JEE Main Online Exam 2023

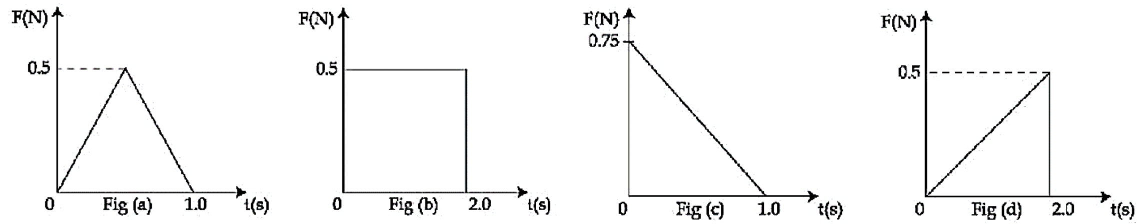
Questions & Solution

1st February 2023 | Evening

PHYSICS

Section-A: This section contains 20 multiple choice questions. Each question has 4 choices(1), (2), (3) and (4), out of which **ONLY ONE** is correct..

Q.1 Figures (a), (b), (c) and (d) show variation of force with time.



The impulse is highest in

(1) Fig. (d) (2) Fig. (c) (3) Fig. (b) (4) Fig (a)

Ans. **[3]**

Sol. Impulse $\propto$ area under (F-t) graph.

Impulse for graph (b) is maximum.

Q.2 Given below are two statements: One is labelled as Assertion A and the other is labelled as Reason R.

Assertion A : Two metallic spheres are charged to the same potential. One of them is hollow and another is solid, and both have the same radii. Solid sphere will have lower charge than the hollow one.

Reason R: Capacitance of metallic spheres depend on the radii of spheres.

In the light of the above statements, choose the correct answer from the options given below:

(1) Both A and R are true but R is not the correct explanation of A

(2) A is false but R is true

(3) Both A and R are true but R is the correct explanation of A

(4) A is true but R is false

Ans. **[2]**

Sol. Assertion (A) is incorrect

Both spheres will have same charge.

Reason (R) is correct.

Q.3 A coil is placed in magnetic field such that plane of coil is perpendicular to the direction of magnetic field. The magnetic flux through a coil can be changed:

A. By changing the magnitude of the magnetic field within the coil.

B. By changing the area of coil within the magnetic field.

C. By changing the angle between the direction of magnetic field and the plane of the coil.

D. By reversing the magnetic field direction abruptly without changing its magnitude.

Choose the most appropriate answer from the options given below:

(1) A and B only (2) A, B and D only (3) A and C only (4) A, B and C only

Ans. **[4]**

Sol. Flux can be changed by changing magnetic field, area of coil, change the angle between B and A.

Q.4 An electron of a hydrogen like atom, having Z = 4, jumps from 4th energy state to 2^{nd} energy state. The energy released in this process, will be: (Given Rch = 13.6 eV)
Where R = Rydberg constant, c = Speed of light in vacuum, h = Planck's constant)

(1) 3.4 eV (2) 10.5 eV (3) 40.8 eV (4) 13.6 eV

Ans. **[3]**

Sol. $E = 13.6Z^2\left[\frac{1}{2^2} - \frac{1}{4^2}\right]$

$= 13.6 \times 16 \times \left[\frac{1}{4} - \frac{1}{16}\right]$

= 40.8 eV

Q.5 Given below are two statements: One is labelled as Assertion A and the other is labelled as Reason R.

Assertion A: For measuring the potential difference across a resistance of 600 Ω, the voltmeter with resistance 1000 Ω will be preferred over voltmeter with resistance 4000 Ω.

Reason R: Voltmeter with higher resistance will draw smaller current than voltmeter with lower resistance.

In the light of the above statements, choose the most appropriate answer from the options given below:

(1) Both **A** and **R** are correct but **R** is **not** the correct explanation of **A**

(2) Both **A** and **R** are correct but **R** is the correct explanation of **A**

(3) **A** is not correct but **R** is correct

(4) **A** is correct but **R** is not correct

Ans. **[3]**

Sol. (A) is not correct because we need higher voltmeter resistance.
(R) is correct based on reason given above.

Q.6 Choose the correct statement about Zener diode

(1) It works as a voltage regulator only in forward bias

(2) It works as a voltage regulator in forward bias and behaves like simple pn junction diode in reverse bias.

(3) It works as a voltage regulator in both forward and reverse bias.

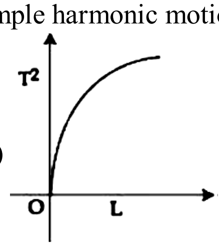
(4) It works as a voltage regulator in reverse bias and behaves like simple pn junction diode in forward bias.

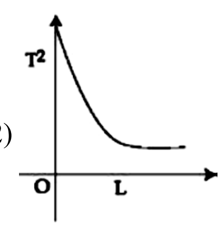
Ans. **[4]**

Sol. Theoretical.

∵ Zener diode is a voltage regulator in reverse bias and behaves as simple pn junction diode in forward bias.

Q.7 Choose the correct length (L) versus square of time period (T2) graph for a simple pendulum executing simple harmonic motion.

(1)


(2)


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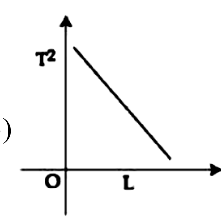
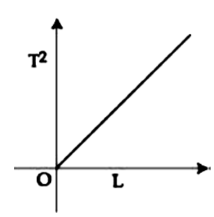


(4)


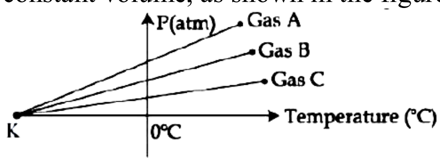
Ans. **[4]**

Sol. $T^2 = \frac{(2\pi)^2 L}{g}$

$y = \frac{(2\pi)^2 L}{g}$ (y axis has T^2 & x axis has L.)

∴ Straight line

Q.8 For three low density gases A, B, C pressure versus temperature graphs are plotted while keeping them at constant volume, as shown in the figure.



The temperature corresponding to the point 'K' is

(1) –373°C (2) –273°C (3) –40°C (4) –100°C

Ans. **[2]**

Sol. PV = nRT

At constant volume

$P \propto T \Rightarrow$ straight line and absolute zero is the common intercept, T = –273ºC

Q.9 A Carnot engine operating between two reservoirs has efficiency $\frac{1}{3}$. When the temperature of cold reservoir raised by x, its efficiency decreases to $\frac{1}{6}$. The value of x, if the temperature of hot reservoir is 99°C, will be

(1) 66 K (2) 33 K (3) 62 K (4) 16.5 K

Ans. **[3]**

Sol. $\eta = 1 - \frac{T_C}{T_H}$

Initially $\frac{T_C}{T_H} = \frac{2}{3}$ ...(1)

Finally $\frac{T_C + x}{T_H} = \frac{5}{6}$ ...(2)

T_H = 99ºC = 372 K

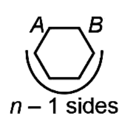
$\therefore$ x = 62 K

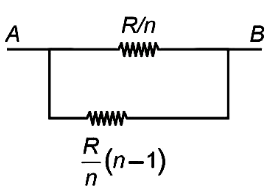
Q.10 Equivalent resistance between the adjacent corners of a regular n-sided polygon of uniform wire of resistance R would be

(1) $\frac{(n-1)R}{(2n-1)}$ (2) $\frac{(n-1)R}{(n^2)}$ (3) $\frac{(n-1)R}{n}$ (4) $\frac{n^2R}{n-1}$

Ans. **[2]**

Sol.





$\frac{R}{n}(n-1)$

$$\frac{1}{R_{eq}} = \frac{n}{R} + \frac{n}{R(n-1)} = \frac{n}{R}\left[1 + \frac{1}{n-1}\right]$$

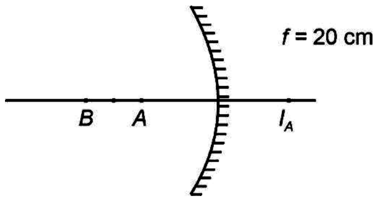
$$R_{eq} = \frac{R(n-1)}{n^2}$$

Q.11 Two objects A and B are placed at 15 cm and 25 cm from the pole in front of a concave mirror having radius of curvature 40 cm. The distance between images formed by the mirror is

(1) 100 cm (2) 40 cm (3) 160 cm (4) 60 cm

Ans. **[3]**

Sol.



$I_A \Rightarrow \frac{1}{v} - \frac{1}{15} = -\frac{1}{20}$

v = 60 (+ve, virtual)

$I_B \Rightarrow \frac{1}{v} - \frac{1}{25} = -\frac{1}{20}$

V = –100 (–ve, Real)

distance b/w images $I_A - I_B = 60 - (-100)$

= 160 cm

Q.12 For a body projected at an angle with the horizontal from the ground, choose the correct statement.

(1) The horizontal component of velocity is zero at the highest point.

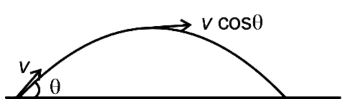
(2) The vertical component of momentum is maximum at the highest point

(3) Gravitational potential energy is maximum at the highest point

(4) The Kinetic Energy (K.E.) is zero at the highest point of projectile motion.

Ans. **[3]**

Sol.



At highest point $KE = \frac{1}{2}mv^2\cos^2\theta \neq 0$

horizontal momentum = mv cosθ ≠ 0

vertical momentum = 0

PE = mgh (Max at maximum height)

Q.13 The threshold frequency of a metal is f_0. When the light of frequency $2f_0$ is incident on the metal plate, the maximum velocity of photoelectrons is v_1. When the frequency of incident radiation is increases to $5f_0$, the maximum velocity of photoelectrons emitted is v_2. The ratio of v_1 to v_2 is:

(1) $\frac{v_1}{v_2} = \frac{1}{2}$ (2) $\frac{v_1}{v_2} = \frac{1}{4}$ (3) $\frac{v_1}{v_2} = \frac{1}{8}$ (4) $\frac{v_1}{v_2} = \frac{1}{16}$

Ans. **[1]**

Sol. $\frac{1}{2}mv^2 = hf - hf_0$

$\Rightarrow \frac{1}{2}mv_2^2 = 5hf_0 - hf_0 = hf_0$ .... (1)

also, $\frac{1}{2}mv_2^2 = 5hf_0 - hf_0 = 4hf_0$ ..... (2)

taking ratio

$$\frac{v_1^2}{v_2^2} = \frac{1}{4} \Rightarrow \frac{v_1}{v_2} = \frac{1}{2}$$

Q.14 The escape velocities of two planets A and B are in the ratio 1 : 2. If the ratio of their radii respectively is 1 : 3, then the ratio of acceleration due to gravity of planet A to the acceleration of gravity of planet B will be:

(1) $\frac{2}{3}$ (2) $\frac{4}{3}$ (3) $\frac{3}{2}$ (4) $\frac{3}{4}$

Ans. **[4]**

Sol. $v_e = \sqrt{2gR}$

$$\frac{(V_e)_A}{(V_e)_B} = \sqrt{\frac{g_A R_A}{g_B R_B}}$$

$$\Rightarrow \frac{1}{4} = \left(\frac{g_A}{g_B}\right) \times \frac{1}{3}$$

$$\frac{g_A}{g_B} = \left(\frac{3}{4}\right)$$

Q.15 The ratio of average electric energy density and total average energy density of electromagnetic wave is

(1) 3 (2) 2 (3) 1 (4) $\frac{1}{2}$

Ans. **[4]**

Sol. Average electric energy density = $\frac{1}{4}\varepsilon_0 E_0^2$

Average energy density = $\frac{1}{2}\varepsilon_0 E_0^2$

Ratio of electric average energy density to the

average energy density = $\left(\frac{1}{2}\right)$.

Q.16 In an amplitude modulation, a modulating signal having amplitude of X V is superimposed with a carrier signal of amplitude Y V in first case. Then, in second case, the same modulating signal is superimposed with different carrier signal of amplitude 2Y V. The ratio of modulation index in the two cases respectively will be:

(1) 2 : 1 (2) 1 : 2 (3) 1 : 1 (4) 4 : 1

Ans. **[1]**

Sol. μ = modulation index

$$\mu_1 = \frac{A_m}{A_{C_1}} = \left(\frac{X}{Y}\right)$$

$$\mu_2 = \frac{A_m}{A_{C_2}} = \left(\frac{X}{2Y}\right)$$

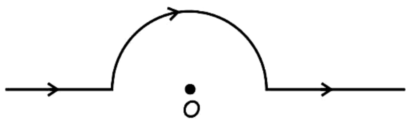
$$\frac{\mu_1}{\mu_2} = \frac{\left(\frac{X}{Y}\right)}{\left(\frac{X}{2Y}\right)} = 2:1$$

Q.17 As shown in the figure, a long straight conductor with semicircular arc of radius $\frac{\pi}{10}$ m is carrying current I = 3 A. The magnitude of the magnetic field at the center O of the arc is: (The permeability of the vacuum $= 4\pi \times 10^{-7}$ NA^{-2})

(1) 6μT (2) 1μT (3) 4μT (4) 3μT

Ans. **[4]**

Sol.



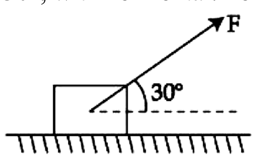
Magnetic field at B will be only due to the semicircular arc.

So, $B = \left(\frac{\mu_0 i}{4r}\right) = \frac{(4\pi \times 10^{-7}) \times 3}{4 \times \left(\frac{\pi}{10}\right)}$

$= 3 \times 10^{-6}$ T

= 3μT

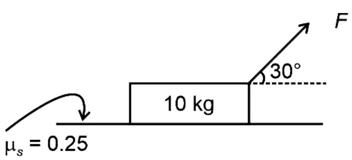
Q.18 As shown in the figure a block of mass 10 kg lying on a horizontal surface is pulled by a force F acting at an angle 30°, with horizontal. For $\mu_s = 0.25$, the block will just start to move for the value of F : [Given g = 10 ms^{-2}]



(1) 25.2 N (2) 33.3 N (3) 20 N (4) 35.7 N

Ans. **[1]**

Sol.



$N = (10g - F\sin 30°) = \left(100 - \frac{F}{2}\right)$

On solving (i) and (ii)

$\Rightarrow \frac{\sqrt{3}}{2}F = 0.25\left(100 - \frac{F}{2}\right)$

$\Rightarrow \left(\frac{\sqrt{3}}{2} + \frac{1}{8}\right)F = 25$

$F = \frac{25 \times 8}{(1 + 4\sqrt{3})} = \frac{200}{(4\sqrt{3} + 1)}N = 25.22$ N

Q.19 The Young's modulus of a steel wire of length 6 m and cross-sectional area 3 mm^2, is 2×10^{11} N/m^2. The wire is suspended from its support on a given planet. A block of mass 4 kg is attached to the free end of the wire. The acceleration due to gravity on the planet is $\frac{1}{4}$ of its value on the earth. The elongation of wire is (Take g on the earth = 10 m/s^2):

(1) 0.1 cm (2) 0.1 mm (3) 1 mm (4) 1 cm

Ans. **[2]**

Sol. $$\Delta\ell = \frac{FL}{AY} = \frac{\frac{Mg}{4} \times L}{AY}$$

$$= \frac{\frac{4\times10}{4}\times 6}{3\times10^{-6}\times2\times10^{11}} = \frac{60}{6\times10^{5}} = 10^{-4}\,\text{m} = 0.1\text{ mm}$$

Q.20 If the velocity of light c, universal gravitational constant G and Planck's constant h are chosen as fundamental quantities. The dimensions of mass in the new system is:

(1) $\left[h^{\frac{1}{2}}c^{\frac{1}{2}}G^{-\frac{1}{2}}\right]$ (2) $\left[h^{\frac{1}{2}}c^{-\frac{1}{2}}G^{1}\right]$ (3) $\left[h^{-\frac{1}{2}}c^{\frac{1}{2}}G^{\frac{1}{2}}\right]$ (4) $[h^{1}c^{1}G^{-1}]$

Ans. **[1]**

Sol. $c = LT^{-1}$

$G \equiv M^{-1}L^{3}T^{-2}$

$h = ML^{2}T^{-1}$

Let $M = c^{x}\, G^{y}\, h^{z}$

$\Rightarrow M^{1}L^{0}T^{0} = M^{z-y}\, L^{x+3y+2z}\, T^{-x-2y-z}$

$\Rightarrow x + 2y + z = 0$ …(i)

$x + 3y + 2z = 0$ …(ii)

$z - y = 1$ …(iii)

$\Rightarrow x = \frac{1}{2},\ y = \frac{-1}{2},\ z = \frac{1}{2}$

$\Rightarrow M = c^{1/2}\, G^{-1/2}\, h^{1/2}$

Section-B: Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer..

Q.21 A force $F = (5 + 3y^2)$ acts on a particle in the y-direction, where F is in newton and y is in meter. The work done by the force during a displacement from y = 2m to y = 5m is __________ J.

Ans. **[132]**

Sol. $$W = \int F dy = \int_{2}^{5} (5+3y^2)\,dy$$

$= (5y + y^3)\big|_2^5$

$= (15 + 125 – 8)$ J

$= 132$ J

Q.22 A block is fastened to a horizontal spring. The block is pulled to a distance x = 10 cm from its equilibrium position (at x = 0) on a frictionless surface from rest. The energy of the block at x = 5 cm is 0.25 J. The spring constant of the spring is _________ Nm^{-1}.

Ans. **[50]**

Sol. $\frac{1}{2}kx^2 = 0.25\,J$

$\Rightarrow k = \frac{0.25 \times 2}{(0.01)} = (50\ N/m)$

Q.23 A square shaped coil of area 70 cm^2 having 600 turns rotates in magnetic field of 0.4 Wbm^{-2}, about an axis which is parallel to one of the side of the coil and perpendicular to the direction of field. If the coil completes 500 revolution in a minute, the instantaneous emf when the plane of the coil is inclined at 60° with the field, will be ___________V. (Take $\pi = \frac{22}{7}$)

Ans. **[44]**

Sol. $\phi = BA\cos\omega t$

$\varepsilon = -\frac{d\phi}{dt} = +BA\omega\sin\omega t$

$= 0.4 \times 600 \times \frac{70}{10^4} \times \frac{500 \times 2\pi}{60}\sin 30°$

$= \frac{0.4 \times 6 \times 7}{600} \times 500 \times \frac{44}{7} \times \frac{1}{2} = 44$ volts

Q.24 For a train engine moving with speed of 20 ms^{-1}, the driver must apply brakes at a distance of 500 m before the station for the train to come to rest at the station. If the brakes were applied at half of this distance, the train engine would cross the station with speed $\sqrt{x}\,ms^{-1}$. The value of x is _______ . (Assuming same retardation is produced by brakes)

Ans. **[200]**

Sol. Distance up to the station = 250 m

Also $0^2 - 20^2 = 2(a)(500)$

$\Rightarrow a = \frac{-400}{1000} = -0.4\ m/s^2$

$\Rightarrow v^2 - 20^2 = 2(-0.4)(250)$

$\Rightarrow v^2 - 400 = 200$

$\Rightarrow v = \sqrt{200}\ m/s$

$\Rightarrow x = 200$

Q.25 Nucleus A having Z = 17 and equal number of protons and neutrons has 1.2 MeV binding energy per nucleon. Another nucleus B of Z = 12 has total 26 nucleons and 1.8 MeV binding energy per nucleons. The difference of binding energy of B and A will be _________ MeV.

Ans. **[6]**

Sol. $BE_A = (17 + 17)1.2$ MeV = 40.8 MeV

$BE_B = 26 \times 1.8$ MeV = 46.8 MeV

BE_B - BEA = 6 Mev

Q.26 A cubical volume is bounded by the surfaces x = 0, x = 0, x = a, y = 0, y = a, z = 0, z = a. The electric field in the region is given by $\vec{E} = E_0 x\hat{i}$. Where E_0 = 4 × 10^4 $NC^{-1}m^{-1}$. If a = 2 cm, the charge contained in the cubical volume is Q × 10^{-14}C. The value of Q is _________. (Take $\epsilon_0 = 9 \times 10^{-12}\ C^2/Nm^2$)

Ans. **[288]**

Sol. $\phi_{net} = E_0aa^3 - E_0(0)(a^2)$

$\frac{q_{end}}{\varepsilon_0} = E_0a^3$

$q_{encl.} = 4 \times 10^4 \times 8 \times 10^{-6} \times 9 \times 10^{-12}$

$= 288 \times 10^{-14}$

$\therefore Q = 288$

Q.27 The surface of water in a water tank of cross-section area 750 cm^2 on the top of a house is h m above the tap level. The speed of water coming out through the tap of cross-section area 500 mm^2 is 30 cm/s. At that instant, $\frac{dh}{dt}$ is $x \times 10^{-3}$ m/s. The value of x will be ________ .

Ans. **[02]**

Sol. AV = av

$750 \times 10^{-4}\left(\frac{dh}{dt}\right) = (500 \times 10^{-6})(30 \times 10^{-2})$

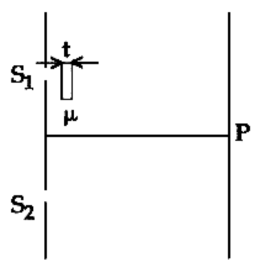
$\frac{dh}{dt} = \frac{15 \times 10^{-5}}{75 \times 10^{-3}}$

$= \frac{1}{5} \times 10^{-2}$

$= 2 \times 10^{-3}$ m/s

x = 2

Q.28 As shown in the figure, in Young's double slit experiment, a thin plate of thickness t = 10 μm and refractive index μ = 1.2 is inserted infront of slit S_1. The experiment is conducted in air (μ = 1) and uses a monochromatic light of wavelength λ = 500 nm. Due to the insertion of the plate, central maxima is shifted by a distance of $x\beta_0$. β_0 is the fringe-width before the insertion of the plate. The value of x is __________.



Ans. **[04]**

Sol. shift due to slab = $\frac{(\mu - 1)tD}{d}$

$\frac{(\mu - 1)tD}{d} = x\left(\frac{\lambda D}{d}\right)$

$x = \frac{(\mu - 1)t}{\lambda}$

$= \frac{(1.2 - 1) \times 10 \times 10^{-6}}{500 \times 10^{-9}}$

$= \frac{0.2}{5} \times \frac{10^{-5}}{10^{-7}}$

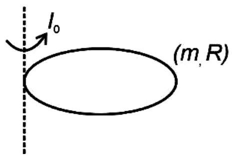
$= \frac{1}{25} \times 100$

$= 4$

Q.29 Moment of inertia of a disc of mass M and radius 'R' about any of its diameter is $\frac{MR^2}{4}$. The moment of inertia of this disc about an axis normal to the disc and passing through a point on its edge will be $\frac{X}{2}MR^2$. the value of x is ________ .

Ans. **[03]**

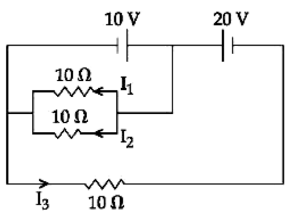
Sol.



$\because \quad I_D = \frac{mR^2}{4}$

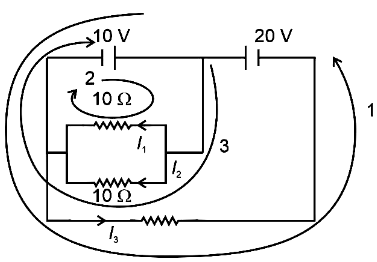
$I_0 = \frac{3}{2}mR^2 \quad \therefore x = 3$

Q.30 In the given circuit, the value of $\left|\frac{I_1 + I_3}{I_2}\right|$ is _________ .



Ans. **[02]**

Sol.



Using Kirchhoff's law in the indicated loop we get

$I_1 = I_2 = I_3 = 1$ A

So, $\left|\frac{I_1 + I_3}{I_2} = 2\right|$

CHEMISTRY

Section-A: Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Q.31 Given below are two statements : one is labelled as **Assertion (A)** and the other is labelled as **Reason (R)**.

Assertion (A) : Cu^{2+} in water is more stable than Cu^{+}.

Reason (R) : Enthalpy of hydration for Cu^{2+} is much less than that of Cu^{+}.

In the light of the above statements, choose the **correct** answer from the options given below :

(1) Both **(A)** and **(R)** are correct and **(R)** is the correct explanation of **(A)**

(2) **(A)** is not correct but **(R)** is correct

(3) **(A)** is correct but **(R)** is not correct.

(4) Both **(A)** and **(R)** are correct but **(R)** is not the correct explanation of **(A)**

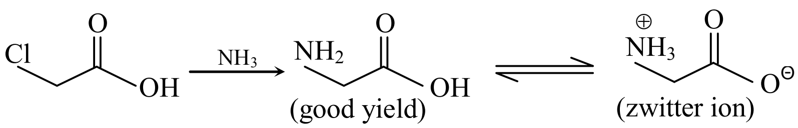
Ans. **[3]**

Sol. Cu^{2+} in water is more stable than Cu^{+} due to much higher hydration enthalpy of Cu^{2+} ion.
Hence correct answer is option (3).

Q.32 Given below are two statements : one is labelled as **Assertion (A)** and the other is labelled as **Reason (R)**.
Assertion (A) : α-halocarboxylic acid on reaction with dil NH_3 gives good yield of α-amino carboxylic acid whereas the yield of amines is very low when prepared from alkyl halides.
Reason (R) : Amino acids exist in zwitter ion form in aqueous medium.
In the light of the above statements, **choose** the correct answer from the options given below :
(1) **(A)** is not correct but **(R)** is correct
(2) **(A)** is correct but **(R)** is not correct
(3) Both **(A)** and **(R)** are correct and **(R)** is the correct explanation of **(A)**
(4) Both **(A)** and **(R)** are correct but **(R)** is **not** the correct explanation of **(A)**

Ans. **[3]**

Sol. **Statement-I :**



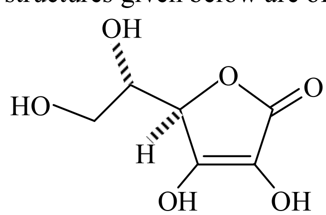
Statement-II : Reason is a correct statement as amino do exist as a zwitter ion. Reason is also a correct explanation.

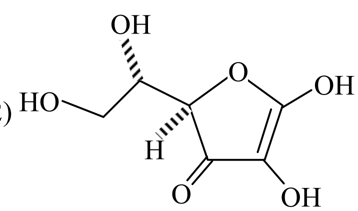
Q.33 Which element is not present in Nessler's reagent ?
(1) Potassium (2) Oxygen (3) Mercury (4) Iodine

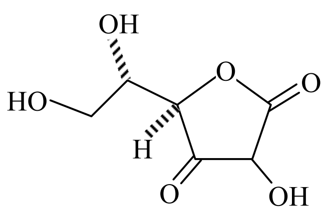
Ans. **[2]**

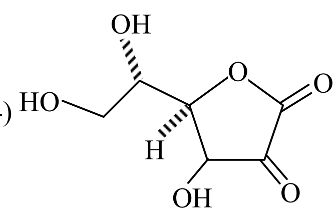
Sol. Nessler's reagent in $K_2[HgI_4]$

Q.34 All structures given below are of vitamin C. Most stable of them is :

(1)


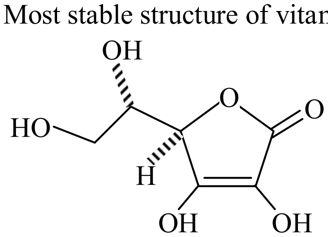
(2)


(3)


(4)


Ans. **[1]**

Sol. Most stable structure of vitamin(C) is :



Q.35 Which one of the following sets of ions represents a collection of isoelectronic species ?
(Given : Atomic Number : F : 9, Cl : 17, Na = 11, Mg = 12, Al = 13, K = 19, Ca = 20, Sc = 21)
(1) Ba^{2+}, Sr^{2+}, K^{+}, Ca^{2+}
(2) N^{3-}, O^{2-}, F^{-}, S^{2-}
(3) K^{+}, Cl^{-}, Ca^{2+}, Sc^{3+}
(4) Li^{+}, Na^{+}, Mg^{2+}, Ca^{2+}

Ans. **[3]**

Sol. Isoelectronic species have same number of electrons.
k^{+}, Cl^{-}, Ca^{2+} and Sc^{3+} all have 18 electrons, hence these are isoelectronic.

Q.36 The correct order of bond enthalpy (kJ mol^{-1}) is
(1) C – C > Si – Si > Sn – Sn > Ge – Ge
(2) Si – Si > C – C > Sn – Sn > Ge – Ge
(3) C – C > Si – Si > Ge – Ge > Sn – Sn
(4) Si – Si > C – C > Ge – Ge > Sn – Sn

Ans. **[3]**

Sol.

Bond	Bond energy (kJ mol^{-1})
C–C	348
S–Si	297
Ge–Ge	260
Sn-Sn	240

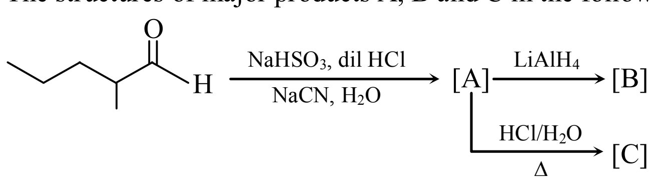
Correct answer will be (3)

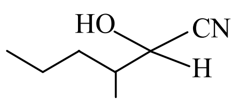
Q.37 The industrial activity held least responsible for global warming is
(1) industrial production of urea
(2) manufacturing of cement
(3) steel manufacturing
(4) Electricity generation in thermal power plants

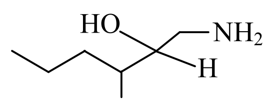
Ans. **[1]**

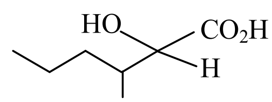
Sol. Industrial production of urea is least responsible for global warming.

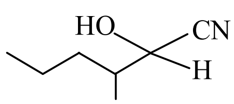
Q.38 The structures of major products A, B and C in the following reaction are sequence.

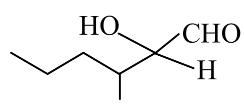


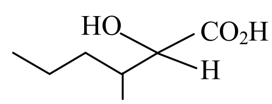
(1) A = 

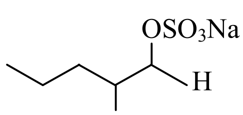
, B = 

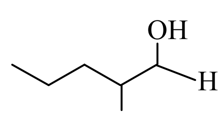
, C = 

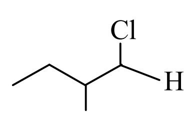
(2) A = 

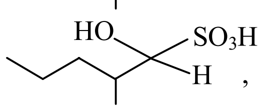
, B = 

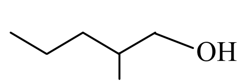
, C = 

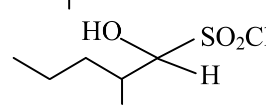
(3) A = 

, B = 

, C = 

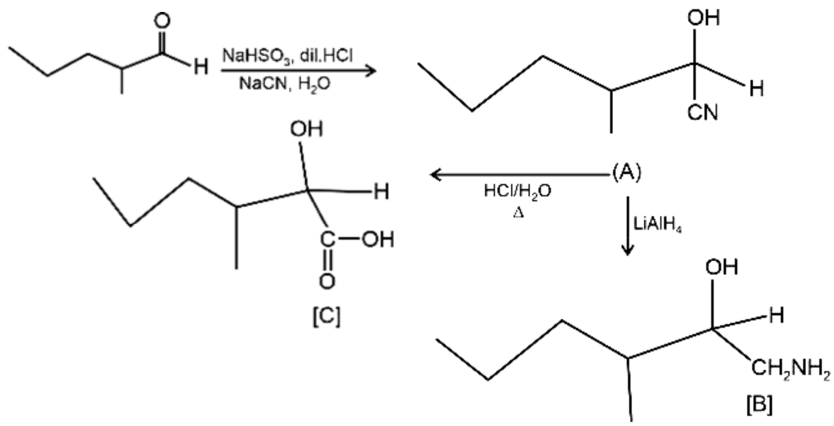
(4) A = 

, B = 

, C = 

Ans. **[1]**

Sol.



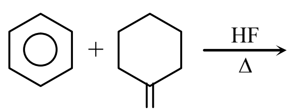
Q.39 The starting material for convenient preparation of deuterated hydrogen peroxide (D_2O_2) in laboratory is

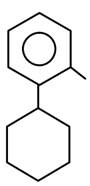
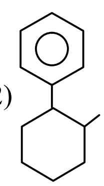
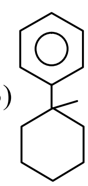
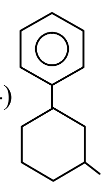
(1) 2-ethylanthraquinol (2) BaO (3) BaO_2 (4) $K_2S_2O_8$

Ans. **[4]**

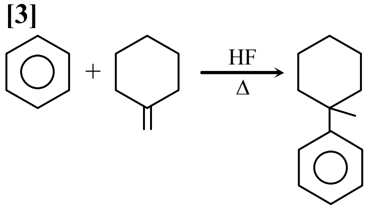
Sol. $K_2S_2O_8$ is used in the laboratory preparation of D_2O_2

$K_2S_2O_8(s) + 2D_2O(l) \rightarrow 2KDSO_4(aq) + D_2O_2(l)$

Q.40 'X' is :  X Major product

(1)  (2)  (3)  (4) 

Ans. **[3]**

Sol. 

Q.41 Given below are two statements : one is labelled as **Assertion (A)** and the other is labelled as **Reason (R)**.

Assertion (A) : An aqueous solution of KOH when used for volumetric analysis, its concentration should be checked before the use.

Reason (R) : On aging, KOH solution absorbs atmospheric CO_2.

In the light of the above statements, choose the **correct** answer from the options given below :

(1) Both **(A)** and **(R)** are correct but **(R)** is **not** the correct explanation of **(A)**

(2) Both **(A)** and **(R)** are correct and **(R)** is the correct explanation of **(A)**

(3) **(A)** is correct but **(R)** is not correct

(4) **(A)** is not correct but **(R)** is correct

Ans. **[2]**

Sol. KOH absorbs CO_2 get converted to K_2CO_3

$KOH + CO_2 \longrightarrow K_2CO_3 + H_2O$

Q.42 Given below are two statements :

Statement-I : Sulphanilic acid gives esterification test for carboxyl group.

Statement-II : Sulphanilic acid gives red colour in Lassigne's test for extra element detection.

In the light of the above statement choose the **most appropriate** answer from the options given below :

(1) **Statement I** is incorrect but **Statement II** is correct

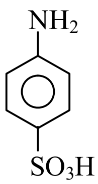
(2) Both **Statement I** and **Statement II** are incorrect

(3) **Statement** I is correct but **Statement II** is incorrect

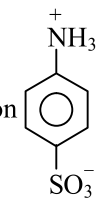
(4) Both **Statement I** and **Statement II** are correct

Ans. **[1]**

Sol. Sulphanilic acid is p-amino benzene sulphonic acid



it exists as zwitter ion



Since it contain both N and S so it give red colour in Lassaigne's test.

Q.43 The complex cation which has two isomers is :

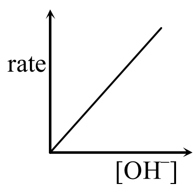
(1) $[Co(NH_3)_5Cl]^+$ (2) $[Co(H_2O)_6]^{3+}$ (3) $[Co(NH_3)_5NO_2]^{2+}$ (4) $[Co(NH_3)_5Cl]^{2+}$

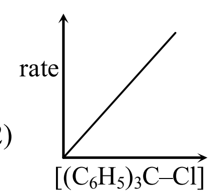
Ans. **[3]**

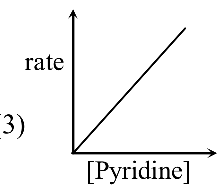
Sol. Complex $[Co(NH_3)_5NO_2]^{2+}$ will have two isomer one linked through N (Nitro) and one through O (Nitrite).

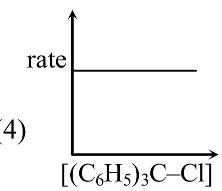
Q.44 The graph which represents the following reaction is :

$$(C_6H_5)_3C\text{–}Cl \xrightarrow[\text{Pyridine}]{OH^-} (C_6H_5)_3C\text{–}OH$$

(1)


(2)


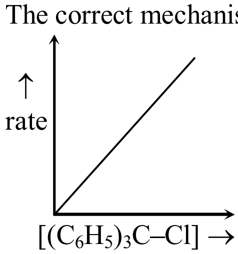
(3)


(4)


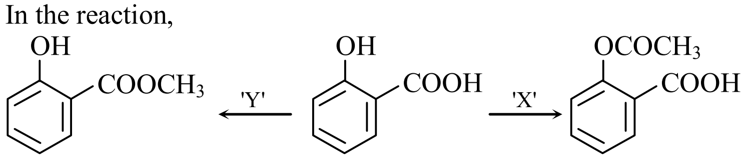
Ans. **[2]**

Sol. Rate = $K[(C_6H_5)_3Cl\text{–}Cl]$

The correct mechanisms is S_N1.



Q.45 In the reaction,



reagents 'X' and 'Y' respectively are :

(1) $(CH_3CO)_2O/H^+$ and $(CH_3CO)_2O/H^+$

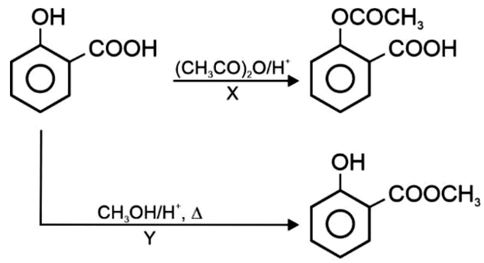
(2) $CH_3OH/H^+, \Delta$ and $CH_3OH/H^+, \Delta$

(3) $(CH_3CO)_2O/H^+$ and $CH_3OH/H^+, \Delta$

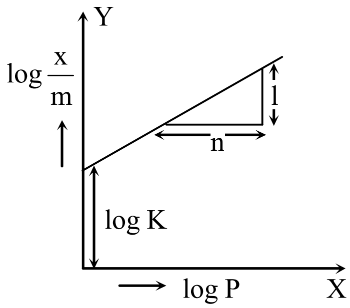
(4) $CH_3OH/H^+, \Delta$ and $(CH_3CO)_2O/H^+$

Ans. **[3]**

Sol.



Q.46 In figure, a straight line is given for Freundrich Adsorption (y = 3x + 2.505). The value of $\frac{1}{n}$ and log K are respectively.



(1) 3 and 2.505 (2) 0.3 and 0.7033 (3) 0.3 and log 2.505 (4) 3 and 0.7033

Ans. **[1]**

Sol. $\log \frac{x}{m} = \log k + \frac{1}{n} \log p$

On comparing, we get

$\frac{1}{n} = 3 \Rightarrow n = 0.3$ and $\log k = 2.505$

Q.47 The effect of addition of helium gas to the following reaction in equilibrium state, is

$PCl_5(g) \rightleftharpoons PCl_3(g) + Cl_2(g)$

(1) the equilibrium will go backward due to suppression of dissociation of PCl_5

(2) addition of helium will not affect the equilibrium

(3) the equilibrium will shift in the forward direction and more of Cl_2 and PCl_3 gases will be produced

(4) helium will deactivate PCl_5 and reaction will strop

Ans. **[3]**

Sol. If we consider addition of He gas at constant pressure, the reaction will shift in forward direction [As rigid container is not given]

Q.48 For electron gain enthalpies of the elements denoted as $\Delta_{eg}H$, the incorrect option is

(1) $\Delta_{eg}H(Cl) < \Delta_{eg}H(F)$ (2) $\Delta_{eg}H(Se) < \Delta_{eg}H(S)$

(3) $\Delta_{eg}H(l) < \Delta_{eg}H(At)$ (4) $\Delta_{eg}H(Te) < \Delta_{eg}H(Po)$

Ans. **[2]**

Sol. $\Delta H_{eg}(Cl) = -349$ kJ/mol $\Delta H_{eg}(F) = -333$ kJ/mol

$\Delta H_{eg}(l) = -296$ kJ/mol

$\Delta H_{eg}(Se) = -195$ kJ/mol

$\Delta H_{eg}(S) = -200$ kJ/mol

$\Delta H_{eg}(Te) = -190$ kJ/mole

$\Delta H_{eg}(Po) = -174$ kJ/mole

Electron gain enthalpy of Se is less negative than that of sulphur.

Q.49 Given below are two statements : one is labelled as **Assertion (A)** and the other is labelled as **Reason (R)**.

Assertion (A) : Gypsum is used for making fireproof wall boards.

Reason (R) : Gypsum is unstable at high temperatures.

In the light of the above statements, choose the **correct** answer from the options given below

(1) Both **(A)** and **(R)** are correct and **(R)** is the correct explanation of **(A)**

(2) **(A)** is correct but **(R)** is not correct

(3) Both **(A)** and **(R)** are correct but **(R)** is not the correct explanation of **(A)**

(4) **(A)** is not correct but **(R)** is correct.

Ans. **[3]**

Sol. Both statements are correct, However, IInd statement has no relation with Ist Statement.

Q.50 O–O bond length in H_2O_2 is $\underline{X}$ than the O–O bond length in F_2O_2.The O–H bond length in H_2O_2 is $\underline{Y}$ than that of the O–F bond in F_2O_2.

Choose the correct option for $\underline{X}$ and $\underline{Y}$ from those given below

(1) X-shorter, Y-Shorter (2) X-shorter, Y-longer

(3) X-longer, Y-Shorter (4) X-longer, Y-longer

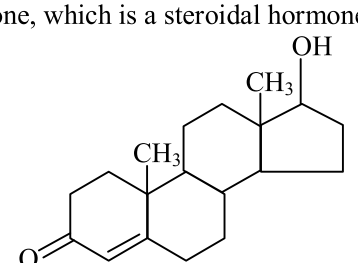
Ans. **[3]**

Sol. X-longer [because of more p-character in O–F bond]

Y-shorter [size of H is very small as compared to F]

Section-B: Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE**. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

Q.51 Testosterone, which is a steroidal hormone, has following structure.

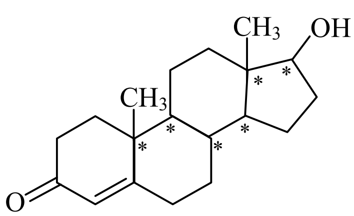


Testosterone

The total number of asymmetric carbon atoms/s in testosterone is ___________.

Ans. **[6]**

Sol.



The total number of asymmetric carbon atoms in testosterone is 6.

Q.52 A metal M crystallizes into two lattices : face centred cubic (fcc) and body centred cubic (bcc) with unit cell edge length of 2.0 and 2.5 Å respectively. The ratio of densities of lattices fcc to bcc for the metal M is _________. (Nearest integer)

Ans. **[4]**

Sol. d_1, Density of fcc lattice of metal M = $\frac{4\times M}{N_0(a_{fcc})^3}$

d_2, Density of bcc lattice of metal M = $\frac{2\times M}{N_0(a_{bcc})^3}$

$$\frac{d_1}{d_2}=\frac{4}{2}\left(\frac{a_{bcc}}{a_{fcc}}\right)^3 = 2\left(\frac{2.5}{2}\right)^3 = 3.90 \simeq 4$$

Q.53 $A \rightarrow B$

The above reaction is of zero order. Half life of this reaction is 50 min. The time taken for the concentration of A to reduce to one-fourth of its initial value is _________ min.

Ans. **[75]**

Sol. $\underset{a-x}{A} \longrightarrow \underset{x}{B}$ (Zero Order reaction)

$a - x = \frac{a}{4} \Rightarrow x = \frac{3a}{4}$

$t_{1/2} = \frac{a}{2K} = 50$ min. $\Rightarrow \frac{a}{K} = 100$ min.

$t = \frac{x}{K} = \frac{3a}{4K} = 75$ min.

Q.54 0.3 g of ethane undergoes combustion at 27ºC in a bomb calorimeter. The temperature of calorimeter system (including the water) is found to rise by 0.5ºC. The heat evolved during combustion of ethane at constant pressure is __________ kJ mol^{-1}. (Nearest integer)

[Given : The heat capacity of the calorimeter system is 20 kJ K^{-1}, R = 8.3 JK^{-1} mol^{-1}.

Assume ideal gas behaviour.

Atomic mass of C and H are 12 and 1 g mol^{-1} respectively]

Ans. **[1006]**

Sol. $C_2H_6(g) + \frac{7}{2}O_2(g) \rightarrow 2CO_2(g) + 3H_2O(\ell)$

No. of moles of ethane = $\frac{0.3}{30} = 0.01$

Heat evolved in Bomb calorimeter = 20 × 0.5 = 10 kJ

$\Delta U = -\frac{10}{0.01} = -1000$ kJ mol^{-1}

$\Delta H = \Delta U + \Delta n_g RT$

$= -1000 + (-2.5) \times \frac{8.3\times 300}{1000}$

$= -1000 - 6.225$

$= -1006.225$

$|\Delta H| \simeq 1006$ kJ mol^{-1}

Q.55 The spin only magnetic moment of $[Mn(H_2O)_6]^{2+}$ complexes is _________ B.M. (Nearest integer)

(Given : Atomic no. of Mn is 25)

Ans. **[6]**

Sol. $[Mn(H_2O)_6]^{2+}$

Mn^{2+} : $3d^5$

No. of unpaired electrons = 5

$\mu = \sqrt{35}$ BM $\simeq$ 6 BM

Q.56 20% of acetic acid is dissociated when its when its 5 g is added to 500 mL of water. The depression in freezing point of such water _______ × 10^{-3} ºC. Atomic mass of C, H and O are 12, 1 and16 a.m.u. respectively.

[Given : Molal depression constant and density of water are 1.86 K Kg mol^{-1} and 1 g cm^{-3} respectively.]

Ans. **[372]**

Sol. $\underset{1-0.2}{CH_3COOH} \rightleftharpoons \underset{0.2}{H^+} + \underset{0.2}{CH_3COO^-}$

i = 1.2

$[CH_3COOH] = \frac{5}{60 \times 0.5} = \frac{5}{30}$ M

$\Delta T_f = i\ K_f\ m$

$1.2 \times 1.86 \times \frac{5}{30} = 0.372$ºC

$= 372 \times 10^{-3}$ ºC

Q.57 1 × 10^{-5} M $AgNO_3$ is added to 1 L of saturated solution of AgBr. The conductivity of this solution at 298 K is _________ × 10^{-8} S m^{-1}.

[Given : $K_{SP}(AgBr) = 4.9 \times 10^{-3}$ at 298 K

$\lambda^0_{Ag^+} = 6 \times 10^{-3}$ S m^2 mol^{-1}

$\lambda^0_{Br^-} = 8 \times 10^{-3}$ S m^2 mol^{-1}

$\lambda^0_{NO_3^-} = 7 \times 10^{-3}$ S m^2 mol^{-1}]

Ans. **[13039.2]**

Sol. $AgBr(S) \rightleftharpoons \underset{(10^{-5}+x)}{Ag^+}\ (aq) + \underset{x}{Br^-}\ (aq)$

$x(x + 10^{-5}) = 4.9 \times 10^{-13}$

$x \simeq 4.9 \times 10^{-8}$ M

$\lambda^0_{Ag^+} = 6 \times 10^{-3}$ S m^2 mol^{-1}

$\lambda^0_{Br^-} = 8 \times 10^{-3}$ S m^2 mol^{-1}

$\lambda^0_{NO_3^-} = 7 \times 10^{-3}$ S m^2 mol^{-1}

$K_{solution} = K_{Ag^+} + K_{Br^-} + K_{NO_3^-}$

$= 6 \times 10^{-3} \times 10^{-5} \times 10^3 + 8 \times 10^{-3} \times 4.9 \times 10^{-8} \times 10^3 + 7 \times 10^{-3} \times 10^{-5} \times 10^3$

$= (6000 + 39.2 + 7000) \times 10^{-8}$

$= 13039.2 \times 10^{-8}$ Sm^{-1}

Q.58 Among the following, the number of tranquilizer/s is/are _____________.

A. Chloroliazepoxide
B. Veronal
C. Valium
D. Salvarsan

Ans. **[3]**

Sol. Chloroliazepoxide
Veronal
Valium
Salvarsan is an antibiotic

Q.59 The molarity of a 10% (v/v) solution of di-bromine solution in CCl_4 (carbon tetrachloride) is 'x'.
x = ____________ $\times 10^{-2}$ M. (Nearest integer)
[Given : molar mass of $Br_2 = 160$ g mol^{-1}
atomic mass of C = 12 g mol^{-1}
atomic mass of Cl = 35.5 mol^{-1}
density of dibromine = 3.2 g cm^{-3}
density of CCl_4 = 1.6 g cm^{-3}]

Ans. **[139]**

Sol. Mass of 10 mL of $Br_2 = 10 \times 3.2 = 32$ gm
Mass of 90 mL of $CCl_4 = 90 \times 1.6 = 144$ gm

Molality of Br_2 solution in CCl_4 $= \frac{32 \times 1000}{160 \times 144}$
$= 1.39$ M
$= 139 \times 10^{-1}$

Q.60 Among following compounds, the number of those present in copper matte is ___________ .
A. $CuCO_3$
B. Cu_2S
C. Cu_2O
D. FeO

Ans. **[3]**

Sol. Copper matte contains
Cu_2S, Cu_2O, FeO

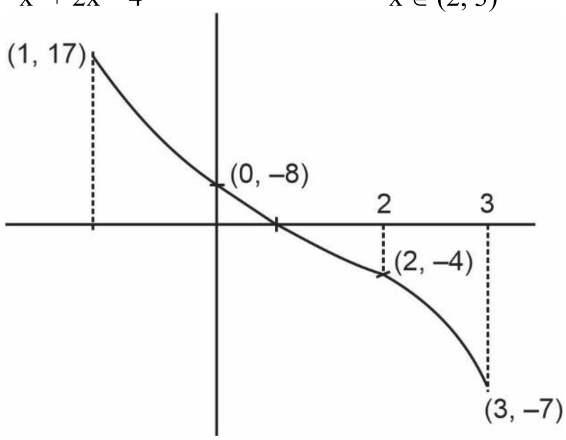
MATHEMATICS

Section-A: This section contains 20 multiple choice questions. Each question has 4 choices(1), (2), (3) and (4), out of which **ONLY ONE** is correct..

Q.61 The sum of absolute maximum and minimum values of the function $f(x) = |x^2 - 5x + 6| - 3x + 2$ in the interval [–1, 3] is equal to :
(1) 24 (2) 13 (3) 12 (4) 10

Ans. **[1]**

Sol. $f(x) = |(x-2)(x-3)| - 3x + 2$ $x \in [-1, 3]$
$\Rightarrow f(x) = x^2 - 8x + 8$ $x \in [-1, 2]$
$-x^2 + 2x - 4$ $x \in (2, 3)$



$\therefore$ Maximum value = 17
Minimum value of –7
$\therefore$ Sum = 24

Q.62 The value of the integral $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{x+\frac{\pi}{4}}{2-\cos 2x}dx$ is :

(1) $\frac{\pi^2}{6}$ (2) $\frac{\pi^2}{6\sqrt{3}}$ (3) $\frac{\pi^2}{12\sqrt{3}}$ (4) $\frac{\pi^2}{3\sqrt{3}}$

Ans. **[2]**

Sol. $I = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{x+\frac{\pi}{4}}{(2-\cos 2x)}dx$

Using $\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$

$$I = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \left(\frac{-x+\frac{\pi}{4}}{2-\cos 2x}\right)dx$$

$$\therefore \quad 2I = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{\pi dx}{2(2-\cos 2x)}$$

$$\Rightarrow \quad I = \frac{2\pi}{4}\int_0^{\frac{\pi}{4}} \left(\frac{dx}{2-\frac{1-\tan^2 x}{1+\tan^2 x}}\right)$$

$$\Rightarrow \quad I = \frac{\pi}{2}\int_0^{\frac{\pi}{4}} \left(\frac{1+\tan^2 x}{1+3\tan^2 x}\right)dx$$

Put tanx = t

$$\Rightarrow \quad I = \frac{\pi}{2}\int_0^1 \frac{dt}{1+3t^2} \qquad \Rightarrow \quad I = \frac{\pi}{6\sqrt{3}}$$

Q.63 The sum $\sum_{n=1}^{\infty} \frac{2n^2+3n+4}{(2n)!}$ is equal to :

(1) $\frac{13e}{4}+\frac{5}{4e}$ (2) $\frac{11e}{2}+\frac{7}{2e}-4$ (3) $\frac{13e}{4}+\frac{5}{4e}-4$ (4) $\frac{11e}{2}+\frac{7}{2e}$

Ans. **[1]**

Sol. $\sum_{n=1}^{\infty} \frac{2n^2+3n+4}{(2n)!}$

Put $2n = t \quad \Rightarrow \quad n = \frac{t}{2}$

$$\therefore \quad \sum_{t\to even} \frac{\frac{t^2}{2}+\frac{3t}{2}+4}{t!}$$

$$\Rightarrow \quad \sum_{t\to even} \frac{t^2+3t+8}{2t!}$$

$$\Rightarrow \quad \frac{1}{2}\sum_{t\to even}\left(\frac{t-1}{(t-1)!}+\frac{1}{(t-1)!}+\frac{3}{(t-1)!}+\frac{8}{t!}\right)$$

$$\Rightarrow \quad \frac{1}{2}\sum_{t\to even}\left(\frac{1}{(t-2)!}+\frac{4}{(t-1)!}+\frac{8}{t!}\right)$$

$$\Rightarrow \quad \frac{1}{2}\left(\frac{e+e^{-1}}{2}+\frac{4(e-e^{-1})}{2}+\frac{8(e+e^{-1})}{2}\right)$$

$$\Rightarrow \quad \frac{1}{4}(13e+5e^{-1})$$

Q.64 Let a, b be two real numbers such that ab < 0. If the complex number $\frac{1+ai}{b+i}$ is of unit modulus and a + ib lies on the circle |z – 1| = |2z|, then a possible value of $\frac{1+[a]}{4b}$, where [t] is greatest integer function is :

(1) –1 (2) 1 (3) $-\frac{1}{2}$ (4) $\frac{1}{2}$

Ans. **[*]**

Sol. |1+ ai| = |b + i|

$\Rightarrow a^2 + 1 = b^2 + 1 \Rightarrow a^2 = b^2$

& |a + ib –1| = |2a + 2ib|

$\Rightarrow a^2 + 1 - 2a + b^2 = 4a^2 + 4b^2$

$\Rightarrow 3a^2 + 3b^2 + 2a - 1 = 0$

$\Rightarrow ba^2 + 2a - 1 = 0$

$$\therefore \quad a = \frac{-2\pm\sqrt{4+24}}{2(6)}$$

$$= \frac{-1\pm\sqrt{7}}{6}$$

$$\therefore \quad (a, b) \equiv \left(\frac{-1+\sqrt{7}}{6}, \frac{-1-\sqrt{7}}{6}\right) \text{ or } \left(\frac{-1-\sqrt{7}}{6}, \frac{-1+\sqrt{7}}{6}\right)$$

$$\therefore \quad \frac{1+[a]}{4b} = 0 \quad \text{or} \quad \frac{3}{2(-1-\sqrt{7})}$$

∴ No option matches

Q.65 Let f : R - {0, 1} → be a function such that $f(x) + f\left(\frac{1}{1-x}\right) = 1 + x$. then f(2) is equal to :

(1) $\frac{9}{2}$ (2) $\frac{9}{4}$ (3) $\frac{7}{3}$ (4) $\frac{7}{4}$

Ans. **[2]**

Sol. $f(x) + f\left(\frac{1}{1-x}\right) = 1 + x$ ….(i)

If $x \to \frac{1}{1-x}$

$$f\left(\frac{1}{1-x}\right)+f\left(\frac{1}{1-\frac{1}{1-x}}\right)=1+\frac{1}{1-x}$$

$$f\left(\frac{1}{1-x}\right)+f\left(\frac{1-x}{-x}\right)=\frac{2-x}{1-x} \quad \text{....(ii)}$$

If $x \to \frac{x-1}{x}$

$$f\left(\frac{x-1}{x}\right)+f(x)=\frac{2x-1}{x} \quad \text{....(iii)}$$

Putting x = 2

f(2) + f(–1) = 3

$f(-1)+f\left(\frac{1}{2}\right)=0$

$f\left(\frac{1}{2}\right)+f(2)=\frac{3}{2}$

Solving these $f(2)=\frac{9}{4}$

Q.66 Let the plane P pass through the intersection of the planes 2x + 3y – z = 2 and x + 2y + 3z = 6, and be perpendicular to the plane 2x + y – z + 1 = 0. If d is the distance of P from the point (–7, 1,1), then d2 is equal to :

(1) $\frac{25}{83}$ (2) $\frac{15}{53}$ (3) $\frac{250}{82}$ (4) $\frac{250}{83}$

Ans. **[4]**

Sol. Let the equation of plane P be

(2x + 3y – z – 2) + λ (x + 2y + 3z – 6) = 0

Now since P is $\perp^r$ to 2x + y – z + 1 = 0

$\therefore$ 2(2 + λ) +1(3 + 2λ) –1(–1+ 3λ) = 0

$\boxed{\lambda=-8}$

$\therefore$ P : 6x +13y + 25z = 46

Now distance from the point (–7, 1, 1)

$$d=\left|\frac{-42+13+25-46}{\sqrt{36+169+625}}\right|$$

$$\therefore d^2=\frac{2500}{830}=\frac{250}{83}$$

Q.67 Let $s=\left\{x\in R: 0<x<1 \text{ and } 2\tan^{-1}\left(\frac{1-x}{1+x}\right)=\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)\right\}$

If n(S) denotes the number of elements in S then :

(1) n(S) = 0

(2) n(S) = 2 and only one element in S is less than $\frac{1}{2}$

(3) n(S) = 1 and the element in S is less than $\frac{1}{2}$

(4) n(S) = 1 and the elements is S is more than $\frac{1}{2}$

Ans. [3]

Sol. $2\tan^{-1}\left(\frac{1-x}{1+x}\right) = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$

Put $\tan^{-1} x = \theta$, $\theta \in \left(0, \frac{\pi}{4}\right)$

$2\tan^{-1}\left(\tan\left(\frac{\pi}{4}-\theta\right)\right) = \cos^{-1}(\cos 2\theta)$

$2\left(\frac{\pi}{4}-\theta\right) = 2\theta$

$\frac{\pi}{2} = 4\theta$ or $\boxed{\theta = \frac{\pi}{8}}$

$x = \tan\left(\frac{\pi}{8}\right) = \sqrt{2} - 1 < \frac{1}{2}$

Q.68 Let $\vec{a} = 2\hat{i} - 7\hat{j} + 5\hat{k}$, $\vec{b} = \hat{i} + \hat{k}$, $\vec{c} = \hat{i} + 2\hat{j} - 3\hat{k}$ be three given vectors. If r is a vector such that $\vec{r} \times \vec{a} = \vec{c} \times \vec{a}$ and $\vec{r} \cdot \vec{b} = 0$, then $|\vec{r}|$ is equal to :

(1) $\frac{11}{7}$ (2) $\frac{11}{5}\sqrt{2}$ (3) $\frac{\sqrt{914}}{7}$ (4) $\frac{11}{7}\sqrt{2}$

Ans. [4]

Sol. $\vec{r} = \vec{c} + \lambda \vec{a}$

$\vec{r} \cdot \vec{b} = \vec{c} \cdot \vec{b} + \lambda \vec{a} \cdot \vec{b} = 0$

$-2 + \lambda(7) = 0 \Rightarrow \lambda = \frac{2}{7}$

$\vec{r} = (\hat{i} + 2\hat{j} - 3\hat{k}) + \frac{2}{7}(2\hat{i} - 7\hat{j} + 5\hat{k})$

$= \frac{11}{7}\hat{i} + 0\hat{j} + \frac{11}{7}\hat{k}$

$|\vec{r}| = \frac{11}{7}\sqrt{2}$

Q.69 Let $P(x_0, y_0)$ be the point on the hyperbola $3x^2 - 4y^2 = 36$, which is nearest to the line $3x + 2y = 1$. Then $\sqrt{2}(y_0 - x_0)$ is equal to :

(1) 9 (2) –3 (3) –9 (4) 3

Ans. [3]

Sol. If (x_0, y_0) is point on hyperbola then tangent at (x_0, y_0) is parallel to $3x + 2y = 1$

Equation of tangent $\rightarrow \frac{xx_0}{12} - \frac{yy_0}{9} = 2$

Slope of tangent = $\frac{-3}{2}$

Equation of tangent in slope form

$y = \frac{-3}{2}x \pm \sqrt{12 \cdot \frac{9}{4} - 9}$

$y = \frac{-3}{2}x \pm 3\sqrt{2}$

Or $3x + 2y = 6\sqrt{2}$

Comparing

$$\frac{\frac{x_0}{12}}{3} = \frac{\frac{-y_0}{9}}{2} = \frac{1}{6\sqrt{2}}$$

$x_0 = 3\sqrt{2}$, $y_0 = \frac{-3}{\sqrt{2}}$

$\sqrt{2}(y_0 - x_0) = -3 - 6 = -9$

Q.70 The number of integral values of k, for which one root of the equation $2x^2 - 8x + k = 0$ lies in the interval (1, 2) and its other root lies in the interval (2, 3) is

(1) 1 (2) 2 (3) 3 (4) 0

Ans. **[1]**

Sol. $f(1) > 0 \Rightarrow k > 6$

$f(2) < 0 \Rightarrow k < 8$

$f(3) > 0 \Rightarrow k > 6$

$k \in (6, 8)$

Only 1 integral value of k is 7

Q.71 Which of the following statements is a tautology?

(1) $(p \wedge (p \rightarrow q)) \rightarrow \sim q$ (2) $p \vee (p \wedge q)$

(3) $(p \wedge q) \rightarrow (\sim (p) \rightarrow q)$ (4) $p \rightarrow (p \wedge (p \rightarrow q))$

Ans. **[3]**

Sol. $\sim p \rightarrow q \equiv \sim(\sim p) \vee q \equiv p \vee q$

$p \wedge q \rightarrow (\sim p \rightarrow q)$

$\equiv p \wedge q \rightarrow (p \vee q)$

$\equiv \sim (p \wedge q) \vee (p \vee q)$

$\equiv (\sim p \vee \sim q) \vee (p \vee q)$

$\equiv (\sim p \vee (p \vee q)) \vee (\sim q \vee (p \vee q))$

$\equiv T \vee T$

$\equiv T$

Q.72 Let P(S) denote the power set of S = {1, 2, 3,…,10}.

Define the relations R_1 and R_2 on P(S) as AR_1B if

$(A \cap B^c) \cup (B \cap A^c) = \phi$ and AR_2B if $A \cup B^c = B \cup A^c$, $\forall$ A B $\in$ P(S). Then :

(1) Only R_2 is an equivalence relation

(2) Both R_1 and R_2 are not equivalence relations

(3) Only R_1 is an equivalence relation

(4) Both R_1 and R_2 are equivalence relations

Ans. **[4]**

Sol. $R_1 : (A \cap B^C) \cup (B \cap A^C) = \phi$

$\Rightarrow \boxed{A = B}$

$R_2 : (A \cup B^C) = (B \cup A^C)$

$\Rightarrow \boxed{A = B}$

Both R_1 and R_2 are equivalence.

Q.73 Let $\alpha x = \exp(x^{\beta}y^{\gamma})$ be the solution of differential equation $2x^2ydy - (1 - xy^2)dx = 0$, $x > 0$, $y(2) = \sqrt{\log_e 2}$. Then $\alpha + \beta - \gamma$ equals :

(1) –1 (2) 1 (3) 3 (4) 0

Ans. **[2]**

Sol. Given differential equation

$2x^2\,ydy - (1 - xy^2)dx = 0,\ x > 0$

$2xy\,dy + y^2\,dx = \frac{1}{x}dx$

$\int d(xy^2) = \int \frac{1}{x}dx\ x$

$xy^2 = \ln x + C$ ....(i)

$y(2) = \sqrt{\log_e 2}$

$2\ln 2 = \ln 2 + C$

$\therefore$ $\boxed{C = \ln 2}$

$\therefore$ by (i)

$xy^2 = \ln 2x$

$2x = e^{xy^2}$

$\therefore$ $\alpha = 2, \beta = 1, \gamma = 2$

$\therefore$ $\alpha + \beta - \gamma = 1$

Q.74 For the system of linear equations $ax + y + z = 1$, $x + ay + z = 1$, $x + y + az = \beta$, which one of the following is **NOT** correct?

(1) It has infinitely many solutions if $\alpha = 1$ and $\beta = 1$

(2) $x + y + z = \frac{3}{4}$ if $\alpha = 2$ and $\beta = 1$

(3) It has no solution if $\alpha = -2$ and $\beta = 1$

(4) It has infinitely many solutions if $\alpha = 2$ and $\beta = -1$

Ans. **[d]**

Sol. For infinite solution $\Delta = \Delta_x = \Delta_y = \Delta_z = 0$

$$\Delta = \begin{vmatrix} \alpha & 1 & 1 \\ 1 & \alpha & 1 \\ 1 & 1 & \alpha \end{vmatrix} = 0 \Rightarrow (\alpha^3 - 3\alpha + 2) = 0 \Rightarrow \alpha = 1, -2$$

If $\beta = 1$, then all planes are overlapping

$\therefore$ Option (1) is correct.

Option (2)

$\alpha = 2, \beta = 1$

$2x + y + z = 1$

$x + 2y + z = 1$

$x + y + 2z = 1$

Adding all three equations

$x + y + z = \frac{3}{4}$

$\therefore$ option (2) is correct.

Option (3)

If $\alpha = -2$ and $\beta = 1$, then $\Delta = 0$, $\Delta_x \neq 0$

$\therefore$ No solution

$\therefore$ Option (3) is correct.

Option (4)

If $\alpha = 2 \Rightarrow \Delta \neq 0$

$\therefore$ Unique solution exist

$\therefore$ Option (4) is incorrect.

$\therefore$ Option (4) is answer.

Q.75 Two dice are thrown independently. Let A be the event that the number appeared on the 1^{st} die is less than the number appeared on the 2^{nd} die, B be the event that the number appeared on the 1^{st} die is even and that one the second die is odd, and C be the event that the number appeared on the 1^{st} die is odd and that on the 2^{nd} is even. Then

(1) A and B are mutually exclusive

(2) The number of favourable cases of the events A, B and C are 15, 6 and 6 respectively

(3) The number of favourable cases of the event $(A \cup B) \cap C$ is 6

(4) B and C are independent

Ans. **[3]**

Sol. $A = \begin{cases} (1,2),(1,3),(1,4),(1,5),(1,6) \\ (2,3),(2,4),(2,5),(2,6) \\ (3,4),(3,5),(3,6) \\ (4,5),(4,6) \\ (5,6) \end{cases}$

n(A) = 15

$B = \begin{cases} (2,1),(2,3),(2,5) \\ (4,1),(4,3),(4,5) \\ (6,1),(6,3),(6,5) \end{cases}$

n(B) = 9

Similarly, n(C) = 9

$(4, 5) \in A$ and $(4, 5) \in B$

$\therefore$ A and B are not exclusive events

$n((A \cup B) \cap C) = n(A \cap C) + n(B \cap C) - n(A \cap B \cap C)$

$= 3 + 3 - 0 = 6$

Option (3) is correct.

$n(B) = \frac{9}{36}$, $n(C) = \frac{9}{36}$, $n(B \cap C) = 0$

$\Rightarrow\ n(B) \cdot n(C) \neq n(B \cap C)$

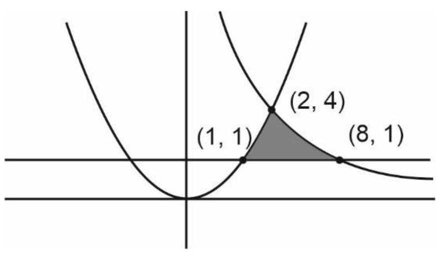
$\therefore$ B and C are not independent

Q.76 The area of the region given by $\{(x, y) : xy \leq 8,\ 1 \leq y \leq x^2\}$ is :

(1) $8\log_e 2 - \frac{13}{3}$ (2) $16\log_e 2 + \frac{7}{3}$ (3) $16 \log_e 2 - \frac{14}{3}$ (4) $8 \log_e 2 + \frac{7}{6}$

Ans. **[3]**

Sol.



Required area = $\int_1^2 (x^2-1)\,dx + \int_2^8 \left(\frac{8}{x}-1\right) dx$

$= \left(\frac{x^3}{3}-x\right)\Big|_1^2 + (8\ln x - x)\Big|_2^8$

$= \left[\left(\frac{8}{3}-2\right)-\left(\frac{1}{3}-1\right)\right] + [8\ln 8 - 8 - (8\ln 2 - 2)]$

$= \frac{4}{3} + 8\ln 4 - 6$

$= 8\ln 4 - \frac{14}{3}$

$= 16\ln 2 - \frac{14}{3}$

Q.77 If $y(x) = x^x$, $x > 0$, then $y''(2) - 2y'(2)$ is equal to:

(1) $4(\log_e 2)^2 - 2$ (2) $4(\log_e 2)^2 + 2$ (3) $4\log_e 2 + 2$ (4) $8\log_e 2 - 2$

Ans. **[1]**

Sol. $y = x^x$

$y' = x^x(1 + \ln x)$

$y'' = x^x(1 + \ln x)^2 + \frac{x^x}{x}$

$f''(2) - 2f'(2) = (4(1+\ln 2)^2 + 2) - (2)(4(1+\ln 2))$

$= 4(1 + (\ln 2)^2) + 2 - 8$

$= 4(\ln 2)^2 - 2$

Q.78 Let $\vec{a} = 5\hat{i} - \hat{j} - 3\hat{k}$ and $\vec{b} = \hat{i} + 3\hat{j} + 5\hat{k}$ be two vectors. Then which one of the following statements is True?

(1) Projection of $\vec{a}$ on $\vec{b}$ is $\frac{-17}{\sqrt{35}}$ and the direction of the projection vector is same as of $\vec{b}$.

(2) Projection of $\vec{a}$ on $\vec{b}$ is $\frac{17}{\sqrt{35}}$ and the direction of the projection vector is opposite to the direction of $\vec{b}$.

(3) Projection of $\vec{a}$ on $\vec{b}$ is $\frac{17}{\sqrt{35}}$ and the direction of the projection vector is same as of $\vec{b}$.

(4) Projection of $\vec{a}$ on $\vec{b}$ is $\frac{-17}{\sqrt{35}}$ and the direction of the projection vector is opposite to the direction of $\vec{b}$.

Ans. **[2]**

Sol. $\vec{a} = 5\hat{i} - \hat{j} - 3\hat{k}$, $\vec{b} = \hat{i} + 3\hat{j} + 5\hat{k}$

projection of $\vec{a}\cdot\vec{b} = \left|\frac{5-3-15}{\sqrt{35}}\right| = \frac{17}{\sqrt{35}}$

$\vec{a}\cdot\vec{b} < 0$

$\therefore$ Option (2) is correct.

Q.79 Let $9 = x_1 < x_2 < \ldots < x_7$ be in an A.P. with common difference d. If the standard deviation of $x_1, x_2, \ldots x_7$ is 4 and the mean is $\bar{x}$, then $\bar{x} + x_6$ is equal to:

(1) $18\left(1+\frac{1}{\sqrt{3}}\right)$ (2) 34 (3) 25 (4) $2\left(9+\frac{8}{\sqrt{7}}\right)$

Ans. [2]

Sol. Let the series be a – 3d, a – 2d, a – d, a, a + d, a + 2d, a + 3d

Given $x_1 = 9 \Rightarrow a - 3d = 9$ …(i)

Variance does not change of shifting origin

∴ Variance and mean of –3d, –2d, –d, 0, d, 2d, 3d is 16 and $\bar{x} - a$

$$\Rightarrow 16 = \frac{2}{7}(9d^2 + 4d^2 + d^2) - (0)^2$$

$$\Rightarrow 16 = \frac{2}{7} \times 14d^2$$

$\Rightarrow d = 2$ (A.P. is increasing)

Using (i)

a = 15

$x_6 = a + 2d$

$= 15 + 4 = 19$

$\bar{x} + x_6 = a + 19$

$= 15 + 19$

$= 34$

∴ option (2) is correct.

Q.80 If $A = \frac{1}{2}\begin{bmatrix} 1 & \sqrt{3} \\ -\sqrt{3} & 1 \end{bmatrix}$, then :

(1) $A^{30} - A^{25} = 2I$ (2) $A^{30} + A^{25} - A = I$ (3) $A^{30} + A^{25} + A = I$ (4) $A^{30} = A^{25}$

Ans. [2]

Sol. $A = \frac{1}{2}\begin{bmatrix} 1 & \sqrt{3} \\ -\sqrt{3} & 1 \end{bmatrix}$

Let $\theta = \frac{\pi}{3}$

$$A^2 = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix}$$

$$A^3 = \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix}\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos 3\theta & \sin 3\theta \\ -\sin 3\theta & \cos 3\theta \end{bmatrix}$$

$$\therefore \quad A^{30} = \begin{bmatrix} \cos 30\theta & \sin 30\theta \\ -\sin 30\theta & \cos 30\theta \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$A^{25} = \begin{bmatrix} \cos 25\theta & \sin 25\theta \\ -\sin 25\theta & \cos 25\theta \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 1 & \sqrt{3} \\ -\sqrt{3} & 1 \end{bmatrix} = A$$

∴ $A^{30} + A^{25} - A = 1$

∴ option (2) is correct.

Section-B: Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer..

Q.81 If the term without x in the expansion of $\left(x^{\frac{2}{3}}+\frac{\alpha}{x^3}\right)^{22}$ is 7315, then $|\alpha|$ is equal to _______.

Ans. **[01]**

Sol. Given expansion $\left(x^{\frac{2}{3}}+\frac{\alpha}{x^3}\right)^{22}$

$$T_{r+1} = {}^{22}C_r \left(x^{\frac{2}{3}}\right)^{22-r}\left(\frac{\alpha}{x^3}\right)^r$$

For constant term

$$\frac{44-2r}{3} - 3r = 0$$

$$\boxed{r = 4}$$

Now ${}^{22}C_4\, \alpha^4 = 7315$

$$\frac{22\times21\times20\times19}{4\times3\times2\times1}\alpha^4 = 7315$$

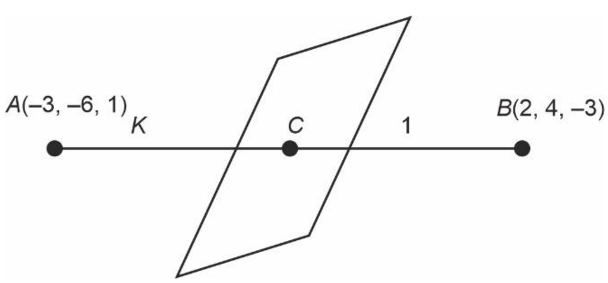
$\therefore\ \alpha^4 = 1$

$\therefore\ |\alpha| = 1$

Q.82 The point of intersection C of the plane $8x + y + 2z = 0$ and the line joining the points A(–3, –6, 1) and B(2, 4, –3) divides the line segment AB internally in the ratio k : 1. If a, b, c (|a|, |b|, |c| are coprime) are the direction ratios of the perpendicular from the point C on the line $\frac{1-x}{1}=\frac{y+4}{2}=\frac{z+2}{3}$, then $|a + b + c|$ is equal to _______.

Ans. **[10]**

Sol.



Then $C = \left(\frac{2K-3}{K+1}, \frac{4K-6}{K+1}, \frac{-3K+1}{K+1}\right)$

It lies on $8x + y + 2z = 0$

$\therefore\ 16K - 24 + 4K - 6 - 6K + 2 = 0$

$\therefore\ \boxed{K = 2}$

$\therefore\ C \equiv \left(\frac{1}{3}, \frac{2}{3}, \frac{-5}{3}\right)$

Given line : $\frac{x-1}{-1}=\frac{y+4}{2}=\frac{z+2}{3}=t$

$x = -t + 1,\ y = 2t - 4,\ z = 3t - 2$

for 1^r

$$-\left(1-t-\frac{1}{3}\right)+2\left(2t-4-\frac{2}{3}\right)+3\left(3t-2+\frac{5}{3}\right)=0$$

$$14t = 11 \Rightarrow t=\frac{11}{14}$$

$\therefore$ $PR'\left\langle \frac{-5}{3\times14},\frac{-130}{3\times14},\frac{85}{3\times14}\right\rangle$

$\therefore$ $|a+b+c| = 10$

Q.83 If the x-intercept of a focal chord of the parabola $y^2 = 8x + 4y + 4$ is 3, then the length of the chord is equal to _______.

Ans. **[16]**

Sol. $y^2 = 8x + 4y + 4$

$(y-2)^2 = 8(x+1)$

Focus $\equiv (1, 2)$

Equation of focal chord : $\frac{x}{3}+\frac{y}{b}=1$ and $\frac{1}{3}+\frac{2}{b}=1$

$\therefore \boxed{b = 3}$

$\therefore x + y = 3$

Intersection with parabola

$y^2 + 4 - 4y = 8(4 - y)$

$y^2 + 4y - 28 = 0$

$\therefore (y_1 - y_2)^2 = 16 + 4 \times 28$

$(x_1 - x_2)^2 = 16 + 4 \times 28$

$\therefore$ length $= \sqrt{2\times16\times8} = 16$

Q.84 Let $\alpha x + \beta y + \gamma z = 1$ be the equation of a plane passing through the point (3, –2, 5) and perpendicular to the line joining the points (1, 2, 3) and (–2, 3, 5). Then the value of $\alpha\beta\gamma$ is equal to _______.

Ans. **[06]**

Sol. Plane :

$a(x-3) + b(y+2) + c(z-5) = 0$

Dr's of plane : $3\hat{i}-\hat{j}-2\hat{k}$

$< 3, -1, -2 >$

$P : 3(x-3) -1(y+2) - 2(z-5) = 0$

$3x - 9 - y - 2 - 2z + 10 = 0$

$3x - y - 2z = 1$

$\therefore \alpha = 3, \beta = -1, \gamma = -2$

$\alpha\beta\gamma = 6$

Q.85 The line x = 8 is the directrix of the ellipse E : $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ with the corresponding focus (2, 0). If the tangent to E at point P in the first quadrant passes through the point (0, $4\sqrt{3}$) and intersects the x-axis at Q, then $(3PQ)^2$ is equal to :

Ans. **[39]**

Sol. $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

$\left.\begin{matrix} F(2,0)\equiv(ae,0) \\ x=8\equiv x=\frac{a}{e}\end{matrix}\right\}\left.\begin{matrix} a=4 \\ e=\frac{1}{2}\end{matrix}\right\}\Rightarrow b^2=12$

$E:\frac{x^2}{16}+\frac{y^2}{12}=1$

$T: y=mx\pm\sqrt{16m^2+12}$

Passes through $(0, 4\sqrt{3})$

$4\sqrt{3}=\pm\sqrt{16m^2+12}$

$\therefore \quad m=\pm\frac{3}{2}$

$T: y=\frac{-3}{2}x+\sqrt{48}$

$T:\frac{3}{2}x+y=\sqrt{48}$ …(i)

$T:\frac{xx_1}{16}+\frac{yy_1}{12}=1$ …(ii)

Comparing (i) and (ii)

$P\left(\frac{\sqrt{48}}{2},\frac{\sqrt{48}}{4}\right)$

$Q\left(\frac{2\sqrt{48}}{3},0\right)$

$(3PQ)^2=9(PQ)^2$

$=9\left(\left(\frac{\sqrt{48}}{2}-\frac{2\sqrt{48}}{3}\right)^2+\left(\frac{\sqrt{48}}{4}\right)^2\right)=39$

Q.86 Number of integral solutions to the equation $x+y+z=21$, where $x\geq 1, y\geq 3, z\geq 4$, is equal to ______.

Ans. **[105]**

Sol. $x+y+z=21$

$\because x\geq 1, y\geq 3, y\geq 4$

$\therefore x_1+y_1+z_1=13$

Number of solutions $=13+3-{}^1C_{3-1}$

$={}^{15}C_2=\frac{151}{21,131}=7\times 15$

$=105$

Q.87 Let the sixth term in the binomial expansion of $\left(\sqrt{2^{\log_2(10-3^x)}}+\sqrt[5]{2^{(x-2)\log_2 3}}\right)^m$, in the increasing powers of $2^{(x-2)\log_2 3}$, be 21. If the binomial coefficients of the second, third and fourth terms in the expansion are respectively the first, third and fifth terms of an A.P., then the sum of the squares of all possible values of x is ________.

Ans. **[04]**

Sol. $^mC_1$, $^mC_2$, $^mC_3$ are first, third and fifth term of AP

$\therefore$ $a = {}^mC_1$

$a + 2d = {}^mC_2$

$a + 4d = {}^mC_3$

$\therefore$ $2\,{}^mC_2 - {}^mC_3 = m$

$\Rightarrow$ $m = 7$ or $m = 2$

$\because$ $m = 2$ is not possible

$\therefore$ $m = 7$

$$\left(\sqrt{2^{\log_2(10-3^x)}} + \sqrt[5]{2^{(x-2)\log_2^3}}\right)^m$$

$T_6 = 21$

$$^7C_5\left((10-3^x)^{\frac{1}{2}}\right)^{7-5}(3)^{x-2} = 21$$

$$\frac{27}{9}3^x(10-3^x) = 27$$

$3^x(10 - 3^x) = 27$

$3^x(10 - 3^x) = 9$

Let $3^x = t$

$t(10 - t) = 9$

$t^2 - 10t + 9 = 0$

$(t - 9)(t - 1) = 0$

$t = 9$ or $t = 1$

$3^x = 9$ or $x_1 = 0$

$\therefore$ $x_1 = 2$ or $x_2 = 0$

$$x_1^2 + x_2^2 = 4$$

Q.88 The sum of the common terms of the following three arithmetic progressions.

3, 7, 11, 15, …, 399,

2, 5, 8, 11, …, 359 and

2, 7, 12, 17, …, 197,

Is equal to ______.

Ans. **[321]**

Sol. $S_1 \rightarrow 3, 7, 11, \ldots, 399$

$S_2 \rightarrow 2, 5, 8, \ldots, 359$

$S_3 \rightarrow 2, 7, 12, \ldots, 197$

Common terms of S_2 and S_3 are given by

$S_4 \rightarrow 2, 17, 32, \ldots, a_n$

$a_n \le 197$

$2 + 15(n - 1) \le 197$

$n \le 14$

$S_4 \rightarrow 2, 17, 32, \ldots, 197$

Common terms of S_4 and S_1 are given by

47, 107, 167

Sum = 47 + 107 + 167 = 321

Q.89 The total number of six digit numbers, formed using the digits 4, 5, 9 only and divisible by 6, is ______.

Ans. **[81]**

Sol. Units, place must be occupied by 4 and hence, at least one 4 must be there.

Possible combination of 4, 5, 9 are as follows

4 5 9		No. of Number
1 1 4	$\rightarrow$	$\frac{5!}{4!} = 5$
1 4 1	$\rightarrow$	$\frac{5!}{4!} = 5$
2 2 2	$\rightarrow$	$\frac{5!}{2!2!} = 30$
3 0 3	$\rightarrow$	$\frac{5!}{2!3!} = 10$
3 3 0	$\rightarrow$	$\frac{5!}{2!3!} = 10$
4 1 1	$\rightarrow$	$\frac{5!}{3!} = 20$
6 0 0	$\rightarrow$	$\frac{5!}{5!} = 1$
		Total = 81

Q.90 If $\int_0^{\pi} \frac{5^{\cos x}(1+\cos x\cos 3x+\cos^2 x+\cos^3 x\cos 3x)\,dx}{1+5^{\cos x}} = \frac{k\pi}{16}$, then k is equal to ________.

Ans. **[13]**

Sol. Let $g(x) = 1 + \cos x \cos 3x + \cos^2 x + \cos^3 x \cos 3x$

Clearly, $g(\pi + x) = g(x)$

$$I = \int_0^{\pi} \frac{5^{\cos x}(g(x))}{1+5^{\cos x}}dx \qquad \ldots.(i)$$

$$I = \int_0^{\pi} \frac{5^{\cos x}\times(g(x))}{1+5^{\cos x}}dx \quad \left(\because \int_0^{\pi} f(x)\,dx = \int_0^{\pi} f(\pi - x)\,dx\right)$$

$$I = \int_0^{\pi} \frac{1}{1+5^{\cos x}}g(x) \qquad \ldots.(ii)$$

$$(i) + (ii) \Rightarrow \quad 2I = \int_0^{\pi} g(x)\,dx$$

$$2I = \int_0^{\pi} 1+\cos x\cos 3x+\cos^2 x+\cos^3 x\cos 3x\,dx$$

$$= \int_0^{\pi} 1+\frac{1}{2}(\cos 4x+\cos 2x)+\frac{1}{2}(1+\cos 2x)+\frac{1}{4}(\cos 3x+3\cos x)\cos 3x\ dx$$

$$= \pi + \frac{1}{2}(0+0) + \frac{\pi}{2} + \frac{1}{2}(0) + \frac{1}{4}\int \cos^2 3x + 3\cos x\cos 3x\ dx$$

$$= \frac{3\pi}{2}+\frac{1}{4}\int\frac{1}{2}(1+\cos 6x)+\frac{3}{2}(\cos 4x+\cos 2x)\ dx$$

$$= \frac{3\pi}{2}+\frac{1}{4}\left(\frac{\pi}{2}+\frac{1}{2}\times 0+\frac{3}{2}(0+0)\right) = \frac{3\pi}{2}+\frac{\pi}{8}$$

$$= \frac{13\pi}{8}\Bigg| \Rightarrow\ I = \frac{13\pi}{16} \Rightarrow k = 13$$