{6}{\alpha} = \frac{3}{2}$$

$$\Rightarrow \alpha = 4$$

Case II

If lines are concurrent

$$\begin{vmatrix} 2 & -1 & 3 \\ 6 & 3 & 1 \\ \alpha & 2 & -2 \end{vmatrix} = 0$$

$$\Rightarrow 8 - 10\alpha = 0$$

$$\Rightarrow \alpha = \frac{4}{5}$$

$$\Rightarrow \alpha_1^2 + \alpha_2^2 + \alpha_3^2 = p = 16 + 16 + \frac{16}{25}$$

Greatest integer less than or equal to p is 32

Q.85 The coefficient of x^{2012} in the expansion of $(1-x)^{2008} (1+x+x^2)^{2007}$ is equal to _____.

Ans. [0]

Sol. coefficient of x^{2012} in $(1-x)^{2008} (1+x+x^2)^{2007}$

$$(1-x)^{2008} (1+x+x^2)^{2007}$$

$$= (1-x) (1-x^3)^{2007}$$

$$= (1-x) \sum_{r=0}^{2007} {}^{2007}C_r (-x^3)^r$$

$$3r = 2012 \text{ or } 3r + 1 = 2012$$

$$r \notin \omega$$

$$r \notin \omega$$

\therefore No term contains x^{2012}

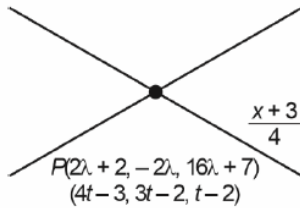
\Rightarrow Coefficient of $x^{2012} = 0$

Q.86 The lines $\frac{x-2}{2} = \frac{y}{-2} = \frac{z-7}{16}$ and $\frac{x+3}{4} = \frac{y+2}{3} = \frac{z+2}{1}$ intersect at the point P. If the distance of P from the line $\frac{x+1}{2} = \frac{y-1}{3} = \frac{z-1}{1}$ is ℓ , then $14\ell^2$ is equal to _____.

Ans. [108]

Sol. $\frac{x-2}{2} = \frac{y}{-2} = \frac{z-7}{16}$

$$\frac{x-2}{2} = \frac{y}{-2} = \frac{z-7}{16} = \lambda$$

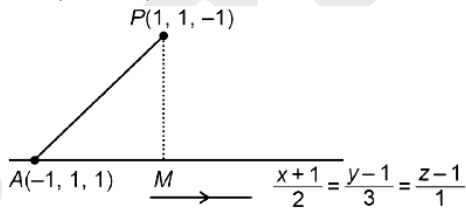


$$\frac{x+3}{4} = \frac{y+2}{3} = \frac{z+2}{1} = t$$

$$\Rightarrow 2\lambda + 2 = 4t - 3, -2\lambda = 3t - 2, 16\lambda + 7 = t - 2$$

$$\Rightarrow \lambda = -\frac{1}{2}, t = 1$$

$$\Rightarrow P(1, 1, -1)$$



$$\vec{AP} = 2\hat{i} - 2\hat{k}$$

$$\vec{b} = 2\hat{i} + 3\hat{j} + \hat{k}$$

$$AP^2 = 4 + 4 = 8$$

$$AM = \frac{|\vec{AP} \cdot \vec{b}|}{\sqrt{14}} \Rightarrow AM^2 = \frac{4}{14} = \frac{2}{7}$$

$$\Rightarrow PM = \sqrt{8 - \frac{2}{7}} = \sqrt{\frac{54}{7}} = \ell$$

$$\Rightarrow \frac{54}{7} = \ell^2 \Rightarrow 14\ell^2 = 108$$

Q.87 The mean and standard deviation of 15 observations were found to be 12 and 3 respectively. On rechecking it was found that an observation was read as 10 in place of 12. If μ and σ^2 denote the mean and variance of the correct observations respectively, then $15(\mu + \mu^2 + \sigma^2)$ is equal to _____.

Ans. [2521]

Sol. Let the incorrect mean be μ' and standard deviation be σ'

$$\mu' = \frac{\sum x_i}{15} = 12 \Rightarrow \sum x_i = 180$$

As per given information correct

$$\sum x_i = 180 - 10 + 12$$

$$\mu = \frac{182}{15} \text{ (correct mean)}$$

Also

$$\sigma' = \sqrt{\frac{\sum x_i^2}{15} - 144} = 3 \Rightarrow \sum x_i^2 = 2295$$

Correct

$$\sum x_i^2 = 2295 - 100 + 144 = 2339$$

$$\sigma^2 \text{ (correct variance)} = \frac{2339}{15} - \mu^2$$

Required value

$$15 \left(\mu + \mu^2 + \frac{2339}{15} - \mu^2 \right)$$

$$15 \left(\frac{182}{15} + \frac{2339}{15} \right) = 2521$$

Q.88 Let A be a 2×2 real matrix and I be the identity matrix of order 2. If the roots of the equation $|A - xI| = 0$ be -1 and 3, then the sum of the diagonal elements of the matrix A^2 is _____.

Ans. [10]

Sol. Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$$|A - xI| = 0 \Rightarrow \begin{vmatrix} a-x & b \\ c & d-x \end{vmatrix} = 0$$

$$(a-x)(d-x) - bc = 0$$

$$x^2 - (a+d)x + ad - bc = 0$$

$$\Rightarrow a+d=2, ad-bc=-3$$

$$A^2 = \begin{bmatrix} a^2+bc & ab+bd \\ ac+cd & bc+d^2 \end{bmatrix}$$

$$\Rightarrow \text{Sum} = a^2 + d^2 + 2bc = (a+d)^2 - 2ad + 2bc \\ = 4 - 2(-3) = 10$$

Q.89 Consider a circle $(x - \alpha)^2 + (y - \beta)^2 = 50$, where $\alpha, \beta > 0$. If the circle touches the line $y + x = 0$ at the point P, whose distance from the origin is $4\sqrt{2}$, then $(\alpha + \beta)^2$ is equal to _____.

Ans. [100]

Sol. Let P $(\alpha + 5\sqrt{2} \cos\theta, \beta + 5\sqrt{2} \sin\theta)$

Equation of tangent to circle $(x - \alpha)^2 + (y - \beta)^2 = 50$ at point P is

$$(x - \alpha) \cos\theta + (y - \beta) \sin\theta = 5\sqrt{2}$$

$$\begin{aligned} \Rightarrow x \cos \theta + y \sin \theta &= \alpha \cos \theta + \beta \sin \theta + 5\sqrt{2} \\ x + y &= 0 \\ \Rightarrow \frac{\cos \theta}{1} &= \frac{\sin \theta}{1} = \frac{\alpha \cos \theta + \beta \sin \theta + 5\sqrt{2}}{0} \\ \Rightarrow \tan \theta &= 1, \alpha \cos \theta + \beta \sin \theta + 5\sqrt{2} = 0 \\ \tan \theta = 1 &\Rightarrow \sin \theta = \frac{1}{\sqrt{2}}, \cos \theta = \frac{1}{\sqrt{2}} \\ \text{or } \sin \theta &= \cos \theta = -\frac{1}{\sqrt{2}} \\ \Rightarrow (\alpha + \beta) &\pm 10 \\ \Rightarrow (\alpha + \beta)^2 &= 100 \end{aligned}$$

Q.90 If the solution curve, of the differential equation $\frac{dy}{dx} = \frac{x+y-2}{x-y}$ passing through the point (2, 1) is

$$\tan^{-1}\left(\frac{y-1}{x-1}\right) - \frac{1}{\beta} \log_e \left(\alpha + \left(\frac{y-1}{x-1}\right)^2 \right) = \log_e |x-1|, \text{ then } 5\beta + \alpha \text{ is equal to.}$$

Ans. [11]

Sol. $\frac{dy}{dx} = \frac{x+y-2}{x-y}$

$x + y - 2 = 0$ and $x - y = 0$ intersect at (1, 1)
 $x = X + 1, y = Y + 1$

$$\Rightarrow \frac{dy}{dx} = \frac{x+y}{x-y}$$

Put $Y = tX \rightarrow \frac{dy}{dx} = t + X \frac{dt}{dX}$

$$\Rightarrow t + X \frac{dt}{dX} = \frac{1+t}{1-t}$$

$$X \frac{dt}{dX} = \frac{1+t^2}{1-t}$$

$$\Rightarrow \frac{dX}{X} = \frac{1-t}{1+t^2} dt$$

$$\ln|X| = \tan^{-1}t - \frac{1}{2} \ln|1+t^2|$$

$$\ln|x-1| = \tan^{-1}\left(\frac{y-1}{x-1}\right) - \frac{1}{2} \ln \left| 1 + \left(\frac{y-1}{x-1}\right)^2 \right| + C$$

Passes through (2, 1) $\Rightarrow C = 0$

$$\Rightarrow \tan^{-1}\left(\frac{y-1}{x-1}\right) - \frac{1}{2} \ln \left(1 + \left(\frac{y-1}{x-1}\right)^2 \right) = \ln|x-1|$$

$$\Rightarrow 5\beta + \alpha = 11$$