JEE Main Online Exam 2023

Questions & Solution

08th April 2023 | Morning

MATHEMATICS

Section-A : Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Q.1 Let $A = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$. If $|\text{adj}(\text{adj}(\text{adj}2A))| = (16)^n$, then n is equal to -

(1) 8 (2) 10 (3) 9 (4) 12

Ans. **[2]**

Sol. $|A| = 2(3) - 1(2) = 4$

$\therefore$ Now $|\text{adj}(\text{adj}(\text{adj}(2A)))|$

$\therefore\ |2A|^{(n-1)^3} = |2A|^8 = 2^{24} \cdot 4^8$

$= 2^4 = 16^{10}$

$\therefore\ \boxed{n = 10}$

Q.2 If the point with position vectors $\alpha\hat{i} + 10\hat{j} + 13\hat{k}$, $6\hat{i} + 11\hat{j} + 11\hat{k}$, $\frac{9}{2}\hat{i} + \beta\hat{j} - 8\hat{k}$ are collinear, then $(19\alpha - 6\beta)^2$ is equal to

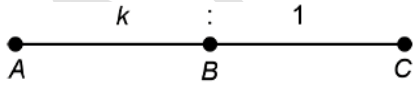
(1) 36 (2) 25 (3) 49 (4) 16

Ans. **[1]**

Sol.

- A(α, 10, 13)
- B(6, 11, 11)
- C $\left(\frac{9}{2}, \beta, -8\right)$

Since A, B, C are collinear



$11 = \frac{-8k + 13}{k + 1}$

$\Rightarrow\ 11k + 11 = -8k + 13$

$\Rightarrow\ 19k = 2$

$\Rightarrow\ k = \frac{2}{19}$

$\therefore$ Ratio = 2 : 19

$\frac{19\alpha + 9}{21} = 6$

$\Rightarrow \quad 19\alpha = 117$

$\Rightarrow \quad \alpha = \dfrac{117}{19}$

$\dfrac{2\beta + 190}{21} = 11$

$\Rightarrow \quad \beta = \dfrac{41}{2}$

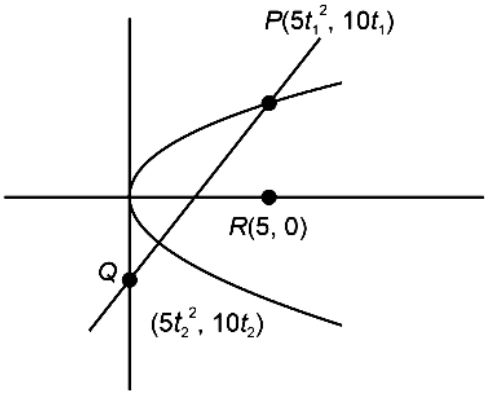
$\therefore \quad (19\alpha - 6\beta)^2 = (117 - 123)^2 = 36$

Q.3 Let R be the focus of the parabola $y^2 = 20x$ and the line $y = mx + c$ intersect the parabola at two points P and Q. Let the points G(10, 10) be the centroid of the triangle PQR. If $c - m = 6$, then $(PQ)^2$ is

(1) 296 (2) 325 (3) 317 (4) 346

Ans. **[2]**

Sol.



$5\left(\dfrac{t_1^2 + t_2^2 + 1}{3}\right) = 10$

$\therefore \quad t_1^2 + t_2^2 = 5$ .....(i)

$\dfrac{10(t_1 + t_2)}{3} = 10$

$\therefore \quad t_1 + t_2 = 3$ .....(ii)

$\therefore \quad t_1 = 1, t_2 = 2$

$\therefore \quad P \equiv (5, 10) \quad Q \equiv (20, 20)$

$\therefore \quad$ Equation of PQ $= y - 10 = \dfrac{10}{15}(x - 5)$

$3y - 30 = 2x - 10$

$y = \dfrac{2}{3}x + \dfrac{20}{3}$

$\therefore \quad PQ^2 = 225 + 100 = 325$

Q.4 The number of arrangements of the letters of the word "INDEPENDENCE" in which all the vowels always occur together is -

(1) 16800 (2) 33600 (3) 18000 (4) 14800

Ans. **[1]**

Sol. Vowels : I, E, E, E, E

Consonants : N N N D D P C

$\boxed{\text{I E E E E}}$ 3N, 2D, P, C

Number of required words = $\frac{8!}{3!\,2!}\times\frac{5!}{4!} = 16800$

Q.5 The shortest distance between the lines $\frac{x-4}{4}=\frac{y+2}{5}=\frac{z+3}{3}$ and $\frac{x-1}{3}=\frac{y-3}{4}=\frac{z-4}{2}$ is -

(1) $6\sqrt{3}$ (2) $2\sqrt{6}$ (3) $6\sqrt{2}$ (4) $3\sqrt{6}$

Ans. **[4]**

Sol. $\vec{\ell}_1\times\vec{\ell}_2 = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 4 & 5 & 3\\ 3 & 4 & 2\end{vmatrix} = -2\hat{i}+\hat{j}+\hat{k}$

$$d = \left|\frac{(\vec{a}-\vec{b}).(\vec{\ell}_1\times\vec{\ell}_2)}{|\vec{\ell}_1\times\vec{\ell}_2|}\right|$$

$$= \left|\frac{(3\hat{i}-5\hat{j}-7\hat{k}).(-2\hat{i}+\hat{j}+\hat{k})}{\sqrt{6}}\right|$$

$$= \left|\frac{-6-5-7}{\sqrt{6}}\right|$$

$$= 3\sqrt{6}$$

Q.6 Let $f(x)=\frac{\sin x+\cos-\sqrt{2}}{\sin x-\cos x}$, $x\in[0,\pi]-\left\{\frac{\pi}{4}\right\}$. Then $f\left(\frac{7\pi}{12}\right)f''\left(\frac{7\pi}{12}\right)$ is equal to -

(1) $\frac{2}{9}$ (2) $\frac{-2}{3}$ (3) $\frac{-1}{3\sqrt{3}}$ (4) $\frac{2}{3\sqrt{3}}$

Ans. **[1]**

Sol. $f(x)=\frac{\sin x+\cos-\sqrt{2}}{\sin x-\cos x}$

$$f(x) = \frac{\sqrt{2}\sin\left(x+\frac{\pi}{4}\right)-\sqrt{2}}{\sqrt{2}\left(\sin\left(x-\frac{\pi}{4}\right)\right)}$$

$$f(x)=\frac{\sin\left(x+\frac{\pi}{4}\right)-1}{\sin\left(x-\frac{\pi}{4}\right)}$$

$$f\left(x+\frac{\pi}{4}\right)=\frac{\cos x-1}{\sin x}=-\tan\frac{x}{2}$$

$$\Rightarrow\ f(x)=-\tan\left(\frac{x}{2}-\frac{\pi}{8}\right)$$

$$f'(x)=\frac{-1}{2}\sec^2\left(\frac{x}{2}-\frac{\pi}{8}\right)$$

$$f''(x)=\frac{1}{2}\left(\sec^2\left(\frac{x}{2}-\frac{\pi}{8}\right)\tan\left(\frac{x}{2}-\frac{\pi}{8}\right)\right)$$

$$f\left(\frac{7\pi}{12}\right)=-\tan\left(\frac{7\pi}{24}-\frac{\pi}{8}\right)$$

$$=-\tan\left(\frac{4\pi}{24}\right)$$

$$=-\tan\left(\frac{\pi}{6}\right)=-\frac{1}{\sqrt{3}}$$

$$f''=\left(\frac{7\pi}{12}\right)=-\frac{1}{2}\times\left(\frac{2}{\sqrt{3}}\right)^2\times\frac{1}{\sqrt{3}}$$

$$=\frac{-2}{3\sqrt{3}}$$

$$f\left(\frac{7\pi}{12}\right)f''\left(\frac{7\pi}{12}\right)=\frac{2}{9}$$

Q.7 In a bolt factory, machines A, B and C manufacture respectively 20%, 30% and 50% of the total bolts. Of their output 3, 4 and 2 percent are respectively defective bolts. A bolts is drawn at random from the product. If the bolt drawn is found the defective, then the probability that it is manufactured by the machine C is -

(1) $\frac{5}{14}$ (2) $\frac{9}{28}$ (3) $\frac{3}{7}$ (4) $\frac{2}{7}$

Ans. **[1]**

Sol. Using Bayes' Theorem

Required probability

$$=\frac{50\times2}{20\times3+30\times4+50\times2}$$

$$=\frac{10}{6+12+10}$$

$$=\frac{10}{28}$$

$$=\frac{5}{14}$$

Q.8 Let $P = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$, $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ and $Q = PAP^T$. If $P^TQ^{2007}P = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then $2a + b - 3c - 4d$ equal to -

(1) 2004 (2) 2005 (3) 2007 (4) 2006

Ans. **[2]**

Sol. $P \times P^T \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ \frac{-1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{-1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Similarly $P^T P = I$

Now, $Q^{2007} = (PAP^T)(PAP^T) \ldots$ 2007 times

$= PA^{2007}P^T$

$A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

$A^2 = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$

$A^3 = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$

.

.

.

$A^{2007} = \begin{bmatrix} 1 & 2007 \\ 0 & 1 \end{bmatrix}$

$P^TQ^{2007}P = A^{2007} = \begin{bmatrix} 1 & 2007 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow$ $a = 1, b = 2007, c = 0, d = 1$

$2a + b - 3c - 4d = 2005$

Q.9 The number of ways, in which 5 girls and 7 boys can be seated at a round table so that no two girls sit together, is -

(1) 720 (2) $126(5!)^2$ (3) $7(360)^2$ (4) $7(720)^2$

Ans. **[2]**

Sol. $6! \times {}^7C_5.5!$

$= 720 \times \frac{7 \times 6}{2} \times 5!$

$\Rightarrow$ $(5!)^2 \times \frac{7 \times 6 \times 6}{2}$

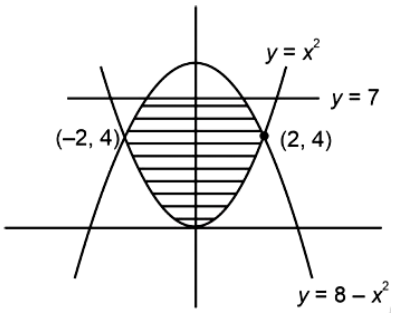
$\Rightarrow$ $126 \times (5!)^2$

Q.10 The area of the region $\{(x, y) : x^2 \le y \le 8 - x^2, y \le 7\}$ is -

(1) 27 (2) 18 (3) 20 (4) 21

Ans. **[3]**

Sol.



$$\text{Required area} = 2\left[\int_0^4 \sqrt{y}dy + \int_4^7 \sqrt{8-y}dy\right]$$

$$= 2\left(\left.\frac{y^{\frac{2}{2}}}{\frac{3}{2}}\right|_0^4 - \left.\frac{(8-y)^{\frac{3}{2}}}{\frac{3}{2}}\right|_4^7\right)$$

$$= \frac{4}{3}(8-(1-8))$$

$$= 20 \text{ sq. units}$$

Q.11 Let $\sum_{j=1}^{n} S_j^2 = \frac{n}{A}(Bn^2 + Cn + D)$, where A, B, C, D $\in$ N and A has least value. Then

(1) A + C + D is not divisible by B (2) A + B = 5(D – C)

(3) A + B + C + D is divisible by 5 (4) A + B is divisible by D

Ans. **[4]**

Sol. $S_K = \frac{K.(K+1)}{2K} = \frac{K+1}{2}$

$$\sum_{j=1}^{n}(S_j)^2 = \sum_{j=1}^{n}\frac{1}{4}(2^2 + 3^2 + 4^2 \ldots\ldots (n+1)^2)$$

$$\Rightarrow \frac{1}{4}\left(\frac{(n+1)(n+2)(2n+3)}{6} - 1\right)$$

$$= \frac{1}{4}\left(\frac{(n^2+3n+2)(2n+3)-6}{6}\right)$$

$$= \frac{1}{4}\left(\frac{2n^3+6n^2+4n+3n^2+9n+6-6}{6}\right)$$

$$= \frac{1}{4}\left(\frac{2n^3+9n^2+13n}{6}\right)$$

$$= \frac{n}{24}(2n^2 + 9n + 13)$$

A = 24, B = 2, C = 9, D = 13

$$\frac{A+B}{D} = \frac{26}{13} = 2$$

Q.12 Negation of $(p \Rightarrow q) \Rightarrow (q \Rightarrow p)$ is

(1) $(\sim p) \vee p$ (2) $q \wedge (\sim p)$ (3) $(\sim q) \wedge p$ (4) $p \vee (\sim q)$

Ans. **[2]**

Sol. $(p \rightarrow q) \rightarrow (q \rightarrow p)$

$\Rightarrow (p' \vee q)' \vee (q' \vee p)$

$\Rightarrow (p \wedge q') \vee (q' \vee p)$

$= p \vee q'$

Now $(p \vee q')'$

$= p' \wedge q$

Q.13 $\lim\limits_{x\to 0}\left(\left(\dfrac{1-\cos^2(3x)}{\cos^3(4x)}\right)\left(\dfrac{\sin^3(4x)}{\log_e(2x+1)^5}\right)\right)$ is equal to .....

(1) 15 (2) 9 (3) 18 (4) 24

Ans. **[3]**

Sol. $\lim\limits_{x\to 0}\left(\left(\dfrac{1-\cos^2(3x)}{\cos^3(4x)}\right)\left(\dfrac{\sin^3(4x)}{\log_e(2x+1)^5}\right)\right)$

$$\lim_{x\to 0}\frac{(1-\cos 3x)(1+\cos 3x)9x^2}{(\cos^3(4x))9x^2}\ \frac{(\sin 4x)^2(64x)^3(2x)^5}{(64x^3)(\log_e(2x+1)^5(2x)^5} = 9\times\frac{1}{2}\times 2\times 64\times\frac{1}{2^5} = 18$$

Q.14 Let α, β, γ be the three roots of the equation $x^3 + bx + c = 0$. If $\beta\gamma = 1 = -\alpha$, then $b^3 + 2c^3 - 3\alpha^3 - 6\beta^3 - 8\gamma^3$ is equal to

(1) $\dfrac{155}{8}$ (2) 21 (3) $\dfrac{169}{8}$ (4) 19

Ans. **[4]**

Sol. Roots of $x^3 + bx + c = 0$ are α, β, γ

$\because \beta\gamma = 1 = -\alpha$

$\therefore \alpha = -1$ …(i)

and $\alpha + \beta + \gamma = 0$ …(ii)

$\alpha\beta\gamma = -c$ …(iii)

$\therefore c = 1$ …(iv)

$\beta + \gamma = 1$ …(v)

$\because \alpha\beta + \beta\gamma + \gamma\alpha = b$

$\Rightarrow \alpha(\beta + \gamma) + \beta\gamma = b$

$\therefore b = 0$ …(vi)

$\therefore \beta = -\omega, \gamma = -\omega^2$ …(vii)

$\therefore b^3 + 2c^3 - 3\alpha^3 - 6\beta^3 - 8\gamma^3$

$= 0 + 2 + 3 + 6 + 8 = 19$

Q.15 Let the number of elements in sets A and B be five and two respectively. Then the number of subsets of A × B each having at least 3 and at most 6 elements is -

(1) 752 (2) 782 (3) 792 (4) 772

Ans. **[3]**

Sol. $n(A) = 5$

$n(B) = 2$

$n(A \times B) = 10$

Number of subset having three elements = $^{10}C_3$

Number of subset having four elements = $^{10}C_4$

Number of subset having five elements = $^{10}C_5$

Number of subset having six elements = $^{10}C_6$

$^{10}C_3 + {}^{10}C_4 + {}^{10}C_5 + {}^{10}C_6$

$= 120 + 210 + 252 + 210 = 792$

Q.16 Let $C(\alpha, \beta)$ be the circumcentre of the triangle formed by the lines

$4x + 3y = 69$,

$4y - 3x = 17$, and

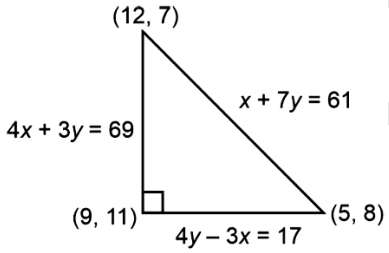
$x + 7y = 61$.

Then $(\alpha - \beta)^2 + \alpha + \beta$ is equal to

(1) 18 (2) 17 (3) 15 (4) 16

Ans. **[2]**

Sol.



$$C\left(\frac{12+5}{2}, \frac{7+8}{2}\right)$$

$$C\left(\frac{17}{2}, \frac{15}{2}\right) \equiv (\alpha, \beta)$$

$(\alpha - \beta)^2 + \alpha + \beta$

$$\left(\frac{17}{2} - \frac{15}{2}\right)^2 + \frac{17}{2} + \frac{15}{2}$$

$1 + 16 = 17$

Q.17 Let $I(x) = \int \frac{(x+1)}{x(1-xe^x)^2}dx$, $x > 0$. If $\lim\limits_{x\to\infty} I(x) = 0$, then $I(1)$ is equal to -

(1) $\frac{e+2}{e+1} - \log_e(e+1)$ (2) $\frac{e+1}{e+2} + \log_e(e+1)$ (3) $\frac{e+1}{e+2} - \log_e(e+1)$ (4) $\frac{e+2}{e+1} + \log_e(e+1)$

Ans. **[1]**

Sol. $I(x) = \int \frac{(x+1)e^x}{ex^x(1+xe^x)^2}dx$

Let $1 + xe^x = t$

$e^x(x + 1)dx = dt$

$I(x) = \int \frac{1}{(t-1)t^2}dt$

$= \int \frac{(1-t^2)+t^2}{(t-1)t^2}dt$

$= \int \frac{-(t+1)}{t^2} + \frac{1}{t-1}dt$

$= \int -\frac{1}{t} - \frac{1}{t^2} + \frac{1}{t-1}dt$

$I(x) = -\text{In } t + \frac{1}{t} + \text{In}(t-1) = C$

$I(x) = \text{In}\left(\frac{xe^x}{xe^x+1}\right) + \frac{1}{xe^x+1} + C$

$\lim\limits_{x\to\infty} I(x) = 0 \Rightarrow C = 0$

$I(x) = \text{In}\left(\frac{e}{e+1}\right) + \frac{1}{e+1} = 1 + \frac{1}{e+1} - \text{In}(e+1) = \frac{e+2}{e+1} - \text{In}(e+1)$

Q.18 If for $z = \alpha + i\beta$, $|z + 2| = z + 4(1 + i)$, then $\alpha + \beta$ and $\alpha\beta$ are the roots of the equation

(1) $x^2 + 3x - 4 = 0$ (2) $x^2 + 7x + 12 = 0$ (3) $x^2 + x - 12 = 0$ (4) $x^2 + 2x - 3 = 0$

Ans. **[2]**

Sol. $z = \alpha + c\beta$

$|z + 2| = z + 4(1 + i)$

$\sqrt{(\alpha+2)^2+\beta^2} = (\alpha + 4) + i(\beta + 4)$

$\sqrt{(\alpha+2)^2+\beta^2} = \alpha + 4$ ....(i)

$\beta + 4 = 0$ ....(ii)

(i) $\Rightarrow \sqrt{(\alpha+2)^2+16} = \alpha + 4$

$\alpha^2 + 4\alpha + 20 = \alpha^2 + 8\alpha + 16$

$\alpha = 1$

$\alpha + \beta = -3$, $\alpha\beta = -4$

Equation with roots –3 and –4 is

$x^2 + 7x + 12 = 0$

Q.19 If the equation of the plane containing the line $x + 2y + 3z - 4 = 0 = 2x + y - z + 5$ and perpendicular to the plane $\vec{r} = (\hat{i} - \hat{j}) + \lambda(\hat{i} + \hat{j} + k) + \mu(\hat{i} - 2\hat{j} + 3k)$ is $ax + by + cz = 4$, then $(a - b + c)$ is equal to

(1) 18 (2) 22 (3) 20 (4) 24

Ans. **[2]**

Sol. Equation of required plane is

$P: (x + 2y + 3z - 4) + \alpha(2x + y - z + 5) = 0$

$\Rightarrow (2\alpha + 1) + (\alpha + 2)y + (3 - \alpha)z = 4 - 5\alpha$

$\vec{n_1} = (2\alpha + 1)\hat{i} + (\alpha + 2)\hat{j} + (3 - \alpha)k$

Normal of the given plane is

$$\vec{n_2} = \begin{vmatrix} \hat{i} & \hat{j} & k \\ 1 & 1 & 1 \\ 1 & -2 & 3 \end{vmatrix} = 5\hat{i} - 2\hat{j} - 3k$$

$\vec{n_1} . \vec{n_2} = 0$

$\Rightarrow 5(2\alpha + 1) - 2(\alpha + 2) - 3(3 - \alpha) = 0$

$11\alpha + (-8) = 0$

$$\alpha = \frac{8}{11}$$

$$P: \frac{27}{11}x + \frac{30}{11}y + \frac{25}{11}z - \frac{4}{11} = 0$$

$27x + 30y + 25z = 4$

$\therefore a = 27, b = 30, c = 25$

$\therefore a - b + c = 22$

Q.20 If the coefficients of three consecutive terms in the expansion of $(1 + x)^n$ are in the ratio 1 : 5 : 20, then the coefficient of the fourth term is

(1) 2436 (2) 5481 (3) 1827 (4) 3654

Ans. **[4]**

Sol. $$\frac{{}^nC_{r-1}}{1} = \frac{{}^nC_r}{5} = \frac{{}^nC_{r+1}}{20}$$

$$\because \frac{{}^nC_r}{{}^nC_{r-1}} = 5 \Rightarrow \frac{\frac{\lfloor n}{\lfloor n-r \rfloor r}}{\frac{\lfloor n}{\lfloor n-r+1 \rfloor r-1}} = 5$$

$$\Rightarrow \frac{n - r + 1}{r} = 5 \Rightarrow n = 6r - 1 \quad \text{.....(i)}$$

$$\because \frac{{}^nC_{r+1}}{{}^nC_r} = 4 \Rightarrow \frac{n - r}{r + 1} = 4 \Rightarrow n = 5r + 4 \quad \text{.....(ii)}$$

from (i) and (ii), $r = 5, n = 29$

$\Rightarrow$ Coefficient of fourth term $= {}^{29}C_3 = 3654$

Section-B: Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer..

Q.21 Let [t] denote the greatest integer ≤ t. If the constant term in the expansion of $\left(3x^2-\frac{1}{2x^5}\right)^7$ is α, then [α] is equal to _____.

Ans. **[1275]**

Sol. General term $(T_{r+1}) = {}^7C_r(3x^2)^{7-r}\left(-\frac{1}{2x^5}\right)^r$

$T_{r+1} = (-1)^r\ {}^7C_r 3^{7-r}.2^{-r}x^{14-7r}$

Constant term = a = $(-1)^2\ {}^7C_2 3^5.2^{-2} = \frac{5103}{4}$

a = 1275.75

$\Rightarrow$ [a] = 1275

Q.22 Let the mean and variance of 8 numbers x, y, 10, 12, 6, 12, 4, 8 be 9 and 9.25 respectively. If x > y, then 3x – 2y is equal to _____.

Ans. **[25]**

Sol. $\because \frac{x+y+10+12+6+12+4+8}{8} = 9$

$x + y = 72 - 52 = 20 \Rightarrow x + y = 20$ …(1)

$(9-x)^2 + (9-y)^2 + (9-10)^2 + (9-12)^2$

$\because \frac{+(9-6)^2+(9-12)^2+(9-4)^2+(9-8)^2}{8} = 9.25$

$\Rightarrow x^2 + y^2 = 218$ …(2)

From (1) & (2)

x = 13, y = 7 or x = 7, y = 13

$\because$ x > y

$\Rightarrow$ x = 13, y = 7 $\Rightarrow$ 3x – 2y = 39 – 14 = 25

Q.23 If a_α is the greatest term in the sequence $a_n = \frac{n^3}{n^4+147}$, n = 1, 2, 3, ... , then α is equal to _____.

Ans. **[5]**

Sol. Let $y = \frac{x^3}{x^4+147} = f(x)$

For increasing function

$\frac{dy}{dx} > 0$

$\frac{-x^2(x^4-441)}{(x^4+147)^2} > 0 \Rightarrow x^4 < 441$

$\because x^4 < 441$

For maxima/minima $\frac{dy}{dx} = 0$

$\Rightarrow x^4 = 441,$

$\Rightarrow x = \alpha, 4 < \alpha < 5$

$\Rightarrow$ Maximum value of f(x) is at x = 4 or x = 5

$f(4) = \frac{64}{403}, f(5) = \frac{125}{772},$

$\because f(5) > f(4) \Rightarrow \alpha = 5$

Q.24 If the solution curve of the differential equation $(y - 2\log_e x)dx + (x \log_e x^2)dy = 0, x > 1$ passes through the points $\left(e, \frac{4}{3}\right)$ and (e^4, α), then α is equal to _____.

Ans. **[03.00]**

Sol. $\because (y - 2\ln x)dx + (2x\ln x)dy = 0.$

$2x \ln x \frac{dy}{dx} + y = 2 \ln x$

$\frac{dy}{dx} + \frac{y}{2x \ln x} = \frac{1}{x}$

$\therefore$ I.F. $= e^{\int \frac{1}{2x \ln x} dx} = \sqrt{\ln x}$

$\therefore$ Solution of the equation is :

$y.\sqrt{\ln x} = \int \frac{\sqrt{\ln x}}{x} dx$

$\therefore y. \sqrt{\ln x} = \frac{2}{3}(\ln x)^{\frac{3}{2}} + C$ .....(i)

$\therefore$ eq.(i) passes through point $\left(e, \frac{4}{3}\right)$

$\therefore C = \frac{2}{3}$

$\therefore y\sqrt{\ln x} = \frac{2}{3}(\ln x)^{\frac{3}{2}} + \frac{2}{3}$ .....(ii)

This equation passes through point (e^4, α)

$\therefore \alpha = 3.$

Q.25 Let λ_1, λ_2 be the values of λ for which the points $\left(\frac{5}{2}, 1, \lambda\right)$ and (–2, 0, 1) are at equal distance from the plane $2x + 3y - 6z + 7 = 0$. If $\lambda_1 > \lambda_2$, then the distance of the point $(\lambda_1 \geq \lambda_2, \lambda_2, \lambda_1)$ from the line $\frac{x-5}{1} = \frac{y-1}{2} = \frac{z+7}{2}$ is _____

Ans. **[09.00]**

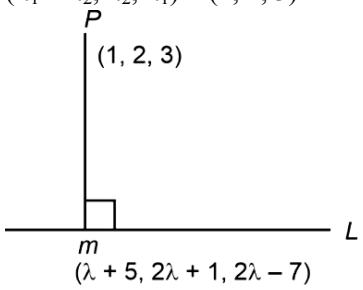
Sol. $\left|\frac{-4+0-6+7}{7}\right| = \left|\frac{15-6\lambda}{7}\right|$

$\frac{3}{7} = \left|\frac{15-6\lambda}{7}\right|$

$\lambda = 2$ or 3

$\lambda_1 = 3, \lambda_2 = 2$

$(\lambda_1 - \lambda_2, \lambda_2, \lambda_1) = (1, 2, 3)$



$\overrightarrow{PM}.(\hat{i}+2\hat{j}+2k) = 0$

$(\lambda + 4) + 2(2\lambda - 1) + 2(2\lambda - 10) = 0$

$\Rightarrow 9\lambda = 18$ or $\lambda = 2$

Distance $= \sqrt{6^2+3^2+6^2} = 9$

Q.26 The largest natural number n such that 3^n divides 66! is _____.

Ans. **[31.00]**

Sol. Exponent of 3 in 66!

$= \left[\frac{66}{3}\right] + \left[\frac{66}{3^2}\right] + \left[\frac{66}{3^3}\right] + ...$

$= 22 + 7 + 2 = 31$

Q.27 Let $\vec{a} = 6\hat{i}+9\hat{j}+12\hat{k}$, $\vec{b} = \alpha\hat{i}+11\hat{j}-2\hat{k}$ and $\vec{c}$ be vectors such that $\vec{a}\times\vec{c} = \vec{a}\times\vec{b}$. If $\vec{a}.\vec{c} = -12$, $\vec{c}.(\hat{i}-2\hat{j}+\hat{k}) = 5$, then $\vec{c}.(\hat{i}+\hat{j}+\hat{k})$ is equal to _____.

Ans. **[11.00]**

Sol. $\vec{a}\times(\vec{c}-\vec{b}) = 0$

$\lambda\vec{a} = \vec{c}-\vec{b}$

$\Rightarrow \vec{c} = \vec{b}+\lambda\vec{a}$ or $\vec{a}.\vec{c} = \vec{a}.\vec{b}+\lambda|\vec{a}|^2$

$\therefore -12 = (6\alpha + 75) + \lambda(261)$

or $2\alpha + 87\lambda = -29$ .....(i)

$\vec{c}.(\hat{i}-2\hat{j}+\hat{k}) = 5$

$\vec{c} = \hat{i}(\alpha+6\lambda)+\hat{j}(11+9\lambda)+\hat{k}(-2+12\lambda)$

$\Rightarrow (\alpha+6\lambda)-2(11+9\lambda)+(-2+12\lambda) = 5$

or $\alpha - 24 = 5$ or $\boxed{\alpha = 29} \Rightarrow \boxed{\lambda = -1}$

$\vec{c} = \hat{i}(23)+\hat{j}(2)+\hat{k}(-14)$

$\vec{c}.(\hat{i}+\hat{j}+\hat{k}) = 11$

Q.28 Let A = {0, 3, 4, 6, 7, 8, 9, 10} and R be the relation defined on A such that R = {(x, y) ∈ A × A : x – y is odd positive integer or x – y = 2}. The minimum number of elements that must be added to the relation R, so that it is a symmetric relation, is equal to _____.

Ans. **[19]**

Sol. A = {10, 9, 8, 7, 6, 4, 3, 0}

R = {(10, 9), (10, 8), (10, 7), (10, 3), (9, 8), (9, 7), (9, 6), (9, 4), (9, 0), (8, 7), (8, 6), (8, 3), (7, 6), (7, 4), (7, 0), (6, 4), (6, 3), (4, 3), (3, 0)}

All the elements of R, (a, b) are of type a > b.

Hence we need to add total of 19 more elements to R to make in symmetric.

Q.29 Consider a circle C_1: $x^2 + y^2 – 4x – 2y = \alpha – 5$. Let its mirror image in the line y = 2x + 1 be another circle C_2 : $5x^2 + 5y^2 – 10fx – 10gy + 36 = 0$. Let r be the radius of C_2. Then α + r is equal to _____.

Ans. **[2]**

Sol. $C_1 : x^2 + y^2 – 4x – 2y + (5 – \alpha) = 0$

centre $O_1 = (2, 1)$, radius = $\sqrt{\alpha}$

$C_2 : x^2 + y^2 – 2fx – 2gy + \frac{36}{5} = 0$

Centre $O_2 = (f, g,)$ radius = $r = \sqrt{f^2 + g^2 - \frac{36}{5}}$

O_2 is reflection of O_1 in 2x – y + 1 = 0

$$\Rightarrow \frac{f-2}{2} = \frac{g-1}{-1} = -2.\left(\frac{2\times 2-1+1}{2^2+1^2}\right) = \frac{-8}{5}$$

$f = \frac{-6}{5}$ and $g = \frac{13}{5}$

⇒ r = 1 and α = 1

Q.30 Let [t] denote the greatest integer ≤ t. Then $\frac{2}{\pi}\int_{\pi/6}^{5\pi/6} [8([\text{cosec}x] – 5[\cot x])dx$ is equal to _____.

Ans. **[14]**

Sol. $I = \int_{\pi/6}^{5\pi/6} (8.[\text{cosec}x] – 5[\cot x])dx = 8.\int_{\pi/6}^{5\pi/6} [\text{cosec}x]dx – 5.\int_{\pi/6}^{5\pi/6} [\cot x]dx = 8I_1 – 5I_2$

$I_1 = \int_{\pi/6}^{5\pi/6} [\text{cosec}x]dx = \int_{\pi/6}^{5\pi/6} 1.dx = \frac{2\pi}{3}$

$I_2 = \int_{\pi/6}^{5\pi/6} [\cot x]dx = \int_{\pi/6}^{\pi/4} 1.dx + \int_{\pi/4}^{\pi/2} 0.dx + \int_{\pi/2}^{3\pi/4} (-1)dx + \int_{3\pi/4}^{5\pi/6} (-2)dx = -\frac{\pi}{3}$

Required value = $\frac{2}{\pi}I = \frac{2}{\pi}\left[8.\frac{2\pi}{3} + 5.\frac{\pi}{3}\right] = 14$

PHYSICS

Section-A: Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Q.31 Two projectiles A and B are thrown with initial velocities of 40 m/s and 60 m/s at angles 30° and 60° with the horizontal respectively. The ratio of their ranges respectively is ($g = 10\ m/s^2$)

(1) 4 : 9 (2) $2 : \sqrt{3}$ (3) $\sqrt{3} : 2$ (4) 1 : 1

Ans. **[1]**

Sol. $R = \dfrac{u^2 \sin 2\theta}{g}$

$\Rightarrow$ Ratio $= \dfrac{40^2 \times \sin 60^\circ}{60^2 \times \sin 120^\circ} = \dfrac{4}{9} \times 1 = \dfrac{4}{9}$

Q.32 The engine of a train moving with speed 10 ms^{-1} towards a platform sounds a whistle at frequency 400 Hz. The frequency heard by a passenger inside the train is: (neglect air speed. Speed of sound in air = 330 ms^{-1})

(1) 400 Hz (2) 200 Hz (3) 412 Hz (4) 388 Hz

Ans. **[1]**

Sol. $f' = f_0\left[\dfrac{v - v_0}{v - v_s}\right] = 400\left[\dfrac{330 - 10}{330 - 10}\right] = 400\ Hz$

Q.33 In a reflecting telescope, a secondary mirror is used to:

(1) reduce the problem of mechanical support (2) make chromatic aberration zero

(3) move the eyepiece outside the telescopic tube (4) remove spherical aberration

Ans. **[3]**

Sol. A secondary mirror is used to move the eyepiece outside the telescopic tube.

Q.34 A charge particle moving in magnetic field B, has the components of velocity along B as well as perpendicular to B. The path of the charge particle will be

(1) helical path with the axis perpendicular to the direction of magnetic field B

(2) helical path with the axis along magnetic field B

(3) circular path

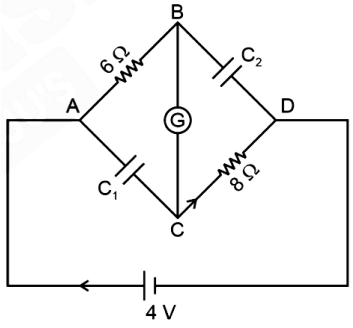
(4) straight along the direction of magnetic field B

Ans. **[2]**

Sol. Perpendicular component results in circular motion. Parallel component results in linear motion.

$\Rightarrow$Helical path with axis along magnetic field.

Q.35 In this figure the resistance of the coil of galvanometer G is 2 Ω. The emf of the cell is 4 V. The ratio of potential difference across C_1 and C_2 is:



(1) 1 (2) $\dfrac{4}{5}$ (3) $\dfrac{5}{4}$ (4) $\dfrac{3}{4}$

Ans. **[2]**

Sol. Capacitors would behave as open circuits

$\Rightarrow \quad V_{C_1} = i[6\Omega + R_G]$

and $V_{C_2} = i[R_G + 8\Omega]$

$\Rightarrow$ Ratio $= \dfrac{8}{10} = \dfrac{4}{5}$

Q.36 A cylindrical wire of mass (0.4 ± 0.01) g has length (8 ± 0.04) cm and radius (6 ± 0.03) mm. The maximum error in its density will be -

(1) 3.5% (2) 5% (3) 1% (4) 4%

Ans. **[4]**

Sol. Density $\rho = \dfrac{M}{\pi R^2 L}$

$\Rightarrow \quad \dfrac{d\rho}{\rho} = \dfrac{dM}{M} + \dfrac{2dR}{R} + \dfrac{dL}{L}$

$= \left[\dfrac{0.01}{0.4} + \dfrac{2\times 0.03}{6} + \dfrac{0.04}{8}\right] \times 100$

$= 2.5 + 1 + 0.5\%$

$= 4\%$

Q.37 Dimension of $\dfrac{1}{\mu_0 \varepsilon_0}$ should be equal to -

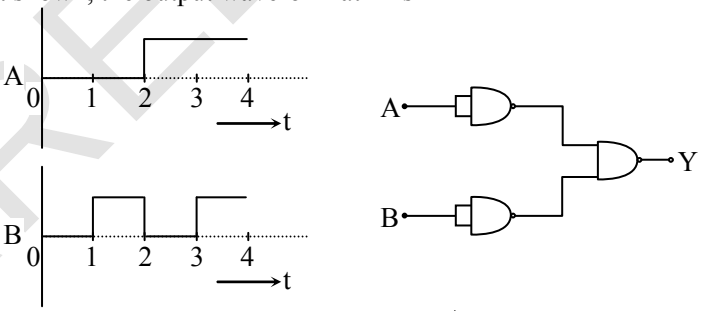
(1) L/T (2) T^2/L^2 (3) L^2/T^2 (4) T/L

Ans. **[3]**

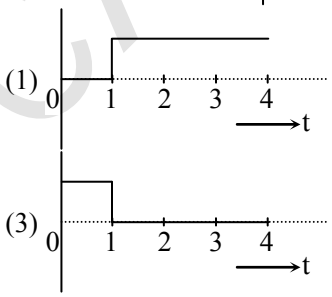
Sol. We know that $c = \dfrac{1}{\sqrt{\mu_0 \varepsilon_0}}$

$\Rightarrow \quad \dfrac{1}{\mu_0 \varepsilon_0} = [LT^{-1}][LT^{-1}] = L^2/T^2$

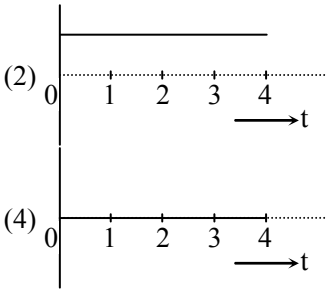
Q.38 For the logic circuit shown, the output waveform at Y is –



(1)



(2)



(3)

(4)

Ans. **[1]**

Sol. $y = (A' . B') = A + B$

$\Rightarrow$ OR gate

$\Rightarrow$ Correct waveform is first

Q.39 An air bubble of volume 1 cm^3 rises from the bottom of a lake 40 m deep to the surface at a temperature of 12°C. The atmospheric pressure is 1×10^5 Pa, the density of water is 1000 kg/m^3 and g = 10 m/s^2. There is no difference of the temperature of water at the depth of 40 m and on the surface. The volume of air bubble when it reaches the surface will be

(1) 2 cm^3 (2) 3 cm^3 (3) 4 cm^3 (4) 5 cm^3

Ans. **[4]**

Sol. $P_{bottom} = P_0 + \rho gh$

$= 10^5 + 1000 \times 10 \times 40 = 5 \times 10^5$

$P_{top} = 10^5$

PV = constant

$\Rightarrow V_{top} = 5 . V_{bottom} = 5\ cm^3$

Q.40 For a nucleus ${}^{A}_{Z}X$ having mass number A and atomic number Z

A. The surface energy per nucleon $(b_s) = -a_1A^{2/3}$

B. The Coulomb contribution to the binding energy $b_c = -a_2\frac{Z(Z-1)}{A^{4/3}}$

C. The volume energy $b_v = a_3A$

D. Decrease in the binding energy is proportional to surface area.

E. While estimating the surface energy, it is assumed that each nucleon interacts with 12 nucleons. (a_1, a_2 and a_3 are constants)

Choose the **most appropriate** answer from the options given below:

(1) B, C, E only (2) C, D only (3) A, B, C, D only (4) B, C only

Ans. **[2]**

Sol. $R = R_0A^{\frac{1}{3}}$

$\Rightarrow$ Surface area $\propto R^2 \propto A^{\frac{2}{3}}$

For coulomb contribution, we choose pairs of protons

$\Rightarrow \propto (Z)(Z-1)$

Volume $\propto R^3 \propto A^1$

$\Rightarrow$ Option (2) is correct

Q.41 Given below are two statements:

Statement I: If heat is added to a system, its temperature must increase.

Statement II: If positive work is done by a system in a thermodynamic process, its volume must increase.

In the light of the above statements, choose the **correct** answer from the options given below

(1) Statement I is true but Statement II is false (2) Both Statement I and Statement II are false

(3) Both Statement I and Statement II are true (4) Statement I is false but Statement II is true

Ans. **[4]**

Sol. Heat capacity can be negative

$\Rightarrow$ Statement I is wrong.

$W = \int PdV$

$\Rightarrow$ If $W > 0$, Volume must increase

Q.42 At any instant the velocity of a particle of mass 500 g is $(2t\hat{i} + 3t^2\hat{j})ms^{-1}$. If the force acting on the particle at t = 1s is $(\hat{i} + x\hat{j})$ N. Then the value of x will be:

(1) 3 (2) 4 (3) 2 (4) 6

Ans. **[1]**

Sol. $\vec{V} = 2t\hat{i} + 3t^2\hat{j}$

$\Rightarrow \quad \vec{a} = 2\hat{i} + 6t\hat{j}$

$\Rightarrow \quad \vec{F} = m\vec{a} = \frac{1}{2}\vec{a} = \hat{i} + 3t\hat{j}$

At $t = 1, \ \vec{F} = \hat{i} + 3\hat{j}$

Q.43 Given below are two statements:

Statement I: If E be the total energy of a satellite moving around the earth, then its potential energy will be $\frac{E}{2}$.

Statement II: The kinetic energy of a satellite revolving in an orbit is equal to the half the magnitude of total energy E.

In the light of the above statements, choose the most appropriate answer from the options given below

(1) Statement I is correct but Statement II is incorrect

(2) Statement I is incorrect but Statement II is correct

(3) Both Statement I and Statement II are correct

(4) Both Statement I and Statement II are incorrect

Ans. **[4]**

Sol. $U = -\frac{GMm}{r}$

$K = \frac{GMm}{2r}$

$T = K + U = -\frac{GMm}{2r}$

$\Rightarrow$ Both statements are false

Q.44 An aluminium rod with Young's modulus $Y = 7.0 \times 10^{10}$ N/m^2 undergoes elastic strain of 0.04%. The energy per unit volume stored in the rod in SI unit is:

(1) 2800 (2) 11200 (3) 5600 (4) 8400

Ans. **[3]**

Sol. Energy/Volume = $\frac{1}{2} \times$ stress $\times$ strain

$= \frac{1}{2} \times Y \times (\text{strain})^2$

$= \frac{1}{2} \times 7 \times 10^{10} \times \left(\frac{0.04}{100}\right)^2$

$= \frac{1}{2} \times 7 \times 10^{10} \times 16 \times 10^{-8}$

$= 5600$

Q.45 A TV transmitting antenna is 98 m high and the receiving antenna is at the ground level. If the radius of the earth is 6400 km, the surface area covered by the transmitting antenna is approximately:

(1) 1240 km (2) 3942 km (3) 4868 km (4) 1549 km

Ans. **[2]**

Sol. $d = \sqrt{2Rh}$

$\Rightarrow$ Area $= \pi d^2 = \pi \times 2 \times R \times h = 3940\ km^2$

Q.46 Two forces having magnitude A and $\frac{A}{2}$ are perpendicular to each other. The magnitude of their resultant is:

(1) $\frac{\sqrt{5}A}{4}$ (2) $\frac{\sqrt{5}A}{2}$ (3) $\frac{5A}{2}$ (4) $\frac{\sqrt{5}A^2}{2}$

Ans. **[2]**

Sol. $A_{net} = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\theta}$

$\Rightarrow F_{net} = \sqrt{A^2 + \frac{A^2}{4}} = \frac{\sqrt{5}A}{2}$

Q.47 Certain galvanometers have a fixed core made of non magnetic metallic material. The function of this metallic material is

(1) to oscillate the coil in magnetic field for longer period of time

(2) to bring the coil to rest quickly

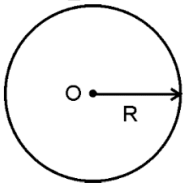
(3) to produce large deflecting torque on the coil

(4) to make the magnetic field radial

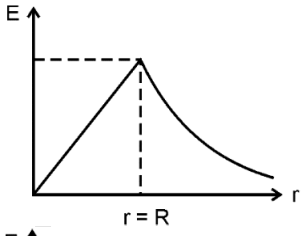
Ans. **[2]**

Sol. Function of magnetic material is to bring the coil to rest quickly through eddy currents.

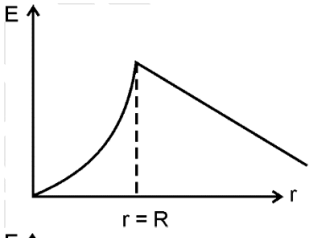
Q.48 Graphical variation of electric field due to a uniformly charged insulating solid sphere of radius R, with distance r from the centre O is represented by:



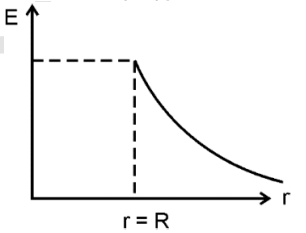
(1)



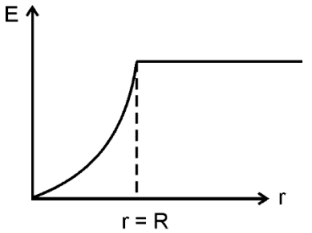
(2)



(3)



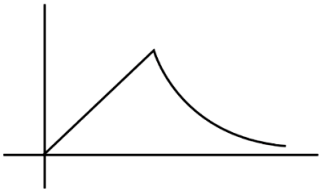
(4)



Ans. **[1]**

Sol. $E_{inside} = \frac{\rho r}{3\varepsilon_0} \propto r$

$E_{outside} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2} \propto \frac{1}{r^2}$

$\Rightarrow$ 

Q.49 Proton (P) and electron (e) will have same de-Broglie wavelength when the ratio of their momentum is (assume, $m_p = 1849\ m_e$) :

(1) 1 : 1 (2) 1 : 1849 (3) 1 : 43 (4) 43 : 1

Ans. **[1]**

Sol. $\lambda = \frac{h}{mv}$

$\Rightarrow$ If $\lambda = \lambda'$ then $p = p'$

$\Rightarrow$ Option 1

Q.50 The weight of a body on the earth is 400 N. Then weight of the body when taken to a depth half of the radius of the earth will be :

(1) 200 N (2) Zero (3) 100 N (4) 300 N

Ans. **[1]**

Sol. $g' = g\left(1 - \frac{d}{R}\right)$

$\Rightarrow W' = W\left(1 - \frac{\frac{R}{2}}{R}\right) = 200\ N$

Section-B: Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE**. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

Q.51 The moment of inertia of a semicircular ring about an axis, passing through the center and perpendicular to the plane of ring, is $\frac{1}{x}MR^2$, where R is the radius and M is the mass of the semicircular ring. The value of x will be _____.

Ans. **[1]**

Sol. $I = MR^2$

$\Rightarrow x = 1$

Q.52 An air bubble of diameter 6 mm rises steadily through a solution of density 1750 kg/m^3 at the rate of 0.35 cm/s. The co-efficient of viscosity of the solution (neglect density of air) is _____ Pas (given, $g = 10\ ms^{-2}$)

Ans. **[10]**

Sol. Buoyancy = $6\pi\eta r v_T$

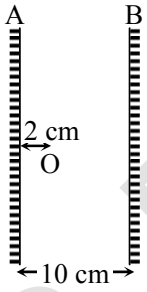
$\Rightarrow \frac{4}{3}\pi r^3 \rho g = 6\pi\eta r v_T$

$\Rightarrow \eta = \frac{\frac{4}{3} r^2 \rho g}{6 v_T}$

$= \frac{2}{9} \times (3\times10^{-3})^2 \times \frac{1750\times10}{\frac{0.35}{100}}$

$= 2\times10^{-6} \times \frac{175\times10000}{0.35} = 10$

Q.53 Two vertical parallel mirrors A and B are separated by 10 cm. A point object O is placed at a distance of 2 cm from mirror A. The distance of the second nearest image behind mirror A from the mirror A is _____ cm.



Ans. **[18]**

Sol. Object distance = 8 cm [For image by B]

⇒Required distance = 8 + 8 + 2 cm

= 18 cm

Q.54 An oscillating LC circuit consists of a 75 mH inductor and a 1.2 μF capacitor. If the maximum charge to the capacitor is 2.7 μC. The maximum current in the circuit will be _____ mA.

Ans. **[9]**

Sol. $i_{max} = q_{max} \cdot \omega$

$= 2.7 \times 10^{-6} \times \frac{1}{\sqrt{LC}}$

$= \frac{2.7\times10^{-6}}{\sqrt{75\times10^{-3}\times1.2\times10^{-6}}} A$

$= \frac{2.7\times10^{-6}}{30\times10^{-5}} A = 9 \text{ mA}$

Q.55 An organ pipe 40 cm long is open at both ends. The speed of sound in air is 360 ms^{-1}. The frequency of the second harmonic is _____ Hz.

Ans. **[900]**

Sol. $f = 2.\frac{v}{2L}$

$= 2.\frac{360}{2\times0.4}$ Hz = 900 Hz

Q.56 An electric dipole of dipole moment is 6.0×10^{-6} Cm placed in a uniform electric field of 1.5×10^{3} NC^{-1} in such a way that dipole moment is along electric field. The work done in rotating dipole by 180° in this field will be_____mJ.

Ans. **[18]**

Sol. $W = \Delta U = U_f - U_i$

Also, $U = -\ \vec{p}.\vec{E}$

$\Rightarrow W = 2 \times p \times E = 2 \times 6 \times 10^{-6} \times 1.5 \times 10^{3}$ J

$= 18$ mJ

Q.57 The magnetic intensity at the center of a long current carrying solenoid is found to be 1.6×10^{3} Am^{-1}. If the number of turns is 8 per cm, then the current flowing through the solenoid is _____A.

Ans. **[2]**

Sol. $\frac{B}{\mu_0} = ni$

$\Rightarrow 1.6 \times 1000 = \frac{8}{1/100} \times i$

$\Rightarrow i = 2A$

Q.58 The momentum of a body is increased by 50%. The percentage increase in the kinetic energy of the body is_____%.

Ans. **[125]**

Sol. $K = \frac{p^2}{2m}$

$\Rightarrow K' = \frac{(1.5p)^2}{2m} = 2.25\ k$

$\Rightarrow \frac{K'-K}{K} \times 100 = 125$

Q.59 A nucleus with mass number 242 and binding energy per nucleon as 7.6 MeV breaks into two fragment each with mass number 121. If each fragment nucleus has binding energy per nucleon as 8.1 MeV, the total gain in binding energy is _____ MeV.

Ans. **[121]**

Sol. BE = (8.1 – 7.6) × 242 MeV = 121 MeV

Q.60 A current of 2 A flows through a wire of cross sectional area 25.0 mm^2. The number of free electrons in a cubic meter are 2.0×10^{28}. The drift velocity of the electrons is _____ $\times 10^{-6}$ ms^{-1} (given, charge on electron = 1.6×10^{-19} C)

Ans. **[25]**

Sol. $I = nAev_d$

$\Rightarrow 2 = 2 \times 10^{28} \times 25 \times 10^{-6} \times 1.6 \times 10^{-19} \times v_d$

$\Rightarrow v_d = \frac{1}{25 \times 1.6 \times 1000}$ m/s $= 25 \times 10^{-6}$ m/s

CHEMISTRY

Section-A: This section contains 20 multiple choice questions. Each question has 4 choices(1), (2), (3) and (4), out of which **ONLY ONE** is correct..

Q.61 In chromyl chloride, the number of d-electrons present on chromium is same as in (Given at no. of Ti : 22, V : 23, Cr : 24, Mn : 25, Fe : 26)

(1) V (IV) (2) Mn (VII) (3) Fe (III) (4) Ti (III)

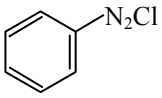
Ans. **[2]**

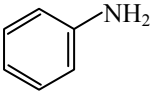
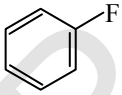
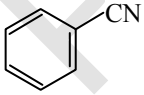
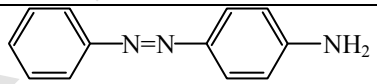
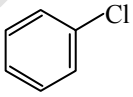
Sol. Chromyl chloride $\Rightarrow CrO_2Cl_2$

O.S. of Cr = +6

$\therefore$ d^0 configuration

Mn (+7) = d^0

Q.62 Match List I with List II :  is reacted with reagents in List I to form products in List II.

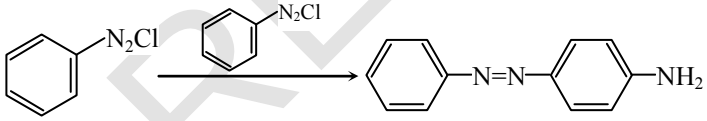
	List-I (Reagent)		List-II (Product)
A.		I.	
B.	HBF_4, Δ	II.	
C.	Cu, HCl	III.	
D.	CuCN/KCN	IV.	

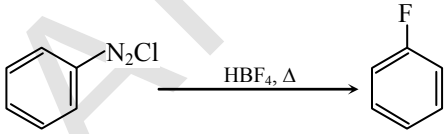
Choose the correct answer from the options given below:

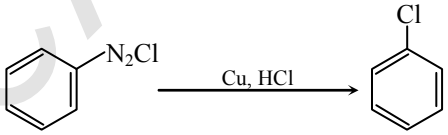
(1) A-IV, B-III, C-II, D-I (2) A-III, B-I, C-II, D-IV (3) A-I, B-III, C-IV, D-II (4) A-III, B-I, C-IV, D-II

Ans. **[4]**

Sol.

A. 

B. 

C. 

D. 

Q.63 The water gas on reacting with cobalt as a catalyst forms

(1) Methanal (2) Methanoic acid (3) Ethanol (4) Methanol

Ans. **[4]**

Sol. $\underbrace{CO(g) + 2H_2(g)}_{\text{Water gas}} \xrightarrow[\text{Catalyst}]{\text{Cobalt}} \underset{\text{Methanol}}{CH_3OH(\ell)}$

Q.64 Which of the following complex is octahedral, diamagnetic and the most stable?

(1) $Na_3[CoCl_6]$ (2) $[Ni(NH_3)_6]Cl_2$ (3) $K_3[Co(CN)_6]$ (4) $[Co(H_2O)_6]Cl_2$

Ans. **[3]**

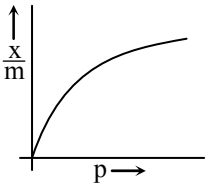
Sol. (1) $Na_3[CoCl_6]$ – Paramagnetic

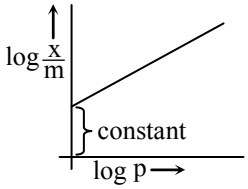
(2) $[Ni(NH_3)_6]Cl_2$ – Paramagnetic

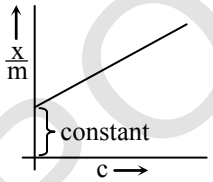
(3) $K_3[Co(CN)_6]$ – Diamagnetic

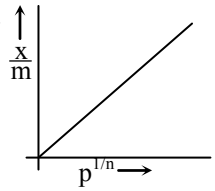
(4) $[Co(H_2O)_6]Cl_2$ – Paramagnetic

Q.65 Which of the following represents the Freundlich adsorption isotherms ?

(A) 

(B) 

(C) 

(D) 

Choose the correct answer from the options given below:

(1) A, C, D only (2) A, B only (3) B, C, D only (4) A, B, D only

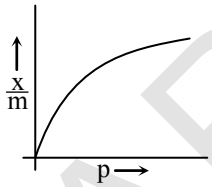
Ans. **[4]**

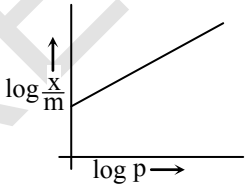
Sol. Freundlich adsorption isotherm equation is

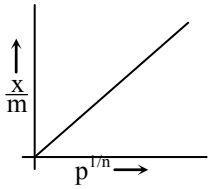
$$\frac{x}{m} = KP^{1/n} \qquad .....(i)$$

$$\Rightarrow \log\frac{x}{m} = \log K + \frac{1}{n}\log P \qquad .....(ii)$$

With the help of the above two equation, following plots are obtained







Q.66 Given below are two statements : One is labelled as Assertion A and the other is labelled as Reason R.

Assertion A : Butan–1–ol has higher boiling point than ethoxyethane.

Reason R : Extensive hydrogen bonding leads to stronger association of molecules.

In the light of the above statements, choose the **correct** answer from the options given below :

(1) A is true but R is false

(2) Both A and R are true and R is the correct explanation of A

(3) Both A and R are true but R is not the correct explanation of A

(4) A is false but R is true

Ans. **[2]**

Sol. Owing to intermolecular hydrogen bonding in butanol, it has higher boiling point than ethoxyethane.

Q.67 Match List I with List II :

	List-I (Reagent)		**List-II (Product)**
A.	Saccharin	I.	High potency sweetener
B.	Aspartame	II.	First artificial sweetening agent
C.	Alitame	III.	Stable at cooking temperature
D.	Sucralose	IV.	Unstable at cooking temperature

Choose the correct answer from the options given below :

(1) A-IV, B-III, C-I, D-II (2) A-II, B-III, C-IV, D-I (3) A-II, B-IV, C-III, D-I (4) A-II, B-IV, C-I, D-III

Ans. **[4]**

Sol. A→II, B→IV, C→I, D→III

Saccharin is the first popular artificial sweetening agent.

Aspartame use is limited to cold foods and soft drinks because it is unstable at cooking temperature.

Alitame is high potency sweetener.

Sucralose is stable at cooking temperature.

Q.68 The reaction

$$\frac{1}{2}H_2(g) + AgCl(s) \rightleftharpoons H^+(aq) + Cl^-(aq) + Ag(s)$$

occurs in which of the given galvanic cell ?

(1) $Pt|H_2(g)|HCl(sol^n)|AgCl(s)|Ag$ (2) $Ag|AgCl(s)|KCl(sol^n)|AgNO_3|Ag$

(3) $Pt|H_2(g)|HCl(sol^n)|AgNO_3(sol^n)|Ag$ (4) $Pt|H_2(g)|KCl(sol^n)|AgCl(s)|Ag$

Ans. **[1]**

Sol. At anode : $\frac{1}{2}H_2(g) \longrightarrow H^+(aq) + e^-$

At cathode : $AgCl_{(s)} + \bar{e} \longrightarrow Ag + Cl^-_{(aq)}$

Overall reaction

$$\frac{1}{2}H_2(g) + AgCl(s) \longrightarrow H^+ + Cl^- + Ag(s)$$

The cell representation for the above cell will be $Pt|H_2(g)|HCl(sol^n)|AgCl(s)|Ag$

Q.69 Given below are two statements :

Statement I : Lithium and Magnesium do not form superoxide

Statement II : The ionic radius of Li^+ is larger than ionic radius of Mg^{2+}

In the light of the above statements, choose the most appropriate answer from the questions given below :

(1) Statement I is incorrect but Statement II is correct

(2) Statement I is correct but Statement II is incorrect

(3) Both statement I and Statement II are incorrect

(4) Both Statement I and Statement II are correct

Ans. **[4]**

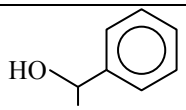
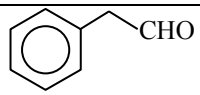
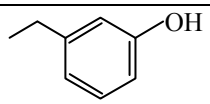
Sol. Ionic radius of Li^+ = 76 pm

Ionic radius of Mg^{2+} = 72 pm

Both Li^+ and Mg^{2+} do not form superoxide because their ionic radius is very small.

Hence, both the given statements are correct.

Q.70 Match List I with List II :

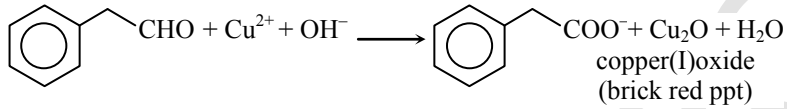
	List-I (Reagents used)		**List-II (Compound with Functional group detected)**
A.	Alkaline solution of copper sulphate and sodium citrate	I.	
B.	Neutral $FeCl_3$ solution	II.	
C.	Alkaline chloroform solution	III.	
D.	Potassium iodide and sodium hypochloride	IV.	

Choose the correct answer from the options given below:

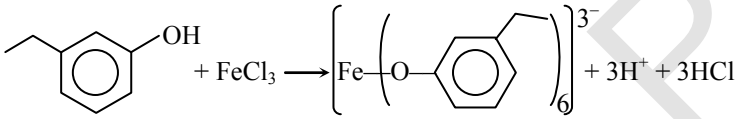
(1) A-III, B-IV, C-I, D-II (2) A-III, B-IV, C-II, D-I (3) A-IV, B-I, C-II, D-III (4) A-II, B-IV, C-III, D–I

Ans. **[2]**

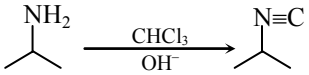
Sol.

A. 

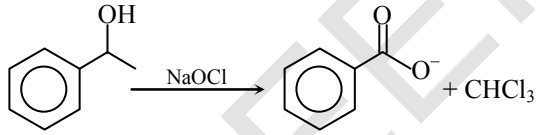
copper(I)oxide
(brick red ppt)

B. 

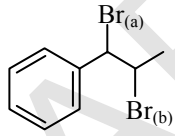
Violet complex

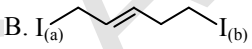
C. 

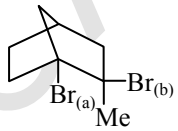
Carbylamine reaction

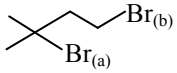
D. 

Q.71 Choose the halogen which is most reactive towards S_N1 reaction in the given compounds (A, B, C & D)

A. 

B. 

C. 

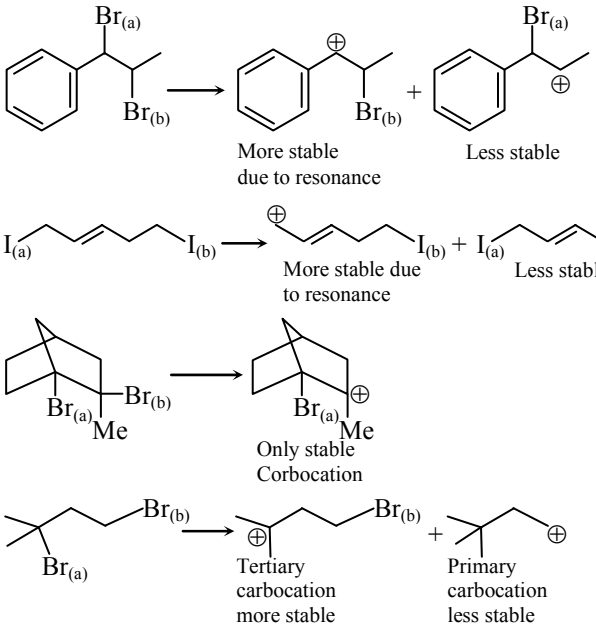
D. 

(1) A – $Br_{(b)}$; B – $I_{(a)}$; C – $Br_{(a)}$; D – $Br_{(a)}$ (2) A – $Br_{(b)}$; B – $I_{(b)}$; C – $Br_{(b)}$; D – $Br_{(b)}$

(3) A – $Br_{(a)}$; B – $I_{(a)}$; C – $Br_{(b)}$; D – $Br_{(a)}$ (4) A – $Br_{(a)}$; B – $I_{(a)}$; C – $Br_{(a)}$; D – $Br_{(a)}$

Ans. **[3]**

Sol. The leaving group which results in the formation of more stable carbocation will be more reactive towards S_N1 reaction.



Hence, the correct answer is option (3).

Q.72 The correct order of spin only magnetic moments for the following complex ions is

(1) $[Fe(CN)_6]^{3-} < [CoF_6]^{3-} < [MnBr_4]^{2-} < [Mn(CN)_6]^{3-}$

(2) $[CoF_6]^{3-} < [MnBr_4]^{2-} < [Fe(CN)_6]^{3-} < [Mn(CN)_6]^{3-}$

(3) $[Fe(CN)_6]^{3-} < [Mn(CN)_6]^{3-} < [CoF_6]^{3-} < [MnBr_4]^{2-}$

(4) $[MnBr_4]^{2-} < [CoF_6]^{3-} < [Fe(CN)_6]^{3-} < [Mn(CN)_6]^{3-}$

Ans. **[3]**

Sol. $[MnBr_4]^{2-} = Mn^{2+}(td) = e^2t_2^3 = 5$ unpaired electrons

$\therefore \mu = \sqrt{5(5+2)} = \sqrt{35}$ B.M.

$[CoF_6]^{3-} = Co^{3+}(Oh)$

(High spin)

$t_{2g}^4 eg^2 = 4$ unpaired electrons

$\therefore \mu = \sqrt{4(4+2)} = \sqrt{24}$ B.M.

$[Mn(CN)_6]^{3-} = Mn^{3+}(Oh) = t_{2g}^4 eg^0$

(Low spin)

= 2 unpaired electrons

$\therefore \mu = \sqrt{2(2+2)} = \sqrt{8}$ B.M.

$[Fe(CN)_6]^{3-} = Fe^{3+}(Oh) = t_{2g}^5 eg^0 = 1$ unpaired electron

(Low spin)

$\therefore \mu = \sqrt{1(1+2)} = 2\sqrt{3}$ B.M

Hence the correct answer is option 3

Q.73 What is the purpose of adding gypsum to cement ?

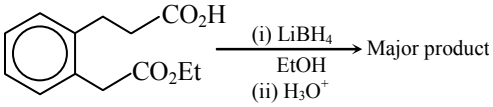
(1) To facilitate the hydration of cement (2) To slow down the process of setting

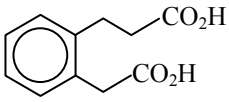
(3) To give a hard mass (4) To speed up the process of setting

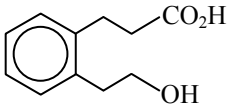
Ans. **[2]**

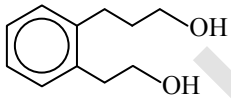
Sol. When mixed with water the setting of cement takes place to give a hard mass. The is due to the hydration of molecules of the constituents and their rearrangement. The purpose of adding gypsum is only to slow down the process of setting of the cement so that it gets sufficiently hardened.

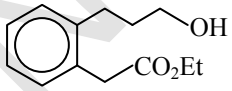
Q.74 The major product formed in the following reaction is



(1) 

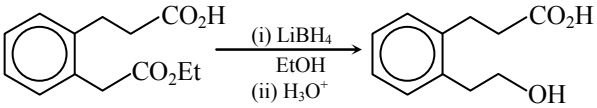
(2) 

(3) 

(4) 

Ans. **[2]**

Sol. Selective reduction of ester group.



Q.75 Match List I with List II :

	List-I (Species)		**List-II (Maximum allowed concentration in ppm in drinking water)**
A.	F^-	I.	< 50 ppm
B.	SO_4^{2-}	II.	< 5 ppm
C.	NO_3^-	III.	< 2 ppm
D.	Zn	IV.	< 500 ppm

Choose the **correct** answer from the options given below:

(1) A-I, B-II, C-III, D-IV (2) A-II, B-I, C-III, D-IV (3) A-IV, B-III, C-II, D-I (4) A-III, B-II, C-I, D-IV

Ans. **[Bonus]**

Sol.

	Species	**Maximum allowed concentration in ppm in drinking water**
A.	F^-	2
B.	SO_4^{2-}	500
C.	NO_3^-	50
D.	Zn	5

$A \rightarrow III$; $B \rightarrow IV$; $C \rightarrow I$, $D \rightarrow II$

Q.76 Sulphur (S) containing amino acids from the following are :

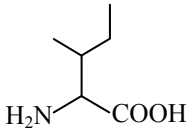
(a) isoleucine (b) cysteine (c) lysine (d) methionine

(e) glutamic acid

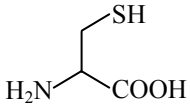
(1) b, c, e (2) a, b, c (3) b, d (4) a, d

Ans. **[3]**

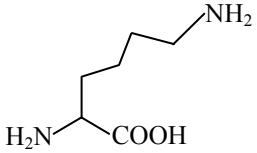
Sol. (a) Isoleucine –



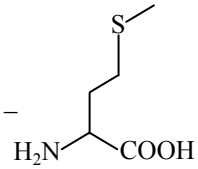
(b) Cystein –



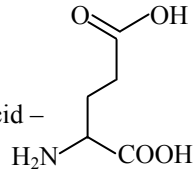
(c) Lysine –



(d) Methionine –



(e) Glutamic acid –



Q.77 Which halogen is known to cause the reaction given below:

$2Cu^{2+} + 4X^- \rightarrow Cu_2X_2(s) + X_2$

(1) All halogens (2) Only Bromine (3) Only Iodine (4) Only Chlorine

Ans. **[3]**

Sol. $2Cu^{2+} + 4I^- \rightarrow Cu_2I_2 + I_2$

Q.78 $2IO_3^- + xI^- + 12H^+ \rightarrow 6I_2 + 6H_2$

What is the value of x ?

(1) 2 (2) 12 (3) 10 (4) 6

Ans. **[3]**

Sol. $2IO_3^- + xI^- + 12H^+ \rightarrow 6I_2 + 6H_2O$

By balancing the charge, we can easily find out that x is 10.

Q.79 Which of the following metals can be extracted through alkali leaching technique ?

(1) Sn (2) Pb (3) Au (4) Cu

Ans. **[1]**

Sol. $SnO_2 + NaOH \rightarrow Na_2[Sn(OH)_6]$

Q.80 The correct order of electronegativity for given elements is

(1) P > Br > C > At (2) Br > P > At > C (3) Br > C > At > P (4) C > P > At > Br

Ans. **[3]**

Sol.

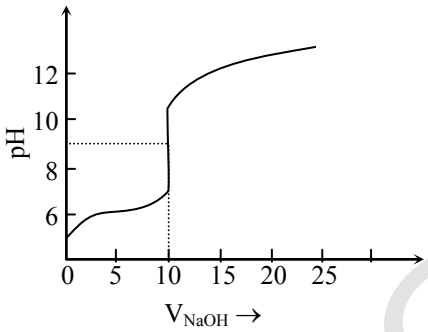
Element	Electronegativity
P	2.1
C	2.5
Br	3.0
At	2.2

Hence, the correct order is Br > C > At > P.

Section-B: Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer..

Q.81 The titration curve of weak acid vs. strong base with phenolphthalein as indicator) is shown below. The $K_{phenolphthalein} = 4 \times 10^{-10}$

Given: log2 = 0.3



The number of following statement/s which is/are correct about phenolphthalein is _____

(A) It can be used as an indicator for the titration of weak acid with weak base.

(B) It begins to change colour at pH = 8.4

(C) It is a weak organic base

(D) It is colourless in acidic medium

Ans. **[02]**

Sol. Phenolphthalein is an organic acid and can be represented as HPh.

$$\underset{\text{(colourless)}}{HPh} \rightleftharpoons H^+ + \underset{\text{(Pink)}}{Ph^-}$$

Using Henderson equation for phenolphthalein

$$pH = pK_{In} + \log\left[\frac{[Ph^-]}{[HPh]}\right]$$

At equivalence point $[Ph^-] = [HPh]$

$\therefore\ pH_2 = pK_{In} = -\log[4] + 10 = -0.6 + 10 = 9.4$

Hence at (9.4 ±1) PH, phenolphthalein starts changing colour.

Phenolphthalein is colourless in acidic medium and pink in basic medium.

Phenolphthalein indicator distinguish the pH change between 8 to 10. Therefore, it is used for strong acid and strong base titration or weak acid and strong base titration.

Hence, A and C are incorrect statements.

Q.82 The number of following statement/s which is/are incorrect is _____

(A) Line emission spectra are used to study the electronic structure

(B) The emission spectra of atoms in the gas phase show a continuous spread of wavelength from red to violet.

(C) An absorption spectrum is like the photographic negative of an emission spectrum

(D) The element helium was discovered in the sun by spectroscopic method

Ans. **[1]**

Sol. Except (B) all other statements are correct.

Q.83 When a 60 W electric heater is immersed in a gas for 100s in a constant volume container with adiabatic walls, the temperature of the gas rises by 5°C. The heat capacity of the given gas is _____ JK^{-1} (Nearest integer)

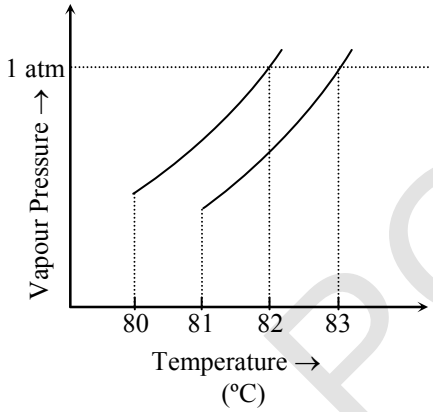
Ans. **[1200]**

Sol. $$\text{Heat capacity} = \frac{\text{Heat absorbed}}{\text{change in temperature}}$$

$$= \frac{60 \times 100\,J}{5}$$

$$= 1200\ JK^{-1}$$

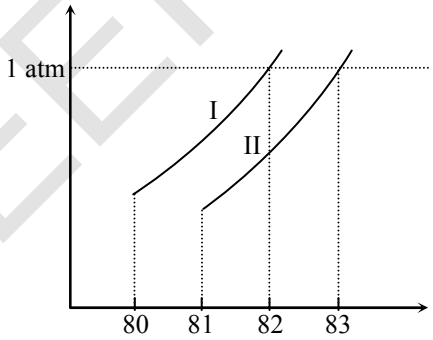
Q.84 The vapour pressure vs. temperature curve for a solution solvent system is shown below.



The boiling point of the solvent is _____°C.

Ans. **[82]**

Sol. We know that the temperature at which vapour pressure of a liquid becomes 1 atm. is called the boiling point of the liquid.



From the figure, we can clearly see that Graph I is for pure solvent whereas graph II is for solution. Hence the boiling point of solvent and solution are 82°C and 83°C respectively.

Q.85 0.5 g of an organic compound (X) with 60% carbon will produce _____ $\times 10^{-1}$ g of CO_2 on complete combustion.

Ans. **[11]**

Sol. $$C\% = \frac{12}{44} \times \frac{\text{Wt. of } CO_2}{\text{Wt. of organic compound}} \times 100$$

$$60 = \frac{12}{44} \times \frac{\text{Wt. of } CO_2}{0.5} \times 100$$

$$\text{Wt. of } CO_2 = 1.1$$

Q.86 The number of following factors which affect the percent covalent character of the ionic bond is_____

(A) Polarising power of cation (B) Extent of distortion of anion

(C) Polarisability of the anion (D) Polarising power of anion

Ans. **[3]**

Sol. Percentage covalent character of an ionic bond depends upon

- Polarising power of cation
- Extent of distortion of anion
- Polarisability of the anion

Q.87 XeF_4 reacts with SbF_5 to form $[XeF_m]^{n+}[SbFy]^{z-}$. m + n + y + z = _____?

Ans. **[11]**

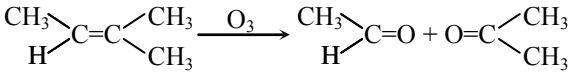
Sol. $XeF_4 + SbF_5 \rightarrow [XeF_3]^+ [SbF_6]^-$

$\therefore$ m = 3, n = 1, y = 6, z = 1

Q.88 Molar mass of the hydrocarbon (X) which on ozonolysis consumes one mole of O_3 per mole of (X) and gives one mole each of ethanal and propanone is _____ g mol^{-1} (Molar mass of C : 12 g mol^{-1}, H : 1 g mol^{-1})

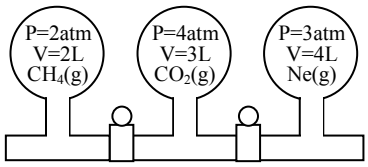
Ans. **[70]**

Sol.



M.wt = 70

Q.89



Three bulbs are filled with CH_4, CO_2 and Ne as shown in the picture. The bulbs are connected through pipes of zero volume. When the stopcocks are opened and the temperature is kept constant throughout, the pressure of the system is found to be_____ atm. (Nearest integer).

Ans. **[3]**

Sol. Using the formula

$P_1V_1 + P_2V_2 + P_3V_3 = P_fV_f$

$2 \times 2 + 4 \times 3 + 3 \times 4 = P_f \times 9$

$P_f = \frac{28}{9} \approx 3$

Q.90 The number of given statement/s which is/are correct is_____

(A) The stronger the temperature dependence of the rate constant, the higher is the activation energy.

(B) If a reaction has zero activation energy, its rate is independent of temperature.

(C) The stronger the temperature dependence of the rate constant, the smaller is the activation energy.

(D) If there is no correlation between the temperature and the rate constant then it means that the reaction has negative activation energy.

Ans. **[2]**

Sol. Rate constant is given by Arrhenius equation

$k = Ae^{-Ea/RT}$

Using the above equation, we can clearly see that only option (A) and (B) are correct.