JEE Main Online Exam 2023

Questions & Solution

01st February 2023 | Morning

PHYSICS

Section-A: This section contains 20 multiple choice questions. Each question has 4 choices(1), (2), (3) and (4), out of which **ONLY ONE** is correct..

Q.1 'n' polarizing sheets are arranged such that each makes an angle 45° with the proceeding sheet. An unpolarized light of intensity I is incident into this arrangement. The output intensity is found to be $\frac{I}{64}$. The value of n will be -

(1) 3 (2) 4 (3) 6 (4) 5

Ans. [3]

Sol. $I_{final} = \frac{I}{2}\left(\frac{1}{2}\right)^{n-1}$

$\frac{I}{2^6} = \frac{I}{2^n}$

n = 6

Q.2 A block of mass 5 kg is placed at rest on a table of rough surface. Now, if a force of 30 N is applied in the direction parallel to surface of the table, the block slides through a distance of 50 m in an interval of time 10 s. Coefficient of kinetic friction is (given, $g = 10\ ms^{-2}$)

(1) 0.50 (2) 0.60 (3) 0.75 (4) 0.25

Ans. [1]

Sol. $a = \frac{30 - 50\mu}{2}$

$\because \quad s = ut + \frac{1}{2}at^2$

$50 = \frac{1}{2}\left(\frac{30 - 50\mu}{5}\right) \times 100$

$5 = 30 - 50\mu$

$\mu = \frac{25}{15} = 0.5$

Q.3 Given below are two statements:

Statement I: Acceleration due to gravity is different at different places on the surface of earth.

Statement II: Acceleration due to gravity increases as we go down below the earth's surface.

In the light of the above statements, choose the correct answer from the options given below

(1) Statement I is false but Statement II is true
(2) Statement I is true but Statement II is false
(3) Both statement I and statement II are false
(4) Both statement I and statement II are true

Ans. [2]

Sol. Statement-I is correct as $g' = g - \omega^2 R\cos^2\phi$

Statement-II is clearly incorrect.

Q.4 Match List I with List II:

	List-I		List-II
(A)	Intrinsic semiconductor	(I)	Fermi-level near the valance band
(B)	n-type semiconductor	(II)	Fermi-level in the middle of the valence and conduction band
(C)	p-type semiconductor	(III)	Fermi-level near the conduction band
(D)	Metals	(IV)	Fermi-level inside the conduction band

Choose the correct answer from the options given below:

(1) A-II, B-III, C-I, D-IV
(2) A-I, B-II, C-III, D-IV
(3) A-II, B-I, C-III, D-IV
(4) A-III, B-I, C-II, D-IV

Ans. **[1]**

Sol. (Theoretical)

(A) Intrinsic semiconductor → II
(B) n-type semiconductor → III
(C) p-type semiconductor → I
(D) Metals → IV

Q.5 $\left(P + \frac{\alpha}{V^2}\right)(V - b) = RT$ represents the equation of state of some gases. Where P is the pressure, V is the volume. T is the temperature and a, b R are the constant. The physical quantity, which has dimensional formula as that of $\frac{b^2}{a}$ will be

(1) Compressibility
(2) Energy density
(3) Modulus of rigidity
(4) Bulk modulus

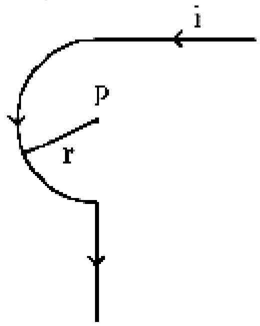
Ans. **[1]**

Sol. $[a] = [ML^5T^{-2}]$

$[b] = [L^3]$

$$\left[\frac{b^2}{a}\right] = \left[\frac{L^6}{ML^5T^2}\right] = [M^{-1}LT^{-2}] = \text{[compressibility]}$$

Q.6 Find the magnetic field at the point P in figure. The curved portion is a semicircle connected to two long straight wires.



(1) $\frac{\mu_0 i}{2r}\left(1 + \frac{2}{\pi}\right)$
(2) $\frac{\mu_0 i}{2r}\left(1 + \frac{1}{\pi}\right)$
(3) $\frac{\mu_0 i}{2r}\left(\frac{1}{2} + \frac{1}{2\pi}\right)$
(4) $\frac{\mu_0 i}{2r}\left(\frac{1}{2} + \frac{1}{\pi}\right)$

Ans. **[3]**

Sol. $B_P = \frac{\mu_0 i}{4\pi r} + \frac{1}{2}\left(\frac{\mu_0 i}{2r}\right)$

Q.7 A steel wire with mass per unit length 7.0×10^{-3} kg m^{-1} is under tension of 70 N. The speed of transverse waves in the wire will be

(1) 200 m/s (2) 100 m/s (3) 50 m/s (4) 10 m/s

Ans. **[2]**

Sol. Speed of transverse wave = $\sqrt{\frac{T}{M}}$

$= \sqrt{\frac{70}{7\times10^{-3}}} = 100 \text{ m/s}$

Q.8 A sample of gas at temperature T is adiabatically expanded to double its volume. The work done by the gas in the process is $\left(\text{given}, \gamma = \frac{3}{2}\right)$

(1) $W = \frac{T}{R}[\sqrt{2} - 2]$ (2) $W = RT[2 - \sqrt{2}]$ (3) $W = TR[\sqrt{2} - 2]$ (4) $W = \frac{R}{T}[2 - \sqrt{2}]$

Ans. **[2]**

Sol. $\gamma = \frac{3}{2}$

$W = \frac{nR\Delta T}{1-\gamma} = \frac{nRT_f - nRT_i}{1-\gamma}$

$= \frac{(PV)_f - (PV_i)}{1-\gamma}$ ... (1)

$PiV_i^{\gamma} = P_f(2V_i)^{\gamma} \Rightarrow P_f = \frac{P_i}{2^{\gamma}} = \frac{P_i}{2\sqrt{2}}$ ... (2)

From (1) and (2)

$W = \frac{\frac{P_i}{2\sqrt{2}} - 2V_iP_iV_i}{1-\gamma} = \frac{P_iV_i}{-1/2}\left(\frac{1}{\sqrt{2}} - 1\right)$

$= -nRT(\sqrt{2} - 2)$

$= nRT(2 - \sqrt{2})$

Q.9 The average kinetic energy of a molecule of the gas is

(1) dependent on the nature of the gas (2) proportional to volume
(3) proportional to absolute temperature (4) proportional to pressure

Ans. **[3]**

Sol. Average kinetic energy of a molecule of gas

$= \frac{f}{2}k_BT$

f is degree of freedom

Q.10 Match List I with List II

	List-I		List-II
(A)	AC generator	(I)	Presence of both L and C
(B)	Transformer	(II)	Electromagnetic Induction
(C)	Resonance phenomenon to occur	(III)	Quality factor
(D)	Sharpness of resonance	(IV)	Mutual Induction

Choose the correct answer from the options given below

(1) A-II, B-I, C-III, D-IV (2) A-II, B-IV, C-I, D-III

(3) A-IV, B-II, C-I, D-III (4) A-IV, B-III, C-I, D-II

Ans. **[2]**

Sol. AC generator works on EMZ principle (A-II) Transformer uses Mutual induction (B-IV) Resonance occurs when both L and C are present (C–Z) and quality factor determines sharpness of resonance (D-III)

Q.11 Which of the following frequencies does not belong to FM broadcast.

(1) 99 MHz (2) 64 MHz (3) 89 Mhz (4) 106 MHz

Ans. **[2]**

Sol. FM broadcast varies from 89 Hz to 108 Hz

Q.12 If earth has a mass nine times and radius twice to that of a planet P. Then $\frac{v_e}{3}\sqrt{x}\text{ms}^{-1}$ will be the minimum velocity required by a rocket to pull out of gravitational force of, P, where v_e is is escape velocity on earth. The value of x is

(1) 2 (2) 18 (3) 1 (4) 3

Ans. **[1]**

Sol. $M_E = 9M_P$

$R_E = 2R_P$

Escape velocity $= \sqrt{\frac{2mG}{R}}$

For earth $v_e = \sqrt{\frac{2GM_E}{R_E}}$

For P, $v_e = \sqrt{\frac{\frac{2GM_E}{9}}{\frac{R_E}{2}}} = \sqrt{\frac{2GM_E}{R_E} \times \frac{2}{9}}$

$= \frac{v_e\sqrt{2}}{3}$

Q.13 The mass of proton, neutron and helium nucleus are respectively 1.0073u, 1.0087u and 4.0015u. The binding energy of helium nucleus is

(1) 56.8 MeV (2) 28.4 MeV (3) 7.1 MeV (4) 14.2 MeV

Ans. **[2]**

Sol. Mass defect = 2 (Mass of p + mass of n) – mass of He nucleus

$\Delta m = 0.0305u$

B.E. $= 931.5 \times \Delta m = 931.5 \times 0.0305$

$= 28.4$ MeV

Q.14 A proton moving with one tenth of velocity of light has a certain de Broglie wavelength of λ. An alpha particle having certain kinetic energy has the same de-Brogle wavelength λ. The ratio of kinetic energy of proton and that of alpha particle is

(1) 1 : 4 (2) 1 : 2 (3) 2 : 1 (4) 4 : 1

Ans. **[4]**

Sol. For same λ_1 momentum should be same

$(P)_P = (P)_\alpha$

$\Rightarrow \sqrt{2k_P m_P} = \sqrt{2K_\alpha m_\alpha}$

$\Rightarrow k_P m_P = k_\alpha m_\alpha$

$\frac{k_P}{k_\alpha} = \left(\frac{m_\alpha}{m_P}\right) = \frac{4}{1} = 4:1$

Q.15 A mercury drop of radius 10^{-3} m is broken into 125 equal size droplets. Surface tension of mercury is 0.45 Nm^{-1}. The gain in surface energy is

(1) 17.5×10^{-5} J (2) 28×10^{-5} J (3) 5×10^{-5} J (4) 2.26×10^{-5} J

Ans. **[4]**

Sol. Initial volume = Final volume

So, $\boxed{R = 5r}$

Gain in surface energy = $[125 \times 4\pi r^2 \times T - 4\pi R^2 T]$

$= 4\pi T\,[125r^2 - R^2]$

$= 16\pi R^2 T$

$= 16\pi \times (10^{-3})2 \times 0.45$

$= 22.6 \times 10^{-6}$ J

$= 2.26 \times 10^{-5}$ J

Q.16 Match List I with List II

	List-I		List-II
(A)	Microwaves	(I)	Radio active decay of the nucleus
(B)	Gamma rays	(II)	Rapid acceleration and deceleration of electron in aerials
(C)	Radio waves	(III)	Inner shell electrons
(D)	X-rays	(IV)	Klystron valve

Choose the correct answer from the options given below:

(1) A-I, B-II, C-III, D-IV (2) A-IV, B-I, C-II, D-III

(3) A-IV, B-III, C-II, D-I (4) A-I, B-III, C-IV, D-II

Ans. **[2]**

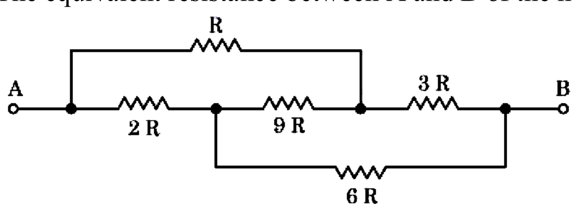
Sol. 1. Klystron valve used to produce Microwave

2. Gamma ray $\rightarrow$ Radioactive decay

3. Radio wave $\rightarrow$ Rapid acceleration and deacceleration of electrons in aerials

4. X-ray $\rightarrow$ Inner shell electrons

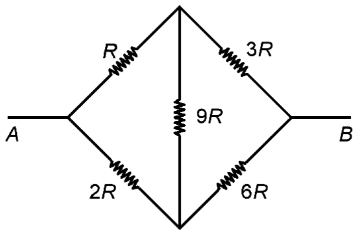
Q.17 The equivalent resistance between A and B of the network shown in figure:



(1) $\frac{8}{3}$R (2) 21 R (3) 14 R (4) $11\frac{2}{3}$R

Ans. **[1]**

Sol.



This is balanced Wheatstone bridge,

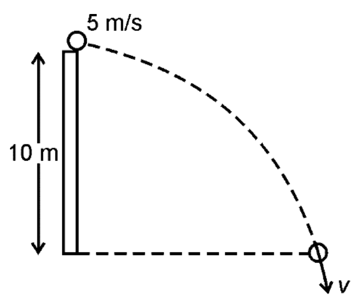
$$R_{eq} = \frac{4R \times 8R}{12R} = \left(\frac{8R}{3}\right)$$

Q.18 A child stands on the edge of the cliff 10 m above the ground and throws a stone horizontally with an initial speed of 5 ms^{-1}. Neglecting the air resistance, the speed with which the stone hits the ground will be ____ ms^{-1} (given, $g = 10\ ms^{-2}$).

(1) 15 (2) 25 (3) 30 (4) 20

Ans. **[1]**

Sol.

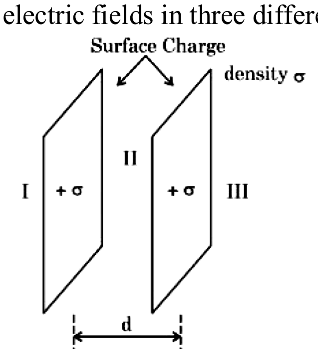


$$v = \sqrt{u^2 + 2gh}$$

$$= \sqrt{25 + 25 \times 10 \times 10}$$

$$= \sqrt{225} = 15 \text{ m/s}$$

Q.19 Let σ be the uniform surface charge density of two infinite thin plane sheets shown in figure. Then the electric fields in three different region E_I, E_{II} and E_{III} are :



(1) $\vec{E}_I = 0,\ \vec{E}_{II} = \frac{\sigma}{\epsilon_0}\hat{n},\ E_{III} = 0$

(2) $\vec{E}_I = -\frac{\sigma}{\epsilon_0}\hat{n},\ \vec{E}_{II} = 0,\ \vec{E}_{III} = \frac{\sigma}{\epsilon 0}\hat{n}$

(3) $\vec{E}_I = -\frac{2\sigma}{2\epsilon_0}\hat{n},\ \vec{E}_{II} = 0,\ \vec{E}_{III} = \frac{2\sigma}{\epsilon_0}\hat{n}$

(4) $\vec{E}_I = -\frac{\sigma}{2\epsilon_0}\hat{n},\ \vec{E}_{II} = 0,\ \vec{E}_{III} = \frac{\sigma}{\epsilon_0}\hat{n}$

Ans. [2]

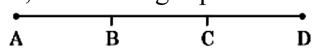
Sol. From the figure:

$$\vec{E}_1 = \frac{\sigma}{2\epsilon_0} + \frac{\sigma}{2\epsilon_0} \qquad \text{(Leftward)}$$

$$\vec{E}_2 = \frac{\sigma}{2\epsilon_0} - \frac{\sigma}{2\epsilon_0}$$

$$\vec{E}_3 = \frac{\sigma}{2\epsilon_0} + \frac{\sigma}{2\epsilon_0} \qquad \text{(Rightward)}$$

Q.20 An object moves with speed v_1, v_2 and v_3 along a line segment AB, BC and CD respectively as shown in figure. Where AB=BC and AD = 3AB, then average speed of the object will be:



(1) $\frac{v_1 v_2 v_3}{3(v_1v_2 + v_2v_3 + v_3v_1)}$ (2) $\frac{(v_1 + v_2 + v_3)}{3}$ (3) $\frac{3v_1v_2v_3}{(v_1v_2 + v_2v_3 + v_3v_1)}$ (4) $\frac{(v_1 + v_2 + v_3)}{3v_1v_2v_3}$

Ans. [3]

Sol. AB = BC = CD

$$\Rightarrow \text{Average speed} = \frac{\text{Distance}}{\text{Time}}$$

$$= \frac{AD}{\frac{AB}{V_1} + \frac{AB}{V_2} + \frac{AB}{V_3}}$$

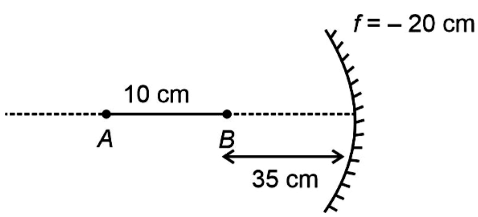
$$= \frac{3V_1V_2V_3}{V_1V_2 + V_2V_3 + V_1V_3}$$

Section-B: Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer..

Q.21 A thin cylindrical rod of length 10 cm is placed horizontally on the principle axis of a concave mirror of focal length 20 cm. The rod is placed in a such a way that mid point of the rod is at 40 cm from the pole of mirror. The length of the image formed by the mirror will be $\frac{x}{3}$ cm. The value of x is ___________.

Ans. [32]

Sol.



$$A : \frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$\Rightarrow \frac{1}{v} + \frac{1}{-45} = \frac{1}{-20}$$

$$\Rightarrow \frac{1}{v} = \frac{1}{45} - \frac{1}{20} = \frac{4-9}{180} = -\frac{1}{36}$$

$\Rightarrow v = -36$ cm

$$B : \frac{1}{v} + \frac{1}{-35} = \frac{1}{-20}$$

$$\Rightarrow \frac{1}{v} = \frac{1}{35} - \frac{1}{20} = \frac{4-7}{140}$$

$$\Rightarrow v = \frac{140}{3}$$

$$\Rightarrow \text{length of image} = \frac{140}{3} - 36 = \frac{32}{3}$$

$\Rightarrow x = 32$

Q.22 The amplitude of a particle executing SHM is 3 cm. The displacement at which its kinetic energy will be 25% more than the potential energy is: _________ cm.

Ans. **[2]**

Sol. A = 3 cm

K = 1.25 U

$$\Rightarrow K + \frac{K}{1.25} = K_{max}$$

$$\Rightarrow \frac{9}{5}K = K_{max}$$

$$\Rightarrow \frac{9}{5}\frac{1}{2}mv^2 = \frac{1}{2}mv_{max}^2$$

$$\Rightarrow \frac{9}{5}\left[\omega\sqrt{A^2 - x^2}\right]^2 = \omega^2 A^2$$

$$\Rightarrow 9(A^2 - x^2) = 5A^2$$

$$\Rightarrow x^2 = \frac{4A^2}{9}$$

$$\Rightarrow x = \frac{2A}{3}$$

$\Rightarrow x = 2$ cm

Q.23 A certain pressure 'P' is applied to 1 litre of water and 2 litre of a liquid separately. Water gets compressed to 0.01% whereas the liquid gets compressed to 0.03%. The bulk modulus of water to that of the liquid is $\frac{3}{x}$. The value of x is ______ is.

Ans. **[1]**

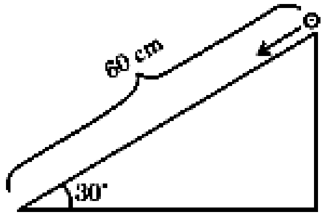
Sol. $$B = \frac{-dp}{\frac{dv}{v}}$$

$$\Rightarrow \frac{B_{water}}{B_{Liquid}} = \frac{\left(\frac{dv}{v}\right)_{liquid}}{\left(\frac{dv}{v}\right)_{water}}$$

$$\frac{0.03}{0.01} = 3$$

$\Rightarrow x = 1$

Q.24 A solid cylinder is released from rest from the top of an inclined plane of inclination 30° and length 60 cm. If the cylinder rolls without slipping, its speed upon reaching the bottom of the inclined plane is_______ ms^{-1}. (Given $g = 10\ ms^{-2}$)



Ans. **[2]**

Sol. Loss in potential energy = mgh = mg[60 sin30° cm]

$$\Rightarrow mg\left[\frac{30}{100}\right] = \frac{1}{2}mv^2 + \frac{1}{2}\frac{mv^2}{2}$$

$$\Rightarrow 0.3 \times 10 = \frac{3}{4}v^2$$

$$\Rightarrow V^2 = 4$$

$$\Rightarrow V = 2\ m/s$$

Q.25 A light of energy 12.75 eV is incident on a hydrogen atom in its ground state. The atom absorbs the radiation and reaches to one of its excited states. The Angular momentum of the atom in the excited state is $\frac{x}{m} \times 10^{-17}$ eVs. The value of x is ________ (use $h = 4.14 \times 10^{-15}$eVs, c = 3 × 108 ms^{-1}).

Ans. **[828]**

Sol. Let the electron jumps to n^{th} orbit so

$$12.75 = 13.6\left[\frac{1}{1^2} - \frac{1}{n^2}\right]$$

$$\Rightarrow n = 4$$

$$\text{So,}\quad L = \frac{nh}{2\pi} = \frac{2h}{\pi}$$

$$= \frac{2 \times 4.14 \times 10^{-15}}{\pi}$$

$$= 8.28 \times 10^{-15}\ eVs$$

Q.26 A small particle moves to position $5\hat{i} - 2\hat{j} + \hat{k}$ from its initial position $2\hat{i} + 3\hat{j} - 4\hat{k}$ under the action of force $5\hat{i} + 2\hat{j} + 7\hat{k}$ N . The value of work done will be ______ J.

Ans. **[40]**

Sol. $W = \vec{F} \cdot (\vec{F}_2 - \vec{F}_1)$

$$= (5\hat{i} + 2\hat{j} + 7\hat{k}) \cdot (3\hat{i} - 5\hat{j} + 5\hat{k})$$

$$= 15 - 10 + 35$$

$$= 40\ J$$

Q.27 A series LCR circuit is connected to an ac source of 220 V, 50 Hz. The circuit contain a resistance R = 100 Ω and an inductor of inductive reactance X_L = 79.6 Ω. The capacitance of the capacitor needed to maximize the average rate at which energy is supplied will be __________ μF.

Ans. **[40]**

Sol. Average rate of energy is maximum at resonance.

$\therefore X_L = X_C$

$79.6 = \frac{1}{2\pi(50) \times C}$

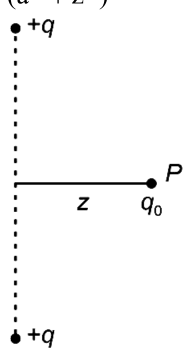
$\approx 40\mu F$

Q.28 Two equal positive point charges are separated by a distance 2a. The distance of a point from the centre of the line joining two charges on the equatorial line (perpendicular bisector) at which force experienced by a test charge q_0 becomes maximum is $\frac{a}{\sqrt{x}}$. The value of x is ______ .

Ans. [2]

Sol. $F_P = q_0 E_p = q_0 \frac{kqz}{(a^2 + z^2)^{3/2}}$

or $F_P = \frac{kq_0 z}{(a^2 + z^2)^{3/2}}$



To maximize $\frac{dF_P}{dz} = 0$

or $kqq_0 = \frac{(a^2 + z^2)^{3/2} - z\frac{3}{2} \times 2z(a^2 + z^2)^{\frac{1}{2}}}{(a^2 + z^2)^3} = 0$

$\Rightarrow z = \frac{a}{\sqrt{2}}$

Q.29 A charge particle of 2 μC accelerated by a potential difference of 100 V enters a region of uniform magnetic field of magnitude 4 mT at right angle to the direction of field. The charge particle completes semicircle of radius 3 cm inside magnetic field. The mass of the charge particle is ________ × 10^{-18} kg.

Ans. [144]

Sol. $R = \sqrt{\frac{2mqV}{qB}}$

$R = \frac{1}{B}\sqrt{\frac{2mV}{q}}$ or $m = \frac{R^2 B^2 q}{2V}$

$= \frac{(3 \times 10^{-2})^2 \times (4 \times 10^{-3})^2 \times 2 \times 10^{-5}}{2 \times 100}$

$= 144 \times 10^{-18}$ kg

Q.30 In an experiment to find emf of a cell using potentiometer, the length of null point for a cell of emf 1.5 V is found to be 60 cm. If this cell is replaced by another cell of emf E, the length of null point increases by 40 cm. The value of E is $\frac{x}{10}$ V. The value of x is _______ .

Ans. **[25]**

Sol. $E \propto \ell$

$$\frac{E_1}{E_2} = \frac{\ell_1}{\ell_2}$$

$$\frac{1.5}{E} = \frac{60}{100}$$

$$E = \frac{150}{60} = \frac{5}{2} = \frac{25}{10}$$

So, $x = 25$

CHEMISTRY

Section-A: Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Q.31 Given below are two statements :

Statement-I : Chlorine can easily combine with oxygen to form oxides; and the product has a tendency to explode.

Statement-II : Chemical reactivity of an element can be determined by its reaction with oxygen and halogens.

In the light of the above statements, choose the **correct** answer from the options given below

(1) Statement I is true but statement-II is false

(2) Both the Statements I and II are true

(3) Statement I is false but Statement II is true

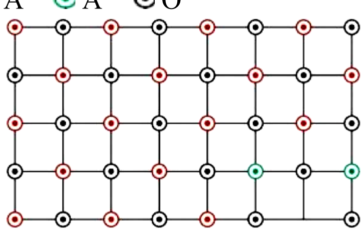
(4) Both the Statements I and II are false

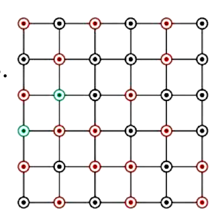
Ans. **[2]**

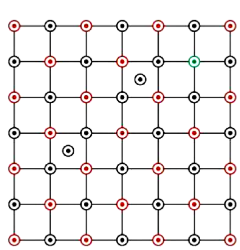
Sol.
- Chlorine can easily combine with oxygen to form oxides, which can explode
- Chemical reactivity of an element can be determined by its reaction with oxygen and Halogens.

Q.32 Which of the following represents the lattice structure of $A_{0.95}O$ containing A^{2+}, A^{3+} and O^{2-} ions ?

⊙ A^{2+} ⊙ A^{3+} ⊙ O^{2-}

A.


B.


C.


(1) A only (2) A and B only (3) B only (4) B and C only

Ans. **[1]**

Sol. $A_{0.95}O$

% of $A^{2+} = \frac{85}{95} \times 100 \approx 90\%$

% of $A^{3+} = \frac{10}{95} \times 100 \approx 10\%$

Option (A) satisfies this condition

Q.33 Given below are two statements : one is labelled as **Assertion A** and the other is labelled as **Reason R**

Assertion A : Hydrogen is an environment friendly fuel.

Reason R : Atomic number of hydrogen is 1 and it is very light element.

In the light of the above statements, choose the **correct** answer from the options given below

(1) Both **A** and **R** are true and **R** is the correct explanation of **A**

(2) **A** is true but **R** is false

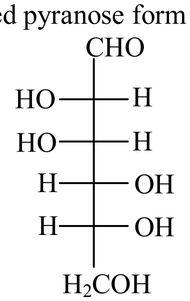
(3) Both **A** and **R** are true but **R** is **NOT** the correct explanation of **A**.

(4) **A** is false but **R** is true

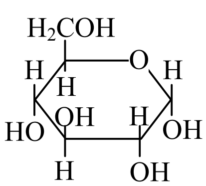
Ans. **[3]**

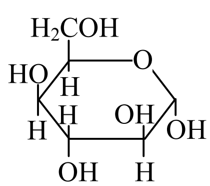
Sol. Hydrogen is an environment friendly fuel as its combustion produces only water vapours.

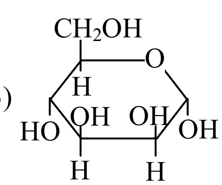
Q.34 The correct representation in six membered pyranose form for the following sugar [X] is

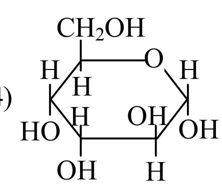


Sugar [X]

(1) 

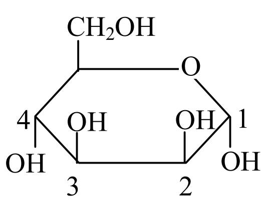
(2) 

(3) 

(4) 

Ans. **[3]**

Sol.



C_2 and C_3OH are cis

C_3 and C_4 are anti to each other.

Q.35 Match **List-I** with **List-II**.

	List-I		**List-II**
A.	Tranquilizers	I.	Anti blood clotting
B.	Aspirin	II.	Salvarsan
C.	Antibiotic	III.	Antidepressant drugs
D.	Antiseptic	IV.	Soframicine

Choose the correct answer from the options given below :

(1) (A) – II, (B) – IV, (C) – I, (D) – III

(2) (A) – II, (B) – I, (C) – III, (D) – IV

(3) (A) – IV, (B) – II, (C) – I, (D) – III

(4) (A) – III, (B) – I, (C) – II, (D) – IV

Ans. **[4]**

Sol. (A) Tranquilizers are antidepressant drugs
(B) Aspirin prevents blood clotting and hence Anti blood clotting
(C) Salvarsan is an antibiotic
(D) Soframicine is antiseptic

Q.36 Given below are two statements : One is labelled as **Assertion A** and the other is labelled as **Reason R**.
Assertion A : In an Ellingham diagram, the oxidation of carbon to carbon monoxide shows a negative slope with respect to temperature.
Reason R : CO tends to get decomposed at higher temperature.
In the light of the above statements, choose the **correct** answer from the options given below.
(1) **A** is not correct but **R** is correct
(2) **A** is correct but **R** is not correct
(3) Both **A** and **R** are correct but **R** is **NOT** the correct explanation of **A**
(4) Both **A** and **R** are correct and **R** is the correct explanation of **A**

Ans. **[2]**

Sol. $C(s) + \frac{1}{2}O_2(g) \longrightarrow CO(g) \qquad (\Delta S > 0)$

Slope = (–ve)

CO doesn't get decompose at high temperature.

Q.37 Match **List-I** with **List-II**.

	List-I **Test**		**List-II** **Functional group/Class of compound**
A.	Molisch's Test	I.	Peptide
B.	Biuret Test	II.	Carbohydrate
C.	Carbylamine	III.	Primary amine
D.	Schiff's Test	IV.	Aldehyde

Choose the correct answer from the options given below :
(1) A - (III), B - (IV), C - (II), D - (I)
(2) A - (I), B - (II), C - (III), D - (IV)
(3) A - (II), B - (I), C - (III), D - (IV)
(4) A - (III), B - (IV), C - (I), D - (II)

Ans. **[3]**

Sol. (A) Moslich test is for carbohydrates
(B) Biuret test is for proteins/peptide
(C) Carbylamine test is for primary amine
(D) Schiff's test is for aldehyde

Q.38 A solution of $FeCl_3$ when treated with $K_4[Fe(CN)_6]$ gives a prussiun blue precipitate due to the formation of
(1) $Fe_3[Fe(CN)_6]_2$
(2) $Fe_4[Fe(CN)_6]_3$
(3) $Fe[Fe(CN)_6]$
(4) $K[Fe_2(CN)_6]$

Ans. **[2]**

Sol. $Fe^{3+} + [Fe(CN)_6]^{4-} \rightarrow \underset{\text{prussian blue}}{Fe_4[Fe(CN)_6]_3}$

Q.39 Given below are two statements : One is labelled as **Assertion A** and the other is labelled as **Reason R**.
Assertion A : Amongst He, Ne, Ar and Kr; 1 g of activated charcoal adsorbs more of Kr.
Reason R : The critical volume $V_c(cm^3\ mol^{-1})$ and critical pressure P_c (atm) is highest for Krypton but the compressibility factor at critical point Z_c is lowest for Krypton.
In the light of the above statements, choose the **correct** answer from the options given below.
(1) Both **A** and **R** are true but **R** is NOT the correct explanation of **A**
(2) **A** is true but **R** is false
(3) **A** is false but **R** is true
(4) Both **A** and **R** are true and **R** is the correct explanation of **A**

Ans. [2]

Sol. MW order, Kr > Ar > Ne > He

Z(at critical point) = $\frac{3}{8}$

Q.40 Which of the following complex will show largest splitting of d-orbitals ?

(1) $[Fe(C_2O_4)_3]^{3-}$ (2) $[FeF_6]^{3-}$

(3) $[Fe(CN)_6]^{3-}$ (4) $[Fe(NH_3)_6]^{3+}$

Ans. [3]

Sol. CN^- is strongest field ligand among given ligands.

Q.41 Which of the following are the example of double salt ?

(A) $FeSO_4.(NH_4)_2SO_4.6H_2O$ (B) $CuSO_4.4NH_3.H_2O$

(C) $K_2SO_4.Al_2(SO_4)_3.24H_2O$ (D) $Fe(CN)_2.4KCN$

Choose the correct answer

(1) B and D only (2) A and C (3) A, B and D only (4) A and B only

Ans. [2]

Sol. (A) $FeSO_4.(NH_4)_2SO_4.6H_2O$ - double salt

(B) $CuSO_4.4NH_3.H_2O = [Cu(NH_3)_4]SO_4.H_2O$ - complex salt

(C) $K_2SO_4.Al_2(SO_4)_3.24H_2O$ - double salt

(D) $Fe(CN)_2.4KCN$

$K_4[Fe(CN)_6]$ - complex salt

Q.42 How can photochemical smog be controlled ?

(1) By complete combustion of fuel

(2) By using catalyst

(3) By using catalytic convertors in the automobiles/industry.

(4) By using tell chimneys.

Ans. [3]

Sol. Photochemical smog is caused by Nitrogen oxides which can be prevented by using catalytic convertors in the automobiles/industry.

Q.43 But-2-yne is reacted separately with one mole of Hydrogen as shown below

$$\underline{B} \xleftarrow[\text{liq } NH_3]{Na} \underset{+H_2}{CH_3{-}C{\equiv}C{-}CH_3} \xrightarrow[\Delta]{Pd/C} \underline{A}$$

(A) A is more than soluble than B.

(B) The boiling point & melting point of A are higher and lower than B respectively.

(C) A is more polar than B because dipole moment of A is zero.

(D) Br_2 adds easily to B than A.

Identify the incorrect statements from the option given below

(1) A, C & D only (2) B, C & D only (3) B and C only (4) A and B only

Ans. [2]

Sol. A : Cis – But-2-ene

B : Trans-But-2-ene

BP : A > B

mp : B > A

μ-order = B > A (μ of A = 0)

Addition of Br_2 is easy in A.

Q.44 Choose the correct statement(s)

(A) Beryllium oxide is purely acidic in nature.

(B) Beryllium carbonate is kept in the atmosphere of CO_2.

(C) Beryllium sulphate is readily soluble in water.

(D) Beryllium shows anamalous behaviour.

Choose the correct answer from the options given below :

(1) A, B & C only (2) A only (3) A and B only (4) B, C and D only

Ans. **[4]**

Sol.

- BeO is amphoteric
- $BeCO_3 \rightleftharpoons BeO + CO_2$

To shift equilibrium in backward direction,
It is kept in atmosphere of CO_2

- $BeSO_4$ is readily soluble in water
- Be shows anomalous behaviour

Q.45 Highest oxidation state of Mn is exhibited in Mn_2O_7. The correct statements about Mn_2O_7 are

(A) Mn is tetrahedrally surrounded by oxygen atoms.

(B) Mn is octahedrally surrounded by oxygen atoms.

(C) Contains Mn-O-Mn bridge.

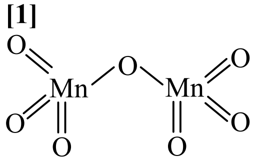
(D) Contains Mn-Mn bond.

Choose the correct answer from the options below :

(1) A and C only (2) B and C only (3) A and D only (4) B and D only

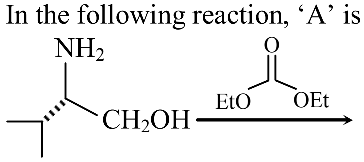
Ans. **[1]**

Sol.

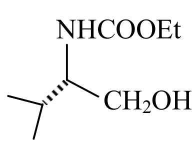


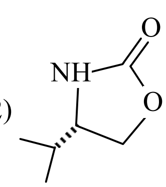
Mn is surrounded tetrahedrally by O-atoms.
Mn_2O_7, contains Mn-O-Mn Bridge.

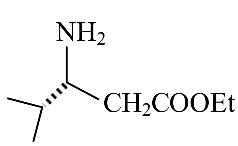
Q.46 In the following reaction, ‘A’ is

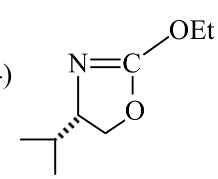


‘A’
Major product

(1)


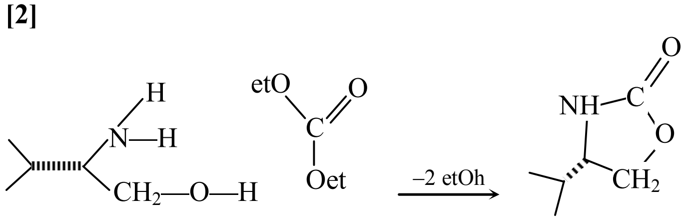
(2)


(3)


(4)


Ans. **[2]**

Sol.



Q.47 Match **List-I** with **List-II**.

	List-I		List-II
A.	Slaked lime	I.	NaOH
B.	Dead burnt plaster	II.	$Ca(OH)_2$
C.	Caustic soda	III.	$Na_2CO_3.10H_2O$
D.	Washing soda	IV.	$CaSO_4$

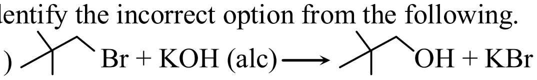
Choose the **correct** answer from the options given below :

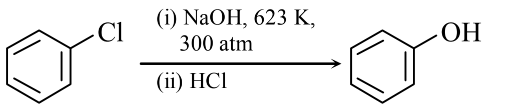
(1) A-II, B-IV, C-I, D-III
(2) A-III, B-IV, C-II, D-I
(3) A-III, B-II, C-IV, D-I
(4) A-I, B-IV, C-II, D-III

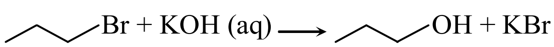
Ans. **[1]**

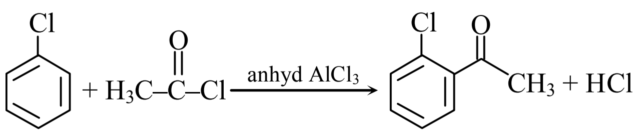
Sol.
A : Slaked lime : $Ca(OH)_2$
B : Dead burnt plaster : $CaSO_4$
C : Caustic Soda : NaOH
D : Washing Soda : $Na_2CO_3 . 10H_2O$

Q.48 Identify the incorrect option from the following.

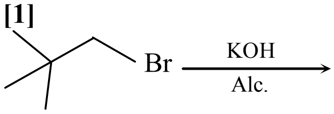
(1) 

(2) 

(3) 

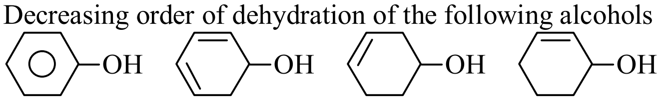
(4) 

Ans. **[1]**

Sol. 

doesn't undergoes
E^2 reaction due to
absence of α-H

Q.49 Decreasing order of dehydration of the following alcohols is



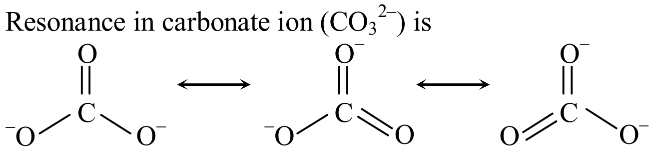
(a) (b) (c) (d)

(1) (b) > (a) > (d) > (c)
(2) (a) > (d) > (b) > (c)
(3) (b) > (d) > (c) > (a)
(4) (d) > (b) > (c) > (a)

Ans. **[3]**

Sol. (b) > (d) > (c) > (a)
(b) will form Aromatic Benzene on dehydration
(d) will form conjugated alkene
(a) will not undergo dehydration easily

Q.50 Resonance in carbonate ion (CO_3^{2-}) is



Which of the following is true ?
(1) CO_3^{2-} has a single structure i.e., resonance hybrid of the above three structures.
(2) It is possible to identify each structure individually by some physical or chemical method.
(3) Each structure exists for equal amount of time.
(4) All these structure are in dynamic equilibrium with each other.

Ans. [1]

Sol. Resonating structures are hypothetical and are assumed to explain properties of Real hybrid.

Section-B: Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE**. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

Q.51 Sum of oxidation states of bromine in bromic acid and perbromic acid is ________ .

Ans. [12]

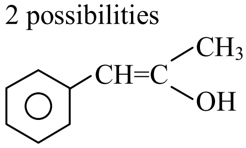
Sol. Bromic Acid $\left(\underset{+5}{HBrO_3}\right)$

Perbromic Acid $\left(\underset{+7}{HBrO_4}\right)$

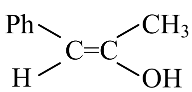
Q.52 Number of isomeric compounds with molecular formula $C_9H_{10}O$ which (i) do not dissolve in NaOH (ii) do not dissolve in HCl. (iii) do not give orange precipitate with 2, 4-DNP (iv) on hydrogenation given identical compound with molecular formula $C_9H_{12}O$ is ___________ .

Ans. [2]

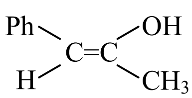
Sol. 2 possibilities



(E/Z isomers)



(E)



(Z)

Q.53 25 mL of an aqueous solution of KCl was found to require 20 mL of 1 M $AgNO_3$ solution when titrated using K_2CrO_4 as an indicator. What is the depression in freezing point of KCl solution of the given concentration ? _________ (Nearest integer).

(Given : $K_f = 2.0$ kg mol^{-1})

Assume (1) 100% ionization and

(2) Density of the aqueous solution as 1 g mol^{-1}

Ans. [3]

Sol. $25 \times M = 20 \times 1$

$M = \frac{20}{25} = \frac{4}{5} = 0.8$

$\Delta T_f = (i)\,(K_f)\,(m)$

$= (2)\,(2)\left(\frac{4}{5}\right) = \frac{16}{5} = 3.2$

Nearest Integer – 3

Q.54 A and B are two substances undergoing radioactive decay in a container. The half life of A is 15 min and that of B is 5 min. If the initial concentration of B is 4 times that of A and they both start decaying at the same time, how much time will it take for the concentration of both of them to be same ? _______ min.

Ans. [15]

Sol.

A: $Co \xrightarrow{15\ min} \frac{Co}{2}$

B: $4Co \xrightarrow{5\ min} 2Co \xrightarrow{5\ min} Co \xrightarrow{5\ min} \frac{Co}{2}$

After 15 min, [A] = [B] = $\frac{CO}{2}$

Q.55 At 25ºC, the enthalpy of the following processes are given

$H_2(g) + O_2(g) \rightarrow 2OH(g) \quad \Delta H^o = 78\ kJ\ mol^{-1}$

$H_2(g) + \frac{1}{2}O_2(g) \rightarrow H_2O(g) \quad \Delta H^o = -242\ kJ\ mol^{-1}$

$H_2(g) \rightarrow 2H(g) \quad \Delta H^o = 436\ kJ\ mol^{-1}$

$\frac{1}{2}O_2(g) \rightarrow O(g) \quad \Delta H^o = 249\ kJ\ mol^{-1}$

What would be the value of X for the following reaction ? ________ (Nearest integer)

$H_2O(g) \rightarrow H(g) + OH(g)\ \Delta H^o = X\ kJ\ mol^{-1}$

Ans. **[499]**

Sol. $\frac{(i)+(iii)}{2} - (ii)$ gives desired reaction

$\Delta H_r = \frac{436+78}{2} - (242)$

$= \frac{436+78}{2} + 242 = 499$

Q.56 The density of 3 solution of NaCl is 1.0 g mL^{-1}. Molality of the solution is _________ × 10^{-2} m. (Nearest integer).
Given : Molar mass of Na and Cl is 23 and 35.5 g mol^{-1} respectively.

Ans. **[364]**

Sol. $m = \frac{1000M}{1000\rho - M.mw} = \frac{1000\times 3}{1000 - 3\times(58.5)}$

$= \frac{3000}{(1000-175.5)} = 3.638$

$= 363.8 \times 10^{-2}$

Nearest integer = 364

Q.57 Electrons in a cathode ray tube have been emitted with a velocity of 1000 m s^{-1}. The number of following statement which is/are true about the emitted radiation is ___________ .
Given : $h = 6 \times 10^{-34}$ J s, $m_e = 9 \times 10^{-31}$ kg

(A) The deBroglie wavelength of the electron emitted is 666.67 nm
(B) The characteristic of electrons emitted depend upon the material of the electrodes of the cathode ray tube.
(C) The cathode rays start from cathode and move towards anode.
(D) The nature of the emitted electrons depends on the nature of the gas present in cathode ray tube.

Ans. [2]

Sol.
- Characteristics of electrons emitted doesn't depend upon material of electrode, nature of gas present.
- Cathode rays start from cathode
- $\lambda = \frac{h}{mv} = \frac{6\times10^{-34}}{(9\times10^{-31})(10^{3})} = .666 \times 10^{-6}$ m

$= 666.67$ nm

A & C are correct.

Q.58 (i) $X(g) \rightleftharpoons Y(g) + Z(g) \quad K_{p_1} = 3$

(ii) $A(g) \rightleftharpoons 2B(g) \quad K_{p_2} = 1$

If the degree of dissociation and initial concentration of both the reactants X(g) and A(g) are equal, then the ratio of the total pressure at equilibrium $\left(\frac{p_1}{p_2}\right)$ is equal to x : 1. The value of x is ________ (Nearest integer)

Ans. [12]

Sol.

$$X_{(g)} \rightleftharpoons Y_{(g)} + Z_{(g)} \quad K_{p_1} = 3$$

$$1 \rightarrow (1-\alpha) \qquad - \rightarrow \alpha \qquad - \rightarrow \alpha$$

mole fraction $\left(\frac{1-\alpha}{1+\alpha}\right) \quad \left(\frac{\alpha}{1+\alpha}\right) \quad \left(\frac{\alpha}{1+\alpha}\right)$

$$K_{p_1} = 3 = \frac{\alpha}{(1+\alpha)}\frac{\alpha}{(1+\alpha)}\frac{(1+\alpha)}{(1-\alpha)}(p_1)^1$$

$$3 = \frac{\alpha^2}{1-\alpha^2}\cdot p_1$$

$$A_{(g)} \rightleftharpoons 2B_{(g)} \quad K_{p_2} = 1$$

$$1 \rightarrow (1-\alpha) \qquad - \rightarrow 2\alpha$$

mole fraction $\left(\frac{1-\alpha}{1+\alpha}\right) \quad \left(\frac{2\alpha}{1+\alpha}\right)$

$$1 = \frac{4\alpha^2}{(1+\alpha^2)}\frac{(1+\alpha)}{(1-\alpha)}.p_2$$

$$1 = \frac{4\alpha^2}{1-\alpha^2}.p_2$$

$$\frac{Kp_1}{Kp_2} = \frac{3}{1} = \frac{p_1}{4p_2} \Rightarrow \frac{p_1}{p_2} = 12$$

Q.59 At what pH, given half cell MnO_4^- (0.1 M) | M^{2+} (0.001 M) will have electrode potential of 1.282 V ? _______ (Nearest Integer)

Given $E^{o}_{MnO_4^-|Mn^{2+}} = 1.54$ V, $\frac{2.303RT}{F} = 0.059$ V

Ans. [3]

Sol. $5e^- + 8H^+ + MnO_4^- \longrightarrow Mn^{2+} + 4H_2O$

$\quad\quad\quad\quad 10^{-1} \quad\quad\quad 10^{-3}$

$$1.282 = 1.54 - \frac{.059}{5}\log\frac{10^{-3}}{10^{-1}(H^+)^8}$$

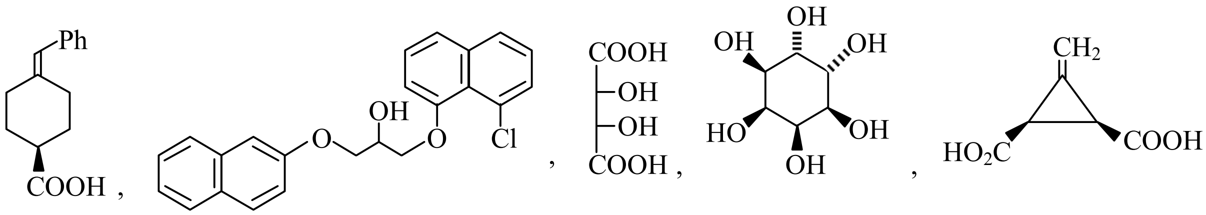
$$-.258 = \frac{-.059}{5}(-2 + 8\text{ pH})$$

$21.8644 = (-2 + 8\text{ pH})$

$23.8644 = 8\text{ pH}$

$\text{pH} = 2.98 \approx 3$

Q.60 The total number of chiral compound/s from the following is __________.



Ans. [2]

Sol. Compound I - achiral

Compound II - chiral

Compound III - achiral

Compound IV - chiral

Compound V - achiral

MATHEMATICS

Section-A: This section contains 20 multiple choice questions. Each question has 4 choices(1), (2), (3) and (4), out of which **ONLY ONE** is correct..

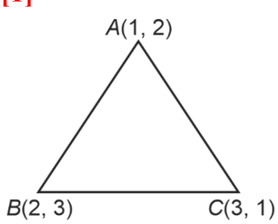
Q.61 If the orthocenter of the triangle, whose vertices are (1, 2), (2, 3) and (3, 1) is (α, β), then the quadratic equation whose roots are $\alpha + 4\beta$ and $4\alpha + \beta$, is

(1) $x^2 - 20x + 99 = 0$ (2) $x^2 - 19x + 90 = 0$

(3) $x^2 - 22x + 120 = 0$ (4) $x^2 - 18x + 99 = 0$

Ans. [1]

Sol.



Altitude of BC is $y - 2 = \frac{1}{2}(x - 1) \Rightarrow x - 2y + 3 = 0$

Altitude of AB is $y - 1 = (-1)(x - 3) \Rightarrow x + y = 4$

$\therefore$ Orthocentre $\left(\frac{5}{3}, \frac{7}{3}\right)$

$\therefore\ \alpha + 4\beta = 11$ and $4\alpha + \beta = 9$

Equation is $x^2 - 20x + 99 = 0$

Q.62 The mean and variance of 5 observations are 5 and 8 respectively. If 3 observations are 1, 3, 5, then the sum of cubes of the remaining two observations is

(1) 1456 (2) 1216 (3) 1792 (4) 1072

Ans. [4]

Sol. Let observations 1, 3, 5, a, b

$\Rightarrow \frac{9+a+b}{5} = 5$ & $\frac{a^2+b^2+35}{5} - 25 = 8$

$\Rightarrow a + b = 16$ & $a^2 + b^2 = 130$

$\therefore$ a & b are 7 & 9

$\therefore a^3 + b^3 = 7^3 + 9^3 = 1072$

Q.63 If the centre and radius of the circle $\left|\frac{z-2}{z-3}\right| = 2$ are respectively (α, β) and γ, then $3(\alpha + \beta + \gamma)$ is equal to

(1) 10 (2) 12 (3) 11 (4) 9

Ans. [2]

Sol. $(x-2)^2 + y^2 = 4(x-3)^2 + 4y^2$

$\Rightarrow 3x^2 + 3y^2 - 20x + 32 = 0$

$\therefore C = \left(\frac{10}{3}, 0\right)$ & $r = \sqrt{\left(\frac{10}{3}\right)^2 - \frac{32}{3}} = \frac{2}{3}$

$\therefore 3(\alpha + \beta + \gamma) = 3\left(\frac{12}{3}\right) = 12$

Q.64 If y = y(x) is the solution curve of the differential equation $\frac{dt}{dx} + y \tan x = x \sec x$, $0 \le x \le \frac{\pi}{3}$, y(0) = 1, then $y\left(\frac{\pi}{6}\right)$ is equal to

(1) $\frac{\pi}{12} - \frac{\sqrt{3}}{2}\log_e\left(\frac{2\sqrt{3}}{e}\right)$ (2) $\frac{\pi}{12} + \frac{\sqrt{3}}{2}\log_e\left(\frac{2}{e\sqrt{3}}\right)$

(3) $\frac{\pi}{12} + \frac{\sqrt{3}}{2}\log_e\left(\frac{2\sqrt{3}}{e}\right)$ (4) $\frac{\pi}{12} - \frac{\sqrt{3}}{2}\log_e\left(\frac{2}{e\sqrt{3}}\right)$

Ans. [4]

Sol. $\frac{dt}{dx} + y \tan x = x \sec x$

$\therefore$ I.F $= e^{\int \tan x dx} = \sec x$

$\Rightarrow y \sec x = \int x \sec^2 x dx$

$\Rightarrow y \sec x = x \tan x - \ln|\sec x| + c \cos x$

$\downarrow$ y(0) = 1

$\Rightarrow 1 = e$

$\therefore y = x\sin x - \cos x \ln|\sec x| + \cos x$

$\therefore y\left(\frac{\pi}{6}\right) = \frac{\pi}{12} - \frac{\sqrt{3}}{2}\ln\left(\frac{2}{\sqrt{3}e}\right)$

Q.65 The sum to 10 terms of the series $\frac{1}{1+1^2+1^4} + \frac{2}{1+2^2+2^4} + \frac{3}{1+3^2+3^4} + \ldots$ is

(1) $\frac{58}{111}$ (2) $\frac{59}{111}$ (3) $\frac{55}{111}$ (4) $\frac{56}{111}$

Ans. [3]

Sol. $S = \sum_{r=1}^{10} \frac{r}{1+r^2+r^4} = \frac{1}{2}\sum\left(\frac{1}{r^2-r+1} - \frac{1}{r^2+r+1}\right)$

$T_1 = \frac{1}{2}\left(\frac{1}{1^2-1+1} - \frac{1}{1^2+1+1}\right)$

$T_2 = \frac{1}{2}\left(\frac{1}{2^2-2+1} - \frac{1}{2^2+2+1}\right)$

$T_3 = \frac{1}{2}\left(\frac{1}{3^2-3+1} - \frac{1}{3^2+3+1}\right)$

.
.
.

$T_{10} = \frac{1}{2}\left(\frac{1}{10^2-10+1} - \frac{1}{10^2+10+1}\right)$

$S = \frac{1}{2}\left(1 - \frac{1}{111}\right) = \frac{55}{111}$

Q.66 The combined equation of the two lines ax + by + c = 0 and a'x + b'y + c' = 0 can be written as (ax + by + c) (a'x + b'y + c') = 0

The equation of the angle bisectors of the lines represented by the equation $2x^2 + xy - 3y^2 = 0$ is

(1) $3x^2 + 5xy + 2y^2 = 0$ (2) $x^2 - y^2 + 10xy = 0$ (3) $3x^2 + xy + 2y^2 = 0$ (4) $x^2 - y^2 - 10xy = 0$

Ans. [4]

Sol. $\frac{x^2 - y^2}{2-(-3)} = \frac{xy}{\frac{1}{2}}$

OR $x^2 - y^2 = 10xy$

Q.67 Let S be the set of all solutions of the equation $\cos^{-1}(2x) - 2\cos^{-1}\left(\sqrt{1-x^2}\right) = \pi$, $x \in \left[-\frac{1}{2}, \frac{1}{2}\right]$. Then $\sum_{x \in S} 2\sin^{-1}\left(x^2 - 1\right)$ is equal to

(1) $\frac{-2\pi}{3}$ (2) 0 (3) $\pi - \sin^{-1}\left(\frac{\sqrt{3}}{4}\right)$ (4) $\pi - 2\sin^{-1}\left(\frac{\sqrt{3}}{4}\right)$

Ans. [*]

Sol. $\cos^{-1}(2x) - 2\cos^{-1}\left(\sqrt{1-x^2}\right) = \pi$

This is possible only when

$\cos^{-1}(2x) = \pi$ ……. (i)

And $2\cos^{-1}\left(\sqrt{1-x^2}\right) = 0$ ……..(ii)

From (i)

$x = -\frac{1}{2}$

Which does not satisfy (ii)

So no such x exist

Q.68 The value of

$$\frac{1}{1!50!}+\frac{1}{3!48!}+\frac{1}{5!46!}+.....+\frac{1}{49!2!}+\frac{1}{5!1!} \text{ is :}$$

(1) $\frac{2^{50}}{51!}$ (2) $\frac{2^{51}}{51!}$ (3) $\frac{2^{50}}{50!}$ (4) $\frac{2^{51}}{50!}$

Ans. [1]

Sol. $\frac{1}{(51)!}\left({}^{51}C_1+{}^{51}C_3+....+{}^{51}C_{51}\right)$

$= \frac{2^{50}}{(51)!}$

Q.69 Let S denote the set of all real value of λ such that the system of equations

$\lambda x + y + z = 1$

$x + \lambda y + z = 1$

$x + y + \lambda z = 1$

is inconsistent, then $\sum_{\lambda \in S}\left(|\lambda|^2+|\lambda|\right)$ is equal to

(1) 4 (2) 2 (3) 6 (4) 12

Ans. [3]

Sol. $\begin{vmatrix} \lambda & 1 & 1 \\ 1 & \lambda & 1 \\ 1 & 1 & \lambda \end{vmatrix} = 0$

$\lambda(\lambda^2 - 1) - 1(\lambda - 1) + 1(1 - \lambda) = 0$

$\lambda^3 - \lambda - \lambda + 1 + 1 - \lambda = 0$

$\lambda^3 - 3\lambda + 2 = 0$

$(\lambda - 1)(\lambda^2 + \lambda - 2) = 0$

$\lambda = 1, -2$

For $\lambda = 1 \Rightarrow \infty$ solution

$\lambda = -2 \Rightarrow$ no solution

$\sum_{\lambda \in S}|\lambda|^2+|\lambda| = 6$

Q.70 For a triangle ABC, the value of cos2A + cos2B + cos2C is least. If its inradius is 3 and incentre is M, then which of the following is NOT correct?

(1) $\overline{MA}\cdot\overline{MB} = -18$ (2) perimeter of ΔABC is $18\sqrt{3}$

(3) area of ΔABC is $\frac{27\sqrt{3}}{2}$ (4) sin2A + sin2B + sin2C = sinA + sinB + sinC

Ans. [3]

Sol. We know that

$\cos 2A + \cos 2B + \cos 2C \geq \frac{-3}{2}$ where equality holds for equilateral triangle

$r = \frac{\Delta}{s} = \frac{\frac{\sqrt{3}}{4}a^2}{\frac{3}{2}a} = \frac{a}{2\sqrt{3}}$

$a = 2\sqrt{3}\,r = 6\sqrt{3}$

$\text{Area} = \frac{\sqrt{3}}{4}a^2 = 27\sqrt{3}$

Q.71 Let $f(x) = \begin{vmatrix} 1+\sin^2 x & \cos^2 x & \sin 2x \\ \sin^2 x & 1+\cos^2 x & \sin 2x \\ \sin^2 x & \cos^2 x & 1+\sin 2x \end{vmatrix}$, $x \in \left[\frac{\pi}{6}, \frac{\pi}{3}\right]$. If α and β respectively are the maximum and the minimum values of f, then

(1) $\beta^2 + 2\sqrt{\alpha} = \frac{19}{4}$ (2) $\alpha^2 + \beta^2 = \frac{9}{2}$ (3) $\alpha^2 - \beta^2 = 4\sqrt{3}$ (4) $\beta^2 - 2\sqrt{\alpha} = \frac{19}{4}$

Ans. [4]

Sol. $C_1 \to C_1 + C_2 + C_3$

$$(2 + \sin2x)\begin{vmatrix} 1 & \cos^2 x & \sin 2x \\ 1 & 1+\cos^2 x & \sin 2x \\ 1 & \cos^2 x & 1+\sin 2x \end{vmatrix}$$

$R_2 \to R_2 - R_1$; $R_3 \to R_3 - R_1$

$$(2 + \sin2x)\begin{vmatrix} 1 & \cos^2 x & \sin 2x \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

$f(x) = 2 + \sin2x$; $x \in \left[\frac{\pi}{6}, \frac{\pi}{3}\right]$

$f(x)_{max} = 2 + 1 = 3$ for $x = \frac{\pi}{4}$

$f(x)_{min} = 2 + \frac{\sqrt{3}}{2}$ for $x = \frac{\pi}{6}, \frac{\pi}{3}$

$\beta^2 - 2\sqrt{\alpha} = 4 + \frac{3}{4} + 2\sqrt{3} - 2\sqrt{3}$

$= \frac{19}{4}$

Q.72 The area enclosed by the closed curve C given by the differential equation $\frac{dy}{dx} + \frac{x+a}{y-2} = 0$, $y(1) = 0$ is 4π.

Let P and Q be the points of intersection of the curve C and the y-axis. If normals at P and Q on the curve C intersect x-axis at points R and S respectively, then the length of the line segment RS is

(1) 2 (2) $\frac{2\sqrt{3}}{3}$ (3) $2\sqrt{3}$ (4) $\frac{4\sqrt{3}}{3}$

Ans. [4]

Sol. $\frac{dy}{dx} + \frac{x+a}{y-2} = 0$

$(y - 2)dy + (x + a)dx = 0$

Integrating

$\frac{y^2}{2} - 2y + \frac{x^2}{2} + ax = C$

Or $x^2 + 2ax + y^2 - 4y = C$

At $x = 1$, $y = 0$

$1 + 2a = C$

Equation of circle

$x^2 + 2ax + y^2 - 4y = 1 + 2a$

$x^2 + y^2 + 2ax - 4y - (1 + 2a) = 0$

$r = \sqrt{a^2 + 4 + 1 + 2a} = 2$

$a^2 + 2a + 5 = 4 \Rightarrow \boxed{a = -1}$

Curve is $x^2 + y^2 - 2x - 4y + 1 = 0$

Intersection with y-axis

$P = (0,\ 2+\sqrt{3})\quad Q = (0,\ 2-\sqrt{3})$

For normal at P & Q

$R = \left(1+\frac{2}{\sqrt{3}}, 0\right), S = \left(1-\frac{2}{\sqrt{3}}, 0\right)$

$RS = \frac{4\sqrt{3}}{3}$

Q.73 Let $f(x) = 2x + \tan^{-1}x$ and $g(x) = \log_e\left(\sqrt{1+x^2}+x\right)$, $x \in [0, 3]$. Then

(1) min $f'(x) = 1 +$ max $g'(x)$

(2) there exist $0 < x_1 < x_2 < 3$ such that $f(x) < g(x), \forall\ x \in (x_1, x_2)$

(3) there exists $\hat{x} \in [0, 3]$ such that $f'(\hat{x}) < g'(\hat{x})$

(4) max $f(x) >$ max $g(x)$

Ans. [4]

Sol. $f'(x) = 2 + \frac{1}{1+x^2}$, $g'(x) = \frac{1}{\sqrt{x^2+1}}$

$f'(x) = -\frac{2x}{(1+x^2)^2} < 0$

$f'(x)\,|_{min} = f'(3) = 2 + \frac{1}{10} = \frac{21}{10}$

$g'(x)\,|_{max} = g'(0) = 1$

$f'(x)\,|_{max} = f(3) = 2 + \tan^{-1}3$

$g(x)\,|_{max} = g(3) = \ln(3 + \sqrt{10}) < \ln < 7 < 2$

Q.74 In a binomial distribution B(n, p), the sum and the product of the mean and the variance are 5 and 6 respectively, then $6(n + p - q)$ is equal to

(1) 52 (2) 50 (3) 53 (4) 51

Ans. [1]

Sol. $np + npq = 5$

$np(1 + q) = 5$ ……(i)

$np(npq) = 6$ ……(ii)

$\Rightarrow np = 3, npq = 2$

$\Rightarrow q = \frac{2}{3}, p = \frac{1}{3}, n = 9$

$6(n + p - q) = 6\left(9+\frac{1}{3}-\frac{2}{3}\right) = 6\left(9-\frac{1}{3}\right)$

$= 52$

Q.75 The shortest distance between the lines

$\frac{x-5}{1} = \frac{y-2}{2} = \frac{z-4}{-3}$ and $\frac{x+3}{1} = \frac{y+5}{4} = \frac{z-1}{-5}$ is

(1) $5\sqrt{3}$ (2) $6\sqrt{3}$ (3) $4\sqrt{3}$ (4) $7\sqrt{3}$

Ans. [2]

Sol. $\overrightarrow{b_1}\times\overrightarrow{b_2} = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 2 & -3\\ 1 & 4 & -5\end{vmatrix} = \hat{i}(2)-\hat{j}(-2)+\hat{k}(2)$

$\therefore\ \overrightarrow{b_1}\times\overrightarrow{b_2} = \hat{i}+\hat{j}+\hat{k}$

$\overrightarrow{a_1}-\overrightarrow{a_2} = 8\hat{i}+7\hat{j}+3\hat{k}$

$$d = \left|\frac{(\overrightarrow{a_1}-\overrightarrow{a_2})\cdot(\overrightarrow{b_1}\times\overrightarrow{b_2})}{|\overrightarrow{b_1}\times\overrightarrow{b_2}|}\right| = \left|\frac{8+7+3}{\sqrt{3}}\right| = \frac{18}{\sqrt{3}} = 6\sqrt{3}$$

Q.76 $\lim\limits_{x\to\infty}\left[\frac{1}{1+n}+\frac{1}{2+n}+\frac{1}{3+n}+....+\frac{1}{2n}\right]$ is equal to

(1) 0 (2) $\log_e\left(\frac{3}{2}\right)$ (3) $\log_e 2$ (4) $\log_e\left(\frac{2}{3}\right)$

Ans. [3]

Sol. $\lim\limits_{x\to\infty}\left(\frac{1}{n+1}+\frac{1}{n+2}.....\frac{1}{n+n}\right)$

$$= \lim_{n\to\infty}\sum_{r=1}^{n}\frac{1}{n}\left(\frac{1}{1+\left(\frac{r}{n}\right)}\right)$$

$$= \int_0^1\frac{dx}{1+x} = \log(1+x)_0^1 = \log 2$$

Q.77 Let R be a relation on R, given by R = {(a, b) : $3a-3b+\sqrt{7}$ is an irrational number Then R is

(1) reflexive but neither symmetric nor transitive

(2) an equivalence relation

(3) reflexive and symmetric but not transitive

(4) reflexive and transitive but not symmetric

Ans. [1]

Sol. For reflexive:

$3a-3a+\sqrt{7}$ is an irrational number $\forall\ a \in R$ R is reflexive

For symmetric

Let $3a-3b+\sqrt{7}$ is an irrational number

For e.g., Let $3a-3b = \sqrt{7}$

$\sqrt{7}+\sqrt{7}$ is irrational but $-\sqrt{7}+\sqrt{7}$ is not

$\therefore$ R is not symmetric

For transitive

Let $3a-3b+\sqrt{7}$ is irrational and $3b-3c+\sqrt{7}$ is irrational

$\Rightarrow 3a-3c+\sqrt{7}$ is irrational

For e.g., take a = 0, $b = -\sqrt{7}$, $c = \frac{\sqrt{7}}{3}$

R is not transitive

Q.78 The negation of the expression $q \vee ((\sim q) \wedge p)$ is equivalent to

(1) $p \wedge (\sim q)$ (2) $(\sim p) \vee (\sim q)$ (3) $(\sim p) \vee q$ (4) $(\sim p) \wedge (\sim q)$

Ans. [4]

Sol. $q \vee (\sim q \wedge p)$

$\Rightarrow (q \vee \sim q) \wedge (q \vee p)$

$\Rightarrow T \wedge (q \vee p)$

$\Rightarrow q \vee p$

Now,

$\sim (q \vee p)$

$= \sim q \wedge \sim p$

Q.79 Let $S = \left\{ \begin{array}{l} x : x \in R \text{ and} (\sqrt{3}+\sqrt{2})^{x^2-4} \\ +(\sqrt{3}-\sqrt{2})^{x^2-4} = 10 \end{array} \right\}$. Then n(S) is equal to

(1) 2 (2) 4 (3) 0 (4) 6

Ans. [4]

Sol. Let $\left(\sqrt{3}+\sqrt{2}\right)^{x^2-4} = t$

$t + \frac{1}{t} = 10$

$\Rightarrow t^2 - 10t + 1 = 0$

$\Rightarrow t = \frac{10 \pm \sqrt{100-4}}{2} = 5 \pm 2\sqrt{6}$

Case-I

$t = 5 + 2\sqrt{6}$

$\Rightarrow \left(\sqrt{3}+\sqrt{2}\right)^{x^2-4} = \left(\sqrt{3}+\sqrt{2}\right)^2$

$\Rightarrow x^2 - 4 = 2 \Rightarrow x^2 = 6 \Rightarrow x = \pm\sqrt{6}$

Case-II

$t = 5 - 2\sqrt{6}$

$\left(\sqrt{3}+\sqrt{2}\right)^{x^2-4} = \left(\sqrt{3}-\sqrt{2}\right)^2$

$\Rightarrow \left((\sqrt{3}-\sqrt{2})^{-1}\right)^{x^2-4} = \left(\sqrt{3}-\sqrt{2}\right)^2$

$\Rightarrow 4 - x^2 = 2$

$\Rightarrow x^2 = 2$

$\Rightarrow x = \pm\sqrt{2}$

Q.80 Let the image of the point P(2, –1,3) in the plane x + 2y – z = 0 be Q. Then the distance of the plane 3x + 2y + z + 29 = 0 from the point Q is

(1) $2\sqrt{14}$ (2) $\frac{22\sqrt{2}}{7}$ (3) $\frac{24\sqrt{2}}{7}$ (4) $3\sqrt{14}$

Ans. [4]

Sol. P(2, -1, 3) Plane: x + 2y – z = 0

Let $Q(\alpha, \beta\ \gamma)$

Then,

$$\frac{\alpha-2}{1}=\frac{\beta+1}{2}=\frac{\gamma-3}{-1}=\frac{-2(-3)}{6}$$

$\therefore\ \alpha = 3, \beta = 1, \gamma = 2$

Now distance of Q from the plane 3x + 2y + z + 29 = 0

$$\left(d=\frac{9+2+2+29}{\sqrt{14}}=\frac{42}{\sqrt{14}}=3\sqrt{14}\right)$$

Section-B: Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer..

Q.81 Let $a_1 = 8, a_2, a_3, \ldots a_n$ be an A.P. If the sum of its first four terms is 50 and the sum of its last four terms is 170, then the product of its middle two terms is ____.

Ans. [754]

Sol. Given, $a_1 = 8, a_2, a_3, \ldots . a_n$ are in A.P.

Now $2(16 + 3d) = 50$

$3d = 9 \Rightarrow \boxed{d = 3}$

Now $2(2a_n - 9) = 170$

$a_n = 47$

$8 + (n - 1)\, 3 = 47$

$\boxed{n = 14}$

Product of middle two terms $= a_7 \times a_8$

$= (8 + 18)(8 + 21)$

$= 26 \times 29$

$= 754$

Q.82 If $\int_0^1 \left(x^{21}+x^{14}+x^7\right)\left(2x^{14}+3x^7+6\right)^{\frac{1}{7}} dx = \frac{1}{l}(11)^{\frac{m}{n}}$ where ℓ, m, n $\in$ N, m and n are coprime then ℓ + m + n is equal to ____.

Ans. [63]

Sol. $\ell = \int_0^1 \left(x^{21}+x^{14}+x^7\right)\left(2x^{14}+3x^7+6\right)^{1/7} dx$

$\ell = \int_0^1 \left(x^{20}+x^{13}+x^6\right)\left(2x^{21}+3x^{14}+6x^7\right)^{1/7} dx$

Let $2x^{21} + 3x^{14} + 6x^7 = t$

$\Rightarrow 42\,(x^{20} + x^{13} + x^6)\,dx = dt$

$\ell = \frac{1}{42}\int_0^{11} t^{1/7} dt = \frac{1}{42}\frac{7}{8}\left[t^{8/7}\right]_0^{11}$

$= \frac{1}{48} 11^{817}$

$\therefore\ \ell = 48, m = 8, n = 7$

$\therefore\ \ell + m + n = 63$

Q.83 Let f : R R → R be a differentiable function such that $f'(x) + f(x) = \int_0^2 f(t)dt$. If $f(0) = e^{-2}$, then $2f(0) - f(2)$ is equal to ____

Ans. [01]

Sol. $f'(x) + f(x) = k$

$\Rightarrow e^x f(x) = ke^x + c$

$f(x) = k + ce^{-x}$

$k = \int_0^2 \left(k + ce^{-t}\right)dt$

$k = 2k + c \cdot \left.\frac{e^{-t}}{-1}\right|_0^2$

$k = 2k + c\left(\frac{e^{-2}}{-1} + 1\right)$

$-k = c\left(1 - \frac{1}{e^2}\right)$

$f(x) = ce^{-x} - c\left(1 - \frac{1}{e^2}\right)$

$f(0) = c - c + \frac{c}{e^2} = \frac{1}{e^2} \Rightarrow c = 1$

$f(2) = e^{-2} - r(1 - e^{-2})$

$= 2e^{-2} - 1$

$2f(0) - f(2) = 1$

Q.84 If $f(x) = x^2 + g'(1)x + g''(2)$ and $g(x) = f(1)x^2 + xf'(x) + f''(x)$, then the value of $f(4) - g(4)$ is equal to ___.

Ans. [14]

Sol. Let $g'(1) = a$ and $g''(2) = b$

$\Rightarrow f(x) = x^2 + ax + b$

Now, $f(1) = 1 + a + b$; $f'(x) = 2x + a$; $f''(x) = 2$

$g(x) = (1 + a + b)x^2 + x(2x + a) + 2$

$\Rightarrow g(x) = (a + b + 3)x^2 + ax + 2$

$\Rightarrow g'(x) = 2x(a + b + 3) + a \Rightarrow g'(1) = 2(a + b + 3) + a = a$

$\Rightarrow a + b + 3 = 0$ .....(i)

$g''(x) = 2(a + b + 3) = b$

$\Rightarrow 2a + b + 6 = 0$ .....(ii)

Solving (i) and (ii), we get

$a = -3$ and $b = 0$

$f(x) = x^2 - 3x$ and $g(x) = -3x + 2$

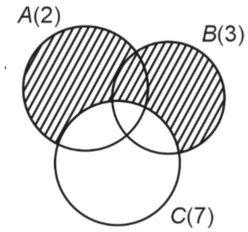
$f(4) = 4$ and $g(4) = -12 + 2 = -10$

$\Rightarrow f(4) - g(4) = 16 - 2 = 14$

Q.85 The number of 3-digit numbers, that are divisible by either 2 or 3 but not divisible by 7, is _____.

Ans. [514]

Sol.



A = Numbers divisible by 2
B = Numbers divisible by 3
C = Numbers divisible by 7
$n(A \cup B) = n(A) + n(B) - n(A \cap B)$
$= n(2) + n(3) - n(6)$
n(A) = n(2) = 100, 102….., 998, = 450
n(B) = n(3) = 102, 105, ……, 999 = 30
$n(A \cap B)$ = n(6) = 102, 108, …., 996 = 150
n(2 or 3) = 450 + 300 – 150 = 600
Now,
$n(A \cap C)$ = n(14) = 112, 126, …., 994 = 64
$n(A \cap B \cap C)$ = n(42) = 126, 168, …. 966 = 21
$n(B \cap C)$ = n(21) = 105, 126, ……. 987, = 43
n(2 or 3 not by 7) = 600 – [64 + 43 – 21]
= 514

Q.86 The remainder, when $19^{200} + 23^{200}$ is divided by 49. is ______

Ans. [29]

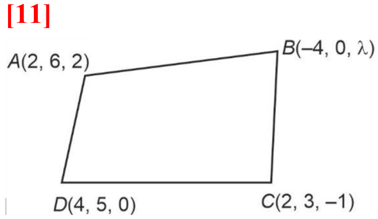
Sol. $19^{200} + 23^{200}$

$(21 - 2)^{200} + (21 + 2)^{200} = 49\lambda + 2^{201}$
$2^{201} = 8^{67} = (7 + 1)^{67} = 49\lambda + 7 \times 67 + 1$
$= 49\lambda + 470$
$= 49(\lambda + 9) + 29$
Remainder = 29

Q.87 A(2, 6, 2), B(–4, 0, λ), C(2, 3, –1) and D(4, 5, 0), $|\lambda| \le 5$ are the vertices of a quadrilateral ABCD. If its area is 18 square units, then $5 - 6\lambda$ is equal to _______.

Ans. [11]

Sol.



$\overrightarrow{d_1} = 3\hat{j} + 3\hat{k}$

$\overrightarrow{d_2} = 8\hat{i} + 5\hat{j} - \lambda\hat{k}$

$$\overrightarrow{d_1} \times \overrightarrow{d_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 3 & 3 \\ 8 & 5 & -\lambda \end{vmatrix}$$

$= (-3\lambda - 15)\hat{i} + 24\hat{j} - 24\hat{k}$

$\frac{1}{2}\left|\overrightarrow{d_1} \times \overrightarrow{d_2}\right| = 18$

$\sqrt{(3\lambda+15)^2+24^2+24^2} = 36$

$(3\lambda + 15)^2 = 1296 - 1152$

$3\lambda + 15 = \pm 12$

$3\lambda = -3$, $\lambda = -1$ | $3\lambda + 15 = -12$, $\lambda = -\frac{27}{3}$, $\lambda = -9$

$\because \lambda \in [-5, 5]$

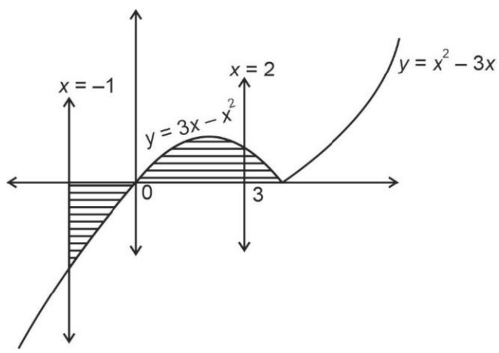
$\therefore \lambda = -1$

$5 - 6(-1) = 11$

Q.88 Let A be the area bounded by the curve $y = x|x - 3|$, the x-axis and the ordinates $x = -1$ and $x = 2$. Then 12A is equal to ____.

Ans. **[62]**

Sol.



$$\text{Area} = \int_{-1}^{2} \left|3x - x^2\right|$$

$$A = \int_{-1}^{0} x^2 - 3x\,dx + \int_{0}^{2} 3x - x^2 dx$$

$$= \left.\frac{x^3}{3} - \frac{3x^2}{2}\right]_{-1}^{0} + \left.\frac{3x^2}{2} - \frac{x^3}{3}\right]_{0}^{2}$$

$$= 0 - \left(\frac{-1}{3} - \frac{3}{2}\right) + \left(6 - \frac{8}{3}\right) - 0$$

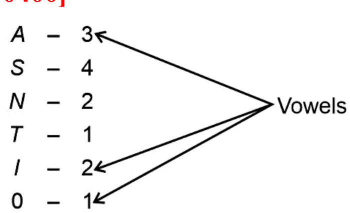
$$= \frac{31}{6}$$

$\therefore 12A = 62$

Q.89 The number of words, with or without meaning, that can be formed using all the letters of word ASSASSINATION so that vowels occur together, is ____.

Ans. **[50400]**

Sol.



$$\text{Number of arrangement} = \frac{8!}{4!2!} \times \frac{6!}{3!2!} = 50400$$

Q.90 Let $\vec{v} = a\hat{i} + 2\hat{j} - 3\hat{k}$, $\vec{w} = 2\alpha\hat{i} + \hat{j} - \hat{k}$ and $\vec{u}$ be a vector such that $|\vec{u}| = \alpha > 0$. If the minimum value of the scalar triple product $[\vec{u}\vec{v}\vec{w}]$ is $-\alpha\sqrt{3401}$, and $|\vec{u}\cdot\hat{i}|^2 = \frac{m}{n}$ where m and n are coprime natural numbers, then m + n is equal to _____.

Ans. **[3501]**

Sol. $\vec{v}\times\vec{w} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \alpha & 2 & -3 \\ 2\alpha & 1 & -1 \end{vmatrix} = \hat{i} - 5\alpha\hat{j} - 3\alpha\hat{k}$

$[\vec{u}\vec{v}\vec{w}] = \vec{u}\cdot(\vec{v}\times\vec{w})$

$= |\vec{u}||\vec{v}\times\vec{w}| \times \cos\theta$

$= \alpha\sqrt{34\alpha^2+1}\cos\theta$

$[\vec{u}\vec{v}\vec{w}]_{min} = -\alpha\sqrt{3401}$

$\alpha\sqrt{34\alpha^2+1} \times (-1) = -\alpha\sqrt{3401}$

(taking cosθ = 1)

$\Rightarrow \alpha = 10$

$\vec{v}\times\vec{w} = \hat{i} - 50\hat{j} - 30\hat{k}$

$\cos\theta = -1 \Rightarrow \vec{u}$ is antiparallel to $\vec{v}\times\vec{w}$

$\vec{u} = -|\vec{u}|\cdot\frac{\vec{v}\times\vec{w}}{|\vec{v}\times\vec{w}|} = \frac{-10(\hat{i}-50\hat{j}-30\hat{k})}{\sqrt{3401}}$

$|\vec{u}\cdot\hat{i}|^2 = \left|\frac{-10}{\sqrt{3401}}\right|^2 = \frac{100}{3401} = \frac{m}{n}$

m + n = 3501