JEE Main Online Exam 2024

Questions & Solution

27th January 2024 | Morning

PHYSICS

Section-A: This section contains 20 multiple choice questions. Each question has 4 choices(1), (2), (3) and (4), out of which **ONLY ONE** is correct..

Q.1 Given below are two statements:

Statement-I : Viscosity of gases is greater than that of liquids.

Statement-II : Surface tension of a liquid decreases due to the presence of insoluble impurities.

In the light of the above statements, choose the **most appropriate** answer from the options given below :

(1) **Statement-I** is incorrect but **statement-II** is correct

(2) Statement-I is correct but **statement-II** is incorrect

(3) Both **Statement-I** and **Statement-II** are incorrect

(4) Both **Statement-I** and **Statement-II** are correct

Ans. **[1]**

Sol. Viscosity of liquids is greater than gas.

Surface tension decreases due to the presence of impurities.

Q.2 0.08 kg air is heated at constant volume through 5°C. The specific heat of air at constant volume is 0.17 kcal/kg°C and J = 4.18 joule/cal. The change in its internal energy is approximately.

(1) 284 J (2) 298 J (3) 318 J (4) 142 J

Ans. **[1]**

Sol. $\Delta U = mC\Delta T$

$= 0.08 \times 0.17 \times 4.18 \times 10^3 \times 5$

$= 284$ J

Q.3 Identify the physical quantity that cannot be measured using spherometer :

(1) Radius of curvature of convex surface (2) Radius of curvature of concave surface

(3) Specific rotation of liquids (4) Thickness of thin plates

Ans. **[3]**

Sol. Spherometer cannot measure specific rotation of liquids.

Q.4 Position of an ant (S in metres) moving in Y-Z plane is given by $S = 2t^2\hat{j} + 5\hat{k}$ (where t is in second). The magnitude and direction of velocity of the ant at t = 1 s will be :

(1) 4 m/s in y-direction (2) 9 m/s in z-direction

(3) 16 m/s in y-direction (4) 4 m/s in x-direction

Ans. **[1]**

Sol. $\vec{v} = \dfrac{d\vec{s}}{dt}$

$\vec{v} = 4t\hat{j}$

At t = 1 s

$\vec{v} = 4\hat{j}$

Q.5 An electric charge 10^{-6} μC is placed at origin (0, 0) m of X – Y co-ordinate system. Two points P and Q are situated at $\left(\sqrt{3},\sqrt{3}\right)$ m and $\left(\sqrt{6},0\right)$ m respectively. The potential difference between the points P and Q will be :

(1) $\sqrt{6}$ V (2) 0 V (3) 3 V (4) $\sqrt{3}$ V

Ans. **[2]**

Sol. $V = \frac{KQ}{r}$

at P : $r_1 = \sqrt{6}$

at Q : $r_2 = \sqrt{6}$

$\therefore$ $V_P = V_Q$

$\Rightarrow$ $\Delta V = 0$

Q.6 A proton moving with a constant velocity passes through a region of space without any change in its velocity. If $\vec{E}$ and $\vec{B}$ represent the electric and magnetic fields respectively, then the region of space may have :

(A) E = 0, B = 0 (B) E = 0, B ≠ 0 (C) E ≠ 0, B = 0 (D) E ≠ 0, B ≠ 0

Choose the **most appropriate** answer from the options given below :

(1) (A), (C) and (D) only (2) (A), (B) and (C) only
(3) (A), (B) and (D) only (4) (B), (C) and (D) only

Ans. **[3]**

Sol. For constant velocity, net force on charge particle should be zero.
That is possible for A, B and D.

Q.7 The radius of third stationary orbit of electron for Bohr's atom is R. The radius of fourth stationary orbit will be :

(1) $\frac{16}{9}$R (2) $\frac{9}{16}$ R (3) $\frac{4}{3}$ R (4) $\frac{3}{4}$ R

Ans. **[1]**

Sol. For 3rd orbit

$R = r_0\frac{(3)^2}{Z}$

$R' = r_0\frac{(4)^2}{Z}$

$\therefore R' = \left(\frac{16}{9}\right) R$

Q.8 The average kinetic energy of a monoatomic molecule is 0.414 eV at temperature : (Use $K_B = 1.38 \times 10^{-23}$ J/mol-K)

(1) 3000 K (2) 3200 K (3) 1500 K (4) 1600 K

Ans. **[2]**

Sol. $E = \frac{3}{2}K_BT$

$0.414 \times 1.6 \times 10^{-19} = \frac{3}{2} \times 1.38 \times 10^{-23}$ T

T = 3200 K

Q.9 A wire of resistance R and length L is cut into 5 equal parts. If these parts are joined parallely, then resultant resistance will be :

(1) $\frac{1}{5}$R (2) $\frac{1}{25}$R (3) 5R (4) 25R

Ans. [2]

Sol. Resistance of each part = $\frac{R}{5}$

$$\therefore R_{eq} = \left(\frac{1}{5}\right)\left(\frac{R}{5}\right) = \frac{R}{25}$$

Q.10 A rectangular loop of length 2.5 m and width 2 m is placed at 60° to a magnetic field of 4T. The loop is removed from the field in 10 s. The average emf induced in the loop during this time is :

(1) + 2 V (2) –2 V (3) –1 V (4) +1 V

Ans. [4]

Sol. $E_{avg} = -\frac{\Delta\phi}{\Delta t}$

$\Delta\phi = -BA\cos 60°$

$= -4 \times 2.5 \times 2\frac{1}{2}$

$= -10$

$E_{avg} = \frac{+10}{10} = 1\text{ V}$

Q.11 A train is moving with a speed of 12 m/s on rails which are 1.5 m apart. To negotiate a curve of radius 400 m, the height by which the outer rail should be raised with respect to the inner rail is (Given $g = 10\text{ m/s}^2$)

(1) 4.8 cm (2) 4.2 cm (3) 6.0 cm (4) 5.4 cm

Ans. [4]

Sol. $\tan\theta = \frac{v^2}{rg} = \frac{h}{w}$

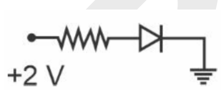
$\therefore h = \frac{v^2}{rg} w$

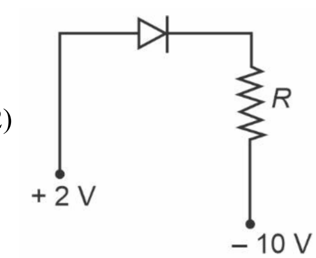
$= \frac{(12)^2 \times 1.5}{400 \times 10}$

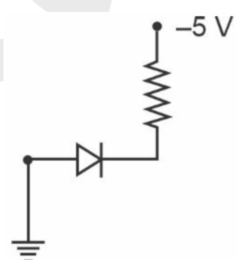
$= 0.054$ m

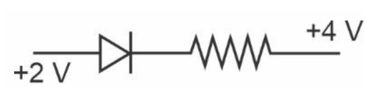
$= 5.4$ cm

Q.12 Which of the following circuits is reverse-biased?

(1) 

(2) 

(3) 

(4) 

Ans. [4]

Sol. For reverse bias $V_P < V_N$, that is true for option (4)

Q.13 If the refractive index of the material of a prism is $\cot\left(\frac{A}{2}\right)$, where A is the angle of prism then angle of minimum deviation will be -

(1) $\frac{\pi}{2} - 2A$ (2) $\pi - 2A$ (3) $\frac{\pi}{2} - A$ (4) $\pi - A$

Ans. **[2]**

Sol. For minimum deviation

$$\mu = \frac{\sin\frac{(\delta + A)}{2}}{\sin\left(\frac{A}{2}\right)} = \cot\left(\frac{A}{2}\right)$$

$$\delta = \pi - 2A$$

Q.14 Two bodies of mass 4 g and 25 g are moving with equal kinetic energies. The ratio of magnitude of their linear momentum is

(1) 5 : 4 (2) 2 : 5 (3) 3 : 5 (4) 4 : 5

Ans. **[2]**

Sol. $K = \frac{P^2}{2m}$

$$\frac{P_1^2}{2m_1} = \frac{P_2^2}{2m_2}$$

$$\frac{P_1}{P_2} = \sqrt{\frac{m_1}{m_2}} = \frac{2}{5}$$

Q.15 A wire of length 10 cm and radius $\sqrt{7} \times 10^{-4}$ m is connected across the right gap of a meter bridge. When a resistance of 4.5 Ω is connected on the left gap by using a resistance box, the balance length is found to be at 60 cm from the left end. If the resistivity of the wire is $R \times 10^{-7}$ Ω m, then value of R is

(1) 35 (2) 63 (3) 70 (4) 66

Ans. **[4]**

Sol. $\frac{4.5}{60} = \frac{S}{40}$

$$\therefore S = 3\ \Omega = \frac{\rho\ell}{A}$$

$$\rho = \frac{SA}{\ell} = 66 \times 10^{-7}$$

$$\therefore R = 66$$

Q.16 The acceleration due to gravity on the surface of earth is g. If the diameter of earth reduces to half of its original value and mass remains constant, then acceleration due to gravity on the surface of earth would be :

(1) 4g (2) $\frac{g}{4}$ (3) 2g (4) $\frac{g}{2}$

Ans. **[1]**

Sol. $\because g = \frac{GM}{R^2}$

If $R' = \frac{R}{2}$

$\Rightarrow g' = 4g$

Q.17 A body of mass 1000 kg is moving horizontally with a velocity 6 m/s. If 200 kg extra mass is added, the final velocity (in m/s) is:

(1) 3 (2) 2 (3) 5 (4) 6

Ans. **[3]**

Sol. By conservation of momentum

$1000 \times 6 = 1200 \times V$

$\Rightarrow V = 5$ m/s

Q.18 A plane electromagnetic wave propagating in x-direction is described by $E_y = (200\ Vm^{-1})\sin[1.5 \times 10^7 t - 0.05\ x]$; The intensity of the wave is: (Use $\varepsilon_0 = 8.85 \times 10^{-12}\ C^2N^{-1}m^{-2}$)

(1) 26.6 Wm^{-2} (2) 35.4 Wm^{-2} (3) 53.1 Wm^{-2} (4) 106.2 Wm^{-2}

Ans. **[3]**

Sol. $I = \frac{1}{2}\epsilon_0 E_0^2 C$

$= \frac{1}{2} \times 8.85 \times 10^{-12} \times (200)^2 \times (3 \times 10^8)$

$= 53.1\ W\ m^{-2}$

Q.19 Given below are two statements:

Statement (I): Planck's constant and angular momentum have same dimensions.

Statement (II): Linear momentum and moment of force have same dimensions.

In the light of the above statements, choose the **correct** answer from the options given below:

(1) Statement I is true but Statement II is false (2) Both Statement I and Statement II are false

(3) Statement I is false but Statement II is true (4) Both Statement I and Statement II are true

Ans. **[1]**

Sol. $[h] = [ML^2T^{-1}]$

$[L] = [ML^2T^{-1}]$

$[P] = [MLT^{-1}]$

$[FL] = [MLT^{-2}] \times [L]$

$= [ML^2T^{-2}]$

Q.20 A convex lens of focal length 40 cm forms an image of an extended source of light on a photoelectric cell. A current I is produced. The lens is replaced by another convex lens having the same diameter but focal length 20 cm. The photoelectric current now is:

(1) 4I (2) I (3) $\frac{I}{2}$ (4) 2I

Ans. **[2]**

Sol. As the diameter is same, amount of incident energy on cell is same, number of photons will be same.

$\therefore I' = I$

Section-B: Numerical Value Type Questions: This section contains 10 Numerical based questions. Attempt any 5 questions out of 10. The answer to each question should be rounded-off to the nearest integer.

Q.21 A particle starts from origin at t = 0 with a velocity $5\hat{i}$ m/s and moves in x-y plane under action of a force which produces a constant acceleration of $(3\hat{i} + 2\hat{j})$ m/s^2. If the x-coordinate of the particle at that instant is 84 m, then the speed of the particle at this time is $\sqrt{\alpha}$ m/s. The value of α is _______.

Ans. **[673]**

Sol. $x = 5t + \frac{1}{2} \times 3t^2 = 84$

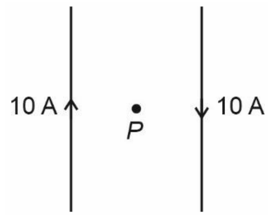
$\therefore t = 6$ s

$\vec{v} = 5\hat{i} + 6\,(3\hat{i} + 2\,\hat{j})$

$\vec{v} = 23\hat{i} + 12\hat{j}$

$v = \sqrt{673}$

Q.22 Two long, straight wires carry equal currents in opposite directions as shown in figure. The separation between the wires is 5.0 cm. The magnitude of the magnetic field at a point P midway between the wires is ______ μT. (Given: $\mu_0 = 4\pi \times 10^{-7}$ TmA^{-1})



Ans. **[160]**

Sol. $B = 2 \times \frac{\mu_0 i}{2\pi\left(\frac{d}{2}\right)} = 160 \times 10^{-6}$ T

Q.23 If average depth of an ocean is 4000 m and the bulk modulus of water is 2 × 109 Nm^{-2}, then fractional compression $\frac{\Delta V}{V}$ of water at the bottom of ocean is $\alpha \times 10^{-2}$. The value of α is __________. (Given, $g = 10$ ms^{-2}, $\rho = 1000$ kg m^{-3})

Ans. **[2]**

Sol. $B = -\frac{\Delta P}{\left(\frac{\Delta V}{V}\right)}$

$\therefore -\frac{\Delta V}{V} = \frac{1000 \times 10 \times 4000}{2 \times 10^9} = 2 \times 10^{-2}$

Q.24 Four particles each of mass 1 kg are placed at four corners of a square of side 2 m. Moment of inertia of system about an axis perpendicular to its plane and passing through one of its vertex is _______ kgm^2.

Ans. **[16]**

Sol. $I = 2(Ma^2) + M(\sqrt{2}\,a)^2$

$I = 4Ma^2$

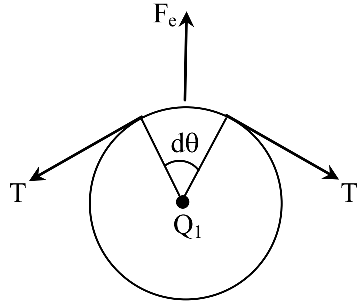
$= 4 \times 1 \times 4 = 16$

Q.25 A thin metallic wire having cross sectional area of 10^{-4} m^2 is used to make a ring of radius 30 cm. A positive charge of 2π C is uniformly distributed over the ring, while another positive charge of 30 pC is kept at the centre of the ring. The tension in the ring is _______ N; provided that the ring does not get deformed (neglect the influence of gravity).

(given, $\frac{1}{4\pi \in_0} = 9 \times 10^9$ SI units)

Ans. [3]

Sol.



$$2T \sin\frac{d\theta}{2} = \frac{kQ_1}{r^2} \times r d\theta$$

$$\because \left(\lambda = \frac{Q_2}{2\pi r}\right)$$

$$T = \frac{kQ_1 Q_2}{r^2 2\pi} = 3N$$

Q.26 A particle executes simple harmonic motion with an amplitude of 4 cm. At the mean position, velocity of the particle is 10 cm/s. The distance of the particle from the mean position when its speed becomes 5 cm/s is $\sqrt{\alpha}$ cm, where α = __________.

Ans. [12]

Sol. $v = \omega\sqrt{A^2 - x^2}$

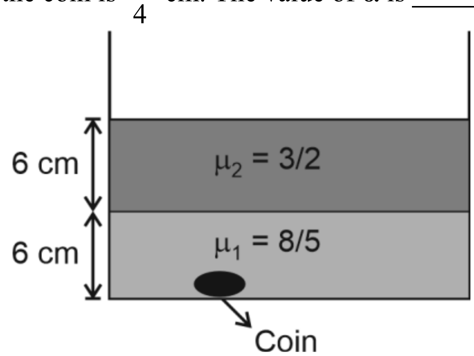
$$\Rightarrow v = \omega A\sqrt{1 - \frac{x^2}{A^2}}$$

$$\Rightarrow 5 = 10\sqrt{1 - \frac{x^2}{4^2}}$$

$$\Rightarrow \frac{1}{4} = 1 - \frac{x^2}{4^2}$$

$$\Rightarrow x = \sqrt{\frac{3}{4} \times 4^2} = \sqrt{12} \text{ cm}$$

Q.27 Two immiscible liquids of refractive indices $\frac{8}{5}$ and $\frac{3}{2}$ respectively are put in a beaker as shown in the figure. The height of each column is 6 cm. A coin is placed at the bottom of the beaker. For near normal vision, the apparent depth of the coin is $\frac{\alpha}{4}$ cm. The value of α is __________.



Ans. **[31]**

Sol. Apparent depth = $\frac{d_1}{\mu_1}+\frac{d_2}{\mu_2}$

$= \frac{6}{\left(\frac{3}{2}\right)}+\frac{6}{\left(\frac{8}{5}\right)}=\frac{31}{4}$

Q.28 In a nuclear fission process, a high mass nuclide (A = 236) with binding energy 7.6 MeV/Nucleon dissociated into middle mass nuclides (A = 118), having binding energy of 8.6 MeV/Nucleon. The energy released in the process would be ________ MeV.

Ans. **[236]**

Sol. $A \rightarrow 2B$

$\Delta E = 2 \times 118 \times 8.6 - 236 \times 7.6$

$\Delta E = 236$ MeV

Q.29 Two coils have mutual inductance 0.002 H. The current changes in the first coil according to the relation $i = i_0\sin\omega t$, where $i_0 = 5A$ and $\omega = 50\pi$ rad/s. The maximum value of emf in the second coil is $\frac{\pi}{\alpha}$ V. The value of α is _________.

Ans. **[2]**

Sol. $\phi_2 = Mi_1$

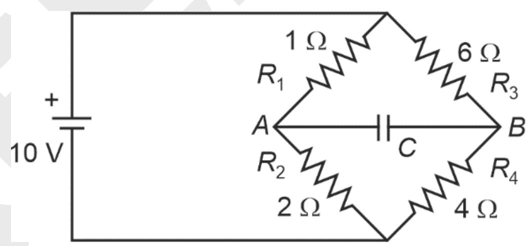
$|E| = \left|\frac{d\phi_2}{dt}\right| = Mi_0\omega\cos\omega t$

$E_{max} = Mi_0\omega$

$= 0.002 \times 5 \times 50\pi$

$= 0.5\pi$

Q.30 The charge accumulated on the capacitor connected in the following circuit is _________ μC. (Given C = 150 μF)



Ans. **[400]**

Sol. $V_A = 10 - \frac{10}{3} = \frac{20}{3}$ V

$V_B = 10 - 6 = 4$ V

$V_A - V_B = \frac{8}{3}$ V

$Q = CV_{AB} = 150 \times 10^{-6} \times \frac{8}{3} = 400\ \mu C$

CHEMISTRY

Section-A: Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Q.31 Given below are two statements :

Statement (I) : p-nitrophenol is more acidic than m-nitrophenol and o-nitrophenol.

Statement (II) : Ethanol will give immediate turbidity with Lucas reagent.

In the light of the above statements, choose the **correct** answer from the options given below.

(1) **Statement I** is false but **Statement II** is true

(2) **Statement I** is true but **Statement II** is false

(3) Both **Statement I** and **Statement II** are true

(4) Both **Statement I** and **Statement II** are false

Ans. **[2]**

Sol. p-Nitrophenol is more acidic than m-nitrophenol and o-nitrophenol.

The electron withdrawing group $–NO_2$ stabilizes phenoxide ion through –R effect at para and ortho position.

$–NO_2$ group in meta position will show –I effect.

Presence of $–NO_2$ group at ortho position it form hydrogen bonding with –OH group.

Hence, acidity is less than p-nitrophenol order of acidic strength.

p-nitrophenol > o-nitrophenol > m-nitrophenol

Ethanol will not give immediate turbidity with Lucas reagent.

3° alcohol will give white turbidity immediately statement I is true but statement II is false.

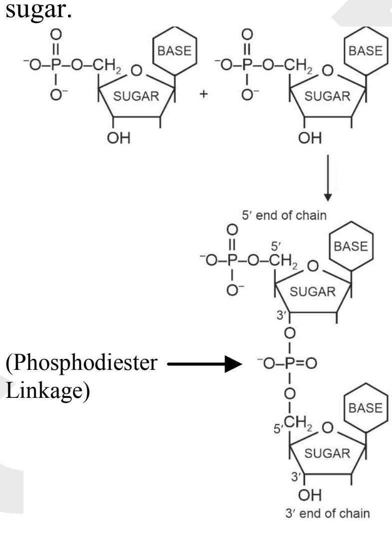
Q.32 Two nucleotides are joined together by a linkage known as

(1) Peptide linkage (2) Disulphide linkage

(3) Glycosidic linkage (4) Phosphodiester linkage

Ans. **[4]**

Sol. Two nucleotides are joined together by phosphodiester linkage between 5′ and 3′ carbon atoms of pentose sugar.



Q.33 Cyclohexene  is ______ type of an organic compound.

(1) Acyclic (2) Benzenoid non-aromatic

(3) Alicyclic (4) Benzenoid aromatic

Ans. **[3]**

Sol. 

The given compound belongs to alicyclic compounds

Q.34 The electronic configuration for Neodymium is [Atomic Number for Neodymium 60]

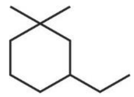
(1) $[Xe]4f^4\ 6s^2$ (2) $[Xe]4f^1\ 5d^1\ 6s^2$ (3) $[Xe]5f^7\ 7s^2$ (4) $[Xe]4f^6\ 6s^2$

Ans. **[1]**

Sol. The electronic configuration of Neodymium is

${}_{60}Nd : [Xe]4f^4\ 6s^2$

Q.35 IUPAC name of following compound (P) is



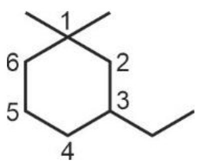
(P)

(1) 1,1-Dimethyl-3-ethylcyclohexane (2) 1-Ethyl-3,3-dimethylcyclohexane

(3) 3-Ethyl-1,1-dimethylcyclohexane (4) 1-Ethyl-5,5-dimethylcyclohexane

Ans. **[3]**

Sol.



Numbering is done in such a way that all locants should get lowest possible number.

Name of substituent written according to alphabetical order

3-Ethyl-1,1-dimethylcyclohexane

Q.36 Given below are two statements : one is labelled as **Assertion (A)** and the other is labelled as **Reason (R)**.

Assertion (A) : Melting point of Boron (2453 K) is unusually high in group 13 elements.

Reason (R) : Solid Boron has very strong crystalline lattice.

In the light of the above statements, choose the **most appropriate** answer from the options given below :

(1) Both **(A)** and **(R)** are correct but **(R)** is not the correct explanation of **(A)**

(2) Both **(A)** and **(R)** are correct and **(R)** is the correct explanation of **(A)**

(3) **(A)** is false but **(R)** is true

(4) **(A)** is true but **(R)** is false

Ans. **[2]**

Sol. Melting point of Boron is unusually high due to very strong crystalline lattice.

Both **(A)** and **(R)** are correct and **(R)** is correct explanation of **(A)**.

Q.37 A solution of two miscible liquids showing negative deviation from Raoult's law will have :

(1) increased vapour pressure, decreased boiling point

(2) increased vapour pressure, increased boiling point

(3) decreased vapour pressure, decreased boiling point

(4) decreased vapour pressure, increased boiling point

Ans. **[4]**

Sol. A solution of two miscible liquids showing negative deviation due to decrease in vapour pressure and increase in boiling point.

Q.38 Consider the following complex ions

$P = [FeF_6]^{3-}$

$Q = [V(H_2O)_6]^{2+}$

$R = [Fe(H_2O)_6]^{2+}$

The correct order of the complex ions, according to their spin only magnetic moment values (in B.M.) is :

(1) R < Q < P (2) R < P < Q (3) Q < R < P (4) Q < P < R

Ans. **[3]**

Sol. $P = [FeF_6]^{3-}$

$Q = [V(H_2O)_6]^{2+}$

$R = [Fe(H_2O)_6]^{2+}$

$F^{\ominus}$ with Fe^{+3}, and H_2O with Fe^{+2} behaves as WFL and pairing does not takes place

$[FeF_6]^{3-} = 3d^5 = t_{2g}^3 e_g^2$

Number of unpaired electron = 5

$\mu = \sqrt{n(n+2)}$ BM

$= \sqrt{5(5+2)}$ BM

$= \sqrt{35}$ BM

$[V(H_2O)_6]^{2+} = 3d^3$ $t_{2g}^3 eg^0$

n = 3

$\mu = \sqrt{15}$ BM

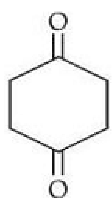
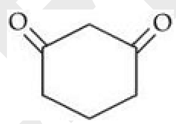
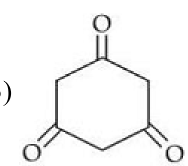
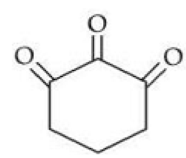
$R = [Fe(H_2O)_6]^{2+} = 3d^6 = t_{2g}^4 e_g^2$

n = 4

$\mu = \sqrt{24}$ BM

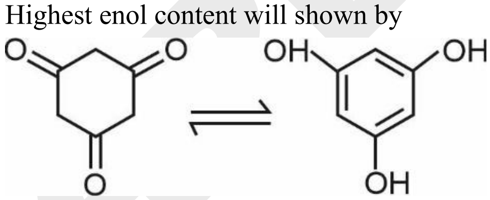
$\boxed{Q < R < P}$

Q.39 Highest enol content will be shown by :

(1)  (2)  (3)  (4) 

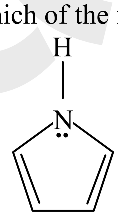
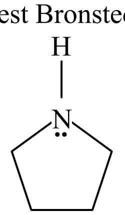
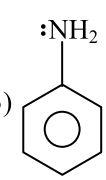
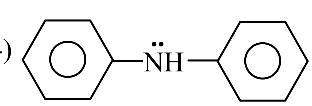
Ans. **[3]**

Sol. Highest enol content will shown by



Due to formation of aromatic compound.

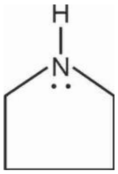
Q.40 Which of the following is strongest Bronsted base?

(1)  (2)  (3)  (4) 

Ans. **[2]**

Sol. Bronsted base are those which can donate its lone pair to H^+ ion.

more easily the compound can donate its lone pair to H^+ strong will be basic character


is strongest Bronsted base due to availability of lone pair.

In other option lone pair is delocalised

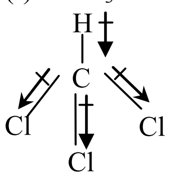
Q.41 Choose the polar molecule from the following :

(1) $CHCl_3$ (2) $CH_2 = CH_2$ (3) CCl_4 (4) CO_2

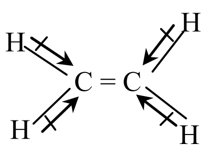
Ans. **[1]**

Sol. Direction of dipole moment is to be electropositive element to electronegative element.

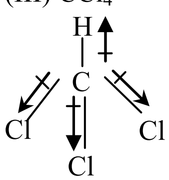
(I) $CHCl_3$


$\mu = 0$

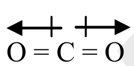
(II) $H_2C = CH_2$


$\mu = 0$

(III) CCl_4


$\mu = 0$

(IV) CO_2


$\mu = 0$

Q.42 Yellow compound of lead chromate gets dissolved on treatment with hot NaOH solution. The product of lead formed is a :

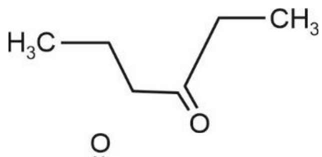
(1) Neutral complex with coordination number four
(2) Dianionic complex with coordination number four
(3) Tetraanionic complex with coordination number six
(4) Dianionic complex with coordination number six

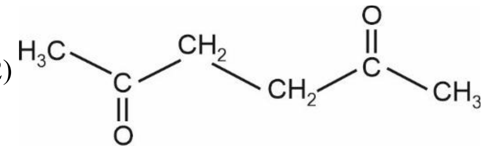
Ans. **[2]**

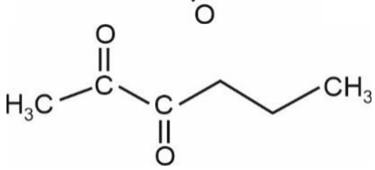
Sol. $\underset{\text{yellow}}{PbCrO_4} \xrightarrow{\text{hot NaOH}} Na_2CrO_4 + \underset{\text{complex}}{[Pb(OH)_4]^{2-}}$

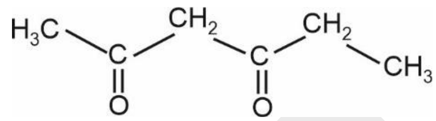
The complex formed is $[Pb(OH)_4]^{2-}$ which has coordination number = 4 and complex is dianionic due to presence of (–2) charge.

Q.43 Which of the following has highly acidic hydrogen?

(1) 

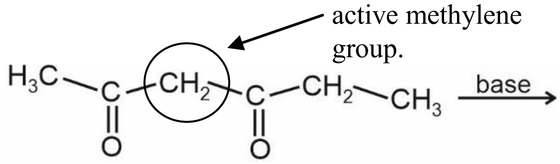
(2) 

(3) 

(4) 

Ans. **[4]**

Sol. After removal of H^+ ion, negative charge is formed, more stable the negative charge more will be the acidic strength.



$$CH_3 - \underset{\underset{O}{\|}}{C} - \overset{\ominus}{\ddot{C}}H - \underset{\underset{O}{\|}}{C} - CH_2 - CH_3$$

Due to cross conjugation of negative charge it is most stabilised anion.
So compound is most acidic.

Q.44 The correct statement regarding nucleophilic substitution reaction in a chiral alkyl halide is :

(1) Retention occurs in S_N1 reaction and inversion occurs in S_N2 reaction
(2) Racemisation occurs in both S_N1 and S_N2 reactions
(3) Racemisation occurs in S_N1 reaction and inversion occurs in S_N2 reaction
(4) Racemisation occurs in S_N1 reaction and retention occurs in S_N2 reaction.

Ans. **[3]**

Sol. In S_N1 reaction carbocation is formed as intermediate. Hence both side attack is possible, racemic product is formed.
In S_N2 reaction, nucleophile attack from back side takes place and hence inversion in configuration takes place.

Q.45 Given below are two statements :

Statement (I) : Aqueous solution of ammonium carbonate is basic
Statement (II) : Acidic/basic nature of salt solution of a salt of weak acid and weak base depends on K_a and K_b value of acid and the base forming it.
In the light of the above statements, choose the **most appropriate** answer from the options given below :
(1) Both **statement-I** and **Statement-II** are correct
(2) Both **statement-I** and **Statement-II** are incorrect
(3) **Statement-I** is correct but **Statement-II** is incorrect
(4) **Statement-I** is incorrect but **Statement-II** is correct

Ans. **[1]**

Sol. Aq solution of ammonium carbonate is basic. The salt is made from weak acid (H_2CO_3) and weak base (NH_4OH), acidic or basic nature depends on value of K_a of weak acid and K_b of weak base
If solution is basic
K_b (weak base) > K_a (weak acid)
$Ka(H_2CO_3) = 4.3 \times 10^{-7}$
$K_b (NH_4OH) = 1.8 \times 10^{-5}$
$K_b > K_a$ so solution is basic
Both statements are correct

Q.46 NaCl reacts with conc. H_2SO_4 and $K_2Cr_2O_7$ to give reddish fumes (B), which react with NaOH to give yellow solution (C). (B) and (C) respectively are :

(1) CrO_2Cl_2, Na_2CrO_4 (2) Na_2CrO_4, CrO_2Cl_2 (3) CrO_2Cl_2, $Na_2Cr_2O_7$ (4) CrO_2Cl_2, $KHSO_4$

Ans. **[1]**

Sol. $4NaCl + K_2Cr_2O_7 + 6H_2SO_4 \longrightarrow \underset{(B)}{2CrO_2Cl_2} + 4NaHSO_4 + 2KHSO_4 + 3H_2O$

$CrO_2Cl_2 + 4NaOH \longrightarrow \underset{\substack{(C)\\ \text{Yellow}}}{Na_2CrO_4} + 2NaCl + 2H_2O$

Q.47 Which of the following electronic configuration would be associated with the highest magnetic moment?

(1) $[Ar]\ 3d^8$ (2) $[Ar]\ 3d^6$ (3) $[Ar]\ 3d^3$ (4) $[Ar]\ 3d^7$

Ans. **[2]**

Sol. $[Ar]\ 3d^6$ have 4 unpaired electrons which has highest magnetic moment.

Q.48 Given below are two statements :

Statement (I) : The 4f and 5f - series of elements are placed separately in the Periodic table to preserve the principle of classification.

Statement (II) : s-block elements can be found in pure form in nature.

In the light of the above statements, choose the **most appropriate** answer from the options given below :

(1) **Statement I** is false but **Statement II** is true
(2) **Statement I** is true but **Statement II** is false
(3) Both **Statement I** and **Statement II** are false
(4) Both **Statement I** and **Statement II** are true

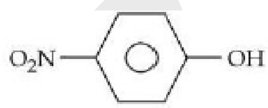
Ans. **[2]**

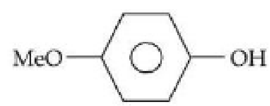
Sol. s-block elements are highly reactive and hence can't be found in pure form in nature.

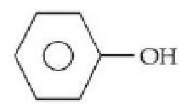
$\Rightarrow$ Statement II is incorrect.

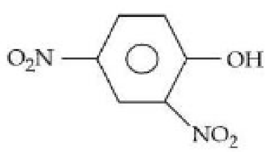
Q.49 The ascending order of acidity of –OH group in the following compounds is :

(A) Bu–OH

(B) 

(C) 

(D) 

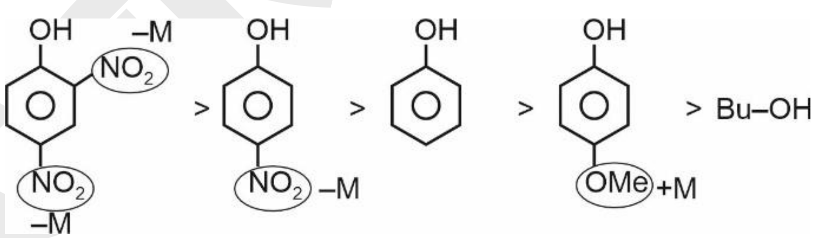
(E) 

Choose the correct answer from the options given below :

(1) (C) < (A) < (D) < (B) < (E)
(2) (A) < (C) < (D) < (B) < (E)
(3) (A) < (D) < (C) < (B) < (E)
(4) (C) < (D) < (B) < (A) < (E)

Ans. **[2]**

Sol.



Acidic strength order

Bu–OH is aliphatic alcohol.

Q.50 Element not showing variable oxidation state is :

(1) Bromine (2) Chlorine (3) Iodine (4) Fluorine

Ans. **[4]**

Sol. Fluorine does not show variable oxidation state due to absence of vacant d-orbital.

Section-B: Numerical Value Type Questions: This section contains 10 Numerical based questions. Attempt any 5 questions out of 10. The answer to each question should be rounded-off to the nearest integer.

Q.51 Sum of bond order of CO and NO^+ is _______.

Ans. **[6]**

Sol. CO and NO^+ both are Isoelectronic to N_2 i.e. both have 14 electrons.
Both have bond order equal to 3
Bond order of CO = 3
Bond order of NO^+ = 3
Sum = 3 + 3 = 6

Q.52 Mass of methane required to produce 22 g of CO_2 after complete combustion is _______ g.
(Given Molar mass in g mol^{-1} C = 12.0
H = 1.0
O = 16.0)

Ans. **[8]**

Sol. $CH_4 + 2O_2 \longrightarrow CO_2 + 2H_2O$

Moles of $CO_2 = \frac{22}{44} = \frac{1}{2}$

Moles of $CH_4 = \frac{1}{2}$

Mass of $CH_4 = \frac{1}{2} \times 16 = 8$ gm

Q.53 From the given list, the number of compounds with +4 oxidation state of Sulphur is _______.
SO_3, H_2SO_3, $SOCl_2$, SF_4, $BaSO_4$, $H_2S_2O_7$

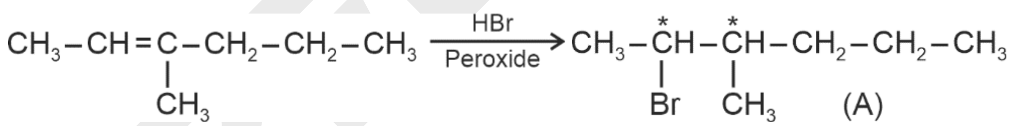
Ans. **[3]**

Sol. H_2SO_3, $SOCl_2$, SF_4 have +4 oxidation state of sulphur.

Q.54 3-Methylhex-2-ene on reaction with HBr in presence of peroxide forms an addition product (A). The number of possible stereoisomers for 'A' is ______.

Ans. **[4]**

Sol.



(A) has 2 chiral carbons
Possible stereoisomers = $(2)^2 = 4$

Q.55 Consider the following data for the given reaction
$2\,HI(g) \rightarrow H_2(g) + I_2(g)$

	1	2	3
HI (mol L^{-1})	0.005	0.01	0.02
Rate (mol $L^{-1}s^{-1}$)	7.5×10^{-4}	3.0×10^{-3}	1.2×10^{-2}

The order of the reaction is ________.

Ans. **[2]**

Sol. Comparing rate values in experiment 2 and 3

$$\frac{r_3}{r_2} = 4 = \left(\frac{0.02}{0.01}\right)^x$$

$\boxed{x = 2}$

Order of reaction is 2

Q.56 If three moles of an ideal gas at 300 K expand isothermally from 30 dm^3 to 45 dm^3 against a constant opposing pressure of 80 kPa, then the amount of heat transferred is ________ J.

Ans. **[1200]**

Sol. $W = -P_{ext}(\Delta V)$

$= -80 \times 10^3 \dfrac{N}{m^2} (45 - 30) \times 10^{-3}\ m^3$

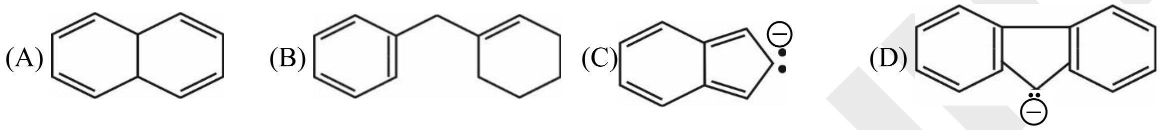
$= -80 \times (15)$ N.m

$= -1200$ J

$q + W = \Delta E = 0$

$q = +1200$ J

Q.57 Among the given organic compounds, the total number of aromatic compounds is _______.



Ans. **[3]**

Sol. Compound (A) is not aromatic due to presence of sp^3 carbon between the rings.

Q.58 The number of electrons present in all the completely filled subshells having n = 4 and $s = +\dfrac{1}{2}$ is ________.

(Where n = principal quantum number and s = spin quantum number)

Ans. **[16]**

Sol. Total (n^2) electrons present with $s = +\dfrac{1}{2}$

$\Rightarrow$ Total 16 electrons with n = 4 and $s = +\dfrac{1}{2}$

Q.59 Among the following, total number of meta directing functional groups is _____. (Integer based)

$-OCH_3$, $-NO_2$, $-CN$, $-CH_3$, $-NHCOCH_3$, $-COR$, $-OH$, $-COOH$, $-Cl$

Ans. **[4]**

Sol. Meta directing groups:

$-NO_2$

$-CN$

$-COR$

$-COOH$

Total 4 groups are meta directing groups.

Q.60 The mass of silver (Molar mass of Ag : 108 $gmol^{-1}$) displaced by a quantity of electricity which displaces 5600 mL of O_2 at S.T.P. will be _______ g.

Ans. **[108]**

Sol. $H_2O \longrightarrow H^+ + OH^-$

at cathode

$H^+ + e \longrightarrow \dfrac{1}{2} H_2$

at anode

$4O^{-2}H^- \longrightarrow 2H_2O + \overset{0}{O_2} + 4e^-$

$VF = 2[\,0 - (-2)] = 4$

mili equivalent of Ag = mili equivalent of O_2

$\left(\dfrac{w}{E}\right)_{Ag} = \left(\dfrac{w}{E}\right)_{O_2}$

$$\frac{\frac{w}{108}}{1} = \left(\frac{w}{MM}\right) \times VF$$

= mole × [VF]

$$\frac{w(gm)}{108} = \frac{V_{STP}(ml)}{22400} \times VF$$

$$\frac{w(gm)}{108} = \frac{5600}{22400} \times 4$$

w = 108 gm

MATHEMATICS

Section-A: This section contains 20 multiple choice questions. Each question has 4 choices(1), (2), (3) and (4), out of which **ONLY ONE** is correct..

Q.61 The function $f : N - \{1\} \rightarrow N$; defined by f(n) = the highest prime factor of *n*, is:

(1) Both one-one and onto (2) Neither one-one nor onto (3) One-one only (4) Onto only

Ans. **[2]**

Sol. Largest prime factors of two different natural numbers can be same so it is many one.

For example f(6) = 3, f(12) = 3

f(10) = 5, f(15) = 5

As range of f(x) is set of prime number so range of f(x) is proper subset of co-domain so f is into.

Q.62 Let x = x(t) and y = y(t) be solutions of the differential equations $\frac{dx}{dt} + ax = 0$ and $\frac{dy}{dt} + by = 0$ respectively, $a, b \in R$. Given that x(0) = 2; y(0) = 1 and 3y(1) = 2x(1), the value of t, for which x(t) = y(t), is :

(1) $\log_3 4$ (2) $\log_{\frac{2}{3}} 2$ (3) $\log_{\frac{4}{3}} 2$ (4) $\log_4 3$

Ans. **[3]**

Sol. $\frac{dx}{dt} + ax = 0$

$\Rightarrow \int \frac{1}{x} dx = -\int a dt$

$\Rightarrow \ln|x| = -at + c$

$\Rightarrow x = \pm e^c . e^{-at} = \lambda e^{-at}$

$\Rightarrow x(0) = 2 \Rightarrow \lambda = 2$

$\Rightarrow x = 2e^{-at}$ … (1)

Now, $\frac{dy}{dt} + by = 0$

$\Rightarrow \int \frac{1}{y} dy = -\int b\,dt \quad \Rightarrow \ln(y) = -bt + c$

$\Rightarrow y = \pm e^c . e^{-bt} = \mu . e^{-bt}$

y(0) = 1

$\therefore \mu = 1$

$\Rightarrow y = e^{-bt}$

3y(1) = 2x(1)

$3e^{-b} = 2(2e^{-a}) = 4e^{-a}$

$e^{-b+a} = \frac{4}{3}$

$\Rightarrow x(t) = y(t)$
$\Rightarrow e^{-bt} = 2e^{-at}$
$\Rightarrow e^{-bt + at} = 2$
$\Rightarrow e^{(-b + a)t} = 2$
$\Rightarrow \left(\frac{4}{3}\right)^t = 2$
$\Rightarrow t \ln\frac{4}{3} = \ln 2$
$\Rightarrow t = \ln_{\frac{4}{3}} 2$

Q.63 Let S = {1, 2, 3, …, 10}. Suppose M is the set of all the subsets of S, then the relation R = {(A, B) : $A \cap B \neq \phi$; A, B $\in$ M} is:

(1) Reflexive only
(2) Symmetric and transitive only
(3) Symmetric only
(4) Symmetric and reflexive only

Ans. **[3]**

Sol. S = {1, 2, 3, …, 10} and R = {(A, B) : $A \cap B \neq \phi$; A, B $\in$ M}
M is the subset of S
$\phi \in M$
$\phi \cap \phi = \phi \Rightarrow$ But Relation is $A \cap B \neq \phi$
So, R is not reflexive
If $(A, B) \in R$
$\Rightarrow A \cap B \neq \phi$
$\Rightarrow (B, A) \in R$
So, R is symmetric
If $(A, B), (B, C) \in R \Rightarrow A \cap B \neq \phi$ and $B \cap C = \phi$
$\Rightarrow A \cap C$ is not necessary ϕ.
Hence, $A \cap C$ not necessarily belongs to R.
So, R is not transitive relation.
eg:- A = {1, 2}, B = {3, 4}, C = {2, 5}
here, $A \cap B = \phi$, $B \cap C = \phi$
but $A \cap C = \{2\} \neq \phi$
So, option (3) is correct.

Q.64 If $a = \lim\limits_{x\to 0}\frac{\sqrt{1+\sqrt{1+x^4}} - \sqrt{2}}{x^4}$ and $b = \lim\limits_{x\to 0}\frac{\sin^2 x}{\sqrt{2} - \sqrt{1+\cos x}}$, then the value of ab^3 is :

(1) 30
(2) 32
(3) 25
(4) 36

Ans. **[2]**

Sol. $a = \lim\limits_{x\to 0}\frac{\sqrt{1+\sqrt{1+x^4}} - \sqrt{2}}{x^4}$

and $b = \lim\limits_{x\to 0}\frac{\sin^2 x}{\sqrt{2} - \sqrt{1+\cos x}}$

$$a = \lim_{x\to 0}\frac{1+\sqrt{1+x^4} - 2}{x^4\left(\sqrt{1+\sqrt{1+x^4}} + \sqrt{2}\right)}$$

$$= \frac{1}{2\sqrt{2}}\lim_{x\to 0}\frac{\sqrt{1+x^4} - 1}{x^4}$$

$$= \frac{1}{2\sqrt{2}} \lim_{x\to 0} \frac{(1+x^4)-1}{x^4(\sqrt{1+x^4}+1)}$$

$$= \frac{1}{2\sqrt{2}}\left(\frac{1}{2}\right) = \frac{1}{4\sqrt{2}}$$

$$\Rightarrow a = \frac{1}{4\sqrt{2}}$$

$$b = \lim_{x\to 0} \frac{(1-\cos^2 x)(\sqrt{2}+\sqrt{1+\cos x})}{2-(1+\cos x)}$$

$$b = \lim_{x\to 0} (1+\cos x)\left(\sqrt{2}+\sqrt{1+\cos x}\right)$$

$$\Rightarrow b = 2\left(2\sqrt{2}\right)$$

$$\Rightarrow \boxed{b = 4\sqrt{2}}$$

So, $ab^3 = \frac{1}{4\sqrt{2}}\left(4\sqrt{2}\right)^3$

$$\boxed{ab^3 = 32}$$

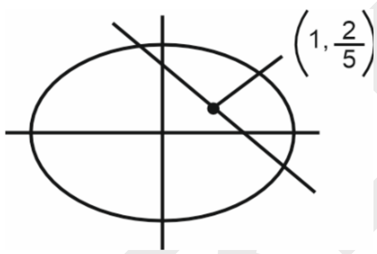
So, option (2) is correct.

Q.65 The length of the chord of the ellipse $\frac{x^2}{25}+\frac{y^2}{16} = 1$, whose mid point is $\left(1, \frac{2}{5}\right)$, is equal to :

(1) $\frac{\sqrt{2009}}{5}$ (2) $\frac{\sqrt{1691}}{5}$ (3) $\frac{\sqrt{1541}}{5}$ (4) $\frac{\sqrt{1741}}{5}$

Ans. **[2]**

Sol.



$$T = S_1 \Rightarrow \frac{x}{25} + \frac{y\left(\frac{2}{5}\right)}{16} = \frac{1}{25} + \frac{4}{16\times 25}$$

$$\Rightarrow \frac{x}{25} + \frac{y}{40} = \frac{5}{100}$$

$$\Rightarrow 4x + \frac{5}{2}y = 5 \quad \Rightarrow 8x + 5y - 10 = 0$$

$$\frac{x^2}{25} + \frac{y^2}{16} = 1 \Rightarrow (4x)^2 + (5y)^2 = 400$$

$$\Rightarrow (4x)^2 + (8x-10)^2 = 400$$

$$\Rightarrow 4x^2 + (4x-5)^2 = 100 \quad \Rightarrow 20x^2 - 40x - 75 = 0$$

$$\Rightarrow 4x^2 - 8x - 15 = 0$$

$$\ell = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$$

$$= \sqrt{(x_2 - x_1)^2 + \left\{\frac{(10 - 8x_2)}{5} - \frac{(10 - 8x_1)}{5}\right\}^2}$$

$$= \sqrt{(x_2 - x_1)^2 + \frac{64}{25}(x_2 - x_1)^2}$$

$$= \sqrt{\frac{89}{25}}\sqrt{(x_2 - x_1)^2}$$

$$= \sqrt{\frac{89}{25}}\sqrt{(x_1 + x_2)^2 - 4x_1x_2}$$

$$= \frac{\sqrt{89}}{5}\sqrt{4 + 4 \times \frac{15}{4}} = \frac{1}{5}\sqrt{89} \times \sqrt{19} = \frac{\sqrt{1691}}{5}$$

Q.66 If $S = \{z \in C : |z - i| = |z + i| = |z - 1|\}$, then n(S) is:

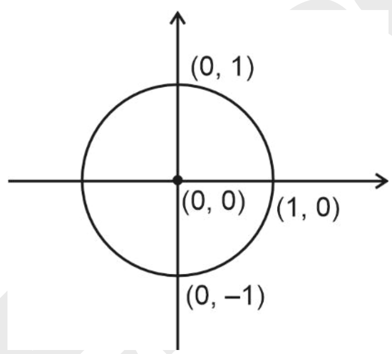
(1) 3 (2) 0 (3) 2 (4) 1

Ans. **[4]**

Sol. z will be circumcentre of the triangle with vertices i, – i and 1(which is unique)

$\Rightarrow z = 0 + 0i$

$\therefore$ Only one element exist in S



Q.67 Consider the matrix $f(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Given below are two statements :

Statement I : f(–x) is the inverse of the matrix f(x).

Statement II : f(x) f(y) = f(x + y).

In the light of the above statements, choose the **correct** answer from the options given below

(1) Both Statement I and Statement II are true

(2) Both Statement I is false but Statement II is true

(3) Statement I is true but Statement II is false

(4) Both Statement I and Statement II are false

Ans. **[1]**

Sol. $f(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$f(x).f(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} \cos y & -\sin y & 0 \\ \sin y & \cos y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos(x+y) & -\sin(x+y) & 0 \\ \sin(x+y) & \cos(x+y) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

= f(x + y)

So, statement II is true.

f(x) f(–x) = I

So, f(–x) is inverse of f(x).

So, statement I is true.

Q.68 $^{n-1}C_r = (k^2 - 8)\ ^nC_{r+1}$ if and only if :

(1) $2\sqrt{3} < k < 3\sqrt{3}$ (2) $2\sqrt{3} < k \le 3\sqrt{2}$ (3) $2\sqrt{2} < k < 2\sqrt{3}$ (4) $2\sqrt{2} < k \le 3$

Ans. **[4]**

Sol. $^{n-1}C_r = (k^2 - 8)\dfrac{n}{r+1}\ ^{n-1}C_r$

$k^2 - 8 = \dfrac{r+1}{n}$

$0 < k^2 - 8 \le 1$

$8 < k^2 \le 9$

$2\sqrt{2} < k \le 3$ or $-3 \le k < 2\sqrt{2}$

Q.69 The distance of the point (7, –2, 11) from the line $\dfrac{x-6}{1} = \dfrac{y-4}{0} = \dfrac{z-8}{3}$ along the line $\dfrac{x-5}{2} = \dfrac{y-1}{-3} = \dfrac{z-5}{6}$ is :

(1) 12 (2) 21 (3) 14 (4) 18

Ans. **[3]**

Sol. Let P (7, –2, 11)

and line $\dfrac{x-6}{1} = \dfrac{y-4}{0} = \dfrac{z-8}{3}$

Let General point be 'Q'

$\therefore\ Q \equiv (\lambda + 6, 4, 3\lambda + 8)$ …(I)

$\therefore\ \overrightarrow{PQ} = (1 - \lambda, -6, -3\lambda - 3)$

As we have to find distance along

$\dfrac{x-5}{2} = \dfrac{y-1}{-3} = \dfrac{z-5}{6}$

$\therefore\ \dfrac{1-\lambda}{2} = \dfrac{-6}{-3} = \dfrac{-3\lambda - 3}{6}$

$\dfrac{1-\lambda}{2} = 2 = \dfrac{-3\lambda - 3}{6}$

$\therefore\ \dfrac{1-\lambda}{2} = 2$

$1 - \lambda = 4$

$\lambda = -3$

Putting in (I)

$Q \equiv (3, 4, -1)$

and $P \equiv (7, -2, 11)$

$\therefore\ PQ = \sqrt{(3-7)^2 + (4+2)^2 + (-1-11)^2}$

$= \sqrt{(-4)^2 + (6)^2 + (-12)^2}$

$= \sqrt{16 + 36 + 144} = \sqrt{196} = 14$

Option (3) is correct

Q.70 Four distinct points (2k, 3k), (1, 0), (0, 1) and (0, 0) lie on a circle for k equal to :

(1) $\frac{3}{13}$ (2) $\frac{2}{13}$ (3) $\frac{5}{13}$ (4) $\frac{1}{13}$

Ans. **[3]**

Sol. General equation of circle is

$x^2 + y^2 + 2gx + 2fy + c = 0$

As it passes through (0, 0) (0, 1) (1, 0)

$\therefore c = 0$

Now $2g + 1 = 0$

$\Rightarrow g = \frac{-1}{2}$

$2f + 1 = 0$

$f = \frac{-1}{2}$

Hence equation of circle is

$x^2 + y^2 - x - y = 0$

Also point (2k, 3k) lies on the circle.

$\therefore 4k^2 + 9k^2 - 5k = 0$

$\Rightarrow 13k^2 - 5k = 0 \Rightarrow k = 0, k = \frac{5}{13}$

$\therefore k = \frac{5}{13}$

Q.71 If $\int_0^1 \frac{1}{\sqrt{3+x}+\sqrt{1+x}}dx = a + b\sqrt{2} + c\sqrt{3}$, where a, b, c are rational numbers, then 2a + 3b – 4c is equal to

(1) 10 (2) 8 (3) 4 (4) 7

Ans. **[2]**

Sol. $\int_0^1 \frac{1}{\sqrt{3+x}+\sqrt{1+x}}dx = a + b\sqrt{2} + c\sqrt{3}$

Taking L.H.S.

Multiply and divide by $\sqrt{3+x} - \sqrt{1+x}$

$= \int_0^1 \frac{\sqrt{3+x}-\sqrt{1+x}}{2}dx$

$= \frac{1}{2}\int_0^1 (\sqrt{3+x} - \sqrt{1+x})\,dx$

$= \frac{1}{2}\left[\frac{2}{3}(3+x)^{3/2} - \frac{2(1+x)^{3/2}}{3}\right]_0^1$

$= \frac{1}{3}[4^{3/2} - 2^{3/2} - 3^{3/2} + 1]$

$= \frac{1}{3}[8 - 2\sqrt{2} - 3\sqrt{3} + 1]$

$= \frac{1}{3}(9 - 2\sqrt{2} - 3\sqrt{3})$

$= 3 - \frac{2\sqrt{2}}{3} - \sqrt{3}$

By comparing, we get :
a = 3
b = –2/3
c = –1
2a + 3b – 4c
= 6 + 3(–2/3) – 4(–1)
= 6 – 2 + 4 = 8

Q.72 Let $\vec{a} = \hat{i} + 2\hat{j} + \hat{k}$, $\vec{b} = 3(\hat{i} - \hat{j} + \hat{k})$. Let $\vec{c}$ be the vector such that $\vec{a} \times \vec{c} = \vec{b}$ and $\vec{a} \cdot \vec{c} = 3$. Then $\vec{a} \cdot ((\vec{c} \times \vec{b}) - \vec{b} - \vec{c})$ is equal to :

(1) 24 (2) 20 (3) 32 (4) 36

Ans. **[1]**

Sol. $\vec{a} = \hat{i} + 2\hat{j} + \hat{k}$, $\vec{b} = 3(\hat{i} - \hat{j} + \hat{k})$

$\vec{a} \cdot \vec{c} = 3$ … (1)

$\vec{a} \times \vec{c} = \vec{b}$ … (2)

Taking cross product of $\vec{a}$ in equation (2),

$\vec{a} \times (\vec{a} \times \vec{c}) = \vec{a} \times \vec{b}$

$(\vec{a} \cdot \vec{c})\vec{a} - (\vec{a} \cdot \vec{a})\vec{c} = \vec{a} \times \vec{b}$ … (3)

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 1 \\ 3 & -3 & 3 \end{vmatrix}$$

$= \hat{i}(6 + 3) - \hat{j}(3 - 3) + \hat{k}(-3 - 6)$

$\vec{a} \times \vec{b} = 9\hat{i} - 9\hat{k}$

$\vec{a} \cdot \vec{a} = (\hat{i} + 2\hat{j} + \hat{k}).(\hat{i} + 2\hat{j} + \hat{k})$

= 1 + 4 + 1 = 6

Putting all these values in equation (3)

$\Rightarrow 3(\hat{i} + 2\hat{j} + \hat{k}) - 6\vec{c} = 9\hat{i} - 9\hat{k}$

$\Rightarrow 6\vec{c} = 3\hat{i} + 6\hat{j} + 3\hat{k} - 9\hat{i} + 9\hat{k}$

$\Rightarrow 6\vec{c} = -6\hat{i} + 6\hat{j} + 12\hat{k}$

$\Rightarrow \vec{c} = -\hat{i} + \hat{j} + 2\hat{k}$

Now, $\vec{a}.((\vec{c} \times \vec{b}) - \vec{b} - \vec{c})$

$$\vec{c} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 1 & 2 \\ 3 & -3 & 3 \end{vmatrix}$$

$= \hat{i}(9) - \hat{j}(-9) + \hat{k}(0) = 9\hat{i} + 9\hat{j}$

$= (\vec{c} \times \vec{b}) - \vec{b} - \vec{c}$

$= 9\hat{i} + 9\hat{j} - 3i + 3\hat{j} - 3\hat{k} + \hat{i} - \hat{j} - 2\hat{k}$

$= 7\hat{i} + 11\hat{j} - 5\hat{k}$

a.$((\vec{c} \times \vec{b}) - \vec{b} - \vec{c})$

$(\hat{i} + 2\hat{j} + \hat{k}).(7\hat{i} + 11\hat{j} - 5\hat{k})$

= 7 + 22 – 5 = 24

Q.73 The number of common terms in the progressions 4, 9, 14, 19, ..., upto 25^{th} term and 3, 6, 9, 12, …, upto to 37^{th} term is :

(1) 7 (2) 9 (3) 8 (4) 5

Ans. **[1]**

Sol. Given A.P.

4, 9, 14, 19, ... Upto 25 terms

Here $d_1 = 5$

$a_n = a + (n - 1)d$

$a_{25} = 4 + (25 - 1)5$

$a_{25} = 4 + 24(5)$

$a_{25} = 124$

So, A.P. is

4, 9, 14, 19, … 124

Second A.P. is given as

3, 6, 9, 12, ... upto 37 term.

Here $d_2 = 3$

$a_n = a(n - 1)d$

$a_n = 3 + (n - 1).3$

$a_{37} = 3 + 36(3)$

$a_{37} = 111$

So, A.P. is

3, 6, 9, 12, ..., 111

L.C.M. of d_1 and $d_2 = 15$

So, A.P. of common terms becomes

9, 24, 39, ….. 99

$a_n = a + (n - 1)d$

$99 = 9 + (n - 1)15$

$\frac{90}{15} = n - 1$

$n = 7$

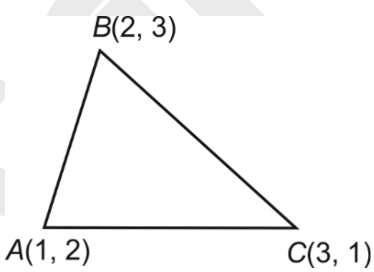
Q.74 If (a, b) be the orthocentre of the triangle whose vertices are (1, 2), (2, 3) and (3, 1), and $I_1 = \int_a^b x\sin(4x - x^2)dx$, $I_2 = \int_a^b \sin(4x - x^2)dx$, then $36\frac{I_1}{I_2}$ is equal to :

(1) 80 (2) 72 (3) 66 (4) 88

Ans. **[2]**

Sol. Slope of AB = 1.

Altitude through C



$y - 1 = -1(x - 3)$

$\Rightarrow x + y = 4.$

Orthocentre (a, b) lies on this

$\Rightarrow a + b = 4$ …(i)

Now, $I_1 = \int_a^b x.\sin(4x - x^2)\,dx$

$= \int_a^b (a + b - x)\sin(4(a + b - x) - (a + b - x)^2)\,dx$

$= \int_a^b (4 - x)\sin(4x - x^2)\,dx$ {using (i)}

$\Rightarrow I_1 = 4.I_2 - I_1$

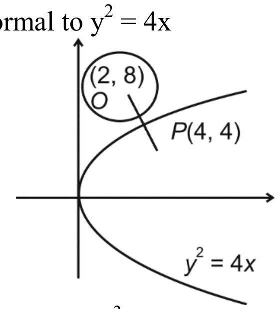
$\Rightarrow \frac{I_1}{I_2} = 2. \quad \Rightarrow 36\frac{I_1}{I_2} = 72$

Q.75 If the shortest distance of the parabola $y^2 = 4x$ from the centre of the circle $x^2 + y^2 - 4x - 16y + 64 = 0$ is d then, d^2 is

(1) 20 (2) 16 (3) 24 (4) 36

Ans. **[1]**

Sol. Any normal to $y^2 = 4x$



$y = mx - 2m - m^3$.

If it passes through (2, 8)

$8 = 2m - 2m - m^3$.

$\Rightarrow m = -2$

Feet of normal on parabola.

$P(am^2, -2am) \equiv (4, 4)$

$d^2 = OP^2 = (4 - 2)^2 + (4 - 8)^2 = 4 + 16 = 20$

Q.76 Consider the function.

$$f(x) = \begin{cases} \dfrac{a(7x - 12 - x^2)}{b|x^2 - 7x + 12|}, & x < 3 \\ 2^{\frac{\sin(x-3)}{x-[x]}}, & x > 3 \\ b, & x = 3 \end{cases}$$

Where [x] denotes greatest integer less than or equal to, if S denotes the set of all ordered pairs (a, b) such that f(x) is continuous at x = 3, then the number of elements in S is :

(1) Infinitely many (2) 2 (3) 1 (4) 4

Ans. **[3]**

Sol. If f(x) is continuous at x = 3

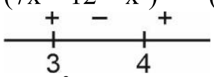
$\Rightarrow f(3) = f(3^+) = f(3^-)$

$\Rightarrow b = \lim_{x \to 3^+} 2^{\frac{\sin(x-3)}{x-[x]}} = \lim_{x \to 3^-} \frac{a}{b}\frac{(7x - 12 - x^2)}{|x^2 - 7x + 12|}$

$\Rightarrow \lim_{x \to 3^+} 2^{\frac{\sin(x-3)}{x-[x]}}$

$\Rightarrow \lim_{h\to 0} 2^{\frac{\sin h}{h}} = 2$

$(7x - 12 - x^2) = -(x^2 - 7x + 12) = -(x - 3)(x - 4)$



$\Rightarrow (x^2 - 7x + 12)$ will open positive

$\Rightarrow \lim_{x\to 3^-} \frac{a}{b}\frac{(7x-12-x^2)}{(x^2-7x+12)} = \lim_{x\to 3^-} \frac{-a}{b} = 2$

$\Rightarrow -a = 2b \Rightarrow a = -2b = -4$

$\Rightarrow (a, b) = \{(-4, 2\}$

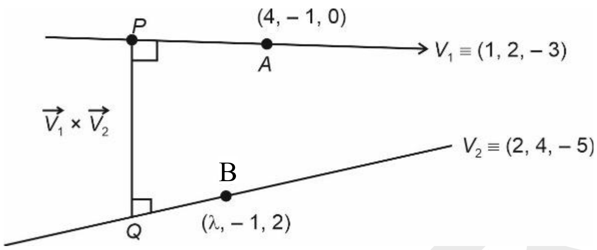
$\Rightarrow$ Only 1 ordered pair

Q.77 If the shortest distance between the lines $\frac{x-4}{1} = \frac{y+1}{2} = \frac{z}{-3}$ and $\frac{x-\lambda}{2} = \frac{y+1}{4} = \frac{z-2}{-5}$ is $\frac{6}{\sqrt{5}}$, then the sum of all possible values of λ is :

(1) 7 (2) 10 (3) 5 (4) 8

Ans. **[4]**

Sol.



Shortest distance PQ is given by

$\left|\frac{\vec{V}_1 \times \vec{V}_2}{|\vec{V}_1 \times \vec{V}_2|}.\overrightarrow{AB}\right| = \frac{6}{\sqrt{5}}$

$\vec{V}_1 \times \vec{V}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -3 \\ 2 & 4 & -5 \end{vmatrix} = 2\hat{i} - \hat{j}$

$\Rightarrow |\vec{V}_1 \times \vec{V}_2| = \sqrt{5}$

$\overrightarrow{AB} = (\lambda - 4)\hat{i} + 2\hat{k}$

$\Rightarrow \left|\frac{\vec{V}_1 \times \vec{V}_2}{\vec{V}_1 \times \vec{V}_2}.\overrightarrow{AB}\right| = \frac{6}{\sqrt{5}} \Rightarrow \left|\frac{2(\lambda-4)}{\sqrt{5}}\right| = \frac{6}{\sqrt{5}}$

$\Rightarrow |\lambda - 4| = 3$

$\Rightarrow \lambda - 4 = 3$ or -3

$\lambda = 1$ or 7

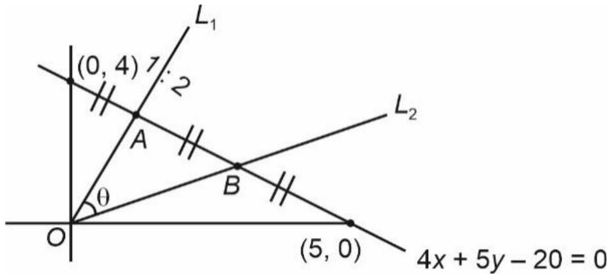
$\Rightarrow$ Sum of $\lambda = 8$

Q.78 The portion of the line $4x + 5y = 20$ in the first quadrant is trisected by the lines, L_1 and L_2 passing through the origin. The tangent of an angle between the lines L_1 and L_2 is:

(1) $\frac{2}{5}$ (2) $\frac{8}{5}$ (3) $\frac{25}{41}$ (4) $\frac{30}{41}$

Ans. **[4]**

Sol.



$A_x \equiv \frac{5(1)+2(0)}{3} = \frac{5}{3}$

$A_y = \frac{0(1)+2(4)}{3} = \frac{8}{3}$

$\Rightarrow A \equiv \left(\frac{5}{3}, \frac{8}{3}\right)$

Similarly, $B \equiv \left(\frac{10}{3}, \frac{4}{3}\right)$

$\Rightarrow$ Slope of AO = $\frac{8}{5}$

Slope of OB = $\frac{2}{5}$

$\tan\theta = \left|\frac{\frac{8}{5}-\frac{2}{5}}{1+\frac{16}{25}}\right| = \left|\frac{\frac{6}{5}}{41/25}\right| = \frac{30}{41}$

Q.79 If A denotes the sum of all, the coefficients in the expansion of $(1 - 3x + 10x^2)^n$ and B denotes the sum of all the coefficients in the expansion of $(1 + x^2)^n$, then:

(1) 3A = B (2) $B = A^3$ (3) A = 3B (4) $A = B^3$

Ans. **[4]**

Sol. A = sum of coefficient = $(1 - 3(1) + 10(1)^2)^n = 8^n$

(Putting x = 1)

B = Sum of coefficient = $(1 + (1)^2)^n = 2^n$

$\Rightarrow 8^n = (2^3)^n = (2^n)^3 = B^3$

$\Rightarrow A = B^3$

Q.80 Let $a_1, a_2, \ldots a_{10}$ be 10 observations such that $\sum_{k=1}^{10} a_k = 50$ and $\sum_{\forall k<j} a_k.a_j = 1100$. Then the standard of deviation of $a_1, a_2, \ldots, a_{10}$ is equal to:

(1) 5 (2) $\sqrt{115}$ (3) 10 (4) $\sqrt{5}$

Ans. **[4]**

Sol. $\sum_{k=1}^{10} a_k = 50$

$a_1 + a_2 + a_3 + \ldots.. + a_{10} = 50$

$\sum_{\forall k<j} a_k a_j = 1100$

If $a_1 + a_2 + a_3 + ... + a_{10} = 50$
$(a_1 + a_2 + a_3 + ... + a_{10})^2 = 2500$

$$\Rightarrow \sum_{i=1}^{10} a_i^2 + \sum_{k<j}^{10} a_k a_j = 2500$$

$$\Rightarrow \sum_{i=1}^{10} a_i^2 = 2500 - 2(1100) = 300$$

Standard deviation (σ)

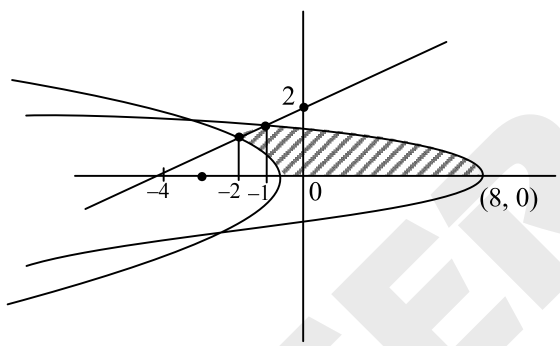
$$\Rightarrow \sigma = \sqrt{\frac{\sum a_x^2}{10} - \left(\frac{\sum a_k}{10}\right)^2} = \sqrt{\frac{300}{10} - \left(\frac{50}{10}\right)^2} = \sqrt{30-25} = \sqrt{5}$$

Section-B: Numerical Value Type Questions: This section contains 10 Numerical based questions. Attempt any 5 questions out of 10. The answer to each question should be rounded-off to the nearest integer.

Q.81 Let the area of region $\{(x, y) : x - 2y + 4 \geq 0, x + 2y^2 \geq 0, x + 4y^2 \leq 8, y \geq 0\}$ be $\frac{m}{n}$, where m and n are coprime numbers. Then m + n is equal to _____.

Ans. **[119]**

Sol.



Area between these curve will be

$$\int_{-2}^{-1}\left(-\sqrt{\frac{-x}{2}} + \frac{x+4}{2}\right)dx + \int_{-1}^{0}\left(\sqrt{\frac{8-x}{4}} - \sqrt{\frac{-x}{2}}\right)dx + \int_{0}^{8}\sqrt{\frac{8-x}{4}}\,dx = \frac{107}{12} = \frac{m}{n}$$

$\Rightarrow m + n = 119$

Q.82 Let $f(x) = x^3 + x^2 f'(1) + x f''(2) + f'''(3)$, $x \in R$. Then $f'(10)$ is equal to ______.

Ans. **[202]**

Sol. $f(x) = x^3 + x^2 f'(1) + x f''(2) + f'''(3)$

$f'(x) = 3x^2 + 2x f'(1) + f''(2)$

$f''(x) = 6x + 2f'(1)$

$f'''(x) = 6$

$\Rightarrow f'''(3) = 6$

$f''(2) = 12 + 2f'(1)$ …(1)

$f'(1) = 3 + 2f'(1) + f''(2)$ …(2)

Adding (1) & (2)

$0 = 15 + 3f'(x) \Rightarrow f'(1) = -5$

$f''(2) = 12 - 10 = 2$

Now

$f'(10) = 300 + 20(-5) + 2$

$= 300 - 100 + 2 = 202$

Q.83 Let the set of all a ∈ R such that the equation cos2x + a sinx = 2a – 7 has a solution be [p, q] and r = tan9° – tan27° – $\frac{1}{\cot 63^\circ}$ + tan81°, then qpr is equal to ______.

Ans. **[48]**

Sol. cos2x + asinx = 2a – 7

$1 - 2\sin^2 x + a\sin x = 2a - 7$

Let sinx = t

$2t^2 - at + 2a - 8 = 0$

$\Rightarrow (t - 2)(2t + 4 - a) = 0$

$\Rightarrow a = 2\sin x + 4$

$\therefore a \in [2, 6] \Rightarrow p = 2$ and $q = 6$.

Now

$\tan\theta + \cot\theta = 2\text{cosec}2\theta$

r = 2cosec 18° – 2 cosec 54°

$= 2\left[\frac{4}{\sqrt{5}-1} - \frac{4}{\sqrt{5}+1}\right] = 8 \times \frac{2}{5-1} = 4, = r$

$\therefore pqr = 2 \times 6 \times 4 = 48.$

Q.84 If the solution of the differential equation (2x + 3y – 2) dx + (4x + 6y – 7) dy = 0, y(0) = 3, is αx + βy + $3\log_e|2x + 3y - \gamma| = 6$, then α + 2β + 3γ is equal to ______.

Ans. **[29]**

Sol. (2x + 3y – 2)dx + (4x + 6y – 7)dy = 0

$\frac{dy}{dx} = -\left(\frac{2x+3y-2}{4x+6y-7}\right)$

Let 2x + 3y = t

$2 + 3\frac{dy}{dx} = \frac{dt}{dx}$

$\frac{1}{3}\left[\frac{dt}{dx} - 2\right] = -\left(\frac{t-2}{2t-7}\right)$

$\frac{dt}{dx} - 2 = -\left(\frac{3t-6}{2t-7}\right)$

$\frac{dt}{dx} = 2 - \frac{3t-6}{2t-7}$

$\frac{dt}{dx} = \frac{t-8}{2t-7}$

$\int\left(\frac{2t-7}{t-8}\right)dt = \int dx$

$= \int 2dt + \int\frac{9}{t-8}dt = \int dx$

= 2t + 9 ln |t – 8| = x + c

= 2(2x + 3y) + 9 |2x + 3y – 8| = x + c

∵ y(0) = 3 ⇒ c = 18

= 3x + 6y + 9 ln(|2x + 3y – 8|) = 18

= x + 2y + 3 ln |2x + 3y – 8| = 6

⇒ α = 1, β = 2, γ = 8

Now

α + 2β + 3γ

= 1 + 4 + 24 = 29

Q.85 A fair die is tossed repeatedly until a six is obtained. Let X denote the number of tosses required and let a = P(X = 3), b = P(X ≥ 3) and c = P(X ≥ 6|X > 3). Then $\frac{b+c}{a}$ is equal to ______.

Ans. **[12]**

Sol. a = P(X = 3) ⇒ third toss obtains 6 at first time

$\Rightarrow P(A) = \frac{1}{6}$ (obtaining a six)

$P(\overline{A}) = \frac{5}{6}$ (not obtaining a six)

$$\Rightarrow a = P(X = 3) = \frac{5}{6}\times\frac{5}{6}\times\frac{1}{6} = \frac{25}{216} = \frac{5^2}{6^3}$$

$$\Rightarrow b = P(X \geq 3) = \frac{5}{6}\times\frac{5}{6}\times\frac{1}{6} + \frac{5}{6}\times\frac{5}{6}\times\frac{5}{6}\times\frac{1}{6} + \frac{5}{6}\times\frac{5}{6}\times\frac{5}{6}\times\frac{5}{6}\times\frac{1}{6} + ...$$

$\Rightarrow$ It is a G.P. with first term $= \frac{25}{216} = \frac{5^2}{6^3}$ and common ratio $\left(\frac{5}{6}\right)$

$$\Rightarrow \text{Sum} = \frac{\frac{25}{216}}{1-\frac{5}{6}} = \frac{25}{36} = \left(\frac{5}{6}\right)^2$$

$$c = P\left(\frac{X \geq 6}{X > 3}\right) = \frac{P(X \geq 6 \cap X > 3)}{P(X > 3)} = \frac{P(X \geq 6)}{P(X > 3)}$$

$$P(X > 3) = \frac{5}{6}\times\frac{5}{6}\times\frac{5}{6}\times\frac{1}{6} + \frac{5}{6}\times\frac{5}{6}\times\frac{5}{6}\times\frac{5}{6}\times\frac{1}{6} + ... = \frac{\frac{5^3}{6^4}}{1-\frac{1}{5}} = \frac{5^3}{6^3}$$

$$P(X \geq 6) = \frac{5}{6}\times\frac{5}{6}\times\frac{5}{6}\times\frac{5}{6}\times\frac{5}{6}\times\frac{1}{6} + \left(\frac{5}{6}\right)^6\times\frac{1}{6} + ... = \frac{\frac{5^5}{6^6}}{1-\frac{5}{6}} = \left(\frac{5^5}{6^5}\right)$$

$$c = \frac{\frac{5^5}{6^5}}{\frac{5^3}{6^3}} = \frac{5^2}{6^2} = \left(\frac{5}{6}\right)^2$$

$$\frac{b+c}{a} = \frac{\frac{25}{36}+\frac{25}{36}}{\frac{25}{216}} = 2\times\frac{25}{36}\times\frac{216}{25} = 12$$

Q.86 The least positive integral value of α, for which the angle between the vectors $\alpha\hat{i} - 2\hat{j} + 2\hat{k}$ and $\alpha\hat{i} + 2\alpha\hat{j} - 2\hat{k}$ is acute, is ______

Ans. **[5]**

Sol. $(\alpha\hat{i} - 2\hat{j} + 2\hat{k}) . (\alpha\hat{i} + 2\alpha\hat{j} - 2\hat{k}) > 0$

$\alpha^2 - 4\alpha - 4 > 0$

$(\alpha - 2)^2 > 8$

$(\alpha - 2 - 2\sqrt{2})(\alpha - 2 + 2\sqrt{2}) > 0$

$\alpha \in (-\infty, 2 - 2\sqrt{2}) \cup (2 + 2\sqrt{2}, \infty)$

⇒ Least integral value of α is 5

Q.87 If α satisfies the equation $x^2 + x + 1 = 0$ and $(1 + \alpha)^7 = A + B\alpha + C\alpha^2$, A, B, C ≥ 0 then $5(3A - 2B - C)$ is equal to _______.

Ans. **[5]**

Sol. $\alpha = \omega, \omega^2$

If $\alpha = \omega$

$(1+ \alpha)^7 = (1+ \omega)^7 = (-\omega^2)^7 = -\omega^{14} = -\omega^2$

$= 1 + \omega = A + B\omega + c\omega^2$

$A = 1, B = 1, C = 0$

$5(3A - 2B - C) = 5(3 - 2 - 0) = 5$

Q.88 Let for a differentiable function $f : (0, \infty) \to R$, $f(x) - f(y) \geq \log_e\left(\frac{x}{y}\right) + x - y$, $\forall$ x, y $\in (0, \infty)$. Then $\sum_{n=1}^{20} f'\left(\frac{1}{n^2}\right)$ is equal to ______

Ans. **[2890]**

Sol. $f(x) - f(y) \geq \ln\left(\frac{x}{y}\right) + x - y$

Put x = x + h and y = x, where h > 0

$\Rightarrow f(x + h) - f(x) \geq \ln\left(\frac{x+h}{x}\right) + (x + h) - x$

$\Rightarrow \frac{f(x+h)-f(x)}{h} \geq \frac{\ln\left(\frac{x+h}{x}\right)+h}{h}$

$\Rightarrow f'(x) \geq \lim_{h\to 0} \frac{\ln\left(1+\frac{h}{x}\right)}{h} + 1$

$\Rightarrow f'(x) \geq \frac{1}{x} + 1$ … (i)

$\because f(y) - f(x) \leq \ln\left(\frac{y}{x}\right) + y - x$

Put $y \to x + h$, $x \to x$ where h > 0

$f(x + h) - f(x) \leq \ln\left(\frac{x+h}{x}\right) + h$

$\frac{f(x+h)-f(x)}{h} \leq \frac{\ln\left(1+\frac{h}{x}\right)}{h} + 1$

$\Rightarrow f'(x) \leq \frac{1}{x} + 1$ … (ii)

From (i) and (ii),

$f'(x) = 1 + \frac{1}{x} = \frac{x+1}{x}$

$\Rightarrow f'\left(\frac{1}{n^2}\right) = n^2 + 1$

$\Rightarrow \sum_{n=1}^{20} f'\left(\frac{1}{n^2}\right) = 20 + \frac{20\times 21\times 41}{6} = 2890$

Q.89 If $8 = 3 + \frac{1}{4}(3 + p) + \frac{1}{4^2}(3 + 2p) + \frac{1}{4^3}(3 + 3p) \ldots \infty$, then the value of p is _____

Ans. **[9]**

Sol. $S = 3 + \frac{1}{4}(3 + p) + \frac{1}{4^2}(3 + 2p) + \frac{1}{4^3}(3 + 3p) + \ldots \infty$

$$8 = \frac{3}{1-\frac{1}{4}} + \frac{p\frac{1}{4}}{\left(1-\frac{1}{4}\right)^2}$$

(sum of infinite terms of A.G.P. = $\frac{a}{1-r} + \frac{ar}{(1-r)^2}$)

$$\Rightarrow \frac{4p}{9} = 4 \Rightarrow p = 9$$

Q.90 Let $A = \begin{bmatrix} 2 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$, $B = [B_1, B_2, B_3]$, where B_1, B_2, B_3 are column matrics and $AB_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $AB_2 = \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix}$, $AB_3 = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$

If $\alpha = |B|$ and β is the sum of all the diagonal elements of B, then $\alpha^3 + \beta^3$ is equal to ______

Ans. **[28]**

Sol. $A = \begin{bmatrix} 2 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$, $|A| = 1$

$$A^{-1} = \text{adj}(A) = \begin{bmatrix} 1 & -1 & -1 \\ 0 & 1 & 0 \\ -1 & 1 & 2 \end{bmatrix}^T = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 1 \\ -1 & 0 & 2 \end{bmatrix}$$

$$\Rightarrow B_1 = A^{-1}\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}$$

$$\Rightarrow B_2 = A^{-1}\begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix},$$

$$\Rightarrow B_3 = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix}$$

$$\Rightarrow B = \begin{bmatrix} 1 & 2 & 2 \\ -1 & 1 & 0 \\ -1 & -2 & -1 \end{bmatrix}$$

$$\Rightarrow \beta = 1, \alpha = 3 \Rightarrow \alpha^3 + \beta^3 = 27 + 1 = 28$$