



JEE Main Online Exam 2023

Questions & Solution

31th January 2023 | Morning

PHYSICS

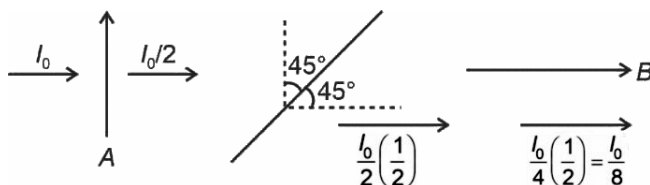
Section-A: This section contains 20 multiple choice questions. Each question has 4 choices(1), (2), (3) and (4), out of which **ONLY ONE** is correct..

Q.1 Two polaroids A and B are placed in such a way that the pass-axis of polaroids are perpendicular to each other. Now, another polaroid C is placed between A and B bisecting angle between them. If intensity of unpolarized light is I_0 then intensity of transmitted light after passing through polaroid B will be :

- (1) $\frac{I_0}{8}$ (2) zero (3) $\frac{I_0}{4}$ (4) $\frac{I_0}{2}$

Ans. [1]

Sol.



Q.2 A bar magnet with a magnetic moment 5.0 Am^2 is placed in parallel position relative to a magnetic field of 0.4 T . The amount of required work done in turning the magnet from parallel to antiparallel position relative to the field direction is :

- (1) Zero (2) 1 J (3) 4 J (4) 2 J

Ans. [3]

Sol.

$$\begin{aligned} W &= -MB(\cos\theta_2 - \cos\theta_1) \\ &= -0.4 \times 5[\cos 180^\circ - \cos 0] \\ &= 4 \text{ J} \end{aligned}$$

Q.3 The maximum potential energy of a block executing simple harmonic motion is 25 J. A is amplitude of oscillation. At $\frac{A}{2}$, the kinetic energy of the block is :

- (1) 18.75 J (2) 12.5 J (3) 37.5 J (4) 9.75 J

Ans. [1]

Sol.

$$\begin{aligned} E_{\text{Total}} &= U_{\text{max}} = 25 \text{ J} \\ K.E_{A/2} + U_{A/2} &= 25 \\ K.E_{A/2} + \left(\frac{1}{2}KA^2\right)\frac{1}{4} &= 25 \\ K.E_{A/2} &= 25 \left[1 - \frac{1}{4}\right] \\ &= \frac{3}{4} \times 25 = \frac{75}{4} \text{ J} = 18.75 \text{ J} \end{aligned}$$

Q.4 A rod with circular cross-section area 2 cm^2 and length 40 cm is wound uniformly with 400 turns of an insulated wire. If a current of 0.4 A flows in the wire windings, the total magnetic flux produced inside windings is $4\pi \times 10^{-6} \text{ Wb}$. The relative permeability of the rod is
(Given: Permeability of vacuum $\mu_0 = 4\pi \times 10^{-7} \text{ N A}^{-2}$)

- (1) 125 (2) $\frac{32}{5}$ (3) 12.5 (4) $\frac{5}{16}$

Ans. [4]

Sol.

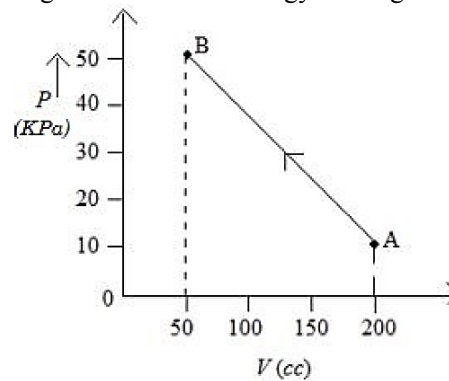
$$\phi = NBA$$

$$(4\pi \times 10^{-6}) = 400[\mu_r \mu_0 ni](2 \times 10^{-4})$$

$$(4\pi \times 10^{-6}) = 400 \left[\mu_r \times 4\pi \times 10^{-7} \times \frac{400}{0.4} \times 0.4 \right] \times 2 \times 10^{-4}$$

$$\mu_r = \frac{5}{16}$$

Q.5 The pressure of a gas changes linearly with volume from A to B as shown in figure. If no heat is supplied to or extracted from the gas then change in the internal energy of the gas will be



- (1) 4.5 J (2) Zero (3) 6 J (4) -4.5 J

Ans. [1]

Sol.

$$\because \Delta Q = 0$$

$$\Delta U = -W$$

$$= - \left[-\frac{1}{2} \times (50 + 10) \times 10^3 \times 150 \times 10^{-6} \right]$$

$$= 4.5 \text{ J}$$

Q.6 100 balls each of mass m moving with speed v simultaneously strike a wall normally and reflected back with same speed, in time t s. The total force exerted by the balls on the wall is

- (1) $\frac{200mv}{t}$ (2) $200mvt$ (3) $\frac{mv}{100t}$ (4) $\frac{100mv}{t}$

Ans. [1]

Sol.

$$\text{Total force exerted} = \frac{\Delta P}{\Delta t}$$

$$= \frac{100(m)(2v)}{t}$$

$$= \frac{200mv}{t}$$

Q.7 The effect of increase in temperature on the number of electrons in conduction band (n_e) and resistance of a semiconductor will be as :

- (1) Both n_e and resistance decrease (2) n_e increases, resistance decreases
(3) n_e decreases, resistance increases (4) Both n_e and resistance increase

Ans. [2]

Sol. As temperature increases n_e increases, this results in increase in conductance
 \therefore T increases, n_e increases and R decreases

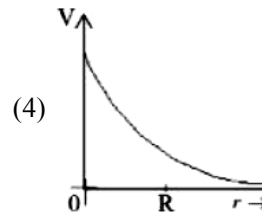
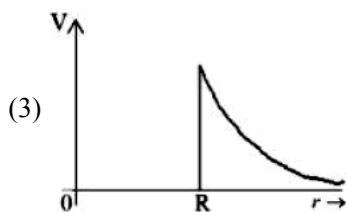
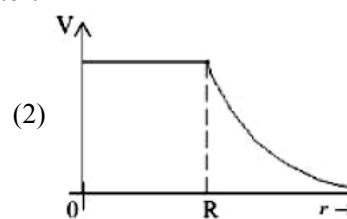
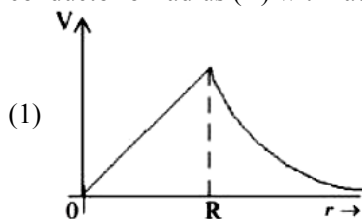
Q.8 A free neutron decays into a proton but a free proton does not decay into neutron. This is because :

- (1) neutron has larger rest mass than proton
(2) neutron is a composite particle made of a proton and an electron
(3) proton is a charged particle
(4) neutron is an uncharged particle

Ans. [1]

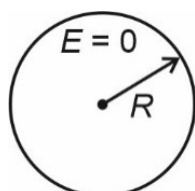
Sol. Rest mass of proton > Rest mass of neutron.

Q.9 Which of the following correctly represents the variation of electric potential(V) of a charged spherical conductor of radius (R) with radial distance (r) from the center?



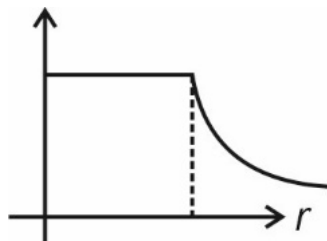
Ans. [2]

Sol.



Electric field inside a conductor = 0

Hence $V = \text{constant}$



- Q.10** The amplitude of $15 \sin(1000\pi t)$ is modulated by $10 \sin(4\pi t)$ signal. The amplitude modulated signal contains frequency(ies) of
- A. 500 Hz
B. 2 Hz
C. 250 Hz
D. 498 Hz
E. 502 Hz

Choose the correct answer from the options given below :

- (1) B only (2) A and B only (3) A, D and E only (4) A only

Ans. [3]

Sol. Frequencies in AM are

$$f_c, f_c + f_m, f_c - f_m$$

$$f_c = 500, f_m = 2$$

500, 498 and 502 are present

- Q.11** If a source of electromagnetic radiation having power 15kW produces 10^{16} photons per second, the radiation belongs to a part of spectrum is.

(Take Planck constant $h = 6 \times 10^{-34} \text{Js}$)

- (1) Gamma rays (2) Ultraviolet rays (3) Micro waves (4) Radio waves

Ans. [1]

Sol. $h\nu = 15\text{kW}$

$$h\nu = \frac{15 \times 10^3}{6 \times 10^{-34} \times 10^{16}} = 2.5 \times 10^{21} \text{ Hz}$$

gamma rays

- Q.12** At a certain depth "d" below surface of earth, value of acceleration due to gravity becomes four times that of its value at a height 3R above earth surface. Where R is Radius of earth (Take $R = 6400 \text{ km}$). The depth d is equal to :

- (1) 4800 km (2) 5260 km (3) 2560 km (4) 640 km

Ans. [1]

Sol. $g_d = \frac{GM}{R^3}(R - d)$ (depth variation)

$$g_h = \frac{GM}{(R + h)^2} \quad (\text{h above surface})$$

$$g_d = 4 g_h$$

$$\frac{GM}{R^3}(R - d) = 4 \frac{GM}{(R + 3R)^2}$$

$$R - d = \frac{R}{4}$$

$$d = \frac{3R}{4}$$

$$d = 4800 \text{ km}$$

- Q.13** Spherical insulating ball and a spherical metallic ball of same size and mass are dropped from the same height. Choose the correct statement out of the following {Assume negligible air friction}
- (1) Time taken by them to reach the earth's surface will be independent of the properties of their materials
(2) Insulating ball will reach the earth's surface earlier than the metal ball
(3) Metal ball will reach the earth's surface earlier than the insulating ball
(4) Both will reach the earth's surface simultaneously.

Ans. [2]

Sol. Consider magnetic force of earth, induced eddy current will develop inside the conducting sphere which retards the conducting sphere.

Q.14 The initial speed of a projectile fired from ground is u . At the highest point during its motion, the speed of projectile is $\frac{\sqrt{3}}{2}u$ The time of flight of the projectile is :

- (1) $\frac{\sqrt{3}u}{g}$ (2) $\frac{u}{g}$ (3) $\frac{2u}{g}$ (4) $\frac{u}{2g}$

Ans. [2]

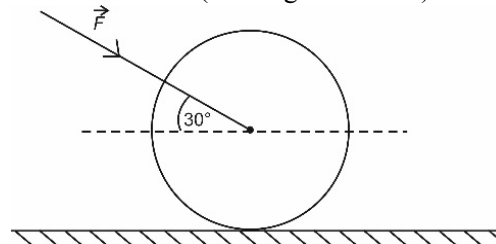
Sol. $u \cos \theta = \frac{\sqrt{3}}{2}u$

$$\cos \theta = \frac{\sqrt{3}}{2}$$

$$\theta = 30^\circ$$

$$\text{Time of flight} = \frac{2u \sin \theta}{g} = \left(\frac{u}{g}\right)$$

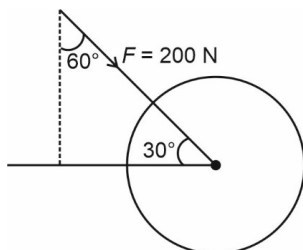
Q.15 As shown in figure, a 70 kg garden roller is pushed with a force of $\vec{F} = 200 \text{ N}$ at an angle of 30° with horizontal. The normal reaction on the roller is (Given $g = 10 \text{ m s}^{-2}$)



- (1) 600 N (2) 800 N (3) $200\sqrt{3}$ N (4) $800\sqrt{2}$ N

Ans. [2]

Sol.



$$\begin{aligned} \text{Normal reaction} &= 70g + F \cos 60 \\ &= 700 + 100 \\ &= 800 \text{ N} \end{aligned}$$

Q.16 If 1000 droplets of water of surface tension 0.07 N/m , having same radius 1 mm each, combine to form a single drop. In the process the released surface energy is-

(Take $\pi = \frac{22}{7}$)

- (1) $7.92 \times 10^{-6} \text{ J}$ (2) $7.92 \times 10^{-4} \text{ J}$ (3) $9.68 \times 10^{-4} \text{ J}$ (4) $8.8 \times 10^{-5} \text{ J}$

Ans. [2]

Sol. Radius of bigger drop = $10r = R$
 (r = radius of smaller droplet)
 $\Delta E = 1000 \times 4\pi r^2 \times T - 4\pi R^2 T$
 $= 4\pi T [1000 \times r^2 - 100r^2]$
 $= 3600 \pi r^2 T$
 $= 3600 \times \frac{22}{7} \times 1 \times 10^{-6} \times \frac{7}{100}$
 $= 22 \times 36 \times 10^{-6}$
 $= 792 \times 10^{-6} \text{ J}$
 $= 7.92 \times 10^{-4} \text{ J}$

Q.17 The drift velocity of electrons for a conductor connected in an electrical circuit is V_d . The conductor is now replaced by another conductor with same material and same length but double the area of cross section. The applied voltage remains same. The new drift velocity of electrons will be :

- (1) V_d (2) $2V_d$ (3) $\frac{V_d}{4}$ (4) $\frac{V_d}{2}$

Ans. [1]

Sol. $i = nAV_d e$
 $i_1 = \left(\frac{V}{R}\right)$
 $i_2 = \left(\frac{2V}{R}\right)$
 So, $\frac{i_1}{i_2} = \frac{1}{2} = \frac{(AV_d)_1}{(AV_d)_2} = \frac{V_d}{(V_d)_2} \times \left(\frac{1}{2}\right)$
 $\frac{1}{2} \times \frac{V_d}{(V_d)_2} = \frac{1}{2}$
 $\Rightarrow (V_d)_2 = V_d$

Q.18 If R , X_L and X_C represent resistance, inductive reactance and capacitive reactance. Then which of the following is dimensionless :

- (1) $R \frac{X_L}{X_C}$ (2) $R X_L X_C$ (3) $\frac{R}{\sqrt{X_L X_C}}$ (4) $\frac{R}{X_L X_C}$

Ans. [3]

Sol. R = Resistance
 $[X_L] = [R]$
 $[X_C] = [R]$
 So, $\frac{R}{\sqrt{X_L X_C}}$ is dimensionless.

Q.19 Given below are two statements: One is labelled as **Assertion A** and the other is labelled as **Reason R**
Assertion A : The beam of electrons show wave nature and exhibit interference and diffraction.

Reason R : Davisson Germer Experimentally verified the wave nature of electrons.

In the light of the above statements, choose the **most appropriate** answer from the options given below:

- (1) A is not correct but R is correct
 (2) Both A and R are correct and R is the correct explanation of A
 (3) Both A and R are correct but R is not the correct explanation of A
 (4) A is correct but R is not correct

Ans. [2]

Sol. Beam of electrons show wave nature and exhibit interference and diffraction as shown by Davisson Germer experiment.

Q.20 The correct relation between $\gamma = \frac{C_p}{C_v}$ and temperature T is :

(1) $\gamma \propto T^\circ$

(2) $\gamma \propto \frac{1}{\sqrt{T}}$

(3) $\gamma \propto \frac{1}{T}$

(4) $\gamma \propto T$

Ans. [1]

Sol. $\gamma = \frac{C_p}{C_v}$

At low temperature (T), γ is independent of T.

Section-B: Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE**. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, -00.33, -00.30, 30.27, -27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer..

Q.21 In a medium the speed of light wave decreases to 0.2 times to its speed in free space. The ratio of relative permittivity to the refractive index of the medium is $x : 1$. The value of x is _____.
(Given speed of light in free space = $3 \times 10^8 \text{ ms}^{-1}$ and for the given medium $\mu_1 = 1$)

Ans. [5]

Sol. We know that $v = \frac{c}{n} = \frac{c}{\sqrt{\epsilon_r}}$

Putting the values:

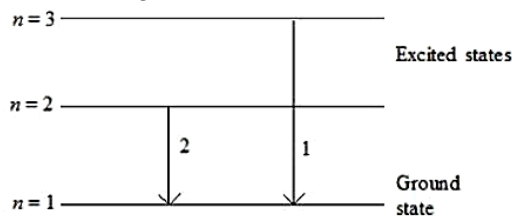
$$0.2c = \frac{c}{\sqrt{\epsilon_r}}$$

$$\Rightarrow \sqrt{\epsilon_r} = 5$$

$$\Rightarrow \text{Required ratio} = \frac{\epsilon_r}{n} = \frac{\epsilon_r}{\sqrt{\epsilon_r}} = \sqrt{\epsilon_r} = 5$$

$$\Rightarrow x = 5$$

Q.22 For hydrogen atom, λ_1 and λ_2 are the wavelengths corresponding to the transitions 1 and 2 respectively as shown in figure. The ratio of λ_1 and λ_2 is $\frac{x}{32}$. The value of x is _____.



Ans. [27]

Sol. $\frac{1}{\lambda} = RZ^2 \left[\frac{1}{n_1^2} - \frac{1}{n_2^2} \right] \Rightarrow \frac{1}{\lambda_1} = R \left[1 - \frac{1}{9} \right] \& \frac{1}{\lambda_2} = R \left[1 - \frac{1}{4} \right]$

$$\Rightarrow \frac{\lambda_1}{\lambda_2} = \frac{R \cdot \frac{3}{4}}{R \cdot \frac{8}{9}} = \frac{27}{32}$$

$$\Rightarrow x = 27$$

Q.23 The speed of a swimmer is 4 km h^{-1} in still water. If the swimmer makes his strokes normal to the flow of river of width 1 km , he reaches a point 750 m down the stream on the opposite bank. The speed of the river water is _____ km h^{-1}

Ans. [3]

Sol. Let speed of river water = v_0

$$\Rightarrow \text{drift} = v_0 \times \Delta t \quad \dots(1)$$

$$\& \Delta t = \frac{1}{4} \text{ h} \quad \dots(2)$$

$$\Rightarrow 0.75 = v_0 \times \frac{1}{4}$$

$$\Rightarrow v_0 = 3 \text{ km/hr}$$

Q.24 Two identical cells, when connected either in parallel or in series gives same current in an external resistance 5Ω . The internal resistance of each cell will be _____ Ω .

Ans. [5]

Sol. $\varepsilon_{\text{series}} = \varepsilon_1 + \varepsilon_2 = 2\varepsilon$

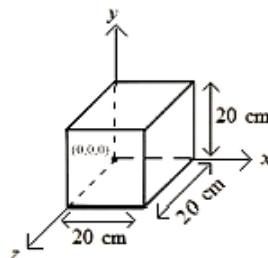
$$r_{\text{series}} = r_1 + r_2 = 2r$$

$$\varepsilon_{\text{parallel}} = \frac{\frac{\varepsilon_1 + \varepsilon_2}{\frac{1}{r_1} + \frac{1}{r_2}}}{\frac{1}{r_1} + \frac{1}{r_2}} = \varepsilon \quad \& \quad r_{\text{parallel}} = \frac{r}{2}$$

$$\Rightarrow \frac{2\varepsilon}{2r + 5} = \frac{\varepsilon}{\frac{r}{2} + 5}$$

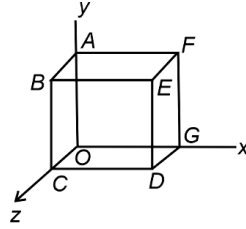
$$\Rightarrow r + 10 = 2r + 5 \Rightarrow r = 5 \Omega$$

Q.25 Expression for an electric field is given by $\vec{E} = 4000x^2 \hat{i} \frac{\text{V}}{\text{m}}$. The electric flux through the cube of side 20 cm when placed in electric field (as shown in the figure) is _____ V cm .



Ans. [640]

Sol. The flux will be only from DEFG surface as on the surface OABC field is 0 and for rest of the surface, area vector is perpendicular to field.



$$\begin{aligned} \text{So } \phi &= EA \\ &= 4000 \times (0.2)^2 \times 0.2 \times 0.2 \\ &= \frac{32}{5} \text{ Vm} = \frac{32}{5} \times 100 \text{ V cm} \\ &= 640 \text{ V cm} \end{aligned}$$

Q.26 An inductor of 0.5 mH, a capacitor of 20 μF and resistance of 20 Ω are connected in series with a 220 V ac source. If the current is in phase with the emf, the amplitude of current of the circuit is \sqrt{x} A. The value of x is

Ans. [242]

Sol. As the current is in phase with emf the circuit is in resonance so

$$\begin{aligned} i_{\text{rms}} &= \frac{V_{\text{rms}}}{R} = \frac{220}{20} = 11 \text{ A} \\ \text{as } i_0 &= \sqrt{2} i_{\text{rms}} = \sqrt{2} \times 11 = \sqrt{242} \end{aligned}$$

Q.27 A lift of mass $M = 500$ kg is descending with speed of 2 ms^{-1} . Its supporting cable begins to slip thus allowing it to fall with a constant acceleration of 2 ms^{-2} . The kinetic energy of the lift at the end of fall through to a distance of 6 m will be _____ kJ.

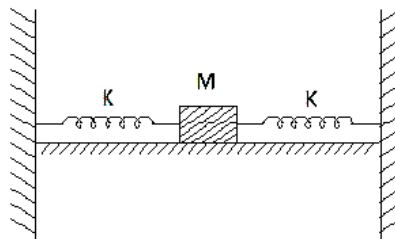
Ans. [7]

Sol.

$$\begin{aligned} u &= 2 \text{ m/s} \\ a &= 2 \text{ m/s}^2 \\ s &= 6 \text{ m} \\ v &= ? \\ v^2 &= u^2 + 2as \\ v^2 &= 4 + 2 \times 2 \times 6 \\ &= 28 \end{aligned}$$

$$\begin{aligned} \text{So KE} &= \frac{1}{2} mv^2 = \frac{1}{2} \times 500 \times 28 \text{ J} \\ &= 7000 \text{ J} \\ &= 7 \text{ kJ} \end{aligned}$$

Q.28 In the figure given below, a block of mass $M = 490$ g placed on a frictionless table is connected with two springs having same spring constant ($K = 2 \text{ N m}^{-1}$). If the block is horizontally displaced through 'X' m then the number of complete oscillations it will make in 14π seconds will be _____.



Ans. [20]

Sol. $k_{\text{net}} = k_1 + k_2 = 4 \text{ N/m}$

$$T = 2\pi \sqrt{\frac{m}{k_{\text{net}}}} = 2\pi \sqrt{\frac{0.49}{4}}$$
$$= \frac{2\pi \times .7}{2}$$
$$= \frac{7\pi}{10}$$

So number of oscillation completed.

$$n = \frac{t}{T} = \frac{14\pi}{7\pi/10} = 20$$

Q.29 A solid sphere of mass 1 kg rolls without slipping on a plane surface. Its kinetic energy is $7 \times 10^{-3} \text{ J}$. The speed of the centre of mass of the sphere is _____ cm s^{-1} .

Ans. [10]

Sol. $K = \frac{1}{2}mv_{\text{cm}}^2 + \frac{1}{2}I\omega^2$

$$7 \times 10^{-3} = \frac{1}{2} \times 1 \times v_{\text{cm}}^2 + \frac{1}{2} \times \frac{2}{5}mR^2 \times \frac{v_{\text{cm}}^2}{R^2}$$
$$7 \times 10^{-3} = \frac{7}{10} \times 1 \times v_{\text{cm}}^2$$
$$v_{\text{cm}} = 0.1 \text{ m/sec}$$
$$= 10 \text{ cm/sec}$$

Q.30 A thin rod having a length of 1 m and area of crosssection $3 \times 10^{-6} \text{ m}^2$ is suspended vertically from one end. The rod is cooled from 210°C to 160°C . After cooling, a mass M is attached at the lower end of the rod such that the length of rod again becomes 1 m. Young's modulus and coefficient of linear expansion of the rod are $2 \times 10^{11} \text{ N m}^{-2}$ and $2 \times 10^{-5} \text{ K}^{-1}$, respectively. The value of M is _____ kg. (Take $g = 10 \text{ m s}^{-2}$)

Ans. [60]

Sol. Stress = $\frac{T}{AY} = \frac{\Delta\ell}{\ell}$

and $\Delta\ell = \ell\alpha\Delta T$ or $\frac{\Delta\ell}{\ell} = \alpha\Delta T$

so $\frac{T}{AY} = \alpha\Delta T$

$$\frac{M \times 10}{2 \times 10^{11} \times 3 \times 10^{-6}} = 2 \times 10^{-5} \times 50$$
$$M = \frac{2 \times 10^{-5} \times 50 \times 3 \times 10^{-6} \times 2 \times 10^{11}}{10}$$
$$= 60 \text{ kg}$$

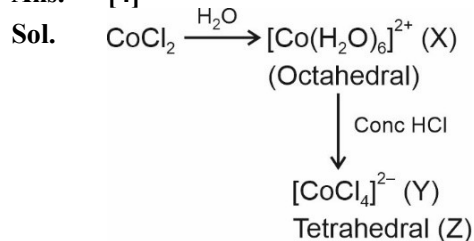
CHEMISTRY

Section-A: This section contains 20 multiple choice questions. Each question has 4 choices(1), (2), (3) and (4), out of which **ONLY ONE** is correct..

Q.31 Cobalt chloride when dissolved in water forms pink coloured complex X which has octahedral geometry. This solution on treating with conc. HCl forms deep blue complex, Y which has a Z geometry. X, Y and Z, respectively, are :

- (1) $X = [\text{Co}(\text{H}_2\text{O})_6]^{3+}$, $Y = [\text{CoCl}_6]^{3-}$, $Z = \text{Octahedral}$
 (2) $X = [\text{Co}(\text{H}_2\text{O})_4\text{Cl}_2]^+$, $Y = [\text{CoCl}_4]^{2-}$, $Z = \text{Tetrahedral}$
 (3) $X = [\text{Co}(\text{H}_2\text{O})_6]^{2+}$, $Y = [\text{CoCl}_6]^{3-}$, $Z = \text{Octahedral}$
 (4) $X = [\text{Co}(\text{H}_2\text{O})_6]^{2+}$, $Y = [\text{CoCl}_4]^{2-}$, $Z = \text{Tetrahedral}$

Ans. [4]



Hence correct answer is option (4).

Q.32 The correct order of basicity of oxides of vanadium is :

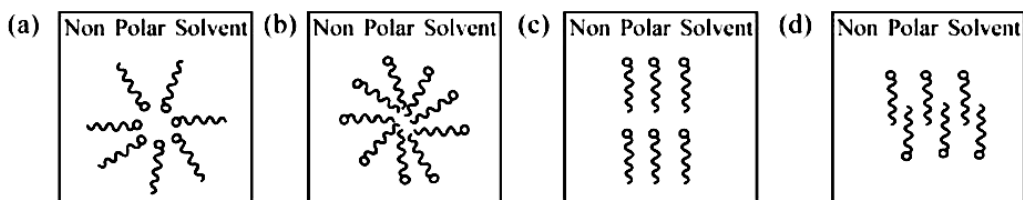
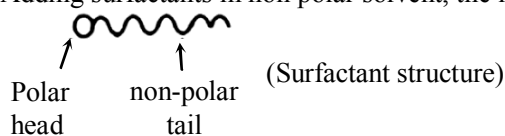
- (1) $\text{V}_2\text{O}_3 > \text{V}_2\text{O}_5 > \text{V}_2\text{O}_4$ (2) $\text{V}_2\text{O}_4 > \text{V}_2\text{O}_3 > \text{V}_2\text{O}_5$ (3) $\text{V}_2\text{O}_3 > \text{V}_2\text{O}_4 > \text{V}_2\text{O}_5$ (4) $\text{V}_2\text{O}_5 > \text{V}_2\text{O}_4 > \text{V}_2\text{O}_3$

Ans. [3]

Sol. $\text{V}_2\text{O}_3 > \text{V}_2\text{O}_4 > \text{V}_2\text{O}_5$

As positive oxidation state increases acidic nature increases and basic nature decreases.

Q.33 Adding surfactants in non polar solvent, the micelles structure will look like



(1) c

(2) a

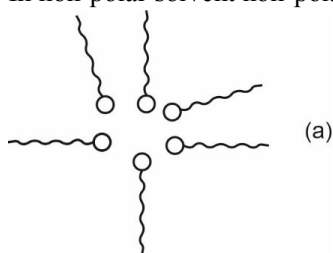
(3) d

(4) b

Ans. [2]

Sol.

Polar
Non-polar
In non-polar solvent non-polar part will point out

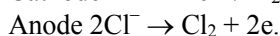
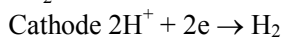
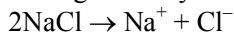


Non-polar part will interact with non-polar solvent.

- Q.34** Which one of the following statements is correct for electrolysis of brine solution?
 (1) Cl_2 is formed at cathode
 (2) H_2 is formed at anode
 (3) O_2 is formed at cathode
 (4) OH^- is formed at cathode

Ans. [4]

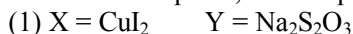
Sol. During electrolysis of Brine



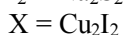
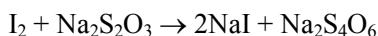
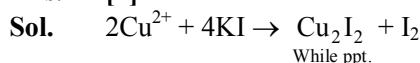
At cathode H_2 is liberated

At anode Cl_2 is formed.

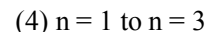
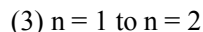
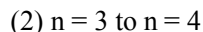
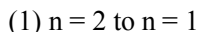
- Q.35** When Cu^{2+} ion is treated with KI, a white precipitate, X appears in solution. The solution is titrated with sodium thiosulphate, the compound Y is formed. X and Y respectively are



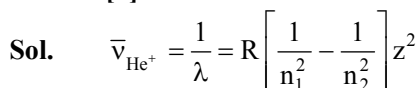
Ans. [4]



- Q.36** Which transition in the hydrogen spectrum would have the same wavelength as the Balmer type transition from $n = 4$ to $n = 2$ of He^+ spectrum :



Ans. [1]



$$= R \left[\frac{1}{(2)^2} - \frac{1}{(4)^2} \right] 4$$

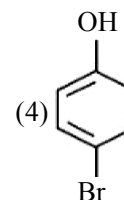
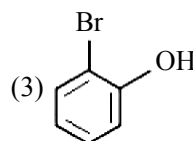
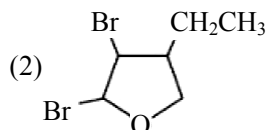
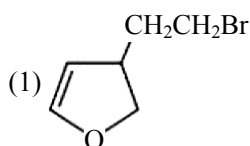
$$= R \left[\frac{1}{1} - \frac{1}{4} \right] = \frac{3}{4} R$$

$$\bar{\nu}_{2 \rightarrow 1} = \frac{1}{\lambda} = R \left[\frac{1}{n_1^2} - \frac{1}{n_2^2} \right]$$

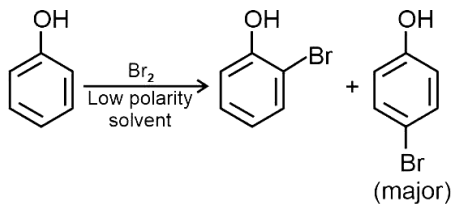
$$= R \left[\frac{1}{1} - \frac{1}{(2)^2} \right]$$

$$= \frac{3}{4} R$$

- Q.37** An organic compound 'A' with empirical formula $\text{C}_6\text{H}_6\text{O}$ gives sooty flame on burning. Its reaction with bromine solution in low polarity solvent results in high yield of B. B is :



Ans. [4]

Sol.

Q.38 Match List I with List II :

List-I		List-II	
A.	XeF ₄	I.	See-saw
B.	SF ₄	II.	Square planar
C.	NH ₄ ⁺	III.	Bent T-shaped
D.	BrF ₃	IV.	Tetrahedral

Choose the correct answer from the options given below:

(1) A - IV, B - III, C - II, D - I

(2) A - II, B - I, C - III, D - IV

(3) A - II, B - I, C - IV, D III

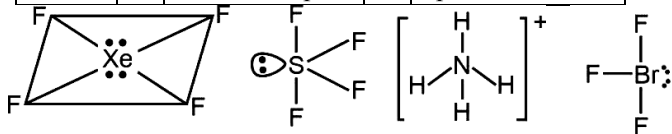
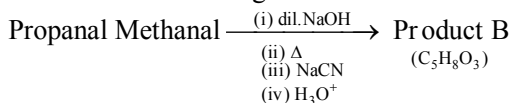
(4) A - IV, B - I, C - II, D - III

Ans. [3]

Sol.

A-II B-I C-IV D-III

			Hybridisation
XeF ₄	-	Square planar	- sp ³ d ²
SF ₄	-	See Saw	- sp ³ d
NH ₄ ⁺	-	Tetrahedral	- sp ³
BrF ₃	-	Bent-T-shape	- sp ³ d


Q.39 Consider the following reaction


The correct statement for product B is. It is

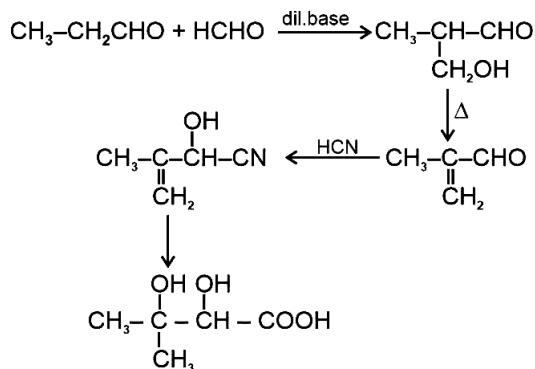
 (1) racemic mixture and gives a gas with saturated NaHCO₃ solution

(2) optically active alcohol and is neutral

(3) optically active and adds one mole of bromine

(4) racemic mixture and is neutral

Ans. [1]

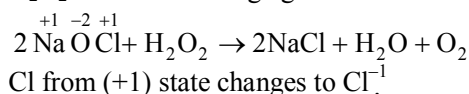
Sol.

 Racemic mixture effervescence with NaHCO₃

- Q.40** H_2O_2 acts as a reducing agent in :
 (1) $2Fe^{2+} + 2H^+ + H_2O_2 \rightarrow 2Fe^{3+} + 2H_2O$
 (3) $Na_2S + 4H_2O_2 \rightarrow Na_2SO_4 + 4H_2O$

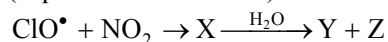
- (2) $Mn^{2+} + 2H_2O_2 \rightarrow MnO_2 + 2H_2O$
 (4) $2NaOCl + H_2O_2 \rightarrow 2NaCl + H_2O + O_2$

Ans. [4]

Sol. H_2O_2 act as a reducing agent



- Q.41** Identify X, Y and Z in the following reaction.
 (Equation not balanced)

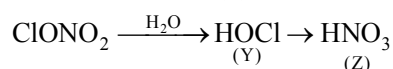


- (1) $X = ClONO_2, Y = HOCl, Z = NO_2$
 (3) $X = ClNO_3, Y = Cl_2, Z = NO_2$

- (2) $X = ClONO_2, Y = HOCl, Z = HNO_3$
 (4) $X = ClNO_2, Y = HCl, Z = HNO_3$

Ans. [2]

Sol. $ClO + NO_2 \rightarrow ClONO_2$
 (X)



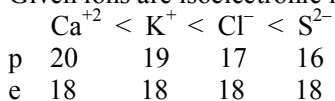
- Q.42** The correct increasing order of the ionic radii is :

- (1) $K^+ < S^{2-} < Ca^{2+} < Cl^-$
 (3) $Ca^{2+} < K^+ < Cl^- < S^{2-}$

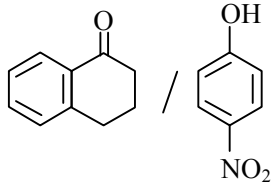
- (2) $Cl^- < Ca^{2+} < K^+ < S^{2-}$
 (4) $S^{2-} < Cl^- < Ca^{2+} < K^+$

Ans. [3]

Sol. Given ions are isoelectronic more is nuclear charge per electron smaller is size



- Q.43** Match items of columns I and II

Column I (Mixture of compounds)		Column II (Separation Technique)	
(A)	H_2O / CH_2Cl_2	(i)	Crystallization
(B)		(ii)	Differential solvent extraction
(C)	Kerosene / Naphthalene	(iii)	Column chromatography
(D)	$C_6H_{12}O_6 / NaCl$	(iv)	Fractional Distillation

Correct match is :

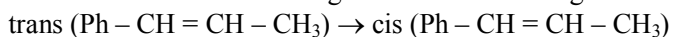
- (1) A-(ii), B-(iii), C-(iv), D-(i) (2) A-(ii), B-(iv), C-(i), D-(iii)
 (3) A-(i), B-(iii), C-(ii), D-(iv) (4) A-(iii), B-(iv), C-(ii), D-(i)

Ans. [1]

Sol. Water and dichloromethane can be separated by differential extraction.

Which $C_6H_{12}O_6$ and $NaCl$ can be separated by crystallization.

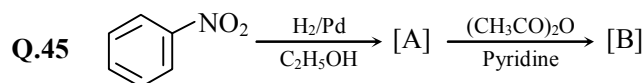
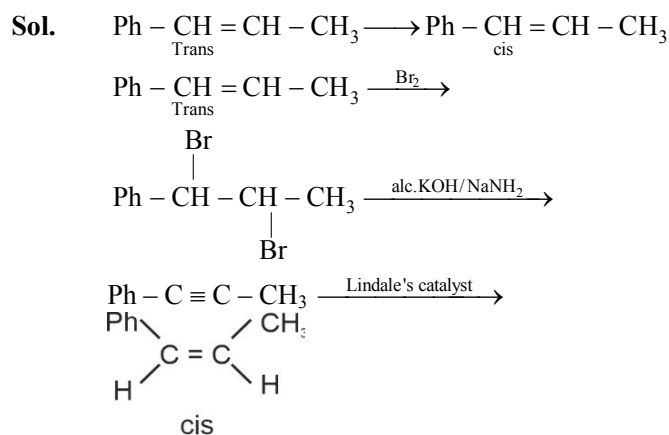
- Q.44** Choose the correct set of reagents for the following conversion.



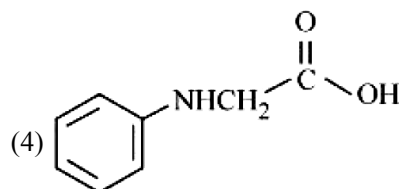
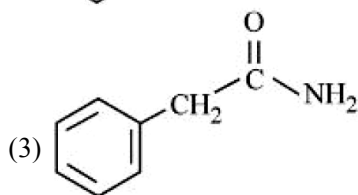
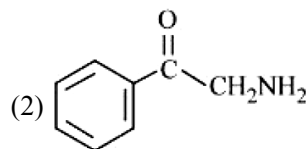
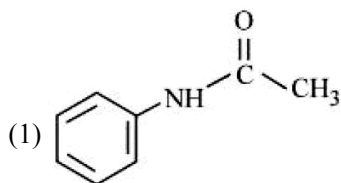
- (1) $Br_2, alc \bullet KOH, NaNH_2, H_2$ Lindlar Catalyst
 (3) $Br_2, alc \bullet KOH, NaNH_2, Na$ (Liq NH_3)

- (2) $Br_2, aq \bullet KOH, NaNH_2, Na$ (Liq NH_3)
 (4) $Br_2, aq \bullet KOH, NaNH_2, H_2$ Lindlar Catalyst

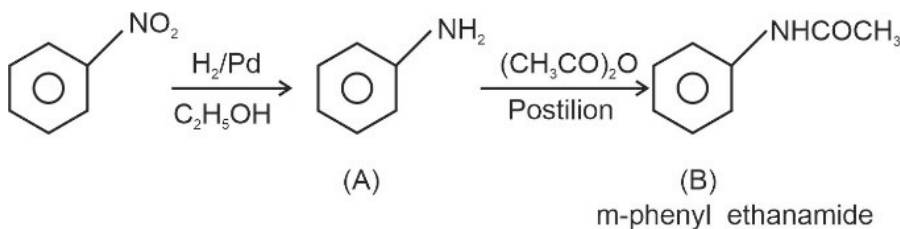
Ans. [1]



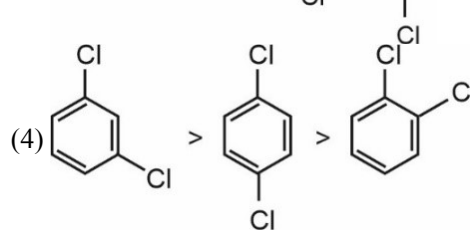
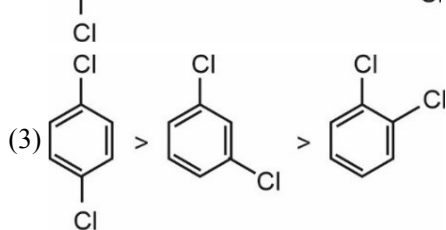
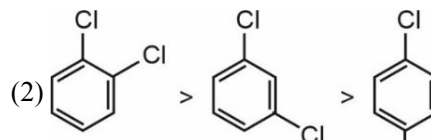
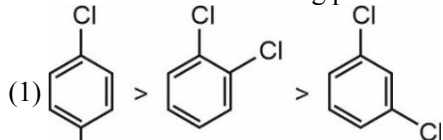
Consider the above reaction and identify the product B.



Ans. [1]
Sol.



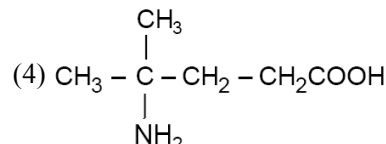
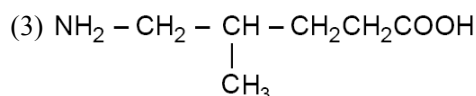
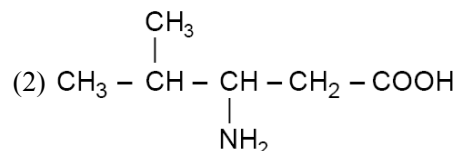
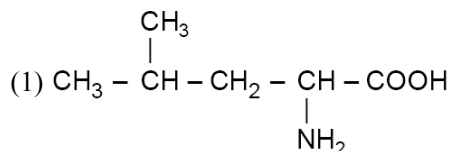
Q.46 The correct order of melting points of dichlorobenzenes is



Ans. [1]

Sol. Out of o, m, p-dichlorobenzene para isomer has maximum melting point due to symmetrical nature.

Q.47 A protein 'X' with molecular weight of 70,000 u, on hydrolysis gives amino acids. One of these amino acid is :



Ans. [1]

Sol. Protein upon hydrolysis gives α -amino acids. Only option (1) contains α -amino acid. Hence the correct answer is (1).

Q.48 $\text{Nd}^{2+} =$ _____

- (1) $4f^3$ (2) $4f^4 6s^2$ (3) $4f^4$ (4) $4f^2 6s^2$

Ans. [3]

Sol. Neodymium $\text{Nd} = 4f^4 6s^2$
 $\text{Nd}^{2+} = 4f^4$.

Q.49 Which of the following artificial sweeteners has the highest sweetness value in comparison to cane sugar?

- (1) Sucralose (2) Aspartame (3) Saccharin (4) Alitame

Ans. [4]

Sol. Highest sweetness value is of Alitame

Sucralose = 600

Aspartame = 100

Saccharin = 550

Alitame = 2000

Q.50 The methods NOT involved in concentration of ore are

- A. Liqation B. Leaching
C. Electrolysis D. Hydraulic washing
E. Froth floatation

Choose the correct answer from the options given below

- (1) B, D and C only (2) B, D and E only (3) C, D and E only (4) A and C only

Ans. [4]

Sol. (A) and (C) only

Liquation is used for purification of metal.

Section-B: Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE**. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, -00.33, -00.30, 30.27, -27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

Q.51 On complete combustion, 0.492 g of an organic compound gave 0.792 g of CO_2 . The % of carbon in the organic compound is _____ (Nearest integer)

Ans. [44]

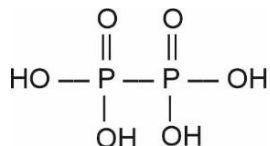
Sol. Percentage of C = $\frac{W_{\text{CO}_2}}{W_{\text{org.comp}}} \times \frac{12}{44} \times 100$
 $= \frac{0.792}{0.492} \times \frac{12}{44} \times 100$
 $= 43.90$

Q.52 The oxidation state of phosphorus in hypophosphoric acid is + _____.

Ans. [4]

Sol. Hypophosphoric acid $H_4P_2O_6$

Oxidation state is +4



Q.53 For reaction : $SO_2(g) + \frac{1}{2}O_2(g) \rightleftharpoons SO_3(g)$

$K_p = 2 \times 10^{12}$ at 27°C and 1 atm pressure. The K_c for the same reaction is _____ $\times 10^{13}$. (Nearest integer) (Given $R = 0.082 \text{ L atm K}^{-1} \text{ mol}^{-1}$)

Ans. [1]

Sol. $SO_{2(g)} + \frac{1}{2}O_{2(g)} \rightleftharpoons SO_{3(g)}$

$$K_p = K_c (RT)^{\Delta n}$$

$$2 \times 10^{12} = K_c (0.082 \times 300)^{-1/2}$$

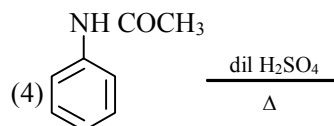
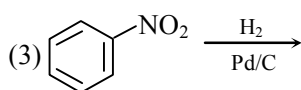
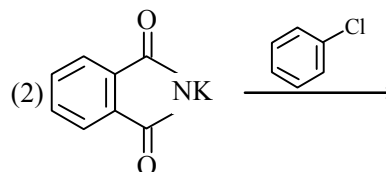
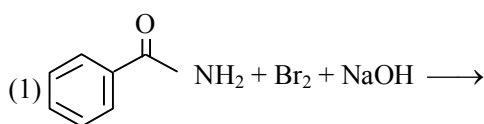
$$K_c = 2 \times 10^{12} \times (0.082 \times 300)^{1/2}$$

$$= 9.9 \times 10^{12}$$

$$= 0.99 \times 10^{13}$$

$$\approx 1 \times 10^{13}$$

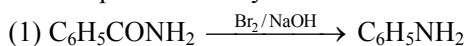
Q.54 How many of the transformations given below would result in aromatic amines?



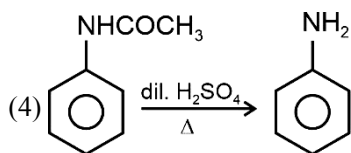
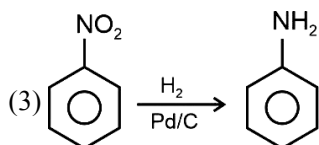
Ans. [3]

Sol. 1, 3, 4 will give Aniline.

Gabriel phthalimide synthesis cannot be used to prepare Aniline.



Hoffmann Bromamide synthesis



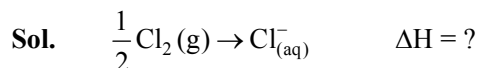
Q.55 The enthalpy change for the conversion of $\frac{1}{2} \text{Cl}_2(\text{g})$ to $\text{Cl}^-(\text{aq})$ is (-) _____ kJ mol^{-1} (Nearest integer)

$$\text{Given : } \Delta_{\text{dis}} H_{\text{Cl}_2(\text{g})}^{\ominus} = 240 \text{ kJ mol}^{-1}$$

$$\Delta_{\text{eg}} H_{\text{Cl}(\text{g})}^{\ominus} = -350 \text{ kJ mol}^{-1},$$

$$\Delta_{\text{hyd}} H_{\text{Cl}(\text{g})}^{\ominus} = -380 \text{ kJ mol}^{-1}$$

Ans. [610]



$$\Delta H = \frac{1}{2} \Delta_{\text{diss}} H_{\text{Cl}_2}^{\ominus} + \Delta_{\text{eg}} \Delta H_{\text{Cl}(\text{g})}^{\ominus} + \Delta_{\text{hyd}} H_{\text{Cl}(\text{g})}^{\ominus}$$

$$= \frac{1}{2} \times 240 + (-350) + (-380)$$

$$= -610 \text{ kJ mol}^{-1}$$

Q.56 The total pressure of a mixture of non-reacting gases X (0.6 g) and Y (0.45 g) in a vessel is 740 mm of Hg. The partial pressure of the gas X is _____ mm of Hg. (Nearest integer)
(Given : molar mass X = 20 and Y = 45 g mol^{-1})

Ans. [555]

Sol. $P_{\text{Total}} = 740 \text{ mm of Hg}$

$$P_X = \text{mole fraction of [X]} P_{\text{Total}}$$

$$n_X = \frac{0.6}{20} = 0.03$$

$$n_Y = \frac{0.45}{45} = 0.01$$

$$\text{Mole fraction of X} = \frac{0.03}{0.01 + 0.03} = \frac{3}{4}$$

$$\text{Partial pressure of X} = \frac{3}{4} \times 740$$

$$= 555 \text{ mm of Hg}$$

Q.57 The logarithm of equilibrium constant for the reaction $\text{Pd}^{2+} + 4\text{Cl}^- \rightleftharpoons \text{PdCl}_4^{2-}$ is _____.
(Nearest integer)

$$\text{Given : } \frac{2.303RT}{F} = 0.06 \text{ V}$$

$$\text{Pd}_{(\text{aq})}^{2+} + 2\text{e}^- \rightleftharpoons \text{Pd}(\text{s}) \quad E^{\ominus} = 0.83 \text{ V}$$

$$\text{PdCl}_4^{2-}(\text{aq}) + 2\text{e}^- \rightleftharpoons \text{Pd}(\text{s}) + 4\text{Cl}^-(\text{aq}) \quad E^{\ominus} = 0.65 \text{ V}$$

Ans. [6]

Sol. $\text{Pd}^{2+} + 4\text{Cl}^- \rightleftharpoons [\text{PdCl}_4]^{2-}$
 $E^{\ominus} = (0.83) - (0.65) = 0.18 \text{ V}$

$$0 = 0.18 - \frac{0.06}{2} \log k_{\text{eq}}$$

$$0.18 = 0.03 \log k_{\text{eq}}$$

$$\boxed{\log k_{\text{eq}} = 6}$$

Q.58 A → B

The rate constants of the above reaction at 200 K and 300 K are 0.03 min^{-1} and 0.05 min^{-1} respectively. The activation energy for the reaction is _____ J (Nearest integer)

(Given : $\ln 10 = 2.3$)

$R = 8.3 \text{ J K}^{-1} \text{ mol}^{-1}$

$\log 5 = 0.70$

$\log 3 = 0.48$

$\log 2 = 0.30$)

Ans. [2520]

Sol. $\log \frac{k_2}{k_1} = \frac{E_a}{2.3 \times 8.3} \left(\frac{1}{200} - \frac{1}{300} \right)$

$$\log \frac{0.05}{0.03} = \frac{E_a}{2.3 \times 8.3} \left(\frac{1}{600} \right)$$

$$(0.70 - 0.48) = \frac{E_a}{2.3 \times 8.3} \times \frac{1}{600}$$

$$\Rightarrow 0.22 = \frac{E_a}{2.3 \times 8.3} \times \frac{1}{600}$$

$$E_a = 2.3 \times 8.3 \times 600 \times 0.22$$

$$= 2519.88$$

$$\approx 2520 \text{ J}$$

Q.59 At 27°C , a solution containing 2.5 g of solute in 250.0 mL of solution exerts an osmotic pressure of 400 Pa. The molar mass of the solute is _____ g mol^{-1} . (Nearest integer)

(Given : $R = 0.083 \text{ L bar K}^{-1} \text{ mol}^{-1}$)

Ans. [62250]

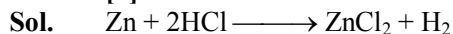
Sol. $400 = \frac{2.5}{\text{mw}} \times 4 \times (0.083 \times 10^5) \times 300$

$$\text{mw} = \frac{10 \times 0.083 \times 3}{4} \times 10^5 = 62250$$

Q.60 Zinc reacts with hydrochloric acid to give hydrogen and zinc chloride. The volume of hydrogen gas produced at STP from the reaction of 11.5 g of zinc with excess HCl is _____ L. (Nearest integer)

(Given : Molar mass of Zn is 65.4 g mol^{-1} and Molar volume of H_2 at STP = 22.7 L)

Ans. [4]



$$n_{\text{Zn}} = \frac{11.5}{65.4} = 0.176$$

$$V_{\text{H}_2} = 0.176 \times 22.7 = 3.99 \text{ litre}$$

MATHEMATICS

Section-A: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct..

Q.61 Let the shortest distance between the line $L: \frac{x-5}{-2} = \frac{y-\lambda}{1} = \frac{z+\lambda}{1}$, $\lambda \geq 0$ and $L_1: x+1 = y-1 = 4-z$ be

$2\sqrt{6}$. If (α, β, γ) lies on L, then which of the following is **NOT** possible?

(1) $\alpha + 2\gamma = 24$

(2) $2\alpha - \gamma = 9$

(3) $2\alpha + \gamma = 7$

(4) $\alpha - 2\gamma = 19$

Ans. [1]

Sol. $\frac{x-5}{-2} = \frac{y-\lambda}{0} = \frac{z+\lambda}{1}, \lambda \geq 0$

$$\frac{x+1}{1} = \frac{y-1}{1} = \frac{z-4}{-1}$$

$$\vec{a}_1 = 5\hat{i} + \lambda\hat{j} - \lambda\hat{k}, \vec{a}_2 = -\hat{i} + \hat{j} + 4\hat{k}$$

$$\vec{a}_1 = \vec{a}_2 = 6\hat{i} + (\lambda-1)\hat{j} - (\lambda+4)\hat{k}$$

$$\vec{b}_1 = -2\hat{i} + \hat{k}, \vec{b}_2 = \hat{i} + \hat{j} - \hat{k}$$

$$\begin{aligned} \vec{b}_1 \times \vec{b}_2 &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & 0 & 1 \\ 1 & 1 & -1 \end{vmatrix} \\ &= -\hat{i} - \hat{j} - 2\hat{k} \end{aligned}$$

$$(\vec{a}_1 - \vec{a}_2) \cdot \vec{b}_1 \times \vec{b}_2 = -6 + 1 - \lambda + 2\lambda + 8 = \lambda + 3$$

$$\text{and } |\vec{b}_1 \times \vec{b}_2| = \sqrt{6}$$

$$\therefore \frac{|\lambda+3|}{\sqrt{6}} = 2\sqrt{6}$$

$$\therefore \lambda = 9, \because \lambda \geq 0$$

$$\therefore L : \frac{x-5}{-2} = \frac{y-9}{0} = \frac{z+9}{1} = k$$

$$\therefore \alpha = -2k + 5, \beta = 9, \gamma = k - 9$$

Here k is real then

$$\alpha + 2\gamma = -13 \neq 24$$

But all other are in terms of k hence possible.

Correct option is (1).

Q.62 Let $\alpha \in (0, 1)$ and $\beta = \log_e(1 - \alpha)$.

$$\text{Let } P_n(x) = x + \frac{x^2}{2} + \frac{x^3}{3} + \dots + \frac{x^n}{n}, x \in (0, 1)$$

Then the integral $\int_0^\alpha \frac{t^{50}}{1-t} dt$ is equal to :

(1) $-(\beta + P_{50}(\alpha))$

(2) $\beta + P_{50}(\alpha)$

(3) $P_{50}(\alpha) - \beta$

(4) $\beta + P_{50}(\alpha)$

Ans. [1]

Sol.

$$\begin{aligned} \int_0^\alpha \frac{t^{50}}{1-t} dt &= -\int_0^\alpha \left(\frac{1-t^{50}}{1-t} - \frac{1}{1-t} \right) dt \\ &= -\left(\int_0^\alpha 1+t+t^2+\dots+t^{49} dt + \ln|1-t| \Big|_0^\alpha \right) \\ &= -\left(\alpha + \frac{\alpha^2}{2} + \frac{\alpha^3}{3} + \dots + \frac{\alpha^{50}}{50} \right) + \ln(1-\alpha) \\ &= -\beta - P_{50}(\alpha) \end{aligned}$$

- Q.63** (S1) $(p \Rightarrow q) \vee (p \wedge (\sim q))$ is a tautology
(S2) $((\sim p) \Rightarrow (\sim q) \wedge ((\sim p) \vee q))$ is a contradiction.

Then

- (1) both (S1) and (S2) are wrong
(2) both (S1) and (S2) are correct
(3) only (S1) is correct
(4) only (S2) is correct

Ans. [3]

Sol. S1

p	q	$\sim q$	$p \rightarrow q$	$p \wedge (\sim q)$	$(p \rightarrow q) \vee p \wedge (\sim q)$
T	T	F	T	F	T
T	F	T	F	T	T
F	T	F	T	F	T
F	F	T	T	F	T

\therefore S1 is correct

S2

p	q	$\sim q$	$\sim q$	$\sim p \rightarrow \sim q$	$\sim p \vee q$	(S2)
T	T	F	F	T	T	T
T	F	F	T	T	F	F
F	T	T	F	F	T	F
F	F	T	T	T	T	T

\therefore S2 is incorrect

Option (3) is correct.

- Q.64** A bag contains 6 balls. Two balls are drawn from it at random and both are found to be black. The probability that the bag contains at least 5 black balls is

- (1) $\frac{3}{7}$ (2) $\frac{5}{6}$ (3) $\frac{2}{7}$ (4) $\frac{5}{7}$

Ans. [4]

Sol. Let $E_i \rightarrow$ Bag have at least i black balls

$E \rightarrow$ 2 balls are drawn & both black

$$\begin{aligned} \therefore P\left(\frac{E_5 \text{ or } E_6}{E}\right) &= \frac{P\left(\frac{E}{E_5}\right) + P\left(\frac{E}{E_6}\right)}{\sum_{i=1}^6 P\left(\frac{E}{E_i}\right)} \\ &= \frac{\frac{{}^5C_2}{{}^6C_2} + \frac{{}^6C_2}{{}^6C_2}}{0 + \frac{{}^2C_2}{{}^6C_2} + \frac{{}^3C_2}{{}^6C_2} + \frac{{}^4C_2}{{}^6C_2} + \frac{{}^5C_2}{{}^6C_2} + \frac{{}^6C_2}{{}^6C_2}} \\ &= \frac{10+15}{1+3+6+10+15} = \frac{25}{35} = \frac{5}{7} \end{aligned}$$

- Q.65** Let R be a relation on $N \times N$ defined by $(a, b) R (c, d)$ if and only if $ad(b - c) = bc(a - d)$. Then R is
- (1) symmetric and transitive but not reflexive
 - (2) reflexive and symmetric but not transitive
 - (3) symmetric but neither reflexive nor transitive
 - (4) transitive but neither reflexive nor symmetric

Ans. [3]

Sol. $(a, b) R (c, d) \Rightarrow ad(b - c) = bc(a - d)$

For Reflexive

$$(a, b) R (a, b) \Rightarrow ab(b - a) = ba(a - b)$$

So not reflexive

For symmetric

$$(c, d) R (a, b) \Rightarrow cb(d - a) = ad(c - b)$$

$$\text{OR } ad(b - c) = bc(a - d)$$

So symmetric

For transitive

$$(a, b) R (c, d) \Rightarrow ad(b - c) = bc(a - d)$$

$$(c, d) R (e, f) \Rightarrow cf(d - e) = de(c - f)$$

$$\text{So } adcf(b - c)(d - e) = bcde(c - d)(c - f)$$

$$af(b - c)(d - e) = be(a - d)(c - f)$$

\Rightarrow Not transitive

- Q.66** Let $\vec{a} = 2\hat{i} + \hat{j} + \hat{k}$ and \vec{b} and \vec{c} be two nonzero vectors such that $\left| \vec{a} + \vec{b} + \vec{c} \right| = \left| \vec{a} + \vec{b} - \vec{c} \right|$ and $\vec{b} \cdot \vec{c} = 0$.

Consider the following two statements.

(A) $\left| \vec{a} + \lambda \vec{c} \right| \geq \left| \vec{a} \right|$ for all $\lambda \in R$

(B) \vec{a} and \vec{c} are always parallel

Then

(1) Neither (A) nor (B) is correct

(2) Both (A) and (B) are correct

(3) Only (B) is correct

(4) Only (A) is correct

Ans. [3]

Sol. $\left| \vec{a} + \vec{b} + \vec{c} \right| = \left| \vec{a} + \vec{b} - \vec{c} \right|$

$$\left| \vec{a} \right|^2 = \left| \vec{b} \right|^2 + \left| \vec{c} \right|^2 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a})$$

$$= \left| \vec{a} \right|^2 + \left| \vec{b} \right|^2 + \left| \vec{c} \right|^2 + 2(\vec{a} \cdot \vec{b} - \vec{b} \cdot \vec{c} - \vec{c} \cdot \vec{a})$$

$$\Rightarrow \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a} = 0 \Rightarrow \vec{c} \cdot \vec{a} = 0$$

$$\left| \vec{a} + \lambda \vec{c} \right|^2 = \left| \vec{a} \right|^2 + \lambda^2 \left| \vec{c} \right|^2 + 0 \geq \left| \vec{a} \right|^2$$

So A is correct

B is incorrect

- Q.67** A wire of length 20 m to be cut into two pieces. A piece of length l_1 is bent to make a square of area A_1 and the other piece of length l_2 is made into a circle of area A_2 . If $2A_1 + 3A_2$ is minimum then $(\pi l_1) : l_2$ is equal to
 (1) 1 : 6 (2) 3 : 1 (3) 6 : 1 (4) 4 : 1

Ans. [3]

Sol. $l_1 = 20 - x, l_2 = x$

$$2A_1 + 3A_2 = 2 \left(\frac{20-x}{4} \right)^2 + 3\pi \left(\frac{x}{2\pi} \right)^2$$

$$f(x) = \frac{(20-x)^2}{8} + \frac{3x^2}{4\pi}$$

$$f'(x_0) = \frac{1}{8} 2(20-x)(-1) + \frac{3}{4\pi} \cdot 2x \Big|_{x_0} = 0$$

$$0 = -\frac{1}{4}(20-x_0) + \frac{6x_0}{4\pi} \Rightarrow \frac{20-x_0}{4} = \frac{6x_0}{4\pi}$$

$$\pi(20-x_0) = 6x_0$$

$$20\pi = (6+\pi)x_0$$

$$x_0 = \frac{20\pi}{\pi+6}$$

$$\frac{\pi l_1}{l_2} = \pi \left(\frac{20-x_0}{x_0} \right) = \pi \left(\frac{x+6}{\pi} - 1 \right)$$

$$= 6$$

- Q.68** Let $y = f(x) = \sin^3 \left(\frac{\pi}{3} \cos \left(\frac{\pi}{3\sqrt{2}} (-4x^3 + 5x^2 + 1)^{\frac{3}{2}} \right) \right)$

Then at $x = 1$,

(1) $2y' + 3\pi^2 y = 0$

(2) $2y' + \sqrt{3} \pi^2 y = 0$

(3) $2y' + 3\pi^2 y = 0$

(4) $\sqrt{2}y' - 3x^2y = 0$

Ans. [1]

Sol. $f(x) = \sin^3 \left(\frac{\pi}{3} \cos \left(\frac{\pi}{3\sqrt{2}} (-4x^3 + 5x^2 + 1)^{\frac{3}{2}} \right) \right)$

$$f'(x) = 3 \sin^2 \left(\frac{\pi}{3} \cos \left(\frac{\pi}{3\sqrt{2}} (-4x^3 + 5x^2 + 1)^{\frac{3}{2}} \right) \right)$$

$$\cos \left(\frac{\pi}{3} \cos \left(\frac{\pi}{3\sqrt{2}} (-4x^3 + 5x^2 + 1)^{\frac{3}{2}} \right) \right)$$

$$\frac{\pi}{3} \left(-\sin \left(\frac{\pi}{3\sqrt{2}} (-4x^3 + 5x^2 + 1)^{\frac{3}{2}} \right) \right)$$

$$\frac{\pi}{3\sqrt{2}} \frac{3}{2} (-4x^3 + 5x^2 + 1)^{1/2} (-12x^2 + 10x)$$

$$f'(1) = \frac{3\pi^2}{16}$$

$$f(1) = \sin^3 \left(\frac{\pi}{3} \cos \left(\frac{\pi}{3\sqrt{2}} 2\sqrt{2} \right) \right) = \sin^3 \left(-\frac{\pi}{6} \right) = \frac{-1}{8}$$

$$\therefore 2f'(1) + 3x^2f(1) = 0$$

Q.69 Let $y = f(x)$ represent a parabola with focus $\left(-\frac{1}{2}, 0\right)$ and directrix $y = -\frac{1}{2}$.

Then

$$S = \left\{ x \in \mathbb{R} : \tan^{-1}(\sqrt{f(x)}) + \sin^{-1}(\sqrt{f(x)+1}) = \frac{\pi}{2} \right\} :$$

(1) Is an empty set

(2) Contains exactly one element

(3) Is an infinite set

(4) Contains exactly two elements

Ans. [4]

Sol. Equation of parabola

$$k^2 + \left(h + \frac{1}{2}\right)^2 = \left|k + \frac{1}{2}\right|^2$$

$$k^2 + h^2 + h + \frac{1}{4} = k^2 + \frac{1}{4} + k$$

$$\boxed{y = x^2 + x}$$

$$\tan^{-1} \sqrt{x^2 + x} + \sin^{-1} \sqrt{x^2 + x + 1} = \frac{\pi}{2}$$

$$\tan^{-1} \sqrt{x^2 + x} = \cos^{-1} \sqrt{x^2 + x + 1}$$

$$\sqrt{x^2 + x + 1} = \frac{1}{\sqrt{x^2 + x + 1}}$$

$$x = 0, -1$$

Q.70 If the domain of the function $f(x) = \frac{[x]}{1+x^2}$, where $[x]$ is greatest integer $\leq x$, is $[2, 6)$, then its range is :

(1) $\left(\frac{5}{37}, \frac{2}{5}\right]$

(2) $\left(\frac{5}{37}, \frac{2}{5}\right] - \left\{\frac{9}{29}, \frac{27}{109}, \frac{18}{89}, \frac{9}{53}\right\}$

(3) $\left(\frac{5}{26}, \frac{2}{5}\right]$

(4) $\left(\frac{5}{26}, \frac{2}{5}\right] - \left\{\frac{9}{29}, \frac{27}{109}, \frac{18}{89}, \frac{9}{53}\right\}$

Ans. [1]

Sol. $f(x) = \frac{k}{1+x^2}$ is a decreasing function

where $k > 0$

$$\therefore x \in [2, 3) \Rightarrow f(x) = \frac{2}{1+x^2} \in \left(\frac{2}{10}, \frac{2}{5}\right] = R_1$$

$$x \in [3, 4) \Rightarrow f(x) = \frac{3}{1+x^2} \in \left(\frac{3}{17}, \frac{3}{10}\right] = R_2$$

$$x \in [4, 5) \Rightarrow f(x) = \frac{4}{1+x^2} \in \left(\frac{4}{26}, \frac{4}{17}\right] = R_3$$

$$x \in [5, 6) \Rightarrow f(x) = \frac{5}{1+x^2} \in \left(\frac{5}{37}, \frac{5}{26}\right] = R_4$$

$$\text{Range} = R_1 \cup R_2 \cup R_3 \cup R_4$$

$$= \left(\frac{5}{37}, \frac{2}{5}\right]$$

- Q.71** The number of real roots of the equation $\sqrt{x^2 - 4x + 3} + \sqrt{x^2 - 9} = \sqrt{4x^2 - 14x + 6}$, is :
 (1) 1 (2) 0 (3) 2 (4) 3

Ans. [1]

Sol. Common domain of functions is $(-\infty, -3] \cup [3, \infty)$

$$\sqrt{x^2 - 4x + 3} + \sqrt{x^2 - 9} = \sqrt{4x^2 - 14x + 6}$$

$$\sqrt{x-3} (\sqrt{x-1} + \sqrt{x+3}) = \sqrt{x-3} \sqrt{4x-2}$$

$$\sqrt{x-3} = 0 \Rightarrow x = 3$$

$$\text{Or } \sqrt{x-1} + \sqrt{x+3} = \sqrt{4x-2}$$

On squaring,

$$x - 1 + x + 3 + 2\sqrt{(x-1)(x+3)} = 4x - 2$$

$$2\sqrt{x^2 + 2x - 3} = 2x - 4$$

$$4(x^2 + 2x - 3) = 4x^2 - 16x + 16$$

$$x = \frac{7}{6} \notin (-\infty, -3] \cup [3, \infty)$$

\therefore Only 1 solution

- Q.72** The value of $\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{(2 + 3\sin x)}{\sin x(1 + \cos x)} dx$ is equal to

(1) $\frac{10}{3} - \sqrt{3} - \log_e \sqrt{3}$

(2) $-2 + 3\sqrt{3} + \log_e \sqrt{3}$

(3) $\frac{7}{2} - \sqrt{3} - \log_e \sqrt{3}$

(4) $\frac{10}{3} - \sqrt{3} + \log_e \sqrt{3}$

Ans. [4]

Sol.

$$\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{2\sin x}{\sin^2 x(1 + \cos x)} dx + \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{3}{1 + \cos x} dx$$

I_1 I_2

$$\cos x = t$$

$$\int_{\frac{1}{2}}^0 \frac{-2dt}{(1-t^2)(1+t)} + \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{3}{2} \sec^2 \frac{x}{2} dx$$

$$2 \int_0^{\frac{1}{2}} \frac{dt}{(1-t^2)(1+t)} + 3 \tan \frac{x}{2} \Big|_{\frac{\pi}{3}}^{\frac{\pi}{2}}$$

$$= \ln \sqrt{3} - \sqrt{3} + \frac{10}{3}$$

Q.73 For the system of linear equations

$$x + y + z = 6$$

$$\alpha x + \beta y + 7z = 3$$

$$x + 2y + 3z = 14,$$

which of the following is **NOT** true?

(1) If $\alpha = \beta$ and $\alpha \neq 7$, then the system has a unique solution

(2) If $\alpha = \beta = 7$, then the system has no solution

(3) There is a unique point (α, β) on the line $x + 2y + 18 = 0$ for which the system has infinitely many solutions

(4) For every point $(\alpha, \beta) \neq (7, 7)$ on the line $x - 2y + 7 = 0$, the system has infinitely many solutions

Ans. [4]

Sol.
$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ \alpha & \beta & 7 \\ 1 & 2 & 3 \end{vmatrix}$$

$$= 1(3\beta - 14) - 1(3\alpha - 7) + 1(2\alpha - \beta)$$

$$= 3\beta - 14 + 7 - 3\alpha + 2\alpha - \beta$$

$$= 2\beta - \alpha - 7$$

So, for $\alpha = \beta \neq 7$, $\Delta \neq 0$ so unique solution

$\alpha = \beta = 7$, equation (i) & (ii) represent 2 parallel planes so no solution.

If $\alpha - 2\beta + 7 = 0$, but $(\alpha, \beta) \neq (7, 7)$, then no solution.

Q.74 Let a differentiable function f satisfy $f(x) + \int_3^x \frac{f(t)}{t} dt = \sqrt{x+1}$, $x \geq 3$. Then $12f(8)$ is equal to :

(1) 1

(2) 34

(3) 17

(4) 19

Ans. [3]

Sol. Differentiating both sides we get

$$f'(x) + \frac{f(x)}{x} = \frac{1}{2\sqrt{x+1}}$$

$$\Rightarrow \frac{dy}{dx} + \frac{y}{x} = \frac{1}{2\sqrt{x+1}}$$

$$\Rightarrow \text{IF} = x$$

$$\Rightarrow yx = \frac{1}{2} \int \frac{x}{\sqrt{x+1}} dx + c$$

$$\Rightarrow yx = \frac{1}{2} \left(\frac{(x+1)^{\frac{3}{2}}}{\frac{3}{2}} - 2(x+1)^{\frac{1}{2}} \right) + c$$

$$xy = \frac{1}{3}(x+1)^{\frac{3}{2}} - (x+1)^{\frac{1}{2}} + c$$

$$f(3) = 2$$

So, $x = 3$ $y = 2$

$$\Rightarrow c = \frac{16}{3}$$

Now, $x = 8$

$$8f(8) = \frac{27}{3} - 3 + \frac{16}{3} = \frac{34}{3}$$

$$12f(8) = \frac{34}{3} \times \frac{12}{8} = 17$$

Option (3) is correct.

Q.75 Let $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 4 & -1 \\ 0 & 12 & -3 \end{pmatrix}$. Then the sum of the diagonal elements of the matrix $(A + I)^{11}$ is equal to :

(1) 2050

(2) 4097

(3) 6144

(4) 4094

Ans. [2]

Sol. $A^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & -1 \\ 0 & 12 & -3 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & -1 \\ 0 & 12 & -3 \end{bmatrix}$
 $= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & -1 \\ 0 & 12 & -3 \end{bmatrix} = A$

Now,

$$(A + I)^{11} = {}^{11}C_0 A^{11} + {}^{11}C_1 A^{10} + \dots + {}^{11}C_{11} I$$
$$= A({}^{11}C_0 + {}^{11}C_1 \dots + {}^{11}C_{10}) + I$$
$$= A(2^{11} - 1) + I$$

Trace of

$$(A+I)^{11} = 2^{11} + 4(2^{11} - 1) + 1 - 3(2^{11} - 1) + 1$$
$$= 2 \times 2^{11} - 4 + 3 + 2$$
$$= 2^{12} + 1$$
$$= 4097$$

Q.76 Let a circle C_1 be obtained on rolling the circle $x^2 + y^2 - 4x - 6y + 11 = 0$ upwards 4 units on the tangent T to it at the point $(3, 2)$. Let C_2 be the image of C_1 in T . Let A and B be the centers of circles C_1 and C_2 respectively, and M and N be respectively the feet of perpendiculars drawn from A and B on the x -axis. Then the area of the trapezium $AMNB$ is:

(1) $2(2 + \sqrt{2})$ (2) $3 + 2\sqrt{2}$ (3) $4(1 + \sqrt{2})$ (4) $2(1 + \sqrt{2})$ **Ans.** [3]

Sol. Given circle is $x^2 + y^2 - 4x - 6y + 11 = 0$, centre $(2, 3)$

Tangent at $(3, 2)$ is $x - y = 1$ After rolling up by 4 units centre of C_1 is

$$A \equiv \left(2 + \frac{4}{\sqrt{2}}, 3 + \frac{4}{\sqrt{2}} \right)$$

$$\Rightarrow A = (2 + 2\sqrt{2}, 3 + 2\sqrt{2})$$

 B is the image of A in $x - y = 1$

$$\frac{x - (2 + 2\sqrt{2})}{1} = \frac{y - (3 + 2\sqrt{2})}{-1} = \frac{-2(-2)}{2} = 2$$

$$\Rightarrow x = 4 + 2\sqrt{2}, y = 1 + 2\sqrt{2}$$

Area of $AMNB$

$$= \frac{1}{2}(4 + 4\sqrt{2})(4 + 2\sqrt{2} - (2 + 2\sqrt{2}))$$

$$= 4(1 + \sqrt{2})$$



Q.77 If $\sin^{-1} \frac{\alpha}{17} + \cos^{-1} \frac{4}{5} - \tan^{-1} \frac{77}{36} = 0$, $0 < \alpha < 13$, then $\sin^{-1}(\sin \alpha) + \cos^{-1}(\cos \alpha)$ is equal to :

- (1) π (2) $16 - 5\pi$ (3) 16 (4) 0

Ans. [1]

Sol. $\sin^{-1} \left(\frac{\alpha}{17} \right) = -\cos^{-1} \left(\frac{4}{5} \right) + \tan^{-1} \left(\frac{77}{36} \right)$

Let $\cos^{-1} \left(\frac{4}{5} \right) = p$ and $\tan^{-1} \left(\frac{77}{36} \right) = q$

$$\Rightarrow \sin \left(\sin^{-1} \frac{\alpha}{17} \right) = \sin(q - p)$$

$$= \sin q \cdot \cos p - \cos q \cdot \sin p$$

$$\Rightarrow \frac{\alpha}{17} = \frac{77}{85} \cdot \frac{4}{5} - \frac{36}{85} \cdot \frac{3}{5}$$

$$\sin^{-1} \sin 8 + \cos^{-1} \cos 8$$

$$\Rightarrow -8 + 3\pi + 8 - 2\pi$$

$$= \pi$$

Q.78 If the sum and product of four positive consecutive terms of a G.P., are 126 and 1296, respectively, then the sum of common ratio of all such GPs is

- (1) 14 (2) 7 (3) 3 (4) $\frac{9}{2}$

Ans. [3]

Sol. Let the terms be $\frac{a}{r^3}, \frac{a}{r}, ar, ar^3$

$$\frac{a}{r^3} \cdot \frac{a}{r} \cdot ar \cdot ar^3 = 1296$$

$$\Rightarrow a = 6$$

Now, $\frac{a}{r^3} + \frac{a}{r} + ar + ar^3 = 126$

$$\Rightarrow \frac{1}{r^3} + \frac{1}{r} + r + r^3 = 21$$

$$\Rightarrow \left(r + \frac{1}{r} \right) \left(\left(r + \frac{1}{r} \right)^2 - 3 \right) + \left(r + \frac{1}{r} \right) = 21$$

Let $r + \frac{1}{r} = t$

$$t^3 - 3t + t = 21$$

$$\Rightarrow t^3 - 2t - 21 = 0$$

$$\Rightarrow (t - 3)(t^2 + 3t + 7) = 0$$

$$\Rightarrow t = 3$$

$$r + \frac{1}{r} = 3$$

$$\Rightarrow r^3 - 3r + 1 = 0$$

$$\Rightarrow r_1 + r_2 = 3$$

Section-B: Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE**. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, -00.33, -00.30, 30.27, -27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer..

Q.81 Let θ be the angle between the planes $P_1 : \vec{r} \cdot (\hat{i} + \hat{j} + 2\hat{k}) = 9$ and $P_2 : \vec{r} \cdot (2\hat{i} - \hat{j} + \hat{k}) = 15$. Let L be the line that meets P_2 at the point $(4, -2, 5)$ and makes an angle θ with the normal of P_2 . If α is the angle between L and P_2 , then $(\tan^2\theta)(\cot^2\alpha)$ is equal to _____.

Ans. [09]

Sol. $P_1 : \vec{r} \cdot (\hat{i} + \hat{j} + 2\hat{k}) = 9$

$$P_2 : \vec{r} \cdot (2\hat{i} - \hat{j} + \hat{k}) = 15$$

$$\text{then } \cos\theta = \frac{3}{\sqrt{6} \cdot \sqrt{6}} = \frac{1}{2}$$

$$\therefore \boxed{\theta = \frac{\pi}{3}} \quad \text{Now, } \alpha = \frac{\pi}{2} - \theta$$

$$\therefore \tan^2\theta \cdot \cot^2\alpha = \tan^4\theta \\ = (\sqrt{3})^4 = 9$$

Q.82 The remainder on dividing 5^{99} by 11 is _____.

Ans. [09]

Sol. $5 \equiv 5 \pmod{11}$
 $5^2 \equiv 3 \pmod{11}$
 $5^4 \equiv -2 \pmod{11}$
 $5^5 \equiv 1 \pmod{11}$
 $5^{99} \equiv -2 \pmod{11}$
 $\therefore \text{remainder} = 9$

Q.83 Number of 4-digit numbers that are less than or equal to 2800 and either divisible by 3 or 11, is equal to _____.

Ans. [710]

Sol. Numbers which are divisible by 3 (4 digit) and less than or equal to 2800

$$= \frac{2799 - 1002}{3} + 1 = 600$$

Numbers which are divisible by 11 (4 digit) and less than or equal to 2800

$$= \frac{2794 - 1001}{11} + 1 = 164$$

Numbers which are divisible by 33 (4 digit) and less than or equal to 2800

$$= \frac{2772 - 1023}{33} + 1 = 54$$

$$\therefore \text{Total no.} = 710$$

Q.84 Let \vec{a} and \vec{b} be two vectors such that $|\vec{a}| = \sqrt{14}$, $|\vec{b}| = \sqrt{6}$ and $|\vec{a} \times \vec{b}| = \sqrt{48}$. Then $(\vec{a} \cdot \vec{b})^2$ is equal to _____.

Ans. [36]

Sol. $|\vec{a}| = \sqrt{14}$, $|\vec{b}| = \sqrt{6}$ and $|\vec{a} \times \vec{b}| = \sqrt{48}$

$$|\vec{a} \times \vec{b}| + (\vec{a} \cdot \vec{b})^2 = |\vec{a}|^2 |\vec{b}|^2$$

$$48 + (\vec{a} \cdot \vec{b})^2 = 6 \times 14$$

$$(\vec{a} \cdot \vec{b})^2 = 84 - 48 = 36$$

Q.85 If the variance of the frequency distribution

x_i	2	3	4	5	6	7	8
Frequency f_i	3	6	16	α	9	5	6

is 3, then α is equal to _____.

Ans. [05.00]

Sol. $3 = \frac{3 \cdot 2^2 + 6 \cdot 3^2 + 16 \cdot 4^2 + \alpha \cdot 5^2 + 9 \cdot 6^2 + 5 \cdot 7^2 + 6 \cdot 8^2}{45 + \alpha} - \left(\frac{225 + 5\alpha}{45 + \alpha} \right)^2$

$$3 = \frac{12 + 54 + 256 + 25\alpha + 324 + 245 + 384}{45 + \alpha} - 25$$

$$28(45 + \alpha) = 1275 + 25\alpha$$

$$\text{OR } 1260 + 28\alpha = 1275 + 25\alpha$$

$$\Rightarrow \alpha = 5$$

Q.86 Let $\alpha > 0$, be the smallest number such that the expansion of $\left(x^{\frac{2}{3}} + \frac{2}{x^3}\right)^{30}$ has a term $\beta x^{-\alpha}$, $\beta \in \mathbb{N}$. Then α is

equal to _____.

Ans. [2.00]

Sol. $\therefore \left(x^{\frac{2}{3}} + \frac{2}{x^3}\right)^{30} = \sum_{r=0}^{30} {}^{30}C_r \left(x^{\frac{2}{3}}\right)^{30-r} \cdot \left(\frac{2}{x^3}\right)^r$

Here $\frac{60 - 2r}{3} - 3r \in \text{integer}$

$\therefore \beta$ is always a natural number.

$$\therefore r = 6$$

$$\text{Thus } \alpha = 2$$

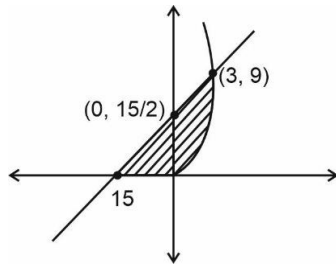
Q.87 Let for $x \in \mathbb{R}$,

$$f(x) = \frac{x + |x|}{2} \text{ and } g(x) = \begin{cases} x, & x < 0 \\ x^2, & x \geq 0 \end{cases}$$

Then area bounded by the curve $y = (f \circ g)(x)$ and the line $y = 0$, $2y - x = 15$ is equal to _____.

Ans. [72]

Sol. $f \circ g(x) = \begin{cases} 0 & x < 0 \\ x^2 & x \geq 0 \end{cases}$



$$\text{Area} = \frac{1}{2} \times 15 \times \frac{15}{2} + \int_0^3 \left(\frac{x+15}{2} - x^2 \right) dx$$

$$\frac{225}{4} + \frac{99}{4} - 9$$

$$\frac{324}{4} - 9$$

$$81 - 9$$

$$= 72$$

Q.88 Let 5 digit numbers be constructed using the digits 0, 2, 3, 4, 7, 9 with repetition allowed, and are arranged in ascending order with serial numbers. Then the serial number of the number 42923 is _____.

Ans. [2997]

Sol. 2 _____ $\rightarrow 6^4 = 1296$

3 _____ $\rightarrow 6^4 = 1296$

4 0 _____ $\rightarrow 6^3 = 216$

$$\left. \begin{array}{l} 4 2 \text{ ---} \\ 4 3 \text{ ---} \\ 5 6 180 \\ 4 4 \text{ ---} \\ 4 7 \text{ ---} \end{array} \right\} \rightarrow 5 \times 6^2 = 180$$

4 2 9 0 $\rightarrow 6$

4 2 9 2 0 $\rightarrow 1$

4 2 9 2 2 $\rightarrow 1$

4 2 9 2 3 $\rightarrow 1$

$$\overline{2997}$$

Q.89 Let a_1, a_2, \dots, a_n be in A.P. If $a_5 = 2a_7$ and $a_{11} = 18$, then

$$12 \left(\frac{1}{\sqrt{a_{10}} + \sqrt{a_{11}}} + \frac{1}{\sqrt{a_{11}} + \sqrt{a_{12}}} + \dots + \frac{1}{\sqrt{a_{17}} + \sqrt{a_{18}}} \right)$$

is equal to _____.

Ans. [08]

Sol. $a_{11} = 18$

$a + 10d = 18 \quad \dots(i)$

$a_5 = 2a_7$

$a + 4d = 2(a + 6d)$

$a = -8d \quad \dots(ii)$

(i) and (ii) $\Rightarrow a = -72, d = 9$.

On rationalising the denominator, given expression

$$\begin{aligned}
 &= 12 \left[\frac{\sqrt{a_{10}} - \sqrt{a_{11}}}{-d} + \frac{\sqrt{a_{11}} - \sqrt{a_{12}}}{-d} + \dots + \frac{\sqrt{a_{17}} - \sqrt{a_{18}}}{-d} \right] \\
 &= 12 \left[\frac{\sqrt{a_{10}} - \sqrt{a_{18}}}{-d} \right] \\
 &= 12 \left[\frac{\sqrt{a_{11} - d} - \sqrt{a_{11} + 7d}}{-d} \right] \\
 &= 12 \left[\frac{\sqrt{18-9} - \sqrt{18+63}}{-9} \right] \\
 &= 12 \times \frac{2}{3} = 8
 \end{aligned}$$

Q.90 Let the line $L: \frac{x-1}{2} = \frac{y+1}{-1} = \frac{z-3}{1}$ intersect the plane $2x + y + 3z = 16$ at the point P. Let the point Q be the foot of perpendicular from the point $R(1, -1, -3)$ on the line L. If α is the area of the triangle PQR then α^2 is equal to _____.

Ans. [180]

Sol. $L: \frac{x-1}{2} = \frac{y+1}{-1} = \frac{z-3}{1} = r$ (say)

Let $P \equiv (2r_1 + 1, -r_1, r_1 + 3)$

P lies on $2x + y + 3z = 16$

$$\therefore 2(2r_1 + 1) + (-r_1 - 1) + 3(r_1 + 3) = 16$$

$$r_1 = 1$$

$P \equiv (3, -2, 4)$

$R \equiv (1, -1, -3)$

Let $Q \equiv (2r_2 + 1, -r_2 - 1, r_2 + 3)$

DRs of QR $\equiv (2r_2 - r_2, r_2 + 6)$

DRs of L $\equiv (2, -1, 1)$

$$QR \perp L \Rightarrow 4r_2 + r_2 + r_2 + 6 = 0$$

$$r_2 = -1$$

$Q \equiv (-1, 0, 2)$

$$\overrightarrow{QP} \times \overrightarrow{RP} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -2 & 2 \\ 2 & -1 & 7 \end{vmatrix} = -12\hat{i} - 24\hat{j} + 0\hat{k}$$

$$\alpha = [PQR] = \frac{1}{2} |\overrightarrow{QP} \times \overrightarrow{RP}| = \frac{1}{2} \times 12\sqrt{5}$$

$$= 6\sqrt{5}$$

$$\alpha^2 = 180$$