CAREER POINT

JEE Main Online Exam 2023

Questions & Solution

25th January 2023 | Evening

PHYSICS

Section-A: This section contains 20 multiple choice questions. Each question has 4 choices(1), (2), (3) and (4), out of which **ONLY ONE** is correct..

Q.1 Every planet revolves around the sun in an elliptical orbit:

A. The force acting on a planet is inversely proportional to square of distance from sun.
B. Force acting on planet is inversely proportional to product of the masses of the planet and the sun.
C. The Centripetal force acting on the planet is directed away from the sun.
D. The square of time period of revolution of planet around sun is directly proportional to cube of semi-major axis of elliptical orbit.

Choose the correct answer from the options given below:

(1) C and D only (2) B and C only (3) A and C only (4) A and D only

Ans. **[4]**

Sol. A is correct because $F = \frac{Gm_1m_2}{r^2}$

D is correct because $T^2 \propto r^3$

$\therefore$ A and D are correct

Q.2 Match List I with List II

	List-I		List-II
(A)	Isothermal Process	(I)	Work done by the gas decreases internal energy
(B)	Adiabatic Process	(II)	No change in internal energy
(C)	Isochoric Process	(III)	The heat absorbed goes partly to increase internal energy and partly to do work
(D)	Isobaric Process	(IV)	No work is done on or by the gas

Choose the correct answer from the options given below:

(1) A-I, B-II, C-III, D-IV (2) A-I, B-II, C-IV, D-III

(3) A-II, B-I, C-IV, D-III (4) A-II, B-I, C-III, D-IV

Ans. **[3]**

Sol. A. Isothermal process $\rightarrow$ II ($\Delta U = 0$)

B. Adiabatic process $\rightarrow$ I ($\Delta Q = 0$)

C. Isochoric process $\rightarrow$ IV ($w = 0$)

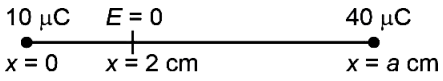
D. Isobaric process $\rightarrow$ III ($\Delta Q = \Delta U + w$)

Q.3 A point charge of 10 μC is placed at the origin. At what location on the X-axis should a point charge of 40 μC be placed so that the net electric field is zero at x = 2 cm on the X-axis?

(1) x = 8 cm (2) x = 6 cm (3) x = – 4 cm (4) x = 4 cm

Ans. **[2]**

Sol.



$$\therefore \quad E_{x=2cm} = 0$$

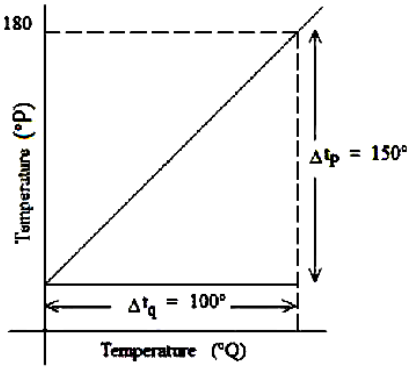
$$\frac{1}{4\pi\varepsilon_0(2cm)^2} = \frac{1}{4\pi\varepsilon_0}\frac{(40\mu C)}{[(a-2)cm]^2}$$

$$\Rightarrow \left(\frac{a-2}{2}\right)^2 = 4$$

$$\Rightarrow \frac{a-2}{2} = 2$$

$$\boxed{a = 6\,cm}$$

Q.4 The graph between two temperature scales P and Q is shown in the figure. Between upper fixed point and lower fixed point there are 150 equal divisions of scale P and 100 divisions on scale Q. The relationship for conversion between the two scales is given by:



(1) $\frac{t_P}{180} = \frac{t_Q - 40}{100}$ (2) $\frac{t_Q}{100} = \frac{t_P - 30}{150}$ (3) $\frac{t_P}{100} = \frac{t_Q - 180}{150}$ (4) $\frac{t_Q}{150} = \frac{t_P - 180}{150}$

Ans. **[2]**

Sol. From graph, we can say that.

$$\frac{t_P - 30}{150} = \frac{t_Q - 0}{100}$$

Q.5 Given below are two statements:

Statement I: Stopping potential in photoelectric effect does not depend on the power of the light source.

Statement II: For a given metal, the maximum kinetic energy of the photoelectron depends on the wavelength of the incident light.

In the light of above statements, choose the most appropriate answer from the options given below.

(1) Both Statement I and statement II are correct

(2) Statement I is correct but statement II is incorrect

(3) Both Statement I and Statement II are incorrect

(4) Statement I is incorrect but statement II is correct

Ans. **[1]**

Sol. Statement I is correct as stopping potential is independent of power of light used.

Statement II is correct as maximum kinetic energy of photoelectron depends on wavelength of light.

Q.6 The light rays from an object have been reflected towards an observer from a standard flat mirror, the image observed by the observer are: -
A. Real
B. Erect
C. Smaller in size than object
D. Laterally inverted
Choose the most appropriate answer from the options given below:
(1) B and D Only (2) A and D Only (3) A, C and D Only (4) B and C Only

Ans. **[1]**

Sol. The image will be erect and laterally inverted.

Q.7 Two objects are projected with same velocity 'u' however at different angles α and β with the horizontal. If $\alpha + \beta = 90°$, the ratio of horizontal range of the first object to the 2nd object will be :
(1) 1 : 2 (2) 4 : 1 (3) 2 : 1 (4) 1 : 1

Ans. **[4]**

Sol. Both the objects will have the same range because, $\alpha + \beta = 90°$. i.e., α, β are complementary angles.

Q.8 Match List I with List II

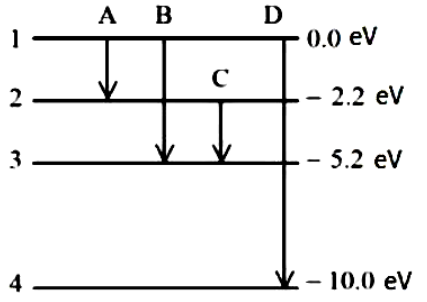
	List-I		List-II
(A)	Troposphere	(I)	Approximate 65-75 km over Earth's surface
(B)	E-Part of Stratosphere	(II)	Approximate 300 km over Earth's surface
(C)	F2-Part of Thermosphere	(III)	Approximate 10 km over Earth's surface
(D)	D-Part of Stratosphere	(IV)	Approximate 100 km over Earth's surface

Choose the correct answer from the options given below:
(1) A-III, B-II, C-I, D-IV (2) A-I, B-II, C-IV, D-III
(3) A-I, B-IV, C-III, D-Il (4) A-III, B-IV, C-II, D-I

Ans. **[4]**

Sol. $\rightarrow$ 10 km over Earth's surface – Troposphere
$\rightarrow$ 100 km over Earth's surface – E-part of stratosphere
$\rightarrow$ 300 km over Earth's surface – F_2-part of thermosphere
$\rightarrow$ 65-75 km over Earth's surface – D-part of stratosphere

Q.9 The energy levels of an atom is shown in figure



Which one of these transitions will result in the emission of a photon of wavelength 124.1 nm?
Given ($h = 6.62 \times 10^{-34}$ Js)
(1) D (2) B (3) C (4) A

Ans. **[1]**

Sol. $\because\ \lambda = \dfrac{hc}{\Delta E}$

$\lambda = \dfrac{1240}{10}\text{nm}$

≈ 124 nm

$\therefore$ D is the transition required

Q.10 For a moving coil galvanometer, the deflection in the coil is 0.05 rad when a current of 10 mA is passed through it. If the torsional constant of suspension wire is 4.0×10^{-5} N m rad^{-1}, the magnetic field is 0.01 T and the number of turns in the coil is 200, the area of each turn (in cm^2) is :

(1) 1.0 (2) 0.5 (3) 2.0 (4) 1.5

Ans. **[1]**

Sol. $\because\ \theta = \left(\dfrac{NBA}{K}\right)I$

$A = \dfrac{\theta K}{NBI}$

$= \dfrac{0.05 \times 4 \times 10^{-5}}{(200) \times (0.01) \times (10 \times 10^{-3})} = 1\ \text{cm}^2$

Q.11 A particle executes simple harmonic motion between x = – A and x = + A. If time taken by particle to go from x = 0 to $\dfrac{A}{2}$ is 2 s; then time taken by particle in going from $x = \dfrac{A}{2}$ to A is

(1) 3s (2) 2s (3) 1.5 s (4) 4s

Ans. **[4]**

Sol. $x = A\sin(\omega t)$

$x = \dfrac{A}{2} = A\sin(\omega t)$

$\dfrac{1}{2}\sin(\omega t)$

$t = \left(\dfrac{\pi}{6\omega}\right) = 2$

$\dfrac{\pi}{\omega} = 12\,\text{sec}$

$x = A = A\sin(\omega t)$

$\omega t = \left(\dfrac{\pi}{2}\right)$

$t = \left(\dfrac{\pi}{2\omega}\right) = 6$ second

time = 6 – 2 = 4 seconds

Q.12 A wire of length 1 m moving with velocity 8 m/s at right angles to a magnetic field of 2 T. The magnitude of induced emf, between the ends of wire will be_____

(1) 20V (2) 16V (3) 8V (4) 12V

Ans. **[2]**

Sol. $\varepsilon = BV\ell$

$= 2 \times 8 \times 1$

$= 16$ V

Q.13 The distance travelled by a particle is related to time t as $x = 4t^2$. The velocity of the particle at t = 5 s is:

(1) 8 ms^{-1} (2) 20 ms^{-1} (3) 40 ms^{-1} (4) 25 ms^{-1}

Ans. **[3]**

Sol. $x = 4t^2$

$$\frac{dx}{dt} = 8t$$

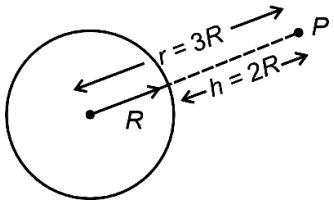
v at t = 5 s = 40 m/s

Q.14 A body of mass is taken from earth surface to the height h equal to twice the radius of earth (R_e), the increase in potential energy will be: (g = acceleration due to gravity on the surface of Earth)

(1) $\frac{1}{3}mgR_e$ (2) $3mgR_e$ (3) $\frac{2}{3}mgR_e$ (4) $\frac{1}{2}mgR_e$

Ans. **[3]**

Sol.



$$V_{surface} = \left(\frac{GMm}{R_e}\right)$$

$$V_P = -\frac{GMm}{3R_e}$$

$$\Delta V = -\frac{GMm}{R_e}\left(1 - \frac{1}{3}\right)$$

$$= \frac{2}{3}\frac{GMm}{(R_e^2)} \times R_e$$

$$\Delta V = \frac{2}{3}mgR_e$$

Q.15 Match List I with List II\

	List-I		List-II
(A)	Young's Modulus (Y)	(I)	$[ML^{-1}T^{-1}]$
(B)	Co-efficient of Viscosity (η)	(II)	$[ML^2T^{-1}]$
(C)	Planck's Constant (h)	(III)	$[ML^{-1}T^{-2}]$
(D)	Work Function (ϕ)	(IV)	$[ML^2T^{-2}]$

Choose the correct answer from the options given below:

(1) A-I, B-II, C-III, D-IV (2) A-III, B-I, C-II, D-IV

(3) A-II, B-III, C-IV, D-I (4) A-I, B-III, C-IV, D-II

Ans. **[2]**

Sol. (A) $[Y]=\left[\frac{MLT^{-2}}{L^2}\right]=[ML^{-1}T^{-2}]$ .... (III)

$$\frac{F}{A}=\eta\left(\frac{dV}{dY}\right)$$

(B) $[\eta]=\left[\frac{MLT^{-2}}{L^2\times LT^{-1}}\right]=[ML^{-1}T^{-1}]$ .... (I)

$h\nu = E = [ML^2T^{-2}]$

(C) $[h] = [ML^2T^{-1}]$ … (II)

(D) $\phi = [ML^2T^{-2}]$ … (IV)

Q.16 Match List I with List II\

	List-I		List-II
(A)	Gauss's Law in Electrostatics	(I)	$\oint\vec{E}\cdot\vec{d\ell}=-\frac{d\phi_B}{dt}$
(B)	Faraday's Law	(II)	$\oint\vec{B}\cdot\vec{dA}=0$
(C)	Gauss's Law in Magnetism	(III)	$\oint\vec{B}\cdot\vec{d\ell}=\mu_0 i_c+\mu_0\in_0\frac{d\phi_E}{dt}$
(D)	Ampere-Maxwell Law	(IV)	$\oint\vec{E}\cdot\vec{ds}=\frac{q}{\in_0}$

Choose the correct answer from the options given below.

(1) A-II, B-III, C-IV, D-I (2) A-III, B-IV, C-I, D-II

(3) A-IV, B-I, C-II, D-III (4) A-I, B-II, C-III, D-IV

Ans. **[3]**

Sol. Gauss's law $\oint\vec{E}\cdot\vec{ds}=\frac{q}{\in_0}$ $(A\rightarrow IV)$

Faraday's law $\oint\vec{E}\cdot\vec{d\ell}=-\frac{d\phi_B}{dt}$ $(B\rightarrow I)$

Gauss's law in $\oint\vec{B}\cdot\vec{dA}=0$ $(C\rightarrow II)$

Ampere's-Maxwell $\oint\vec{B}\cdot\vec{d\ell}=\mu_0 i_c+\mu_0\in_0\frac{d\phi_E}{dt}$ $(D\rightarrow III)$

Q.17 According to law of equipartition of energy the molar specific heat of a diatomic gas at constant volume where the molecule has one additional vibrational mode is

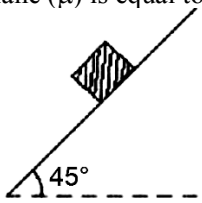
(1) $\frac{3}{2}R$ (2) $\frac{7}{2}R$ (3) $\frac{9}{2}R$ (4) $\frac{5}{2}R$

Ans. **[2]**

Sol. According to the equipartition of energy degree of freedom of diatomic gas is f = 7, (2 degree of freedom is added for every vibrational mode) So,

$$C_V=\frac{i}{2}R=\frac{7R}{2}$$

Q.18 Consider a block kept on an inclined plane (inclined at 45°) as shown in the figure. If the force required to just push it up the incline is 2 times the force required to just prevent it from sliding down, the coefficient of friction between the block and inclined plane (μ) is equal to



(1) 0.60 (2) 0.25 (3) 0.50 (4) 0.33

Ans. **[4]**

Sol. Force required to push

$F_1 = mg\sin\theta + \mu mg\cos\theta = \frac{mg}{\sqrt{2}}(1+\mu)$

Force required to prevent from sliding

$F_2 = (mg\sin\theta - \mu mg\cos\theta) = \frac{mg}{\sqrt{2}}(1-\mu)$

Given $F_1 = 2F_2$

$1 + \mu = 2(1 - \mu)$

$\mu = \frac{1}{3} = 0.33$

Q.19 **Statement I:** When a Si sample is doped with Boron, it becomes P type and when doped by Arsenic it becomes N-type semiconductor such that P-type has excess holes and N-type has excess electrons.
Statement II: When such P-type and N-type semiconductors, are fused to make a junction, a current will automatically flow which can be detected with an externally connected ammeter.
In the light of above statements, choose the most appropriate answer from the options given below.
(1) Both statement I and statement II are correct
(2) Statement I is correct but statement II is incorrect
(3) Both statement I and statement II are incorrect
(4) Statement I is incorrect but statement II is correct

Ans. **[2]**

Sol. Statement I is correct but in statement II we cannot detect the current through ammeter thus the statement II is incorrect.

Q.20 The resistance of a wire is 5 Ω. It's new resistance in ohm if stretched to 5 times of it's original length will be
(1) 25 (2) 5 (3) 125 (4) 625

Ans. **[3]**

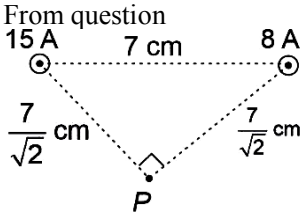
Sol. $R' = n^2R$
$= 52 \times 5\ \Omega$
$= 125\ \Omega$

Section-B: Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer..

Q.21 Two long parallel wires carrying currents 8 A and 15 A in opposite directions are placed at a distance of 7 cm from each other. A point P is at equidistant from both the wires such that the lines joining the point P to the wires are perpendicular to each other. The magnitude of magnetic field at P is _____ $\times 10^{-6}$ T.
(Given : $\sqrt{2} = 1.4$)

Ans. **[68]**

Sol. From question



$$\vec{B}_{15A} = \frac{\mu_0 \times 15}{2\pi\left(\frac{7}{\sqrt{2}}\text{cm}\right)}, \quad \vec{B}_{BA} \frac{\mu_0 \times 8}{2\pi\left(\frac{7}{\sqrt{2}}\text{cm}\right)}$$

$\vec{B}_{15A}$ & $\vec{B}_{8A}$ are perpendicular to each other

Hence $\vec{B}_P = \vec{B}_{15A} + \vec{B}_{8A}$

$$= \sqrt{(B_{15A})^2 + (B_{8A})^2}$$

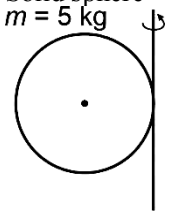
$$= \frac{\mu_0}{2\pi\left(\frac{7}{\sqrt{2}}\right)\text{cm}}\sqrt{18^2 + 8^2} = \frac{\mu_0 17}{2\pi \times \left(\frac{7}{\sqrt{2}}\right) \times 10^{-2}}$$

$$= \frac{4\pi \times 17 \times 10^{-7}}{2\pi \times \frac{7}{\sqrt{2}} \times 10^{-2}} = 68 \times 10^{-6}$$

Q.22 If a solid sphere of mass 5 kg and a disc of mass 4 kg have the same radius. Then the ratio of moment of inertia of the disc about a tangent in its plane to the moment of inertia of the sphere about its tangent will be $\frac{x}{7}$. Then the value of x is _____

Ans. **[5]**

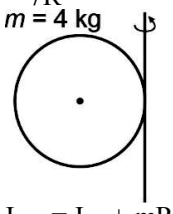
Sol. Solid sphere



$$I_{tangent} = I_{cm} + mR^2$$

$$= \frac{2}{5}mR^2 + mR^2 = \frac{7}{5}mR^2$$

$$= 7R^2 \qquad (m = 5 \text{ kg})$$

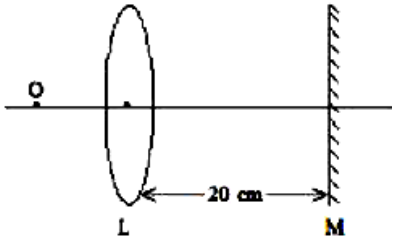


$$I_{disc} = I_{cm} + mR^2$$

$$= \frac{mR^2}{4} + mR^2 = \frac{5}{4}mR^2 = 5R^2$$

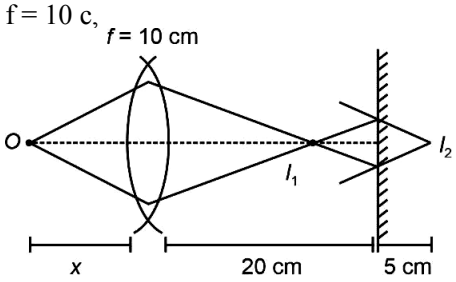
$$\frac{I_{disc}}{I_{tangent}} = \frac{5}{7}$$

Q.23 An object is placed on the principal axis of convex lens of focal length 10 cm as shown. A plane mirror is placed on the other side of lens at a distance of 20 cm. The image produced by the plane mirror is 5 cm inside the mirror. The distance of the object from the lens is _____ cm



Ans. **[30]**

Sol. f = 10 c,



From diagram

I_1 is image formed by lens and I_2 is image formed by mirror.

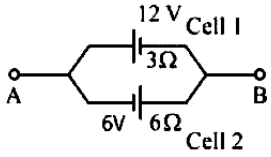
Location of I_1 and I_2 from mirror will be equal = 5 cm

Hence I_1 = 15 cm from lens

From $\frac{1}{v}-\frac{1}{u}=\frac{1}{10}$; u = –x, v = 15

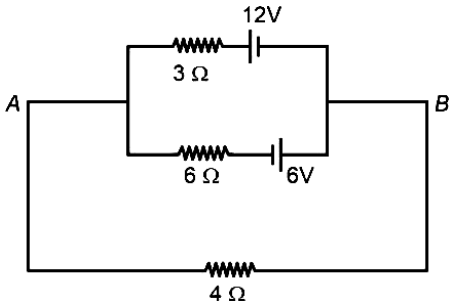
$\frac{1}{x}=\frac{1}{10}-\frac{1}{15} \Rightarrow x = 30$ cm

Q.24 Two cells are connected between points A and B as shown. Cell 1 has emf of 12 V and internal resistance of 3 Ω. Cell 2 has emf of 6 V and internal resistance of 6 Ω. An external resistor R of 4 Ω is connected across A and B. The current flowing through R will be _____A.



Ans. **[1]**

Sol.



KCL at A gives $\frac{6-V_A}{4}+\frac{0-V_A}{6}+\frac{18-V_A}{3}=0$

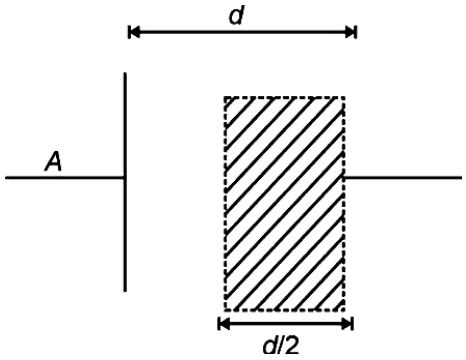
$V_A = 10$

So, current through $4\Omega = \frac{10-6}{4} = 1A$

Q.25 A capacitor has capacitance 5 μF when it's parallel plates are separated by air medium of thickness d. A slab of material of dielectric constant 1.5 having area equal to that of plates but thickness $\frac{d}{2}$ is inserted between the plates. Capacitance of the capacitor in the presence of slab will be _____μF

Ans. **[6]**

Sol.



$C = \frac{\varepsilon_0 A}{d} K$

When completely air filled

$C = 5\mu F = \frac{\varepsilon_0 A}{d}$ ... (1)

When half filled with K = 1.5

$$\frac{1}{C_{eq}} = \frac{\frac{d}{2}}{\varepsilon_0 A} + \frac{\frac{d}{2}}{\varepsilon_0 AK}$$

$$C_{eq} = \left(\frac{2K}{K+1}\right)\frac{\varepsilon_0 A}{d}$$ ... (2)

From (1) & (2)

$$C_{eq} = \left(\frac{2\times 1.5}{1.5+1}\right)5\mu F = 6\mu F$$

Q.26 A train blowing a whistle of frequency 320 Hz approaches an observer standing on the platform at a speed of 66 m/s. The frequency observed by the observer will be (given speed of sound = 330 ms^{-1}) Hz.

Ans. **[400]**

Sol. $f = f_0\left(\frac{v}{v - v_s}\right)$

$$f = 320\left(\frac{330}{330-66}\right)$$

$$= 320\times\frac{330}{264}$$

= 400 Hz

Q.27 A nucleus disintegrates into two smaller parts, which have their velocities in the ratio 3:2. The ratio of their nuclear sizes will be $\left(\frac{X}{3}\right)^{\frac{1}{3}}$ The value of 'x' is

Ans. **[02]**

Sol. $m_1 v_1 = m_2 v_2$

$$\Rightarrow \left(\frac{m_1}{m_2}\right) = \frac{v_2}{v_1} = \left(\frac{2}{3}\right)$$

$m \propto A$

$$\frac{A_1}{A_2} = \left(\frac{2}{3}\right)$$

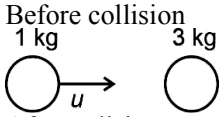
$$\frac{R_1}{R_2} = \left(\frac{A_1}{A_2}\right)^{\frac{1}{3}} = \left(\frac{2}{3}\right)^{\frac{1}{3}}$$

$x = 2$

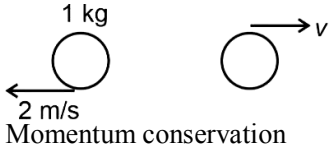
Q.28 A body of mass 1 kg collides head on elastically with a stationary body of mass 3 kg. After collision, the smaller body reverses its direction of motion and moves with a speed of 2 m/s. The initial speed of the smaller body before collision is _____ ms^{-1}.

Ans. **[04]**

Sol. Before collision



After colision



Momentum conservation

$u + 0 = 3v - 2$

$\boxed{3v - u = 2}$ …(1)

also

$$\frac{v+2}{u} = 1 \Rightarrow v + 2u$$

$u - v = 2$ .... (2)

adding (1) and (2)

$2v = 4$

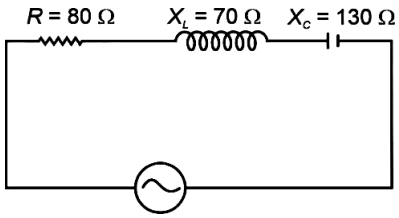
$v = 2$ m/s

$u = 4$ m/s

Q.29 A series LCR circuit is connected to an AC source of 220 V, 50 Hz. The circuit contains a resistance R = 80 Ω, an inductor of inductive reactance X_L = 70 Ω, and a capacitor of capacitive reactance X_C = 130Ω. The power factor of circuit is $\frac{x}{10}$. The value of x is:

Ans. **[08]**

Sol.



Power factor = $\cos\phi = \frac{R}{Z} = \frac{80}{\sqrt{80^2 + 60^2}}$

$$\frac{8}{10} = \frac{x}{10} \Rightarrow \boxed{X = 8}$$

Q.30 A spherical drop of liquid splits into 1000 identical spherical drops. If U_i is the surface energy of the original drop and U_f is the total surface energy of the resulting drops, the (ignoring evaporation), $\frac{U_f}{U_i}=\left(\frac{10}{x}\right)$ Then value of x is _____ :

Ans. **[01]**

Sol. $\frac{U_f}{U_i}=\frac{\text{Area of final drop}}{\text{Area of Initial drop}}$

$\frac{U_f}{U_i}=\frac{1000\times 4\pi r_f^2}{4\pi r_i^2}=\frac{1000(r_f^2)}{(f_i^2)}$

$1000\times\frac{4}{3}\pi r_f^3=\frac{4}{3}\pi r_i^3$

$r_i = 10r_f$

$\frac{U_f}{U_i}=\frac{1000r_f^2}{100r_f^2}=10$

$\frac{10}{x}=10 \Rightarrow x = 1$

CHEMISTRY

Section-A: Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Q.31 Which of the following represents the correct order of metallic character of the given elements?

(1) Be < Si < K < Mg (2) K < Mg < Be < Si (3) Be < Si < Mg < K (4) Si < Be < Mg < K

Ans. **[4]**

Sol. Metallic character of an element is directly proportional to its electropositivity. Of the given elements silicon is least electro positive and potassium is most electropositive whereas beryllium and magnesium have intermediate values in the increasing order. Therefore, correct order of metallic character is Si < Be < Mg < K.

Q.32 Match List-I with List-II.

	List-I (Amines)		**List-II (pK_b)**
A.	Aniline	I.	3.25
B.	Ethanamine	II.	3.00
C.	N-Ethylethanamine	III.	9.38
D.	N, N-Diethylethanamine	IV.	3.29

Choose the correct answer from the options given below

(1) A-III, B-II, C-I, D-IV (2) A-III, B-II, C-IV, D-I

(3) A-I, B-IV, C-II, D-III (4) A-III, B-IV, C-II, D-I

Ans. **[4]**

Sol. Aromatic amines are less basic than aliphatic amines. Among given aliphatic amines, 2° amine is most basic, followed by 3° amine and 1° amine. Therefore the correct basic strength (Kb) order of the given amines is

$CH_3CH_2NHCH_2CH_3 > (CH_3CH_2)_3N > \underset{(B)}{CH_3CH_2NH_2} > \underset{(A)}{C_6H_5NH_2}$

The pK_b order of the given amines will be just the opposite of their basic strength order. The correct matching is

A – III, B – IV, C – II, D – I

Q.33 Given below are two statements, one is labelled as **Assertion A** and the other is labelled as **Reason R**.
Assertion A : Carbon forms two important oxides – CO and CO_2. CO is neutral whereas CO_2 is acidic in nature.
Reason R : CO_2 can combine with water in a limited way to form carbonic acid, while CO is sparingly soluble in water.
In the light of the above statements, choose the most appropriate answer from the options given below.
(1) Both A and R are correct and R is the correct explanation of A
(2) A is not correct but R is correct
(3) A is correct but R is not correct
(4) Both A and R are correct but R is NOT the correct explanation of A

Ans. **[1]**

Sol. Carbon monoxide is neutral and CO_2 is acidic in nature because with the increase in oxidation state of carbon acidic strength increases. So, Assertion is correct.
CO_2 combines with water to form carbonic acid while CO is sparingly soluble in water. So, Reason is also correct and is the correct explanation of Assertion.

Q.34 **Statement I :** Dipole moment is a vector quantity and by convention it is depicted by a small arrow with tail on the negative centre and head pointing towards the positive centre.
Statement II : The crossed arrow of the dipole moment symbolized the direction of the shift of charges in the molecules.
In the light of the above statements, choose the most appropriate answer from the options given below.
(1) Statement I is incorrect but Statement II is correct
(2) Both Statement I and Statement II are incorrect
(3) Both Statement I and Statement II are correct
(4) Statement I is correct but Statement II is incorrect

Ans. **[1]**

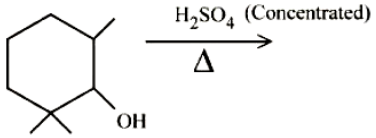
Sol. The dipole moment is a vector quantity and is depicted by an arrow with tail on the positive centre and head pointing towards the negative centre. So, Statement-I is incorrect. The crossed arrow of the dipole moment symbolizes the direction of the shift of charges in the molecules. So, Statement-II is correct.

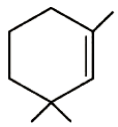
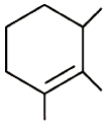
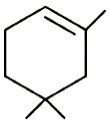
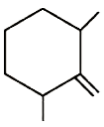
Q.35 Which one among the following metals is the weakest reducing agent?
(1) Li (2) Rb (3) Na (4) K

Ans. **[3]**

Sol. Among the given alkali metals, sodium metal is the weakest reducing agent as its standard reduction potential $(E^{\circ}_{Na^+/Na} = -2.719)$ is least negative

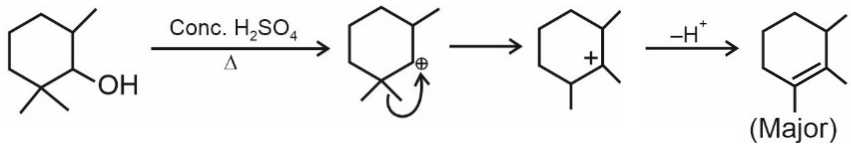
Q.36 Find out the major product from the following reaction.



(1) 
(2) 
(3) 
(4) 

Ans. **[2]**

Sol.



Q.37 A. Ammonium salts produce haze in atmosphere.
B. Ozone gets produced when atmospheric oxygen reacts with chlorine radicals.
C. Polychlorinated biphenyls act as cleansing solvents.
D. 'Blue baby" syndrome occurs due to the presence of excess of sulphate ions in water.
Choose the correct answer from the options given below.
(1) A and C only (2) A, B and C only (3) A and D only (4) B and C only

Ans. **[1]**

Sol. Ammonium salts produce haze in atmosphere. Ozone is produced when atmospheric oxygen reacts with oxygen atoms and not chlorine atoms. Polychlorinated biphenyls have number of applications including their use as cleansing solvents.
'Blue baby' syndrome occurs due to the presence of excess of nitrate ions and not sulphate ions in water.

Q.38 A chloride salt solution acidified with dil.HNO_3 gives a curdy white precipitate, [A], on addition of $AgNO_3$. [A] on treatment with NH_4OH gives a clear solution, B. A and B are respectively
(1) AgCl & $(NH_4)[Ag(OH)_2]$ (2) $H[AgCl_3]$ & $[Ag(NH_3)_2]Cl$
(3) AgCl & $[Ag(NH_3)_2]Cl$ (4) $H[AgCl_3]$ & $(NH_4)[Ag(OH)_2]$

Ans. **[3]**

Sol. $Cl^-(aq) + AgNO_3(aq) \xrightarrow{\text{dil } HNO_3} \underset{(A)}{AgCl}\downarrow$

$AgCl\downarrow + 2NH_4OH(aq) \rightarrow \underset{(B)}{[Ag(NH_3)_2]}Cl(aq) + 2H_2O$

$\therefore$ (A) is AgCl and (B) is $[Ag(NH_3)_2]Cl$

Q.39 Match List-I with List-II.

	List-I (Name of Polymer)		**List-II (Uses)**
A.	Glyptal	I.	Flexible pipes
B.	Neoprene	II.	Synthetic wool
C.	Acrilan	III.	Paints and Lacquers
D.	LDP	IV.	Gaskets

Choose the correct answer from the options given below
(1) A-III, B-I, C-IV, D-II (2) A-III, B-IV, C-II, D-I
(3) A-III, B-IV, C-I, D-II (4) A-III, B-II, C-IV, D-I

Ans. **[2]**

Sol. (A) Glyptal — (III) Paints and Lacquers
(B) Neoprene — (IV) Gaskets
(C) Acrilan — (II) Synthetic wool
(D) LDP — (I) Flexible pipes

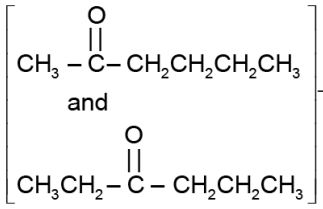
Q.40 Match List-I with List-II.

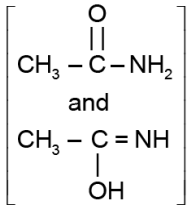
	List-I (Isomeric pairs)		**List-II (Type of isomers)**
A.	Propanamine and N-Methylethanamine	I.	Metamers
B.	Hexan-2-one and Hexan-3-one	II.	Positional isomers
C.	Ethanamide and Hydroxyethanimine	III.	Functional isomers
D.	o-nitrophenol and p-nitrophenol	IV.	Tautomers

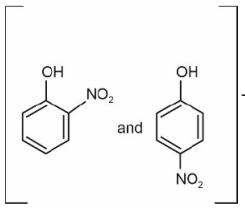
Choose the correct answer from the options given below
(1) A-III, B-I, C-IV, D-II (2) A-IV, B-III, C-I, D-II
(3) A-II, B-III, C-I, D-IV (4) A-III, B-IV, C-I, D-II

Ans. **[1]**

Sol. (A) $\left[\begin{array}{c} CH_3CH_2CH_2NH_2 \\ \text{and} \\ CH_3CH_2NHCH_3 \end{array}\right]$ — (III) Functional isomers

(B)  — (I) Metamers

(C)  — (IV) Tautomers

(D)  — (II) Positional isomers

Q.41 Potassium dichromate acts as a strong oxidizing agent in acidic solution. During this process, the oxidation state changes from

(1) +2 to +1 (2) +6 to +2 (3) +3 to +1 (4) +6 to +3

Ans. **[4]**

Sol. $K_2Cr_2O_7$ acts as a strong oxidising agent in acidic medium. During this process, oxidation state of Cr changes from +6 to +3.

$$Cr_2O_7^{2-} + 14H^+ + 6e^- \rightarrow 2Cr^{3+} + 7H_2O$$

Q.42 Given below are two statements, one is labelled as **Assertion A** and the other is labelled as **Reason R**

Assertion A : Butylated hydroxy anisole when added to butter increases its shelf life.

Reason R : Butylated hydroxy anisole is more reactive towards oxygen than food.

In the light of the above statements, choose the most appropriate answer from the options given below

(1) Both A and R are correct but R is NOT the correct explanation of A

(2) A is not correct but R is correct

(3) Both A and R are correct and R is the correct explanation of A

(4) A is correct but R is not correct

Ans. **[3]**

Sol. Butylated hydroxy anisole is added to butter to increase its shelf life from months to years as it is more reactive towards oxygen than food. Therefore, both Assertion and Reason are correct and Reason is the correct explanation of Assertion.

Q.43 When the hydrogen ion concentration $[H^+]$ changes by a factor of 1000, the value of pH of the solution _____

(1) increases by 1000 units (2) decreases by 2 units
(3) increases by 2 units (4) decreases by 3 units

Ans. **[4]**

Sol. Let the initial concentration of H^+ be 1

$\therefore [H^+]_i = 1 \Rightarrow pH = 0$

It changes by 1000 units

$\therefore [H^+]_f = 10^3 \Rightarrow pH = -3$

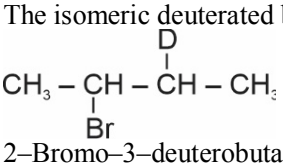
$\therefore$ pH decreases by 3 units

Q.44 The isomeric deuterated bromide with molecular formula C_4H_8DBr having two chiral carbon atoms is

(1) 2–Bromo–2–deuterobutane (2) 2–Bromo–1–deutero–2–methylpropane
(3) 2–Bromo–3–deuterobutane (4) 2–Bromo–1–deuterobutane

Ans. **[3]**

Sol. The isomeric deuterated bromide with molecular formula C_4H_8DBr having two chiral carbon atoms is



2–Bromo–3–deuterobutane

Q.45 Match List-I with List-II.

	List-I (Coordination entity)		**List-II (Wavelength of light absorbed in nm)**
A.	$[CoCl(NH_3)_5]^{2+}$	I.	310
B.	$[Co(NH_3)_6]^{3+}$	II.	475
C.	$[Co(CN)_6]^{3-}$	III.	535
D.	$[Cu(H_2O)_4]^{2+}$	IV.	600

Choose the correct answer from the options given below

(1) A-III, B-I, C-II, D-IV (2) A-IV, B-I, C-III, D-II
(3) A-II, B-III, C-IV, D-I (4) A-III, B-II, C-I, D-IV

Ans. **[4]**

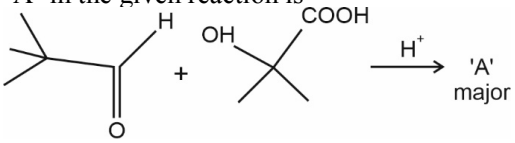
Sol. Co-ordination compounds absorb a particular wavelength following certain rules.

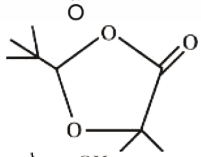
$$\text{Wavelength of light absorbed} \propto \frac{1}{\text{Oxidation state of metal ion}} \propto \frac{1}{\text{Strength of ligand}}$$

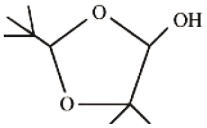
Ligand field strength : $CN^- > NH_3 > H_2O > Cl^-$

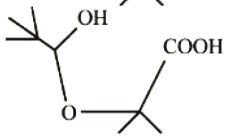
C.	$[Co^{III}(CN)_6]^{3-}$	I.	310
B.	$[Co^{III}(NH_3)_6]^{3+}$	II.	475
A.	$[Co^{III}Cl(NH_3)_5]^{2+}$	III.	535
D.	$[Cu^{II}(H_2O)_4]^{2+}$	IV.	600

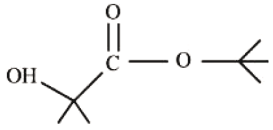
Q.46 'A' in the given reaction is



(1) 

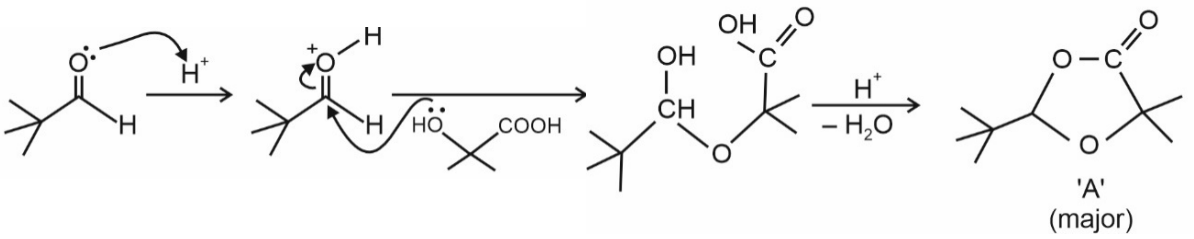
(2) 

(3) 

(4) 

Ans. **[1]**

Sol.



Q.47 Given below are two statements, one is labelled as **Assertion A** and the other is labelled as **Reason R**
Assertion A: The alkali metals and their salts impart characteristic colour to reducing flame.
Reason R: Alkali metals can be detected using flame tests.
In the light of the above statements, choose the most appropriate answer from the options given below.
(1) Both A and R are correct and R is the correct explanation of A
(2) Both A and R are correct but R is NOT the correct explanation of A
(3) A is not correct but R is correct
(4) A is correct but R is not correct

Ans. **[3]**

Sol. Assertion is not correct because alkali metals and their salts impart characteristic colour to oxidising part of flame and not reducing part of flame. Reason is correct because all alkali metals can be detected by their flame tests.

Q.48 Match List-I with List-II.

	List-I		**List-II**
A.	Cobalt catalyst	I.	$(H_2 + Cl_2)$ production
B.	Syngas	II.	Water gas production
C.	Nickel catalyst	III.	Coal gasification
D.	Brine solution	IV.	Methanol production

Choose the correct answer from the options given below
(1) A-IV, B-I, C-II, D-III
(2) A-IV, B-III, C-I, D-II
(3) A-II, B-III, C-IV, D-I
(4) A-IV, B-III, C-II, D-I

Ans. **[4]**

Sol. (A) Hydrogen reacts with carbon monoxide in presence of cobalt catalyst to give methanol

$$2H\,(g) + CO(g) \xrightarrow{\text{cobalt catalyst}} CH_3OH(l)$$

(B) Syn gas is produced from coal and the process is called coal gasification.

$$C(s) + H_2O(g) \xrightarrow{1270\ K} \underset{\text{(syn gas)}}{CO(g) + H_2(g)}$$

(C) Reaction of steam with hydrocarbons or coke at high temperature in presence of nickel catalyst gives a mixture of CO and H_2, called water gas

$$CH_4(g) + H_2O(g) \xrightarrow[Ni]{1270\ K} \underset{\text{(water gas)}}{CO(g) + 3H_2(g)}$$

(D) Electrolysis of brine solution produces H_2 gas at cathode and Cl_2 gas at anode

$$NaCl(aq) \longrightarrow Na^+(aq) + Cl^-(aq)$$

Cathode : $2H_2O(l) + 2e^- \longrightarrow H_2(g) + 2OH^-(aq)$

Anode : $2Cl^-(aq) \longrightarrow Cl_2(g) + 2e^-$

Q.49 Given below are two statements:
Statement I : In froth floatation method a rotating paddle agitates the mixture to drive air out of it.
Statement II : Iron pyrites are generally avoided for extraction of iron due to environmental reasons.
In the light of the above statements, choose the correct answer from the options given below:
(1) Statement I is false but Statement II is true
(2) Statement I is true but Statement II is false
(3) Both Statement I and Statement II are false
(4) Both Statement I and Statement II are true

Ans. **[1]**

Sol. Statement I is false because the rotating paddle in froth floatation method agitates the mixture to generate froth and not to drive air out of it.
Statement II is true because iron is commercially extracted from haematite ore and not from iron pyrites to minimize environmental pollution.

Q.50 What is the mass ratio of ethylene glycol ($C_2H_6O_2$, molar mass = 62 g/mol) required for making 500 g of 0.25 molal aqueous solution and 250 mL of 0.25 molal aqueous solution?
(1) 2 : 1 (2) 1 : 2 (3) 1 : 1 (4) 3 : 1

Ans. **[1]**

Sol. Molality of aq. ethylene glycol solution = 0.25 m

Mass of ethylene glycol required for 1000 g water = $\frac{62}{4} = 15.5$ gm

Mass of solution = 1015.5 gm

Mass of ethylene glycol in 500 gm solution = $\frac{15.5 \times 500}{1015.5} = 7.63$ gm

Assuming density of solution as 1 gm/mL.

Mass of ethylene glycol in 250 mL = $\frac{7.63}{2} = 3.815$ gm

$\therefore$ Mass ratio of ethylene glycol for making 500 gm of 0.25 m solution and 250 mL of 0.25 m solution = 2 : 1

Section-B: Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE**. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

Q.51 A first order reaction has the rate constant, $k = 4.6 \times 10^{-3}\ s^{-1}$. The number of correct statement/s from the following is/are_____
Given: log 3 = 0.48
A. Reaction completes in 1000 s.
B. The reaction has a half-life of 500 s.
C. The time required for 10% completion is 25 times the time required for 90% completion.
D. The degree of dissociation is equal to $(1 - e^{-kt})$
E. The rate and the rate constant have the same unit.

Ans. **[1]**

Sol. $A \underset{1-\alpha}{A} \longrightarrow$ Products

$k = 4.6 \times 10^{-3} s^{-1}$

$kt = \ln \frac{1}{1-\alpha}$

$\alpha = 1 - e^{-kt}$

Reaction completes at infinite time

Half-life = $\frac{0.693}{4.6 \times 10^{-3}} = 150.65$ s

$t_{10\%} = \frac{2.303}{k} \log \frac{100}{90} = \frac{2.303 \times 0.04}{k}$

$t_{90\%} = \frac{2.303}{k} \log \frac{100}{10} = \frac{2.303}{k}$

$t_{10\%} = 0.04 \times t_{90\%}$

Units of rate and rate constant are different

$\therefore$ Number of correct statements = 1

Q.52 The number of incorrect statement/s from the following is/are_____

A. Water vapours are adsorbed by anhydrous calcium chloride.
B. There is a decrease in surface energy during adsorption.
C. As the adsorption proceeds, ΔH becomes more and more negative.
D. Adsorption is accompanied by decrease in entropy of the system.

Ans. **[2]**

Sol. The correct statements are :
(A) Water vapours are adsorbed by anhydrous calcium chloride
(D) Adsorption is accompanied by decrease in entropy of the system.

Q.53 Number of hydrogen atoms per molecule of a hydrocarbon A having 85.8% carbon is _____
(Given: Molar mass of A = 84 g mol^{-1})

Ans. **[12]**

Sol. Molar mass of a hydrocarbon (A) = 84 g/mol

Mass of carbon in 1 mol of (A) = $\frac{85.8}{100} \times 84 = 72$ gm

Mass of hydrogen in 1 mol of (A) = 12 gm

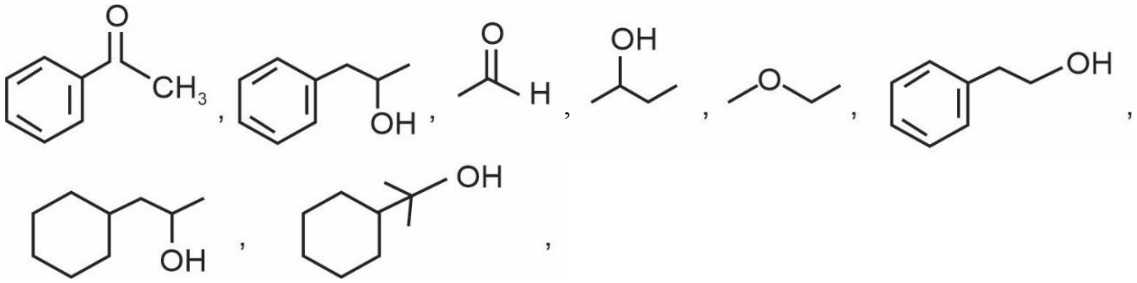
$\therefore$ Number of H-atoms in a molecules of (A) = 12.

Q.54 The number of given orbitals which have electron density along the axis is _____
p_x, p_y, p_z, d_{xy}, d_{yz}, d_{xz}, d_{z^2} , $d_{x^2-y^2}$

Ans. **[5]**

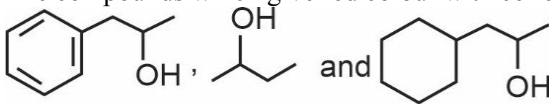
Sol. The orbitals having electron density along the axis are p_x, p_y, p_z, $d_{x^2-y^2}$ and d_{z^2} .

Q.55 Number of compounds giving (i) red colouration with ceric ammonium nitrate and also (ii) positive iodoform test from the following is ____



Ans. **[3]**

Sol. The compounds which give red colour with ceric ammonium nitrate and also give positive iodoform test are



Q.56 The number of pairs of the solutions having the same value of the osmotic pressure from the following is _____.
(Assume 100% ionization)

A. 0.500 M C_2H_5OH (aq) and 0.25 M KBr (aq)
B. 0.100 M $K_4[Fe(CN)_6]$ (aq) and 0.100 M $FeSO_4(NH_4)_2SO_4$(aq)
C. 0.05 M $K_4[Fe(CN)_6]$ (aq) and 0.25 M NaCl (aq)
D. 0.15 M NaCl (aq) and 0.1 M $BaCl_2$ (aq)
E. 0.02 M KCl. $MgCl_2.6H_2O$ (aq) and 0.05 M KCl(aq)

Ans. **[4]**

Sol. The following pairs of solutions have same value of osmotic pressure
(A) 0.500 M C_2H_5OH (aq) i = 1 and 0.25 M KBr(aq) i = 2
(B) 0.100 M $K_4[Fe(CN)_6]$ (aq) i = 5 and 0.100 M $FeSO_4(NH_4)_2$ (aq) i = 5
(D) 0.15 M NaCl (aq) i = 2 and 0.10 M $BaCl_2$ (aq) i = 3
(E) 0.02 M $KCl.MgCl_2.6H_2O$ (aq) i = 5 and 0.05 M KCl (aq) i = 2

Q.57 28.0 L of CO_2 is produced on complete combustion of 16.8 L gaseous mixture of ethene and methane at 25ºC and 1 atm. Heat evolved during the combustion process is _____ kJ.
Given: $\Delta H_C(CH_4) = -900$ kJ mol^{-1}
$\Delta H_C(C_2H_4) = -1400$ kJ mol^{-1}

Ans. **[925]**

Sol. $\underset{(16.8-x)}{CH_4(g)} + 2O_2(g) \rightarrow \underset{(16.8-x)}{CO_2(g)} + 2H_2O(l)$

$\underset{x}{C_2H_4(g)} + 3O_2(g) \rightarrow \underset{2x}{2CO_2(g)} + \underset{2x}{2H_2O(l)}$

$16.8 + x = 28 \quad \Rightarrow x = 11.2$ L

No. of moles of $CH_4 = 0.25$ and that of $C_2H_4 = 0.50$

$$|\text{Total heat evolved}| = \left|-\frac{900}{4} - \frac{1400}{2}\right| = 925 \text{ kJ mol}^{-1}$$

Q.58 $Pt(s)|H_2(g)(1bar)|H^+(aq)(1M)||M^{3+}(aq),M^+(aq)|Pt(s)$

The E_{cell} for the given cell is 0.1115 V at 298 K when $\frac{[M^+(aq)]}{[M^{3+}(aq)]} = 10^a$

The value of a is _____

Given : $E^{\theta}_{M^{3+}/M^+} = 0.2$ V

$\frac{2.303}{F} = 0.059$ V

Ans. **[3]**

Sol. $Pt(s)|H_2(g)(1bar)|H^+(aq)(1M)||M^{3+}(aq),M^+(aq)|Pt(s)$

$E_{cell} = 0.1115$ V at 298 K ; $E^{o}_{M^{3+}/M^+} = 0.2$ V

Cell reaction is $H_2 + M^{3+} \rightarrow 2H^+ + M^+$

$$E_{cell} = E^{o}_{cell} - \frac{0.059}{2}\frac{[H^+]^2[M^+]}{[M^{3+}]}$$

$$0.1115 = 0.2 - \frac{0.059}{2}\log 10^a$$

$a = 3$

Q.59 Total number of moles of AgCl precipitated on addition of excess of $AgNO_3$ to one mole each of the following complexes $[Co(NH_3)_4Cl_2]Cl$, $[Ni(H_2O)_6]Cl_2$, $[Pt(NH_3)_2Cl_2]$ and $[Pd(NH_3)_4]Cl_2$ is _____

Ans. **[5]**

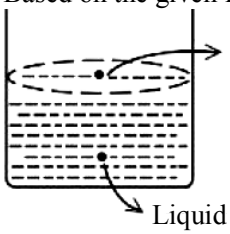
Sol. $[Co(NH_3)_4Cl_2]Cl \xrightarrow{AgNO_3} AgCl$

$[Ni(H_2O)_6]Cl_2 \xrightarrow{AgNO_3} 2AgCl$

$[Pd(NH_3)_4]Cl_2 \xrightarrow{AgNO_3} 2AgCl$

Total moles of AgCl precipitated = 5

Q.60 Based on the given figure, the number of correct statement/s is/are _____



A. Surface tension is the outcome of equal attractive and repulsive forces acting on the liquid molecule in bulk.
B. Surface tension is due to uneven force acting on the molecules present on the surface.
C. The molecule in the bulk can never come to the liquid surface.
D. The molecules on the surface are responsible for vapour pressure if the system is a closed system.

Ans. **[2]**

Sol. The correct statements are
(B) Surface tension is due to uneven forces acting on the molecules present on the surface
(D) The molecules on the surface are responsible for vapour pressure if the system is a closed system

MATHEMATICS

Section-A: Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Q.61 Let $A = \begin{bmatrix} \frac{1}{\sqrt{10}} & \frac{3}{\sqrt{10}} \\ \frac{-3}{\sqrt{10}} & \frac{1}{\sqrt{10}} \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -i \\ 0 & 1 \end{bmatrix}$, where $i = \sqrt{-1}$. If $M = A^T B A$, then the inverse of the matrix $AM^{2023} A^T$ is

(1) $\begin{bmatrix} 0 & 0 \\ -2023i & 1 \end{bmatrix}$ (2) $\begin{bmatrix} 1 & -2023i \\ 0 & 1 \end{bmatrix}$

(3) $\begin{bmatrix} 1 & 2023i \\ 0 & 1 \end{bmatrix}$ (4) $\begin{bmatrix} 1 & 0 \\ 2023i & 1 \end{bmatrix}$

Ans. **[2]**

Sol. $A = \begin{bmatrix} \frac{1}{\sqrt{10}} & \frac{3}{\sqrt{10}} \\ \frac{-3}{\sqrt{10}} & \frac{1}{\sqrt{10}} \end{bmatrix}$

$\therefore A \cdot A^T = A^T \cdot A = I$

$\therefore AM^{2023} AT = B^{2023}$

$= \begin{bmatrix} 1 & -2023i \\ 0 & 1 \end{bmatrix}$

Q.62 Let $\vec{a} = -\hat{i} - \hat{j} + \hat{k}$ and $\vec{a}.\vec{b} = 1$ and $\vec{a} \times \vec{b} = \hat{i} - \hat{j}$. Then $\vec{a} - 6\vec{b}$ is equal to

(1) $3(\hat{i} + \hat{j} - \hat{k})$ (2) $3(\hat{i} - \hat{j} - \hat{k})$

(3) $3(\hat{i} - \hat{j} + \hat{k})$ (4) $3(\hat{i} + \hat{j} + \hat{k})$

Ans. **[4]**

Sol. $\vec{a} = -\hat{i} - \hat{j} + \hat{k}$ Let $\hat{b} = b_1\hat{i} + b_2\hat{j} - b_3\hat{k}$

$\Rightarrow -b_1 - b_2 + b_3 = 1$ $(\vec{a}.\vec{b} = 1)$

&

$\vec{a} \times \vec{b} = -(b_2 + b_3)\hat{i} + (b_1 + b_3)\hat{j} - (b_1 + b_2)\hat{k} = \hat{i} - \hat{j}$

$\Rightarrow b_1 + b_2 = 0,\ b_2 + b_3 = -1,\ b_1 + b_3 = -1$

also $|\vec{b}| = 1$ (by Lami's Theorem)

$\Rightarrow b_1 = \frac{-2}{3},\ b_2 = \frac{-2}{3}$ & $b_3 = \frac{1}{3}$

$\Rightarrow -6\hat{b} = 4\hat{i} + 4\hat{j} + 2\hat{k}$

$\Rightarrow \hat{a} - 6\hat{b} = 3(\hat{i} + \hat{j} + \hat{k})$

Q.63 Let $\Delta, \nabla \in \{\wedge \vee\}$ be such that $(p \to q)\ \Delta\ (p\ \nabla\ q)$ is a tautology. Then

(1) $\Delta = \vee, \nabla = \vee$ (2) $\Delta = \vee, \nabla = \wedge$ (3) $\Delta = \wedge, \nabla = \vee$ (4) $\Delta = \wedge, \nabla = \wedge$

Ans. **[2]**

Sol. $(p \to q)\ \Delta\ (p\ \nabla\ q)$

$\Rightarrow (p' \vee q)\ \Delta\ (p\ \nabla\ q)$ …(i)

If $\Delta = \vee, \nabla = \vee$

(i) becomes $(p' \vee q) \vee (p \vee q) \equiv T$

Q.64 The equations of two sides of a variable triangle are $x = 0$ and $y = 3$, and its third side is a tangent to the parabola $y^2 = 6x$. The locus of its circumcentre is

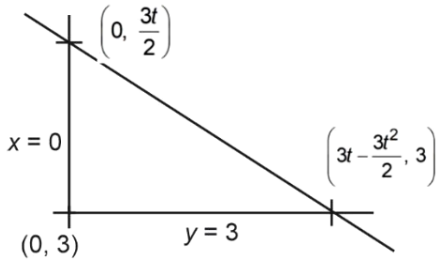
(1) $4y^2 + 18y + 3x + 18 = 0$ (2) $4y^2 - 18y - 3x - 18 = 0$

(3) $4y^2 - 18y + 3x + 18 = 0$ (4) $4y^2 - 18y - 3x + 18 = 0$

Ans. **[3]**

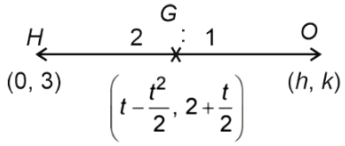
Sol. Third side of triangle

$ty = x + \frac{3}{2}t^2$



$\therefore H \equiv (0, 3)$ $G \equiv \left(t - \frac{t^2}{2}, 2 + \frac{t}{2},\right)$

Let $O\ (h, k)$



$\Rightarrow \frac{2h}{3} = t - \frac{t^2}{2}$ & $\frac{2k+3}{3} = 2 + \frac{t}{2}$

$\Rightarrow 4h = 6t - 3t^2$ & $4k = 6 + 3t$

$\Rightarrow 4h = 2(4k-6)\,3\left(\frac{(4k-6)^2}{2}\right)$

$\Rightarrow 3h = 6k - 9 - (4k^2 + 9 - 12k)$

$\Rightarrow 4k^2 - 18k + 3h + 18 = 0$

$\Rightarrow 4y^2 - 18y + 3x + 18 = 0$

Q.65 The shortest distance between the lines $x + 1 = 2y = -12z$ and $x = y + 2 = 6z - 6$ is

(1) 2 (2) $\frac{3}{2}$ (3) 3 (4) $\frac{5}{2}$

Ans. **[1]**

Sol. $L_1 : \frac{x+1}{1} = \frac{y}{\frac{1}{2}} = \frac{z}{\frac{-1}{12}}$

$L_2 : \frac{x}{1} = \frac{y+2}{1} = \frac{z-1}{\frac{1}{6}}$

$S.D = \left|\frac{(-\hat{i}+2\hat{j}-\hat{k}).(2\hat{i}-3\hat{j}+6\hat{k})}{7}\right|$

$= \left|\frac{-2-6-6}{7}\right| = 2$ units

66. If the function $f(x) = \begin{cases} 1+|\cos x|\frac{\lambda}{|\cos x|} & , \quad 0 < x < \frac{\pi}{2} \\ \mu & , \quad x = \frac{\pi}{2} \\ \frac{\cot 6x}{e^{\cot 4x}} & , \quad \frac{\pi}{2} < x < \pi \end{cases}$ is continuous at $x = \frac{\pi}{2}$, then $9\lambda + 6\log_e\mu = \mu^6 - e^{6\lambda}$ is equal to

(1) 11 (2) 8 (3) 10 (4) $2e^4 + 8$

Ans. **[3]**

Sol. $\lim\limits_{x\to\frac{\pi}{2}^-}(1+|\cos x|)\frac{\lambda}{|\cos x|} = e^{\lambda}$

$\lim\limits_{x\to\frac{\pi}{2}^+} e^{\frac{\cot 6x}{\cot 4x}} = \lim\limits_{e^{x\to\frac{\pi}{2}^+}} \frac{\tan 4x}{\tan 6x} = e^{\frac{2}{3}}$

$\lambda = \frac{2}{3}, \mu = e^{\frac{2}{3}}$

$9\lambda + 6\ln\mu + \mu^6 - e^{6\lambda}$

$= 6 + 4 + e^4 - e^4 = 10$

Q.67 If the four points, whose position vectors are $3\hat{i} - 4\hat{j} + 2k$, $\hat{i} + 2\hat{j} - \hat{k}$, $-2\hat{i} - \hat{j} + 3\hat{k}$ and $5\hat{i} - 2\alpha\hat{j} + 4\hat{k}$ are coplanar, then α is equal to

(1) $\frac{107}{17}$ (2) $-\frac{107}{17}$ (3) $\frac{73}{17}$ (4) $-\frac{73}{17}$

Ans. **[3]**

Sol. $\overrightarrow{AB} = -2\hat{i} + 6\hat{j} - 3\hat{k}$

$\overrightarrow{AC} = -5\hat{i} + 3\hat{j} + \hat{k}$

$\overrightarrow{AD} = 2\hat{i} + (4 - 2\alpha)\hat{j} + 2\hat{k}$

$$\begin{vmatrix} -2 & 6 & -3 \\ -5 & 3 & 1 \\ 2 & 4-2\alpha & 2 \end{vmatrix} = 0$$

$\Rightarrow 14b - 34\alpha = 0$

Or $\alpha = \dfrac{73}{17}$

Q.68 Let T and C respectively be the transverse and conjugate axes of the hyperbola $16x^2 - y^2 + 64x + 4y + 44 = 0$. Then the area of the region above the parabola $x^2 = y + 4$, below the transverse axis T and on the right of the conjugate axis C is :

(1) $4\sqrt{6} + \dfrac{28}{3}$ (2) $4\sqrt{6} - \dfrac{44}{3}$ (3) $4\sqrt{6} + \dfrac{44}{3}$ (4) $4\sqrt{6} - \dfrac{28}{3}$

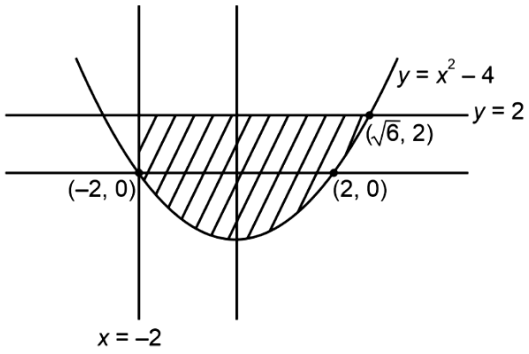
Ans. **[1]**

Sol. $16(x+2)^2 - (y-2)^2 = 16$

$$\frac{(x+2)^2}{1} - \frac{(y-2)^2}{16} = 1$$

TA : $y = 2$

CA : $x = -2$



$$A = \left| \int_{-2}^{\sqrt{6}} \left(2 - (x^2 - 4)\right) dx \right|$$

$$= 6x - \frac{x^3}{3}\Bigg|_{-2}^{\sqrt{6}}$$

$$= \left(6\sqrt{6} - \frac{6\sqrt{6}}{3}\right) - \left(-12 + \frac{8}{3}\right)$$

$$= \frac{12\sqrt{6}}{3} + \frac{28}{3}$$

Q.69 The number of numbers, strictly between 5000 and 10000 can be formed using the digits 1, 3, 5, 7, 9 without repetition, is

(1) 6 (2) 12 (3) 72 (4) 120

Ans. **[3]**

Sol. $\underline{5}\_\_\_ \Rightarrow {}^4C_3 \cdot 3!$ 24 ways

$\underline{7}\_\_\_ \Rightarrow {}^4C_3 \cdot 3!$ 24 ways

$\underline{9}\_\_\_ \Rightarrow {}^4C_3 \cdot 3!$ 24 ways

Total ways = 72

Q.70 Let $y = y(t)$ be a solution of the differential equation

$$\frac{dy}{dt} + \alpha y = \gamma e^{-\beta t}$$

where, $\alpha > 0$, $\beta > 0$ and $\gamma > 0$. Then $\lim\limits_{x\to\infty} y(t)$

(1) Does not exist (2) Is 0 (3) Is –1 (4) Is 1

Ans. **[2]**

Sol. $\frac{dy}{dt} + \alpha y = \gamma e^{-\beta t}$

I.F. $e^{\int \alpha dt} = e^{\alpha t}$

$$\Rightarrow y.e^{\alpha t} = \gamma \int e^{(\alpha-\beta)t} dt = \frac{\gamma^{(\alpha-\beta)t}}{(\alpha-\beta)} + C$$

$$\Rightarrow y = \frac{\gamma}{(\alpha-\beta)} e^{-\beta t} + Ce^{-\alpha t}$$

$$\lim_{x\to\infty} y(t) = \lim_{x\to\infty} \left[\frac{\gamma}{(\alpha-\beta)} e^{-\beta t} + Ce^{-\alpha t} \right] = 0$$

Q.71 $\sum\limits_{k=0}^{6} {}^{51-k}C_3$ is equal to

(1) ${}^{52}C_3 - {}^{45}C_3$ (2) ${}^{52}C_4 - 45C_4$ (3) ${}^{51}C_3 - 45C_3$ (4) ${}^{51}C_4 - {}^{45}C_4$

Ans. **[2]**

Sol. $\sum\limits_{k=0}^{6} {}^{51-k}C_3 = {}^{51}C_3 + {}^{50}C_3 + {}^{49}C_3 + {}^{48}C_3 + {}^{47}C_3 + {}^{46}C_3 + ({}^{45}C_3 + {}^{45}C_4) - {}^{45}C_4$

$S = {}^{51}C_3 + {}^{50}C_3 + {}^{49}C_3 + {}^{48}C_3 + {}^{47}C_3 + ({}^{46}C_3 + {}^{46}C_4) - {}^{45}C_4$

$\Rightarrow S = {}^{52}C_4 - {}^{45}C_4$

Q.72 Let A, B, C be 3×3 matrices such that A is symmetric and B and C are skew-symmetric. Consider the statements

(S1) $A^{13}B^{26} - B^{26}A^{13}$ is symmetric

(S2) $A^{26}C^{13} - C^{13}A^{26}$ is symmetric

Then,

(1) Both S1 and S2 are true (2) Only S2 is true

(3) Only S1 is true (4) Both S1 and S2 are false

Ans. **[2]**

Sol. $A^T = A, B^T = -B, C^T = -C$

$P = A^{13}B^{26} - B^{26}A^{13}$

$P^T = (A^{13}B^{26} - B^{26}A^{13})^T = (A^{13}B^{26})^T - (B^{26}A^B)^T$

$= (B^{26})^T (A^{13})^T - (A^{13})^T (B^{26})^T$

$= (B^T)^{26} (A^T)^{13} - (A^T)^{13} (A^T)^{26}$

$= B^{26}A^{13} - A^{13}B^{26}$

$= -(A^{13}B^{26} - B^{26}A^{13}) = -P$

P is skew-symmetric matrix $\Rightarrow S_1$ is false

$Q = A^{26}C^{13} - C^{13}A^{26} = Q^T = (A^{26}C^{13} - C^{13}A^{26})^T$

$Q = (A^{26}C^{13})^T - (C^{13}A^{26})^T = (C^{13})^T(A^{26})^T - (A^{26})^T(C^{13})^T$

$= (C^T)^{13} (A^T)^{26} - (A^T)^{26} (C^T)^{13} = -C^{13}A^{26} + A^{26}C^{13}$

$= A^{26}C^{13} + C^{13}A^{26}$

$\Rightarrow Q^T = Q \Rightarrow$ Q is symmetric matrix $\Rightarrow S_2$ is true.

Q.73 Let $f(x)= 2x^n + \lambda, \lambda \in R, n \in N$ and $f(4) = 133, f(5) = 255$. Then the sum of all the positive integer divisors of $(f(3) - f(2))$ is

(1) 61 (2) 59 (3) 60 (4) 58

Ans. **[3]**

Sol. $f(x) = 2x^n + \lambda, \lambda \in R, n \in N$

$f(4) = 2.4^n + \lambda = 133, f(5) = 2.5^n + \lambda = 255$

$f(5) - f(4) = 2.(5^n . 4^n) = 122 \Rightarrow n = 3$

$\Rightarrow f(3) - f(2) = 2.(3^n . 2^n) = 2. (3^3 - 2^3) = 2 \times 19$

Required sum = 1 + 2 + 19 + 38 = 60

Q.74 The number of functions

$f: \{1, 2, 3, 4\} \rightarrow \{a \in : Z \mid \alpha \mid \leq 8\}$

satisfying $f(n) + \frac{1}{n} f(n + 1) = 1, \forall n \in \{1, 2, 3\}$ is

(1) 3 (2) 2 (3) 4 (4) 1

Ans. **[2]**

Sol. $\because f: \{1, 2, 3, 4\} \rightarrow \{a \in Z : |9| \leq 8\}$

and $f(n) + \frac{1}{n} f(n + 1) = 1$

$\Rightarrow n f(n) + f(n +1) = n$ …(i)

$\therefore f(1) + f(2) = 1 \Rightarrow f(2) = 1 - f(1)$

But $f(1) \in [-8, 8]$

Hence, $f(2) \in [-8, 8] \Rightarrow f(1) \in [-7, 8]$ …(A)

and $2f(2) + f(3) = 2 \Rightarrow f(3) = 2f(1)$

$\therefore 2f(1) \in [-8, 8] \Rightarrow f(1) \in [-4, 4]$ …(B)

and $3f(3) + f(4) = 3 \Rightarrow f(4) = 3 - 6f(1)$

$\therefore f(1) \in \left[-\frac{5}{6}, \frac{11}{6}\right]$ …(C)

From (A), (B) and (C) : f(1) = 0 or 1

$\therefore$ Only two functions are possible.

Q.75 Let the function $f(x) = 2x^3 + (2p-7)x^2 + 3(2p-9)x - 6$ have a maxima for some value of $x < 0$ and a minima for some value of $x > 0$. Then, the set of all values of p is

(1) $\left(0, \frac{9}{2}\right)$ (2) $\left(\frac{9}{2}, \infty\right)$ (3) $\left(-\frac{9}{2}, \frac{9}{2}\right)$ (4) $\left(-\infty, \frac{9}{2}\right)$

Ans. **[4]**

Sol. $f'(x) = 6x^2 + 2x(2p-7) + 3(2p-9)$

$x_1 < 0, x_2 > 0$

$\Rightarrow f'(0) < 0$

$\Rightarrow p < \frac{9}{2}$

Q.76 The integral $16\int_1^2 \frac{dx}{x^3(x^2+2)}$ is equal to

(1) $\frac{11}{12} + \log_e 4$ (2) $\frac{11}{6} - \log_e 4$ (3) $\frac{11}{12} - \log_e 4$ (4) $\frac{11}{6} + \log_e 4$

Ans. **[2]**

Sol. $I = \int \frac{dx}{x^3(x^2+2)^2}$

$= \frac{1}{4}\int \frac{x}{x^2+2}dx + \frac{1}{4}\int \frac{x}{(x^2+2)^2} - \frac{1}{4}\int \frac{dx}{x} + \frac{1}{4}\int \frac{dx}{x^3}$

$= \frac{1}{8}\ln(x^2+2) - \frac{\ln x}{4} - \frac{1}{8(x^2+2)} - \frac{1}{8x^3}$

Now, $16\int_1^2 \frac{dx}{x^3(x^2+2)^2} = 2\ln 6 - 2\ln 3 - 4\ln 2 + \frac{11}{6} = \frac{11}{6} - \ln 4$

Q.77 Let $f: R \to R$ be a function defined by $f(x) = \log_{\sqrt{M}}\left\{\sqrt{2}(\sin x - \cos x) + m - 2\right\}$, for some m, such that the range of f is $[0, 2]$. Then the value of m is _____

(1) 4 (2) 2 (3) 3 (4) 5

Ans. **[4]**

Sol. We know that $\sin x - \cos x \in [-\sqrt{2}, \sqrt{2}]$

$\log_{\sqrt{M}}(\sqrt{2}(\sin x - \cos x) + M - 2) \in [\log_{\sqrt{M}}(M-4), \log_{\sqrt{M}} M]$

$\Rightarrow \log_{\sqrt{M}}(M-4) = 0 \Rightarrow M = 5$

Q.78 Let N be the sum of the numbers appeared when two fair dice are rolled and let the probability that $N-2$, $\sqrt{3N}$, $N+2$ are in geometric progression be $\frac{k}{48}$ Then the value of k is

(1) 16 (2) 8 (3) 4 (4) 2

Ans. **[3]**

Sol. $n-2, \sqrt{3n}, n+2 \to$ G.P.

$3n = n^2 - 4$

$\Rightarrow n^2 - 3n - 4 = 0$

$\Rightarrow n = 4, -1$ (rejected)

$P(S=4) = \frac{3}{36} = \frac{1}{12} = \frac{4}{48}$

$\therefore k = 4$

Option (3) is correct.

Q.79 Let z be a complex number such that $\left|\frac{z-2i}{x+i}\right| = 2$, $z \neq -i$. then z lies on the circle of radius 2 and centre

(1) (0, 2) (2) (2, 0) (3) (0, 0) (4) (0, –2)

Ans. **[4]**

Sol. $\left|\frac{z-2i}{x+i}\right| = 2$

$\Rightarrow (z-2i)(\bar{z}+2i) = 4(z+i)(\bar{z}-i)$

$\Rightarrow 2\bar{z} + 2iz - 2i\bar{z} + 4 = 4(z\bar{z} - zi + \bar{z}i + 1)$

$\Rightarrow 3z\bar{z} - 6iz + 6i\bar{z} = 0$

$\Rightarrow 2\bar{z} - 2iz + 2i\bar{z} = 0$

$\therefore$ Centre (–2i) or (0, –2)

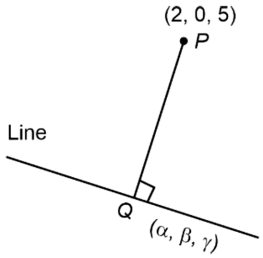
Option (4) is correct.

80. The foot of perpendicular of the point (2, 0, 5) on $\frac{x+1}{2} = \frac{y-1}{5} = \frac{z+1}{-1}$ is (α, β, γ). Then, which of the following is NOT correct ?

(1) $\frac{\alpha\beta}{\gamma} = \frac{4}{15}$ (2) $\frac{\beta}{\gamma} = -5$ (3) $\frac{\gamma}{\alpha} = \frac{5}{8}$ (4) $\frac{\alpha}{\beta} = -8$

Ans. **[2]**

Sol. $L : \frac{x+1}{2} = \frac{y-1}{5} = \frac{z+1}{-1} = t$



then,

$\alpha = 2t - 1$

$\beta = 5t + 1$

$\gamma = -t - 1$

for foot of $\perp^r$ $2(2t-3) + 5(5t+1) - (-t-6) = 0$

$\Rightarrow 30t + 5 = 0$

$\therefore t = -\frac{1}{6}$

$\therefore \alpha = -\frac{1}{3} - 1 = \frac{-4}{3}, \beta = \frac{1}{6}, \gamma = \frac{-5}{6}$

So, $\frac{\beta}{\gamma} = \frac{\frac{1}{6}}{-\frac{5}{6}} = \frac{-1}{5}$

So, option (2) is correct.

Section-B: Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

Q.81 Let $a \in R$ and let α, β be the roots of the equation $x^2 + 60^{\frac{1}{4}} x + a = 0$. If $\alpha^4 + \beta^4 = -30$, then the product of all possible values of a is _____

Ans. **[45]**

Sol. $x^2 + 60^{\frac{1}{4}} x + a = 0$

$\therefore \alpha + \beta = -60^{\frac{1}{4}}$, $\alpha\beta = a$

Now $\alpha^4 + \beta^4 = -30$

$\Rightarrow (\alpha^2 + \beta^2)^2 - 2a^2 = -30$

$\Rightarrow [(\alpha + \beta)^2 - 2a]^2 - 2a^2 = -30$

$\Rightarrow \left(60^{\frac{1}{2}} - 2a\right)^2 - 2a^2 = -30$

$\Rightarrow 60 + 4a^2 - 4 \cdot 60^{\frac{1}{2}} a - 2a^2 + 30 = 0$

$\Rightarrow 2a^2 - 8\sqrt{15}\, a + 90 = 0$

Product of value of a = 45

Q.82 For the two positive number a, b, if a, b and $\frac{1}{18}$ are in a geometric progression, while $\frac{1}{a}$, 10 and $\frac{1}{b}$ are in arithmetic progression, then 16a + 12b is equal to _____.

Ans. **[03]**

Sol. $\because$ a, b, $\frac{1}{18} \rightarrow$ G.P.

$\therefore b^2 = \frac{a}{18}$ …(1)

And $\frac{1}{a}$, 10, $\frac{1}{b} \rightarrow$ A.P.

$\therefore 20 = \frac{1}{a} + \frac{1}{b}$

$20ab = a + b$

By (1) $a = 18\, b^2$

$\therefore 20 \times 18b^3 = 18b^2 + b$

$\because a, b > 0$

$360b^2 - 18b - 1 = 0$

$\Rightarrow 360b^2 - 30b + 12b - 1 = 0$

$\Rightarrow 30b(12b - 1) + 1(12b - 1) = 0$

$b = \frac{1}{12}, b = \frac{-1}{30}x$

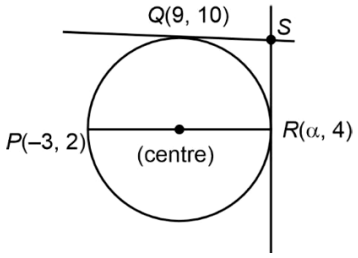
$\therefore 12b = 1, a = 18 \times \frac{1}{144} = \frac{2}{16}$

$\therefore 16a + 12b = 3$

Q.83 Points P(–3, 2), Q(9, 10) and R(α, 4) lie on a circle C with PR as its diameter. The tangents to C at the points Q and R intersect at the point S. If S lies on the line 2x – ky = 1, then K is equal to

Ans. **[03]**

Sol.



Now, $\frac{10-2}{9+3}\times\frac{10-4}{9\alpha}=-1$

$\Rightarrow \frac{8}{12}.6=\alpha-9 \Rightarrow \boxed{\alpha=13}$

$\therefore$ 0 = (5, 3) So, $m_{OQ}=\frac{7}{4}$, $m_{OR}=\frac{1}{8}$

$\therefore$ Q : $y-10=\frac{-4}{7}(x-9)$

$\Rightarrow 4x+7y=106$ …(i)

Tangent at R : $y-4=-8(x-13)$

$8x+y=108$ …(ii)

By (i) and (ii) S $\equiv \left(\frac{25}{2}, 8\right)$, satisfies with the line

$\therefore$ $\boxed{K=3}$

Q.84 If the shortest distance between the line joining the points (1, 2, 3) and (2, 3, 4), and the line $\frac{x-1}{2}=\frac{y+1}{-1}=\frac{z-2}{0}$ is α, then $28\alpha^2$ is equal to _____.

Ans. **[18]**

Sol. Points (1, 2, 3) and (2, 3, 4)

$L_1: \frac{(x-1)}{1}=\frac{(y-2)}{1}=\frac{(2-3)}{1}$

$L_2: \frac{x-1}{2}=\frac{y+1}{-1}=\frac{z-2}{0}$

$\vec{b}_1=\hat{i}+\hat{j}+\hat{k}$

$\vec{b}_2=2\hat{i}-\hat{j}+0\hat{k}$

$\vec{a}_1-\vec{a}_2=0\hat{i}-3\hat{j}-\hat{k}$

$d=\left|\frac{(\vec{a}_1-\vec{a}_2).(n_1\times n_2)}{|n_1\times n_2|}\right|=\left|\frac{6-3}{\sqrt{9+1+4}}\right|=\frac{3}{\sqrt{14}}=\alpha$

$28\alpha^2=\frac{28\times 9}{14}=18$

Q.85 Suppose Anil's mother wants to give 5 whole fruits to Anil from a basket of 7 red apples, 5 white apples and 8 oranges. If in the selected 5 fruits, at least 2 oranges, at least one red apple and at least one white apple must be given, then the number of ways, Anil's mother can offer 5 fruits to Anil is ____

Ans. **[6860]**

Sol. Total 8 oranges, 5 white apple and 7 red apple.
5 fruits needs to be selected.
Case I : 3 orange + 1 red apple + 1white apple
$= {}^8C_3 \times {}^7C_1 \times {}^5C_1 = 1960$
Case II : 2 oranges + 2 red apples + 1 white apple.
$= {}^8C_2 \times {}^7C_2 \times {}^5C_1 = 2940$
Case III : 2 oranges + 1 red apples + 2 white apple.
$= {}^8C_2 \times {}^7C_1 \times {}^5C_2 = 1960$
Total = 1960 + 2940 + 1960 = 6860

Q.86 25% of the population are smokers. A smoker has 27 times more chances to develop lung cancer than a non-smoker. A person is diagnosed with lung cancer and the probability that this person is a smoker is $\frac{k}{10}$. Then the value of k is _____.

Ans. **[09]**

Sol. Probability of a person being smoker = $\frac{1}{4}$

Probability of a person being non-smoker = $\frac{3}{4}$

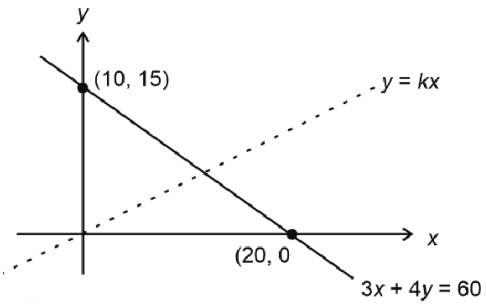
$$P\left(\frac{\text{Person is smoker}}{\text{Person diagonsed with cancer}}\right) = \frac{\frac{1}{4}.27P}{\frac{1}{4}.27P + \frac{3P}{4}} = \frac{9}{10} = \frac{k}{10}$$

$\Rightarrow \boxed{k = 9}$

Q.87 A triangle is formed by X-axis and the line $3x + 4y = 60$. Then the number of points $P(a, b)$, which lie strictly inside the triangle, where a is integer and b is a multiple of a, is _____.

Ans. **[31]**

Sol.



As b is multiple of a the required point lie on
Line y = kx (k ∈ z)
∴ 3x + 4kx = 60

$$x = \frac{60}{3 + 4k}$$

If k = 1 8 integral points
k = 2 5 integral points
k = 3 3 integral points
k = 4 3 integral points
k = 5 2 integral points

k = 6	2 integral points
k = 7	1 integral point
k = 8	1 integral point
k = 14	1 integral point
⋮	⋮

∴ Total 31 points

Q.88 The remainder when $(2023)^{2023}$ is divided by 35 is _____.

Ans. **[07]**

Sol. Let $N = 2023$

2023 is divisible by 7

$\therefore 2023^{2023}$ is divisible by 7

$\therefore$ Let $N = 7\alpha$

$N = 2023^{2023} \equiv 3^{2023} \pmod 5$

$\equiv 3^3 \pmod 5 \equiv 2 \pmod 5$

$\therefore N = 5\beta + 2$

$\Rightarrow 7\alpha = 5\beta + 2$

$7\alpha = 5\beta + 7 - 5$

$7(\alpha - 1) = 5(\beta - 1)$

$\alpha - 1$ is divisible by 5

$\alpha = 5p + 1$

$N = 7\alpha = 7(5p + 1) = 35p + 7$

Q.89 If m and n respectively are the numbers of positive and negative values of θ in the interval $[-\pi, \pi]$ that satisfy the equation $\cos2\theta\cos\frac{\theta}{2} = \cos3\theta\cos\frac{9\theta}{2}$, then mn is equal to _____.

Ans. **[25]**

Sol. $2\cos2\theta\cos\frac{\theta}{2} = 2\cos3\theta\cos\frac{9\theta}{2}$

$\cos\frac{5\theta}{2} + \cos\frac{3\theta}{2} = \cos\frac{15\theta}{2} + \cos\frac{3\theta}{2}$

$\cos\frac{5\theta}{2} = \cos\frac{15\theta}{2}$

$\frac{15\theta}{2} = 2n\pi \pm \frac{5\theta}{2}$

$\frac{15\theta}{2} \pm \frac{5\theta}{2} = 2n\pi$

$10\theta = 2n\pi$ or $5\theta = 2n\pi$

$\theta = \frac{n\pi}{5}$ or $\theta = \frac{2n\pi}{5}$

$\Rightarrow \theta = \frac{n\pi}{5}$

$\theta = \pm\pi, \pm\frac{4\pi}{5}, \pm\frac{3\pi}{5}, \pm\frac{2\pi}{5}, \pm\frac{\pi}{5}$

$m = 5, n = 5$

$mn = 25$

Q.90 If $\int_{\frac{1}{3}}^{3} |\log_e x| dx = \frac{m}{n}\log e\left(\frac{n^2}{e}\right)$, where m and n are coprime natural numbers, then $m^2 + n^2 - 5$ is equal to _____.

Ans. **[20]**

Sol. $I = \int_{\frac{1}{3}}^{3} |\ln x| dx = -\int_{\frac{1}{3}}^{1} \ln x dx + \int_{1}^{3} \ln x dx$

$= -[x \ln x - x]_{\frac{1}{3}}^{1} + x \ln x - x]_1^3$

$= -\left[(0-1) - \left(\frac{1}{3}\ln 3 - \frac{1}{3}\right)\right] + [(3 \ln 3 - 3) - (0-1)]$

$= -\frac{2}{3} - \frac{1}{3}\ln 3 + 3 \ln 3 - 2$

$= \frac{8}{3}\ln 3 - \frac{4}{3}$

$= \frac{4}{3}(2 \ln 3 + \ln e)$

$= \frac{4}{3}\ln\left(\frac{3^2}{e}\right)$

$m = 4, m = 3$

$m^2 + n^2 - 5 = 20$