



## JEE Main Online Exam 2023

### Questions & Solution

12<sup>th</sup> April 2023 | Morning

#### MATHEMATICS

**Section-A:** This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct..

- Q.1** The number of five-digit numbers, greater than 40000 and divisible by 5, which can be formed using the digits 0, 1, 3, 5, 7 and 9 without repetition, is equal to  
 (1) 132 (2) 120 (3) 72 (4) 96

**Ans.** [2]

**Sol.** Case I: Numbers start with 5

$$\begin{array}{ccc} \underline{5} & & \underline{0} \\ \downarrow & & \downarrow \\ & & \end{array} = 4 \times 3 \times 2 = 24$$

4 ways 3 ways 2 ways

Case II :

$$\begin{array}{ccc} \underline{7} & & \\ \downarrow & & \downarrow \\ 4 & & 2 \end{array} = 48$$

Case III :

$$\begin{array}{ccc} \underline{9} & & \\ \downarrow & & \downarrow \\ 4 & & 2 \end{array} = 48$$

Total numbers = 120

- Q.2** Let a, b, c be three distinct real numbers, none equal to one. If the vectors  $a\hat{i} + \hat{j} + \hat{k}$ ,  $\hat{i} + b\hat{j} + \hat{k}$  and  $\hat{i} + \hat{j} + c\hat{k}$  are coplanar, then  $\frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c}$  is equal to

- (1) 2 (2) -1 (3) -2 (4) 1

**Ans.** [4]

**Sol.** 
$$\begin{vmatrix} a & 1 & 1 \\ 1 & b & 1 \\ 1 & 1 & c \end{vmatrix} = 0$$

$$\begin{vmatrix} a-1 & 1-b & 0 \\ 0 & b-1 & 1-c \\ 1 & 1 & c \end{vmatrix} = 0$$

$R_1 \rightarrow R_1 - R_2$

$R_2 \rightarrow R_2 - R_3$

$\Rightarrow (a-1)[c(b-1) - (1-c)] + 1[(1-b)(1-c)] = 0$

$\Rightarrow c(a-1)(b-1) - (a-1)(1-c) + (1-b)(1-c) = 0$

$\Rightarrow \frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c} = 1$

- Q.3** Let  $\langle a_n \rangle$  be a sequence such that  $a_1 + a_2 + \dots + a_n = \frac{n^2 + 3n}{(n+1)(n+2)}$ . If  $28 \sum_{k=1}^{10} \frac{1}{ak} = P_1 P_2 P_3 \dots P_m$ , where  $P_1, P_2, \dots, P_m$  are the first  $m$  prime numbers, then  $m$  is equal to  
 (1) 5 (2) 8 (3) 6 (4) 7

**Ans.** [3]

**Sol.** 
$$a_n = \frac{n^2 + 3n}{(n+1)(n+2)} - \frac{(n-1)^2 + 3(n-1)}{n(n+1)}$$

$$= \frac{4}{n(n+1)(n+2)}$$

$$\sum_{k=1}^{10} \frac{1}{ak} = \frac{1}{4} \sum_{k=1}^{10} k(k+1)(k+2)$$

$$= \frac{1}{16} \sum_{k=1}^{10} k(k+1)(k+2)(k+3) - (k-1)k(k+1)(k+2)$$

$$\frac{1}{16} [(1 \cdot 2 \cdot 3 \cdot 4 \cdot 0) + (2 \cdot 3 \cdot 4 \cdot 5 - 1 \cdot 2 \cdot 3 \cdot 4) (10 \cdot 11 \cdot 12 \cdot 13 - 9 \cdot 10 \cdot 11 \cdot 12)]$$

$$= \frac{1}{16} [10 \cdot 11 \cdot 12 \cdot 13]$$

$$\therefore 28 \sum_{k=1}^{10} \frac{1}{ak} = \frac{28 \times 5 \times 11 \times 3 \times 13}{2} = 2 \cdot 3 \cdot 5 \cdot 7 \cdot 11 \cdot 13$$

$$\therefore m = 6$$

- Q.4** Let  $A = \begin{bmatrix} 1 & 1 \\ 0 & 51 \\ 0 & 1 \end{bmatrix}$ . If  $B = \begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix}$ ,  $A = \begin{bmatrix} -1 & -2 \\ 1 & 1 \end{bmatrix}$ , then the sum of all the elements of the matrix  $\sum_{n=1}^{50} B^n$  is

equal to

- (1) 75 (2) 125 (3) 50 (4) 100

**Ans.** [4]

**Sol.** 
$$\therefore \begin{matrix} (M) \\ \begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix} \end{matrix} \begin{matrix} (N) \\ \begin{bmatrix} -1 & -2 \\ 1 & 1 \end{bmatrix} \end{matrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\therefore MN = I = NM$$

$$B = MAN$$

$$B^n = (MAN)^n = (MAN)(MAN)$$

$$= (MA^2N)MAN$$

$$= MA^nN$$

$$\text{Now } A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ 0 & 51 \\ 0 & 0 \end{bmatrix} = I + E$$

$$E^2 = 0$$

$$\therefore An = (I + E)^n = I + nE + \underbrace{C_2 E^2}_0$$

$$= I + nE$$

$$= \begin{bmatrix} 1 & n \\ 0 & 51 \\ & & 1 \end{bmatrix}$$

$$\text{So, } B^n = MA^nN = \begin{bmatrix} 1 + \frac{n}{51} & \frac{n}{51} \\ -\frac{n}{51} & 1 - \frac{n}{51} \end{bmatrix}$$

$$\sum_{n=1}^{50} B^n = \begin{bmatrix} 50 + \frac{50 \cdot 51}{2 \cdot 51} & \frac{50 \cdot 51}{2 \cdot 51} \\ -\frac{50 \cdot 51}{2 \cdot 51} & 50 - \frac{50 \cdot 51}{2 \cdot 51} \end{bmatrix} = \begin{bmatrix} 75 & 25 \\ -25 & 25 \end{bmatrix}$$

$$\therefore \text{Sum} = 100$$

**Q.5** If  $\frac{1}{n+1} {}^n C_n + \frac{1}{n} {}^n C_{n-1} + \dots + \frac{1}{2} {}^n C_1 + {}^n C_0 = \frac{1023}{10}$  then n is equal to

(1) 9

(2) 8

(3) 7

(4) 6

**Ans.** [1]

**Sol.** 
$$\sum_{r=0}^n \frac{1}{r+1} {}^n C_r = \frac{1023}{10} \left( \because {}^{n+1} C_{r+1} = \frac{n+1}{r+1} {}^n C_r \right)$$

$$\Rightarrow \sum_{r=0}^n \frac{1}{r+1} {}^{n+1} C_{r+1} = \frac{1023}{10}$$

$$\Rightarrow \frac{1}{n+1} [{}^{n+1} C_1 + {}^{n+1} C_2 + \dots + {}^{n+1} C_{n+1}] = \frac{1023}{10}$$

$$\Rightarrow \frac{2^{n+1} - 1}{n+1} = \frac{1023}{10} = \frac{2^{10} - 1}{10}$$

$$\Rightarrow n+1 = 10$$

$$\Rightarrow n = 9$$

**Q.6** The area of the region enclosed by the curve  $y = x^3$  and its tangent at the point  $(-1, -1)$  is

(1)  $\frac{19}{4}$

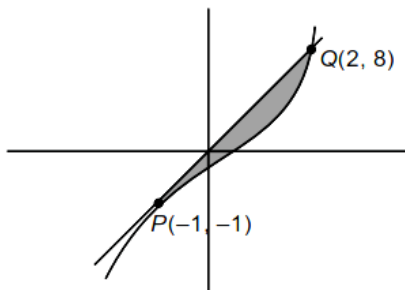
(2)  $\frac{23}{4}$

(3)  $\frac{31}{4}$

(4)  $\frac{27}{4}$

**Ans.** [4]

**Sol.**



Given,  $y = x^3$

.....(i)

$$\Rightarrow \frac{dy}{dx} = 3x^2$$

$$\left(\frac{dy}{dx}\right)_{(-1,-1)} = 3$$

Equation of tangent at  $(-1, -1)$

$$(y + 1) = 3(x + 1)$$

$$y = 3x + 2$$

....(ii)

Solving (i) and (ii)

$$x^3 = 3x + 2$$

$$x^3 - 3x - 2 = 0 \begin{cases} -1 \\ -1 \\ \alpha = 2 \end{cases}$$

Q (2, 8)

$$\text{Required area} = \int_{-1}^2 (3x + 2 - x^3) dx$$

$$= \frac{3}{2} (4 - 1) + 2(2 + 1) - \frac{1}{4} (16 - 1)$$

$$= \frac{27}{4}$$

**Q.7** If the total maximum value of the function  $f(X) = \left(\frac{\sqrt{3e}}{2\sin x}\right)^{\sin^2 x}$ ,  $x \in \left(0, \frac{\pi}{2}\right)$ , is  $\frac{k}{e}$ , then  $\left(\frac{k}{e}\right)^8 + \frac{k^8}{e^5} + k^8$  is

equal to

(1)  $e^3 + e^6 + e^{11}$

(2)  $e^5 + e^6 + e^{11}$

(3)  $e^3 + e^6 + e^{10}$

(4)  $e^3 + e^5 + e^{11}$

**Ans.** [1]

**Sol.**  $F(X) = \left(\frac{\sqrt{3e}}{2\sin x}\right)^{\sin^2 x}$

$$f'(X) = f(x) \left[ \begin{array}{l} 2\sin x \cos x \times \ln\left(\frac{\sqrt{3e}}{2\sin x}\right) + \\ \sin^2 x \times \frac{1 \times 2\sin x}{\sqrt{3e}} \times \frac{\sqrt{3e}}{2} - \frac{1}{\sin^2 x} \times \cos x \end{array} \right]$$

$$= f(X) \left[ \sin 2x \ln\left(\frac{\sqrt{3e}}{2\sin x}\right) - \sin x \cos x \right] = 0$$

$\sin 2x = 0$  (not possible)

$$\ln\left(\frac{\sqrt{3e}}{2\sin x}\right) = +\frac{1}{2}$$

$$\frac{\sqrt{3e}}{2\sin x} = e + \frac{1}{2}$$

$$\sin x = \frac{\sqrt{3}}{2}$$

$$f_{\max} = (e)^{\frac{3}{8}} = \frac{e^{\frac{11}{8}}}{e} \Rightarrow k = e^{\frac{17}{8}}$$

$$\left(\frac{k}{e}\right)^8 + \frac{k^8}{e^5} + k^8 = e^3 + e^6 + e^{11}$$

- Q.8** Let  $\alpha, \beta$  be the roots of the quadratic equation  $x^2 + \sqrt{6}x + 3 = 0$ . The  $\frac{\alpha^{23} + \beta^{23} + \alpha^{14} + \beta^{14}}{\alpha^{15} + \beta^{15} + \alpha^{10} + \beta^{10}}$  is equal to
- (1) 81                                      (2) 9                                      (3) 72                                      (4) 729

**Ans.** [2]

**Sol.** 
$$\frac{\alpha^{23} + \beta^{23} + \alpha^{14} + \beta^{14}}{\alpha^{15} + \beta^{15} + \alpha^{10} + \beta^{10}}$$

Let  $a_n = \alpha^n + \beta^n$

$$\frac{a_{23} + a_{14}}{a_{15} + a_{10}}$$

$$x^2 + \sqrt{6}x + 3 = 0 \begin{cases} \alpha \\ \beta \end{cases}$$

$$x = \frac{-\sqrt{6} \pm \sqrt{-6}}{2}$$

$$= \sqrt{6} \left( \frac{-1 \pm i}{2} \right)$$

$$= \sqrt{3} \left( \frac{-1 \pm i}{\sqrt{2}} \right)$$

$$\alpha = \sqrt{3} e^{i\frac{3\pi}{4}}$$

$$\beta = \sqrt{3} e^{i\frac{5\pi}{4}}$$

$$\frac{\alpha^{23} + \beta^{23} + \alpha^{14} + \beta^{14}}{\alpha^{15} + \beta^{15} + \alpha^{10} + \beta^{10}}$$

$$= 9 \frac{\left[ \begin{aligned} & \left( \sqrt{3} \right)^9 \left( e^{i23 \times \frac{3\pi}{4}} + e^{i23 \times \frac{5\pi}{4}} \right) \\ & + \left( \sqrt{3} \right)^9 \left( e^{i14 \times \frac{3\pi}{4}} + e^{i14 \times \frac{5\pi}{4}} \right) \end{aligned} \right]}{\left[ \begin{aligned} & \left( \sqrt{3} \right)^5 \left( e^{i15 \times \frac{3\pi}{4}} + e^{i15 \times \frac{5\pi}{4}} \right) \\ & + \left( \sqrt{3} \right)^5 \left( e^{i10 \times \frac{3\pi}{4}} + e^{i10 \times \frac{5\pi}{4}} \right) \end{aligned} \right]}$$

$$= \frac{\left( \sqrt{3} \right)^9 \left( \frac{1+i-1+i}{\sqrt{2}} \right) + 0}{\left( \sqrt{3} \right)^5 \left( \frac{1+i-1+i}{\sqrt{2}} \right) + 0} \times 9 = 81$$

- Q.9** Let the lines  $l_1 : \frac{x+5}{3} = \frac{y+4}{1} = \frac{z-a}{-2}$  and  $l_2 : 3x + 2y + z - 2 = 0 = x - 3y + 2z - 13$  be coplanar. If the point  $P(a, b, c)$  on  $l_1$  is nearest to the point  $Q(-4, -3, 2)$ , then  $|a| + |b| + |c|$  is equal to
- (1) 12                                      (2) 14                                      (3) 8                                      (4) 10

Ans. [4]

Sol. for  $P_2: n_2$   $\begin{vmatrix} \hat{i} & \hat{j} & k \\ 3 & 2 & 1 \\ 1 & -3 & 2 \end{vmatrix}$

$$= \hat{i}(7) - \hat{j}(5) + k(-11)$$

Let R lies on  $l_2$

Let  $Z = 0$

$$\therefore 3x + 2y = 2$$

$$x - 3y = 13$$

$$\therefore 11y = -39 + 2 = -37$$

$$\therefore y = \frac{-37}{11}$$

$$\therefore x = 13 + 3y$$

$$= 13 - \frac{37 \times 3}{11}$$

$$= \frac{143 - 111}{11} = \frac{32}{11}$$

$$\therefore R \left( \frac{32}{11}, \frac{37}{11}, 0 \right)$$

$l_1$  &  $l_2$  are coplanar

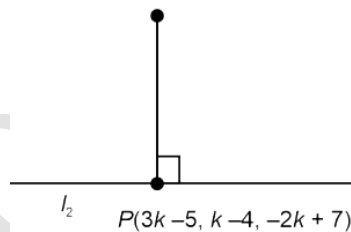
$$\therefore \begin{vmatrix} -5 - \frac{32}{11} & -4 + \frac{37}{11} & a \\ 3 & 1 & -2 \\ 7 & -5 & -11 \end{vmatrix} = 0$$

$$= \begin{vmatrix} -\frac{87}{11} & -\frac{7}{11} & a \\ 3 & 1 & -2 \\ 7 & -5 & -11 \end{vmatrix} = 0$$

$$\Rightarrow a = 7$$

$\therefore P(3k - 5, k - 4, -2k + 7)$  is nearest to  $(-4, -3, 2)$

$Q(-4, -3, 2)$



$$\vec{QP} \cdot \vec{n}_1 = 0$$

$$\left( (3k-1)\hat{i} + (k-1)\hat{j} + (-2k+5)\hat{k} \right) \cdot (3\hat{i} + \hat{j} - 2\hat{k}) = 0$$

$$\Rightarrow 9k - 3 + k - 1 + 4k - 10 = 0$$

$$= 14k - 14 \Rightarrow k = 1$$

$$\therefore P(-2, -3, 5) \equiv (a, b, c)$$

$$|a| + |b| + |c| = 2 + 3 + 5 = 10$$

∴ option (d) is correct

**Q.10** Let  $P \left( \frac{2\sqrt{3}}{\sqrt{7}}, \frac{6}{\sqrt{7}} \right)$ , Q, R and S be four points on the ellipse  $9x^2 + 4y^2 = 36$ . Let PQ and RS be mutually perpendicular and pass through the origin. If  $\frac{1}{(PQ)^2} + \frac{1}{(RS)^2} = \frac{p}{q}$ , where p and q are coprime, then p + q is

equal to

(1) 147

(2) 143

(3) 137

(4) 157

**Ans.** [4]

**Sol.**  $OP = r_1 = \sqrt{\left(\frac{2\sqrt{3}}{\sqrt{7}}\right)^2 + \left(\frac{6}{\sqrt{7}}\right)^2} = \sqrt{\frac{48}{7}}$

Let P be  $(r_1 \cos \theta, r_1 \sin \theta)$

P lies on ellipse

$$\frac{r_1^2 \cos^2 \theta}{4} + \frac{r_1^2 \sin^2 \theta}{9} = 1$$

$$\Rightarrow \frac{\cos^2 \theta}{4} + \frac{\sin^2 \theta}{9} = \frac{7}{48} \quad \dots(i)$$

Let R be  $(-r_2 \sin \theta, r_2 \cos \theta)$

$$\frac{r_2^2 \sin^2 \theta}{4} + \frac{r_2^2 \cos^2 \theta}{9} = 1$$

$$\Rightarrow \frac{\sin^2 \theta}{4} + \frac{\cos^2 \theta}{9} = \frac{1}{r_2^2}$$

From (i)

$$\frac{1}{r_2^2} = \frac{1}{4} + \frac{1}{9} - \frac{7}{48} = \frac{31}{144}$$

$$\frac{1}{PQ^2} + \frac{1}{RS^2} = \frac{1}{4} \left( \frac{1}{r_1^2} + \frac{1}{r_2^2} \right)$$

$$= \frac{1}{4} \left( \frac{7}{48} + \frac{31}{144} \right) = \frac{13}{144} = \frac{p}{m}$$

$$\therefore p + m = 157$$

Option (d) is correct.

**Q.11** Two dice A and B are rolled. Let the numbers obtained on A and B be  $\alpha$  and  $\beta$  respectively. If the variance of  $\alpha - \beta$  is  $\frac{p}{q}$ , where p and q are co-prime, then the sum of the positive divisors of p is equal to

(1) 72

(2) 36

(3) 48

(4) 31

**Ans.** [3]

**Sol.**  $\alpha \in \{1, 2, 3, 4, 5, 6\}$

$$\beta \in \{1, 2, 3, 4, 5, 6\}$$

$$(\alpha - \beta) = 0 \quad (6 \text{ case})$$

$$(\alpha - \beta) = -1 \quad (5 \text{ case})$$

$$(\alpha - \beta) = -2 \quad (4 \text{ case})$$

- $(\alpha - \beta) = -3$  (3 case)
- $(\alpha - \beta) = -4$  (2 case)
- $(\alpha - \beta) = -5$  (1 case)
- $(\alpha - \beta) = 1$  (5 case)
- $(\alpha - \beta) = 2$  (4 case)
- $(\alpha - \beta) = 3$  (3 case)
- $(\alpha - \beta) = 4$  (2 case)
- $(\alpha - \beta) = 5$  (1 case)
- Mean = 0

$$\text{Volume} = \sigma^2 = \frac{0^2 \times 6 + 2 \times 1^2 \times 5 + 2 \times 2^2 \times 4 + 2 \times 3^2 \times 3 + 2 \times 4^2 \times 2 + 5^2 \times 1}{36}$$

$$= \frac{2}{36} \times (5 + 16 + 27 + 32 + 25) = \frac{105}{18} = \frac{35}{6}$$

$$\therefore p = 35$$

$$\text{Sum of divisors of } p = 1 + 5 + 7 + 35 = 48$$

Option (c) is correct.

- Q.12** Let the plane  $P : 4x - y + z = 10$  be rotated by an angle  $\frac{\pi}{2}$  about its line of intersection with the plane  $x + y - z = 4$ . If  $a$  is the distance of the point  $(2, 3, -4)$  from the new position of the plane  $P$ , then  $35a$  is equal to
- (1) 85                                      (2) 105                                      (3) 126                                      (4) 90

**Ans.** [3]

**Sol.** Equation of plane after rotation :

$$(4x - y + z - 10) + \lambda(x + y - z - 4) = 0$$

$$\Rightarrow (4 + \lambda)x + y(\lambda - 1) + z(1 - \lambda) - 4\lambda - 10 = 0$$

$$\vec{n}_1 \cdot \vec{n}_2 = 0$$

$$\Rightarrow (4 + \lambda)4 + (\lambda - 1)(-1) + (1 - \lambda)1 = 0$$

$$\Rightarrow 16 + 4\lambda - \lambda + 1 + 1 - \lambda = 0$$

$$\Rightarrow 2\lambda = -18$$

$$\Rightarrow \lambda = -9$$

$$\therefore \text{equation of plane : } -5x - 10y + 10z + 26 = 0$$

Distance of plane from  $(2, 3, -4)$

$$= \frac{|-10 - 30 - 40 + 26|}{\sqrt{100 + 100 + 26}} = \frac{54}{15} = \alpha$$

$$35\alpha = 35 \cdot \frac{54}{15} = 7 \times \frac{54}{3} = 7 \times 18 = 126$$

Option (3) is correct.

- Q.13** Let  $\lambda \in \mathbb{Z}$ ,  $\vec{a} = \lambda\hat{i} + \hat{j} - \hat{k}$  and  $\vec{b} = 3\hat{i} - \hat{j} + 2\hat{k}$ . Let  $\vec{c}$  be a vector such that  $(\vec{a} + \vec{b} + \vec{c}) \times \vec{c} = \vec{0}$ ,  $\vec{a} \cdot \vec{c} = -17$  and  $\vec{b} \cdot \vec{c} = -20$ . Then  $|\vec{c} \times (\lambda\hat{i} + \hat{j} + \hat{k})|^2$  is equal to
- (1) 46                                      (2) 53                                      (3) 62                                      (4) 49

**Ans.** [1]

**Sol.**  $\vec{a} = \lambda\hat{i} + \hat{j} - \hat{k}$



$$\vec{b} = 3\hat{i} - \hat{j} + 2\hat{k}$$

$$k(\vec{a} + \vec{b}) = \vec{c}$$

$$\vec{a} \cdot \vec{c} = -17$$

$$\vec{b} \cdot \vec{c} = -20$$

$$K(\lambda^2 + 3\lambda - 1) = -17$$

$$K(3\lambda + 11) = -20$$

$$\lambda = 3, \frac{-69}{20}$$

$$\text{For } \lambda = 3, k = -1$$

$$\vec{c} = -1(\vec{a} + \vec{b})$$

$$= -((\lambda + 3)\hat{i} + \hat{j}) = -6\hat{i} - \hat{k}$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -6 & 0 & -1 \\ 3 & 1 & 0 \end{vmatrix} = \hat{i} - 3\hat{j} + 6\hat{k}$$

$$|\vec{c} \times (\lambda\hat{i} + \hat{j} + \hat{k})|^2 = 46$$

**Q.14** Let D be the domain of the function  $f(x) = \sin^{-1} \left( \log_{3x} \left( \frac{6 + 2\log_3 x}{-5x} \right) \right)$ . If the range of the function  $g : D \rightarrow \mathbb{R}$

defined by  $g(x) = x - [x]$ , ( $[x]$  is the greatest integer function), is  $(\alpha, \beta)$ , then  $\alpha^2 + \frac{5}{\beta}$  is equal to

(1) 135

(2) 45

(3) 46

(4) 136

**Ans.** [\*]

**Sol.**  $f(x) = \sin^{-1} \left( \log_{3x} \left( \frac{6 + 2\log_3 x}{-5x} \right) \right)$

$$\text{Log}_{3x} \left( \frac{6 + 2\log_3 x}{-5x} \right) \in \begin{matrix} [-1, 1], & 3x > 0, & 3x \neq 1, & \frac{6 + 2\log_3 x}{-5x} > 0 \\ - (1) & - (2) & - (3) & - (4) \end{matrix}$$

$$\Rightarrow \text{From (2), } x > 0, \text{ from (3), } x \neq \frac{1}{3}$$

$$\text{From (4), } 6 + 2\log_3 x < 0 \text{ (} \therefore x > 0 \text{)}$$

$$\text{Log}_{3x} < -3$$

$$\Rightarrow x < 3^{-3}$$

$$\Rightarrow x < \frac{1}{27}$$

$$\Rightarrow \text{From (2), (3), (4), } 0 < x < \frac{1}{27} \quad \dots (5)$$

$$\text{From (1), } -1 \leq \log_{3x} \left( \frac{6 + 2\log_3 x}{-5x} \right) \leq 1$$

$$\therefore 3x \in \left( 0, \frac{1}{9} \right)$$

$$\begin{aligned} \Rightarrow \frac{1}{3x} &\geq \frac{6+2\log_3 x}{-5x} \geq 3x \\ \Rightarrow -\frac{5}{3} &\leq 6+2\log_3 x \leq -15x^2 \\ \Rightarrow -\frac{23}{6} &\leq \log_3 x \leq \frac{-15x^2-6}{2} \\ \Rightarrow 3^{\frac{23}{6}} &\leq x \leq 3^{\frac{-15x^2-6}{2}} \\ \Rightarrow \frac{1}{3^{\frac{23}{6}}} &\leq x \leq \frac{1}{3^{\frac{15x^2+6}{2}}} \end{aligned} \quad \dots (6)$$

From (5) & (6),  $0 < x < \frac{1}{27}$ ,  $[x] = 0$

$$\Rightarrow g(x) = x$$

$\Rightarrow$  Range  $g(x)$  is domain of  $f(x)$

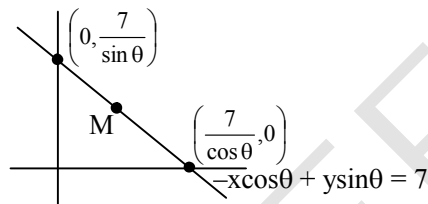
**Q.15** If the point  $\left(\alpha, \frac{7\sqrt{3}}{3}\right)$  lies on the curve traced by the mid-points of the line segments of the lines

$x \cos \theta + y \sin \theta = 7$ ,  $\theta \in \left(0, \frac{\pi}{2}\right)$  between the co-ordinates axes, then  $\alpha$  is equal to

- (1)  $-7$                       (2)  $-7\sqrt{3}$                       (3)  $7\sqrt{3}$                       (4)  $7$

**Ans.** [4]

**Sol.**



$$M\left(\frac{7}{2\cos\theta}, \frac{7}{2\sin\theta}\right) = (h, k)$$

$$\therefore \cos\theta = \frac{7}{2h} \quad \sin\theta = \frac{7}{2k}$$

$$\therefore \theta \in \left(0, \frac{\pi}{2}\right)$$

$$\frac{49}{4h^2} + \frac{49}{4k^2} = 1 \quad \therefore x \text{ \& \& } y \text{ are positive}$$

$$\therefore \text{Locus} = \frac{1}{x^2} + \frac{1}{y^2} = \frac{4}{49}$$

$$\left(\alpha, \frac{7\sqrt{3}}{3}\right) \text{ lies on curve}$$

$$\frac{1}{\alpha^2} + \frac{9}{147} = \frac{4}{49}$$

$$\frac{1}{\alpha^2} = \frac{1}{49}$$

$$\therefore \alpha = \pm 7$$

$$\therefore \alpha = 7$$

**Q.16** Let  $y = y(x)$ ,  $y > 0$ , be a solution curve of the differential equation  $(1 + x^2) dy = y(x - y) dx$ . If  $y(0) = 1$  and  $y(2\sqrt{2}) = \beta$ , then

$$(1) e^{3\beta^{-1}} = e(3 + 2\sqrt{2}) \quad (2) e^{3\beta^{-1}} = e(5 + \sqrt{2}) \quad (3) e^{\beta^{-1}} = e^{-2}(3 + 2\sqrt{2}) \quad (4) e^{\beta^{-1}} = e^{-2}(5 + \sqrt{2})$$

**Ans.** [1]

**Sol.**  $(1 + x^2) dy = y(x - y) dx$

$$\frac{dy}{dx} = \frac{x}{1+x^2}y - \frac{y^2}{1+x^2}$$
$$\frac{-1}{y^2} \frac{dy}{dx} + \frac{x}{1+x^2} \cdot \frac{1}{y} = \frac{1}{1+x^2}$$

$$\text{Let } \frac{1}{y} = t$$

$$\frac{-1}{y^2} y' = \frac{dt}{dx}$$

$$\therefore \frac{dt}{dx} + \frac{x}{1+x^2} t = \frac{1}{1+x^2}$$

$$\text{IF} = e^{\int \frac{x}{1+x^2} dx} = \sqrt{1+x^2}$$

$$t \sqrt{1+x^2} = \int \frac{\sqrt{1+x^2}}{1+x^2} dx$$

$$\frac{1}{y} \sqrt{1+x^2} = \int \frac{1}{\sqrt{1+x^2}} dx$$

$$\frac{1}{y} \sqrt{1+x^2} = \ln(x + \sqrt{x^2 + 1}) + c$$

$$\therefore y(0) = 1 \Rightarrow C = 1$$

$$\frac{1}{y} \sqrt{1+x^2} = \ln(x + \sqrt{x^2 + 1}) + 1$$

$$\text{For } y = 2\sqrt{2}$$

$$\frac{3}{y} = \ln|2\sqrt{2} + 3| + 1$$

$$y = \beta = \frac{3}{1 + \ln|2\sqrt{2} + 3|}$$

$$\Rightarrow 3\beta^{-1} = 1 + \ln|2\sqrt{2} + 3|$$

$$E3\beta^{-1} = e^{|2\sqrt{2} + 3|}$$

**Q.17** In a triangle  $ABC$ , if  $\cos A + 2 \cos B + \cos C = 2$  and the lengths of the sides opposite to the angles  $A$  and  $C$  are 3 and 7 respectively, then  $\cos A - \cos C$  is equal to

- (1)  $\frac{9}{7}$                       (2)  $\frac{10}{7}$                       (3)  $\frac{5}{7}$                       (4)  $\frac{3}{7}$

**Ans.** [2]

**Sol.**  $\cos A + 2 \cos B + \cos C = 2$

$$\cos A + \cos C = 2(1 - \cos B)$$

$$2 \cos \frac{A+C}{2} \cos \left( \frac{A-C}{2} \right) = 2 \times 2 \sin^2 \frac{B}{2}$$

$$\cos \frac{A-C}{2} = 2 \sin \frac{B}{2}$$

$$2 \cos \frac{B}{2} \cos \frac{A-C}{2} = 4 \sin \frac{B}{2} \cos \frac{B}{2}$$

$$2 \sin \left( \frac{A+C}{2} \right) \cos \left( \frac{A-C}{2} \right) = 2 \sin B$$

$$\sin A + \sin C = 2 \sin B$$

$$a + c = 2b$$

$$\Rightarrow b = 5 \quad (\because a = 3, c = 7)$$

$$\cos A - \cos C = \frac{25 + 49 - 9}{70} - \frac{9 + 25 - 49}{30} = \frac{10}{7}$$

**Q.18** Let  $C$  be the circle in the complex plane with centre  $z_0 = \frac{1}{2}(1 + 3i)$  and radius  $r = 1$ . Let  $z_1 = 1 + i$  and the complex number  $z_2$  be outside circle  $C$  such that  $|z_1 - z_0||z_2 - z_0| = 1$ . If  $z_0, z_1$  and  $z_2$  are collinear, then the smaller value of  $|z_2|^2$  is equal to

- (1)  $\frac{5}{2}$                       (2)  $\frac{7}{2}$                       (3)  $\frac{13}{2}$                       (4)  $\frac{3}{2}$

**Ans.** [1]

**Sol.**  $z_0 = \frac{1+3i}{2}, z_1 = (1+i)$

$$|z_1 - z_0||z_2 - z_0| = 1$$

$$\frac{1}{\sqrt{2}}|z_2 - z_0| = 1$$

$$\Rightarrow |z_2 - z_0| = \sqrt{2}$$

$$\frac{z_2 - z_0}{z_1 - z_0} = \frac{|z_2 - z_0|}{|z_1 - z_0|} (\pm 1) = \pm 2$$

$$Z_2 = z_0 \pm 2(z_1 - z_0)$$

$$z_2 = 2z_1 - z_0 = \frac{3}{2} + \frac{1}{2}i \Rightarrow |z_2|^2 = \frac{5}{2}$$

OR

$$z_2 = 3z_0 - 2z_1 = \frac{-1}{2} + \frac{5}{2}i \Rightarrow |z_2|^2 = \frac{13}{2}$$

**Q.19** Among the two statements

(S<sub>1</sub>) :  $(p \Rightarrow q) \wedge (p \wedge (\sim q))$  is a contradiction and

(S<sub>2</sub>) :  $(p \wedge q) \vee ((\sim p) \wedge q) \vee (p \wedge (\sim q)) \vee ((\sim p) \wedge (\sim q))$  is a tautology

(1) only (S<sub>2</sub>) is true      (2) only (S<sub>1</sub>) is true      (3) both are false      (4) both are true

**Ans.** [4]

**Sol.** S<sub>1</sub> :  $(p \Rightarrow q) \wedge (p \wedge \sim q)$

$$\equiv (\sim p \vee q) \wedge (p \wedge \sim q)$$

$$\equiv (\sim p \wedge p \wedge \sim q) \vee (q \wedge p \wedge \sim q)$$

$$\equiv (f \wedge \sim q) \vee (f \wedge p)$$

$$\equiv f \vee f \equiv f$$

S<sub>2</sub> :  $(p \wedge q) \vee (\sim p \wedge q) \vee (p \wedge \sim q) \vee (\sim p \wedge \sim q)$

$$\equiv ((p \vee \sim p) \wedge q) \vee ((p \vee \sim p) \wedge \sim q)$$

$$\equiv (t \wedge q) \vee (t \wedge \sim q) \equiv q \vee \sim q \equiv t$$

**Q.20** The sum, of the coefficients of the first 50 terms in the binomial expansion of  $(1-x)^{100}$ , is equal to

(1)  $^{101}C_{50}$       (2)  $^{99}C_{49}$       (3)  $^{-101}C_{50}$       (4)  $^{-99}C_{49}$

**Ans.** [4]

**Sol.**  $(^{100}C_0 - ^{100}C_1 + ^{100}C_2 - \dots - ^{100}C_{49}) + ^{100}C_{50} + (-^{100}C_{51} + ^{100}C_{52} - \dots + ^{100}C_{100}) = 0$

$$\lambda_1 = -^{100}C_{50} + \lambda_2 = 0$$

$$\lambda_1 = -\frac{1}{2} ^{100}C_{50} (\because \lambda_1 = \lambda_2)$$

$$= -^{99}C_{49}$$

**Section-B: Numerical Value Type Questions:** This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE**. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, -00.33, -00.30, 30.27, -27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer..

**Q.21** Let  $I(x) = \int \sqrt{\frac{x+7}{x}} dx$  and  $I(9) = 12 + 7 \log_e 7$ . If  $I(1) = \alpha + 7 \log_e (1 + 2\sqrt{2})$ , then  $\alpha^4$  is equal to \_\_\_\_\_.

**Ans.** [64]

**Sol.**  $I(X) = \int \sqrt{\frac{x+7}{x}} dx$

$$\frac{x+7}{x} = t^2 \Rightarrow -\frac{7}{x^2} dx = 2t dt$$

$$dx = \frac{-14t}{(t^2-1)^2} dt$$

$$I(X) = -14 \int \frac{t^2}{(t^2-1)^2} dt$$

$$= -14 \int \frac{dt}{\left(t^2 + \frac{1}{t^2} - 2\right)} = \frac{-14}{2} \int \left[ \frac{\left(1 - \frac{1}{t^2}\right)}{\left(t + \frac{1}{t}\right) - 4} + \frac{\left(1 + \frac{1}{t^2}\right)}{\left(t - \frac{1}{t}\right)} \right] dt$$

$$I(X) = -7 \left( \frac{1}{4} \ln \left| \frac{t + \frac{1}{t} - 2}{t + \frac{1}{t} + 2} \right| - \frac{1}{t - \frac{1}{t}} \right) + c$$

When  $x = 9$ ,  $t = \frac{4}{3}$

$$\Rightarrow 12 + 7 \ln 7 = \frac{-7}{4} \ln \left( \frac{1}{7} \right)^2 + 7 \times \frac{12}{7} + c$$

$$\Rightarrow c = \frac{7}{2} \ln 7$$

when  $x = 1$ ,  $t = 2\sqrt{2}$

$$\Rightarrow I(1) = + \frac{7}{4} \ln \left( \frac{2\sqrt{2} + 1}{2\sqrt{2} - 1} \right)^2 + 7 \times \frac{2\sqrt{2}}{7} + \frac{7}{2} \ln 7$$

$$= \frac{7}{2} \ln \left( \frac{(2\sqrt{2} + 1)^2}{7} \right) + 2\sqrt{2} + \frac{7}{2} \ln 7$$

$$= 7 \ln (2\sqrt{2} + 1) - \frac{7}{2} \ln 7 + 2\sqrt{2} + \frac{7}{2} \ln 7$$

$$\Rightarrow \alpha = 2\sqrt{2} \Rightarrow \alpha^4 = 64$$

**Q.22** If  $\int_{-0.15}^{0.15} |100x^2 - 1| dx = \frac{k}{3000}$ , then  $k$  is equal to \_\_\_\_\_.

**Ans.** [575]

**Sol.**

$$\begin{aligned} \int_{-0.15}^{0.15} |100x^2 - 1| dx &= 2 \int_0^{0.15} |100x^2 - 1| dx \\ &= 2 \int_0^{0.1} -100x^2 - 1 dx + 2 \int_{0.1}^{0.15} (100x^2 - 1) dx \\ &= 2 \cdot \left( \frac{-100x^3}{3} + x \right)_0^{0.1} + 2 \left( \frac{100x^3}{3} - x \right)_{0.1}^{0.15} \\ &= 4 \left( -\frac{1}{30} + \frac{1}{10} \right) + 2 \left( \frac{9}{80} - \frac{3}{20} \right) \\ &= \frac{8}{30} - \frac{3}{40} = \frac{575}{3000} \Rightarrow k = 575 \end{aligned}$$

**Q.23** Let  $[x]$  be the greatest integer  $\leq x$ . Then the number of points in the interval  $(-2, 1)$  where the function  $f(x) = |[x]| + \sqrt{x - [x]}$  is discontinuous, is \_\_\_\_\_.

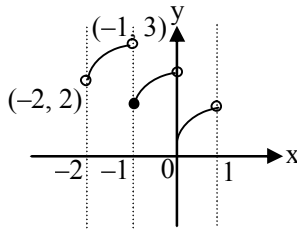
**Ans.** [2]

**Sol.**

$$f(x) = |[x]| + \sqrt{x - [x]}$$

$$x - [x] \geq 0 \Rightarrow x \in \mathbb{R}$$

$$f(x) = \begin{cases} 2 + \sqrt{x+2}, & -2 < x < -1 \\ 1 + \sqrt{x+1}, & -1 \leq x < 0 \\ \sqrt{x}, & 0 \leq x < 1 \end{cases}$$



$f(x)$  is discontinuous at two points  $x = \{-1, 0\}$

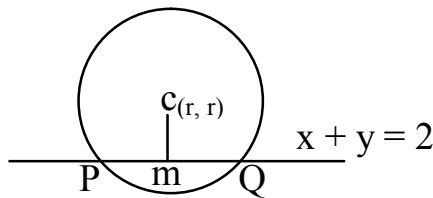
- Q.24** Two circles in the first quadrant of radii  $r_1$  and  $r_2$  touch the coordinate axes. Each of them cuts off an intercept of 2 units with the line  $x + y = 2$ . Then  $r_1^2 + r_2^2 - r_1 r_2$  is equal to \_\_\_\_\_.

**Ans.** [07.00]

**Sol.**

$$(x-r)^2 + (y-r)^2 = r^2$$

$$PQ = 2 \sqrt{r^2 - \left(\frac{2r-2}{\sqrt{2}}\right)^2} = 2$$



$$r^2 - \frac{4(r-1)^2}{2} = 1$$

$$r^2 - 1 = 2(r-1)^2$$

$$r^2 - 1 = 2r^2 - 4r + 2$$

$$\text{or } r^2 - 4r + 3 = 0$$

$$r = 1, 3$$

$$r_1^2 + r_2^2 - r_1 r_2 = 7$$

- Q.25** Let the positive numbers  $a_1, a_2, a_3, a_4,$  and  $a_5$  be in a G.P. Let their mean and variance be  $\frac{31}{10}$  and  $\frac{m}{n}$  respectively, where  $m$  and  $n$  are co-prime. If the mean of their reciprocals is  $\frac{31}{10}$  and  $a_3 + a_4 + a_5 = 14$ , then  $m + n$  is equal to \_\_\_\_\_.

**Ans.** [211.00]

**Sol.**  $a + ar + ar^2 + ar^3 + ar^4 = \frac{31}{10} \cdot 5 = \frac{31}{2}$

$$\frac{a(r^5 - 1)}{r - 1} = \frac{31}{2} \quad \dots\dots (i)$$

$$\frac{1}{a} \left( 1 + \frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3} + \frac{1}{r^4} \right) = \frac{31}{40} \cdot 5 = \frac{31}{8}$$

$$\frac{1}{a} \left( \frac{1 - \left(\frac{1}{r}\right)^5}{1 - \frac{1}{r}} \right) = \frac{31}{8}$$

Or  $\frac{1}{a} \left( \frac{r^5 - 1}{r - 1} \right) \frac{1}{r^4} = \frac{31}{8} \quad \dots\dots (ii)$

From (i) and (ii)

$$\frac{1}{a} \cdot \frac{31}{2a} \cdot \frac{1}{r^4} = \frac{31}{8}$$

$$Ar^2 = 2$$

From (i)

$$\frac{2}{r^2} \left( \frac{r^5 - 1}{r - 1} \right) = \frac{31}{2}$$

$$\frac{1 + r + r^2 + r^3 + r^4}{r^2} = \frac{31}{4}$$

$$\left( r^2 + \frac{1}{r^2} \right) + \left( r + \frac{1}{r} \right) = \frac{27}{4}$$

$$t^2 - 2 + t = \frac{27}{4}$$

$$4t^2 + 4t - 35 = 0$$

$$4t^2 + 14t - 10t - 35 = 0$$

$$(2t - 5)(2t + 7) = 0$$

$$t = \frac{5}{2}, \frac{-7}{2} \Rightarrow r = 2$$

$$r = 2, a = \frac{1}{2}$$

Variance of data set  $\left\{ \frac{1}{2}, 1, 2, 4, 8 \right\}$

$$= \frac{\left( \frac{341}{4} \right)}{5} - \left( \frac{31}{10} \right)^2$$

$$= \frac{341}{20} - \frac{961}{100} = \frac{1705 - 961}{100}$$

$$= \frac{744}{100} = \frac{186}{25}$$

**Q.26** Let  $D_k = \begin{vmatrix} 1 & 2k & 2k-1 \\ n & n^2+n+2 & n^2 \\ n & n^2+n & n^2+n+2 \end{vmatrix}$ . If  $\sum_{k=1}^n D_k = 96$ , then  $n$  is equal to \_\_\_\_\_.

**Ans.** [06.00]

**Sol.**  $\sum_{k=1}^n D_k = \begin{vmatrix} \sum 1 & 2 \sum k & 2 \sum k - \sum 1 \\ n & n^2+n+2 & n^2 \\ n & n^2+n & n^2+n+2 \end{vmatrix}$

$$= \begin{vmatrix} n & n(n+1) & n^2 \\ n & n^2+n+2 & n^2 \\ n & n^2+n & n^2+n+2 \end{vmatrix}$$



$$= \begin{vmatrix} 0 & -2 & 0 \\ 0 & 2 & -n-2 \\ n & n^2+n & n^2+n+2 \end{vmatrix}$$

$$= 2((-n)(-n-2)) = 96$$

$$n^2 + 2n = 48$$

$$n = 6, -8$$

$$n = 6$$

**Q.27** A fair  $n$  ( $n > 1$ ) faces die is rolled repeatedly until a number less than  $n$  appears. If the mean of the number of tosses required is  $\frac{n}{9}$ , then  $n$  is equal to \_\_\_\_\_.

**Ans.** [10]

**Sol.**  $x_i$  1 2 3 4 .....

$$p_i \frac{n-1}{n} \cdot \frac{1}{n} \cdot \left(\frac{n-1}{n}\right) \frac{1}{n^2} \cdot \frac{n-1}{n} \cdot \frac{1}{n^3} \cdot \left(\frac{n-1}{n}\right) \dots\dots$$

$$\text{Mean} = \sum_{i=1}^{\infty} p_i x_i = 1 \cdot \frac{n-1}{n} + \frac{2}{n} \cdot \left(\frac{n-1}{n}\right) + \frac{3}{n^2} \left(\frac{n-1}{n}\right) + \dots\dots$$

$$\frac{n}{9} = \left(1 - \frac{1}{n}\right)S \quad \dots\dots(i)$$

$$\text{where } S = 1 + \frac{2}{n} + \frac{3}{n^2} + \frac{4}{n^3} + \dots\dots$$

$$\frac{1}{n}S = \frac{1}{n} + \frac{2}{n^2} + \frac{3}{n^3} + \dots\dots$$

$$\left(1 - \frac{1}{n}\right)S = 1 + \frac{1}{n} + \frac{1}{n^2} + \frac{1}{n^3} + \dots\dots$$

$$\left(1 - \frac{1}{n}\right)S = \frac{1}{1 - \frac{1}{n}}$$

$$\Rightarrow \frac{n}{9} = \left(1 - \frac{1}{n}\right) \times \frac{1}{\left(1 - \frac{1}{n}\right)^2} = \frac{n}{n-1}$$

$$\Rightarrow n = 10$$

**Q.28** The number of relations, on the set  $\{1, 2, 3\}$  containing  $(1, 2)$  and  $(2, 3)$ , which are reflexive and transitive but not symmetric, is \_\_\_\_\_.

**Ans.** [4]

**Sol.**  $(1, 1), (2, 2), (3, 3) \in R$

Since  $(1, 2), (2, 3) \in R$ ,  $(1, 3)$  must  $\in R$

**Possible cases :**

**Case-1 :** All of  $(2, 1), (3, 2), (3, 1) \notin R \rightarrow 1$  relation.

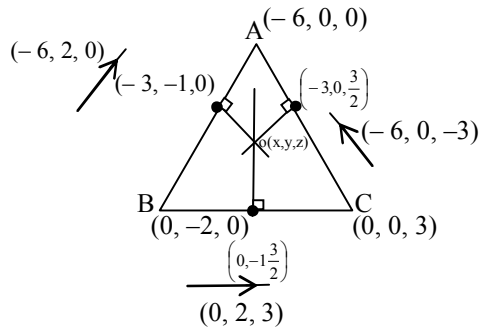
**Case-2 :** Only one of  $(2, 1), (3, 2), (3, 1) \in R \rightarrow 3$  relations.

Note that exactly two of  $(2, 1), (3, 2), (3, 1) \in R$  is not possible because if two of these  $\in R$ , third must  $\in R$  to make relation transitive.

Total number of relations = 4

**Q.29** Let the plane  $x + 3y - 2z + 6 = 0$  meet the co-ordinate axes at the points A, B, C. If the orthocenter of the triangle ABC is  $\left(\alpha, \beta, \frac{6}{7}\right)$ , then  $98(\alpha + \beta)^2$  is equal to \_\_\_\_\_.

**Ans.** [288]  
**Sol.**



$$(x - 0) \times 0 + (y + 1) \times 2 + \left(z - \frac{3}{2}\right) \times 3 = 0$$

$$\Rightarrow 4y + 6z - 5 = 0 \quad \dots(i)$$

$$(x + 3) \times (-6) + (y - 0) \times 0 + \left(z - \frac{3}{2}\right) \times (-3) = 0$$

$$\Rightarrow 4x + 2z + 9 = 0 \quad \dots(ii)$$

$$\Rightarrow (x + 3) \times (-6) + (y + 1) \times 2 + (z - 0) \times 0 = 0$$

$$\Rightarrow 3x - y + 8 = 0 \quad \dots(iii)$$

From (i), (ii), (iii),

$$x = \frac{-9}{4} - \frac{z}{2}, y = \frac{5}{4} - \frac{3}{2}z, z = z$$

$$\left(-\frac{9}{4} - \frac{z}{2}, \frac{5}{4} - \frac{3}{2}z, z\right) \quad \left(-2, \frac{-2}{3}, 1\right) \quad \left(\alpha, \beta, \frac{6}{7}\right)$$

$$\Rightarrow G \equiv \left(\frac{\alpha - \frac{9}{2} - z}{3}, \frac{\beta + \frac{5}{2} - 3z}{3}, \frac{\frac{6}{7} + 2z}{3}\right)$$

$$\Rightarrow z = \frac{15}{14}, \alpha = -\frac{3}{7}, \beta = -\frac{9}{7}$$

$$\Rightarrow 98(\alpha + \beta)^2 = 98 \times \frac{144}{49} = 288$$

**Q.30** Let the digits  $a, b, c$  be in A.P. Nine-digit numbers are to be formed using each of these three digits thrice such that three consecutive digits are in A.P. at least once. How many such numbers can be formed?

**Ans.** [1260]

**Sol.** Digits are in A. P. at least once Possible cases a b c or c b a  
Total ways of selecting 3 consecutive places out of 9 places is 7.

$$\text{Total ways} = \frac{{}^7C_1 \times 2 \times 6!}{2! \times 2! \times 2!} = 1260$$

---

**PHYSICS**

---

**Section-A:** Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

---

**Q.31** A proton and an  $\alpha$ -particle are accelerated from rest by 2 V and 4 V potentials, respectively. The ratio of their de-Broglie wavelength is :

- (1) 8 : 1                      (2) 2 : 1                      (3) 4 : 1                      (4) 16 : 1

**Ans.** [3]

**Sol.** 
$$\frac{\lambda_p}{\lambda_\alpha} = \frac{\sqrt{m_\alpha}}{\sqrt{m_p}} \times \frac{\sqrt{q_\alpha 4}}{\sqrt{q_p 2}} = 2 \times 2$$

**Q.32** Given below are two statements: one is labelled as Assertion A and the other is labelled as Reason R  
**Assertion A:** EM waves used for optical communication have longer wavelengths than that of microwave, employed in Radar technology.

**Reason R :** Infrared EM waves are more energetic than microwaves, (used in Radar).

In the light of given statements, choose the correct answer from the options given below.

- (1) Both A and R are true but R is NOT the correct explanation of A  
(2) A is false but R is true  
(3) A is true but R is false  
(4) Both A and R are true and R is the correct explanation of A

**Ans.** [1]

**Sol.** Assertion is false

as  $\lambda_{\text{optical}} < \lambda_{\text{microwave}}$

**Q.33** If the r.m.s speed of chlorine molecule is 490 m/s at 27°C, the r.m.s speed of argon molecules at the same temperature will be (Atomic mass of argon = 39.9 u, molecular mass of chlorine = 70.9 u)

- (1) 551.7 m/s                      (2) 651.7 m/s                      (3) 451.7 m/s                      (4) 751.7 m/s

**Ans.** [2]

**Sol.** 
$$\frac{V_{\text{Ar}}}{V_{\text{Cl}}} = \sqrt{\frac{M_{\text{Cl}}}{M_{\text{Ar}}}}$$
$$\Rightarrow V_{\text{Ar}} = \sqrt{\frac{71}{40}} \times 490$$
$$\cong 651.7$$

**Q.34** Two satellites A and B move round the earth in the same orbit. The mass of A is twice the mass of B. The quantity which is same for the two satellites will be

- (1) Speed                      (2) Kinetic energy                      (3) Total energy                      (4) Potential energy

**Ans.** [1]

**Sol.** Speed will be independent of mass of satellite.

**Q.35** A particle is executing simple harmonic motion (SHM). The ratio of potential energy and kinetic energy of the particle when its displacement is half of its amplitude will be

- (1) 1 : 1                      (2) 1 : 3                      (3) 2 : 1                      (4) 1 : 4

**Ans.** [2]

**Sol.** 
$$\frac{\text{P.E.}}{\text{K.E.}} = \frac{X^2}{A^2 - X^2} = \frac{\frac{1}{4}}{\frac{3}{4}} = \frac{1}{3}$$

- Q.36** Given below are two statements:  
Statement I : The diamagnetic property depends on temperature.  
Statement II : The induced magnetic dipole moment in a diamagnetic sample is always opposite to the magnetizing field.  
In the light of given statements, choose the correct answer from the options given below  
(1) Both Statement I and Statement II are False (2) Statement I is incorrect but Statement II is true  
(3) Statement I is correct but Statement II is false (4) Both Statement I and Statement II are true

**Ans.** [2]

**Sol.** Diamagnetic materials oppose external field.

- Q.37** A wire of resistance  $160\Omega$  is melted and drawn in a wire of one-fourth of its length. The new resistance of the wire will be  
(1)  $16\Omega$  (2)  $10\Omega$  (3)  $640\Omega$  (4)  $40\Omega$

**Ans.** [2]

**Sol.**  $R' = R/16$   
 $R' = 10\Omega$

- Q.38** A body cools from  $80^\circ\text{C}$  to  $60^\circ$  in 5 minutes. The temperature for the surrounding is  $20^\circ\text{C}$ . The time it takes to cool from  $60^\circ\text{C}$  to  $40^\circ\text{C}$  is  
(1) 450 s (2) 420 s (3) 500 s (4)  $\frac{25}{3}$  s

**Ans.** [3]

**Sol.**  $\frac{20}{5} = c(50)$   
 $\frac{20}{x} = c(30)$   
 $\Rightarrow x = \frac{25}{3}$  minutes  
 $= 500$  s

- Q.39** Match List I with List II

	List-I		List-II
A.	Spring constant	I.	$[\text{T}^{-1}]$
B.	Angular speed	II.	$[\text{MT}^{-2}]$
C.	Angular momentum	III.	$[\text{ML}^2]$
D.	Moment of Inertia	IV.	$[\text{ML}^2\text{T}^{-1}]$

Choose the correct answer from the options given below:

- (1) A-I, B-III, C-II, D-IV (2) A-IV, B-I, C-III, D-II (3) A-II, B-I, C-IV, D-III (4) A-II, B-III, C-I, D-IV

**Ans.** [3]

**Sol.**  $[\text{K}] = [\text{M}^1\text{L}^0\text{T}^{-2}]$   
 $\omega = [\text{T}^{-1}]$

- Q.40** In an n-p-n common emitter (CE) transistor the collector current changes from 5 mA to 16 mA for the change in base current from  $10\mu\text{A}$  and  $200\mu\text{A}$ , respectively. The current gain of transistor is \_\_\_\_\_  
(1) 110 (2) 210 (3) 0.9 (4) 9

**Ans.** [1]

**Sol.**  $\Delta I_c = 11$  mA  
 $\Delta I_b = 100\mu\text{A}$   
 $\beta = \frac{\Delta I_c}{\Delta I_b} = 110$

**Q.41** An ice cube has a bubble inside. When viewed from one side the apparent distance of the bubble is 12 cm. When viewed from the opposite side, the apparent distance of the bubble is observed as 4 cm. If the side of the ice cube is 24 cm, the refractive index of the ice cube is

- (1)  $\frac{3}{2}$                       (2)  $\frac{2}{3}$                       (3)  $\frac{6}{5}$                       (4)  $\frac{4}{3}$

**Ans.** [1]

**Sol.**  $\frac{x}{\mu} = 12$   
 $\frac{a-x}{\mu} = 4$   
 $24 - 12\mu = 4\mu$   
 $\mu = \frac{24}{16} = 1.5$

**Q.42** The amplitude of  $15 \sin(1000 \pi t)$  is modulated by  $10 \sin(4 \pi t)$  signal. The amplitude modulated signal contains frequencies of

- A. 500 Hz                      B. 2 Hz                      C. 250 Hz                      D. 498 Hz                      E. 502 Hz

Choose the correct answer from the options given below

- (1) A and B only                      (2) A and C only                      (3) A and D only                      (4) A, D and E only

**Ans.** [4]

**Sol.** Frequencies in the system

$$= \frac{1000\pi}{2\pi}, \frac{996\pi}{2\pi}, \frac{1004\pi}{2\pi}$$
$$= 500 \text{ Hz}, 498 \text{ Hz}, 502 \text{ Hz}$$

**Q.43** An engine operating between the boiling and freezing points of water will have

- A. Efficiency more than 27%  
B. Efficiency less than the efficiency of a Carnot engine operating between the same two temperatures.  
C. Efficiency equal to 27%  
D. Efficiency less than 27%

Choose the correct answer from the options given below

- (1) B, C and D only                      (2) A and B only                      (3) B and D only                      (4) B and C only

**Ans.** [3]

**Sol.**  $\eta_c = 1 - \frac{273}{373} = 26.8\%$   
 $\Rightarrow \eta_{\text{engine}} < \eta_{\text{carnot}} = \eta_c$

**Q.44** Three forces  $F_1 = 10 \text{ N}$ ,  $F_2 = 8 \text{ N}$ ,  $F_3 = 6 \text{ N}$  are acting on a particle of mass 5 kg. The forces  $F_2$  and  $F_3$  are applied perpendicularly so that particle remains at rest. If the force  $F_1$  is removed, then the acceleration of the particle is

- (1)  $7 \text{ ms}^{-2}$                       (2)  $0.5 \text{ ms}^{-2}$                       (3)  $4.8 \text{ ms}^{-2}$                       (4)  $2 \text{ ms}^{-2}$

**Ans.** [4]

**Sol.**  $\sqrt{F_2^2 + F_3^2} = 10 \text{ N}$   
 $\Rightarrow a = \frac{10}{5} = 2 \text{ m/s}^2$

- Q.45** Given below are two statements: one is labelled as Assertion A and the other is labelled as Reason R.  
**Assertion A :** If an electric dipole of dipole moment  $30 \times 10^{-5}$  Cm is enclosed by a closed surface, the net flux coming out of the surface will be zero.  
**Reason R :** Electric dipole consists of two equal and opposite charges.  
In the light of above, statements, choose the correct answer from the options given below.  
(1) Both A and R are true and R is the correct explanation of A  
(2) A is false but R is true  
(3) A is true but R is false  
(4) Both A and R are true and R is NOT the correct explanation of A

**Ans.** [1]

**Sol.**  $\phi = \frac{q_{in}}{\epsilon_0} = 0$

- Q.46** Given below are two statements:  
**Statement I :** When the frequency of an a.c source in a series LCR circuit increases, the current the circuit first increases, attains a maximum value and then decreases.  
**Statement II:** In a series LCR circuit, the value of power factor at resonance is one.  
In the light of given statements, choose the most appropriate answer from the options given below.  
(1) Statement I is incorrect but Statement II is true. (2) Both Statement I and Statement II are false.  
(3) Both Statement I and Statement II are true. (4) Statement I is correct but Statement II is false.

**Ans.** [3]

**Sol.** Z first decreases and then increases  $\phi_{resonance} = 1$  as  $Z = R$

- Q.47** A 12.5 eV electron beam is used to bombard gaseous hydrogen at room temperature. The number of spectral lines emitted will be:  
(1) 1 (2) 4 (3) 2 (4) 3

**Ans.** [4]

**Sol.**  $h\nu_{max} = 12.5$  eV  
 $\Rightarrow$  maximum excitation level = 3  
 $\Rightarrow$  spectral lines possible =  $3C_2 = 3$

- Q.48** A ball is thrown vertically upward with an initial velocity of 150 m/s. The ratio of velocity after 3 s and 5 s is  $\frac{x+1}{x}$ . The value of x is \_\_\_\_\_. {take,  $g = 10$  m/s<sup>2</sup>}

(1) 10 (2) -5 (3) 6 (4) 5

**Ans.** [4]

**Sol.**  $\frac{v_3}{v_4} = \frac{150 - 30}{150 - 50} = \frac{120}{100} = \frac{6}{5}$   
 $x = 5$

- Q.49** Given below are two statements:  
**Statement I :** A truck and a car moving with same kinetic energy are brought to rest by applying breaks which provide equal retarding forces. Both come to rest in equal distance.  
**Statement II:** A car moving towards east takes a turn and moves towards north, the speed remains unchanged. The acceleration of the car is zero.  
In the light of given statements, choose the most appropriate answer from the options given below  
(1) Statement I is correct but statement II is incorrect.  
(2) Statement I is incorrect but statement II is correct.  
(3) Both statement I and Statement II are correct.  
(4) Both statement I and statement II are incorrect.

**Ans.** [1]

Sol.  $WD = Fx = \Delta KE$   
 $x_1 = x_2$

**Q.50** The ratio of escape velocity of a planet to the escape velocity of earth will be :-  
Given: Mass of the planet is 16 times mass of earth and radius of the planet is 4 times the radius of earth.

- (1) 4 : 1                      (2) 1 : 4                      (3) 1 :  $\sqrt{2}$                       (4) 2 : 1

Ans. [4]

Sol.  $v = \sqrt{\frac{2GM}{R}}$   
 $\Rightarrow \frac{V_p}{V_e} = \sqrt{\frac{M_p}{M_e}} \times \sqrt{\frac{R_e}{R_p}} = 4 \times \frac{1}{2}$   
 $= 2 : 1$

---

**Section-B: Numerical Value Type Questions:** This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE**. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, -00.33, -00.30, 30.27, -27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

---

**Q.51** 64 identical drops each charged upto potential of 10mV are combined to form a bigger drop. The potential of the bigger drop will be \_\_\_\_\_ mV.

Ans. [160]

Sol.  $V' = \frac{kQ'}{R'}$   
 $V' = \frac{k \times 64 \times Q}{4R} = 16 \times V = 160 \text{ mV}$

**Q.52** A common example of alpha decay is  ${}_{92}^{238}\text{U} \rightarrow {}_{90}^{234}\text{Th} + {}_2^4\text{He} + Q$

Given:

$${}_{92}^{238}\text{U} = 238.05060 \text{ u}$$

$${}_{90}^{234}\text{Th} = 234.04360 \text{ u}$$

$${}_2^4\text{He} = 4.00260 \text{ u and } 1\text{u} = 931.5 \frac{\text{MeV}}{c^2}$$

The energy released (Q) during the alpha decay of

$${}_{92}^{238}\text{U} \text{ is } \underline{\hspace{2cm}} \text{ MeV.}$$

Ans. [4]

Sol.  $Q = 931.5 \times 0.0044 = 4.0986$

**Q.53** Glycerin of density  $1.25 \times 10^3 \text{ kg m}^{-3}$  is flowing through the conical section of pipe. The area of cross-section of the pipe at its ends are  $10 \text{ cm}^2$  and  $5 \text{ cm}^2$  and pressure drop across its length is  $3 \text{ Nm}^{-2}$ . The rate of flow of glycerin through the pipe is  $x \times 10^{-5} \text{ m}^2 \text{ s}^{-1}$ . The value of x is \_\_\_\_\_.

Ans. [4]

Sol.  $A_1 = 10 \text{ cm}^2; A_2 = 5 \text{ cm}^2$   
 $\Delta P = -3 \text{ N/m}^2$   
 $-\frac{1}{2}\rho v_1^2 + \frac{1}{2}\rho v_2^2 = \Delta P$

$$= v_1^2(3) = \frac{3 \times 2}{1.25 \times 10^3}$$
$$\Rightarrow v_1 = \frac{\sqrt{2}}{\sqrt{1250}}$$
$$Q = A_2 v_2 = \frac{10 \times \sqrt{2}}{\sqrt{1250}} = 4 \times 10^{-5}$$

**Q.54** A conducting circular loop is placed in a uniform magnetic field of 0.4 T with its plane perpendicular to the field. Somehow, the radius of the loop starts expanding at a constant rate of 1 mm/s. The magnitude of induced emf in the loop at an instant when the radius of the loop is 2 cm will be \_\_\_\_\_  $\mu\text{V}$ .

**Ans.** [50]

**Sol.**  $\phi = 2\pi R \frac{dR}{dt} \times B$

$$= 2 \times \frac{22}{7} \times 2 \times 10^{-2} \times 10^{-3} \times 0.4$$
$$= 5 \times 10^{-5} \text{ V} = 50.28 \mu\text{V}$$

**Q.55** A compass needle oscillates 20 times per minute at a place where the dip is  $30^\circ$  and 30 times per minute where the dip is  $60^\circ$ . The ratio of total magnetic field due to the earth at two places respectively is  $\frac{4}{\sqrt{x}}$ . The

value of x is

**Ans.** [243]

**Sol.**  $T = 2\pi \sqrt{\frac{I}{MB_H}}$

$$T = 2\pi \sqrt{\frac{I}{MB \cos \theta}}$$
$$\frac{20}{30} = \sqrt{\frac{B_1 \sqrt{3}/2}{B_2 \cdot 1/2}}$$
$$\frac{4}{9} = \frac{B_1}{B_2} \sqrt{3}$$
$$\frac{B_1}{B_2} = \frac{4}{\sqrt{243}}$$

**Q.56** Two convex lenses of focal length 20 cm each are placed coaxially with a separation of 60 cm between them. The image of the distant object formed by the combination is at \_\_\_\_\_ cm from the first lens.

**Ans.** [100]

**Sol.**  $u_1 = \infty, f_1 = 20 \text{ cm}, v_1 = 20 \text{ cm}$

$$u_2 = 40 \text{ cm}, f_2 = 20 \text{ cm}, v_2 = 40 \text{ cm}$$
$$\Rightarrow x = 40 \text{ cm} + 60 \text{ cm}$$
$$= 100 \text{ cm}$$

**Q.57** To maintain a speed of 80 km/h by a bus of mass 500 kg on a plane rough road for 4 km distance, the work done by the engine of the bus will be \_\_\_ kJ. [The coefficient of friction between tyre of bus and road is 0.04]

**Ans.** [784]

**Sol.**  $v = 80 \text{ kph}$



$$m = 500$$

$$S = 4 \text{ km}$$

$$P_{\text{friction}} = (0.04) (500\text{g}) \times 80 \times \frac{5}{18} \text{ watts}$$

$$P = \frac{20 \times 80 \times 5 \times 9.8}{18}$$

$$W = \frac{20 \times 80 \times 5}{18} \times \frac{4 \times 10^3}{80 \times \frac{5}{18}} \times 9.8 = 784 \text{ kJ}$$

**Q.58** For a certain organ pipe, the first three resonance frequencies are in the ratio of 1:3:5 respectively. If the frequency of fifth harmonic is 405 Hz and the speed of sound in air is  $324 \text{ ms}^{-1}$  the length of the organ pipe is \_\_\_\_\_ m.

**Ans.** [1]

**Sol.**  $f_5 = 405$

$$\Rightarrow \frac{5v}{4\ell} = 405$$

$$\Rightarrow \ell = \frac{405 \times 4}{5 \times 324} = 1 \text{ m}$$

**Q.59** For rolling spherical shell, the ratio of rotational kinetic energy and total kinetic energy is  $\frac{x}{5}$ . The value of x is \_\_\_\_\_.

**Ans.** [2]

**Sol.**  $\frac{K.E_R}{K.E_T} = \frac{\frac{2}{3}MR^2\omega^2}{\frac{5}{3}MR^2\omega^2} = \frac{2}{5}$

**Q.60** The current flowing through a conductor connected across a source is 2 A and 1.2 A at  $0^\circ\text{C}$  and  $100^\circ\text{C}$  respectively. The current flowing through the conductor at  $50^\circ\text{C}$  will be \_\_\_\_\_  $\times 10^2$  mA.

**Ans.** [15]

**Sol.**  $\frac{R_1}{R_2} = \frac{1.2}{2} = \frac{1}{1 + \alpha 100}$

$$1 + \alpha 100 = \frac{2}{1.2}$$

$$\alpha 100 = \frac{0.8}{1.2} = \frac{2}{3}$$

$$\alpha 50 = \frac{1}{3}$$

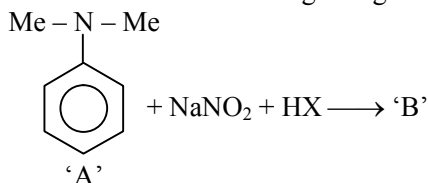
$$1 + \alpha 50 = \frac{4}{3}$$

$$\Rightarrow \frac{I_3}{I_1} = \frac{3}{4} \times 2\text{A} = 1.5\text{A}$$
$$= 15 \times 10^2 \text{ mA}$$

**CHEMISTRY**

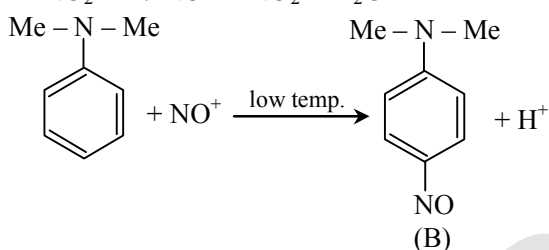
**Section-A:** This section contains 20 multiple choice questions. Each question has 4 choices(1), (2), (3) and (4), out of which **ONLY ONE** is correct..

**Q.61** The incorrect statement regarding the reaction given below is –



- (1) The product ‘B’ formed in the above reaction is p-nitroso compound at low temperature
- (2) ‘B’ is N-nitroso ammonium compound
- (3) The reaction occurs at low temperature
- (4) The electrophile involved in the reaction is  $\text{NO}^+$

**Ans.** [2]



$\therefore$  Statement (2) is incorrect

**Q.62** The bond order and magnetic property of acetylide ion are same as that of -

- (1)  $\text{O}_2^+$
- (2)  $\text{N}_2^+$
- (3)  $\text{NO}^+$
- (4)  $\text{O}_2^-$

**Ans.** [3]

Species	Bond Order	Magnetic moment
$\text{HC} \equiv \text{C}^-$	3	0
$\text{O}_2^+$	2.5	$\sqrt{3}$ B.M
$\text{N}_2^+$	2.5	$\sqrt{3}$ B.M
$\text{NO}^+$	3	0
$\text{O}_2^-$	1.5	$\sqrt{3}$ B.M

**Q.63** Match List I with List II.

List I Complex	List II CFSE ( $\Delta_0$ )
A. $[\text{Cu}(\text{NH}_3)_6]^{2+}$	I. -0.6
B. $[\text{Ti}(\text{H}_2\text{O})_6]^{3+}$	II. -2.0
C. $[\text{Fe}(\text{CN})_6]^{3-}$	III. -1.2
D. $[\text{NiF}_6]^{4-}$	IV. -0.4

Choose the correct answer from the options given below :

- (1) A(III), B(IV), C(I), D(II)
- (2) A(I), B(IV), C(II), D(III)
- (3) A(I), B(II), C(IV), D(III)
- (4) A(II), B(III), C(I), D(IV)

**Ans.** [2]

**Sol.**

(A)	$[\text{Cu}(\text{NH}_3)_6]^{2+}$ $\text{Cu}^{2+} : 3d^6, t_{2g}^6 e_g^3$	CFSE $= (-6 \times 0.4 + 3 \times 0.6)\Delta_0$ $= -0.6 \Delta_0$
(B)	$[\text{Ti}(\text{H}_2\text{O})_6]^{3+}$ $\text{Ti}^{3+} : 3d^1, t_{2g}^1 e_g^0$	CFSE $= -1 \times 0.4\Delta_0$ $= -0.4 \Delta_0$
(C)	$[\text{Fe}(\text{CN})_6]^{3-}$ $\text{Fe}^{3+} : 3d^5, t_{2g}^5 e_g^0$	CFSE $= -5 \times 0.4\Delta_0$ $= -2.0 \Delta_0$
(D)	$[\text{NiF}_6]^{4-}$ $\text{Ni}^{2+} : 3d^8, t_{2g}^6 e_g^2$	CFSE $= (-6 \times 0.4 + 2 \times 0.6)\Delta_0$ $= -1.2 \Delta_0$

A – I; B – IV; C – II; D – III

**Q.64** Given below are two statements: one is labelled as **Assertion A** and the other is labelled as **Reason R**.  
**Assertion A:** In the Ellingham diagram, a sharp change in slope of the line is observed from  $\text{Mg} \rightarrow \text{MgO}$  at  $\sim 1120^\circ\text{C}$

**Reason R:** There is a large change of entropy associated with the change of state.

In the light of the above statements, choose the correct answer from the options given below :

- (1) Both A and R are true but R is NOT the correct explanation of A
- (2) A is false but R is true
- (3) Both A and R are true and R is the correct explanation of A
- (4) A is true but R is false

**Ans.** [3]

**Sol.** In the Ellingham diagram, a sharp change in slope of the line is observed for  $\text{Mg} - \text{MgO}$  at  $\sim 1120^\circ\text{C}$  because that is the boiling point of magnesium.

There is a large increase in entropy associated with the change of state of magnesium. So, both Assertion (A) and Reason (R) are true and (R) is the correct explanation of (A).

**Q.65** In the given reaction cycle



X, Y and Z respectively are

- | X                   | Y                           | Z             |
|---------------------|-----------------------------|---------------|
| (1) $\text{CaCO}_3$ | $\text{NaCl}$               | $\text{KCl}$  |
| (2) $\text{CaCO}_3$ | $\text{NaCl}$               | $\text{HCl}$  |
| (3) $\text{CaO}$    | $\text{NaCl} + \text{CO}_2$ | $\text{NaCl}$ |
| (4) $\text{CaO}$    | $\text{NaCl} + \text{CO}_2$ | $\text{KCl}$  |

**Ans.** [2]



**Q.68** Given below are two statements:

**Statement I:**  $\text{SbCl}_5$  is more covalent than  $\text{SbCl}_3$ .

**Statement II:** The higher oxides of halogens also tend to be more stable than the lower ones.

In the light of the above statements, choose the most appropriate answer from the options given below.

- (1) Statement I is incorrect but statement II is correct  
 (2) Both Statement I and Statement II are incorrect  
 (3) Both Statement I and Statement II are correct  
 (4) Statement I is correct but statement II is incorrect

**Ans.** [3]

**Sol.**  $\text{SbCl}_5$  is more covalent than  $\text{SbCl}_3$  due to higher polarisation power of  $\text{Sb}^{\text{V}}$  than that of  $\text{Sb}^{\text{III}}$ . Higher oxides of halogens are more stable than lower ones due to the formation of more number of bonds, which results in higher release of energy. So, both the statements I and II are correct.

**Q.69** Four gases, A, B, C and D have critical temperatures 5.3, 33.2, 126.0 and 154.3K respectively. For their adsorption on a fixed amount of charcoal, the correct order is :

- (1)  $C > D > B > A$       (2)  $C > B > D > A$       (3)  $D > C > B > A$       (4)  $D > C > A > B$

**Ans.** [3]

**Sol.** The extent of adsorption of different gases on the surface of a fixed amount of charcoal is directly proportional to their critical temperatures.

Order of  $T_c$                        $D > C > B > A$

Order of adsorption          $D > C > B > A$

**Q.70** Match List I with List II.

List I	List II
A. Nitrogen oxides in air	I. Eutrophication
B. Methane in air	II. pH of rain water becomes 5.6
C. Carbon dioxide	III. Global warming
D. Phosphate fertilisers in water	IV. Acid rain

Choose the correct answer from the options given below :

- (1) A-II, B-III, C-I, D-IV                      (2) A-I, B-II, C-III, D-IV  
 (3) A-IV, B-III, C-II, D-I                      (4) A-IV, B-II, C-III, D-I

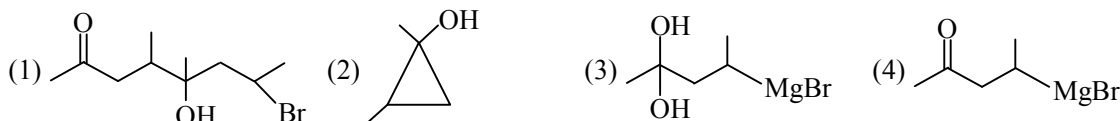
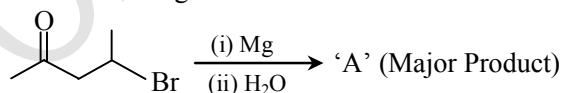
**Ans.** [3]

**Sol.**

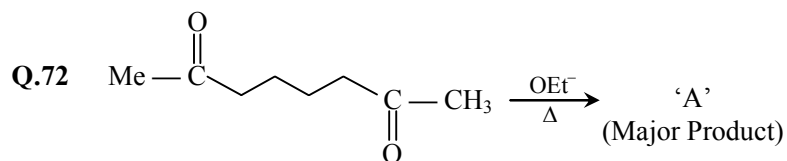
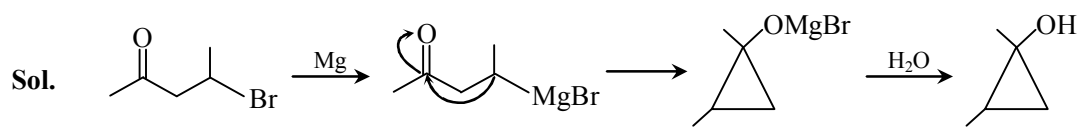
A. Nitrogen oxides in air	IV. Acid rain
B. Methane in air	III. Global warming
C. Carbon dioxide	II. pH of rain water becomes 5.6
D. Phosphate fertilisers in water	I. Eutrophication

$\therefore$  A-IV; B-III; C-II; D-I

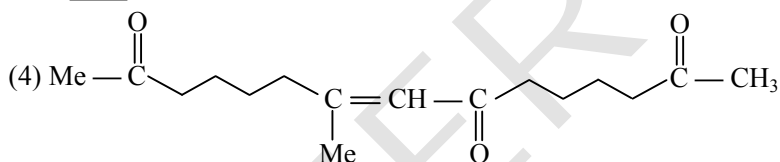
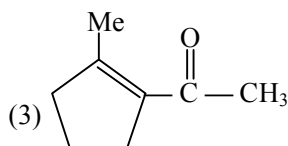
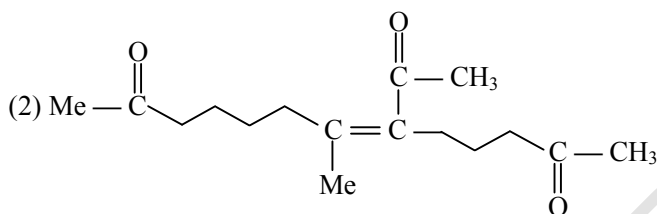
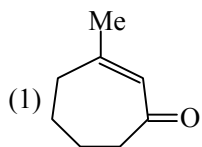
**Q.71** In the following reaction



**Ans.** [2]

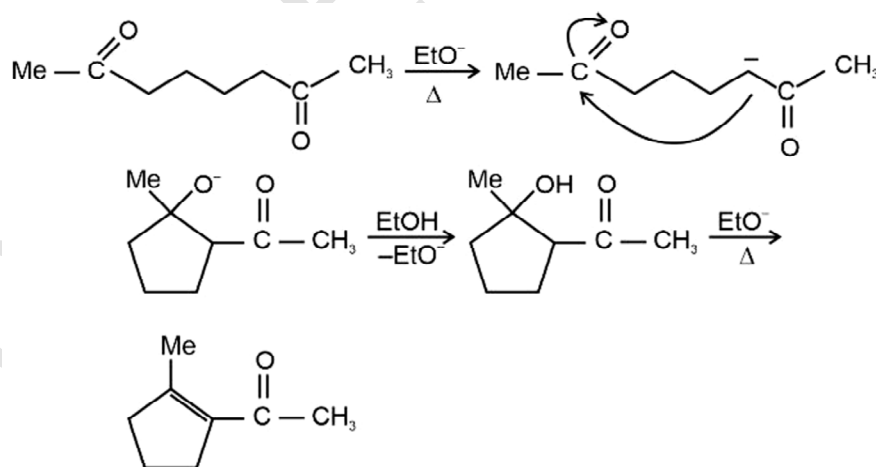


A in the above reaction is :



Ans. [3]

Sol.



**Q.73** Given below are two statement : one is labelled as Assertion A and the other is labelled as Reason R.

**Assertion A :** 5f electron can participate in bonding to a far greater extent than 4f electrons.

**Reason R :** 5f orbitals are not as buried as 4f orbitals.

In the light of the above statements, choose the correct answer from the options given below

- (1) A is false but R is true
- (2) Both A and R are true and R is the correct explanation of A
- (3) A is true but R is false
- (4) Both A and R are true but R is NOT the correct explanation of A

**Ans.** [2]

**Sol.** 5f electron can participate in bonding to a far greater extent than 4f electron because 5f orbitals are more exposed than 4f orbitals.

So, Assertion (A) is true 5f orbitals are not as buried as 4f orbitals.

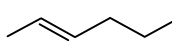
So, reason (R) is also true and (R) is the correct explanation of (A)

**Q.74** 2-hexene  $\xrightarrow[\text{(ii) H}_2\text{O}]{\text{(i) O}_3}$  Products

The two products formed in above reaction are-

- (1) Butanal and acetaldehyde
- (2) Butanoic acid and acetaldehyde
- (3) Butanal and acetic acid
- (4) Butanoic acid and acetic acid

**Ans.** [4]

**Sol.**   $\xrightarrow[\text{(ii) H}_2\text{O}]{\text{(i) O}_3}$  CH<sub>3</sub>COOH + CH<sub>3</sub>-CH<sub>2</sub>-CH<sub>2</sub>-COOH  
(Acetic acid) (Butanoic acid)

**Q.75** Match List I with List II.

List I (Type of Hydride)	List II (Example)
A. Electron deficient hydride	I. MgH <sub>2</sub>
B. Electron rich hydride	II. HF
C. Electron precise hydride	III. B <sub>2</sub> H <sub>6</sub>
D. Saline hydride	IV. CH <sub>4</sub>

Choose the correct answer from the options given below :

- (1) A-III, B-II, C-IV, D-I
- (2) A-II, B-III, C-I, D-IV
- (3) A-II, B-III, C-IV, D-I
- (4) A-III, B-II, C-I, D-IV

**Ans.** [1]

**Sol.** A. Electron deficient hydride III. B<sub>2</sub>H<sub>6</sub>  
 B. Electron rich hydride II. HF  
 C. Electron precise hydride IV. CH<sub>4</sub>  
 D. Saline hydride I. MgH<sub>2</sub>

**Q.76** The density of alkali metals is in the order -

- (1) K < Cs < Na < Rb
- (2) Na < Rb < K < Cs
- (3) Na < K < Cs < Rb
- (4) K < Na < Rb < Cs

**Ans.** [4]

**Sol.** Density of alkali metals increases down the group with the exception of potassium. Therefore, correct order of density of given alkali metals is

K < Na < Rb < Cs

**Q.77** Given below are two statements:

**Statement I:** Boron is extremely hard indicating its high lattice energy.

**Statement II:** Boron has highest melting and boiling point compared to its other group members.

In the light of the above statements, choose the most appropriate answer from the options given below

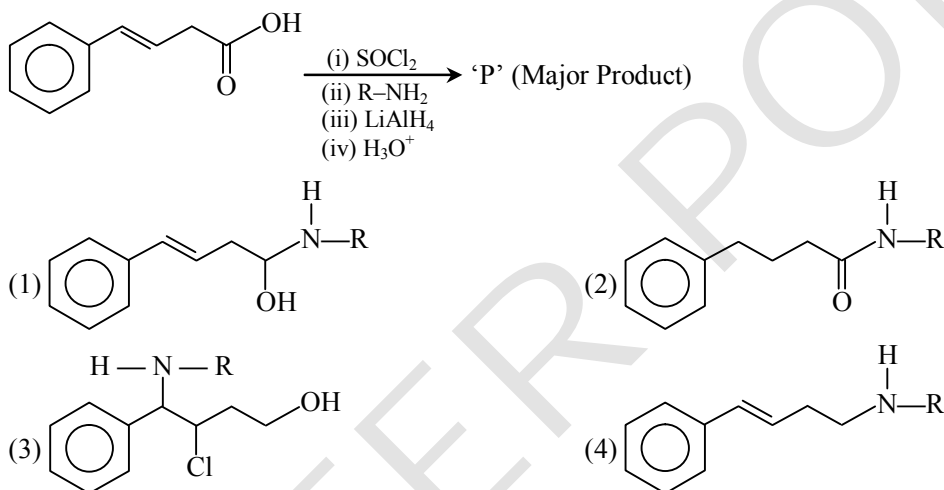
- (1) Both Statement I and Statement II are incorrect
- (2) Statement I is correct but Statement II is incorrect
- (3) Both statement I and Statement II are correct
- (4) Statement I is incorrect but Statement II is correct

**Ans.** [3]

**Sol.** Boron is extremely hard as it exists as  $B_{12}$  icosahedral molecular units which accounts for its high lattice energy. So, statement-I is correct.

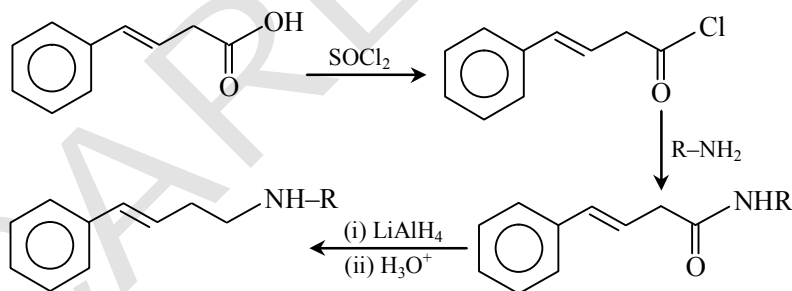
Boron has the highest melting point and boiling point than other members of its group due to its network structure. So, statement-II is also correct

**Q.78** The major product 'P' formed in the following sequence of reactions is :



**Ans.** [4]

**Sol.**



**Q.79** A metal chloride contains 55.0% of chlorine by weight. 100 mL vapours of the metal chloride at STP weigh 0.57 g. The molecular formula of the metal chloride is (Given: Atomic mass of chlorine is 35.5 u)

- (1)  $MCl_4$
- (2)  $MCl_3$
- (3)  $MCl_2$
- (4)  $MCl$

**Ans.** [3]

**Sol.** Let the formula of metal chloride be  $MCl_x$

Mass of 100 mL of vapours of  $MCl_x$  at



$$\text{STP} = 0.57 \text{ g}$$

$$\therefore \frac{100 \times M}{22400} = 0.57$$

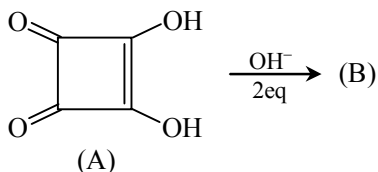
$$\therefore \text{Molar mass of } \text{MCl}_x = 127.68 \text{ g mol}^{-1}$$

$$\% \text{ of Cl} = \frac{35.5 \times 100}{127.68} = 55$$

$$\Rightarrow x = 2$$

$\therefore$  Formula of metal chloride is  $\text{MCl}_2$

**Q.80** Correct statements for the given reaction are:

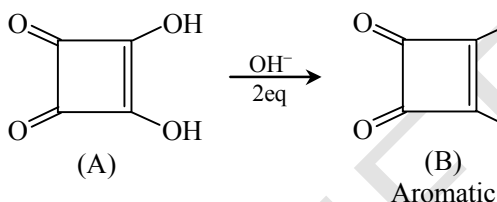


- A. Compound 'B' is aromatic  
 B. The completion of above reaction is very slow  
 C. 'A' shows tautomerism  
 D. The bond lengths of C-C in compound B are found to be same  
 Choose the correct answer from the options given below.

(1) B, C and D only    (2) A, B and C only    (3) A, C and D only    (4) A, B and D only

**Ans.** [3]

**Sol.**



Compound (A), also called squaric acid is a strong acid. So, above reaction proceeds very fast. All C - C bond lengths in (B) are same due to resonance. (A) shows tautomerism.

**Section-B: Numerical Value Type Questions:** This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE**. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, -00.33, -00.30, 30.27, -27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

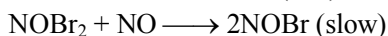
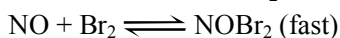
**Q.81** At 600 K, the root mean square (rms) speed of gas X (molar mass = 40) is equal to the most probable speed of gas Y at 90 K. The molar mass of the gas Y is \_\_\_\_\_  $\text{g mol}^{-1}$ . (Nearest integer)

**Ans.** [4]

**Sol.** 
$$\frac{V_{\text{rms}} \text{ of X}}{V_{\text{mp}} \text{ of Y}} = \sqrt{\frac{3T_X M_Y}{2T_Y M_X}}$$

$$1 = \sqrt{\frac{3 \times 600 \times M_Y}{2 \times 90 \times 40}} = \sqrt{\frac{M_Y}{4}}$$

**Q.82** The reaction  $2\text{NO} + \text{Br}_2 \longrightarrow 2\text{NOBr}$  takes place through the mechanism given below



The overall order of the reaction is \_\_\_\_\_.

**Ans. [3]**

**Sol.** On applying LOMA over slowest step (r.d.s.)

$$R = k[\text{NOBr}_2][\text{NO}]$$

On applying LOMA on equilibrium of Step I

$$k = \frac{[\text{NOBr}_2]}{[\text{NO}][\text{Br}_2]}$$

$$[\text{NOBr}_2] = k [\text{NO}][\text{Br}_2]$$

Hence overall rate equation will be

$$R = k [\text{NO}]^2 [\text{Br}_2]$$

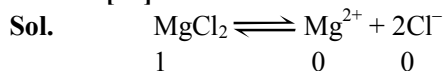
$$= k' [\text{NO}]^2 [\text{Br}_2]$$

Hence, overall order will be 3.

**Q.83** 80 mole percent of  $\text{MgCl}_2$  is dissociated in aqueous solution. The vapour pressure of 1.0 molal aqueous solution of  $\text{MgCl}_2$  at  $38^\circ\text{C}$  is \_\_\_\_\_ mm Hg. (Nearest integer)

Given: Vapour pressure of water at  $38^\circ\text{C}$  is 50 mm Hg

**Ans. [48]**



$$\begin{array}{ccc} 1 & 0 & 0 \\ 1 - 0.8 & 0.8 & 1.6 \end{array}$$

Hence overall molality will be equal to = 2.6

$$\frac{p^\circ - p}{p^\circ} = \frac{2.6}{\frac{1000}{18} + 2.6}$$

For dil solution

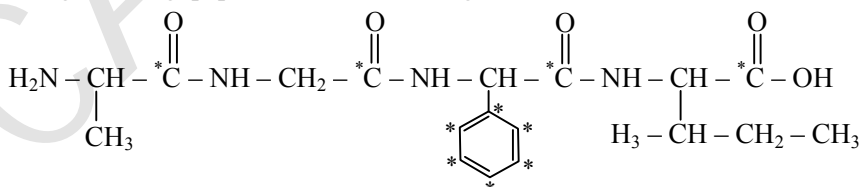
$$\frac{p^\circ - p}{p^\circ} = \frac{2.6}{\frac{1000}{18}}$$

$$p = 47.66 \approx 48 \text{ mm Hg}$$

**Q.84** In an oligopeptide named Alanylglycylphenyl alanyl isoleucine, the number of  $\text{sp}^2$  hybridised carbons is \_\_\_\_\_.

**Ans. [10]**

**Sol.** The given Oligopeptide has the following structure



It has 10  $\text{sp}^2$  hybridised C atoms given with star in the above structure.

- Q.85** One mole of an ideal gas at 350 K is in a 2.0 L vessel of thermally conducting walls, which are in contact with the surroundings. It undergoes isothermal reversible expansion from 2.0 L to 3.0 L against a constant pressure of 4 atm. The change in entropy of the surroundings ( $\Delta S$ ) is \_\_\_\_\_  $\text{J K}^{-1}$  (Nearest integer)  
Given:  $R = 8.314 \text{ J K}^{-1} \text{ mol}^{-1}$ .

**Ans.** [3]

**Sol.** For isothermal process  $Q = -W$

The  $W$  for isothermal reversible process is

$$W = nRT \ln \frac{V_2}{V_1}$$

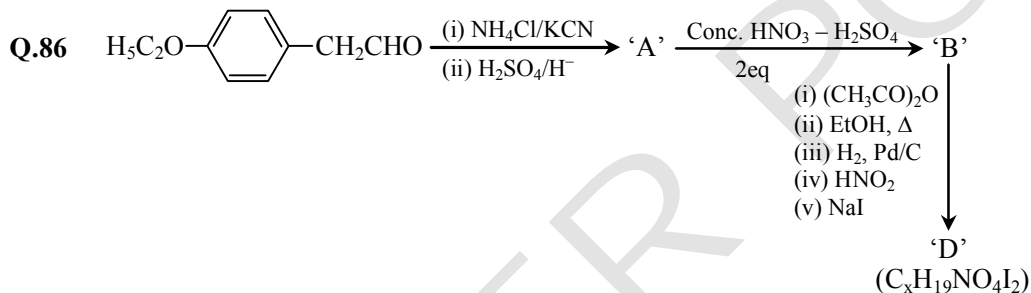
$$\Delta S_{\text{surrounding}} = -nR \ln \frac{V_2}{V_1}$$

$$= -1 \times 8.314 \times 2.303 \times \log \frac{3}{2}$$

$$= -3.37 \text{ JK}^{-1}$$

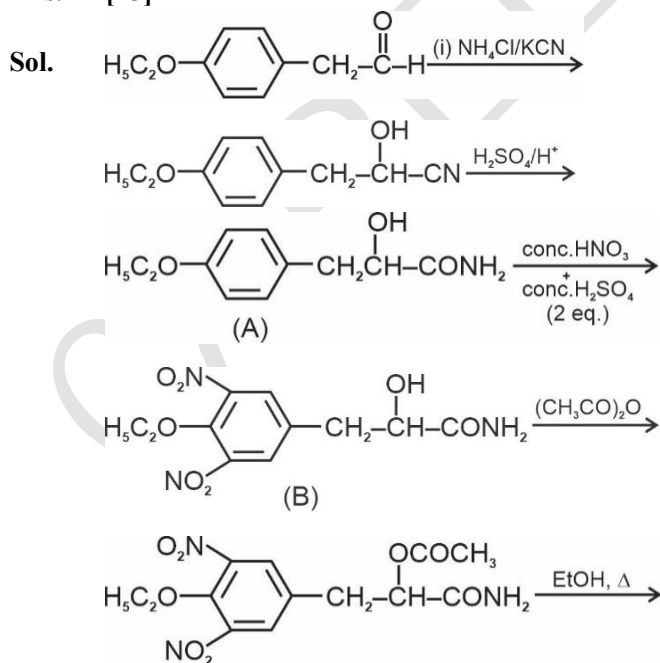
$$= -3 \text{ JK}^{-1}$$

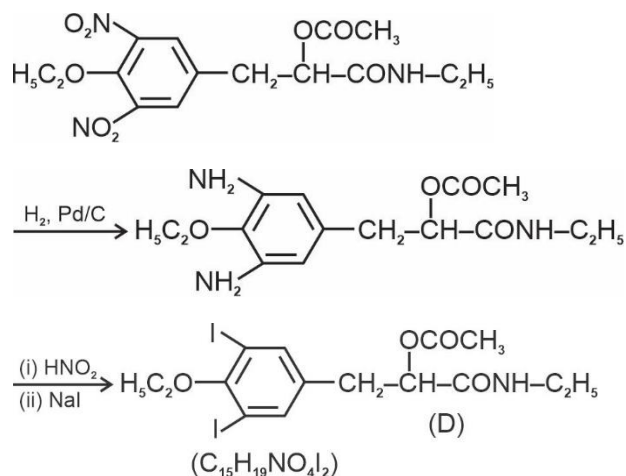
**Note :** The given process is mentioned as isothermal reversible process hence the  $\Delta S_{\text{surrounding}}$  is calculated accordingly.



The value of  $x$  in compound 'D' is \_\_\_\_\_.

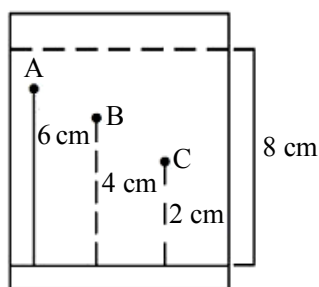
**Ans.** [15]





$$x = 15$$

- Q.87** Three organic compounds A, B and C were allowed to run in thin layer chromatography using hexane and gave the following result (see figure). The R<sub>F</sub> value of the most polar compound is  $\underline{\hspace{2cm}} \times 10^{-2}$



**Ans.** [25]

**Sol.** In thin layer chromatography using hexane, the least polar compound will rise to maximum height and most polar compound will rise to minimum height.

∴ R<sub>F</sub> value for most polar compound

$$= \frac{2}{8} = 0.25$$

$$= 25 \times 10^{-2}$$

- Q.88** Values of work function (W<sub>0</sub>) for a few metals are given below.

Metal	Li	Na	K	Mg	Cu	Ag
W <sub>0</sub> /eV	2.42	2.3	2.25	3.7	4.8	4.3

The number of metals which will show photoelectric effect when light of wavelength 400 nm falls on it is  $\underline{\hspace{2cm}}$ .

Given:  $h = 6.6 \times 10^{-34}$  Js

$c = 3 \times 10^8$  ms<sup>-1</sup>

$e = 1.6 \times 10^{-19}$  C

**Ans.** [3]

**Sol.** Energy of incident photon =  $\frac{hc}{\lambda}$

$$\begin{aligned} &= \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{400 \times 10^{-9}} \text{ J} \\ &= \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{400 \times 10^{-9} \times 1.6 \times 10^{-19}} \text{ eV} \\ &= 3.1 \text{ eV} \end{aligned}$$

∴ Li, Na and K will show photoelectric effect.

**Q.89** The mass of  $\text{NH}_3$  produced when 131.8 kg of cyclohexane carbaldehyde undergoes Tollen's test is \_\_\_\_\_ kg. (Nearest Integer)

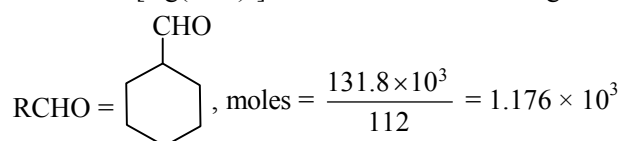
Molar mass of C = 12 g/mol

N = 14 g/mol

O = 16 g/mol

**Ans.** [60]

**Sol.**  $\text{RCHO} + 2[\text{Ag}(\text{NH}_3)_2] \text{OH} \rightarrow \text{RCOONH}_4 + 2\text{Ag} + 3\text{NH}_3 + \text{H}_2\text{O}$



1 mole of aldehyde produces 3 moles of  $\text{NH}_3$

$1.176 \times 10^3$  mol aldehyde will produce

$$= 3 \times 1.176 \times 10^3 \text{ moles of } \text{NH}_3$$

Mass of  $\text{NH}_3$  produced =  $3 \times 1.176 \times 10^3 \times 17$

$$= 59.97 \times 10^3 \text{ g}$$

$$\approx 60 \text{ kg}$$

**Q.90** An analyst wants to convert 1 L HCl of pH = 1 to a solution of HCl of pH 2. The volume of water needed to do this dilution is \_\_\_\_\_ mL. (Nearest integer)

**Ans.** [9000]

**Sol.** The concentration of  $\text{H}^+$  or say HCl in both the given solution is  $10^{-1}$  and  $10^{-2}$  mole  $\text{L}^{-1}$  respectively.

For the given amount of HCl the concentration depends over dilution hence

$$\frac{C_1}{C_2} = \frac{V_2}{V_1}$$

$$\frac{10^{-1}}{10^{-2}} = \frac{V_2}{1\text{L}}$$

$$V_2 = 10 \text{ L} = 10000 \text{ mL}$$

$$\Delta V = 10000 - 1000 \text{ mL} = 9000 \text{ mL}$$