JEE Main Online Exam 2023

Questions & Solution

11th April 2023 | Evening

MATHEMATICS

Section-A: This section contains 20 multiple choice questions. Each question has 4 choices(1), (2), (3) and (4), out of which **ONLY ONE** is correct..

Q.1 The domain of the function $f(x) = \frac{1}{\sqrt{[x]^2 - 3[x] - 10}}$ is (where [x] denotes the greatest integer less or equal to x)

(1) $(-\infty, -3] \cup (5, \infty)$ (2) $(-\infty, -2) \cup [6, \infty)$ (3) $(-\infty, -2) \cup (5, \infty)$ (4) $(-\infty, -3] \cup [6, \infty)$

Ans. **[2]**

Sol. $f(x) = \frac{1}{\sqrt{[x]^2 - 3[x] - 10}}$

For Domain $[x]^2 - 3[x] - 10 > 0$

$\Rightarrow ([x] - 5)([x] + 2) > 0$

$\Rightarrow [x] \in (-\infty, -2) \cup (5, \infty)$

$\therefore x \in (-\infty, -2) \cup [6, \infty)$

Q.2 If the system of linear equations $7x + 11y + \alpha z = 13$; $5x + 4y + 7z = \beta$; $175x + 194y + 57z = 361$ has infinitely many solutions, then $\alpha + \beta + 2$ is equal to

(1) 4 (2) 3 (3) 5 (4) 6

Ans. **[1]**

Sol. $7x + 11y + \alpha z = 13$;

$5x + 4y + 7z = \beta$;

$175x + 194y + 57z = 361$

For infinite solution, $\begin{vmatrix} 7 & 11 & \alpha \\ 5 & 4 & 7 \\ 175 & 194 & 57 \end{vmatrix} = 0$

$\Rightarrow \begin{vmatrix} 7 & 11 & \alpha \\ 5 & 4 & 7 \\ 0 & -81 & 57 - 25\alpha \end{vmatrix} = 0$

$\Rightarrow 81(49 - 5\alpha) + (57 - 25\alpha)(-27) = 0$

$\Rightarrow 270\alpha = -2430 \Rightarrow \alpha = -9$

and $\Delta_1 = 0$

$\begin{vmatrix} 13 & 11 & -9 \\ \beta & 4 & 7 \\ 361 & 194 & 57 \end{vmatrix} = 0$

$\Rightarrow \beta = 11$

$\therefore \alpha + \beta + 2 = 4$

Q.3 Let A = {1, 3, 4, 6, 9} and B = {2, 4, 5, 8, 10}. Let R be a relation defined on A × B such that R = {((a$_1$, b$_1$), (a$_2$, b$_2$)) : a$_1$ ≤ b$_2$ and b$_1$ ≤ a$_2$}. Then the number of elements in the set R is

(1) 160 (2) 52 (3) 26 (4) 180

Ans. **[1]**

Sol. A = {1, 3, 4, 6, 9}; B = {2, 4, 5, 8, 10}

R = {((a$_1$, b$_1$), (a$_2$, b$_2$)) : a$_1$ ≤ b$_2$ and b$_1$ ≤ a$_2$}

For		Similarly for (b$_1$, a$_2$) we will have	
a$_1$ = 1	b$_2$ → 5	b$_1$ → 2	a$_2$ = 4
a$_1$ = 3	b$_2$ → 4	b$_1$ → 4	a$_2$ = 3
a$_1$ = 4	b$_2$ → 4	b$_1$ → 5	a$_2$ = 2
a$_1$ = 6	b$_2$ → 2	b$_1$ → 8	a$_2$ = 1
a$_1$ = 9	b$_2$ → 1	b$_1$ → 10	a$_2$ = 0
	16 cases		**10 cases**

∴ Total elements in relation = 16 × 10 = 160

Q.4 Let the mean of 6 observations 1, 2, 4, 5, x and y be 5 and their variance be 10. Then their mean deviation about the mean is equal to

(1) 7/3 (2) 3 (3) 8/3 (4) 10/3

Ans. **[3]**

Sol. $12 + x + y = 30$

$\Rightarrow x + y = 18$

and $\frac{x^2 + y^2 + 46}{6} - 25 = 10$

∴ x = 10, y = 8

Now, mean deviation about mean

$= \frac{4+3+1+0+5+3}{6} = \frac{8}{3}$

Q.5 If four distinct points with position vectors $\vec{a}$, $\vec{b}$, $\vec{c}$ and $\vec{d}$ are coplanar, then $[\vec{a}\ \vec{b}\ \vec{c}]$ is equal to

(1) $[\vec{d}\ \vec{b}\ \vec{a}]+[\vec{a}\ \vec{c}\ \vec{d}]+[\vec{d}\ \vec{b}\ \vec{c}]$

(2) $[\vec{a}\ \vec{d}\ \vec{b}]+[\vec{d}\ \vec{c}\ \vec{a}]+[\vec{d}\ \vec{b}\ \vec{c}]$

(3) $[\vec{d}\ \vec{c}\ \vec{a}]+[\vec{b}\ \vec{d}\ \vec{a}]+[\vec{c}\ \vec{d}\ \vec{b}]$

(4) $[\vec{b}\ \vec{c}\ \vec{d}]+[\vec{d}\ \vec{a}\ \vec{c}]+[\vec{d}\ \vec{b}\ \vec{a}]$

Ans. **[3]**

Sol. $[\vec{b}-\vec{a}\ \ \vec{c}-\vec{a}\ \ \vec{d}-\vec{a}] = 0$

$(\vec{b}-\vec{a})\cdot[(\vec{c}-\vec{a})\times(\vec{d}-\vec{a})] = 0$

$(\vec{b}-\vec{a})\cdot(\vec{c}\times\vec{d}-\vec{c}\times\vec{a}-\vec{a}\times\vec{d}) = 0$

$[\vec{b}\ \vec{c}\ \vec{d}]-[\vec{b}\ \vec{c}\ \vec{a}]-[\vec{b}\ \vec{a}\ \vec{d}]-[\vec{a}\ \vec{c}\ \vec{d}] = 0$

$\therefore\ [\vec{a}\ \vec{b}\ \vec{c}] = [\vec{b}\ \vec{c}\ \vec{d}]+[\vec{a}\ \vec{b}\ \vec{d}]+[\vec{a}\ \vec{d}\ \vec{c}]$

$= [\vec{d}\ \vec{c}\ \vec{a}]+[\vec{b}\ \vec{d}\ \vec{a}]+[\vec{c}\ \vec{d}\ \vec{b}]$

Q.6 Let y = y(x) be the solution of the differential equation $\frac{dy}{dx}+\frac{5}{x(x^5+1)}y=\frac{(x^5+1)^2}{x^7}$, x > 0. If y(1) = 2, then y(2) is equal to

(1) $\frac{637}{128}$ (2) $\frac{679}{128}$ (3) $\frac{693}{128}$ (4) $\frac{697}{128}$

Ans. **[3]**

Sol. I. F. $= e^{\int\frac{5}{x(1+x^5)}dx} = e^{\int\frac{5}{x^6\left(\frac{1}{x^5}+1\right)}dx}$

Put $\frac{1}{x^5}+1=t$

$\frac{-5}{x^6}dx=dt$

$e^{\int-\frac{dt}{t}}=e^{-\ln t}=\frac{1}{t}=\frac{x^5}{1+x^5}$

$y\left(\frac{x^5}{1+x^5}\right)=\int\frac{x^5}{1+x^5}\frac{(1+x^5)^2}{x^7}dx+c$

$y\left(\frac{x^5}{1+x^5}\right)=\frac{-1}{x}+\frac{x^4}{4}+c$

Now, y(1) = 2, then $c=\frac{7}{4}$

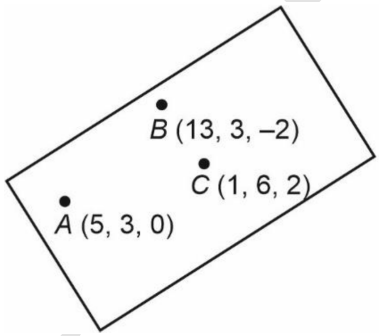
$\therefore\ y(2)=\frac{693}{128}$

Q.7 Let P be the plane passing through the points (5, 3, 0), (13, 3, –2) and (1, 6, 2). For $\alpha \in N$, if the distances of the points A(3, 4, α) and B(2, α, a) from the plane P are 2 and 3 respectively, then the positive value of a is

(1) 6 (2) 3 (3) 5 (4) 4

Ans. **[4]**

Sol.



$\overrightarrow{AB}=8\hat{i}-2\hat{k}$

$\overrightarrow{AC}=-4\hat{i}+3\hat{j}+2\hat{k}$

$\overrightarrow{AB}\times\overrightarrow{AC}=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\8&0&-2\\-4&3&2\end{vmatrix}$

$= 6\hat{i} - 8\hat{j} + 24\hat{k}$

Equation of plane : $6x - 8y + 24z = d$ passes through $(5, 3, 0)$

$6 \times 5 - 8 \times 3 + 24 \times 0 = d$

$d = 6$

$6x - 8y + 24z = 6 \Rightarrow 3x - 4y + 12z = 3$

Distance of point $(3, 4, \alpha)$

$$\frac{9 - 16 + 12\alpha - 3}{\sqrt{9 + 16 + 144}} = 2 \Rightarrow \alpha = 3$$

Distance of point $(2, \alpha, a)$

$$\frac{3 \times 2 - 4 \times 3 + 12 \times a - 3}{13} = 3$$

$12a - 9 = 39$

$12a = 48$

$a = 4$

Q.8 The converse of $((\sim p) \wedge q) \Rightarrow r$ is

(1) $((\sim p) \vee q) \Rightarrow r$ (2) $(\sim r) \Rightarrow p \wedge q$

(3) $(\sim r) \Rightarrow ((\sim p) \wedge q)$ (4) $(p \vee (\sim q)) \Rightarrow (\sim r)$

Ans. **[4]**

Sol. Converse of $(\sim p \wedge q) \Rightarrow r$ is

$r \Rightarrow (\sim p \wedge q)$

$(p \vee (\sim q)) \Rightarrow (\sim r)$

Q.9 Let the function $f : [0, 2] \to R$ be defined as $f(x) = \begin{cases} e^{\min\{x^2, x-[x]\}}, & x \in [0, 1) \\ e^{[x - \log_e x]}, & x \in [1, 2] \end{cases}$

Where $[t]$ denotes the greatest integer less than or equal to t. Then the value of the integral $\int_0^2 xf(x)\,dx$ is

(1) $1 + \frac{3e}{2}$ (2) $(e - 1)\left(e^2 + \frac{1}{2}\right)$ (3) $2e - 1$ (4) $2e - \frac{1}{2}$

Ans. **[4]**

Sol. $f(x) = \begin{cases} e^{\min\{x^2, x\}} & x \in [0, 1) \\ e & x \in [1, 2] \end{cases} = \begin{cases} e^{x^2} & x \in [0, 1) \\ e & x \in [1, 2] \end{cases}$

$\because x - \ln x \in [1, 2)$ for $x \in [1, 2]$

$\therefore [x - \ln x] = 1$

$$\int_0^2 xf(x) = \int_0^1 x.e^{x^2}dx + \int_1^2 x.e\,dx$$

$x^2 = t \Rightarrow 2xdx = dt$

$$= \frac{1}{2}\int_0^1 e^t dt + e\frac{x^2}{2}\Big|_1^2$$

$\Rightarrow \left(\frac{e-1}{2}\right) + \frac{3e}{2} \Rightarrow 2e - \frac{1}{2}$

Option (4) is correct

Q.10 If $f : R \to R$ be a continuous function satisfying $\int_0^{\pi/2} f(\sin 2x)\sin x\, dx + \alpha \int_0^{\pi/4} f(\cos 2x)\cos x\, dx = 0$, then the value of α is

(1) $\sqrt{2}$ (2) $-\sqrt{3}$ (3) $\sqrt{3}$ (4) $-\sqrt{2}$

Ans. **[4]**

Sol. $\int_0^{\pi/2} f(\sin 2x)\sin x\, dx + \alpha \int_0^{\pi/4} f(\cos 2x)\cos x\, dx$

let $I = \int_0^{\pi/2} f(\sin 2x).\sin x\, dx$

$= \int_0^{\pi/4} f(\sin 2x)\sin x\, dx + \int_{\pi/4}^{\pi/2} f(\cos 2x)\cos x\, dx$

$= \int_0^{\pi/4} f(\cos 2x).\sin\left(\frac{\pi}{4} - x\right) dx + \int_0^{\pi/4} f\left(\sin 2\left(\frac{\pi}{4} + x\right)\right) \sin\left(\frac{\pi}{4} + x\right) dx$

$= \int_0^{\pi/4} f(\cos 2x)\left(\frac{1}{\sqrt{2}}\cos x - \frac{1}{\sqrt{2}}\sin x\right) dx + \int_0^{\pi/4} f(\cos 2x)\left(\frac{1}{\sqrt{2}}\cos x + \frac{1}{\sqrt{2}}\sin x\right) dx$

$= \int_0^{\pi/4} f(\cos 2x)(\sqrt{2}\cos x)\, dx$

$\therefore \alpha = -\sqrt{2}$

Option (4) is correct

Q.11 If the radius of the largest circle with centre (2, 0) inscribed in the ellipse $x^2 + 4y^2 = 36$ is r, then $12r^2$ is equal to

(1) 115 (2) 92 (3) 69 (4) 72

Ans. **[2]**

Sol. Equation of normal at P(6 cos θ, 3 sinθ) is

6 secθ x – 3 cosecθ y = 27

If passes through (2, 0)

$\therefore$ 12 secθ = 27

$\therefore \cos\theta = \frac{4}{9}, \sin\theta = \frac{\sqrt{65}}{9}$

$P\left(\frac{8}{3}, \frac{\sqrt{65}}{3}\right)$

r = OP (O = (2, 0))

$= \sqrt{\left(\frac{8}{3} - 2\right)^2 + \left(\frac{\sqrt{65}}{3}\right)^2} = \frac{\sqrt{69}}{3}$

$\therefore\ 12r^2 = 12 \times \frac{69}{9} = 92$

$\therefore$ Option (2) is correct

Q.12 Let the line passing through the points P(2, –1, 2) and Q(5, 3, 4) meet the plane x – y + z = 4 at the point R. Then the distance of the point R from the plane x + 2y + 3z + 2 = 0 measured parallel to the line $\frac{x-7}{2} = \frac{y+3}{2} = \frac{z-2}{1}$ is

(1) $\sqrt{61}$ (2) $\sqrt{189}$ (3) $\sqrt{31}$ (4) 3

Ans. **[4]**

Sol. Equation of line PQ.

$\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{2} = \lambda$

Let R be $(3\lambda + 2, 4\lambda - 1, 2\lambda + 2)$

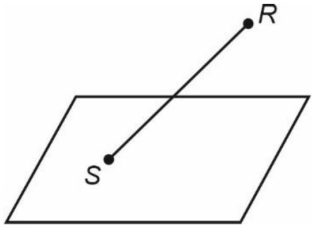
R lies on plane x – y + z = 4

$\therefore\ 3\lambda + 2 - 4\lambda + 1 + 2\lambda + 2 = 4$

$\Rightarrow \lambda = -1$

$\therefore$ R(–1, –5, 0)

Let SR be : $\frac{x+1}{2} = \frac{y+5}{2} = \frac{z}{1} = k$



S : (2k – 1, 2k – 5, k)

S lies on plane : x + 2y + 3z + 2 = 0

$\Rightarrow$ (2k – 1) + (4k – 10) + 3k + 2 = 0

$\Rightarrow$ 9k – 9 = 0 $\Rightarrow$ k = 1

S(1, –3, 1) $\therefore$ SR = $\sqrt{4+4+1} = 3$

$\therefore$ Option (4) is correct

Q.13 If $\begin{vmatrix} x+1 & x & x \\ x & x+\lambda & x \\ x & x & x+\lambda^2 \end{vmatrix} = \frac{9}{8}(103x + 81)$, then λ, $\frac{\lambda}{3}$ are the roots of the equation

(1) $4x^2 + 24x - 27 = 0$ (2) $4x^2 - 24x - 27 = 0$

(3) $4x^2 + 24x + 27 = 0$ (4) $4x^2 - 24x + 27 = 0$

Ans. **[4]**

Sol. $\begin{vmatrix} x+1 & x & x \\ x & x+\lambda & x \\ x & x & x+\lambda^2 \end{vmatrix} = \frac{9}{8}(103x + 81)$

Put x = 0

$$\begin{vmatrix}1 & 0 & 0\\ 0 & \lambda & 0\\ 0 & 0 & \lambda^2\end{vmatrix} = \frac{9}{8} \times 81$$

$$\lambda^3 = \frac{3^6}{2^3}$$

$$\lambda = \frac{9}{2}$$

$$\frac{\lambda}{3} = \frac{3}{2}$$

Equation : $x^2 - \left(\frac{9}{2}+\frac{3}{2}\right)x + \frac{9}{2} \times \frac{3}{2} = 0$

$4x^2 - 24x + 27 = 0$

Q.14 If the letters of the word MATHS are permuted and all possible words so formed are arranged as in a dictionary with serial numbers, then the serial number of the word THAMS is

(1) 103 (2) 102 (3) 101 (4) 104

Ans. **[1]**

Sol.

5	2	1	3	4
T	H	A	M	S
4	1	0	0	0
4!	3!	2!	1!	0!

Rank = $4 + 4! + 1 \times 3! + 1$

$= 96 + 6 + 1$

$= 103$

Q.15 Let f and g be two functions defined by

$f(x) = \begin{cases} x+1, & x<0 \\ |x-1|, & x \geq 0 \end{cases}$ and $g(x) = \begin{cases} x+1, & x<0 \\ 1. & x \geq 0 \end{cases}$. Then (gof)(x) is

(1) Continuous everywhere but not differentiable exactly at one point

(2) Continuous everywhere but not differentiable at x = 1

(3) Differentiable everywhere

(4) Not continuous at x = –1

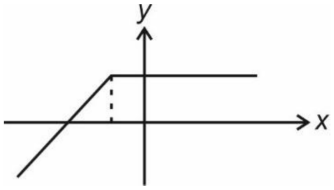
Ans. **[1]**

Sol.

$f(x) = \begin{cases} x+1, & x<0 \\ 1-x, & 0 \leq x<1 \\ x-1, & x>1 \end{cases}$, $g(x) = \begin{cases} 1+x, & x<0 \\ 1, & x \geq 0 \end{cases}$

For gof(x)

$gof\, x = \begin{cases} x+2, & x<-1 \\ 1, & x \geq -1 \end{cases}$



$\therefore$ Continuous but not differentiable at x = –1

Q.16 Let a, b, c and d be positive real numbers such that a + b + c + d = 11. If the maximum value of $a^5 b^3c^2d$ is 3750β, then the value of β is

(1) 90 (2) 110 (3) 55 (4) 108

Ans. **[1]**

Sol. $a + b + c + d = 1$

$a^5b^3c^2d\big|_{min} = 3750\beta$

Assume numbers be $\frac{a}{5},\frac{a}{5},\frac{a}{5},\frac{a}{5},\frac{a}{5},\frac{b}{3},\frac{b}{3},\frac{b}{3},\frac{c}{2},\frac{c}{2},d$

Apply AM ≥ GM

$$\frac{\frac{a}{5}+\frac{a}{5}+\frac{a}{5}+\frac{a}{5}+\frac{a}{5}+\frac{b}{3}+\frac{b}{3}+\frac{b}{3}+\frac{c}{2}+\frac{c}{2}+d}{11} \geq \left(\frac{(a^5b^3c^2d}{5^5 3^3 2^2 1}\right)^{1/11}$$

$\therefore\ a^5 b^3 c^2 d \leq 90 \times 3750$

$\therefore\ \beta = 90$

Q.17 For $a \in C$, let $A = \{z \in C : Re(a + \bar{z}) > Im(\bar{a} + z)\}$ and $B = \{z \in C : Re(a + \bar{z}) < Im(\bar{a} + z)\}$. Then among the two statements :

(S1) : If Re(a), Im(a) > 0, then the set A contains all the real numbers

(S2) : If Re(a), Im(a) < 0, then the set B contains all the real numbers

(1) Only (S2) is true (2) Only (S1) is true (3) Both are true (4) Both are false

Ans. **[4]**

Sol. Let $a = x_1 + iy_1$, $\bar{a} = x_1 - iy_1$

$z = x_2 + iy_2$, $\bar{z} = x_2 - iy_2$

$A = \{z : x_1 + x_2 > y_2 - y_1\} = \{z : x_1 + y_1 + x_2 > y_2\}$

$B = \{z : x_1 + x_2 < y_2 - y_1\} = \{z : x_1 + y_1 + x_2 < y_2\}$

If $y_2 = 0$ and $x_1, y_1 > 0$, then

$A = \{z : x_2 > -(x_1 + y_1)\}$

A covers a part of negative real axis and therefore, does not contain whole real axis

Similarly if $y_2 = 0$, and $x_1, y_1 < 0$, then

$B = \{z : x_2 < -(x_1 + y_1)\}$

$\therefore$ B covers part of positive real axis and therefore does not cover whole real axis.

Q.18 If the 1011^{th} term from the end in the binomial expansion of $\left(\frac{4x}{5} - \frac{5}{2x}\right)^{2022}$ is 1024 times 1011^{th} term from the beginning, then |x| is equal to

(1) 15 (2) 10 (3) 12 (4) 8

Ans. **[*]**

Sol. 1011^{th} term from end = a_{1013} = 1013^{th} term from beginning

$$\frac{a_{1013}}{a_{1011}} = 1024$$

$$\frac{{}^{2022}C_{1012}\left(\frac{4x}{5}\right)^{1010}\cdot\left(\frac{-5}{2x}\right)^{1012}}{{}^{2022}C_{1010}\left(\frac{4x}{5}\right)^{1012}\left(\frac{-5}{2x}\right)^{1010}} = 1024$$

$$\frac{\left(\frac{-5}{2x}\right)^2}{\left(\frac{4x}{5}\right)^2} = 1024$$

$$\frac{5^4}{2^6 x^4} = 2^{10}$$

$$\left(\frac{5}{x}\right)^4 = 2^{16}$$

$$|x| = \frac{5}{16}$$

Note : If the binomial expansion of $(x + a)^n$ is written in increasing power of x (and decreasing power of a), then answer matches to one of the option.

Q.19 The sum of the coefficients of three consecutive terms in the binomial expansion of $(1 + x)^{n+2}$, which are in the ratio 1 : 3 : 5, is equal to

(1) 92 (2) 63 (3) 41 (4) 25

Ans. **[2]**

Sol. Let the three terms be T_r, T_{r+1}, T_{r+2}

$$\frac{T_{r+1}}{T_r} = \frac{{}^{n+2}C_r}{{}^{n+2}C_{r-1}} = \frac{n+2-r+1}{r} = \frac{n+3-r}{r} = 3$$

$n - 4r + 3 = 0$ ...(1)

$$\frac{T_{r+2}}{T_{r+1}} = \frac{{}^{n+2}C_{r+1}}{{}^{n+2}C_r} = \frac{(n+2)-(r+1)+1}{r+1} = \frac{n-r+2}{r+1} = \frac{5}{3}$$

$3n - 8r + 1 = 0$ ...(2)

(1) and (2) $\Rightarrow n = 5, r = 2$

$T_r + T_{r+1} + T_{r+2} = {}^7C_1 + {}^7C_2 + {}^7C_3$

$= 7 + 21 + 35$

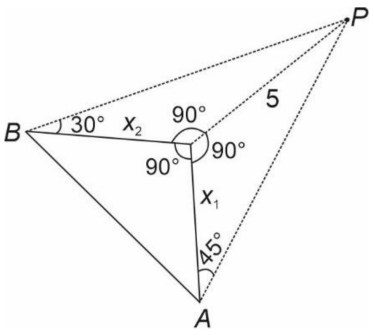
$= 63$

Q.20 The angle of elevation of the top P of a tower from the feet of one person standing due south of the tower is 45º and from the feet of another person standing due west of the tower is 30º. If the height of the tower is 5 meters, then the distance (in meters) between the two persons is equal to

(1) $\frac{5}{2}\sqrt{5}$ (2) 10 (3) 5 (4) $5\sqrt{5}$

Ans. **[2]**

Sol.



$\tan 45^\circ = \frac{5}{x_1}$, $\tan 30^\circ = \frac{5}{x_2}$

$x_1 = 5$, $x_2 = 5\sqrt{3}$, AB = 10

Section-B: Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer..

Q.21 The number of points, where the curve $f(x) = e^{8x} - e^{6x} - 3e^{4x} - e^{2x} + 1$, $x \in R$ cuts x-axis, is equal to

Ans. **[2]**

Sol. $f(x) = e^{8x} - e^{6x} - 3e^{4x} - e^{2x} + 1$, $x \in R$

$\lim\limits_{x\to-\infty} f(x) = 1$, $f(0) = -3$, $\lim\limits_{x\to\infty} f(x) \to \infty$

One root of $f(x) = 0$ is negative and other root of $f(x) = 0$ is positive

Curve $y = f(x)$ cuts x-axis at two points

Q.22 Let $S = \left\{ z \in C - \{i, 2i\} : \frac{z^2 + 8iz - 15}{z^2 - 3iz - 2} \in R \right\}$. If $\alpha - \frac{13}{11} i \in S$, $\alpha \in R - \{0\}$, then $242\alpha^2$ is equal to

Ans. **[1680]**

Sol. Put $z = x + iy$

$$\text{Im}\left(\frac{z^2 + 8iz - 15}{z^2 - 3iz - 2}\right) = 0$$

$\Rightarrow -(x^2 - y^2 - 8y - 15)(2xy - 3x) + (2xy + 8x)(x^2 - y^2 + 3y - 2) = 0$

$\Rightarrow (x^2 - y^2)(2xy + 8x - 2xy + 3x) + (8y + 15)(2xy - 3x) + (2xy + 8x)(3y - 2) = 0$

$\Rightarrow 11x^3 - 11xy^2 + 16xy^2 - 24xy + 30xy - 45x + 6xy^2 - 4xy + 24xy - 16x = 0$

$\Rightarrow 11x^3 + 11xy^2 + 26xy - 61x = 0$

$\Rightarrow (11x^2 + 11y^2 + 26y - 61 = 0$

$\because \alpha \neq 0, \Rightarrow x = 0$ (neglected)

Put $y = -\frac{13}{11}$, $x = \alpha$

$$11\alpha^2 + 11.\frac{13^2}{11^2} - 26.\frac{13}{11} - 61 = 0$$

$\Rightarrow 121\alpha^2 = 840$

$\Rightarrow 242\alpha^2 = 1680$

Q.23 Let the probability of getting head for a biased coin be 1/4 . It is tossed repeatedly until a head appears. Let N be the number of tosses required. If the probability that the equation $64x^2 + 5Nx + 1 = 0$ has no real root is p/q, where p and q are co-prime, then q – p is equal to .....

Ans. **[27]**

Sol. $64x^2 + 5Nx + 1 = 0$ has no real root

$\Rightarrow 25N^2 - 4 \times 64 < 0$

$\Rightarrow N < \frac{16}{5}$

$\Rightarrow N = 1, 2, 3$

$\Rightarrow \frac{1}{4}+\frac{3}{4}\times\frac{1}{4}+\frac{3}{4}.\frac{3}{4}.\frac{1}{4} = \frac{p}{q}$

$\Rightarrow \frac{p}{q} = \frac{37}{64}$

$\Rightarrow q - p = 64 - 37 = 27$

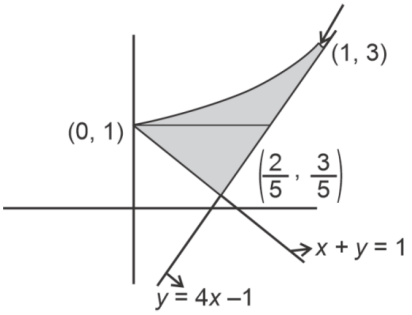
Q.24 If A is the area in the first quadrant enclosed by the curve C : $2x^2 - y + 1 = 0$, the tangent to C at the point (1, 3) and the line $x + y = 1$, then the value of 60A is .....

Ans. **[16.00]**

Sol. $y = 2x^2 + 1$

Tangent : $y - 3 = 4(x - 1)$

$y = 4x - 1$



Required area $= \frac{1}{2}\times\frac{1}{2}\times\frac{2}{5}+\int_1^3\left(\frac{y+1}{4}-\left(\frac{y-1}{2}\right)^{1/2}\right)dy$

$= \frac{4}{15}$

$= 60A = 16$

Q.25 Let the tangent to the parabola $y^2 = 12x$ at the point $(3, \alpha)$ be perpendicular to the line $2x + 2y = 3$. Then the square of distance of the point (6, –4) from the normal to the hyperbola $\alpha^2x^2 - 9y^2 = 9\alpha^2$ at its point $(\alpha - 1, \alpha + 2)$ is equal to .....

Ans. **[116.00]**

Sol. Slope of tangent $= 1 = \frac{6}{\alpha} \Rightarrow \alpha = 6$

$36x^2 - 9y^2 = 324$

$\frac{x^2}{9} - \frac{y^2}{36} = 1$

Tangent at (5, 8)

$\frac{5x}{9} - \frac{8y}{36} = 1$

$5x - 2y = 9$

Slope of normal $= \frac{-2}{5}$

Equation of normal

$y - 8 = \frac{-2}{5}(x - 5)$

$5y - 40 = -2x + 10$

$5y + 2x = 50$

Distance from $(6, -4)$

$= \left|\frac{12 - 20 - 50}{\sqrt{29}}\right| = \frac{58}{\sqrt{29}}$

$= 2\sqrt{29}$

Q.26 Let $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$ and $\vec{b} = \hat{i} + \hat{j} - \hat{k}$. If $\vec{c}$ is a vector such that $\vec{a}\cdot\vec{c} = 11$, $\vec{b}.(\vec{a}\times\vec{c}) = 27$ and $\vec{b}\cdot\vec{c} = -\sqrt{3}\,|\vec{b}|$, then $|\vec{a}\times\vec{c}|^2$ is equal to .....a

Ans. **[285]**

Sol. $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$

$\vec{b} = \hat{i} + \hat{j} - \hat{k}$

$\vec{a}\cdot\vec{c} = 11$

$\vec{b}\cdot(\vec{a}\times\vec{c}) = 27$

$\vec{b}\cdot\vec{c} = -\sqrt{3}\,|\vec{b}|$

$(\vec{b}\times\vec{a})\cdot\vec{c} = 27$

Let $\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$

$c_1 + 2c_2 + 3c_3 = 11$ ...(i)

$c_1 + c_2 - c_3 = -\sqrt{3}\sqrt{3}$

$c_1 + c_2 - c_3 = -3$ ...(ii)

$5c_1 - 4c_2 + c_3 = 27$ ...(iii)

From (i), (ii) & (iii)

$\vec{c} = 3\hat{i} - 2\hat{j} + 4\hat{k}$

$$|\vec{a}\times\vec{c}|^2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 3 & -2 & 4 \end{vmatrix}^2$$

$= |14\hat{i} + 5\hat{j} - 8\hat{k}|^2$

$= 14^2 + 5^2 + 8^2$

$= 285$

Q.27 Let A = {1, 2, 3, 4, 5} and B = {1, 2, 3, 4, 5, 6}. Then the number of functions f : A → B satisfying f(1) + f(2) = f(4) – 1 is equal to

Ans. **[360.00]**

Sol. $f(1) + f(2) + 1 = f(4)$

$\Rightarrow f(1) + f(2) \le 5$

Let $f(4) = 6$, then $f(1) + f(2) = 5$

So (f_1, f_2) may be (1, 4), (4, 1), (2, 3), (3, 2)

Total ways = 4 . 1 . 6 . 6 = 144

Let f(4) = 5, f(1) + f(2) = 4
So (f_1, f_2) may be (1, 3), (3, 1), (2, 2)
Total ways = 3.1.6.6 = 108
Let f(4) = 4, f(1) + f(2) = 3
So (f_1, f_2) may be (1, 2), (2, 1)
Total ways = 2.1.6.6 = 72
Let f(4) = 3, f(1) + f(2) = 2
Only way (1, 1)
Total ways = 1.1.36 = 36

Q.28 For $k \in N$, if the sum of the series $1+\frac{4}{k}+\frac{8}{k^2}+\frac{13}{k^3}+\frac{19}{k^4}+....$ is 10, then the value of k is .....

Ans. **[2]**

Sol. $S = 1+\frac{4}{k}+\frac{8}{k^2}+\frac{13}{k^3}+\frac{19}{k^4}+....$ ...(i)

$\frac{1}{k}S = \frac{1}{k}+\frac{4}{k^2}+\frac{8}{k^3}+....$ ...(ii)

(i) – (ii)

$\Rightarrow \left(1-\frac{1}{k}\right)S = 1+\frac{3}{k}+\frac{4}{k^2}+\frac{5}{k^3}+....$ ...(iii)

Let $S' = \frac{3}{k}+\frac{4}{k^2}+\frac{5}{k^3}+....\infty$ ...(iv)

$\frac{S'}{k} = \frac{3}{k}+\frac{4}{k^2}+....\infty$ ...(v)

(iv) – (v)

$\Rightarrow \left(1-\frac{1}{k}\right)S' = \frac{3}{k}+\frac{1}{k^2}+\frac{1}{k^3}+\frac{1}{k^4}+....\infty$

$= \frac{3}{k}+\frac{\frac{1}{k^2}}{1-\frac{1}{k}}$

$\Rightarrow S' = \frac{3}{k-1}+\frac{1}{(k-1)^2}$ ...(vi)

From (iii) and (vi)

$\frac{k-1}{k}S = 1+\frac{3}{k-1}+\frac{1}{(k-1)^2}$

$S = 10 \Rightarrow 9k^3 - 31k^2 + 31k - 10 = 0$

$\Rightarrow k = 2$

Q.29 Let the line ℓ : $x = \frac{1-y}{-2} = \frac{z-3}{\lambda}$, $\lambda \in R$ meet the plane P : x + 2y + 3z = 4 at the point (α, β, γ). if the angle between the line ℓ and the plane P is $\cos^{-1}\left(\sqrt{\frac{5}{14}}\right)$, then $\alpha + 2\beta + 6\gamma$ is equal to .....

Ans. **[11]**

Sol. $L: \frac{x-0}{1}=\frac{y-1}{2}=\frac{z-3}{\lambda}$ & $P: x+2y+3z=4$

Vector parallel to line : $<1, 2, \lambda> = \bar{b}$

Normal vector to plane P : $<1, 2, 3> = \bar{n}$

Angle between plane & line is θ

Then, $\sin\theta = \frac{<1, 2, \lambda>\cdot<1, 2, 3>}{\sqrt{1^2+2^2+\lambda^2}\cdot\sqrt{1^2+2^2+3^2}}$

$\Rightarrow \frac{3}{\sqrt{14}} = \frac{1+4+3\lambda}{\sqrt{\lambda^2+5}\sqrt{14}} \Rightarrow \lambda = \frac{2}{3}$

$L_1 \equiv \frac{x-0}{3}=\frac{y-1}{6}=\frac{z-3}{2}=\mu$

Any point on line : $(3\mu, 6\mu + 1, 2\mu + 3)$

It lies on P $\Rightarrow 3\mu + 12\mu + 2 + 6\mu + 9 = 4$

$\Rightarrow \mu = \frac{-1}{3}$

Hence, $\alpha = 3\mu = -1$, $\beta = 6\mu + 1 = -1$, $\gamma = 2\mu + 3 = \frac{7}{3}$

Now, $\alpha + 2\beta + 6\gamma = 11$

Q.30 If the line $\ell_1 = 3y - 2x = 3$ is the angular bisector of the lines $\ell_2 : x - y + 1 = 0$ and $\ell_3 : \alpha x + \beta y + 17 = 0$, then $\alpha^2 + \beta^2 - \alpha - \beta$ is equal to .....

Ans. **[348]**

Sol. $L_1 : 3y - 2x = 3$

$L_2 : x - y + 1 = 0$

$L_3 : \alpha x + \beta y + 17 = 0$

Point of intersection of L_1 & L_2 is (0, 1), should lie on $L_3 \Rightarrow \beta = -17$

Any point, say $\left(\frac{-3}{2}, 0\right)$ on L_1 should be equidistant from the lines L_2 & L_3

$\Rightarrow \left|\frac{\frac{-3}{2}-0+1}{\sqrt{1^2+1^2}}\right| = \left|\frac{\frac{-3\alpha}{2}+0+17}{\sqrt{\alpha^2+(-17)^2}}\right|$

$\Rightarrow (\alpha - 7)(\alpha - 17) = 0$

For $\alpha = 17$, L_2 & L_3 coincides $\Rightarrow \alpha = 7$

$\alpha^2 + \beta^2 - \alpha - \beta = (-17)^2 + 7^2 - 7 + 17$

$= 348$

PHYSICS

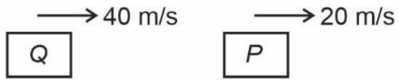
Section-A: Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Q.31 A car P travelling at 20 ms^{-1} sounds its horn at a frequency of 400 Hz. Another car Q is travelling behind the first car in the same direction with a velocity 40 ms^{-1}. The frequency heard by the passenger of the car Q is approximately [Take, velocity of sound = 360 ms^{-1}]

(1) 421 Hz (2) 471 Hz (3) 485 Hz (4) 514 Hz

Ans. **[2]**

Sol.



$$f = f_0\left(\frac{C+40}{C-20}\right)$$

$$= 400\left(\frac{360+40}{340}\right)$$

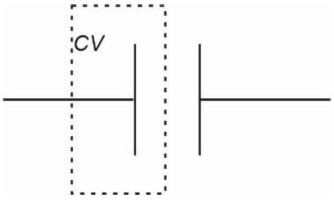
$$\approx 471 \text{ Hz}$$

Q.32 A capacitor of capacitance C is charged to a potential V. The flux of the electric field through a closed surface enclosing the positive plate of the capacitor is :

(1) $\frac{CV}{\varepsilon_0}$ (2) Zero (3) $\frac{2CV}{\varepsilon_0}$ (4) $\frac{CV}{2\varepsilon_0}$

Ans. **[1]**

Sol.



$$\phi = \frac{q_{inc}}{\varepsilon_0} = \left(\frac{CV}{\varepsilon_0}\right)$$

Q.33 Given below are two statements : one is labelled as Assertion A and the other is labelled as Resaon R

Assertion A : A bar magnet dropped through a metallic cylindrical pipe takes more time to come down compared to a non-magnetic bar with same geometry and mass.

Reason R : For the magnetic bar, Eddy currenrts are produced in the metallic pipe which oppose the motion of the magnetic bar.

In the light of the above statements, choose the correct answer from the options given below.

(1) A is false but R is true

(2) Both A and R are true and R is the correct explanation of A

(3) A is true but R is false

(4) Both A and R are true but R is NOT the correct explanation of A

Ans. **[2]**

Sol. Due to change in flux, eddy current is produced which opposes the motion.

Q.34 When one light ray is reflected from a plane mirror with 30º angle of reflection, the angle of deviation of the ray after reflection is :

(1) 120º (2) 110º (3) 140º (4) 130º

Ans. **[1]**

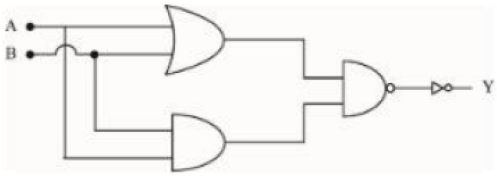
Sol. As i = 30º

$\delta = 180º - 2i$

$= 180º - 60º$

$= 120º$

Q.35 The logic operations performed by the given digital circuit is equivalent to :



(1) NOR (2) AND (3) OR (4) NAND

Ans. **[2]**

Sol. (A + B) . (A.B) = A.(AB) + B.(AB)

= AB + AB

= (AB)

Q.36 If V is the gravitational potential due to sphere of uniform density on it's surface, then it's value at the centre of sphere will be :

(1) $\frac{4}{3}V$ (2) $\frac{V}{2}$ (3) V (4) $\frac{3V}{2}$

Ans. **[4]**

Sol. $V = \frac{GM}{R}$

$V_{centre} = -\frac{3}{2}\left(\frac{GM}{R}\right) = \left(\frac{3}{2}V\right)$

Q.37 In satellite communication, the uplink frequency band used is :

(1) 420 – 890 MHz (2) 5.925 – 6.425 GHz (3) 76 – 88 MHz (4) 3.7 – 4.2 GHz

Ans. **[2]**

Sol. Uplink frequency band for satellite communication is 5.925 – 6.425 GHz.

Q.38 When vector $\vec{A} = 2\hat{i} + 3\hat{j} + 2\hat{k}$ is subtracted from vector $\vec{B}$, it gives a vector equal to $2\hat{j}$. Then the magnitude of vector $\vec{B}$ will be :

(1) $\sqrt{5}$ (2) 3 (3) $\sqrt{6}$ (4) $\sqrt{13}$

Ans. **[*]**

Sol. $\vec{B} - (2\hat{i} + 3\hat{j} + 2\hat{k}) = 2\hat{j}$

$\vec{B} = 2\hat{i} + 5\hat{j} + 2\hat{k}$

$|\vec{B}| = \sqrt{4 + 25 + 4} = \sqrt{33}$

Q.39 A projectile is projected at 30º from horizontal with initial velocity 40 ms^{-1}. The velocity of the projectile at t = 2s from the start will be : (Given g = 10 m/s^2)

(1) $40\sqrt{3}$ ms^{-1} (2) zero (3) 20 ms^{-1} (4) $20\sqrt{3}$ ms^{-1}

Ans. **[4]**

Sol. $V_H = 40 \cos 30^\circ = 20\sqrt{3}$

$V_v = 20 - 10 \times 2 = 0$ m/s

$V = V_H = 20\sqrt{3}$ m/s

Q.40 A body of mass 500 g moves along x-axis such that it's velocity varies with displacement x according to the relation $v = 10\sqrt{x}$ m/s the force acting on the body is :

(1) 125 N (2) 25 N (3) 166 N (4) 5N

Ans. **[2]**

Sol. $V = 10\sqrt{x}$

$$\frac{dv}{dx} = \frac{10}{2\sqrt{x}} = \frac{5}{\sqrt{x}}$$

$$v\frac{dv}{dx} = a = 10\sqrt{x} \times \frac{5}{\sqrt{x}} = 50 \text{ m/s}^2$$

$$F = \frac{1}{2} \times 50 = 25N$$

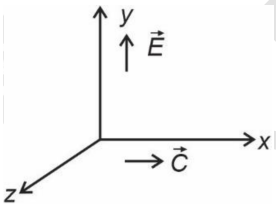
Q.41 A plane electromagnetic wave of frequency 20 MHz propagates in free space along x-direction. At a particular space and time $\vec{E} = 6.6\hat{j}$ v/m. What is $\vec{B}$ at this point?

(1) 2.2×10^{-8} $\hat{k}$ T (2) -2.2×10^{-8} $\hat{i}$ T (3) -2.2×10^{-8} $\hat{k}$ T (4) 2.2×10^{-8} $\hat{i}$ T

Ans. **[1]**

Sol. $\vec{E} = 6.6\hat{j}$

$\vec{E} \times \vec{B} = \vec{C}$



$\vec{B} = B_0\hat{k}$

$CB_0 = E$

$$B_0 = \left(\frac{E}{C}\right) = \frac{6.6}{3\times10^8} = 2.2 \times 10^{-8} \hat{k} \text{ T}$$

Q.42 The root mean square speed of molecules of nitrogen gas at 27ºC is approximately. (Given mass of a nitrogen molecule = 4.6×10^{-26} kg and take Boltzmann constant $k_B = 1.4 \times 10^{-23}$ JK^{-1})

(1) 27.4 m/s (2) 91 m/s (3) 1260 m/s (4) 523 m/s

Ans. **[4]**

Sol. $$V_{rms} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3KT}{m}} = \sqrt{\frac{3\times1.4\times10^{-23}\times300}{4.6\times10^{-26}}} \approx 523 \text{ m/s}$$

Q.43 Eight equal drops of water are falling through air with a steady of 10 cm/s. If the drops coalesce, the new velocity is

(1) 16 cm/s (2) 40 cm/s (3) 5 cm/s (4) 10 cm/s

Ans. **[2]**

Sol. $V_T \propto r^2$

as, $8r_1^3 = r_2^3$

$$\frac{(V_T)_1}{(V_T)_2} = \left(\frac{r_1}{r_2}\right)^2$$

$$\Rightarrow \frac{r_1}{r_2} = \left(\frac{1}{2}\right)$$

$$\Rightarrow \frac{10}{V_T} = \frac{1}{4}$$

$\Rightarrow V_T = 40$ cm/s

Q.44 An electron is allowed to move with constant velocity along the axis of current carrying straight solenoid.

(A) The electron will experience magnetic force along the axis of the solenoid

(B) The electron will not experience magnetic force

(C) The electron will continue to move the axis of the solenoid

(D) The electron will be accelerated along the axis of the solenoid

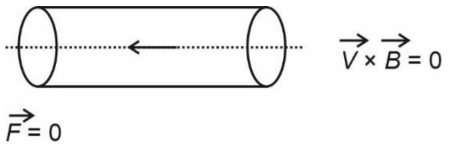
(E) The electron will follow parabolic path-inside the solenoid

Choose the correct answer from the option given below :

(1) B, C and D only (2) A and D only (C) B and C only (D) B and E only

Ans. **[3]**

Sol.



Q.45 A space ship of mass 2×10^4 kg is launched into a circular orbit close to the earth surface. The additional velocity to be imparted to the space ship in the orbit to overcome the gravitational pull will be (if g = 10 m/s^2 and radius of earth = 6400 km) :

(1) $11.2(\sqrt{2}-1)$ km/s (2) $8(\sqrt{2}-1)$ km/s

(3) $7.9(\sqrt{2}-1)$ km/s (4) $7.4(\sqrt{2}-1)$ km/s

Ans. **[2]**

Sol. $$\Delta V = \sqrt{\frac{2GM}{R}} - \sqrt{\frac{GM}{R}} = (\sqrt{2}-1)\sqrt{\frac{GM}{R}}$$

$= (\sqrt{2}-1)\sqrt{gR}$

$= (\sqrt{2}-1)\sqrt{10 \times 6400 \times 10^3}$

$= 8(\sqrt{2}-1)$ km/s

Q.46 The ratio of the de-Broglie wavelengths of proton and electron having same kinetic energy : (Assume $m_p = m_e \times 1849$)

(1) 1 : 43 (2) 1 : 30 (3) 1 : 62 (4) 2 : 43

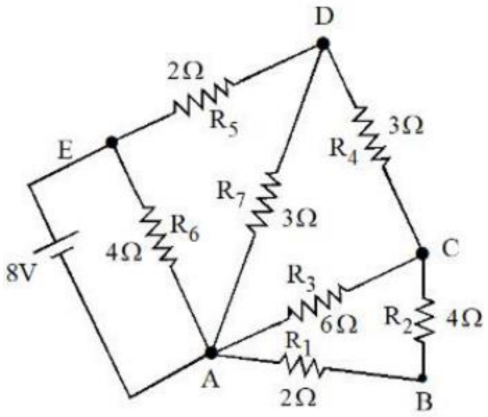
Ans. **[1]**

Sol. $(K)_e = (K)_P$

$$\lambda = \frac{h}{\sqrt{2km}}$$

$$\frac{\lambda_p}{\lambda_e} = \frac{\sqrt{2k_e m_e}}{\sqrt{2k_e m_p}} = \sqrt{\frac{m_e}{m_p}} = \sqrt{\frac{1}{1849}} = \frac{1}{43}$$

Q.47



The current flowing through R_2 is :

(1) $\frac{2}{3}$A (2) $\frac{1}{2}$A (3) $\frac{1}{3}$A (4) $\frac{1}{4}$A

Ans. **[3]**

Sol. I through battery $\frac{8}{2} = 4A$

I through CD $= \frac{3}{9} \times 2 = \frac{2}{3}$A

I through $R_2 = \frac{1}{2}\left(\frac{2}{3}\right) = \frac{1}{3}$A

Q.48 The thermodynamic process, in which internal energy of the system remains constant is

(1) Isochoric (2) Adiabatic (3) Isothermal (4) Isobaric

Ans. **[3]**

Sol. $\Delta T = 0, \Delta U = 0$

Q.49 If force (F), velocity (V) and time (T) are considered as fundamental physical quantity, then dimensional formula of density will be :

(1) FV^4T^{-6} (2) $FV^{-4}T^{-2}$ (3) $F^2V^{-2}T^6$ (4) $FV^{-2}T^2$

Ans. **[2]**

Sol. Density $= \frac{M}{L^3} = ML^{-3} = [MLT^{-2}]^x [LT^{-1}]^y [T]^z$

$ML^{-3} = M^xL^{(x+y)}T^{-2x-y+z}$

$x = 1$, $x + y = -3$, $y = -4$, $-2x - y + z = 0$, $-2 + 4 + z = 0$, $z = -2$

density $= FV^{-4}T^{-2}$

Q.50 The energy of He^+ ion in its first state is, (The ground state energy for the Hydrogen atom –13.6eV) :

(1) –27.2 eV (2) –3.4 eV (3) –13.6 eV (4) –54.4 eV

Ans. **[3]**

Sol. $E = \frac{13.6 \times 2^2}{2^2} = -13.6$ eV

Section-B: Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE**. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

Q.51 A nucleus disintegrates into two nuclear parts, in such a way that ratio of their sizes is $1 : 2^{1/3}$. Their respective speed have a ratio of n : 1. The value of n is .....

Ans. **[2]**

Sol. Momentum conservation

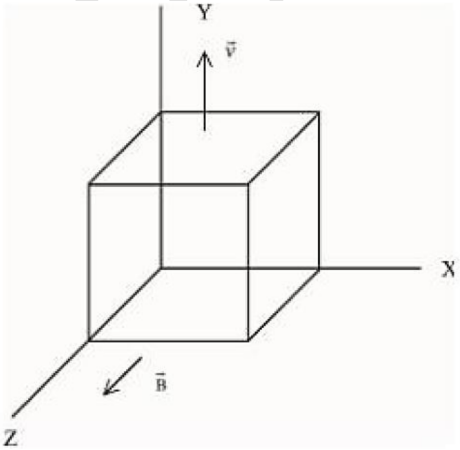
$m_1v_1 = m_2v_2$

$$\Rightarrow \left(\frac{m_1}{m_2}\right) = \left(\frac{v_2}{v_1}\right)$$

$$\Rightarrow \left(\frac{r_1}{r_2}\right)^3 = \left(\frac{v_2}{v_1}\right) \Rightarrow \left(\frac{1}{2}\right) = \left(\frac{v_2}{v_1}\right)$$

$$\frac{v_2}{v_1} = \frac{2}{1}$$

Q.52 A metallic cube of side 15 cm moving along y-axis at a uniform velocity of 2 ms^{-1}. In a region of uniform magnetic field of magnitude 0.5 T directed along z-axis. In equilibrium the potential difference between the faces of higher and lower potential developed because of the motion through the field will be ..... mV.



Ans. **[150]**

Sol. $qE = eVB \Rightarrow \Delta V = (E.d)$

$E = VB = (VB) \times 0.15$

$= 2 \times \frac{1}{2} \times 0.15$ V

$= 0.15$ V $= 150$ mV

Q.53 A wire of density 8×10^3 kg/m^3 is stretched between two clamps 0.5 m apart. The extension developed in the wire is 3.2×10^{-4} m. If $Y = 8 \times 10^{10}$ N/m^2, the fundamental frequency of vibration in the wire will be ..... Hz.

Ans. **[80]**

Sol. $\frac{T}{A} = Y\left(\frac{\Delta L}{L}\right)$ $\mu = \left(\frac{m}{L}\right)$

$T = \left(\frac{Y\Delta L}{L} \times A\right)$

$\frac{T}{\mu} = \frac{Y\Delta LA}{L\left(\frac{m}{L}\right)} = \frac{Y(\Delta L) \times LA}{L(m)} = \left(\frac{Y\Delta L}{L}\right) \times \left(\frac{1}{\rho}\right)$

$\frac{T}{\mu} = \frac{8\times10^{10} \times 3.2\times10^{-4}}{0.5} \times \left(\frac{1}{8\times10^3}\right) = 6.4 \times 10^3$

$\frac{T}{\mu} = 64 \times 10^2$

$\sqrt{\frac{T}{\mu}} = 8 \times 10 = 80$ m/s

$f = \left(\frac{V}{2L}\right) = \left(\frac{80}{1}\right) = 80$ Hz

Q.54 A coil has an inductance of 2H and resistance of 4Ω. A 10V is applied across the coil. The energy stored in the magnetic field after the current has built up its equilibrium value will be ..... $\times 10^{-2}$ J.

Ans. **[625]**

Sol. $I = \frac{10}{4} = \frac{5}{2}$ A

$E = \frac{1}{2}Li^2 = \frac{1}{2} \times 2 \times \left(\frac{25}{4}\right) = 625 \times 10^{-2}$ J

Q.55 A circular plate is rotating in horizontal plane, about an axis passing through its centre and perpendicular to the plate, with an angular velocity ω. A person sits at the centre having two dumbbells in his hands. When he stretched out his hands, the moment of inertia of the system becomes triple. If E be the initial Kinetic energy of the system, then final Kinetic energy will be $\frac{E}{x}$. The value of x is

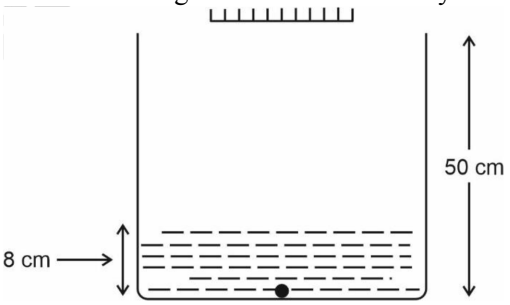
Ans. **[3]**

Sol. $E = \frac{1}{2}I\omega^2 = \left(\frac{L^2}{2I}\right)$

as $I' = 3I$

$E_f = \frac{L^2}{6I} \Rightarrow E_f = \left(\frac{E}{3}\right)$

Q.56 As shown in the figure, a plane mirror is fixed at a height of 50 cm from the bottom of tank containing water $\left(\mu = \frac{4}{3}\right)$. The height of water in the tank is 8 cm. A small bulb is placed at the bottom of the water tank. The distance of image of the bulb formed by mirror from the bottom of the tank is ..... cm.



Ans. **[98]**

Sol. $h' = \frac{8}{4/3} = 6$ cm

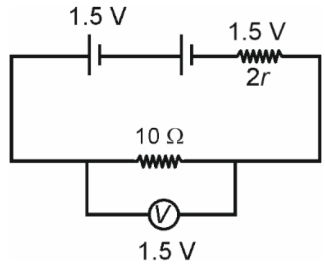
Distance of object from mirror = 42 + 6 = 48 m

Distance of image from bottom of tank = 48 + 50 = 98 m

Q.57 Two identical cells each of emf 1.5 V are connected in series across a 10Ω resistance. An ideal voltmeter connected across 10Ω resistance reads 1.5 V. The internal resistance of each cell is ..... Ω.

Ans. **[5]**

Sol.



$i = \left(\frac{3}{10+2r}\right)$

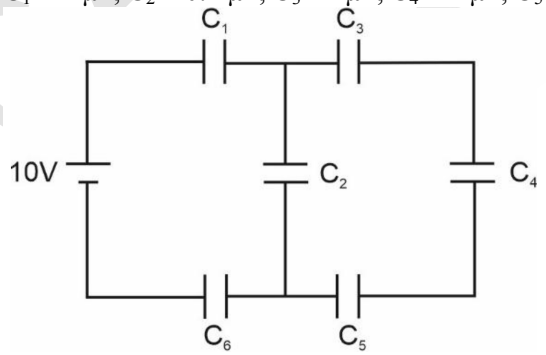
$10 \times \frac{3}{10+2r} = 1.5$

$20 = 10 + 2r \Rightarrow 2r = 10$

$r = 5\Omega$

Q.58 In the given circuit.

$C_1 = 2\ \mu F$, $C_2 = 0.2\ \mu F$, $C_3 = 2\mu F$, $C_4 = 4\ \mu F$, $C_5 = 2\mu F$, $C_6 = 2\mu F$. The charge stored on capacitor C_4 ..... μC



Ans. **[4]**

Sol. q through battery = $10 \times \frac{1}{2} = 5\mu C$

$$\frac{q}{0.8C} = \frac{5-q}{0.2}$$

$\Rightarrow q = 20 - 4q$

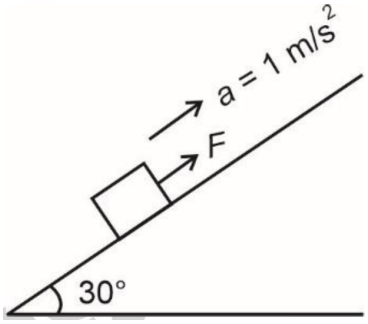
$5q = 20$

$q = 4\mu C$

Q.59 A block of mass 5 kg starting from rest pulled up on a smooth incline plane making an angle of 30º with horizontal with an affective acceleration of 1 ms^{-2}. The power delivered by the puling force at t = 10 s from the start is ..... W. [Use g = 10 ms^{-2}] (Calculate the nearest integer value)

Ans. **[300]**

Sol.



$F - 50 \sin 30 = 5 \times 1$

$F = 5 + 25 = 30N$

V at t = 10 sec = $0 + 1 \times 10 = 10$ m/s

Power = $30 \times 10 = 300$ watt

Q.60 The surface tension of soap solution is 3.5×10^{-2} Nm^{-1}. The amount of work done required to increase the radius of soap bubble from 10 cm to 20 cm is ..... $\times 10^{-4}$J. (take $\pi = 22/7$)

Ans. **[264]**

Sol. $\Delta U = \Delta w = (T \times \Delta A) \times 2$

$= 3.5 \times 10^{-2} \times 4\pi[(0.2)^2 - (0.1)^2] \times 2$

$= 3.5 \times 10^{-2} \times 4 \times \frac{22}{7} \times 0.03 \times 2$

$= 2 \times 1.32 \times 10^{-2}$

$= 2 \times 132 \times 10^{-4}$ J

$= 264 \times 10^{-4}$J

CHEMISTRY

Section-A: This section contains 20 multiple choice questions. Each question has 4 choices(1), (2), (3) and (4), out of which **ONLY ONE** is correct..

Q.61 For a chemical reaction A + B → Product, the order is 1 with respect to A and B

Rate mol L^{-1} S^{-1}	[A] mol L^{-1}	[B] mol L^{-1}
0.10	20	0.5
0.40	x	0.5
0.80	40	y

What is the value of x and y?

(1) 160 and 4 (2) 80 and 4 (3) 80 and 2 (4) 40 and 4

Ans. **[3]**

Sol. Rate of Reaction α(A) (B)

From experiment II x = 80

from experiment III y = 2

Q.62 Which of the following compounds is an example of freon?

(1) $C_2H_2F_2$ (2) C_2F_4 (3) C_2HF_3 (4) $C_2Cl_2F_2$

Ans. **[4]**

Sol. From is chlorofluoro carbons, Hence the correct answer is (4)

Q.63 Alkali metal from the following with least melting points is

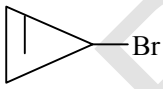
(1) Cs (2) Rb (3) Na (4) K

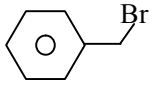
Ans. **[1]**

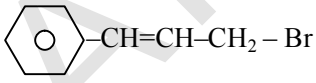
Sol.

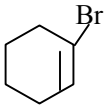
Element	**M.P. (k)**
Na	371
K	336
Rb	312
Cl	302

Q.64 Compound from the following that will not produce precipitate on reaction with $AgNO_3$ is

(1) 

(2) 

(3) 

(4) 

Ans. **[4]**

Sol. Compounds which results in the formation of stable carbocation upon rating with $AgNO_3$ will produce precipitate.

Q.65 Which one of the following pairs is an example of polar molecular solids ?

(1) $SO_2(s)$, $CO_2(s)$ (2) $SO_2(s)$, $NH_3(s)$ (3) MgO(s), $SO_2(s)$ (4) HCl(s), AIN(s)

Ans. **[2]**

Sol.

Solid	**type**
$SO_2(s)$	Polar molecular
$CO_2(s)$	Non polar molecular
$NH_3(s)$	Polar molecular

MgO	Ionic Solid
HCl	Polar molecular solid
AIN	Covalent solid

Q.66 Which hydride among the following is less stable ?

(1) BeH_2 (2) HF (3) NH_3 (4) LiH

Ans. **[1]**

Sol. BeH_2 is the least stable among the given hydrides

Q.67 If Ni^{2+} is replaced by Pt^{2+} in the complex $[NiCl_2Br_2]^{2-}$, which of the following properties are expected to get changed ?

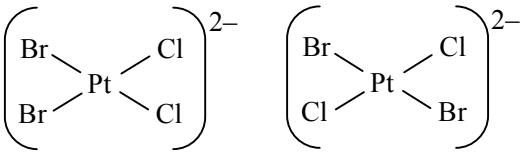
A. Geometry B. Geometrical isomerism

C. Optical isomerism D. Magnetic properties

(1) A and D (2) A, B and C (3) B and C (4) A, B and D

Ans. **[4]**

Sol. $[P + Cl_2Br_2]^{2-}$ — Square planar



Cis trans

$[PtCl_2Br_2]^{2-}$ — Diamagnetic

Q.68 Match List I with List II

List I Complex	**List II Colour**
A. $Mg(NH_4)PO_4$	I. brown
B. $K_3[Co(NO_2)_6]$	II. white
C. $MnO(OH)_2$	III. yellow
D. $Fe_4[Fe(CN)_6]_3$	IV. Blue

Choose the correct answer from the options given below :

(1) A-III, B-IV, C-II, D-I (2) A-II, B-III, C-I, D-IV

(3) A-II, B-IV, C-I, D-III (4) A-II, B-III, C-IV, D-I

Ans. **[2]**

Sol.

Complex	Colour
$Mg(NH_4)PO_4$	White
$K_3[Co(NO_2)_6]$	Yellow
$MnO(OH)_2$	Brown
$Fe_4[Fe(CN)_6]_3$	Blue

Q.69 Given below are two statements :

Statement I : In the metallurgy process, sulphide ore is converted to oxide before reduction.

Statement II : Oxide ores in general are easier to reduce.

In the light of the above statements, choose the most appropriate answer from the options given below :

(1) Statement I is correct but Statement II is incorrect

(2) Both Statement I and Statement II are incorrect

(3) Statement I is incorrect but Statement II is correct

(4) Both Statement I and Statement II are correct

Ans. **[4]**

Sol. Both Statements (I) and (II) are cottect

Q.70 A solution is prepared by adding 2 g of "X" to 1 mole of water. Mass percent of X" in the solution is

(1) 5% (2) 20% (3) 2% (4) 10%

Ans. **[4]**

Sol. Mass percent of x = $\frac{2}{2+18} \times 100 = 10$

Q.71 What weight of glucose must be dissolved in 100 g of water to lower the vapour pressure by 0.20 mm Hg? (Assume dilute solution is being formed)

Given : Vapour pressure of pure water is 54.2 mm Hg at room temperature. Molar mass of glucose is 180 g mol^{-1}

(1) 3.59 g (2) 3.69 g (3) 4.69 g (4) 2.59 g

Ans. **[2]**

Sol. Using the formula

$$\frac{P_0 - P}{P} = \frac{\text{Moles of solute}}{\text{Moles of solvent}}$$

$$\frac{54.2 - 54}{54} = \frac{w/180}{100/18}$$

w = 3.69 g

Q.72 Given below are two statements :

Statement I : Ethane at 333 to 343 K and 6-7 atm pressure in the presence of $AlEt_3$ and $TiCl_4$ undergoes addition polymerization to give LDP.

Statement II : Caprolactam at 533-543K in H_2O through step growth polymerizes to give Nylon 6. In the light of the above statements, choose the correct answer from the options given below :

(1) Statement I is false but Statement II is true

(2) Statement I is true but Statement II is false

(3) Both Statement I and Statement II are true

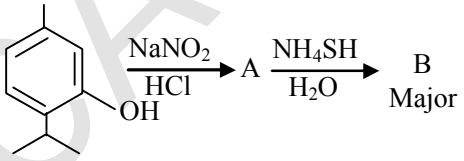
(4) Both Statement I and Statement II are false

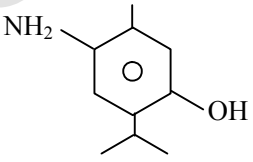
Ans. **[1]**

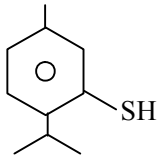
Sol. Ethene at 333 to 343 K and 6-7 atm pressure in the presence of $AlEt_3$ and $TiCl_4$ undergo addition polymerization to give HDP.

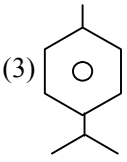
Caprolactam at 533-543 K in H_2O through step growth polymerization gives Nylon 6.

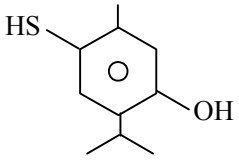
Q.73 Compound 'B' is



(1) 

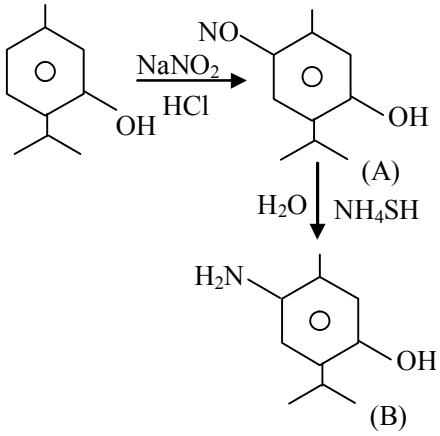
(2) 

(3) 

(4) 

Ans. **[1]**

Sol.



Q.74 Given below are two statements, one is labelled as Assertion A and the other is labelled as Reason R.

Assertion A : A solution of the product obtained by heating a mole of glycine with a mole of chlorine in presence of red phosphorous generates chiral carbon atom.

Reason R : A molecule with 2 chiral carbons is always optically active.

In the light of the above statements, choose the correct answer from the options given below:

(1) A is true but R is false

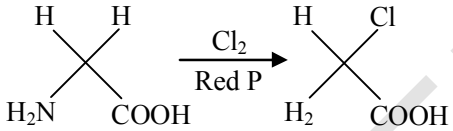
(2) Both A and R are true but R is the correct explanation of A

(3) A is false but R is true

(4) Both A and B are true but R is NOT the correct explanation of A

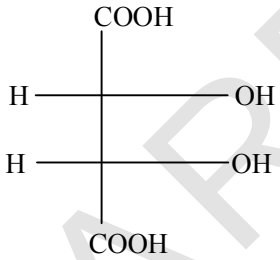
Ans. **[1]**

Sol.



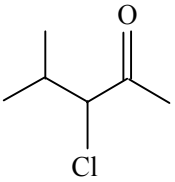
Chiral carbon

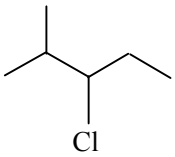
All molecules with 2 chiral carbons are not always optically active. E.g.

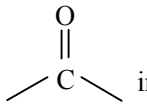
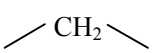


Optically inactive

Q.75 Given below are two statements, one is labelled as Assertion A and the other is labelled as Reason R.

Assertion A : 

can be subjected to Wolff-Kishner reduction to give 

Reason R : Wolff-Kishner reduction is used to convert  into 

In the light of the above statements, choose the correct answer from the options given below :

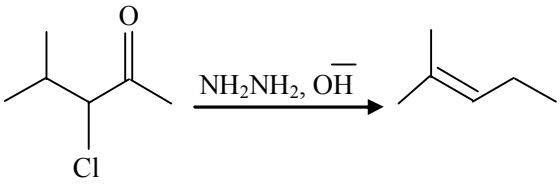
(1) A is true but R is false

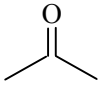
(2) A is false but R is true

(3) Both A and R are true and R is the correct explanation of A

(4) Both A and R are true but R is NOT the correct explanation of A

Ans. **[2]**

Sol. 

Wolff-Kishner reduction is used to convert  into 

Q.76 Given below are two statements, one is labelled as Assertion A and the other is labelled as Reason R.

Assertion A : $[CoCl(NH_3)_5]^{2+}$ absorbs at lower wavelength of light with respect to $[Co(NH_3)_5(H_2O)]^{3+}$

Reason R : It is because the wavelength of the light absorbed depends on the oxidation state of the metal ion.

In the light of the above statements, choose the correct answer from the options given below :

(1) A is false but R is true

(2) A is true but R is false

(3) Both A and R are true and R is the correct explanation of A

(4) Both A and R are true but R is NOT the correct explanation of A

Ans. **[1]**

Sol. $[CoCl(NH_3)_5]^{2+}$ absorbs at higher wavelength of light with respect to $[Co(NH_3)_5H_2O]^{3+}$ because Cl^- is weaker ligand than H_2O.

Wavelength of light absorbed depends upon the oxidation state of the metal ion.

Hence, A is false but R is true.

Q.77 $H_3C - CH_2 - \underset{\displaystyle OH}{\underset{|}{CH}} - CH_3 \xrightarrow[\text{(iii) } D_2O]{\text{(i) NaI, } H_3PO_4 \text{ (ii) Mg, Dry ether}} \underset{\text{Product}}{[X]}$

Product [X] formed in the above reaction is :

(1) $H_3C - CH_2 - \underset{\displaystyle OH}{\overset{\displaystyle H}{\underset{|}{\overset{|}{C}}}} - CH_3$

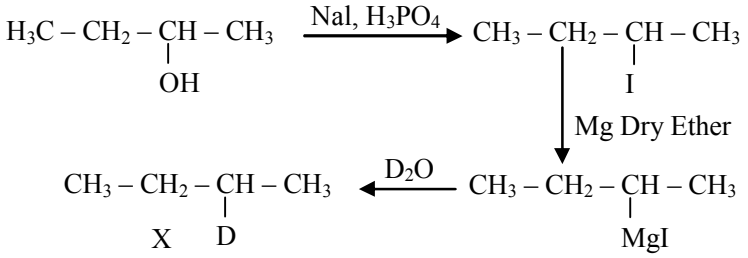
(2) $H_3C - CH = CH - CH_3$

(3) $H_3C - CH_2 - \underset{\displaystyle D}{\underset{|}{CH}} - CH_3$

(4) $H_3C - CH_2 - CH = CH_2$

Ans. **[3]**

Sol.



Q.78 One mole of P_4 reacts with 8 moles of $SOCl_2$ to give 4 moles of A, x mole of SO_2 and 2 moles of B. A, B and x respectively are

(1) $POCl_3$, S_2Cl_2 and 2 (2) PCl_3, S_2Cl_2 and 4

(3) PCl_3, S_2Cl_2 and 2 (4) $POCl_3$, S_2Cl_2 and 4

Ans. **[2]**

Sol. $P_4 + 8SOCl_2 \rightarrow 4PCl_3 + 4SO_2 + 2S_2Cl_2$

Hence, A = PCl_3, x = 4, B = S_2Cl_2

Q.79 The magnetic moment is measured in Bohr Magneton (BM).

Spin only magnetic moment of Fe in $[Fe(H_2O)_6]^{3+}$ and $[Fe(CN)_6]^{3-}$ complexes respectively is :

(1) 6.92 B.M. in both (2) 3.87 B.M. and 1.732 B.M

(3) 5.92 B.M. and 1.732 B.M (4) 4.89 B.M. and 6.92 B.M.

Ans. **[3]**

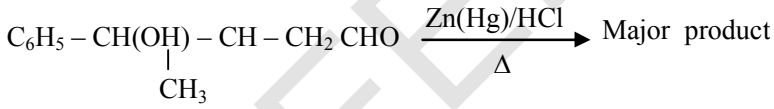
Sol. $[Fe(H_2O)_6]^{3+} = d^5$(H.S.) = 5 unpaired electrons

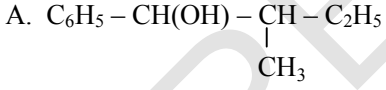
$\mu = \sqrt{5\times 7} = \sqrt{35} = 5.92$ B.M.

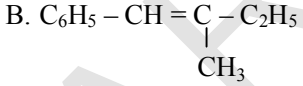
$[Fe(CN)_6]^{3-} = d^5$ (L.S.) = 1 unpaired electrons

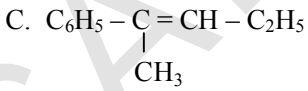
$\mu = \sqrt{1\times 3} = \sqrt{3} = 1.732$ B.M.

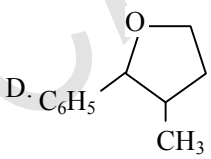
Q.80 The major product formed in the following reaction is



A. 

B. 

C. 

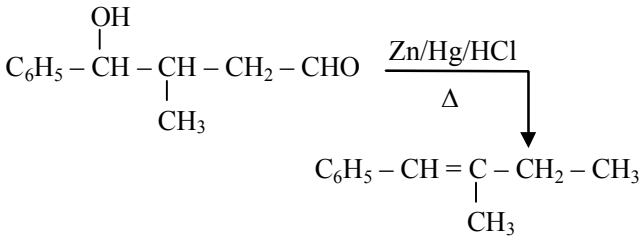
D. 

Choose the correct answer from the options given below :

(1) B only (2) A only (3) C only (4) D only

Ans. **[2]**

Sol.



Section-B: Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer..

Q.81 Number of compounds from the following which will not produce orange red precipitate with Benedict solutions is .....

Glucose, maltose, sucrose, ribose, 2-deoxyribose, amylose, lactose

Ans. **[2]**

Sol. Except sucrose and amylose all other give orange red precipitate with Benedict solution.

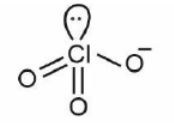
Sucrose being non-reducing disaccharide and amylose being polysaccharides do not give positive Benedict's test.

Q.82 The maximum number of lone pairs of electron on the central atom from the following species is .....

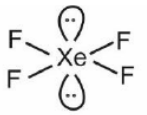
ClO_3^-, XeF_4, SF_4 and I_3^-

Ans. **[3]**

Sol.

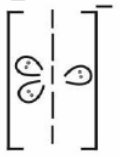


1. L.P/ on central atom



2. L.P/ on central atom

1. L.P/ on central atom



3. L.P/ on central atom

Q.83 The number of correct statements from the following is .....

A. For 1s orbital, the probability density is maximum at the nucleus

B. For 2s orbital, the probability density first increases to maximum and then decreases sharply to zero.

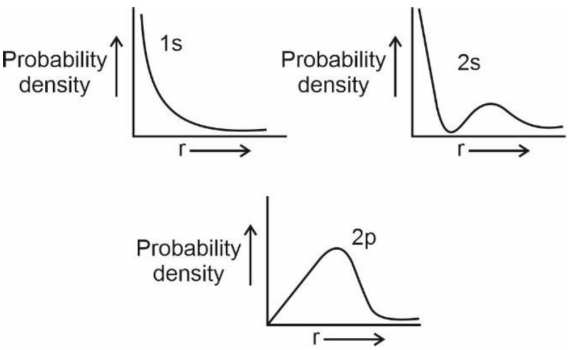
C. Boundary surface diagrams of the orbitals encloses a region of 100% probability of finding the electron.

D. p and d-orbitals have 1 and 2 angular nodes respectively

E. probability density of p-orbital is zero at the nucleus

Ans. **[3]**

Sol. Except B and C all statements are correct. Probability density curves for 1s, 2s and 2p orbitals are given below.



Q.84 The total number of intensive properties from the following is ..... .
Volume, Molar heat capacity, molarity, E° cell, gibbs free energy change, Molar mass, Mole

Ans. **[4]**

Sol. Molar heat capacity, molarity, E^{o}_{cell} and molar mass are intensive properties.

Q.85 4.5 moles each of hydrogen and iodine is heated in a sealed ten litre vessel. At equilibrium, 3 moles of HI were found. The equilibrium, 3 moles of HI were found. The equilibrium constant for
$H_2(g) + I_2(g) \rightleftharpoons 2HI(g)$ is ..... .

Ans. **[1]**

Sol.

	H_2	+	I_2	$\rightleftharpoons$	2HI
at initial	4.5		4.5		0
at equilibrium	3		3		3

$$k_{eq} = \frac{[HI]^2}{[H_2][I_2]} = \frac{(3)^2}{3\times 3} = 1$$

Q.86 The volume of hydrogen liberated at STP by treating 2.4 g of magnesium with excess of hydrochloric acid is ..... $\times 10^{-2}$L. Given : Molar volume of gas is 22.4 L at STP. Molar mass of magnesium is 24g mol^{-1}

Ans. **[224]**

Sol. $\underset{\substack{2.4g\\=0.1\text{ mole}}}{Mg} + 2HCl \rightarrow MgCl_2 + \underset{0.1\text{ mole}}{H_2}$

Volume of 1 mole H_2 at STP = 22.4 Litre

$\therefore$ 0.1 mole H_2 at STP will occupy = 2.24 Litre

Q.87 The number of correct statements about modern adsorption theory of heterogeneous catalysis from the following is .....

A. The catalyst is diffused over the surface of reactants.
B. Reactants are adsorbed on the surface of the catalyst.
C. Occurrence of chemical reaction on the catalyst's surface through formation of an intermediate.
D. It is a combination of intermediate compound formation theory and the old adsorption theory.
E. It explains the action of the catalyst as well as those of catalytic promoters and poisons.

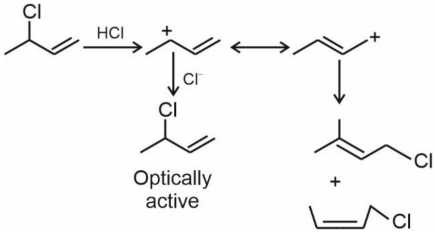
Ans. **[3]**

Sol. Except (A) and (E) all statements are correct.

Q.88 The number of possible isomeric products formed when 3-chloro-1-butene reacts with HCl through carbocation formed is .....

Ans. **[4]**

Sol.



Q.89 $Mg(NO_3)_2 \cdot XH_2O$ and $Ba(NO_3)_2 \cdot YH_2O$, represent formula of the crystalline forms of nitrate salts. Sum of X and Y is .....

Ans. **[6]**

Sol. Magnesium nitrate crystallises with six molecules of water, whereas barium nitrate crystallises as the anhydrous salt as $Mg(NO_3)_2.6H_2O$

$Ba(NO_3)_2$ respectively

$\therefore$ x = 6, y = 0

Q.90 The number of correct statements from the following is .....

A. E_{cell} is an intensive parameter

B. A negative E^o means that the redox couple is a stronger reducing agent than the H^+/H_2 couple.

C. The amount of electricity required for oxidation or reduction depends on the stoichiometry of the electrode reaction.

D. The amount of chemical reaction which occurs at any electrode during electrolysis by a current is proportional to the quantity of electricity passed through the electrolyte.

Ans. **[4]**

Sol. All statements are correct