JEE MAIN ONLINE PAPER 2021

Held on August 31, 2021 (Morning)

Instructions

1. This test will be a 3 hours Test.
2. This test consists of Physics, Chemistry and Mathematics questions with equal weightage of 100 marks.
3. Each question is of 4 marks.
4. In the question paper consisting of Physics (Q.no. 1 to 30), Chemistry (Q.no. 31 to 60) and Mathematics (Q.no. 61 to 90). There are two sections for each subject (Section-A : MCQ Type & Section-B : Numerical Response Type). Section-A consists of 20 multiple choice questions & Section-B consists of 10 Numerical Value type Questions. **Candidates have a choice to Answer 5 out of the 10 numerical value answer based questions per section.**
5. There will be only one correct choice in the given four choices in Section-A. For each question 4 marks will be awarded for correct choice, 1 mark will be deducted for incorrect choice and zero mark will be awarded for not attempted question. For Section-B questions 4 marks will be awarded for correct answer and zero for unattempted and incorrect answer.
6. Any textual, printed or written material, mobile phones, calculator etc. is not allowed for the students appearing for the test.
7. All calculations/written work should be done in the rough sheet provided.

PHYSICS

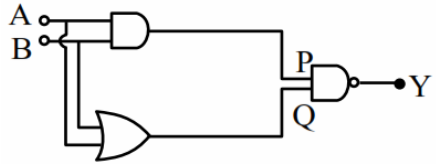
Section -A

Q.1 A helicopter is flying horizontally with a speed 'v' at an altitude 'h' has to drop a food packet for a man on the ground. What is the distance of helicopter from the man when the food packet is dropped?

(1) $\sqrt{\frac{2ghv^2+1}{h^2}}$ (2) $\sqrt{2ghv^2+h^2}$

(3) $\sqrt{\frac{2v^2h}{g}+h^2}$ (4) $\sqrt{\frac{2gh}{v^2}+h^2}$

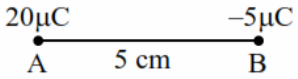
Q.2 In the following logic circuit the sequence of the inputs A, B are (0, 0), (0,1), (1, 0) and (1, 1). The output Y for this sequence will be :



(1) 1, 0, 1, 0 (2) 0, 1, 0, 1

(3) 1, 1, 1, 0 (4) 0, 0, 1, 1

Q.3 Two particles A and B having charges 20 μC and –5 μC respectively are held fixed with a separation of 5 cm. At what position a third charged particle should be placed so that it does not experience a net electric force?



(1) At 5 cm from 20 μC on the left side of system

(2) At 5 cm from – 5 μC on the right side

(3) At 1.25 cm from – 5 μC between two charges

(4) At midpoint between two charges

Q.4 A reversible engine has an efficiency of $\frac{1}{4}$ .If the temperature of the sink is reduced by 58°C, its efficiency becomes double. Calculate the temperature of the sink :

(1) 174°C (2) 280°C

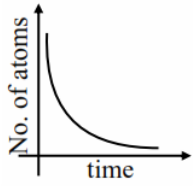
(3) 180.4°C (4) 382°C

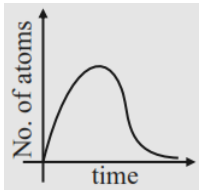
Q.5 An object is placed at the focus of concave lens having focal length *f*. What is the magnification and distance of the image from the optical centre of the lens ?

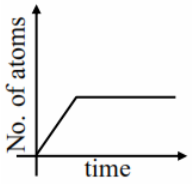
(1) 1, ∞ (2) Very high, ∞

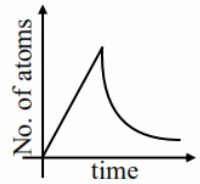
(3) $\frac{1}{2},\frac{f}{2}$ (4) $\frac{1}{4},\frac{f}{4}$

Q.6 A sample of a radioactive nucleus A disintegrates to another radioactive nucleus B, which in turn disintegrates to some other stable nucleus C. Plot of a graph showing the variation of number of atoms of nucleus B vesus time is : (Assume that at t = 0, there are no B atoms in the sample)

(1) 

(2) 

(3) 

(4) 

Q.7 A coil having N turns is wound tightly in the form of a spiral with inner and outer radii 'a' and 'b' respectively. Find the magnetic field at centre, when a current I passes through coil :

(1) $\frac{\mu_0 IN}{2(b-a)}\log_e\left(\frac{b}{a}\right)$ (2) $\frac{\mu_0 I}{8}\left[\frac{a+b}{a-b}\right]$

(3) $\frac{\mu_0 I}{4(a-b)}\left[\frac{1}{a}-\frac{1}{b}\right]$ (4) $\frac{\mu_0 I}{8}\left(\frac{a-b}{a+b}\right)$

Q.8 A boy of mass M moving at speed V_0 collides elastically with a mass 'm' at rest. After the collision, the two masses move at angles θ_1 and θ_2 with respect to the initial direction of motion of the body of mass M. The largest possible value of the ratio M/m, for which the angles θ_1 and θ_2 will be equal, is :

(1) 4 (2) 1 (3) 3 (4) 2

Q.9 The masses and radii of the earth and moon are (M_1, R_1) and (M_2, R_2) respectively. Their centres are at a distance 'r' apart. Find the minimum escape velocity for a particle of mass 'm' to be projected from the middle of there two masses:

(1) $V = \frac{1}{2}\sqrt{\frac{4G(M_1+M_2)}{r}}$

(2) $V = \sqrt{\frac{4G(M_1+M_2)}{r}}$

(3) $V = \frac{1}{2}\sqrt{\frac{2G(M_1+M_2)}{r}}$

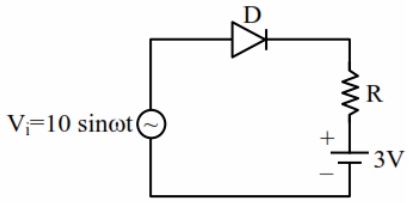
(4) $V = \frac{\sqrt{2G}(M_1+M_2)}{r}$

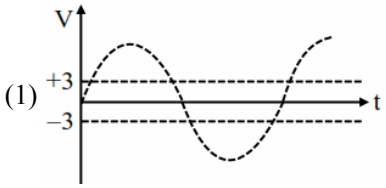
Q.10 A small square loop of side 'a' and one turn is placed inside a larger square loop of side b and one turn (b >> a). The two loops are coplanar with their centres coinciding. If a current I is passed in the square loop of side 'b', then the coefficient of mutual inductance between the two loops is :

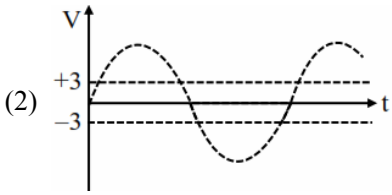
(1) $\frac{\mu_0}{4\pi}8\sqrt{2}\frac{a^2}{b}$ (2) $\frac{\mu_0}{4\pi}\frac{8\sqrt{2}}{a}$

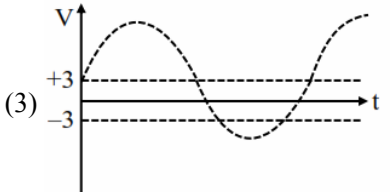
(3) $\frac{\mu_0}{4\pi}8\sqrt{2}\frac{b^2}{a}$ (4) $\frac{\mu_0}{4\pi}\frac{8\sqrt{2}}{b}$

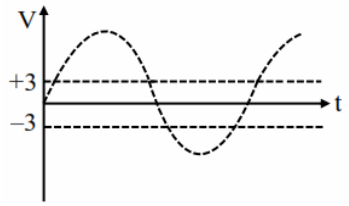
Q.11 Choose the correct waveform that can represent the voltage across R of the following circuit, assuming the diode is ideal one:



(1) 

(2) 

(3) 

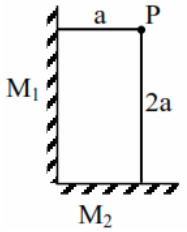
(4) 

Q.12 A uniform heavy rod of weight 10 kg ms^{-2}, cross-sectional area 100 cm^2 and length 20 cm is hanging from a fixed support. Young modulus of the material of the rod is 2×10^{11} Nm^{-2}. Neglecting the lateral contraction, find the elongation of rod due to its own weight.

(1) 2×10^{-9} m (2) 5×10^{-8} m

(3) 4×10^{-8} m (4) 5×10^{-10} m

Q.13 Two place mirrors M_1 and M_2 are at right angle to each other shown. A point source 'P' is placed at 'a' and '2a' meter away from M_1 and M_2 respectively. The shortest distance between the images thus formed is : (Take $\sqrt{5} = 2.3$)



(1) 3a (2) 4.6 a (3) 2.3 a (4) $2\sqrt{10}a$

Q.14 Match **List-I** with **List-II**.

List-I	**List-II**
(a) Torque	(i) MLT^{-1}
(b) Impulse	(ii) MT^{-2}
(c) Tension	(iii) ML^2T^{-2}
(d) Surface Tension	(iv) MLT^{-2}

Choose the **most appropriate** answer from the option given below :

(1) (a)–(iii), (b)–(i), (c)–(iv), (d)–(ii)

(2) (a)–(ii), (b)–(i), (c)–(iv), (d)–(iii)

(3) (a)–(i), (b)–(iii), (c)–(iv), (d)–(ii)

(4) (a)–(iii), (b)–(iv), (c)–(i), (d)–(ii)

Q.15 For an ideal gas the instantaneous change in pressure 'p' with volume 'v' is given by the equation $\frac{dp}{dv} = -ap$. If $p = p_0$ at $v = 0$ is the given boundary condition, then the maximum temperature one mole of gas can attain is :

(Here R if the gas constant)

(1) $\frac{p_0}{aeR}$ (2) $\frac{ap_0}{aR}$

(3) infinity (4) 0°C

Q.16 Which of the following equations is dimensionally incorrect ?

Where t = time, h = height, s = surface tension, θ = angle, ρ = density, a, r = radius, g = acceleration due to gravity, v = volume, p = pressure, W = work done, Γ = torque, $\in$ = permittivity, E = electric field, J = current density, L = length.

(1) $v = \frac{\pi pa^4}{8\eta L}$ (2) $h = \frac{2s\cos\theta}{\rho rg}$

(3) $J = \in \frac{\partial E}{\partial t}$ (4) $W = \Gamma\theta$

Q.17 Angular momentum of a single particle moving with constant speed along circular path :

(1) changes in magnitude but remains same in the direction

(2) remains same in magnitude and direction

(3) remains same in magnitude but changes in the direction

(4) is zero

Q.18 In an ac circuit, an inductor, a capacitor and a resistor are connected in series with $X_L = R = X_C$. Impedance of this circuit is :

(1) $2R^2$ (2) Zero (3) R (4) $R\sqrt{2}$

Q.19 A moving proton and electron have the same de- Broglie wavelength. If K and P denote the K.E. and momentum respectively. Then choose the correct option :

(1) $K_p < K_e$ and $P_p = P_e$

(2) $K_p = K_e$ and $P_p = P_e$

(3) $K_p < K_e$ and $P_e < P_e$

(4) $K_p > K_e$ and $P_p = P_e$

Q.20 Consider a galvanometer shunted with 5Ω resistance and 2% of current passes through it. What is the resistance of the given galvanometer ?

(1) 300 Ω (2) 344 Ω

(3) 245 Ω (4) 226 Ω

Section –B

Q.21 When a rubber ball is taken to a depth of ___ m in deep sea, its volume decreases by 0.5%. (The bulk modulus of rubber = 9.8×10^8 NM^{-2} Density of sea water = $10g^3$ kgm^{-3} g = 9.8 m/s^2)

Q.22 A particle of mass 1 kg is hanging from a spring of force constant 100 Nm^{-1}. The mass is pulled slightly downward and released so that it executes free simple harmonic motion with time period T.

The time when the kinetic energy and potential energy of the system will become equal, is $\frac{T}{x}$.

The value of x is _____.

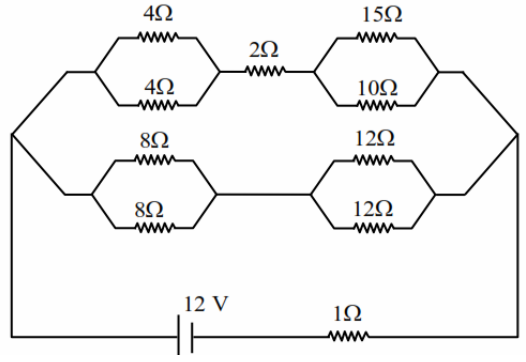
Q.23 If the sum of the heights of transmitting and receiving antennas in the line of sight of communication is fixed at 160 m, then the maximum range of LOS communication is _____km.

(Take radius of Earth = 6400 km)

Q.24 A square shaped wire with resistance of each side 3Ω is bent to form a complete circle. The resistance between two diametrically opposite points of the circle in unit of Ω will be _____.

Q.25 A wire having a linear mass density 9.0×10^{-4} kg/m is stretched between two rigid supports with a tension of 900 N. The wire resonates at a frequency of 500 Hz. The next higher frequency at which the same wire resonates is 550 Hz. The length of the wire is _____m.

Q.26 The voltage drop across 15Ω resistance in the given figure will be_____V.

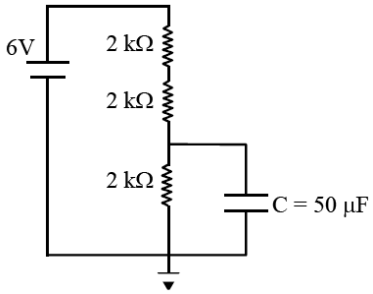


Q.27 A block moving horizontally on a smooth surface with a speed of 40 ms^{-1} splits into two equal parts. If one of the parts moves at 60 ms^{-1} in the same direction, then the fractional change in the kinetic energy will be x : 4 where x =_____.

Q.28 The electric field in an electromagnetic wave is given by $E = (50\ NC^{-1})\ \sin\omega\ (t - x/c)$

The energy contained in a cylinder of volume V is 5.5×10^{-12} J. The value of V is _____ cm^3. (given $\in_0 = 8.8 \times 10^{-12}$ $C^2N^{-1}m^{-2}$)

Q.29 A capacitor of 50 μF is connected in a circuit as shown in figure. The charge on the upper plate of the capacitor is_____μC.

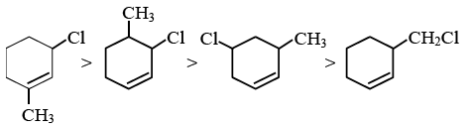


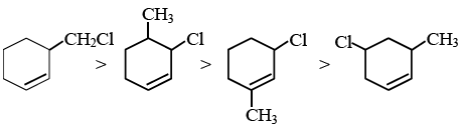
Q.30 A car is moving on a plane inclined at 30º to the horizontal with an acceleration of 10 ms^{-2} parallel to the plane upward. A bob is suspended by a string from the roof of the car. The angle in degrees which the string makes with the vertical is___. (Take g = 10 ms^{-2})

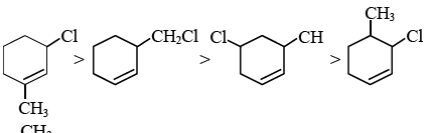
CHEMISTRY

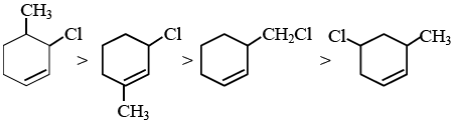
Section -A

Q.31 The correct order of reactivity of the given chlorides with acetate in acetic acid is :

(1) 

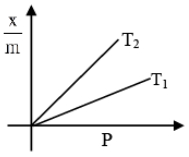
(2) 

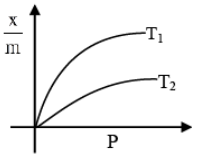
(3) 

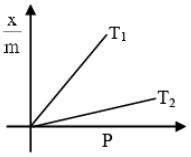
(4) 

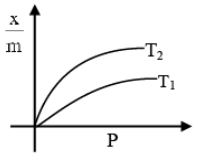
Q.32 Select the graph that correctly describes the adsorption isotherms at two temperatures T_1 and T_2 $(T_1 > T_2)$ for a gas :

(x – mass of the gas adsorbed ; m – mass of adsorbent ; P – pressure)

(1) 

(2) 

(3) 

(4) 

Q.33 The major component/ingredient of Portland Cement is :

(1) tricalcium aluminate (2) tricalcium silicate

(3) dicalcium aluminate (4) dicalcium silicate

Q.34 In the structure of the dichromate ion, there is a :

(1) linear symmetrical Cr–O–Cr bond.

(2) non-linear symmetrical Cr–O–Cr bond.

(3) linear unsymmetrical Cr–O–Cr bond.

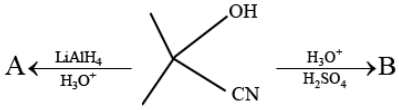
(4) non-linear unsymmetrical Cr–O–Cr bond.

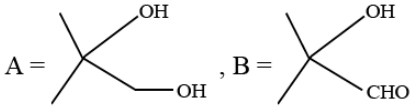
Q.35 Which one of the following compounds contains β-C_1-C_4 glycosidic linkage ?

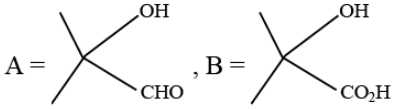
(1) Lactose (2) Sucrose

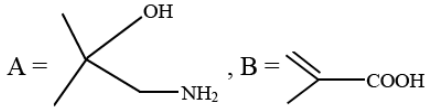
(3) Maltose (4) Amylose

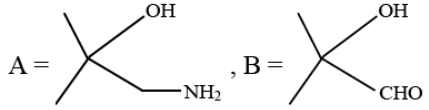
Q.36 The major products A and B in the following set of reactions are :



(1) 

(2) 

(3) 

(4) 

Q.37 Which one of the following lanthanides exhibits +2 oxidation state with diamagnetic nature ?

(Given Z for Nd = 60, Yb = 70, La = 57, Ce =58)

(1) Nd (2) Yb

(3) La (4) Ce

Q.38 Given below are two statements : one is labelled as **Assertion (A)** and the other is labelled as **Reason (R).**

Assertion (A) : Aluminium is extracted from bauxite by the electrolysis of molten mixture of Al_2O_3 with cryolite.

Reason (R) : The oxidation state of Al in cryolite is +3.

In the light of the above statements, choose the **most appropriat**e answer from the options given below :

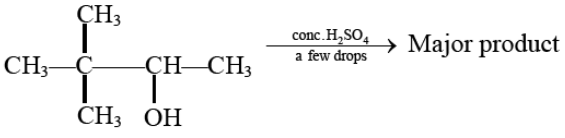
(1) (A) is true but (R) is false

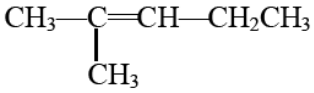
(2) (A) is false but (R) is true.

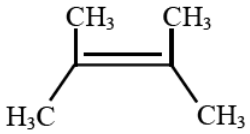
(3) Both (A) and (R) are correct and (R) is the correct explanation of (A)

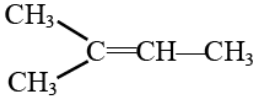
(4) Both (A) and (R) are correct but (R) is not the correct explanation of (A)

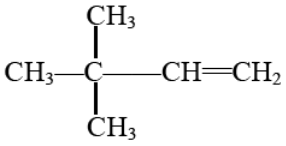
Q.39 The major product formed in the following reaction is :



(1) 

(2) 

(3) 

(4) 

Q.40 Monomer of Novolac is :
(1) 3-Hydroxybutanoic acid
(2) phenol and melamine
(3) o-Hydroxymethylphenol
(4) 1,3-Butadiene and styrene

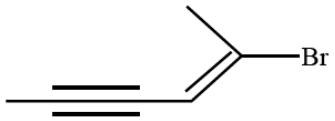
Q.41 Given below are two statements :
Statement-I : The process of producing syn-gas is called gasification of coal.
Statement-II : The composition of syn-gas is $CO + CO_2 + H_2$ (1 : 1 : 1)
In the light of the above statements, choose the **most appropriate** answer from the options given below :
(1) **Statement-I** is false but **Statement-II** is true
(2) **Statement-I** is true but **Statement-II** is false
(3) Both **Statement-I** and **Statement-II** are false
(4) Both **Statement-I** and **Statement-II** are true

Q.42 Given below are two statements : one is labelled as
Assertion (A) and the other is labelled as **Reason (R).**
Assertion (A) : Treatment of bromine water with propene yields 1-bromopropan-2-ol.
Reason (R) : Attack of water on bromonium ion follows Markovnikov rule and results in 1-bromopropan-2-ol.
In the light of the above statements, choose the **most appropriate** answer from the options given below :
(1) Both **(A)** and **(R)** are true but **(R)** is NOT the correct explanation of **(A)**
(2) **(A)** is false but **(R)** is true.
(3) Both **(A)** and **(R)** are true and **(R)** is the correct explanation of **(A)**
(4) **(A)** is true but **(R)** is false

Q.43 The denticity of an organic ligand, biuret is :
(1) 2 (2) 4 (3) 3 (4) 6

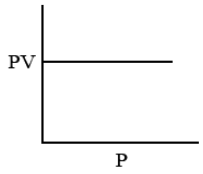
Q.44 Given below are two statements: one is labelled as
Assertion (A) and the other is labelled as **Reason (R).**
Assertion (A) : Metallic character decreases and non-metallic character increases on moving from left to right in a period.
Reason (R) : It is due to increase in ionisation enthalpy and decrease in electron gain enthalpy, when one moves from left to right in a period.
In the light of the above statements, choose the most appropriate answer from the options given below :
(1) **(A)** is false but **(R)** is true.
(2) **(A)** is true but **(R)** is false
(3) Both **(A)** and **(R)** are correct and **(R)** is the correct explanation of **(A)**
(4) Both **(A)** and **(R)** are correct but **(R)** is not the correct explanation of **(A)**

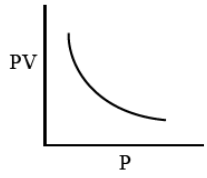
Q.45 Choose the correct name for compound given below :

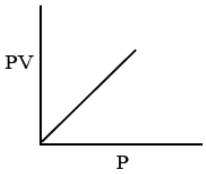


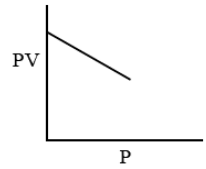
(1) (4E)-5-Bromo-hex-4-en-2-yne
(2) (2E)-2-Bromo-hex-4-yn-2-ene
(3) (2E)-2-Bromo-hex-2-en-4-yne
(4) (4E)-5-Bromo-hex-2-en-4-yne

Q.46 Which one of the following is the correct PV vs P plot at constant temperature for an ideal gas ? (P and V stand for pressure and volume of the gas respectively)

(1) 

(2) 

(3) 

(4) 

Q.47 Given below are two statements : one is labelled as
Assertion (A) and the other is labelled as **Reason (R) :**
Assertion (A) : A simple distillation can be used to separate a mixture of propanol and propanone.
Reason (R) : Two liquids with a difference of more than 20°C in their boiling points can be separated by simple distillations.
In the light of the above statements, choose the **most appropriate** answer from the options given below :
(1) **(A)** is false but **(R)** is true.
(2) Both **(A)** and **(R)** are correct but **(R)** is not the correct explanation of **(A)**
(3) **(A)** is true but **(R)** is false
(4) Both **(A)** and **(R)** are correct and **(R)** is the correct explanation of **(A)**

Q.48 Which one of the following 0.10 M aqueous solutions will exhibit the largest freezing point depression ?

(1) hydrazine (2) glucose

(3) glycine (4) $KHSO_4$

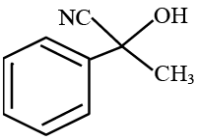
Q.49 BOD values (in ppm) for clean water (A) and polluted water (B) are expected respectively :

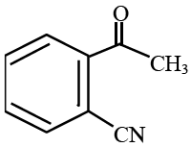
(1) A > 50, B < 27 (2) A > 25, B < 17

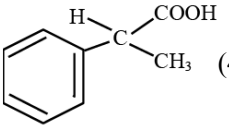
(3) A < 5, B > 17 (4) A > 15, B > 47

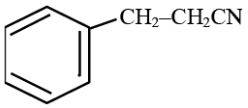
Q.50 The structure of product C, formed by the following sequence of reactions is :

$$CH_3COOH+SOCl_2 \longrightarrow A \xrightarrow[AlCl_3]{Benzene} B \xrightarrow[-OH]{KCN} C$$

(1) 

(2) 

(3) 

(4) 

Section -B

Q.51 Consider the following cell reaction

$$Cd_{(s)}+Hg_2SO_{4(s)}+\frac{9}{5}H_2O_{(l)} \rightleftharpoons CdSO_4.\frac{9}{5}H_2O_{(s)}+2Hg_{(l)}$$

The value of E^0_{cell} is 4.315 V at 25°C. If $\Delta H° = -825.2$ kJ mol^{-1}, the standard entropy

[Given : Faraday constant = 96487 C mol^{-1}]

Q.52 The molarity of the solution prepared by dissolving 6.3 g of oxalic acid ($H_2C_2O_4.2H_2O$) in 250 mL of water in mol L^{-1} is $x \times 10^{-2}$. The value of x is _____. (Nearest integer)

[Atomic mass : H :1.0, C : 12.0, O : 16.0]

Q.53 Consider the sulphides HgS, PbS, CuS, Sb_2S_3, AS_2S_3 and CdS Number of these sulphides soluble in 50% HNO_3 is _____.

Q.54 The total number of reagents from those given below that can convert nitrobenzene into aniline is _____. (Integer answer)

I. Sn – HCl

II. Sn – NH_4OH

III. Fe – HCl

IV. Zn – HCl

V. H_2 – Pd

VI. H_2 – Raney Nickel

Q.55 The number of halogen/(s) forming halic (V) acid is _____.

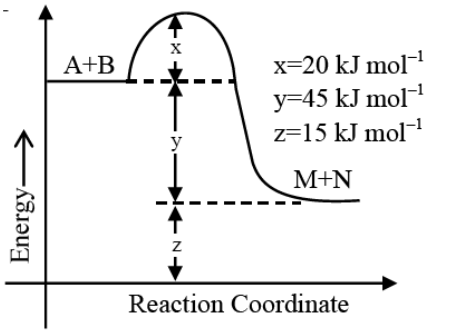
Q.56 For a first order reaction, the ratio of the time for 75% completion of a reaction to the time for 50% completion is _____. (Integer answer)

Q.57 The number of hydrogen bonded water molecule(s) associated with stoichiometry $CuSO_4.5H_2O$ is _____.

Q.58 According to the following figure, the magnitude of the enthalpy change of the reaction

$A + B \rightarrow M + N$ in kJ mol^{-1}

is equal to _____. (Integer answer)



Q.59 Ge(Z = 32) in its ground state electronic configuration has x completely filled orbitals with $m\ell = 0$. The value of x is _____.

Q.60 A_3B_2 is a sparingly soluble salt of molar mass M (g mol^{-1}) and solubility x g L^{-1}. The solubility product satisfies $K_{sp} = \alpha\left(\frac{x}{M}\right)^5$. The value of α is _____. (Integer answer)

MATHEMATICS

Section -A

Q.61 Let *, $\square \in \{\wedge, \vee\}$ be such that the Boolean expression $(p * \sim q) \Rightarrow (p \square q)$ is a tautology. Then :

(1) $* = \vee, \square = \vee$ (2) $* = \wedge, \square = \wedge$

(3) $* = \wedge, \square = \vee$ (4) $* = \vee, \square = \wedge$

Q.62 The number of real roots of the equation $e^{4x} + 2e^{3x} - e^x - 6 = 0$ is :

(1) 2 (2) 4

(3) 1 (4) 0

Q.63 The sum of 10 terms of the series

$\frac{3}{1^2 \times 2^2} + \frac{5}{2^2 \times 3^2} + \frac{7}{3^2 \times 4^2} + \ldots$ is :

(1) 1 (2) $\frac{120}{121}$ (3) $\frac{99}{100}$ (4) $\frac{143}{144}$

Q.64 Let the equation of the plane, that passes through the point (1, 4, –3) and contains the line of intersection of the planes $3x - 2y + 4z - 7 = 0$ and $x + 5y - 2z + 9 = 0$, be $\alpha x + \beta y + yz + 3 = 0$, then $\alpha + \beta + \gamma$ is equal to :

(1) –23 (2) –15 (3) 23 (4) 15

Q.65 Let f be a non-negative function in [0, 1] and twice differentiable in (0, 1). If $\int_0^x \sqrt{1-(f'(t)^2)}\, dt = \int_0^x f(t)dt$, $0 \le x \le 1$ and $f(0) = 0$, then $\lim\limits_{x\to 0} \frac{1}{x^2} \int_0^x f(t)\, dt$:

(1) equals 0 (2) equals 1

(3) does not exist (4) equals $\frac{1}{2}$

Q.66 Let $\vec{a}$ and $\vec{b}$ be two vectors such that $|2\vec{a} + 3\vec{b}| = |3\vec{a} + \vec{b}|$ and the angle between $\vec{a}$ and $\vec{b}$ is 60°. If $\frac{1}{8}\vec{a}$ is a unit vector, then $|\vec{b}|$ is equal to :

(1) 4 (2) 6 (3) 5 (4) 8

Q.67 The function $f(x) = |x^2 - 2x - 3|\ e^{|9x^2 - 12x + 4|}$ is not differentiable at exactly :

(1) four points (2) three points

(3) two points (4) one point

Q.68 Three numbers are in an increasing geometric progression with common ratio r. If the middle number is doubled. then the new numbers are in an arithmetic progression with common difference d. If the fourth term of GP is $3r^2$, then $r^2 - d$ is equal to :

(1) $7 - 7\sqrt{3}$ (2) $7 + \sqrt{3}$

(3) $7 - \sqrt{3}$ (4) $7 + 3\sqrt{3}$

Q.69 Which of the following is not correct for relation R on the set of real numbers ?

(1) $(x, y) \in R \Leftrightarrow 0 < |x| - |y| \le 1$ is neither transitive nor symmetric.

(2) $(x, y) \in R \Leftrightarrow 0 < |x-y| \le 1$ is symmetric and transitive.

(3) $(x, y) \in R \Leftrightarrow |x| - |y| \le 1$ is reflexive but not symmetric.

(4) $(x, y) \in R \Leftrightarrow |x-y| \le 1$ is reflexive and symmetric.

Q.70 The integral $\int \frac{1}{\sqrt[4]{(x-1)^3(x+2)^5}}\, dx$ is equal to :

(where C is constant of integration)

(1) $\frac{3}{4}\left(\frac{x+2}{x-1}\right)^{\frac{1}{4}} + C$ (2) $\frac{3}{4}\left(\frac{x+2}{x-1}\right)^{\frac{5}{4}} + C$

(3) $\frac{4}{3}\left(\frac{x-1}{x+2}\right)^{\frac{1}{4}} + C$ (4) $\frac{4}{3}\left(\frac{x-1}{x+2}\right)^{\frac{5}{4}} + C$

Q.71 If p and q are the lengths of the perpendiculars from the origin on the lines,

$x \operatorname{cosec}\alpha - y \sec\alpha = k\cot 2\alpha$ and

$x \sin\alpha + y \cos\alpha = k \sin 2\alpha$

respectively, then k^2 is equal to :

(1) $4p^2 + q^2$ (2) $2p^2 + q^2$

(3) $p^2 + 2q^2$ (4) $p^2 + 4q^2$

Q.72 $\operatorname{cosec}18°$ is a root of the equation :

(1) $x^2 + 2x - 4 = 0$ (2) $4x^2 + 2x - 1 = 0$

(3) $x^2 - 2x + 4 = 0$ (4) $x^2 - 2x - 4 = 0$

Q.73 If the following system of linear equations
$2x + y + z = 5$
$x - y + z = 3$
$x + y + az = b$
has no solution, then :

(1) $a = -\frac{1}{3}, b \neq \frac{7}{3}$ (2) $a \neq \frac{1}{3}, b = \frac{7}{3}$

(3) $a \neq -\frac{1}{3}, b = \frac{7}{3}$ (4) $a = \frac{1}{3}, b \neq \frac{7}{3}$

Q.74 The length of the latus rectum of a parabola, whose vertex and focus are on the positive x–axis at a distance R and S (>R) respectively from the origin, is :

(1) 4(S + R) (2) 2(S – R)

(3) 4(S – R) (4) 2(S + R)

Q.75 If the function $f(x) = \begin{cases} \frac{1}{x}\log_e\left(\frac{1+\frac{x}{a}}{1-\frac{x}{b}}\right) & , x < 0 \\ k & , x = 0 \\ \frac{\cos^2 x - \sin^2 x - 1}{\sqrt{x^2+1}-1} & , x > 0 \end{cases}$

is continuous at x = 0, then $\frac{1}{2} + \frac{1}{b} + \frac{4}{k}$ is equal to :

(1) – 5 (2) 5 (3) –4 (4) 4

Q.76 If $\frac{dy}{dx} = \frac{2^{x+y} - 2^x}{2^y}$, y(0) = 1, then y(1) is equal to :

(1) $\log_2(2 + e)$ (2) $\log_2(1 + e)$

(3) $\log_2(2e)$ (4) $\log_2(1 + e^2)$

Q.77 $\lim\limits_{x\to 0} \frac{\sin^2(\pi\cos^4 x)}{x^4}$ is equal to :

(1) π^2 (2) $2\pi^2$ (3) $4\pi^2$ (4) 4π

Q.78 A vertical pole fixed to the horizontal ground is divided in the ratio 3 : 7 by a mark on it with lower part shorter than the upper part. If the two parts subtend equal angles at a point on the ground 18 m away from the base of the pole, then the height of the pole (in meters) is :

(1) $12\sqrt{15}$ (2) $12\sqrt{10}$

(3) $8\sqrt{10}$ (4) $6\sqrt{10}$

Q.79 If $a_r = \cos\frac{2r\pi}{9} + i \sin\frac{2r\pi}{9}$, r = 1, 2, 3 …. $i = \sqrt{-1}$, then the determinant $\begin{vmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{vmatrix}$ is equal to :

(1) $a_2a_6 - a_4a_8$ (2) a_9

(3) $a_1a_9 - a_3a_7$ (4) a_5

Q.80 The line $12x\cos\theta + 5y\sin\theta = 60$ is tangent to which of the following curves ?

(1) $x^2 + y^2 = 169$

(2) $144x^2 + 25y^2 = 3600$

(3) $25x^2 + 12y^2 = 3600$

(4) $x^2 + y^2 = 60$

Section -B

Q.81 Let [t] denote the greatest integer ≤ t. Then the value of $8.\int_{\frac{1}{2}}^{1}([2x] + |x|\,dx)$ is _____

Q.82 A point z moves in the complex plane such that $\arg\left(\frac{z-2}{z+2}\right) = \frac{\pi}{4}$, then the minimum value of $\left|z - 9\sqrt{2} - 2i\right|^2$ is equal to _____.

Q.83 The square of the distance of the point of intersection of the line $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z+1}{6}$ and the plane 2x – y + z = 6 from the point (–1, –1, 2) id _____.

Q.84 If 'R' is the least value of 'a' such that the function $f(x) = x^2 + ax + 1$ is increasing on [1, 2] and 'S' is the greatest value of 'a' such that the function $f(x) = x^2 + ax + 1$ is decreasing on [1, 2], then the value of |R – S| is ____.

Q.85 The mean of 10 numbers
$7 \times 8, 10 \times 10, 13 \times 12, 16 \times 14, \ldots\ldots$ is _____

Q.86 If the variable line $3x + 4y = \alpha$ lies between the two circles $(x - 1)^2 + (y - 1)^2 = 1$ and $(x - 9)^2 + (y - 1)^2 = 4$, without intercepting a chord on either circle, then the sum of all the integral values of α is _____.

Q.87 The number of six letter words (with or without meaning), formed using all the letters of the word 'VOWELS', so that all the consonants never come together, is _____.

Q.88 If $x\,\phi(x) = \int_5^x (3t^2 - 2\phi'(t)\,dt,)\ x > -2$, and $\phi(0) = 4$, then $\phi(2)$ is _____.

Q.89 If $\left(\frac{3^6}{4^4}\right)k$ is the term, independent of x, in the binomial expansion of $\left(\frac{x}{4} - \frac{12}{x^2}\right)^{12}$ then k is equal to _____.

Q.90 An electric instrument consists of two units. Each unit must function independently for the instrument to operate. The probability that the first unit functions is 0.9 and that of the second unit is 0.8. The instrument is switched on and it fails to operate. If the probability that only the first unit failed and second unit is functioning is p, then 98 p is equal to _____.

JEE MAIN ONLINE PAPER 2021

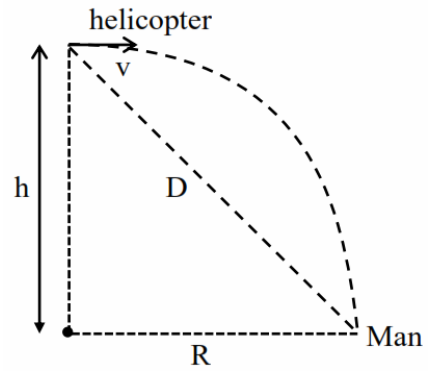
Held on August 31, 2021 (Morning)

Hints & Solutions

PHYSICS

Section -A

1.[3]

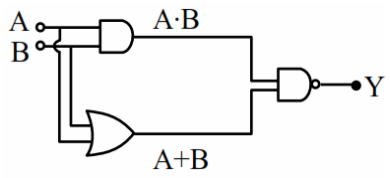


$$R = \sqrt{\frac{2h}{g}}.v$$

$$D = \sqrt{R^2 + h^2} = \sqrt{\left(\sqrt{\frac{2h}{g}}.v\right)^2 + h^2}$$

$$D = \sqrt{\frac{2hv^2}{g} + h^2}$$

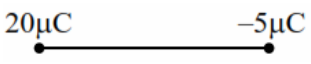
2.[3]



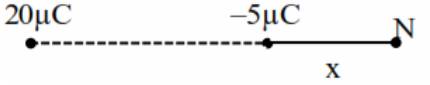
$$Y = \overline{(A.B).(A+B)}$$

$Y)_{(0,0)} = 1$
$Y)_{(0,1)} = 1$
$Y)_{(1,0)} = 1$
$Y)_{(1,1)} = 0$

3.[2]



Null point is possible only right side of –5μC



$$E_N = + \frac{k(-5\mu C)}{x^2} + \frac{k(20\mu C)}{(5+x)^2} = 0$$

$x = 5$

$\therefore$ option (2) is correct

4.[Bonus]

Official Ans. by NTA (1)

T_2 = sink temperature

$$\eta = 1 - \frac{T_2}{T_1}$$

$$\frac{1}{4} = 1 - \frac{T_2}{T_1}$$

$$\frac{T_2}{T_1} = \frac{3}{4} \quad \ldots.(i)$$

$$\frac{1}{2} = 1 - \frac{T_2 - 58}{T_1}$$

$$\frac{T_2}{T_1} - \frac{58}{T_1} = \frac{1}{2}$$

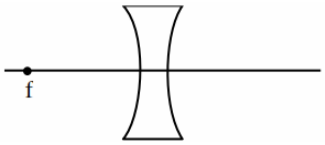
$$\frac{3}{4} = \frac{58}{T_1} + \frac{1}{2}$$

$$\frac{1}{4} = \frac{58}{T_1} \Rightarrow T_1 = 232$$

$$T_2 = \frac{3}{4} \times 232$$

$T_2 = 174$ K

5.[3]



$U = -f$

$$\frac{1}{V} - \frac{1}{U} = \frac{1}{-f} \Rightarrow \frac{1}{V} = \frac{2}{f}$$

$$V = \frac{-f}{2}$$

$$m = \frac{V}{U} = \frac{1}{2}$$

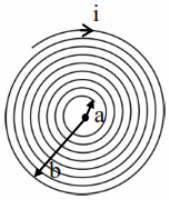
$$\text{distance} = \frac{f}{2}$$

6.[2]

A ⟶ B ⟶ C (stable)

Initially no. of atoms of B = 0 after t = 0, no. of atoms of B will starts increasing & reaches maximum value when rate of decay of B = rate of formation of B. After that maximum value, no. of atoms will starts decreasing as growth & decay both are exponential functions, so best possible graph is (2)

7.[1]

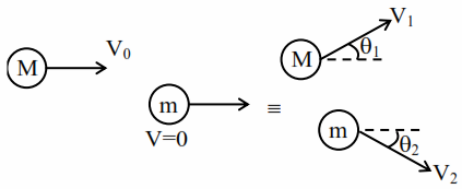


No. of turns in dx width = $\frac{N}{b-a}dx$

$$\int dB = \int_a^b \left(\frac{N}{b-a}\right) dx \ \frac{\mu_0 i}{2x}$$

$$B = \frac{N\mu_0 i}{2(b-a)} \ \ell n\left(\frac{b}{a}\right)$$

8.[3]



given $\theta_1 = \theta_2 = \theta$

from momentum conservation

in x-direction $MV_0 = MV_1 \cos\theta + mV_2 \cos\theta$

in y-direction $0 = MV_1 \sin\theta - mV_2 \sin\theta$

Solving above equations

$V_2 = \frac{MV_1}{m}$, $V_0 = 2V_1 \cos\theta$

From energy conservation

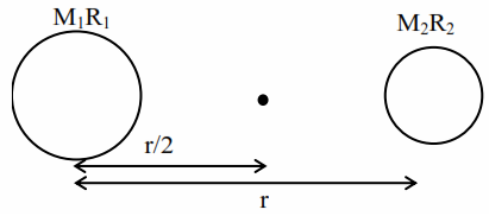
$$\frac{1}{2}MV_0^2 = \frac{1}{2}MV_1^2 + \frac{1}{2}MV_2^2$$

Substituting value of V_2 & V_0, we will get

$$\frac{M}{m} + 1 = 4\cos^2\theta \le 4$$

$$\frac{M}{m} \le 3$$

9.[2]

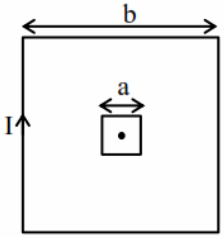


$$\frac{1}{2}mV^2 - \frac{GM_1m}{r/2} - \frac{GM_2m}{r/2} = 0$$

$$\frac{1}{2}mV^2 = \frac{2GM}{r}(M_1 + M_2)$$

$$V = \sqrt{\frac{4G(M_1+M_2)}{r}}$$ Option (2)

10.[1]



$$B = \left[\frac{\mu_0}{4\pi}\frac{I}{b/2} \times 2\sin 45\right] \times 4$$

$$\phi = 2\sqrt{2}\ \frac{\mu_0}{\pi}\ \frac{I}{b} \times a^2$$

$$\therefore M\ \frac{\phi}{I} = \frac{2\sqrt{2}\mu_0 a^2}{\pi b} = \frac{\mu_0}{4\pi}\ 8\sqrt{2}\ \frac{a^2}{b}$$

11.[1] **Official Ans. by NTA (1)**

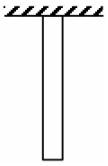
When $V_i > 3$ volt, $V_R > 0$

Because diode will be in reverse biased state

When $V_i \le 3$ Volt ; $V_R > 0$

Because diode will be in reverse biased state.

12.[4]



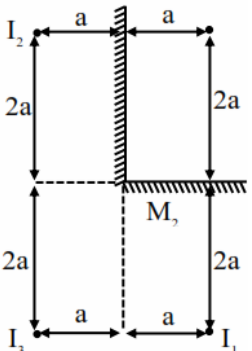
We know,

$$\Delta\ell = \frac{WL}{2AY}$$

$$\Delta\ell = \frac{10\times1}{2\times5} \times 100 \times 10^{-4} \times 2 \times 10^{11}$$

$$\Delta\ell = \frac{1}{2} \times 10^{-9} = 5 \times 10^{-10}\ \text{m}$$

13.[2]



Shortest distance is 2a between I_1 & I_3

But answer given is for I_1 & I_2

$\sqrt{(4a)^2 + (2a)^2}$

$a\sqrt{20}$

4.47 a

14.[1] torque $\tau \rightarrow ML^2T^{-2}$ (III)
Impulse $I \Rightarrow MLT^{-1}$ (I)
Tension face $\Rightarrow MLT^2$ (IV)
Surface tension $\Rightarrow MT^{-2}$ (II)
Option (1)

15.[1] $\int_{p_0}^{p} \frac{dp}{p} = -a \int_0^v dv$

$\ell n \left(\frac{p}{p_0}\right) = -av$

$p = p_0 e^{-av}$

For temperature maximum p-v product should be maximum

$T = \frac{pv}{nR} = \frac{p_0 v e^{-av}}{R}$

$\frac{dT}{dv} = 0 \Rightarrow \frac{p_0}{R}\{e^{-av} + ve^{-av}(-a)\}$

$\frac{p_0 e^{-av}}{R}\{1 - av\} = 0$

$v = \frac{1}{a}, \infty$

$T = \frac{p_0 l}{R a e} = \frac{p_0}{R a e}$

at $v = \infty$

$T = 0$

16.[1] (i) $\frac{\pi p a^4}{8\eta L} = \frac{dv}{dt}$ = Volumetric flow rate (poiseuille's law)

(ii) $h \rho g = \frac{2s}{r}\cos\theta$

(iii) RHS $\Rightarrow \varepsilon \times \frac{1}{4\pi\varepsilon_0} \frac{a}{r^2} \times \frac{1}{\varepsilon} = \frac{q}{t} \times \frac{1}{r^2}$

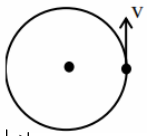
$= \frac{I}{A} IL^{-2}$

LHS

$T = \frac{I}{A} = IL^{-2}$

(iv) $W = \tau\theta$

17.[2]



$|\vec{L}| = mvr$

And direction will be upward & remain constant

18.[3] $Z = \sqrt{(X_L - X_C)^2 + R^2} = R \because X_L = X_C$

19.[1] $\lambda_p = \frac{h}{P_p} \quad \lambda_e = \frac{h}{P_e}$

$\because \lambda_p = \lambda_e$

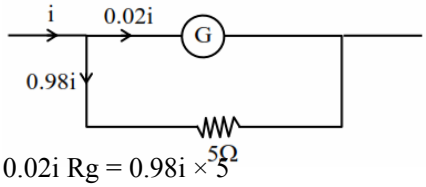
$\Rightarrow P_p = P_e$

$(K)_P = \frac{P_p^2}{2m_e}$

$(K)_e = \frac{P_e^2}{2m_e}$

$K_P < K_e$ as $m_p > m_e$

20.[3]



$0.02i\ Rg = 0.98i \times 5$

$Rg = 245\ \Omega$

Section –B

21.[500]

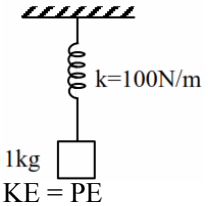
$B = -\frac{\Delta P}{\left(\frac{\Delta V}{V}\right)} = -\frac{\rho g h}{\left(\frac{\Delta V}{V}\right)}$

$-\frac{B\frac{\Delta V}{V}}{\rho g} = h$

$\frac{9.8\times10^8\times0.5}{100\times10^3\times9.8} = h$

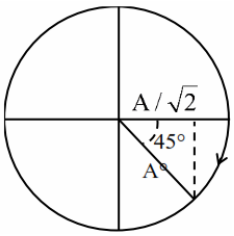
$h = 500$

22.[8]



KE = PE

$y = \frac{A}{\sqrt{2}} = A\sin\omega t$



$t = \frac{T}{8} = \frac{T}{x}$

$x = 8$

23.[64] $h_T = h_R = 160$ …. (i)

$d = \sqrt{2Rh_T} + \sqrt{2Rh_R}$

$d = \sqrt{2R}\left[\sqrt{h_T} + \sqrt{h_R}\right]$

$d = \sqrt{2R}\left[\sqrt{x} + \sqrt{160 - x}\right]$

$\frac{d(d)}{dx} = 0$

$\frac{1}{2\sqrt{x}} - \frac{1(-1)}{2\sqrt{160-x}} = 0$

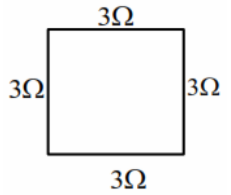
$\frac{1}{\sqrt{x}} + \frac{1}{\sqrt{160-x}} = 0$

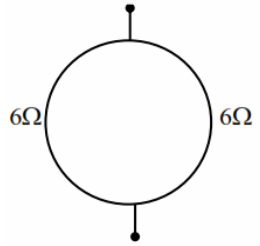
$x = 80$ m

$d_{max} = \sqrt{2\times 6400}\left[\frac{\sqrt{80}}{1000} + \frac{\sqrt{20}}{1000}\right]$

$= \frac{80\sqrt{2}\times 2\sqrt{80}}{10\sqrt{10}} = 8 \times 2 \times \sqrt{2} \times 2\sqrt{2} = 64$ km

24.[3]





$R_{eq} = 3\Omega$

25.[10] $\mu = 9.0\ 10^{-4}\ \frac{kg}{m}$

$T = 900$ N

$V = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{900}{9\times 10^{-4}}} = 1000$m/s

$f_l = 500$ Hz

$f = 550$

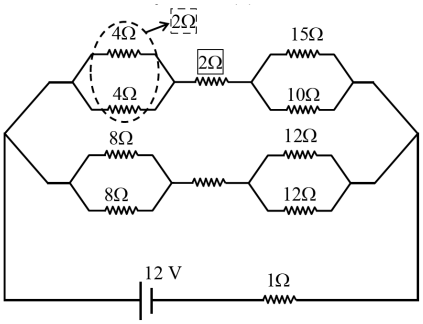
$\frac{nV}{2\ell} = 500$ … (i)

$\frac{(n+1)V}{2\ell} = 500$ …(ii)

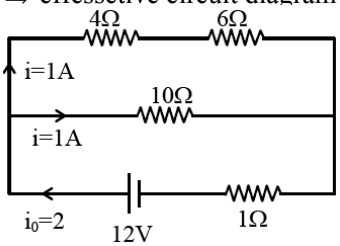
(ii) (i) $\frac{V}{2\ell} = 50$

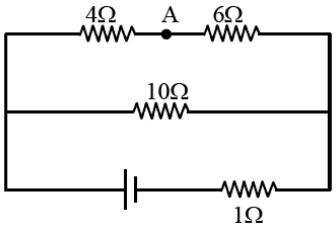
$\ell = \frac{1000}{2\times 50} = 10$

26.[6]

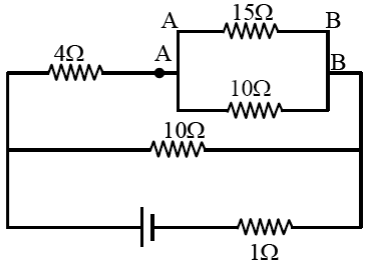


⇒ effessctive circuit diagram will be





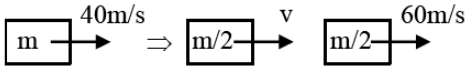
⇒



Point drop across $6\Omega = 1 \times 6 = 6 = V_{AB}$

⇒ Hence point drop across $15\Omega = 6$ volt $= V_{AB}$

27.[1]



$P_i = P_f$

$m \times 40 = \frac{m}{2} \times v + \frac{m}{2} \times 60$

$40 = \frac{v}{2} + 30$

$\Rightarrow v = 20$

$(K.E.)_I = \frac{1}{2}\ m \times (40)^2 = 800$m

$(K.E.)_f = \frac{1}{2}\ \frac{m}{2}.(20)^2 + \frac{1}{2}.\frac{m}{2}\ (60)^2 = 1000$ m

$|\Delta K.E.| = |1000\text{m} - 800\text{m}| = 200$m

$\frac{\Delta K.E}{(K.E.)_i} = \frac{200m}{800m} = \frac{1}{4} = \frac{x}{4}$

$x = 1$

28.[500] $E = 50 \sin\left(\omega t - \frac{\omega}{c}.x\right)$

Energy density $= \frac{1}{2} \in_0 E_0^2$

Energy for volume $V = \frac{1}{2} \in_0 E_0^2 .V$

$= 5.5 \times 10^{-12}$

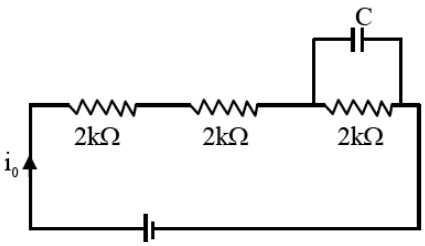
$\frac{1}{2} 8.8 \times 10^{-12} \times 2500\ V = 5.5 \times 10^{-12}$

$V = \frac{5.5 \times 2}{2500 \times 8.8} = 0.005 m^2$

$= .0005 \times 10^6$ (c. m)^3

$= 500$ (c. m)^3

29.[100]



$i_0 = 1$ mA 6V

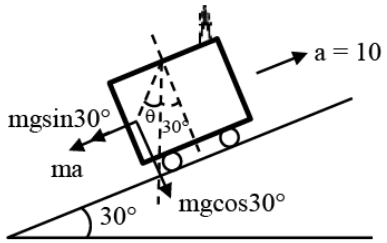
Pot. Diff. across each resistor = 2V

$q = CV$

$= 50 \times 10^{-6} \times 2$

$= 100 \times 10^{-6} = 100\ \mu C$

30.[30]



$\tan(30 + \theta) = \frac{mg\sin 30° + ma}{mg\cos 30°}$

$\tan(30 + \theta) = \frac{5+10}{5\sqrt{3}} = \frac{1+2}{\sqrt{3}}$

$\frac{\tan\theta + \frac{1}{\sqrt{3}}}{1 - \frac{1}{\sqrt{3}}\tan\theta} = \sqrt{3}$

$\sqrt{3}\tan\theta + 1 = 3 - \sqrt{3}\tan\theta$

$2\sqrt{3}\tan\theta = 2$

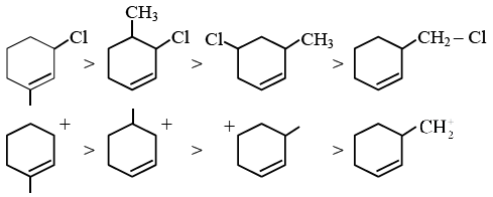
$\tan\theta = \frac{1}{\sqrt{3}}$

$\theta = 30°$

CHEMISTRY

Section -A

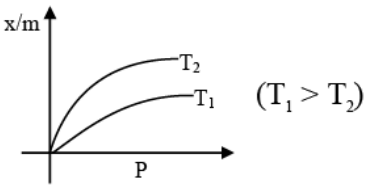
31.[1] As it is example of SN^1.
so carbocation stability ↑, reaction rate ↑



32.[4] $\frac{x}{m} \alpha P^{1/n} \left(0 < \frac{1}{n} < 1\right)$

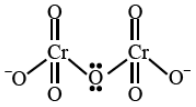
On Increasing temperature $\frac{x}{m}$ decreases.

∵ adsorption is generally exothermic



33.[2] Major component of portland cement is "Tricalcium silicate (51% , 3CaO.SiO2)

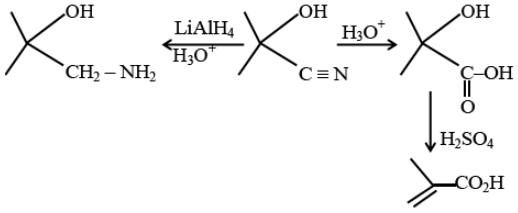
34.[2]



dichromate ion contain non-linear symmetrical Cr–O–Cr Bond

35.[1] In Lactose it is β $C_1 - C_4$ glycosidic linkage.
In Maltose, Amylose α $C_1 - C_4$ glycosidic linkage is present

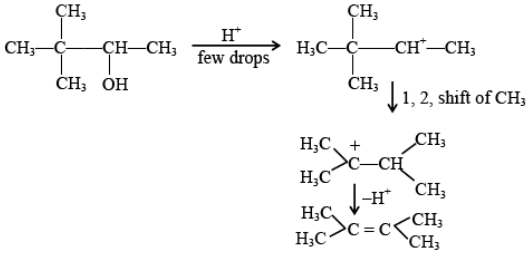
36.[3]



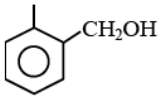
37.[2] Ytterbium shows +2 oxidation state with diamagnetic nature
So ans is 2

38.[4] (A) Aluminium is reactive metal so Alusminium is extracted by electrolysis of Alumina with molten mixture of Cryolite
(B) Cryolite, Na_3AlF_6
Here Al is in +3 O.S.
So Answer is 4

39.[2]



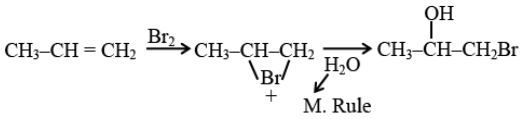
40.[3] Monomer of Novolac is



O-hydroxy methyl phenol

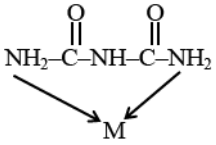
41.[2] The process of producing syn-gas from coal is called gasification of coal.
Syn-gas having composition of CO & H_2 in 1 : 1

42.[3]



Its IUPAC name 1-bromopropan–2-ol
A and R are true and (R) is the correct explanation of (A)

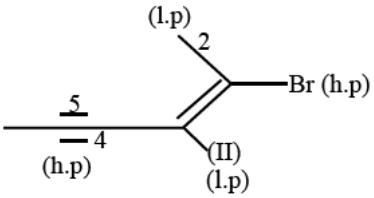
43.[1]



Biuret :- Bidentate ligand
The denticity of organic ligand is 2.

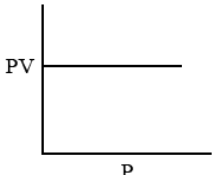
44.[2] From left to right in periodic table :-
Metallic character decreases
Non-metallic character increases
$\Rightarrow$ It is due to increase in ionization enthalpy and increase in electron gain enthalpy.

45.[3]



h.p. $\Rightarrow$higher priority
l.p. $\Rightarrow$ lower priority
2E –2- bromo hex –2- en–4-yne

46.[1] PV = nRT (n,T constant)
PV = constant

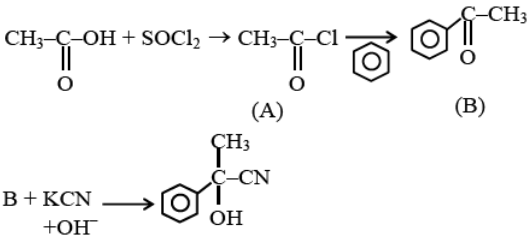


47.[4] Both assertion & reason are correct & **(R)** is the correct explanation of **(A)**

48.[4] $\because$ Van't Hoff factor is highest for $KHSO_4$
$\therefore$ colligative property (ΔT_f) will be highest for $KHSO_4$

49.[3] BOD values of clean water (A) is less than 5 ppm
So A < 5
BOD values of polluted water (B is greater than 17 ppm
So B > 17
So Ans. is 3

50.[1]



Section -B

51.[25] $\Delta G^\circ = -nFE^\circ = \Delta H^\circ - T\Delta S^\circ$

$$= \frac{\Delta H^\circ + nFE^\circ}{T}$$

$$= \frac{(-825.2\times10^3) + (2\times96487\times4.315)}{298}$$

$$= \frac{-825.2\times10^3 + 832.682\times10^3}{298}$$

$$= \frac{7.483\times10^3}{298}$$

$= 25.11\ JK^{-1}\ mol^{-1}$

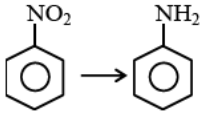
$\therefore$ Nearest integer answer is 25

52.[20] $[H_2C_2O_4.2H_2O] = \frac{weight/M_w}{V(L)}$

$\Rightarrow x \times 10^{-2} = \frac{6.3/126}{250/1000}$

x = 20

53.[4] Pbs, CuS_3 As_2S_3, CdS are soluble in 50% HNO_3 HgS, Sb_2S_3 are insoluble in 50% HNO_3
So answer is 4 _____.

54.[5]  Reagents used can be

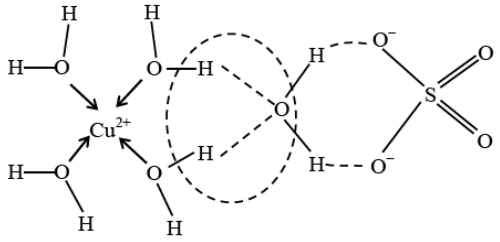
(i) Sn + HCl
(ii) Fe + HCl
(iii) Zn + HCl
(iv) H_2 – Pd
(v) H_2 (Raney Ni)

55.[3] The number of halogen forming halic (V) acid
$HClO_3$
$HBrO_3$
HIO_3
So Answer is 3

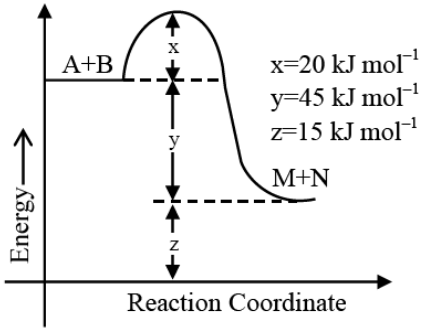
56.[2] $k = \frac{2.303}{t}\log\frac{a}{a-x}$

$$\frac{2.303}{t_{50\%}}\log\frac{100}{100-50} = \frac{2.303}{t_{75\%}}\log\frac{100}{100-50}$$

$t_{75\%} = 2t_{50\%}$

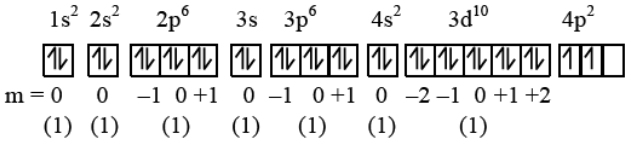
57.[1] 

One hydrogen bonded H_2O molecule

58.[45] 

$\Delta H = E_{a_f} - E_{a_b}$
$= 20 - 65$
$= -45$ KJ/ mol
$|\Delta H| = 45$ KJ/mol

59.[7]



Completely filled orbital with $m_\ell = 0$ are
$= 1+1+1+1+1+1+1$
$= 7$
So Answer is 7

60.[108] $A_3B_2\,(s) \rightleftharpoons 3\,A^{+2}_{(aq)} + 2\,B^{-3}_{(aq)}$

3s 2s

$K_{sp} = (3s)^3\,(2s)^2$
$K_{sp} = 108\,S^5$ & $s = (X/M)$

$$K_{sp} = 108\left(\frac{x}{m}\right)^5$$

given $K_{sp} = a\left(\frac{x}{m}\right)^5$

comparing a = 108

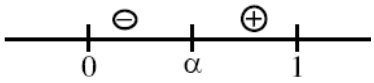
MATHEMATICS

Section -A

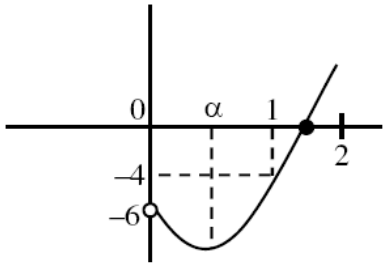
61.[3] $(p \wedge \sim q) \rightarrow (p \vee q)$ is tautology

p	q	~q	p∧~ q	p ∨ q	(p ∧ ~ q) → (p ∨ q)
T	T	F	F	T	T
T	F	T	T	T	T
F	T	F	F	T	T
F	F	T	F	F	T

62.[3] Let $e^x = t > 0$
$f(t) = t^4 + 2t^3 - t - 6 = 0$
$f'(t) = 4t^3 + 6t^2 - 1$



$f''(t) = 12t^2 + 12t > 0$



$f(0) = -6, f(1) = -4, f(2) = 24$
$\Rightarrow$ Number of real roots = 1

63.[2] $S\ \frac{2^2-1^2}{1^2\times 2^2}+\frac{3^2-2^2}{2^2\times 3^2}+\frac{4^2-3^2}{3^2\times 4^2}+\ldots.$

$=\left[\frac{1}{1^2}-\frac{1}{2^2}\right]+\left[\frac{1}{2^2}-\frac{1}{3^2}\right]+\left[\frac{1}{3^2}-\frac{1}{4^2}\right]+\ldots$

$\ldots\ldots+\left[\frac{1}{10^2}-\frac{1}{11^2}\right]$

$=1-\frac{1}{121}$

$=\frac{120}{121}$

64.[1] Equation of plane is

$3x-2y+4z-7+\lambda(x+5y-2z+9)=0$

$(3+\lambda)x+(5\lambda-2)y+(4-2\lambda)z+9\lambda-7=0$

passing through $(1, 4, -3)$

$\Rightarrow 3+\lambda+20\lambda-8-12+6\lambda+9\lambda-7=0$

$\Rightarrow \lambda=\frac{2}{3}$

$\Rightarrow$ equation of plane is

$-11x-4y-8z+3=0$

$\Rightarrow \alpha+\beta+\gamma=-23$

65.[4] $\int_0^x\sqrt{1-(f'(t)^2)}\,dt=\int_0^x f(t)dt\ 0\le x\le 1$

differentiating both the sides

$\sqrt{1-(f'(x)^2)}=f(x)$

$\Rightarrow 1-(f'(x))^2=f^2(x)$

$\frac{f'(x)}{\sqrt{1-f^2(x)}}=1$

$\sin^{-1}f(x)=x+C$

$\because f(0)=0\Rightarrow C=0\Rightarrow f(x)=\sin x$

Now $\lim\limits_{x\to 0}\frac{\int_0^x \sin t\,dt}{x^2}\left(\frac{0}{0}\right)=\frac{1}{2}$

66.[3] $\left|3\vec{a}+\vec{b}\right|^2=\left|2\vec{a}+3\vec{b}\right|^2$

$\left(3\vec{a}+\vec{b}\right).\left(3\vec{a}+\vec{b}\right)=\left(2\vec{a}+3\vec{b}\right).\left(2\vec{a}+3\vec{b}\right)$

$9\vec{a}.\vec{a}+6\vec{a}.\vec{b}+\vec{b}.\vec{b}=4\vec{a}.\vec{a}+12\,\vec{a}.\vec{b}+9\vec{b}.\vec{b}$

$5|\vec{a}|^2-6\vec{a}.\vec{b}=8|\vec{b}|^2$

$5(8)^2-6.8.|\vec{b}|\cos 60^\circ=8|\vec{b}|^2\quad\left(\because \frac{1}{8}|\vec{a}|=1\Rightarrow|\vec{a}|=8\right)$

$40-3|\vec{b}|=|\vec{b}|^2$

$\Rightarrow |\vec{b}|^2+3|\vec{b}|-40=0$

$|\vec{b}|=-8,\ |\vec{b}|=5$

(rejected)

57.[3] $f(x)=|(x-3)(x+1)|.\ e^{(3x-2^2)}$

$$f(x)=\begin{cases}(x-3)(x-1).e^{(3x-2^2)} & ;x\in(3,\infty)\\ -(x-3)(x-1).e^{(3x-2^2)} & ;x\in[-1,3]\\ (x-3).(x+1).e^{(3x-2^2)} & ;x\in(-\infty,-1)\end{cases}$$

Clearly, non-differentiable at $x=-1$ & $x=3$.

68.[2] Let numbers be $\frac{a}{r}$, a, ar $\rightarrow$ G.P

$\frac{a}{r}$, 2a, ar $\rightarrow$ A.P $\Rightarrow 4a=\frac{a}{r}+ar\Rightarrow r+\frac{1}{r}=4$

$r=2\pm\sqrt{3}$

4^{th} form of G.P $=3r^2\Rightarrow ar^2=3r^2\Rightarrow a=3$

$r=2+\sqrt{3}$, $a=3$, $d=2a-\frac{a}{r}=3\sqrt{3}$

$r^2-d=\left(2+\sqrt{3}\right)^2-3\sqrt{3}$

$=7+4\sqrt{3}-3\sqrt{3}$

$=7+\sqrt{3}$

69.[2] Note that (1,2) and (2,3) satisfy $0<|x-y|\le 1$ but (1,3) does not satisfy it so

$0\le|x-y|\le 1$ is symmetric but not transitive

So, (2) is correct.

70.[3] $\int\frac{dx}{(x-1)^{3/4}(x+2)^{5/4}}$

$=\int\frac{dx}{\left(\frac{x+2}{x-1}\right)^{5/4}.(x-1)^2}$

put $\frac{x+2}{x-1}=t$

$=-\frac{1}{3}\int\frac{dt}{t^{5/4}}$

$= \frac{4}{3}\cdot\frac{1}{t^{1/4}} + C$

$= \frac{4}{3}\left(\frac{x-1}{x+2}\right)^{1/4} + C$

71.[1] First line is $\frac{x}{\sin\alpha} - \frac{y}{\cos\alpha} = \frac{k\cos 2\alpha}{\sin 2\alpha}$

$\Rightarrow x\cos\alpha - y\sin\alpha = \frac{k}{2}\cos 2\alpha$

$\Rightarrow p = \left|\frac{k}{2}\cos\alpha\right| \Rightarrow 2p = |k\cos 2\alpha| \quad ....(i)$

second line is $x\sin\alpha + y\cos\alpha = k\sin 2\alpha$
$\Rightarrow q = |k\sin 2\alpha l| \quad .... (ii)$
Hence $4p^2 + q^2 = k^2$ (From (i) & (ii))

72.[4] $\text{cosec } 18° = \frac{1}{\sin 18°} = \frac{4}{\sqrt{5}-1} = \sqrt{5}+1$

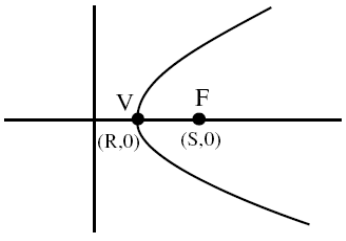
Let $\text{cosec } 18° = x = \sqrt{5} + 1$
$\Rightarrow x - 1 = \sqrt{5}$
Squaring both sides, we get
$x^2 - 2x + 1 = 5$
$\Rightarrow x^2 - 2x - 4 = 0$

73.[4] Here $D = \begin{vmatrix} 2 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & a \end{vmatrix} = 2(-a-1) - 1(a-1) + 1 + 1 = 1 - 3b$

$D_3 = \begin{vmatrix} 2 & 1 & 5 \\ 1 & -1 & 3 \\ 1 & 1 & b \end{vmatrix} = 2(-b-3) - 1(b-3) + 5(1+1) = 7 - 3b$

for $a \frac{1}{3}$, $b \neq \frac{7}{3}$, system has no solutions

74.[3]



$V \rightarrow$ Vertex
$F \rightarrow$ focus
$VF = S - R$
So latus rectum $= 4(S - R)$

75.[1] If $f(x)$ is continuous at $x = 0$,
RHL = LHL = $f(0)$

$\lim\limits_{x\to 0^+} f(x) = \lim\limits_{x\to 0^+} \frac{\cos^2 x - \sin^2 x - 1}{\sqrt{x^2+1}-1}\cdot\frac{\sqrt{x^2+1}+1}{\sqrt{x^2+1}+1}$

(Rationalisation)

$\lim\limits_{x\to 0^+} -\frac{2\sin^2 x}{x^2}\cdot\left(\sqrt{x^2+1}+1\right) = -4$

$\lim\limits_{x\to 0^-} f(x) = \lim\limits_{x\to 0^-} \frac{1}{x}\,\ell n\left(\frac{1+\frac{x}{a}}{1-\frac{x}{b}}\right)$

$\lim\limits_{x\to 0^-} \frac{\ell n\left(1+\frac{x}{a}\right)}{\left(\frac{x}{a}\right).a} + \frac{\ell n\left(1+\frac{x}{b}\right)}{\left(-\frac{x}{b}\right).b} = \frac{1}{a} + \frac{1}{b}$

So $\frac{1}{a} + \frac{1}{b} = -4 = k$

$\frac{1}{a} + \frac{1}{b} + \frac{4}{k} = -4 - 1 = -5$

76.[2] $\frac{dy}{dx} = \frac{2^x 2^y - 2^x}{2y}$

$2^y\frac{dy}{dx} = 2^x(2^y - 1)$

$\int\frac{2^y}{2^y-1}dy = \int 2^x\,dx$

$\frac{\ell n(2^y-1)}{\ell n 2} = \frac{2^x}{\ell n 2} + C$

$\Rightarrow \log_2(2^y - 1) = 2^x\log_2 e + C$
$\because y(0) = 1 \Rightarrow 0 = \log_2 e + C$
$C = -\log_2 e$
$\Rightarrow \log_2(2^y - 1) = (2^x - 1)\log_2 e$
put $x = 1$, $\log_2(2^y - 1) = \log_2 e$
$2^y = e + 1$
$y = \log_2(e + 1)$ Ans.

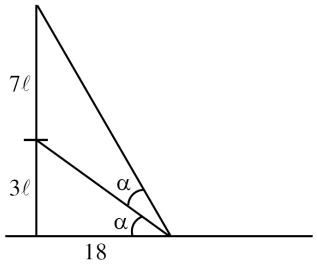
77.[3] $\lim\limits_{x\to 0}\frac{\sin^2(\pi\cos^4 x)}{x^4}$

$\lim\limits_{x\to 0}\frac{1-\cos(2\pi\cos^4 x)}{2x^4}$

$\lim\limits_{x\to 0}\frac{1-\cos(2\pi - 2\pi\cos^4 x)}{[2\pi(1-\cos^4 x]^2)}\,4\pi^2.\,\frac{\sin^4 x}{2x^4}(1+\cos^2 x^2)$

$= \frac{1}{2}\cdot 4\pi^2\cdot\frac{1}{2}(2)^2 = 4\pi^2$

78.[2]



Let height of pole = l0ℓ

$\tan\alpha = \frac{3\ell}{18} = \frac{\ell}{6}$

$\tan 2\alpha = \frac{10\ell}{18}$

$\frac{2\tan\alpha}{1-\tan^2\alpha} = \frac{10\ell}{18}$

use $\tan\alpha = \frac{\ell}{6} \Rightarrow \sqrt{\frac{72}{5}}$

height of pole = $10\ell = 12\sqrt{10}$

79.[3] $a_r = e^{\frac{i2\pi r}{9}}$, r = 1, 2, 3, …. a_1, a_2, a_3 …. are in G.P.

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ a_n & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{vmatrix} = \begin{vmatrix} a_1 & a_2^2 & a_1^3 \\ a_1^4 & a_1^5 & a_1^6 \\ a_1^7 & a_1^8 & a_1^9 \end{vmatrix} = a_1.a_1^4.a_1^7$$

$$\begin{vmatrix} 1 & a_1 & a_1^2 \\ 1 & a_1 & a_1^2 \\ 1 & a_1 & a_1^2 \end{vmatrix} = 0$$

Now $a_1a_9 - a_3a_7 = a_1^{10} - a_1^{10} = 0$

80.[2] $12x\cos\theta + 5y\sin\theta = 60$

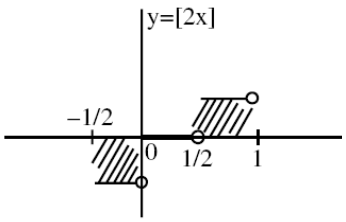
$\frac{x\cos\theta}{5} + \frac{y\sin\theta}{12} = 1$

is tangent to $\frac{x^2}{25} + \frac{y^2}{144} = 1$

$144x^2 + 25y^2 = 3600$

Section –B

81.[5] $I = \int_{-\frac{1}{2}}^{1} ([2x] + |x|\,dx)$



$= \int_{-1/2}^{1} [2x]dx + \int_{-1/2}^{1} [x]dx\,dx + \int_{0}^{1} x\,dx$

$0 + \int_{-1/2}^{0} (-x)\,dx + \int_{0}^{1} x\,dxss$

$= \left(-\frac{x^2}{2}\right)_{-1/2}^{0} + \left(\frac{x^2}{2}\right)_{0}^{1}$

$= \left(0 + \frac{1}{8}\right) + \frac{1}{2} = \frac{5}{8}$

$8I = 5$

82.[98] Let z = x + iy

$\arg\left(\frac{x-2+iy}{x+2+iy}\right) = \frac{\pi}{4}$

are (x –2 + iy) – age (x + 2 + iy) = $\frac{\pi}{4}$

$\tan^{-1}\left(\frac{y}{x-2}\right) - \tan^{-1}\left(\frac{y}{x+2}\right) = \frac{\pi}{4}$

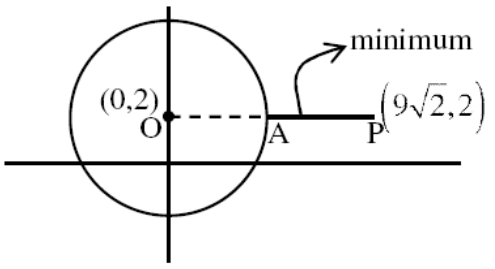
$\frac{\frac{y}{x-2} - \frac{y}{x+2}}{1+\left(\frac{y}{x-2}\right)\left(\frac{y}{x+2}\right)} = \tan\frac{\pi}{4} = 1$

$\frac{xy + 2y - xy + 2y}{x^2 - 4 + y^2} = 1$

$4y = x^2 - 4 + y^2$

$x^2 + y^2 - 4y - 4 = 0$

locus is a circle with center (0, 2) & radius = $2\sqrt{2}$



min. value = $(AP)^2 = (OP - OA)^2$

$= (9\sqrt{2} - 2\sqrt{2})^2$

$= (7\sqrt{2})^2 = 98$

83.[61] $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z+1}{6} = \lambda$

$x = 2\lambda + 1$, $y = 3\lambda + 2$, $z = 6\lambda - 1$

for point of intersection of line & plane

$2(2\lambda + 1) - (3\lambda + 2) + (6\lambda - 1) = 6$

$7\lambda = 7 \Rightarrow \lambda = 1$

point : (3. 5, 5)
$(\text{distance})^2 = (3+1)^2 + (5+1)^2 + (5-2)^2$
$= 16 + 36 + 9 = 61$

84.[2] $f(x) = x^2 + ax + 1$
$f'(x) = 2x + a$
when f(x) is increasing on [1, 2]
$2x + a \geq 0 \quad \forall\ x \in [1, 2]$
$a \geq -2x\ \forall\ x \in [1, 2]$
$R = -4$
when f(x) is decreasing on [1, 2]
$2x + a \leq 0 \quad \forall\ x \in [1, 2]$
$a \leq -2x\ \forall\ x \in [1, 2]$
$S = -2$
$|R - S| = |-4 + 2| = 2$

85.[398] 7 x 8, 10 × 10, 13 × 12, 16 × 14 …..
$T_n = (3n + 4)(2n + 6) = 2(3n + 4)(n + 3)$
$= 2(3n^2 + 13n + 12) = 6n^2 + 26n + 24$

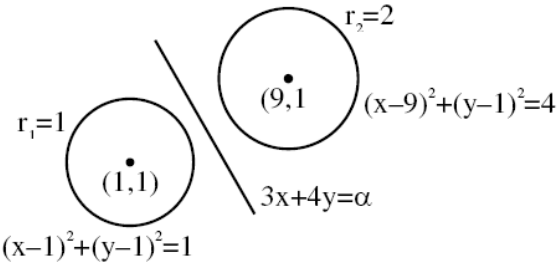
$$S_{10} = \sum_{n=1}^{10} T_n = 6\sum_{n=1}^{10} n^2 + 26\sum_{n=1}^{10} n + 24\sum_{n=1}^{10} 1$$

$$= \frac{6(10\times11\times21)}{6} + 26 \times \frac{10\times11}{2} + 24 \times 10$$

$= 10 \times 11\ (21 + 13) + 240$
$= 3980$

$$\text{Mean} = \frac{S_{10}}{10} = \frac{3980}{10} = 398$$

86.[165]



Both centres should lie on either side of the line as well as line can be tangent to circle.
$(3 + 4 - \alpha).(27 + 4 - \alpha) < 0$
$(7 - \alpha).(31 - \alpha) < 0 \Rightarrow \alpha \in (9, 31) \quad \ldots (1)$
d_1 = distance of (1, 1) from line
d_2 = distance of (9, 1) from line

$$d_1 \geq r_1 \Rightarrow \frac{|7-\alpha|}{5}\ 1$$

$\Rightarrow \alpha \in (-\infty, 2] \cup [12, \infty) \quad \ldots (2)$

$$d_2 \geq r_2 \Rightarrow \frac{|31-\alpha|}{5} \geq 2$$

$\Rightarrow \alpha \in (-\infty, 21] \cup [41, \infty) \quad \ldots.(3)$
$(1) \cap (2) \cap (3) \Rightarrow \alpha \in [12, 21]$
Sum of integers = 165

87.[576] VOWELS → 2 Vowels, 4 Consonants
All Consonants should not be together
= Total – All consonants together,
$= 6! - 3!\ 4! = 576$

88.[4] $x\,\phi(x) = \int_5^x (3t^2 - 2\phi'(t)\,dt,)$
$x\,\phi(x) = x^2 - 125 - 2[\phi(x) - \phi(5)]$
$x\,\phi(x) = x^3 - 125 - 2\phi(x) - 2\phi(5)$

$$\phi(0) = 4 \Rightarrow \phi(5) = \frac{133}{2}$$

$$\phi(x) = \frac{x^3+8}{x+2}$$

$\phi(2) = 4$

89.[55] $\left(\frac{x}{4} - \frac{12}{x^2}\right)^{12}$

$$T_{r+1} = (-1)^r\ .\ {}^{12}C_r \left(\frac{x}{4}\right)^{12-r} \left(\frac{12}{x^2}\right)^r$$

$$T_{r+1} = (-1)^r\ .\ {}^{12}C_r \left(\frac{1}{4}\right)^{12-r} (12)^r.\ (x)^{12-3r}$$

Term independent of $x \Rightarrow 12 - 3r = 0 \Rightarrow r = 4$

$$T_5 = (-1)^4\ .\ {}^{12}C_4 \left(\frac{1}{4}\right)^8 (12)^4 = \frac{3^6}{4^4}.k$$

$\Rightarrow k = 55$

90.[28] I_1 first unit is functioning
I_2 = second unit is functioning
$P(I_1) = 0.9,\ P(I_2) = 0.8$
$P(\bar{I}_1) = 0.1,\ P(\bar{I}_2) = 0.2$

$$P = \frac{0.8\times0.1}{0.1\times0.2 + 0.9\times0.2 + 0.1\times0.8} = \frac{8}{28}$$

$$98P = \frac{8}{28} \times 98 = 28$$