JEE MAIN ONLINE PAPER 2021

Held on July 27, 2021 (Morning)

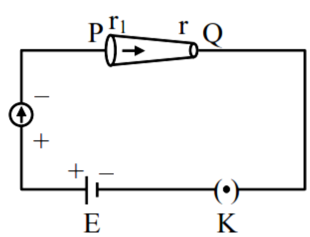
Instructions

1. This test will be a 3 hours Test.
2. This test consists of Physics, Chemistry and Mathematics questions with equal weightage of 100 marks.
3. Each question is of 4 marks.
4. In the question paper consisting of Physics (Q.no. 1 to 30), Chemistry (Q.no. 31 to 60) and Mathematics (Q.no. 61 to 90). There are two sections for each subject (Section-A : MCQ Type & Section-B : Numerical Response Type). Section-A consists of 20 multiple choice questions & Section-B consists of 10 Numerical Value type Questions. **Candidates have a choice to Answer 5 out of the 10 numerical value answer based questions per section.**
5. There will be only one correct choice in the given four choices in Section-A. For each question 4 marks will be awarded for correct choice, 1 mark will be deducted for incorrect choice and zero mark will be awarded for not attempted question. For Section-B questions 4 marks will be awarded for correct answer and zero for unattempted and incorrect answer.
6. Any textual, printed or written material, mobile phones, calculator etc. is not allowed for the students appearing for the test.
7. All calculations/written work should be done in the rough sheet provided.

PHYSICS

Section -A

Q.1 In the given figure, a battery of emf E is connected across a conductor PQ of length '*l*' and different area of cross-sections having radii r_1 and r_2 ($r_2 < r_1$).



Choose the correct option as one moves from P to Q :

(1) Drift velocity of electron increases.
(2) Electric field decreases.
(3) Electron current decreases.
(4) All of these

Q.2 The number of molecules in one liitre of an ideal gas at 300 K and 2 atmospheric pressure with mean kinetic energy 2×10^{-9} J per molecules is :

(1) 0.75×10^{11} (2) 3×10^{11}
(3) 1.5×10^{11} (4) 6×10^{11}

Q.3 The relative permittivity of distilled water is 81. The velocity of light in it will be :
(Given $\mu_r = 1$)

(1) 4.33×10^7 m/s (2) 2.33×10^7 m/s
(3) 3.33×10^7 m/s (4) 5.33×10^7 m/s

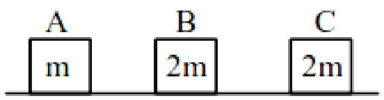
Q.4

List – I	List – II
(a) MI of the rod (length L, Mass M, about an axis ⊥ to rod passing through the midpoint)	(i) $8\ ML^2/3$
(b) MI of the rod (length L, Mass 2M, about an axis ⊥ to the rod passing through one of its end)	(iii) $ML^2/3$
(c) MI of the rod (length 2L, Mass M, about an axis ⊥ to the rod passing through its midpoint one of its end)	(iii) $ML^2/12$
(d) MI of the rod (Length 2 L, Mass 2 M, about an axis ⊥ to the rod passing through one of its end)	(iv) $2\ ML^2/3$

Choose the correct answer from the options given below :

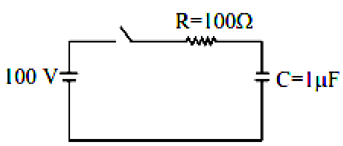
(1) (a)–(ii), (b)–(iii), (c)– (i), (d)–(iv)
(2) (a)–(ii), (b)–(i), (c)– (iii), (d)–(iv)
(3) (a)–(iii), (b)–(iv), (c)– (ii), (d)–(i)
(4) (a)–(iii), (b)–(iv), (c)– (i), (d)–(ii)

Q.5 Three objects A, B and C are kept in a straight line on a frictionless horizontal surface. The masses of A, B and C are m, 2 m and 2 m respectively. A moves towards B with a speed of 9 m/s and makes an elastic collision with it. Thereafter B makes a completely inelastic collision with C. All motions occur along same straight line. The final speed of C is :



(1) 6 m/s (2) 9 m/s
(3) 4 m/s (4) 3 m/s

Q.6

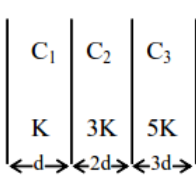


A capacitor of capacitance C = 1 μF is suddenly connected to a battery of 100 volt through a resistance R = 100 Ω. The time taken for the capacitor to be charged to get 50 V is :

[Take ln 2 = 0.69]

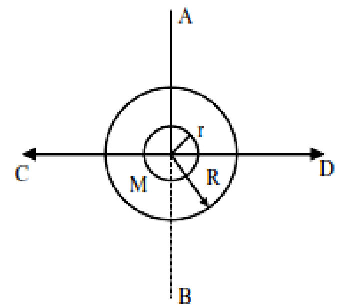
(1) 1.44×10^{-4} s (2) 3.33×10^{-4} s
(3) 0.69×10^{-4} s (4) 0.30×10^{-4} s

Q.7 In the reported figure, a capacitor is formed by placing a compound dielectric between the plates of parallel plate capacitor. The expression for the capacity of the said capacitor will be : (Given area of plate = A)



(1) $\frac{15}{34}\frac{K\varepsilon_0 A}{d}$ (2) $\frac{15}{6}\frac{K\varepsilon_0 A}{d}$
(3) $\frac{25}{6}\frac{K\varepsilon_0 A}{d}$ (4) $\frac{9}{6}\frac{K\varepsilon_0 A}{d}$

Q.8 The figure shows two solid discs with radius R and r respectively. If mass per unit area is same for both, what is the ratio of MI of bigger disc around axis AB (Which is ⊥ to the plane of the disc and passing through its centre) of MI of smaller disc around one of its diameters lying on its plane ? Given 'M' is the mass of the larger disc. (MI stands for moment of inertia)

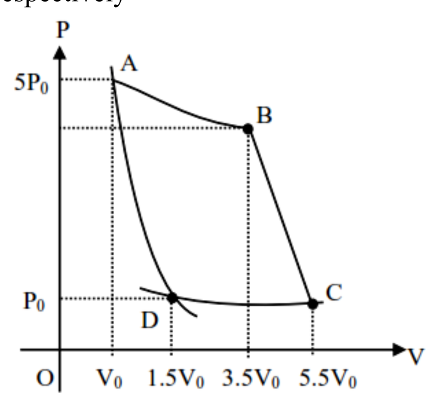


(1) $R^2 : r^2$ (2) $2r^4 : R^4$
(3) $2R^2 : r^2$ (4) $2R^4 : r^4$

Q.9 In Young's double slit experiment, if the source of light changes from orange to blue then :

(1) the central bright fringe will become a dark fringe.
(2) the distance between consecutive fringes will decrease.
(3) the distance between consecutive fringes will decrease.
(4) the intensity of the minima will increase.

Q.10 In the reported figure, there is a cyclic process ABCDA on a sample of 1 mol of a diatomic gas. The temperature of the gas during the process A → B and C → D are T_1 and T_2 ($T_1 > T_2$) respectively

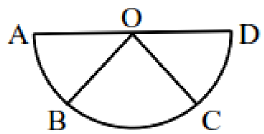


(1) $W_{AB} = W_{DC}$ (2) $W_{AD} = W_{BC}$
(3) $W_{BC} = W_{DA} > 0$ (4) $W_{AB} < W_{CD}$

Q.11 **Assertion A :** If A, B, C, D are four points on a semi-circular arc with center at 'O' such that $|\overrightarrow{AB}| = |\overrightarrow{BC}| = |\overrightarrow{CD}|$, then

$\overrightarrow{AB} + \overrightarrow{AC} + \overrightarrow{AD} = 4\overrightarrow{AO} + \overrightarrow{OB} + \overrightarrow{OC}$

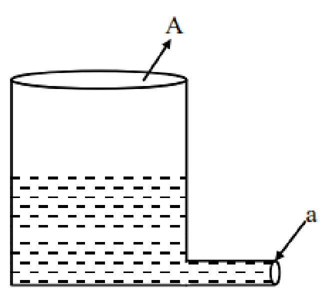
Reason R : Polygon law of vector addition yields $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} + \overrightarrow{AD} = 2\overrightarrow{AO}$



In the light of the above statements choose the **most appropriate** answer from the options given below :

(1) **A** is correct but **R** in not correct
(2) **A** is not correct but **R** is correct.
(3) Both **A** and **R** are correct and **R** is the correct explanation of **A**.
(4) Both **A** and **R** are correct but **R** is not the correct explanation of **A**.

Q.12 A light cylindrical vessel is kept on a horizontal surface. Area of base is A. A hole of cross-sectional area 'a' is made just at its bottom side. The minimum coefficient of friction necessary to prevent sliding the vessel due to the impact force of the emerging liquid is (a < < A) :



(1) $\frac{A}{2a}$ (2) None of these

(3) $\frac{2a}{A}$ (4) $\frac{a}{A}$

Q.13 A particle starts executing simple harmonic motion (SHM) of amplitude 'a' and total energy E. At any instant, its kinetic energy is $\frac{3E}{4}$ then its displacement 'y' is given by :

(1) y = a (2) $y = \frac{a}{\sqrt{2}}$

(3) $y = \frac{a\sqrt{3}}{2}$ (4) $y = \frac{a}{2}$

Q.14 If 'f' denotes the ratio of the number of nuclei decayed (N_d) to the number of nuclei at t = 0 (N_0) then for a collection of radioactive nuclei, the rate of change of 'f' with respect to time is given as :

[λ is the radioactive decay constant]

(1) $-\lambda(1 - e^{-\lambda t})$ (2) $\lambda(1 - e^{-\lambda t})$

(3) $\lambda e^{-\lambda t}$ (4) $-\lambda e^{-\lambda t}$

Q.15 Two capacitors of capacities 2C and C are joined in parallel and charged up to potential V. The battery is removed and the capacitor of capacity C is filled completely with a medium of dielectric constant K. the potential difference across the capacitors will now be :

(1) $\frac{V}{K+2}$ (2) $\frac{V}{K}$

(3) $\frac{3V}{K+2}$ (4) $\frac{3V}{K}$

Q.16 A ball is thrown up with a certain velocity so that it reaches a height 'h'. Find the ratio of the two different times of the ball reaching $\frac{h}{3}$ in both the directions.

(1) $\frac{\sqrt{2}-1}{\sqrt{2}+1}$ (2) $\frac{1}{3}$

(3) $\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$ (4) $\frac{\sqrt{3}-1}{\sqrt{3}+1}$

Q.17 A 0.07 H inductor and a 12 Ω resistor are connected in series to a 220. V, 50 Hz ac source. The approximate current in the circuit and the phase angle between current and source voltage are respectively. [Take π as $\frac{22}{7}$]

(1) 8.8 A and $\tan^{-1}\left(\frac{11}{6}\right)$

(2) 88 A and $\tan^{-1}\left(\frac{11}{6}\right)$

(3) 0.88 A and $\tan^{-1}\left(\frac{11}{6}\right)$

(4) 8.8 A and $\tan^{-1}\left(\frac{6}{11}\right)$

Q.18 Two identical tennis balls each having mass 'm' and charge 'q' are suspended from a fixed point by threads of length '*l*'. What is the equilibrium separation when each thread makes al small angle 'θ' with the vertical ?

(1) $x = \left(\frac{q^2 l}{2\pi\varepsilon_0 mg}\right)^{\frac{1}{2}}$ (2) $x = \left(\frac{q^2 l}{2\pi\varepsilon_0 mg}\right)^{\frac{1}{3}}$

(3) $x = \left(\frac{q^2 l^2}{2\pi\varepsilon_0 m^2 g}\right)^{\frac{1}{3}}$ (4) $x = \left(\frac{q^2 l^2}{2\pi\varepsilon_0 m^2 g^2}\right)^{\frac{1}{3}}$

Q.19 **Assertion A :** If in five complete rotations of the circular scale, the distance travelled on main scale of the screw gauge is 5 mm and there are 50 total divisions on circular scale, then least count is 0.001 cm.

Reason R :

$$\text{Least Count} = \frac{\text{Pitch}}{\text{Total divisions on circular scale}}$$

In the light of the above statements, choose the most appropriate answer from the options given below :

(1) **A** is not correct but **R** is correct.

(2) Both **A** and **R** are correct and **R** is the correct explanation of **A**.

(3) **A** is correct but **R** is not correct.

(4) Both **A** and **R** are correct and **R** is NOT the correct explanation of **A**.

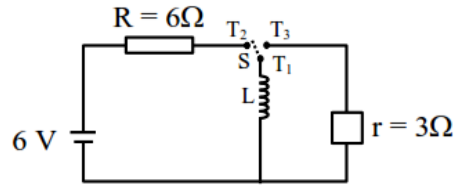
Q.20 A body takes 4 min. to cool from 61° C to 59°C. If the temperature of the surroundings is 30°C, the time taken by the body to cool from 51°C to 49° C is :

(1) 4 min. (2) 3 min.

(3) 8 min. (4) 6 min.

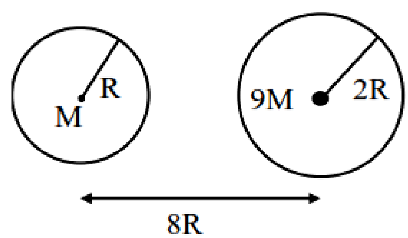
Section -B

Q.21 Consider an electrical circuit containing a two way switch 'S'. Initially S is open and then T_1 is connected to T_2. As the current in R = 6 Ω attains a maximum value of steady state level, T_1 is disconnected from T_2 and immediately connected to T_3. Potential drop across r = 3 Ω resistor immediately after T_1 is connected to T_3 is _____ V. (Round off to the Nearest Integer)



Q.22 Suppose two planets (spherical in shape) of radii R and 2R, but mass M and 9 M respectively have a centre to centre separation 8 R as shown in the figure. A satellite of mass 'm' is projected from the surface of the planet of mass 'M' directly towards the centre of the second planet. The minimum speed 'v' required for the satellite to reach the surface of the second planet is $\sqrt{\frac{a}{7}\frac{GM}{R}}$ then the value of 'a' is ________.

[Given : The two planets are fixed in their position]



Q.23 In Bohr's atomic model, the electoron is assumed to revolve in a circular orbit of radius 0.5 Å . If the speed of electron is 2.2×16^6 m/s, then the current associated with the electron will be ______ $\times 10^{-2}$mA. [Take π as $\frac{22}{7}$].

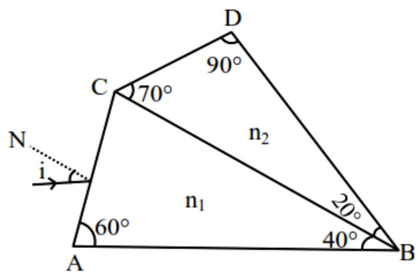
Q.24 A radioactive sample has an average life of 30 m and is decaying. A capacitor of capacitance 200 μF is first charged and later connected with resistor 'R'. If the ratio of charge on capacitor to the activity of radioactive sample is fixed with respect to time then the value of 'R' should be ______Ω.

Q.25 A particle of mass 9.1×10^{-31} kg travels in a medium with a speed of 10^6 m/s and a photon of a radiation of linear momentum 10^{-27} kg m/s travels in vacuum. The wavelength of photon is _______times the wavelength of the particle.

Q.26 A prism of refractive index n_1 and another prism of refractive index n_2 are stuck together (as shown in the figure). n_1 and n_2 depend on λ, the wavelength of light, according to the relation

$$n_1 = 1.2 + \frac{10.8\times10^{-14}}{\lambda_2} \text{ and } n_2 = 1.45 + \frac{1.8\times10^{-14}}{\lambda_2}$$

The wavelength for which rays incident at any angle on the interface BC pass through without bending at that interface will be _________ nm.



Q.27 A stone of mass 20 g is projected from a rubber catapult of length 0.1 m and area of cross section 10^{-6} m^2 stretched by an amount 0.04 m. The velocity of the projected stone is __________ m/s.
(Young's modulus of rubber = 0.5×10^9 N/m^2)

Q.28 A transistor is connected in common emitter circuit configuration, the collector supply voltage is 10 V and the voltage drop across a resistor of 1000 Ω in the collector circuit is 0.6 V. If the current gain factor (β) is 24, then the base current is
_______ μA. (Round off to the Nearest Integer)

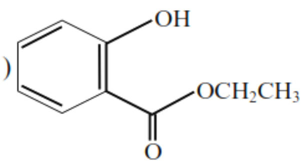
Q.29 The amplitude of upper and lower side bands of A.M. wave where a carrier signal with frequency 11.21 MHz, peak voltage 15 V is amplitude modulated by a 7.7 kHz sine wave of 5V amplitude are $\frac{a}{10}$ V and $\frac{b}{10}$ V respectively.
Then the value of $\frac{a}{b}$ is _______.

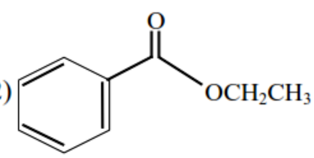
Q.30 In a uniform magnetic field, the magnetic needle has a magnetic moment 9.85×10^{-2} A/m^2 and moment of inertia 5×10^{-6} kg m^2.
If it performs 10 complete oscillations in 5 seconds then the magnitude of the magnetic field is ________ mT. [Take π^2 as 9.85]

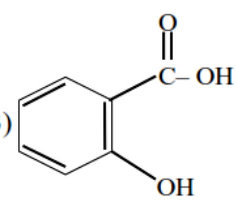
CHEMISTRY

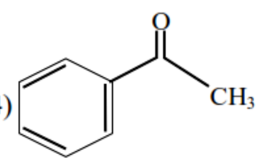
Section -A

Q.31 Which one of the following compounds will give orange precipitate when treated with 2,4-dinitrophenyl hydrazine ?

(1) 

(2) 

(3) 

(4) 

Q.32 The product obtained from the electrolytic oxidation of acidified sulphate solutions, is :
(1) HSO_4^- (2) HO_3SOOSO_3H
(3) HO_2SOSO_2H (4) HO_3SOSO_3H

Q.33 The parameters of the unit cell of a substance are a = 2.5, b = 3.0, c = 4.0, $\alpha = 90°$, $\beta = 120°$. $\gamma = 90°$. The crystal system of the substance is :
(1) Hexagonal (2) Orthorhombic
(3) Monoclinic (4) Triclinic

Q.34 The oxidation states of 'P' in $H_4P_2O^7$, $H_4P_2O_5$ and $H_4P_2O_6$, respectively, are :
(1) 7, 5 and 6 (2) 5, 4 and 3
(3) 5, 3 and 4 (4) 6, 4 and 5

Q.35 For a reaction of order n, the unit of the rate constant is :
(1) $mol^{1-n}\,L^{1-n}\,s$ (2) $mol^{1-n}\,L^{2n}\,s^{-1}$
(3) $mol^{1-n}\,L^{n-1}\,s^{-1}$ (4) $mol^{1-n}\,L^{1-n}\,s^{-1}$

Q.36 Given below are two statements :

Statement I : Aniline is less basic than acetamide.

Statement II : In aniline, the lone pair of electrons on nitrogen atom is delocalised over benzene ring due to resonance and hence less available to a proton.

Choose the most appropriate option ;

(1) Statement I is true but statement II is false.

(2) Statement I is false but statement II is true.

(3) Both statement I and statement II are true.

(4) Both statement I and statement II are false.

Q.37 The type of hybridisation and magnetic property of the complex $[MnCl_6]^{3-}$, respectively, are :

(1) $sp^3 d^2$ and diamagnetic

(2) d^2sp^3 and diamagnetic

(3) d^2sp^3 and paramagnetic

(4) sp^3d^2 and paramagnetic

Q.38 The number of geometrical isomers found in the metal complexes $[PtCl_2(NH_3)_2]$, $[Ni(CO)_4]$, $[Ru(H_2O)_3Cl_3]$ and $[CoCl_2(NH_3)_4]^+$ respectively, are :

(1) 1, 1, 1, 1 (2) 2, 1, 2, 2

(3) 2, 0, 2, 2 (4) 2, 1, 2, 1

Q.39 Which one of the following statements is NOT correct ?

(1) Eutrophication indicates that water body is polluted ?

(2) The dissolved oxygen concentration below 6 ppm inhibits fish growth

(3) Eutrophication leads to increase in the oxygen level in water

(4) Eutrophication leads to anaerobic conditions

Q.40 Given below are two statements :

Statement I : Rutherford's gold foil experiment cannot explain the line spectrum of hydrogen atom.

Statement II : Bohr's model of hydrogen atom contradicts Heisenberg's uncertainty principle. In the light of the above statements, choose the most appropriate answer from the options given below :

(1) **Statement I** is false but **statement II** is true.

(2) **Statement I** is true but **statement II** is false.

(3) Both **statement I** and **statement II** are false.

(4) Both **statement I** and **statement II** are true.

Q.41 Presence of which reagent will affect the reversibility of the following reaction, and change it to a irreversible reaction :

$$CH_4 + I_2 \underset{\text{Reversible}}{\overset{h\nu}{\rightleftharpoons}} CH_3 - I + HI$$

(1) HOCl (2) dilute HNO_2

(3) Liquid NH_3 (4) Concentrated HIO_3

Q.42 Which one among the following chemical tests is used to distinguish monosaccharide from disaccharide ?

(1) Seliwanoff's test (2) Iodine test

(3) Barfoed test (4) Tollen's test

Q.43 Match List-I with List-II :

List-I (Drug)	List-II (Class of Drug)
(a) Furacin	(i) Antibiotic
(b) Arsphenamine	(ii) Tranquilizers
(c) Dimetone	(iii) Antiseptic
(d) Valium	(iv) Synthetic antihistamines

Choose the most appropriate match :

(1) (a)-(i), (b)-(iii), (c)-(iv), (d)-(ii)

(2) (a)-(iii), (b)-(iv), (c)-(ii), (d)-(i)

(3) (a)-(ii), (b)-(i), (c)-(iii), (d)-(iv)

(4) (a)-(iii), (b)-(i), (c)-(iv), (d)-(ii)

Q.44 The statement that is INCORRECT about Ellingham diagram is

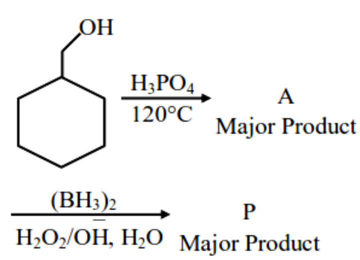
(1) provides idea about the reaction rate.

(2) provides idea about free energy change.

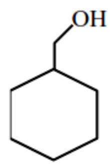
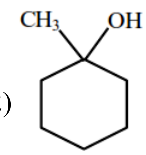
(3) provides idea about changes in the phases during the reaction.

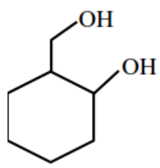
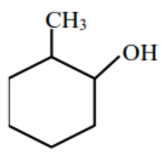
(4) provides idea about reduction of metal oxide.

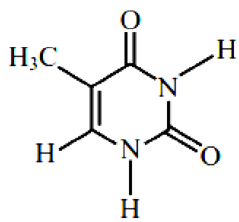
Q.45



Consider the above reaction and identify the Product P :

(1) 
(2) 

(3) 
(4) 

Q.46 

(A)

The compound 'A' is a complementary base of _______ in DNA stands.

(1) Uracil (2) Guanine
(3) Adenine (4) Cytosine

Q.47 Staggered and eclipsed conformers of ethane are :

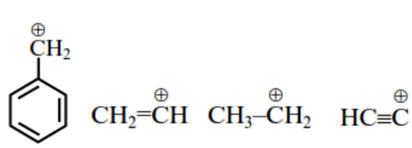
(1) Polymers (2) Rotamers
(3) Enantiomers (4) Mirror images

Q.48 Match **List - I** with **List - II :**

List - I	**List - II**
(a) NaOH	(i) Acidic
(b) $Be(OH)_2$	(ii) Basic
(c) $Ca(OH)_2$	(iii) Amphoteric
(d) $B(OH)_3$	
(e) $Al(OH)_3$	

Choose the most appropriate answer from the options given below

(1) (a)-(ii), (b)-(ii), (c)-(iii), (d)-(ii), (e)-(iii)
(2) (a)-(ii), (b)-(iii), (c)-(ii), (d)-(i), (e)-(iii)
(3) (a)-(ii), (b)-(ii), (c)-(iii), (d)-(i), (e)-(iii)
(4) (a)-(ii), (b)-(i), (c)-(ii), (d)-(iii), (e)-(iii)

Q.49 

A B C D

The correct order of stability of given carbocation is :

(1) A > C > B > D (2) D >B > C > A
(3) D > B > A > C (4) C > A > D > B

Q.50 Given below are two statements : One is labelled as **Assertion A** and the other labelled as **Reason R.**

Assertion A : Lithium halides are some what covalent in nature.

Reason R : Lithium possess high polarisation capability.

In the light of the above statements, choose the most appropriate answer from the options given below:

(1) **A** is true but **R** is false
(2) **A** is false but **R** is true
(3) Both **A** and **R** are true but **R** is NOT the correct explanation of **A**
(4) Both **A** and **R** are true and **R** is the correct explanation of **A**

Section -B

Q.51 The density of NaOH solution is 1.2 g cm^{-3}. The molality of this solution is ______ m.
(Round off to the Nearest Integer)
[Use : Atomic masses : Na : 23.0 u O : 16.0 u H : 1.0 u Density of H_2O : 1.0 g cm^{-3}]

Q.52 CO_2 gas adsorbs on charcoal following Freundlich adsorption isotherm. For a given amount of charcoal, the mass of CO_2 is doubled becomes 64 times when the pressure of CO_2 is doubled. The value of n in the Freundlich isotherm equation is ______ × 10^{-2}. (Rouund off to Nearest Integer)

Q.53 The conductivity of a weak acid HA of concentration 0.001 mol L^{-1} is 2.0 × 10^{-5} S cm^{-1}. If Λ_m° (HA) = 190 S cm^2 mol^{-1}, the ionization constant (K_a) of HA is equal to ______ × 10^{-6} (Round off to the Nearest Integer)

Q.54 1.46 g of a biopolymer dissolved in a 100 mL water at 300 K exerted an osmotic pressure of 2.42 × 10^{-3} bar.
The molar mass of the biopolymer is _____ × 10^4 g mol^{-1}. (Round off to the Nearest Integer)
[Use : R = 0.083 L bar mol^{-} K^{-1}]

Q.55 An organic compound is subjected to chlorination to get compound A using 5.0 g of chlorine. When 0.5 g of compound A is reacted with $AgNO_3$ [Carius Method], the percentage of chlorine in compound A is _____ when it forms 0.3849 g of AgCl. (Round off to the Nearest Integer)

(Atomic masses of Ag and Cl are 107.87 and 35.5 respectively)

Q.56 The number of geometrical isomers possible in triamminetrinitrocobalt (III) is X and in trioxalatochromate (III) is Y. Then the value of X + Y is ______.

Q.57 In gaseous triethyl amine the "–C–N–C" bond angle is ______ degree.

Q.58 For water at 100°C and 1 bar,

$\Delta_{vap} H - \Delta_{vap} U$ = _______ $\times 10^2$ J mol^{-1}.

(Round off to the Nearest Integer)

[Assume volume of $H_2O(l)$ is much smaller than volume of $H_2O(g)$. Assume $H_2O(g)$ treated as an ideal gas]

Q.59 $PCl_5 \rightleftharpoons PCl_3 + Cl_3$ $K_c = 1.844$

3.0 moles of PCl_5 is introduced in a 1 L closed reaction vessel at 380 K. The number of moles of PCl_5 at equilibrium is ______ $\times 10^{-3}$

(Round off to the Nearest Integer)

(1) T (2) T (3) T (4) T

Q.60 The difference between bond orders of CO and $NO^{\oplus}$ is $\frac{x}{2}$ where x = ________.

(Round off to the Nearest Integer)

MATHEMATICS

Section -A

Q.61 If the mean and variance of the following data : 6, 10, 7, 13, a, 12, b, 12 are 9 and $\frac{37}{4}$ respectively, then $(a-b)^2$ is equal to :

(1) 24 (2) 12 (3) 32 (4) 16

Q.62 The value of $\lim_{n\to\infty} \frac{1}{n} \sum_{j=1}^{n} \frac{(2j-1)+8n}{(2j-1)+4n}$ is equal to :

(1) $5 + \log_e\left(\frac{3}{2}\right)$ (2) $2 - \log_e\left(\frac{2}{3}\right)$

(3) $3 + 2\log_e\left(\frac{2}{3}\right)$ (4) $1 + 2\log_e\left(\frac{3}{2}\right)$

Q.63 Let $\vec{a} = \hat{i} + \hat{j} + 2\vec{k}$ and $\vec{b} = -\hat{i} + 2\hat{j} + 3\hat{k}$. Then the vector product $\left(\vec{a} + \vec{b}\right) \times ((\vec{a} \times ((\vec{a} - \vec{b}) \times \vec{b})) \times \vec{b})$ is equal to :

(1) $5(34\hat{i} - 5\hat{j} + 3\hat{k})$ (2) $7(34\hat{i} - 5\hat{j} + 3\hat{k})$

(3) $7(30\hat{i} - 5\hat{j} + 7\hat{k})$ (4) $5(30\hat{i} - 5\hat{j} + 7\hat{k})$

Q.64 The value of the definite integral

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{dx}{(1+e^{x\cos x})(\sin^4 x + \cos^4 x)}$$ is equal to :

(1) $-\frac{\pi}{2}$ (2) $\frac{\pi}{2\sqrt{2}}$ (3) $-\frac{\pi}{4}$ (4) $\frac{\pi}{\sqrt{2}}$

Q.65 Let C be the set of all complex numbers Let

$S_1 = \{z \in C \mid |z - 3 - 2i|^2 = 8\}$,

$S_2 = \{z \in C \mid \text{Re}(z) \geq 5\}$ and

$S_3 = \{z \in C \mid |z - \bar{z}| \geq 8\}$.

Then the number of elements in $S_1 \cap S_2 \cap S_3$ is equal to

(1) 1 (2) 0 (3) 2 (4) Infinite

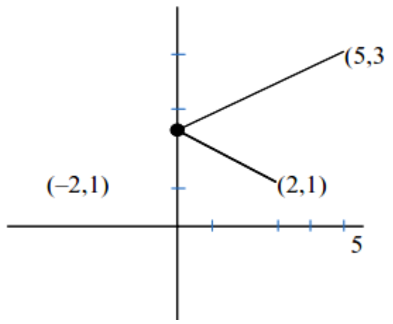
Q.66 If the area of the bounded region

$R = \{(x,y) : \max\{0, \log_e x\} \leq y \leq 2^x, \frac{1}{2} \leq x \leq 2\}$

is, $\alpha(\log_e 2)^{-1} + \beta(\log_c 2) + \gamma$, then the value of $(\alpha + \beta - 2\gamma)^2$ is equal to :

(1) 8 (2) 2 (3) 4 (4) 1

Q.67 A ray of light through (2, 1) is reflected at a point P on the y-axis and then passes through the point (5, 3). If this reflected ray is the directrix of an ellipse with eccentricity $\frac{1}{3}$ and the distance of the nearer focus from this directrix is $\frac{8}{\sqrt{53}}$, then the equation of the other directrix can be :



(1) $11x + 7y + 8 = 0$ or $11x + 7y - 15 = 0$
(2) $11x - 7y - 8 = 0$ or $11x + 7y + 15 = 0$
(3) $2x - 7y + 29 = 0$ or $2x - 7y - 7 = 0$
(4) $2x - 7y - 39 = 0$ or $2x - 7y - 7 = 0$

Q.68 If the coefficients of x^7 in $\left(x^2 + \frac{1}{bx}\right)^{11}$ and x^{-7} in $\left(x - \frac{1}{bx^2}\right)^{11}$, $b \neq 0$, are equal, then the value of b is equal to :
(1) 2 (2) – 1 (3) 1 (4) –2

Q.69 The compound statement $(P \vee Q) \wedge (\sim P) \Rightarrow Q$ is equivalent to :
(1) $P \vee Q$ (2) $P \wedge \sim Q$
(3) $\sim (P \Rightarrow Q)$ (4) $\sim (P \Rightarrow Q) \Leftrightarrow P \wedge Q$

Q.70 If $\sin\theta + \cos\theta = \frac{1}{2}$, then
$16(\sin(2\theta) + \cos(4\theta) + \sin(6\theta))$ is equal to :
(1) 23 (2) –27 (3) –23 (4) 27

Q.71 Let $A = \begin{bmatrix} 1 & 2 \\ -1 & 4 \end{bmatrix}$. If $A^{-1} = \alpha I + \beta A$, $\alpha, \beta \in R$, I is a 2 × 2 identity matrix, then $4(\alpha - \beta)$ is equal to :
(1) 5 (2) $\frac{8}{3}$ (3) 2 (4) 4

Q.72 Let $f : \left(-\frac{\pi}{4}, \frac{\pi}{4}\right) \to R$ be defined as

$$f(x) = \begin{cases} (1 + |\sin x|)^{\frac{3a}{|\sin x|}} & , -\frac{\pi}{4} < x < 0 \\ b & , \quad x = 0 \\ e^{\cot 4x / \cot 2x} & , \quad 0 < x < \frac{\pi}{4} \end{cases}$$

If f is continuous at x = 0, then the value of $6a + b^2$ is equal to :
(1) $1 - e$ (2) $e - 1$ (3) $1 + e$ (4) e

Q.73 Let $y = y(x)$ be solution of the differential equation $\log_e\left(\frac{dy}{dx}\right) = 3x + 4y$, with $y(0) = 0$.

If $y\left(-\frac{2}{3}\log_e 2\right) = \alpha \log_e 2$, then the value of α is equal to :
(1) $-\frac{1}{4}$ (2) $\frac{1}{4}$ (3) 2 (4) $-\frac{1}{2}$

Q.74 Let the plane passing through the point (–1, 0, –2) and perpendicular to each of the planes $2x + y - z = 2$ and $x - y - z = 3$ be $ax + by + cz + 8 = 0$. Then the value of a + b + c is equal to :
(1) 3 (2) 8 (3) 5 (4) 4

Q.75 The tangents are drawn from the point P(–1, 1) to the circle $x^2 + y^2 - 2x - 6y + 6 = 0$. If these tangents touch the circle at points A and B, and if D is a point on the circle such that length of the segments AB and AD are equal then the area of the triangle ABD is equal to :
(1) 2 (2) $(3\sqrt{2} + 2)$
(3) 4 (4) $3(\sqrt{2} - 1)$

Q.76 Let $f : R \to R$ be a function such that f(2) = 4 and f'(2) = 1. Then, the value of $\lim_{x \to 2} \frac{x^2 f(2) - 4f(x)}{x - 2}$ is equal to :
(1) 4 (2) 8 (3) 16 (4) 12

Q.77 Let P and Q be two distinct points on a circle which has center at C(2, 3) and which passes through origin O. If OC is perpendicular to both the line segments CP and CQ, then the set {P, Q} is equal to
(1) {(4,0), (0,6)}
(2) $\{(2 + 2\sqrt{2}, 3 - \sqrt{5}), (2 - 2\sqrt{2}, 3 + \sqrt{5})\}$
(3) $\{(2 + 2\sqrt{2}, 3 + \sqrt{5}), (2 - 2\sqrt{2}, 3 - \sqrt{5})\}$
(4) {(–1,5), (5,1)}

Q.78 Let α, β be two roots of the equation $x^2 + (20)^{1/4} x + (5)^{1/2} = 0$.
Then $\alpha^8 + \beta^8$ is equal to
(1) 10 (2) 100 (3) 50 (4) 160

Q.79 The probability that a randomly selected 2-digit number belongs to the set $\{n \in N : (2^n - 2)$ is a multiple of 3$\}$ is equal to

(1) $\frac{1}{6}$ (2) $\frac{2}{3}$ (3) $\frac{1}{2}$ (4) $\frac{1}{3}$

Q.80 Let

$A = \{(x,y) \in R \times R \mid 2x^2 + 2y^2 - 2x - 2y = 1\}$,

$B = \{(x,y) \in R \times R \mid 4x^2 + 4y^2 - 16y - 7 = 0\}$ and

$C = \{(x,y) \in R \times R \mid x^2 + y^2 - 4x - 2y + 5 \le r^2\}$.

Then the minimum value of $|r|$ such that $A \cup B \subseteq C$ is equal to

(1) $\frac{3+\sqrt{10}}{2}$ (2) $\frac{2+\sqrt{10}}{2}$

(3) $\frac{3+2\sqrt{5}}{2}$ (4) $1+\sqrt{5}$

Section -B

Q.81 For real numbers α and β, consider the following system of linear equations :

$x + y - z = 2,\ x + 2y + \alpha z = 1,\ 2x - y + z = \beta$.

If the system has infinite solutions, then $\alpha + \beta$ is equal to ________

Q.82 Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}, \vec{b}$ and $\vec{c} = \hat{j} - \hat{k}$ be three vectors such that $\vec{a} \times \vec{b} = \vec{c}$ and $\vec{a}.\vec{b} = 1$. If the length of projection vector of the vector $\vec{b}$ on the vector $\vec{a} \times \vec{c}$ is l, then the value of $3l^2$ is equal to ____.

Q.83 If $\log_3 2$, $\log_3(2^x - 5)$, $\log_3\left(2^x \frac{7}{2}\right)$ are in an arithmetic progression, then the value of x is equal to _____.

Q.84 Let the domain of the function $f(X) = \log_4(\log_5(\log_3(18x - x^2 - 77)))$ be (a, b). Then the value of the integral

$$\int_a^b \frac{\sin^3 x}{(\sin^3 x + \sin^3(a+b-x))} dx$$ is equal to ____.

Q.85 Let

$$f(X) = \begin{vmatrix} \sin^2 x & -2+\cos^2 x & \cos 2x \\ 2+\sin^2 x & \cos^2 x & \cos 2x \\ \sin^2 x & \cos^2 x & 1+\cos 2x \end{vmatrix}, x \in [0, \pi]$$

Then the maximum value of f(X) is equal to ___.

Q.86 Let $F : [3, 5] \to R$ be a twice differentiable function on (3, 5) such that

$$F(X) = e^{-x}\int_3^x (3t^2 + 2t + 4F'(t))dt.$$

If $F'(4) = \frac{\alpha e^{\beta} - 224}{(e^{\beta} - 4)^2}$, then $\alpha + \beta$ is equal to ___.

Q.87 Let a plane P pass through the point (3, 7, – 7) and contain the line, $\frac{x-2}{-3} = \frac{y-3}{-3} = \frac{z+2}{1}$. If distance of the plane P from the origin is d, then d^2 then is equal to ______.

Q.88 Let S = {1, 2, 3, 4, 5, 6, 7}. Then the number of possible functions $f : S \to S$ such that $f(m \cdot n) = f(m) \cdot f(n)$ for every $m, n \in S$ and $m \cdot n \in S$ is equal to______.

Q.89 If $y = y(x)$, $y \in \left[0, \frac{\pi}{2}\right)$ is the solution of the differential equation

$\sec y \frac{dy}{dx} - \sin(x + y) - \sin(x - y) = 0$, with $y(0) = 0$, then $5y'\left(\frac{\pi}{2}\right)$ is equal to _______.

Q.90 Let $f : [0, 3] \to R$ be defined by

$f(x) = \min\{x - [x], 1 + [x] - x\}$

where [x] is the greatest integer less than or equal to x. Let P denote the set containing all $x \in [0, 3]$ where f is discontinuous, and Q denote the set containing all $x \in (0, 3)$ where f is not differentiable. Then the sum of number of elements in P and Q is equal to _________.

JEE MAIN ONLINE PAPER 2021

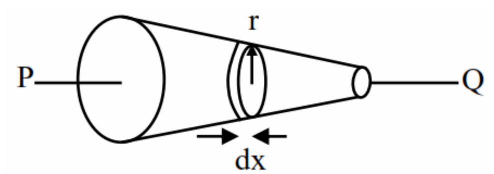
Held on JULY 27, 2021 (Morning)

Hints & Solutions

PHYSICS

Section -A

1.[1]



Current is constant in conductor i = constant

Resistance of element $dR = \frac{\rho dx}{\pi r^2}$

$dV = idR = \frac{i\rho dx}{\pi r^2}$

$E = \frac{dV}{dx} = \frac{i\rho}{\pi r^2}$

& $V_d = \frac{eE\tau}{m}$

$\therefore V_d \propto E$

$\rightarrow E \propto \frac{1}{r^2}$

It decreases, E will increases $\therefore V_d$ will increase

2.[3] $KE = \frac{3}{2}kT$

$PV = \frac{N}{N_A}RT$

$N = \frac{PV}{kT}$

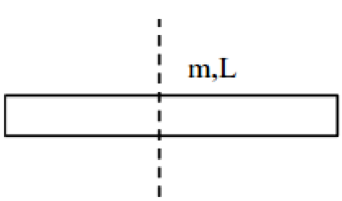
$= N = 1.5 \times 10^{11}$

3.[3] $V = \frac{c}{\sqrt{\mu_r \varepsilon_r}}$

$= 3.33 \times 10^7$ m/sec

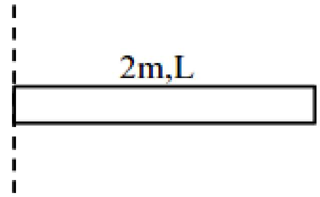
4.[3]

(a)



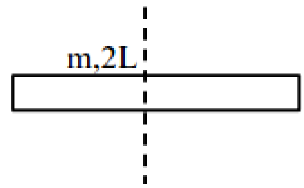
$I = \frac{mL^2}{12}$

(b)



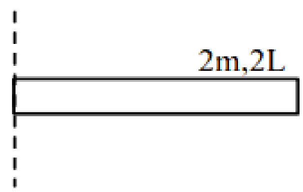
$I = \frac{(2m)(L^2)}{3}$

(c)



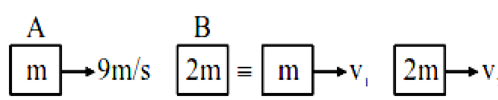
$I = \frac{m(2L)^2}{12} = \frac{mL^2}{3}$

(d)



$I = \frac{2m(2L)^2}{3} = \frac{8}{3}mL^2$

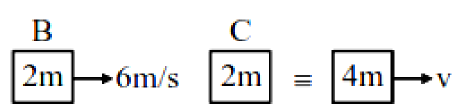
5.[4] Collision between A and B



$m \times 9 = mv_1 + 2m\ v_2$ (from momentum conservation) $e = 1 = \frac{v_2 - v_1}{9}$

$\Rightarrow v_2 = 6$ m/sec. , $v_1 = 3$ m/sec.

Collision between B and C



$2m \times 6 = 4mv$ (from momentum conservation)

$v = 3$ m/s

6.[3] $V = V_0\left(1 - e^{\frac{t}{RC}}\right)$

$50 = 100\left(1 - e^{\frac{t}{RC}}\right)$

$t = 0.69 \times 10^{-4}$ sec.

7.[1] $\frac{1}{C_{eff}} = \frac{d}{K \in_0 A} + \frac{2d}{3K \in_0 A} + \frac{3d}{5K \in_0 A}$

$C_{eff} = \frac{15K \in_0 A}{34d}$

8.[4] Ratio of moment of inertia = $\frac{\frac{1}{2}MR^2}{\frac{1}{4}mr^2}$

$= \frac{2\sigma\pi R^2 R^2}{\sigma\pi r^2 r^2} = \frac{2R^2}{r^4}$

9.[2] Fringe width = $\lambda D/d$

as λ decreases, fringe width also decreases

10.[2] Work done in adiabatic process = $\frac{-nR}{\lambda - 1}(T_f - T_i)$

$\therefore W_{AD} = \frac{-nR}{\lambda - 1}(T_2 - T_1)$

and $W_{BC} = \frac{-nR}{\gamma - 1}(T_2 - T_1)$

$\therefore W_{AD} = W_{BC}$

11.[4] Polygon law is applicable in both but the equation given in the reason is not useful in explaining the assertion.

12.[3] For no sliding

$f \geq \rho a v^2$

$\mu mg \geq \rho a v^2$

$\mu \rho A h g \geq \rho a 2gh$

$$\boxed{\mu \geq \frac{2a}{A}}$$

13. **[4]**

$E = \frac{1}{2}Ka^2$

$\frac{3E}{4} = \frac{1}{2}K(a^2 - y^2)$

$\frac{3}{4} \times \frac{1}{2}Ka^2 = \frac{1}{2}K(a^2 - y^2)$

$y^2 = a^2 - \frac{3a^2}{4}$

$y = \frac{a}{2}$

14.[3] $N = N_0 e^{-\lambda t}$

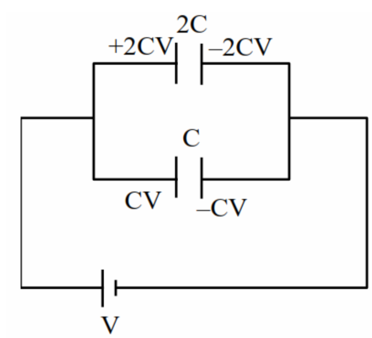
$N_d = N_0 - N$

$N_d = N_0(1 - e^{-\lambda t})$

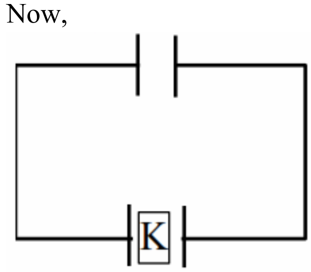
$\frac{N_d}{N_0} = f = 1 - e^{-\lambda t}$

$\frac{df}{dt} = \lambda e^{-\lambda t}$

15.[3]



Now,



$V_C = \frac{2CV + CV}{KC + 2C}$

$= \frac{3V}{K + 2}$

16.[3] $u = \sqrt{2gh}$

Now, $S = \frac{h}{3}$ $a = -g$

$S = ut + \frac{1}{2}at^2$

$\frac{h}{3} = \sqrt{2gh}\,t + \frac{1}{2}(-g)t^2$

$$t^2\left(\frac{g}{2}\right) - \sqrt{2gh}\ t + \frac{h}{3} = 0$$

From quadratic equation

$$t_1, t_2 = \frac{\sqrt{2gh} \pm \sqrt{2gh - \frac{4g}{2}\frac{h}{3}}}{g}$$

$$\frac{t_1}{t_2} = \frac{\sqrt{2gh} - \sqrt{\frac{4gh}{3}}}{\sqrt{2gh} + \sqrt{\frac{4gh}{3}}}$$

$$= \frac{\sqrt{3} - \sqrt{2}}{\sqrt{3} + \sqrt{2}}$$

17.[1] $\phi = \tan^{-1}\left(\frac{X_L}{R}\right)$ $X_L = \omega L$

$$X_L = 2 \times \frac{22}{7} \times 50 \times 0.07 = 22\Omega$$

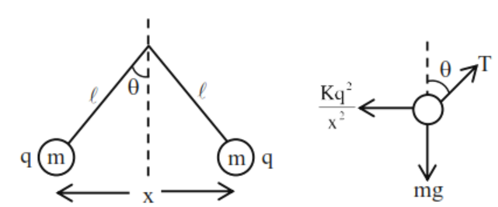
$$\phi = \tan^{-1}\left(\frac{22}{12}\right) \quad R = 12\ \Omega$$

$$\phi = \tan^{-1}\left(\frac{11}{6}\right)$$

$$Z = \sqrt{X_L^2 + R^2} = 25.059$$

$$I = \frac{V}{Z} = \frac{220}{25.059} = 8.77\ A$$

18.[2]



$$T\cos\theta = mg$$

$$T\sin\theta = \frac{kq^2}{x^2}$$

$$\tan\theta = \frac{kq^2}{x^2 mg}$$

as $\tan\theta \approx \sin\theta \approx \frac{X}{2L}$

$$\frac{X}{2L} = \frac{Kq^2}{x^2 mg}$$

$$x = \left(\frac{q^2 L}{2\pi\varepsilon_0 mg}\right)^{1/3}$$

19.[1] Least count = $\frac{\text{Pitch}}{\text{Total divisions on circular scale}}$

In 5 revolution, distance travel, 5 mm
In 1 revolution, it will travel 1 mm.

So least count = $\frac{1}{50} = 0.02$

20.[4] $\frac{\Delta T}{\Delta T} = K(T_t - T_S)$ T_t = average temp.

T_S = surrounding temp.

$$\frac{61-59}{4} = K\left(\frac{61+59}{2} - 30\right) \quad \text{.... (1)}$$

$$\frac{51-49}{t} = K\left(\frac{51+49}{2} - 30\right) \quad \text{.... (2)}$$

Divide (1) & (2)

$$\frac{t}{4} = \frac{60-30}{50-30} = \frac{30}{20}$$

so, t = 6 minutes

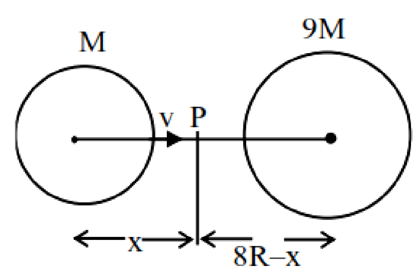
Section -B

21.[3] When T_1 and T_2 are connected then the steady state current in the inductor $I = \frac{6}{6} = 1$ A

When T_1 and T_3 are connected then current through inductor remains same. So potential difference across 3Ω

V = Ir = 1 × 3 = 3 volt

22.[4]



Acceleration due to gravity will be zero at P therefore,

$$\frac{GM}{x^2} = \frac{G9M}{(8R-x)^2}$$

8R – x = 3x

x = 2R

Apply conservation of energy and consider velocity at P is zero.

$$\frac{1}{2}mv^2 - \frac{GMm}{R} - \frac{G9Mm}{7R} = 0 \quad \frac{GMm}{2R} - \frac{G9Mm}{6R}$$

$$\therefore V = \sqrt{\frac{4}{7}\frac{GM}{R}}$$

23.[112] $I = \frac{e}{T} = \frac{e\omega}{2\pi} = \frac{eV}{2\pi r}$

$$I = \frac{1.6\times10^{-19}\times2.2\times10^{6}\times7}{2\times22\times0.5\times10^{-10}}$$

$= 1.12$ mA

112×10^{-2} mA

24.[150] $T_m = 30$ ms

$C = 200\ \mu F$

$$\frac{q}{N} = \frac{Q_0e^{-t/RC}}{N_0e^{-\lambda t}} = \frac{Q_0}{N_0}e^{t\left(\lambda - \frac{1}{RC}\right)}$$

Since q/N is constant hence

$$\lambda = \frac{1}{RC}$$

$$R = \frac{1}{\lambda C} = \frac{T_m}{C} = \frac{30\times10^{-3}}{200\times10^{-6}} = 150\ \Omega$$

25.[910] For photon $\lambda_1 = \frac{h}{P} = \frac{6.6\times10^{-34}}{10^{-27}}$

For particle $\lambda_2 = \frac{h}{mv} = \frac{6.6\times10^{-34}}{9.1\times10^{-31}\times10^{6}}$

$$\therefore \frac{\lambda_1}{\lambda_2} = 910$$

26.[600] For no bending, $n_1 = n_2$

$$1.2 + \frac{10.8\times10^{-14}}{\lambda_2} = 1.45 + \frac{1.8\times10^{-4}}{\lambda_2}$$

On solving,

$9 \times 10^{-14} = 25\ \lambda^2$

$\lambda = 6 \times 10^{-7}$

$\lambda = 600$ nm

27.[20] By energy conservation

$$\frac{1}{2}.\frac{YA}{L}.x^2 = \frac{1}{2}mv^2$$

$$\frac{0.5\times10^{9}\times10^{-6}\times(0.04)^2}{0.1} = \frac{20}{1000}v^2$$

$\therefore v^2 = 400$

$\boxed{v = 20m/s}$

28.[25] $\beta = \frac{I_C}{I_B} = 24$; $R_C = 1000$

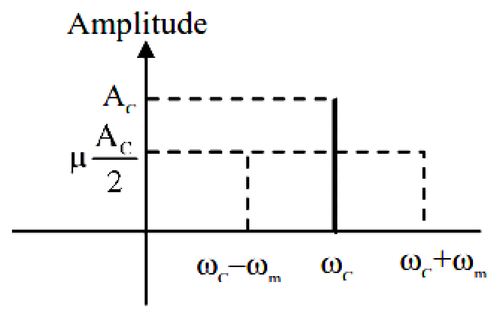
$\Delta V = 0.6$

$I_C = \frac{0.6}{1000}$

$I_C = 6 \times 10^{-4}$

$$I_B = \frac{I_C}{\beta} = \frac{6\times10^{-4}}{24} = 25\mu A$$

29.[1]



$$\frac{a}{10} = \frac{b}{10} = \frac{\mu A_C}{2}$$

$$\Rightarrow \frac{a}{b} = 1$$

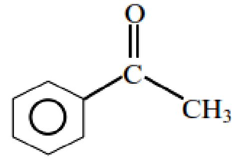
30.[8] $T = 2\pi\sqrt{\frac{I}{MB}}$

$B = 80 \times 10^{-4} = 8$m T

CHEMISTRY

Section -A

31.[4]



Explanation ⇒ 2-4-D.N.P test is used for carbonyl compound (aldehyde & ketone)

32.[2] Electrolysis of concentrated solution of acidified sulphate solution yields $H_2S_2O_8$.

33.[3] $a \neq b \neq c$ and $\alpha = \gamma = 90° \neq \beta$

are parameters of monoclinic unit cell.

34.[3] Oxidation state of P in $H_4P_2O_5$ and $H_4P_2O_5$ and $H_4P_2O_6$ is 5, 3 & 4 respectively

$H_4P_2O_7$

$2x + 4(+1) + 7(-2) = 0$

$x = +5$

$H_4\ \underline{P_2}\ O_5$

$2x + 4(+1) + 5(-2) = 0$

$x = +3$

$H_4\ \underline{P_2}\ O_6$

$2x + 4(+1) + 6(-2) = 0$

$x = +4$

35.[3] Rate = $k[A]^n$

comparing units

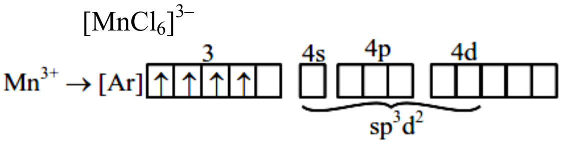
$$\frac{(\text{mol}/\ell)}{\text{Sec}} = k\left(\frac{\text{mol}}{\ell}\right)^n$$

$\Rightarrow k = \text{mol}^{(1-n)}\ l^{(n-1)} s^{-1}$

36.[2] **Explanation :-** aniline is more basic than acetamide because in acetamide, lone pair of nitrogen is delocalised to more electronegative element oxygen. In Aniline lone pair of nitrogen delocalised over benzene ring.

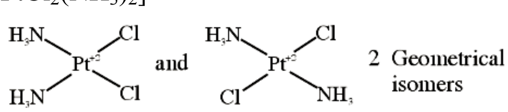
37. [4]

$[MnCl_6]^{3-}$



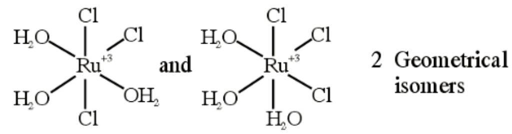
Paramagnetic and having 4 unpaired electrons.

38.[2] $[PtCl_2(NH_3)_2]$

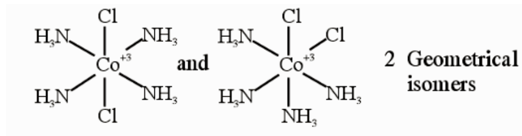


$[Ni(CO)_4]$ → All ligands are same Zero Geometrical isomers

$[Ru(H_2O)_3Cl_3]$



$[CoCl_2(NH_3)_4]^+$



39.[3] Eutrophication leads to decrease in oxygen level of water.

3rd statement is incorrect

40.[4] Rutherford's gold foil experiment only proved that electrons are held towards nucleus by electrostatic forces of attraction and move in circular orbits with very high speeds. Bohr's model gave exact formula for simultaneous calculation of speed & distance of electron from the nucleus, something which was deemed impossible according to Heisenberg.

41.[4] Iodination of alkane is reversible reaction. It can be irreversible in the presence of strong oxidizing agent like conc. HNO_3 or conc. HIO_3

42.[3] Barford test is used for distinguish monosaccharide from disaccharide

43.[4] → furacine acts as Antiseptic

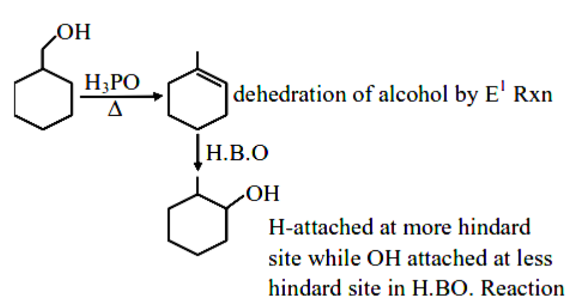
→ Arsphenamine also known as salvarsan acts as antibiotic

→ Dimetone is synthetic histamine

→ valium is a Tranquilizer

44.[1] Ellingham diagram is a plot between $\Delta G°$ and T and does not give any information regarding rate of reaction

45.[4]



46.[3] Given structure is Thymine and Thymine being paired with adenine

47.[2] Staggered and eclipsed conformist of ethane also known as rotamers

48.[2] $NaOH \rightarrow$ Basic

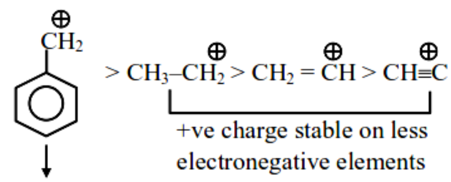
$Be(OH)_2 \rightarrow$ Amphoteric

$Ca(OH)_2 \rightarrow$ Basic

$B(OH)_3 \rightarrow$ Acidic

$Al(OH)_3 \rightarrow$ Amphoteric

49.[1]



Stable due to Resonance

50.[4] Lithium due to small size has very high polarization capability and thus increases covalent nature in Halides.

Section -B

51.[5] Consider 1 ℓ solution

mass of solution = (1.2×1000)g = 1200 gm

Neglecting volume = NaOH

Mass of water = 1000 gm

$\Rightarrow$ Mass of NaOH = (1200 – 1000) gm

= 200 gm

$$\Rightarrow \text{Moles of NaOH} = \frac{200\text{g}}{50\text{g / mol}} = 5 \text{ mol}$$

$$\Rightarrow \text{molality} = \frac{5\text{mol}}{\text{lkg}} = 5 \text{ m}$$

52.[17] Freundlich isotherm. ;

$$\frac{x}{m} = k.p^{\frac{1}{n}}$$

Substituting values ;

$$\left(\frac{64}{1}\right) = (2)^{\frac{1}{2}} \Rightarrow n = \frac{1}{6} = 0.166$$

$\cong 17 \times 10^{-2}$

53.[12] $\Lambda_m = 1000 \times \dfrac{\kappa}{m}$

$$= 1000 \times \frac{2 \times 10^{-5}}{0.001} = 20 \text{ S cm}^2 \text{ mol}^{-1}$$

$$\Rightarrow \alpha = \frac{\Lambda_m}{\Lambda_m^{\infty}} = \frac{20}{190} = \left(\frac{2}{19}\right)$$

$$HA \rightleftharpoons H^+ + A^-$$

$0.001\ (1 - \alpha)\ 0.001\alpha\ 0.001\ \alpha$

$$\Rightarrow k_a = 0.001 \left(\frac{\alpha^2}{1-\alpha}\right) = \frac{0.001 \times \left(\frac{2}{19}\right)^2}{1 - \left(\frac{2}{19}\right)}$$

$= 12.3 \times 10^{-6}$

54.[15] $\pi = CRT$; π = osmotic pressure

C = molarity

T = Temperature of solution

let the molar mass be M gm/mol

2.42×10^{-3} bar =

$$\frac{\left(\frac{1.46\text{g}}{\text{Mgm / mol}}\right)}{01\ell} \times \left(\frac{0.083\ell - \text{bar}}{\text{mol} - \text{K}}\right) \times (300\text{K})$$

$\Rightarrow M = 15.02 \times 10^4$ g/mol

55.[19] $n_{e\ell}$ in compound $= n_{AgCl} = \dfrac{0.3849\text{g}}{(107.87 + 35.5)}$ g/mol

$\Rightarrow$ mass of chlorine $= n_{Cl} \times 35.5 = 0.0953$ gm

$$\Rightarrow \% \text{ wt of chlorine} = \frac{0.0953}{0.5} \times 100$$

19.06%

OR

Mass of organic compound = 0.5 gm.

mass of formed AgCl = 0.3849 gm

% of Cl

$$= \frac{\text{atomic mass of Cl} \times \text{mass formed AgCl}}{\text{molecular mass of AgCl} \times \text{mass of organic compound}} \times 100$$

$$= \frac{35.5 \times 0.3849}{143.37 \times 0.5} \times 100$$

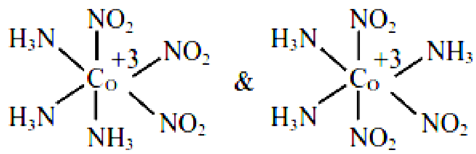
= 19.06

≈ 19

56.[2] Triamminetrinitrocobalt (III) → $[Co(NO_2)_3(NH_3)_3]$

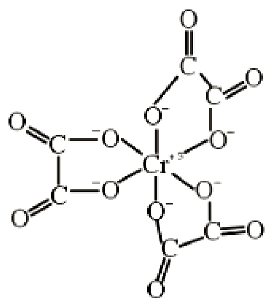
trioxalatochromate(III) ion →$[Cr(C_2O_4)_3]^{3-}$

$[Co(NO_2)_3(NH_3)_3]$



Two geometrical isomers (X)

$[Cr(C_2O_4)_3]^{3-}$



Zero geometrical isomer (Y)

X + Y = 2 + 0 = 2.0

57.[108] In gaseous triethyl amine the "–C–N–C" bond angle is 108 degree.

58.[31] $H_2O_{(\ell)} \rightleftharpoons H_2O_{(V)}$

$\Delta H = \Delta U + \Delta n_g RT$

for 1 mole waters ;

$\Delta ng = 1$

$\therefore \Delta n_g RT = 1 \text{ mol} \times 8.31 \text{ J/mol-k} \times 373 \text{ K}$

$= 3099.63 \text{ J} \cong 31 \times 10^2 \text{ J}$

59. [11396]

Official Ans. by NTA (1400)

$PCl_{5(g)} \rightleftharpoons PCl_{3(g)} + Cl_{2(g)}$ $K_2 = 1.844$

t = 0 3 moles

t = ∞ x x

$\Rightarrow \frac{[PCl_3][Cl_2]}{[PCl_5]} = \frac{x^2}{3-x} = 1.844$

$\Rightarrow x^2 + 1.844 + 5.532 = 0$

$\Rightarrow x = \frac{-1.844 + \sqrt{(1.844)^2 + 4 \times 5.532}}{2}$

$\cong 1.604$

$\Rightarrow$ Moles of $PCl_5 = 3 - 1.604 \cong 1.396$

60.[0] Bond order of CO = 3

Bond order of $NO^+ = 3$

Difference = 0 $\frac{x}{2}$

x = 0

MATHEMATICS

Section -A

61.[4] Mean $= \frac{6+10+7+13+a+12+b+12}{8} = 9$

$60 + a + b = 72$

$a + b = 12$ … (1)

variance $= \frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n}\right)^2 = \frac{37}{4}$

$\sum x_i^2 = 6^2 + 10^2 + 7^2 + 13^2 + a^2 + b^2 + 12^2 + 12^2 = a^2 + b^2 + 642$

$\frac{a^2 + b^2 + 642}{8} - (9)^2 = \frac{37}{4}$

$\frac{a^2+b^2}{8} + \frac{321}{4} - 81 = \frac{37}{4}$

$\frac{a^2+b^2}{8} = 81 + \frac{37}{4} - \frac{321}{4}$

$\frac{a^2+b^2}{8} = 81 - 71$

$\therefore a^2 + b^2 = 80$ …(2)

From (1) $a^2 + b^2 + 2ab = 144$

$80 + 2ab = 144$ $\therefore 2ab = 64$

$(a-b)^2 = a^2 + b^2 - 2ab = 80 - 64 = 16$

62.[4] $\lim_{n\to\infty} \frac{1}{n} \sum_{j=1}^{n} \frac{\left(\frac{2j}{n} - \frac{1}{n} + 8\right)}{\left(\frac{2j}{n} - \frac{1}{n} + 4\right)}$

$\int_0^1 \frac{2x+8}{2x+4} dx = \int_0^1 dx + \int_0^1 \frac{4}{2x+4} dx$

$= 1 + 4\ \frac{1}{2}\ \frac{1}{2}(\ell n\,|\,2x+4\,|)_0^1$

$= 1 + 2\ell n\left(\frac{3}{2}\right)$

63.[2] $\vec{a} = \hat{i} + \hat{j} + 2\hat{k}$

$\vec{b} = -\vec{i} + 2\vec{j} + 3\vec{k}$

$\vec{a} + \vec{b} = 3\vec{j} + 5\vec{k}$; $\vec{a}.\vec{b} = -1 + 2 + 6 = 7$

$((\vec{a} \times ((\vec{a} - \vec{b}) \times \vec{b})) \times \vec{b})$

$((\vec{a} \times (\vec{a} \times \vec{b} - \vec{b} \times \vec{b})) \times \vec{b})$

$(\vec{a} \times (\vec{a} \times \vec{b} - 0)) \times \vec{b}$

$(\vec{a} \times (\vec{a} \times \vec{b})) \times \vec{b}$

$((\vec{a}.\vec{b})\vec{a} - (\vec{a}.\vec{a})\vec{b} \times \vec{b}$

$(\vec{a}.\vec{b})\vec{a} \times \vec{b} - (\vec{a}.\vec{a})(\vec{b} \times \vec{b})$

$(\vec{a}.\vec{b})(\vec{a} \times \vec{b})$

$$\vec{a} \times \vec{b} = \begin{vmatrix} i & j & k \\ 1 & 1 & 2 \\ -1 & 2 & 3 \end{vmatrix} = -\hat{i} - 5\hat{j} + 3\hat{k}$$

$\therefore\ 7(-\hat{i} - 5\hat{j} + 3\hat{k}))$

$(\vec{a} + \vec{b}) \times (7(-\hat{i} - 5\hat{j} + 3\hat{k}))$

$7(0\hat{i} + 3\hat{j} + 5\hat{k}) \times (-\hat{i} - 5\hat{j} + 3\hat{k})$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 3 & 5 \\ -1 & -5 & 3 \end{vmatrix}$$

$\Rightarrow 34\hat{i} - (5)\ \hat{j} + (3\hat{k})$

$\Rightarrow 34\hat{i} - 5\hat{j} + 3\hat{k}$

$\therefore\ 7(34\hat{i} - 5\hat{j} + 3\hat{k})$

64.[2] $$I = \int_{\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{dx}{(1 + e^{x\cos x})(\sin^4 x + \cos^4 x)} \quad \ldots(1)$$

Using $\int_a^b f(x)dx = \int_a^b f(a + b - x)dx$

$$I = \int_{-\pi/4}^{\pi/4} \frac{dx}{(1 + e^{-x\cos x})(\sin^4 x + \cos^4 x)}$$

Add (1) and (2)

$$2I = \int_{\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{dx}{\sin^4 x + \cos^4 x}$$

$$2I = 2\int_0^{\frac{\pi}{4}} \frac{dx}{\sin^4 x + \cos^4 x}$$

$$I = \int_0^{\frac{\pi}{4}} \frac{(1 + \tan^2 x)\sec^2 x}{\tan^4 x + 1} dx$$

$$I = \int_0^{\frac{\pi}{4}} \frac{\left(1 + \frac{1}{\tan^2 x}\right)\text{sex}^2 x}{\left(\tan x - \frac{1}{\tan x}\right)^2 + 2} dx$$

$$\tan x - \frac{1}{\tan x} = t$$

$$\left(1 + \frac{1}{\tan^2 x}\right)\sec^2 x dx = dt$$

$$I = \int_{-\infty}^{0} \frac{dt}{t^2 + 2} = \left[\frac{1}{\sqrt{2}} \tan^{-1}\left(\frac{t}{\sqrt{2}}\right)\right]_{-\infty}^{0}$$

$$I = 0 - \frac{1}{\sqrt{2}}\left(-\frac{\pi}{2}\right) = \frac{\pi}{2\sqrt{2}}$$

65.[1] $S_1 : |z - 3 - 2i|^2 = 8$

$|z - 3 - 2i| = 2\sqrt{2}$

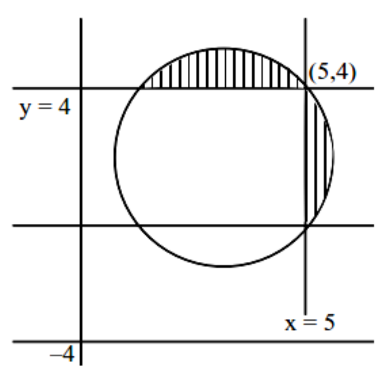
$(x-3)^2 + (y - 2)^2 = (2\sqrt{2})^2$

$S_2 : x \geq 5$

$S_3 : |z - \bar{z}| \geq 8$

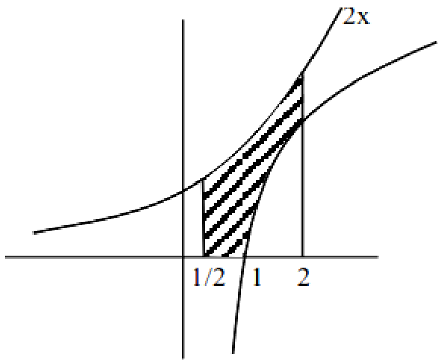
$|2iy| \geq 8$

$2|y| \geq 8 \ \therefore\ y \geq 4, y \leq -4$



$n(S_1 \cap S_2 \cap S_3) = 1$

66.[2] $R = \{(x, y) : \max(0, \log_e x) \leq y \leq 2^x, \frac{1}{2} \leq x \leq 2\}$



$$\int_{\frac{1}{2}}^{22} 2^x dx - \int_1^2 \ell nx dx$$

$$\Rightarrow \left[\frac{2^x}{\ln 2}\right]_{1/2}^{2} [x \ln x - x]_1^2$$

$$\Rightarrow \frac{(2^2) - 2^{1/2}}{\log_e 2} - (2 \ln 2 - 1)$$

$$\Rightarrow \frac{(2^2 - \sqrt{2})}{\log_e 2} - 2 \ln 2 + 1$$

$\therefore \alpha = 2^2 - \sqrt{2}, \beta = -2, \gamma = 1$

$\Rightarrow (\alpha + \beta + 2\gamma)^2$

$\Rightarrow (2^2 - \sqrt{2} - 2 - 2)^2$

$\Rightarrow (\sqrt{2})^2 = 2$

67.[3] Equation of reflected Ray

$y - 1 = \frac{2}{7}(x + 2)$

$7y - 7 = 2x + 4$

$2x - 7y + 11 = 0$

Let the equation of other directrix is $2x - 7y + \lambda$

Distance of directrix from Focub

$\frac{a}{c} - ae = \frac{8}{\sqrt{53}}$

$3a - \frac{a}{3} = \frac{8}{\sqrt{53}}$ or $a = \frac{3}{\sqrt{53}}$

Distance from other focus $= \frac{a}{e} + ae$

$3a + \frac{a}{3} = \frac{10a}{3} = \frac{10}{3} \times \frac{3}{\sqrt{53}} = \frac{10}{\sqrt{53}}$

between two directix $= \frac{2a}{e}$

$= 2 \times 3 \times \frac{3}{\sqrt{53}} = \frac{18}{\sqrt{53}}$

$\left|\frac{\lambda - 11}{\sqrt{53}}\right| = \frac{18}{\sqrt{53}}$

$\lambda - 11 = 18$ or -18

$\lambda = 29$ or -7

$2x - 7y - 7 = 0$ or $2x - 7y + 29 = 0$

68.[3] Coefficient of x^7 in $\left(x^2 + \frac{1}{bx}\right)^{11}$

${}^{11}C_r (x^2)^{11-r} \cdot \left(\frac{1}{bx}\right)^r$

${}^{11}C_r\ x^{22-3r} \cdot \frac{1}{b^r}$

$22 - 3r = 7$

$r = 5$

$\therefore\ {}^{11}C_5 \cdot \frac{1}{b^5} . x^7$

Coefficient of x^{-7} in $\left(x - \frac{b}{bx^2}\right)^{11}$

${}^{11}C_r (x)^{11-r} \cdot \left(-\frac{1}{bx^2}\right)^r$

${}^{11}C_r x^{11-3r} \cdot \frac{(-1)r}{b^r}$

$11 - 3r = -7 \therefore r = 6$

${}^{11}C_6 . \frac{1}{b^6} x^{-}7$

${}^{11}C_5 . \frac{1}{b^5} = {}^{11}C_6 . \frac{1}{b^6}$

Since $b \neq 0 \therefore b = 1$

69.[4] Using Truth Table

p	Q	$P \vee Q$	~P	$(P \vee Q) \wedge P$	$(P \vee Q) \wedge \sim P \to Q$
T	T	T	F	F	T
T	F	T	F	F	T
F	T	T	T	T	T
F	F	F	T	F	T

p	Q	~Q	$P \wedge \sim Q$	$P \to Q$	$(P \to Q)$
T	T	F	F	T	F
T	F	T	T	F	T
F	T	F	F	T	F
F	F	T	F	T	F

$\sim(P \rightarrow Q)$	$P \wedge \sim Q$	$\sim(P \rightarrow Q) \Leftrightarrow P \wedge \sim Q$
F	F	T
T	T	T
F	F	T
F	F	T

70.[3] $\sin\theta + \cos\theta = \frac{1}{2}$

$\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = \frac{1}{2}$

$\sin 2\theta = -\frac{3}{4}$

Now :

$\cos 4\theta = 1 - 2\sin^2 2\theta$

$= 1 - 2\left(-\frac{3}{4}\right)^2$

$= 1 - 2 \times \frac{9}{16} = -\frac{1}{8}$

$\sin 6\theta = 3\sin 2\theta - 4\sin^3 2\theta$

$= (3 - 4\sin^2 2\theta).\sin 2\theta$

$= \left[3 - 4\left(\frac{9}{16}\right)\right].\left(-\frac{3}{4}\right)$

$\Rightarrow \left[\frac{3}{4}\right] \times \left(-\frac{3}{4}\right) = -\frac{9}{16}$

$16[\sin 2\theta + \cos 4\theta + \sin 6\theta]$

$16\left(-\frac{3}{4} - \frac{1}{8} - \frac{9}{16}\right) = -23$

71.[4] $A = \begin{bmatrix} 1 & 2 \\ -1 & 4 \end{bmatrix}, |A| = 6$

$A^{-1} = \frac{\text{adj}A}{|A|} = \frac{1}{6}\begin{bmatrix} 4 & -2 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} \frac{2}{3} & -\frac{1}{3} \\ \frac{1}{6} & \frac{1}{6} \end{bmatrix}$

$\begin{bmatrix} \frac{2}{3} & -\frac{1}{3} \\ \frac{1}{6} & \frac{1}{6} \end{bmatrix} = \begin{bmatrix} \alpha & 0 \\ 0 & \alpha \end{bmatrix} + \begin{bmatrix} \beta & 2\beta \\ -\beta & 4\beta \end{bmatrix}$

$\left.\begin{array}{l} \alpha + \beta = \frac{2}{3} \\ \beta = -\frac{1}{6} \end{array}\right\} \Rightarrow \alpha = \frac{2}{3} + \frac{1}{3} = \frac{5}{6}$

$4(\alpha - \beta) = 4(1) = 4$

72.[3] $\lim_{x\to 0} f(x) = 0$

$\lim_{x\to 0^+} xe^{\frac{\cot 4x}{\cot 2x}} = e^{\frac{1}{2}} = b$

$\lim_{x\to 0^-} (1 + |\sin x|)^{\frac{3a}{|\sin x|}} = e^{3a} = e^{\frac{1}{2}}$

$\lim_{x\to 0^-} (1 + |\sin x|)^{\frac{3a}{|\sin x|}} = e^{3a} = e^{3a} = e^{\frac{1}{2}}$

$a = \frac{1}{6} \Rightarrow 6a = 1$

$(6a + b^2) = (1 + e)$

73.[1] $\frac{dy}{dx} = e^{3x}.e^{4y} \Rightarrow \int e^{-4y}dy = \int e^{3x}dx$

$\frac{e^{-4y}}{-4} = \frac{e^{3x}}{3} + C \Rightarrow -\frac{1}{4} - \frac{1}{3} = C \Rightarrow C = -\frac{7}{12}$

$\frac{e^{-4y}}{-4} = \frac{e^{3x}}{3} - \frac{7}{12} \Rightarrow e^{-4y} = \frac{4e^{3x} - 7}{-3}$

$e^{4y} = \frac{3}{7 - 4e^{3x}} \Rightarrow 4y = \ell n\left(\frac{3}{7 - 4e^{3x}}\right)$

$4y = \ell n\left(\frac{3}{6}\right)$ when $x = -\frac{2}{3}\ell n 2$

$y = \frac{1}{4}\ell n\left(\frac{1}{2}\right) = -\frac{1}{4}\ell n 2$

74.[4] Normal of req. plane $(2\hat{i} + \hat{j} - \hat{k}) \times (\hat{i} - \hat{j} - \hat{k})$

$= -2\hat{i} + \hat{j} - 3\hat{k}$

Equation of plane

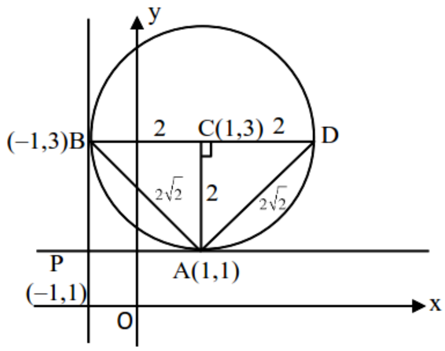
$-2(x + 1) + 1(y - 0) - 3(z + 2) = 0$

$-2x + y - 3z - 8 = 0$

$2x - y + 3z + 8 = 0$

$a + b + c = 4$

76.[3]



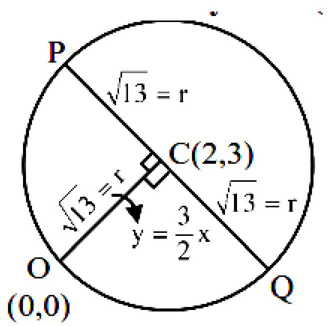
$\Delta ABD = \frac{1}{2} \times 2 \times 4 = 4$

76.[4] Apply L' Hopital Rule

$$\lim_{x \to 2}\left(\frac{2xf(2) - 4f'(x)}{1}\right)$$

$$= \frac{4(4) - 4}{1} = 12$$

77.[4]



$\tan\theta = -\frac{2}{3}$

Using symmetric from of line

P,Q : $(2 \pm \sqrt{13}\cos\theta, 3 \pm \sqrt{13}\sin\theta)$

$$\left(2 \pm \sqrt{13}.\left(-\frac{3}{\sqrt{13}}\right), 3 \pm \left(\frac{2}{\sqrt{13}}\right)\right)$$

(–1,5) & (5, 1)

78.[3] $\left(x^2 + \sqrt{5}\right)^2 = \sqrt{20}x^2$

$x^4 = -5 \Rightarrow x^8 = 25$

$\alpha^8 + \beta^8 = 50$

79.[3] Total number of cases = ${}^{90}C_1 = 90$

Now, $2^n - 2 = (3 - 1)^n - 2$

${}^nC_0 3^n - {}^nC_1 . 3^{n-2} + \ldots + (-1)^{n-1} . {}^nC_{n-1}\ 3 + (-1)^n . {}^nC_n - 2$

$3(3\,n^{-1} - n\,3^{n-2} + \ldots.. + (-1)^{n-1}\,n) + (-1)^n - 2$

$(2^n - 2)$ is multiply of 3 only when n is odd

Req. Probability = $\frac{45}{90} = \frac{1}{2}$

80.[3] For infinite solutions

$S_1 : x^2 + y^2 - x - y - \frac{1}{2} = 0\ ;\ C_1\left(\frac{1}{2}, \frac{1}{2}\right)$

$r_1 = \sqrt{\frac{1}{4} + \frac{1}{4} + \frac{1}{2}} = 1$

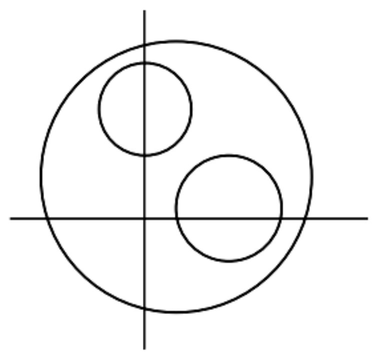
$S_2 : x^2 + y^2 - 4y + \frac{7}{4} = 0\ ;\ C_2 : (0,2)$

$r_2 = \sqrt{4 - \frac{7}{4}} = \frac{3}{2}$

$S_3 : x^2 + y^2 - 4x - 2y + 5 - r^2 = 0$

$C_3 : (2,1)$

$r_3 = \sqrt{4 + 1 - 5 + r^2} = |r|$



$C_1C_3 = \sqrt{\frac{5}{2}}$

$$\sqrt{\frac{5}{2}} \le |r - 1| \Rightarrow \left.\begin{array}{l} r \le 1 + \sqrt{\frac{5}{2}} \\ r \ge \frac{3}{2} + \sqrt{5} \end{array}\right\}$$

$C_2C_3 = \sqrt{5} \le \left|r - \frac{3}{2}\right|$

$$\left.\begin{array}{l} r - \frac{3}{2} \ge \sqrt{5} \\ r - \frac{3}{2} \le -\sqrt{5} \end{array}\right\}$$

Section -B

81.[5] $\Delta = \Delta_1 = \Delta_2 = \Delta_3 = 0$

$$\Delta = \begin{vmatrix} 1 & 1 & -1 \\ 1 & 2 & \alpha \\ 2 & -1 & 1 \end{vmatrix} = 0$$

$$\Delta = \begin{vmatrix} 3 & 0 & 0 \\ 1 & 2 & \alpha \\ 2 & -1 & 1 \end{vmatrix} = 0$$

$\Delta = 3(2 + \alpha) = 0$

$\Rightarrow \alpha = -2$

$$\Delta_2 = \begin{vmatrix} 1 & 2 & -1 \\ 1 & 1 & -2 \\ 2 & \beta & 1 \end{vmatrix} = 0$$

$1(1 + 2\beta) - 2(1 + 4) - (\beta - 2) = 0$

$\beta - 7 = 0$

$\beta = 7$

$\therefore \alpha + \beta = 5$ Ans.

82.[2] $\vec{a} \times \vec{b} = c$

Take Dot with $\vec{c}$

$(\vec{a} \times \vec{b}) \cdot \vec{c} = |\vec{c}|^2 = 2$

Projection of $\vec{b}$ or $\vec{a} \times \vec{c} = \ell$

$$\frac{|\vec{b}.(\vec{a} \times \vec{c})|}{|\vec{a} \times \vec{c}|} \ell$$

$\therefore \ell = \frac{2}{\sqrt{6}} \Rightarrow \ell^2 = \frac{4}{6}$

$3\ell^2 = 2$

83.[3] $2\log_3 (2^x - 5) = \log_3 2 + \log_3 \left(2^x - \frac{7}{2}\right)$

Let $2^x = t$

$\log_3 (t - 5)^2 = \log_3 2\left(t - \frac{7}{2}\right)$

$(t - 5)^2 = 2t - 7$

$t^2 - 12t + 32 = 0$

$(t - 4)(t - 8) = 0$

$\Rightarrow 2^x = 4$ or $2^x = 8$

X = 2 (Rejected)

Or x = 3

84.[1] For domain

$\log_5 (\log_3 (18x - x^2 - 77)) > 0$

$\log_3 (18x - x^2 - 77) > 1$

$18x - x^2 - 77 > 3$

$x^2 - 18x + 80 < 0$

$x \in (8,10)$

$\Rightarrow a = 8$ and $b = 10$

$$I = \int_a^b \frac{\sin^3 x}{(\sin^3 x + \sin^3(a + b - x))} dx$$

$$I = \int_a^b \frac{\sin^3(a + b - x)}{(\sin^3 x + \sin^3(a + b - x))}$$

$2I = (b - a) \Rightarrow I = \frac{b - a}{2}$ ($\because$ a = 8 and b = 10)

$I = \frac{10 - 8}{2} = 1$

85.[6] $$\begin{vmatrix} -2 & -2 & 0 \\ 2 & 0 & -1 \\ \sin^2 x & \cos^2 x & 1 + \cos 2x \end{vmatrix} \begin{pmatrix} R_1 \to R_1 - R_2 \\ \& R_2 \to R_2 - R_3 \end{pmatrix}$$

$-2(\cos^2 x) + 2(2 + 2\cos 2x + \sin^2 x)$

$4 + 4\cos 2x - 2(\cos^2 x - \sin^2 x)$

$f(X) = 4 + \underbrace{2\cos 2x}_{max=1}$

$f(X)_{max} = 4 + 2 = 6$

86.[16] $F(3) = 0$

$$e^x F(X) = \int_3^x (3t^2 + 2t + 4F'(t))dt$$

$e^x F(X) + e^x F'(X) = 3x^2 + 2x + 4F'(X)$

$(e^x - 4)\frac{dy}{dx} + e^x y = (3x^2 + 2x)$

$$\frac{dy}{dx} + \frac{e^x}{(e^x - 4)} y = \frac{(3x^2 + 2x)}{(e^x - 4)}$$

$$y e^{\int \frac{e^x}{(e^x - 4)} dx} = \int \frac{(3x^2 + 2x)}{(e^x - 4)} e^{\int \frac{e^x}{e^x - 4}} dx$$

$y.(e^x - 4) = \int (3x^2 + 2x)dx + c$

$y(e^x - 4) = x^3 + x^2 + c$

Put $x = 3 \Rightarrow c = -36$

$F(X) = \dfrac{(x^3 + x^2 - 36)}{(e^x - 4)}$

$F'(X) = \dfrac{(3x^2 + 2x)(e^x - 4) - (x^3 + x^2 - 36)e^x}{(e^x - 4)^2}$

Now put value of x = 4 we will get $\alpha = 12$ & $\beta = 4$

87.[3] $\overrightarrow{BA} = (\hat{i} + 4\hat{j} - 5\hat{k})$

$\overrightarrow{BA} = (\hat{i} + 4\hat{j} - 5\hat{k})$

$\overrightarrow{BA} \times \vec{\ell} = \vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -3 & 2 & 1 \\ 1 & 4 & -5 \end{vmatrix}$

$a\hat{i} + b\hat{j} + c\hat{k} = -14\hat{i} - \hat{j}(14) + \hat{k}(-14)$

a = 1, b = 1, c = 1

Plane is (x –2) + (y –3) + (z + 2) = 0

x + y + z – 3 = 0

$d = \sqrt{3} \Rightarrow d^2 = 3$

88.[490] F(mn) = f(m). f(n)

Put m = 1 f(n) = f(1). f(n) ⇒ f(1) = 1Put m = n = 2

$f(4) = f(2).f(2) \begin{cases} f(2) = 1 \Rightarrow f(4) = 1 \\ \text{or} \\ f(2) = 2 \Rightarrow f(4) = 4 \end{cases}$

Put m = 2, n = 3

$f(6) = f(2).f(3) \begin{cases} \text{when } f(2) = 1 \\ f(3) = 1 \text{ to } 7 \\ f(2) = 2 \\ f(3) = 1 \text{ or } 2 \text{ or } 3 \end{cases}$

f(5), f(7) can take any value

Total = (1 × 1 × 7 × 1 × 7 × 1 × 7)

+ (1 × 1 × 3 × 1 × 7 × 1 × 7)

= 490

89.[2] $\sec y \dfrac{dy}{dx} = 2 \sin x \cos y$

$\sec^2 y dx = 2 \sin x dx$

tan y = –2cos x + c

c = 2

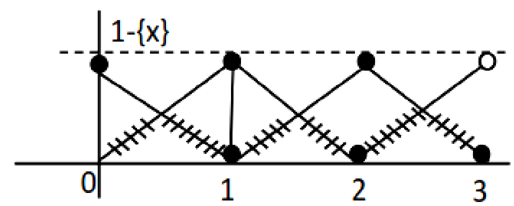
$\tan y = -2\cos x + 2 \Rightarrow$ at $x = \dfrac{\pi}{2}$

tan y = 2

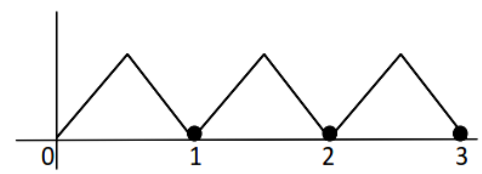
$\sec^2 y \dfrac{dy}{dx} = 2\sin x$

$5\dfrac{dy}{dx} = 2$

90.[5]



$1 - \{x\} = 1 - x; 0 \le x < 1$



Non differentiable at

$x = \dfrac{1}{2}, 1, \dfrac{3}{2}, 2, \dfrac{5}{2}$