JEE MAIN ONLINE PAPER 2021

Held on July 25, 2021 (Evening)

Instructions

1. This test will be a 3 hours Test.
2. This test consists of Physics, Chemistry and Mathematics questions with equal weightage of 100 marks.
3. Each question is of 4 marks.
4. In the question paper consisting of Physics (Q.no. 1 to 30), Chemistry (Q.no. 31 to 60) and Mathematics (Q.no. 61 to 90). There are two sections for each subject (Section-A : MCQ Type & Section-B : Numerical Response Type). Section-A consists of 20 multiple choice questions & Section-B consists of 10 Numerical Value type Questions. **Candidates have a choice to Answer 5 out of the 10 numerical value answer based questions per section.**
5. There will be only one correct choice in the given four choices in Section-A. For each question 4 marks will be awarded for correct choice, 1 mark will be deducted for incorrect choice and zero mark will be awarded for not attempted question. For Section-B questions 4 marks will be awarded for correct answer and zero for unattempted and incorrect answer.
6. Any textual, printed or written material, mobile phones, calculator etc. is not allowed for the students appearing for the test.
7. All calculations/written work should be done in the rough sheet provided.

PHYSICS

Section -A

Q.1 The relation between time t and distance x for a moving body is given as $t = mx^2 + nx$, here m and n are constants. The retardation of the motion is : (When v stands for velocity)

(1) $2mv^3$ (2) $2mnv^3$ (3) $2nv^3$ (4) $2n^2v^3$

Q.2 In a simple harmonic oscillation, what fraction of total mechanical energy is in the form of kinetic energy, when the particle is midway between mean and extreme position.

(1) $\frac{1}{2}$ (2) $\frac{3}{4}$ (3) $\frac{1}{3}$ (4) $\frac{1}{4}$

Q.3 A force $\vec{F} = (40\hat{i} + 10\hat{j})$ N acts on a body of mass 5 kg. If the body starts from rest, its position vector $\vec{r}$ at time t = 10 s, will be :

(1) $(100\hat{i} + 400\hat{j})$ m (2) $(100\hat{i} + 100\hat{j})$ m

(3) $(400\hat{i} + 100\hat{j})$ m (4) $(400\hat{i} + 400\hat{j})$ m

Q.4 A prism of refractive index μ and angle of prism A is placed in the position of minimum angle of deviation. If minimum angle of deviation is also A, then in terms of refractive index

(1) $2\cos^{-1}\left(\frac{\mu}{2}\right)$ (2) $\sin^{-1}\left(\frac{\mu}{2}\right)$

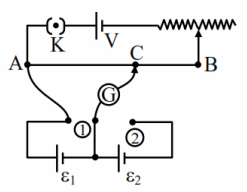
(3) $\sin^{-1}\left(\sqrt{\frac{\mu-1}{2}}\right)$ (4) $\cos^{-1}\left(\frac{\mu}{2}\right)$

Q.5 A heat engine has an efficiency of $\frac{1}{6}$. When the temperature of sink is reduced by 62°C, its efficiency get doubled. The temperature of the source is :

(1) 124°C (2) 37°C

(3) 62°C (4) 99°C

Q.6 In the given potentiometer circuit arangement, the balancing length AC is measured to be 50 cm. When the galvanometer connection is shifted from point (1) to point (2) in the given diagram, the balancing length becomes 400 cm. The ratio of the emf of two cells, $\frac{\varepsilon_1}{\varepsilon_2}$ is :



(1) $\frac{5}{3}$ (2) $\frac{8}{5}$ (3) $\frac{4}{3}$ (4) $\frac{3}{2}$

Q.7 Two ions having same mass have charges in the ratio 1 : 2. They are projected normally in a uniform magnetic field with their speeds in the ratio 2 : 3. The ratio of the radii of their circular trajectories is :

(1) 1 : 4 (2) 4 : 3 (3) 3 : 1 (4) 2 : 3

Q.8 A 10Ω resistance is connected across 220V – 50HzAC supply. The time taken by the current to change from its maximum value to the rms value is:

(1) 2.5 ms (2) 1.5 ms (3) 3.0 ms (4) 4.5 ms

Q.9 A balloon was moving upwards with a uniform velocity of 10 m/s. An object of finite mass is dropped from the balloon when it was at a height of 75 m from the ground level. The height of the balloon from the ground when object strikes the ground was around :
(takes the value of g as 10 m/s^2)

(1) 300 m (2) 200 m (3) 125 m (4) 250 m

Q.10 If q_f is the bound charge on the capacitor plates and q_b is the bound charge on the dielectric slab of dielectric constant k placed between the capacitor plates, then bound charge q_b can be expressed as

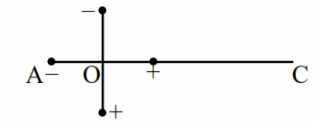
(1) $q_b = q_f\left(1-\frac{1}{\sqrt{k}}\right)$ (2) $q_b = q_f\left(1-\frac{1}{k}\right)$

(3) $q_b = q_f\left(1+\frac{1}{\sqrt{k}}\right)$ (4) $q_b = q_f\left(1+\frac{1}{k}\right)$

Q.11 Consider a planet in some solar system which has a mass double the mass of earth and density equal to the average density of earth. If the weight of an object on earth is W, the weight of the same object on that planet will be :

(1) 2W (2) W (3) $2^{\frac{1}{3}}$W (4) $\sqrt{2}$ W

Q.12 Two ideal electric dipoles A and B, having their dipole moment p1 and p2 respectively are placed on a plane with their centres at O as shown in the figure. At point C on the axis of dipole A, the resultant electric field is making an angle of 37°with the axis. The ratio of the dipole moment of A and B, $\frac{p_1}{p_2}$ is : (take $\sin 37^\circ = \frac{3}{5}$)



(1) $\frac{3}{8}$ (2) $\frac{3}{2}$ (3) $\frac{2}{3}$ (4) $\frac{4}{3}$

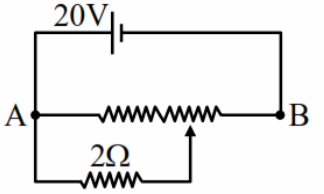
Q.13 Two spherical soap bubbles of radii r_1 and r_2 in vacuum combine under isothermal conditions. The resulting bubble has a radius equal to :

(1) $\frac{r_1 r_2}{r_1 + r_2}$ (2) $\sqrt{r_1 r_2}$ (3) $\sqrt{r_1^2 + r_2^2}$ (4) $\frac{r_1 + r_2}{2}$

Q.14 The force is given in terms of time t and displacement x by the equation F = A cos Bx + C sin Dt The dimensional formula of $\frac{AD}{B}$ is :

(1) $[M^0 L T^{-1}]$ (2) $[M L^2 T^{-3}]$
(3) $[M^1 L^1 T^{-2}]$ (4) $[M]^2 L^2 T^{-3}]$

Q.15 The given potentiometer has its wire of resistance 10Ω. When the sliding contact is in the middle of the potentiometer wire, the potential drop across 2Ω resistor is :



(1) 10V (2) 5V (3) $\frac{40}{9}$V (4) $\frac{40}{11}$ V

Q.16 An electron moving with speed v and a photon moving with speed c, have same D-Broglie wavelength. The ratio of kinetic energy of electron to that of photon is :

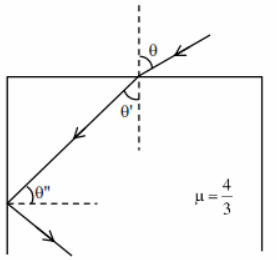
(1) $\frac{3c}{v}$ (2) $\frac{v}{3c}$ (3) $\frac{v}{2c}$ (4) $\frac{2c}{v}$

Q.17 The instantaneous velocity of a particle moving in a straight line is given as $v = \alpha t + \beta t^2$, where α and β are constants. The distance travelled by the particle between 1s and 2s is :

(1) $3\alpha + 7\beta$ (2) $\frac{3}{2}\alpha + \frac{7}{3}\beta$

(3) $\frac{\alpha}{2} + \frac{\beta}{3}$ (4) $\frac{3}{2}\alpha + \frac{7}{2}\beta$

Q.18 A ray of light entering from air into a denser medium of refractive index $\frac{4}{3}$, as shown in figure. The light ray suffers total internal reflection at the adjacent surface as shown. The maximum value of angle θ should be equal to :



(1) $\sin^{-1}\frac{\sqrt{7}}{3}$ (2) $\sin^{-1}\frac{\sqrt{5}}{4}$

(3) $\sin^{-1}\frac{\sqrt{7}}{4}$ (4) $\sin^{-1}\frac{\sqrt{5}}{3}$

Q.19 When radiation of wavelength λ is incident on a metallic surface, the stopping potential of ejected photoelectrons is 4.8 V. If the same surface is illuminated by radiation of double the previous wavelength, then the stopping potential becomes 1.6 V. The threshold wavelength of the metal is :

(1) 2λ (2) 4λ (3) 8λ (4) 6λ

Q.20 Two vectors $\vec{X}$ and $\vec{Y}$ have equal magnitude. The magnitude of $(\vec{X} - \vec{Y})$ is n times the magnitude of $(\vec{X} + \vec{Y})$. The angle between $\vec{X}$ and $\vec{Y}$ is .

(1) $\cos^{-1}\left(\frac{-n^2-1}{n^2-1}\right)$ (2) $\cos^{-1}\left(\frac{n^2-1}{-n^2-1}\right)$

(3) $\cos^{-1}\left(\frac{n^2+1}{-n^2-1}\right)$ (4) $\cos^{-1}\left(\frac{n^2+1}{n^2-1}\right)$

Section -B

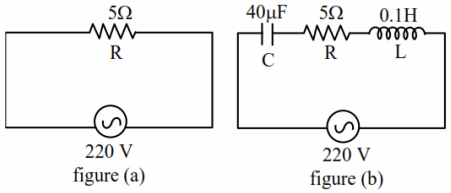
Q.21 A system consists of two types of gas molecules A and B having same number density $2 \times 10^{25}/m^3$. The diameter of A and B are 10 Å and 5 Å respectively. They suffer collision at room temperature. The ratio of average distance covered by the molecule A to that of B between two successive collision is _____ $\times 10^{-2}$

Q.22 A light beam of wavelength 500 nm is incident on a metal having work function of 1.25 eV, placed in a magnetic field of intensity B. The electrons emitted perpendicular to the magnetic field B, with maximum kinetic energy are bent into circular arc of radius 30 cm. The value of B is ___ $\times 10^{-7}$ T

Q.23 A message signal of frequency 20 kHz and peak voltage of 20 volt is used to modulate a carrier wave of frequency 1 MHz and peak voltage of 20 volt. The modulation index will be :

Q.24 A 16 Ω wire is bend to form a square loop. A 9V supply having internal resistance of 1 Ω is connected across one of its sides. The potential drop across the diagonals of the square loop is _____ $\times 10^{-1}$ V

Q.25 Two circuits are shown in the figure (a) & (b). At a frequency of _____ rad/s the average power dissipated in one cycle will be same in both the circuits.



Q.26 From the given data, the amount of energy required to break the nucleus of aluminium $\frac{27}{13}$Al is —— x $\times 10^{-3}$ J.

Mass of neutron = 1.00866 u
Mass of proton = 1.00726 u
Mass of Aluminium nucleus = 27.18846 u
(Assume 1 u corresponds to x J of energy)
(Round off to the nearest integer)

Q.27 A force of $F = (5y + 20)\ \hat{j}$ N acts on a particle. The workdone by this force when the particle is moved from y = 0 m to y = 10 m is _____ J.

Q.28 A solid disc of radius 20 cm and mass 10 kg is rotating with an angular velocity of 600 rpm, about an axis normal to its circular plane and passing through its centre of mass. The retarding torque required to bring the disc at rest in 10 s is _____ $\pi \times 10^{-1}$ Nm.

Q.29 In a semiconductor, the number density of intrinsic charge carriers at 27°C is $1.5 \times 10^{16}/\ m^3$. If the semiconductor is doped with impurity atom, the hole density increases to $4.5 \times 10^{22}\ /\ m^3$. The electron density in the doped semiconductor is —— $\times\ 10^9\ /m^3$.

Q.30 The nuclear activity of a radioactive element becomes $\left(\frac{1}{8}\right)^{th}$ of its initial value in 30 years. The half-life of radioactive element is _____ years.

CHEMISTRY

Section -A

Q.31 In the following the correct bond order sequence is:

(1) $O_2^{2-} > O_2^{+} > O_2^{-} > O_2$

(2) $O_2^{+} > O_2^{-} > O_2^{2-} > O_2$

(3) $O_2^{+} > O_2 > O_2^{-} > O_2^{2-}$

(4) $O_2 > O_2^{-} > O_2^{2-} > O_2^{+}$

Q.32 A biodegradable polyamide can be made from:

(1) Glycine and isoprene

(2) Hexamethylene diamine and adipic acid

(3) Glycine and aminocaproic acid

(4) Styrene and caproic acid

Q.33 Match **List I** with **List II** :

	List-I Elements		**List-II** Properties
(a)	Li	(i)	Poor water solubility of I^- salt
(b)	Na	(ii)	Most abundant element in cell fluid
(c)	K	(iii)	Bicarbonate salt used in fire extinguisher
(d)	Cs	(iv)	Carbonate salt decomposes easily on heating

Choose the correct answer from the options given below :

(1) (a)-(iv), (b)-(iii), (c)-(ii), (d)-(i)

(2) (a)-(i), (b)-(iii), (c)-(ii), (d)-(iv)

(3) (a)-(iv), (b)-(ii), (c)-(iii), (d)-(i)

(4) (a)-(i), (b)-(ii), (c)-(iii), (d)-(iv)

Q.34 Which one of the following metal complexes is most stable?

(1) $[Co(en)(NH_3)_4Cl_2$ (2) $[Co(en)_3]Cl_2$

(3) $[Co(en)_2(NH_3)_2Cl_2$ (4) $[Co(NH_3)_6Cl_2$

Q.35 Match **List I** with **List II** : (Both having metallurgical terms)

	List-I		**List-II**
(a)	Concentration of Ag ore	(i)	Reverberatory furnace
(b)	Blast furnace	(ii)	Pig iron
(c)	Blister copper	(iii)	Leaching with dilute NaCN solution
(d)	Froth floatation method	(iv)	Sulfide ores

Choose the correct answer from the options given below :

(1) (a)–(iii), (b)–(ii), (c)–(i), (d)–(iv)

(2) (a)–(iii), (b)–(iv), (c)–(i), (d)–(ii)

(3) (a)–(iv), (b)–(i), (c)–(iii), (d)–(ii)

(4) (a)–(iv), (b)–(iii), (c)–(ii), (d)–(i)

Q.36 The ionic radii of F^- and O^{2-} respectively are 1.33 Å and 1.4 Å, while the covalent radius of N is0.74 Å.

The correct statement for the ionic radius of N^{3-} from the following is :

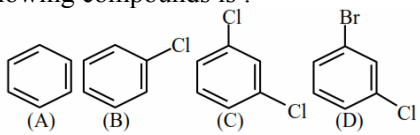
(1) It is smaller than F^- and N

(2) It is bigger than O^{2-} and F^-

(3) It is bigger than F^- and N, but smaller than of O^{2-}

(4) It is smaller than O^{2-} and F^-, but bigger than of N

Q.37 The correct decreasing order of densities of the following compounds is :



(1) (D) > (C) > (B) > (A)

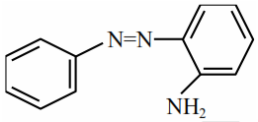
(2) (C) > (D) > (A) > (B)

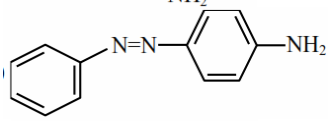
(3) (C) > (B) > (A) > (D)

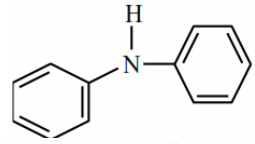
(4) (A) > (B) > (C) > (D)

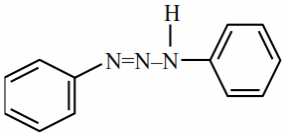
Q.38 $C_6H_5NO_2 \xrightarrow{Sn + HCl}$ "A" $\xrightarrow[H^{\oplus}]{C_6H_5\overset{\oplus}{N_2}\overset{\ominus}{Cl}}$ P (Yellow coloured compound)

Consider the above reaction, the Product "P" is :

(1) 

(2) 

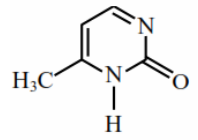
(3) 

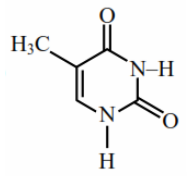
(4) 

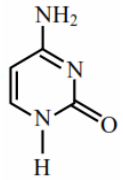
Q.39 A reaction of benzonitrile with one equivalent CH_3MgBr followed by hydrolysis produces a yellow liquid "P". The compound "P" will give positive_____.
(1) Iodoform test (2) Schiff's test
(3) Ninhydrin's test (4) Tollen's test

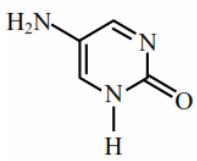
Q.40 The spin only magnetic moments (in BM) for free Ti^{3+}, V^{2+} and Sc^{3+} ions respectively are
(At.No. Sc : 21, Ti : 22, V : 23)
(1) 3.87, 1.73, 0 (2) 1.73, 3.87, 0
(3) 1.73, 0, 3.87 (4) 0, 3.87, 1.73

Q.41 Which one of the following is correct structure for cytosine ?

(1) 

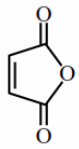
(2) 

(3) 

(4) 

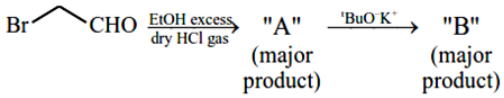
Q.42 Identify the species having one π-bond and maximum number of canonical forms from the following :
(1) SO_3 (2) O_2 (3) SO_2 (4) CO_3^{2-}

Q.43 Which one of the following metals forms interstitial hydride easily ?
(1) Cr (2) Fe (3) Mn (4) Co

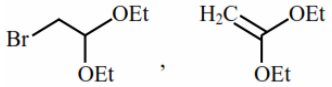
Q.44  Maleic anhydride

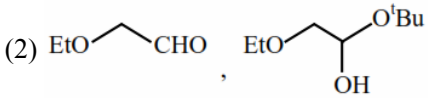
Maleic anhydride can be prepared by :
(1) Heating trans-but-2-enedioic acid
(2) Heating cis-but-2-enedioic acid
(3) Treating cis-but-2-enedioic acid with alcohol and acid
(4) Treating trans-but-2-enedioic acid with alcohol and acid

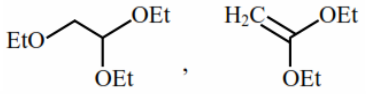
Q.45 Given below are two statements :
Statement I : Chlorofluoro carbons breakdown by radiation in the visible energy region and release chlorine gas in the atmosphere which then reacts with stratospheric ozone.
Statement II : Atmospheric ozone reacts with nitric oxide to give nitrogen and oxygen gases, which add to the atmosphere.
For the above statements choose the correct answerfrom the options given below :
(1) **Statement I** is incorrect but **statement II** is true
(2) Both **statement I** and **II** are false
(3) **Statement I** is correct but **statement II** is false
(4) Both **statement I** and **II** are correct

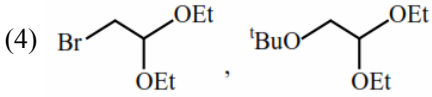
Q.46 

[where Et ⇒ $-C_2H_5$ ^tBu ⇒ $(CH_3)_3C-$]
Consider the above reaction sequence, Product "A"and Product "B" formed respectively are :

(1) 

(2) 

(3) 

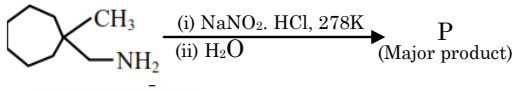
(4) 

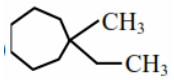
Q.47 Match **List I** with **List II** :

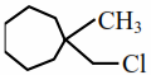
List-I **Example of colloids**	**List-II** **Classification**
(a) Cheese	(i) dispersion of liquid in liquid
(b) Pumice stone	(ii) dispersion of liquid in gas
(c) Hair cream	(iii) dispersion of gas in solid
(d) Cloud	(iv) dispersion of liquid in solid

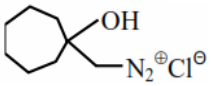
Choose the most appropriate answer from the options given below
(1) (a)-(iv), (b)-(iii), (c)-(ii), (d)-(i)
(2) (a)-(iv), (b)-(i), (c)-(iii), (d)-(ii)
(3) (a)-(iii), (b)-(iv), (c)-(i), (d)-(ii)
(4) (a)-(iv), (b)-(iii), (c)-(i), (d)-(ii)

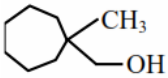
Q.48 What is the major product "P" of the following reaction ?



(1) 

(2) 

(3) 

(4) 

Q.49 Identify the process in which change in the oxidation state is five :

(1) $Cr_2O_7^{2-} \rightarrow 2Cr^{3+}$ (2) $MnO_4^- \rightarrow Mn^{2+}$

(3) $CrO_4^{2-} \rightarrow Cr^{3+}$ (4) $C_2O_4^{2-} \rightarrow 2CO_2$

Q.50 Which among the following is the strongest acid ?

(1) $CH_3CH_2CH_2CH_3$ (2) 

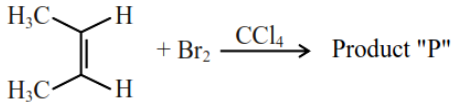
(3)  (4) 

Section -B

Q.51 A system does 200 J of work and at the same time absorbs 150 J of heat. The magnitude of the change in internal energy is _____ J. (Nearest integer)

Q.52 An accelerated electron has a speed of $5 \times 10^6 ms^{-1}$ with an uncertainty of 0.02%. The uncertainty in finding its location while in motion is $x \times 10^{-9}$ m.
The value of x is _____ . (Nearest integer)
[Use mass of electron = 9.1×10^{-31} kg,
$h = 6.63 \times 10^{-34}$ Js, $\pi = 3.14$]

Q.53 Number of electrons present in 4f orbital of Ho^{3+} ion is _____ . (Given Atomic No. of Ho = 67)

Q.54 

Consider the above chemical reaction. The total number of stereoisomers possible for Product 'P' is_____ .

Q.55 For a chemical reaction A → B, it was found that concentration of B is increased by 0.2 mol L^{-1} in30 min. The average rate of the reaction is _____ $\times 10^{-1}$ mol L^{-1} h^{-1}.(in nearest integer)

Q.56 The number of significant figures in 0.00340 is _____ .

Q.57 Assuming that $Ba(OH)_2$ is completely ionised in aqueous solution under the given conditions the concentration of H_3O^+ ions in 0.005 M aqueous solution of $Ba(OH)_2$ at 298 K is _____ $\times 10^{-12}$ mol L^{-1}. (Nearest integer)

Q.58 0.8 g of an organic compound was analysed by Kjeldahl's method for the estimation of nitrogen. If the percentage of nitrogen in the compound was found to be 42%, then _____ mL of 1 MH_2SO_4 = would have been neutralized by the ammonia evolved during the analysis.

Q.59 When 3.00 g of a substance 'X' is dissolved in 100 g of CCl_4, it raises the boiling point by 0.60 K. The molar mass of the substance 'X' is _____ g mol^{-1}. (Nearest integer).
[Given Kb for CCl4 is 5.0 K kg mol^{-1}]

Q.60 An LPG cylinder contains gas at a pressure of 300 kPa at 27°C. The cylinder can withstand the pressure of 1.2×10^6 Pa. The room in which the cylinder is kept catches fire. The minimum temperature at which the bursting of cylinder will take place is _____ °C. (Nearest integer)

MATHEMATICS

Section -A

Q.61 The sum of all those terms which are rational numbers in the expansion of $(2^{1/3} + 3^{1/4})^{12}$ is:
(1) 89 (2) 27 (3) 35 (4) 43

Q.62 The first of the two samples in a group has 100 items with mean 15 and standard deviation 3. If the whole group has 250 items with mean 15.6 and standard deviation $\sqrt{13.44}$, then the standard deviation of the second sample is :
(1) 8 (2) 6 (3) 4 (4) 5

Q.63 If $f(x) = \begin{cases} \int_0^x (5+|1-t|)dt, & x > 2 \\ 5x+1, & x \le 2 \end{cases}$, then
(1) f(x) is not continuous at x = 2
(2) f(x) is everywhere differentiable
(3) f(x) is continuous but not differentiable at x = 2
(4) f(x) is not differentiable at x = 1

Q.64 If the greatest value of the term independent of 'x' in the expansion of $\left(x\sin\alpha + a\frac{\cos\alpha}{x}\right)^{10}$ is $\frac{10!}{(5!)^2}$, then the value of 'a' is equal to:
(1) –1 (2) 1 (3) –2 (4) 2

Q.65 Consider the statement "The match will be played only if the weather is good and ground is not wet".
Select the correct negation from the following:
(1) The match will not be played and weather is not good and ground is wet.
(2) If the match will not be played, then either weather is not good or ground is wet.
(3) The match will be played and weather is not good or ground is wet.
(4) The match will not be played or weather is good and ground is not wet.

Q.66 The value of $\cot\frac{\pi}{24}$ is :
(1) $\sqrt{2}+\sqrt{3}+2-\sqrt{6}$
(2) $\sqrt{2}+\sqrt{3}+2+\sqrt{6}$
(3) $\sqrt{2}-\sqrt{3}-2+\sqrt{6}$
(4) $3\sqrt{2}-\sqrt{3}-\sqrt{6}$

Q.67 The lowest integer which is greater than $\left(1+\frac{1}{10^{100}}\right)^{10^{100}}$ is ——.
(1) 3 (2) 4 (3) 2 (4) 1

Q.68 The value of the integral $\int_{-1}^{1}\log(x+\sqrt{x^2+1})\,dx$ is :
(1) 2 (2) 0 (3) –1 (4) 1

Q.69 Let a, b and c be distinct positive numbers. If the vectors $a\hat{i}+a\hat{j}+c\hat{k}$, $\hat{i}+\hat{k}$ and $c\hat{i}+c\hat{j}+b\hat{k}$ are co-planar, then c is equal to :
(1) $\frac{2}{\frac{1}{a}+\frac{1}{b}}$ (2) $\frac{a+b}{2}$
(3) $\frac{1}{a}+\frac{1}{b}$ (4) $\sqrt{ab}$

Q.70 If [x] be the greatest integer less than or equal to x,then $\sum_{n=8}^{100}\left[\frac{(-1)^n.n}{2}\right]$ is equal to :
(1) 0 (2) 4 (3) –2 (4) 2

Q.71 The number of distinct real roots of $\begin{vmatrix}\sin x & \cos x & \cos x\\ \cos x & \sin x & \cos x\\ \cos x & \cos x & \sin x\end{vmatrix} = 0$ in the interval $-\frac{\pi}{4}\le x\le\frac{\pi}{4}$ is :
(1) 4 (2) 1 (3) 2 (4) 3

Q.72 If $|\vec{a}| = 2$, $|\vec{b}| = 5$ and $|\vec{a}\times\vec{b}| = 8$, then $|\vec{a}.\vec{b}|$ is equal to
(1) 6 (2) 4 (3) 3 (4) 5

Q.73 The number of real solutions of the equation, $x^2 - |x| - 12 = 0$ is:
(1) 2 (2) 3 (3) 1 (4) 4

Q.74 Consider function $f: A \to B$ and $g: B \to C$ (A, B, $C \subseteq \mathbf{R}$) such that $(gof)^{-1}$ exists, then:
(1) f and g both are one-one
(2) f and g both are onto
(3) f is one-one and g is onto
(4) f is onto and g is one-one

Q.75 If $P = \begin{bmatrix}1 & 0\\ 1/2 & 1\end{bmatrix}$, then P^{50} is :
(1) $\begin{bmatrix}1 & 0\\ 25 & 1\end{bmatrix}$ (2) $\begin{bmatrix}1 & 50\\ 0 & 1\end{bmatrix}$
(3) $\begin{bmatrix}1 & 25\\ 0 & 1\end{bmatrix}$ (4) $\begin{bmatrix}1 & 0\\ 50 & 1\end{bmatrix}$

Q.76 Let x be a random variable such that the probability function of a distribution is given by $P(X) = 0) \frac{1}{2}$, $P(X = j) = \frac{1}{3}$ (j = 1, 2, 3...., ∞). Then the mean of the distribution and P(X is positive and even) respectively are:
(1) $\frac{3}{8}$ and $\frac{1}{8}$ (2) $\frac{3}{4}$ and $\frac{1}{8}$
(3) $\frac{3}{4}$ and $\frac{1}{9}$ (4) $\frac{3}{4}$ and $\frac{1}{16}$

Q.77 If a tangent to the ellipse $x^2 + 4y^2 = 4$ meets the tangents at the extremities of its major axis at B and C, then the circle with BC as diameter passes through the point :
(1) $(\sqrt{3},0)$ (2) $(\sqrt{2},0)$ (3) (1 , 1) (4) (– 1, 1)

Q.78 Let the equation of the pair of lines, y = px and y = qx, can be written as (y – px) (y – qx) = 0. Then the equation of the pair of the angle bisectors of the lines $x^2 - 4xy - 5y^2 = 0$ is:
(1) $x^2 - 3xy + y^2 = 0$ (2) $x^2 + 4xy - y^2 = 0$
(3) $x^2 + 3xy - y^2 = 0$ (4) $x^2 - 3xy - y^2 = 0$

Q.79 If ${}^nP_r = {}^nP_{r+1}$ and ${}^nC_r = {}^nC_{r-1}$ then the value of r is equal to :

(1) 1 (2) 4 (3) 2 (4) 3

Q.80 Let y = y (x) be the solution of the differential equation $xdy = (y + x^3 \cos x)dx$ with $y(\pi) = 0$, then $y\left(\frac{\pi}{2}\right)$ is equal to

(1) $\frac{\pi^2}{4}+\frac{\pi}{2}$ (2) $\frac{\pi^2}{2}+\frac{\pi}{4}$

(3) $\frac{\pi^2}{2}-\frac{\pi}{4}$ (4) $\frac{\pi^2}{4}-\frac{\pi}{2}$

Section -B

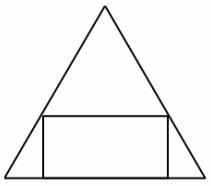
Q.81 Let $n \in N$ and [x] denote the greatest integer less than or equal to x. If the sum of (n + 1) terms ${}^nC_0$, 3. ${}^nC_1$, 5. ${}^nC_2$, 7 . ${}^nC_3$, ... is equal to 2^{100} . 101 then $2\left[\frac{n-1}{2}\right]$ is equal to ——

Q.82 Consider the function $f(x)\begin{cases}\frac{P(x)}{\sin(x-2)}, & x \neq 2\\ 7, & x = 2\end{cases}$

Where P(x) is a polynomial such that P″ (x) is always a constant and P(3) = 9. If f(x) is continuous at x = 2, then P(5) is equal to _____.

Q.83 The equation of a circle is $Re(z^2) + 2 (Im(z))^2 + 2Re(z) = 0$, where $z = x + iy$. A line which passes through the center of the given circle and the vertex of the parabola, $x^2 - 6x - y + 13 = 0$, has y-intercept equal to ____.

Q.84 If a rectangle is inscribed in an equilateral triangle of side length $2\sqrt{2}$ as shown in the figure, then the square of the largest area of such a rectangle is ____



Q.85 If $(\vec{a}+3\vec{b})$ is perpendicular to $(7\vec{a}-5\vec{b})$ and $(\vec{a}-4\vec{b})$ is perpendicular to $(7\vec{a}-2\vec{b})$, then the angle between $\vec{a}$ and $\vec{b}$ (in degrees) is _____.

Q.86 Let a curve y = f(x) pass through the point $(2, (\log_e 2)^2)$ and have slope $\frac{2y}{x \log_e x}$ for all positive real value of x. Then the value of f(e) is equal to_____.

Q.87 If a + b + c = 1, ab + bc + ca = 2 and abc = 3, then the value of $a^4 + b^4 + c^4$ is equal to _____.

Q.88 A fair coin is tossed n-times such that the probability of getting at least one head is at least 0.9. Then the minimum value of n is _____.

Q.89 If the co-efficient of x^7 and x^8 in the expansion of $\left(2+\frac{x}{3}\right)^n$ | are equal, then the value of n is equal to

Q.90 If the lines $\frac{x-k}{1} = \frac{y+2}{2} = \frac{z+3}{3}$ are co-planar, then the value of k is _____.

JEE MAIN ONLINE PAPER 2021

Held on JULY 25, 2021 (Evening)

Hints & Solutions

PHYSICS

Section -A

1.[1] $t = mx^2 + nx$

$\frac{1}{v} = \frac{dt}{dx} = 2mx + n$

$v = \frac{1}{2mx+n}$

$\frac{dv}{dt} = -\frac{2m}{(2mx+n)^2}\left(\frac{dx}{dt}\right)$

$a = -(2m)v^3$

2.[2] $K = \frac{1}{2}m\omega^2(A^2 - x^2)$

$= \frac{1}{2}m\omega^2\left(A^2 - \frac{A^2}{4}\right)$

$= \frac{1}{2}m\omega^2\left(\frac{3A^2}{4}\right)$

$K = \frac{3}{4}\left(\frac{1}{2}m\omega^2A^2\right)$

3.[3] $\frac{d\vec{v}}{dt} = \vec{a} = \frac{\vec{F}}{m} = (8\hat{i} + 2\hat{j})\ m/s^2$

$\frac{d\vec{r}}{dt} = \vec{v} = (8t\hat{i} + 2t\hat{j})\ m/s$

$\vec{r} = \left(8\hat{i} + 2\hat{j}\right)\frac{t^2}{2}m$

At t = 10 sec

$\vec{r} = [(8\hat{i} + 2\hat{j})50]m$

$\Rightarrow \vec{r} = (400\hat{i} + 100\hat{j})\ m$

4.[1] $\mu = \frac{\sin\left(\frac{A+\delta_{min}}{2}\right)}{\sin\left(\frac{A}{2}\right)}$

$\mu = \frac{\sin\left(\frac{A+A}{2}\right)}{\sin\left(\frac{A}{2}\right)}$

$\mu = \frac{\sin A}{\sin\frac{A}{2}} = 2\cos\frac{A}{2}$

$A = 2\cos^{-1}\left(\frac{\mu}{2}\right)$

5.[4] $\eta = 1 - \frac{T_L}{T_H}$ ...(i)

$2\eta = 1 - \frac{(T_L - 62)}{T_H} = 1 - \frac{T_L}{T_H} + \frac{62}{T_H}$

$\Rightarrow \eta = \frac{62}{T_H} \Rightarrow \frac{1}{6} = \frac{62}{T_H}$

$\Rightarrow T_H = 6 \times 62 = 372\ K$

In °C $\Rightarrow 372 - 273 = 99°C$

6.[1] $E_1 = k\ell_1$ ... (i)

$E_1 + E_2 = k\ell_2$ ... (ii)

$\frac{E_1}{E_1 + E_2} = \frac{\ell_1}{\ell_2} = \frac{250}{400} = \frac{5}{8}$

$8E_1 = 5E_1 + 5E_2$

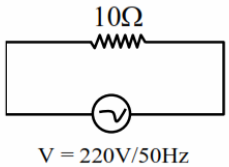
$3E_1 = 5E_2$

$\frac{E_1}{E_2} = \frac{5}{3}$

7.[2] $R = \frac{mv}{qB} \Rightarrow \frac{R_1}{R_2} = \frac{\frac{mv_1}{q_1B}}{\frac{mv_2}{q_2B}} = \frac{v_1}{q_1} \times \frac{q_2}{v_2} = \frac{q_2}{q_1} \times$

$\frac{v_1}{v_2} = \frac{2}{1} \times \left(\frac{2}{3}\right) = \frac{4}{3}$

8.[1]



$\Rightarrow i = i_0\sin\omega t$

When $i = i_0$

$i_0 = i_0 \sin\omega t_1 \Rightarrow \omega t_1 = \frac{\pi}{2}$ ....(i)

When $i = \frac{i_0}{\sqrt{2}}$

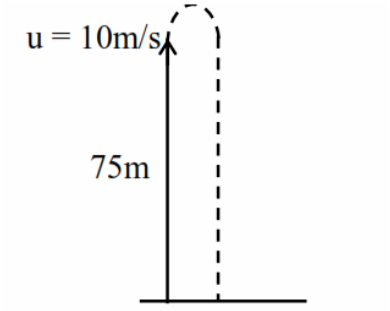
$\frac{i_0}{\sqrt{2}} = i_0 \sin \omega t_2 \Rightarrow \omega t_2 = \frac{\pi}{4}$ ....(ii)

Time taken by current from maximum value to rms value

$\Rightarrow (t_1 - t_2) = \frac{\pi}{2\omega} - \frac{\pi}{4\omega} = \frac{\pi}{4\omega} = \frac{\pi}{4 \times 2\pi f}$

$= \frac{1}{8 \times 50} = \frac{1}{400}$ sec = 2.5 ms

9.[3]



Object is projected as shown so as per motionunder gravity

$S = ut + \frac{1}{2} at^2$

$-75 = +10t + \frac{1}{2}(-10)t^2 \Rightarrow t = 5$ sec

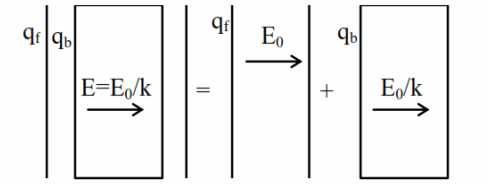
Object takes t = 5 s to fall on ground

Height of balloon from ground

$H = 75 + ut$

$= 75 + 10 \times 5 = 125$ m

10.[0]



When a dielectric is inserted in a capacitor

Due to free charge $\vec{E} = \vec{E}_0$ only

After dielectric $E' = \frac{E_0}{k}$

$q_B = q_f \left(1 - \frac{1}{k}\right)$

11.[3] Density is same

$M = \frac{4}{3} \pi R^3 \rho$, $2m = \frac{4}{3} \pi R'^3 \rho$

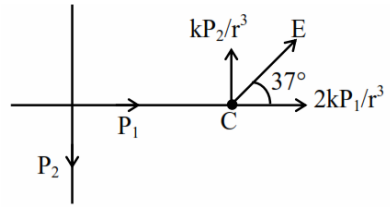
$R' = 2^{1/3} R$

$\omega = \frac{GMm}{R^2}$

$\omega_2 = \frac{G2Mm}{R'^2}$

$\omega_2 = 2^{1/3} \omega$

12.[3]

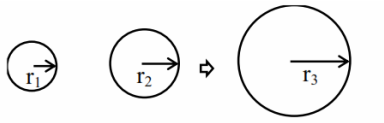


$\tan 37^\circ = \frac{4}{3} = \frac{\frac{kP_2}{r^3}}{\frac{2KP_1}{r^3}} = \frac{P_2}{2P_1} = \frac{3}{4}$

$\frac{P_2}{P_1} = \frac{3}{2}$

$\frac{P_1}{P_2} = \frac{2}{3}$

13.[3]



no. of moles is conserved

$n_1 + n_2 = n_3$

$P_1V_1 + P_2V_2 = P_3V$

$\frac{4S}{r_1}\left(\frac{4}{3}\pi r_1^3\right) + \frac{4S}{r_2}\left(\frac{4}{3}\pi r_2^3\right) = \frac{4S}{r_3}\left(\frac{4}{3}\pi r_3^3\right)$

$r_1^2 + r_2^2 = r_3^2$

$r_3 = \sqrt{r_1^2 + r_2^2}$

14.[2] $[A] = [MLT^{-2}]$

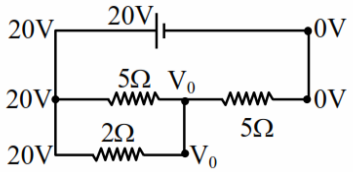
$[B] = [L^{-1}]$

$[D] = [T^{-1}]$

$\left[\frac{AD}{B}\right] = \frac{[MLT^{-2}][T^{-1}]}{[L^{-1}]}$

$\left[\frac{AD}{B}\right] = [ML^2T^{-3}]$

15.[3]



$\frac{20 - V_0}{5} + \frac{0 - V_0}{5} + \frac{20 - V_0}{2} = 0$

$$4+10=\frac{2V_0}{5}+\frac{V_0}{2}$$

$$14=\frac{4V_0+5V_0}{10}$$

$$V_0=\frac{140}{9}\text{ Volt}$$

Potential difference across 2Ω resistor is $20-V_0$

That is $\left(20-\frac{140}{9}\right)$ Volt

Hence answer is $\left(\frac{40}{9}\right)$ Volt

16.[3] $\lambda_e=\lambda_{ph}$

$$\frac{h}{p_e}=\frac{h}{p_{ph}}$$

$$\sqrt{2mk_e}=\frac{E_{ph}}{c}$$

$$2mk_e=\frac{(E_{ph})^2}{c^2}$$

$$\frac{k_e}{E_{ph}}\ \frac{E_{ph}}{c^2}\ \left(\frac{1}{2m}\right)$$

$$=\frac{P_{ph}}{c}\left(\frac{1}{2m}\right)$$

$$=\frac{P_e}{c}\left(\frac{1}{2m}\right)$$

$$=\frac{mv}{c}\ \frac{1}{2m}$$

$$=\frac{v}{2c}$$

17.[2] $V\alpha t+\beta t^2$

$$\frac{ds}{dt}=\alpha t+\beta t^2$$

$$\int_{S_1}^{S_2}ds=\int_1^2(\alpha t+\beta t^2)dt$$

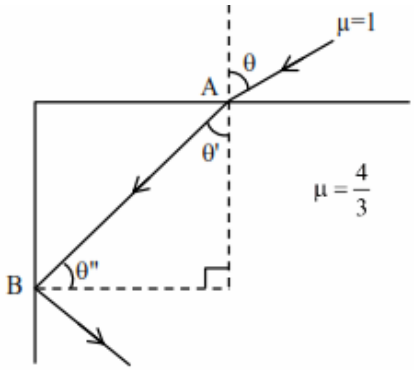
$$S_2-S_1=\left[\frac{\alpha t^2}{2}+\frac{\beta t^3}{3}\right]_1^2$$

As particle is not changing direction
So distance = displacement.

$$\text{Distance}=\left[\frac{\alpha[4-1]}{2}+\frac{\beta[8-1]}{3}\right]$$

$$=\frac{3\alpha}{2}+\frac{7\beta}{3}$$

18.[1]



At maximum angle θ ray at point B goes in gazing emergence, at all less values of θ, TIR occurs. At point B

$$\frac{4}{3}\times\sin\theta''=1\times\sin90^\circ$$

$$\theta''=\sin^{-1}\left(\frac{3}{4}\right)$$

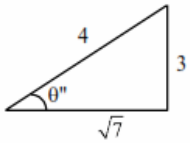
$$\theta'=\left(\frac{\pi}{2}-\theta''\right)$$

At point A

$$1\times\sin\theta=\frac{4}{3}\times\sin\theta'$$

$$\sin\theta=\frac{4}{3}\times\sin\left(\frac{\pi}{2}-\theta''\right)$$

$$\sin\theta=\frac{4}{3}\cos\left[\cos^{-1}\frac{\sqrt{7}}{4}\right]$$



$$\sin\theta=\frac{4}{3}\times\frac{\sqrt{7}}{4}$$

$$\theta=\sin^{-1}\left(\frac{\sqrt{7}}{3}\right)$$

19.[2] $V_S=h\nu-\phi$

$$4.8=\frac{hc}{\lambda}-\phi \qquad ...(i)$$

$$1.6=\frac{hc}{2\lambda}-\phi \qquad ....(ii)$$

Using above equation (i) – (ii)

$$3.2=\frac{hc}{2\lambda}-\frac{hc}{2\lambda}$$

$$3.2=\frac{hc}{2\lambda} \qquad ....(iii)$$

$\left[\lambda = \frac{hc}{6.4}\right]$

Put in equation (ii)

$\phi = 1.6$

$\frac{hc}{\lambda_{th}} = -1.6$

$\lambda_{th} = \frac{hc}{1.6}$

$\left(\frac{hc}{6.4}\right) \times 4 = 4\lambda$

20.[2] Given X = Y

$\sqrt{X^2 + Y^2 - 2 \times Y\cos\theta}$

$= n\sqrt{X^2 + Y^2 + 2 \times Y\cos\theta}$

square both sides

$2X^2(1 - \cos\theta) = n^2 . 2X^2(1 + \cos\theta)$

$1 - \cos\theta = n^2 + n^2\cos\theta$

$\cos\theta = \frac{1-n^2}{1+n^2}$

$\theta = \cos^{-1}\left[\frac{n^2-1}{-n^2-1}\right]$

Section -B

21.[25] $\because$ mean free path

$\lambda = \frac{1}{\sqrt{2}\pi d^2 n}$

$\frac{\lambda_1}{\lambda_2} = \frac{d_2^2 n_2}{d_1^2 n_1}$

$= \left(\frac{5}{10}\right)^2 = 0.25 = 25 \times 10^{-2}$

22.[125] By photoelectric equation

$\frac{hc}{\lambda} - \phi = k_{max}$

$k_{max} = \frac{1240}{500} - 1.25 \approx 1.25$

$r = \frac{\sqrt{2mk}}{eB}$

$B = \frac{\sqrt{2mk}}{er}$

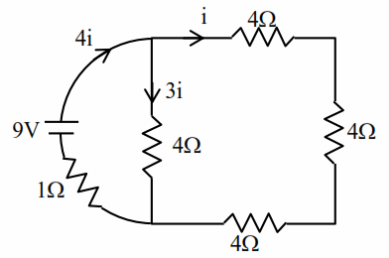
$= 125 \times 10^{-7}$ T

23.[1] Modulation index

$\mu = \frac{A_m}{A_c} = \frac{20}{20} = 1$

_____ $\times 10^{-1}$ V

24.[45] here assume current as



By KVL in outer loop

$9 - 12i - 4i = 0$

$16i = 9$

$8i = \frac{9}{2} = 4.5$

$= 45 \times 10^{-1}$

25.[500] For figure (a) $P_{avg} = \frac{V^2_{rms}}{R}$

$\frac{V^2_{rms}}{Z^2} \times R = \frac{V^2_{rms}}{R} \times 1$

$R^2 = Z^2$

$25 = \left(\sqrt{(x_C - x_L)^2 + 5^2}\right)^2$

$25 = (x_C - X_L)^2 + 25$

$x_C = x_L \Rightarrow \frac{1}{\omega C} = \omega L$

$\omega^2 = \frac{1}{LC} = \frac{10^6}{0.1 \times 40}$

$\omega = 500$

26.[27] $\Delta m = (Zm_P + (A - Z)m_n) - M_{A\ell}$

$= (13 \times 1.00726 + 14 \times 1.00866) - 27.18846$

$= 27.21562 - 27.18846$

$= 0.02716$ u

$E = 27.16 \text{ x} \times 10^{-3}$J

27.[450] $F = (5y + 20)\ \hat{j}$

$\omega = \int F dy = \int_0^{10} (5y + 20)dy$

$= \left(\frac{5y^2}{2} + 20y\right)_0^{10}$

$= \frac{5}{2} \times 100 + 20 \times 10$

$= 250 + 200$

$= 450$ J

28.[4] $\tau = \frac{\Delta L}{\Delta t} = \frac{I(\omega_f - \omega_i)}{\Delta t}$

$\tau = \frac{\frac{mR^2}{2} \times [0 - \omega]}{\Delta t}$

$= \frac{10 \times (20 \times 10^{-2})^2}{2} \times \frac{600 \times \pi}{30 \times 10}$

$= 0.4\pi = 4\pi \times 10^{-2}$

29.[5] $n_e n_h = n_i^2$

$n_e = \frac{n_i^2}{n_h} = \frac{(1.5 \times 10^{16})^2}{4.5 \times 10^{22}} = \frac{1.5 \times 1.5 \times 10^{32}}{4.5 \times 10^{22}}$

$5 \times 10^9/m^3$

30.[10] $A = A_0 e^{-\lambda t}$

$\frac{A_0}{8} = A_0 e^{-\lambda t} \Rightarrow \lambda t = in8$

$\lambda t = 3In_2$

$\frac{In2}{\lambda} = \frac{t}{3} = \frac{30}{3} = 10$ years

CHEMISTRY

Section -A

31.[3] $\sigma_{1s}^2, \sigma_{1s}^{*2}, \sigma_{2s}^2, \sigma_{2s}^{*2}, \sigma_{2p_z}^2$

$\pi_{2p_x}^2 = \pi_{2p_y}^2, \pi_{2p_x}^{*1} = \pi_{2p_y}^{*1}, \sigma_{2p_z}^*$

Bond order of $O_2 \Rightarrow 2$

Bond order of $O_2^- \Rightarrow 1.5$

Bond order of $O_2^{2-} \Rightarrow 1$

Bond order of $O_2^+ \Rightarrow 2.5$

32.[3] A biodegradable polyamide nylon-2-Nylon-6 in made from glycine and amino caproic acid

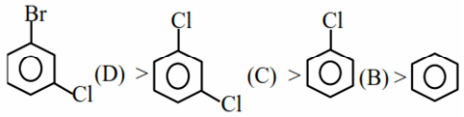
33.[1] (a) CsI salt is poor water soluble due to it's low hydration energy
(b) $NaHCO_3$ is used in fire extinguisher
(c) K is most abundant element in cell fluid (d) Li_2CO_3 decomposes easily due to high covalent character caused by small size Li^+ cation.

34.[2] Complex $[Co(en)_3]Cl_2$ is most stable complex among the given complex compounds because more number of chelate rings are present in this complex as compare to others.
(1) $[Co(en)(NH_3)_4]Cl_2$ 1 chelate ring
(2) $[Co(en)_3]Cl_3$ 3 chelate ring
(3) $[Co(en)_2(NH_3)_2]Cl_2$ 2 chelate ring
(4) $[Co(NH_3)_6]Cl_2$ 0 chelate ring

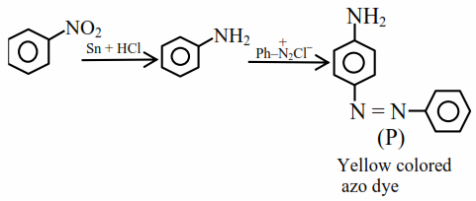
35.[1] (a) Concentration of Ag is performed by leaching with dilute NaCN solution
(b) Pig iron is formed in blast furnace
(c) Blister Cu is produced in Bessemer converter
(d) Froth floatation method is used for sulphide ores.
Note : During extraction of Cu reverberatory furnace is involved.

36.[2] F^-, O^{2-} and N^{3-} all are isoelectronic species in which N^{3-} have least number of protons due to which it's size increases as least nuclear attraction is experienced by the outer shell electrons.
Size order $N^{3-} > O^{2-} > F^-$

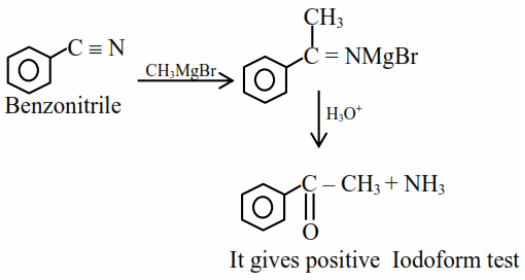
37.[1]



38.[2]



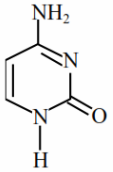
39.[1]



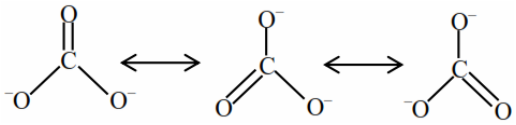
40.[2] $\mu = \sqrt{n(n+2)}BM$

$Ti^{+3} = [Ar]3d^1$ n = 1 μ = 1.73 BM
$V^{+2} = [Ar]3d^3$ n = 3 μ = 3.87 BM
$Sc^{+3} = [Ar]3d^0 4s^0$ n = 0 μ = 0

41.[3] The correct structure of cytosine

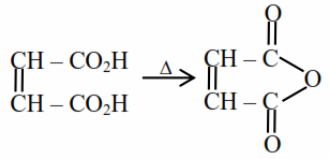


42.[4] Among SO_3, O_2, SO_2 and CO_3^{2-}, only O_2 and CO_3^{2-} has only one π-bond



43.[1] Elements of group 7,8,9 do not form hydrides thus Cr will only form hydride among the given elements (Fe, Mn, Co)

44.[2] Cis but 2-enoic acid



Maleic anhydride

45.[2] CFCs are broken down by powerful UV radiation and releases chlorine free radical which reacts with ozone and start chain reaction.

$$CF_2Cl_{2(g)} \xrightarrow{UV} \dot{C}l_{(g)} + \dot{C}F_2Cl_{(g)}$$

$$\dot{C}l_{(g)} + O_{3(g)} \rightarrow \dot{C}lO_{(g)} + O_{2(g)}$$

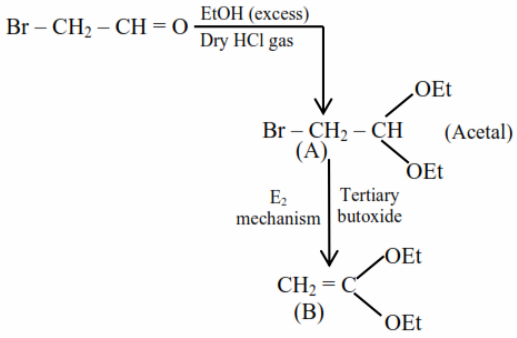
$$\dot{C}lO_{(g)} + O_{(g)} \rightarrow \dot{C}l_{(g)} + O_{2(g)}$$

Statement (2)

Atmosphere ozone reacts with nitric oxide to produce nitrogen dioxide and oxygen.

$NO(g) + O_3(g) \rightarrow NO_2(g) + O_{2(g)}$

46.[1]



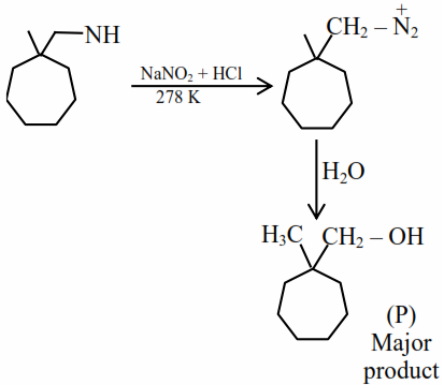
47.[4] Cheese → liquid in solid

Pumice stone → gas in solid

Hair cream → liquid in liquid

Cloud → liquid in gas

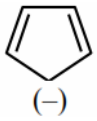
48.[4]



49.[2] $MnO_4^- + 5e \rightarrow Mn^{+2}$

50.[4]  ; because its conjugate base is

Strongest acid aromatic



(–)

Section -B

51.[50] w = –200 J, q = +150 : ΔU= q + w

ΔU = 150 – 200 = –50 J : magnitude = 50 J = |ΔU |

52.[58] $\Delta v = \frac{0.02}{100} \times 5 \times 10^6 = 10^3$ m/s

$\Delta x . \Delta v = \frac{h}{4\pi m}$

$x \times 10^{-9} \times 10^3 = \frac{6.63 \times 10^{-34}}{4 \times 3.14 \times 9.1 \times 10^{-31}}$

$x \times 10^{-9} \times 10^3 = 0.058 \times 10^{-3}$

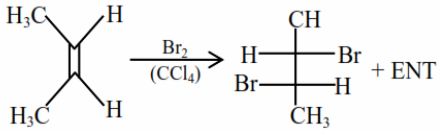
$x = \frac{0.058 \times 10^{-6}}{10^{-9}} = 58$

53.[10] Ho = $[Xe]4f^{11}6s^2$

Ho^{3+} $[Xe]\ 4f^{10}$

so number of e^- present in 4f is 10.

54.[2]



The total number of products possible = 2

55.[4]

	A ⟶	B
t = 0		0
t = 30 min		0.2M

Av. rate of reaction

$$-\frac{\Delta[A]}{\Delta t}=\frac{\Delta[B]}{\Delta t}=\frac{(0.2-0)}{\frac{1}{2}}$$

$= 0.4 = 4 \times 10^{-1}$ mol / L × hr

56.[3] Number of significant figures = 3

57.[1] $Ba(OH)_2 \rightarrow Ba^{+2} + 2OH^-$

$\downarrow$

$2 \times 0.005 = 0.01 = 10^{-2}$

At 298 K : in aq. solution $[H_3O^+]\,[OH^-] = 10^{-14}$

$$[H_3O^+] = \frac{10^{-14}}{10^{-2}} = 10^{-12}$$

58.[12] Organic compound : 0.8 gm

$$\text{wt. of N} = \left(\frac{42}{100}\times 0.8\right)\text{gm}$$

$$\text{mole of N} = \frac{42\times 0.8}{100\times 14} = \frac{2.4}{100}\text{ mol}$$

$$\text{moles of } NH_3 = \frac{2.4}{100}$$

$2NH_3 + H_2SO_4 \rightarrow (NH_4)_2SO_4$

$\downarrow$

$\frac{2.4}{100}$ mole $\qquad$ $\frac{1.2}{100}$ mole

$$\frac{1.2}{100} = 1 \times v\,(\ell)$$

$$\Rightarrow V_{H_2SO_4} = \frac{1.2}{100}\,\ell = 12m\ell$$

59.[250] $\Delta T_b = K_b \times$ molality

$$0.60 = 5 \times \left(\frac{3/M}{100/100}\right)$$

$M = 250$

60.[927]

$$\frac{P_1}{T_1}=\frac{P_2}{T_2} \Rightarrow \frac{300\times 10^3}{300} = \frac{1.2\times 10^6}{T_2}$$

$\Rightarrow T_2 = 1200$ K

$T_2 = 927°C$

MATHEMATICS

Section -A

61.[4] $T_{r+1} = {}^{12}C_r\,(2^{1/3})^r \,.\, (3^{1/4})^{12-r}$

T_{r+1} will be rational number

when r = 0, 3, 6, 9, 12

& r = 0, 4, 8, 12

$\Rightarrow$ r = 0, 12

$T_1 + T_{13} = 1 \times 3^3 + 1 \times 2^4 \times 1$

$= 24 + 16 = 43$

62.[3] $n_1 = 100$ $\qquad$ $m = 250$

$\overline{X}_1 = 15$ $\qquad$ $\overline{X} = 15.6$

$V_1(x) = 9$ $\qquad$ $Var(x) = 13.44$

$$\sigma^2 = \frac{n_1\sigma_1^2 + n_2\sigma_2^2}{n_1+n_2} + \frac{n_1 n_2}{(n_1+n_2)^2}\left(\overline{X}_1 - \overline{X}_2\right)^2$$

$n_2 = 150$, $\overline{X}_2 = 16$, $V_2(x) = \sigma_2$

$$13.44 = \frac{100\times 9 + 150\times\sigma_2^2}{250} + \frac{100\times 150}{(250)^2}\times 1$$

$\Rightarrow \sigma_2^2 = 16$

$\Rightarrow \sigma_2 = 4$

63.[3] $$f(x) = \begin{cases} \int_0^1 (5+(1-t))\,dt + \int_1^x (5+(t-1))\,dt \end{cases}$$

$$= 6 - \frac{1}{2} + \left(4t + \frac{t^2}{2}\right)\Big|_1^x$$

$$= \frac{11}{2} + 4x + \frac{x^2}{2} - 4 - \frac{1}{2}$$

$$= \frac{x^2}{2} + 4x + 1$$

$f(2^+) = 2 + 8 + 1 = 11$

$f(2) = f(2^-) = 5 \times 2 + 1 = 11$

$\Rightarrow$ continuous at x = 2

Clearly differentiable at x = 1

$Lf'(2) = 5$

$Rf'(2) = 6$

$\Rightarrow$ not differentiable at x = 2

64.[4] $$T_{r+1} = {}^{10}C_r\,(x\sin\alpha)\,10-r\left(\frac{a\cos\alpha}{x}\right)^r$$

r = 0, 1, 2 10

T_{r+1} will be independent of x

when $10 - 2r = 0 \Rightarrow r = 5$

$$T_6 = {}^{10}C_5\,(x\sin\alpha)^5 \times \left(\frac{a\cos\alpha}{x}\right)^5$$

$$= {}^{10}C_5 \times a^5 \times \frac{1}{2^5}(\sin 2\alpha)^5$$

will be greatest when $\sin 2\alpha = 1$

$\Rightarrow {}^{10}C_5 \frac{a^5}{2^5} = {}^{10}C_5 = a = 2$

65.[3] p : weather is food
q : ground is not wet
$\sim (p \wedge q) \equiv \sim p \vee \sim q$
$\equiv$ weather is not good or ground is wet

66.[2] $\cot\theta = \frac{1+\cos 2\theta}{\sin 2\theta} = \frac{1+\left(\frac{\sqrt{3}+1}{2\sqrt{2}}\right)}{\left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right)}$

$\theta = \frac{\pi}{24}$

$\Rightarrow \cot\left(\frac{\pi}{24}\right) = \frac{1+\left(\frac{\sqrt{3}+1}{2\sqrt{2}}\right)}{\left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right)}$

$= \frac{\left(2\sqrt{2}+\sqrt{3}+1\right)}{\left(\sqrt{3}-1\right)} \times \frac{\left(\sqrt{3}+1\right)}{\left(\sqrt{3}+1\right)}$

$= \frac{2\sqrt{6}+2\sqrt{2}+3+\sqrt{3}+\sqrt{3}+1}{2}$

$= \sqrt{6}+\sqrt{2}+\sqrt{3}+2$

67.[1] Let $P = \left(1+\frac{1}{10^{100}}\right)^{10^{100}}$

Let $x = 10^{100}$

$\Rightarrow P = \left(1+\frac{1}{x}\right)^x$

$\Rightarrow P = 1 + (x)\left(\frac{1}{x}\right) + \frac{(x)(x-1)}{\lfloor 2}\cdot\frac{1}{x^2} + \ldots$

$+ \frac{(x)(x-1)(x-2)}{\lfloor 3}\frac{1}{x^3} + \ldots$

(upto $10^{100}+1$ terms)

$\Rightarrow P = 1 + 1 + \left(\frac{1}{\lfloor 2} - \frac{1}{\lfloor 2 x^2}\right) + \left(\frac{1}{\lfloor 3}\ldots\right)$

$+\ldots$ so on

$\Rightarrow P = 2 +$

$\left(\text{Positive value less then } \frac{1}{\lfloor 2}+\frac{1}{\lfloor 3}+\frac{1}{\lfloor 4}+\ldots\right)$

Also $e = 1+\frac{1}{\lfloor 1}+\frac{1}{\lfloor 2}+\frac{1}{\lfloor 3}+\frac{1}{\lfloor 4}+\ldots$

$\Rightarrow \frac{1}{\lfloor 2}+\frac{1}{\lfloor 3}+\frac{1}{\lfloor 4}+\ldots e-2$

$\Rightarrow P = 2 +$ (Positive value less then $e-2$)

$\Rightarrow P \in (2, 3)$

$\Rightarrow$ least integer value of P is 3

68.[2] Let $I = \int_{-1}^{1} \log(x+\sqrt{x^2+1})\,dx$

$\because \log(x+\sqrt{x^2+1})$ is an odd function

$\therefore I = 0$

69.[4] Because vectors are coplanar

Hence $\begin{vmatrix} a & a & c \\ 1 & 0 & 1 \\ c & c & b \end{vmatrix} = 0$

$\Rightarrow c^2 = ab \Rightarrow C = \sqrt{ab}$

70.[2] $\sum_{n=8}^{100}\left[\frac{(-1)^n.n}{2}\right]$

$= 4 - 5 + 5 - 6 + 6 + \ldots. - 50 + 50 = 4$

71.[2] $\begin{vmatrix} \sin x & \cos x & \cos x \\ \cos x & \sin x & \cos x \\ \cos x & \cos x & \sin x \end{vmatrix} = 0, -\frac{\pi}{4} \le x \le \frac{\pi}{4}$

Apply : $R_1 \to R_1 - R_2$ & $R_2 \to R_2 - R_3$

$\begin{vmatrix} \sin x-\cos x & \cos x-\sin x & 0 \\ 0 & \sin x-\cos x & \cos x-\sin x \\ \cos x & \cos x & \sin x \end{vmatrix} = 0$

$(\sin x - \cos x)^2 \begin{vmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ \cos x & \cos x & \sin x \end{vmatrix} = 0$

$(\sin x - \cos x)^2 (\sin x + 2\cos x) = 0$

$\therefore x = \frac{\pi}{4}$

72.[1] $|\vec{a}| = 2,\ |\vec{b}| = 5$

$|\vec{a}\times\vec{b}| = |\vec{a}||\vec{b}|\sin\theta = \pm 8$

$\sin\theta = \pm\frac{4}{5}$

$\therefore \vec{a}.\vec{b} = |\vec{a}||\vec{b}|\cos\theta$

$= 10.\left(\pm\frac{3}{5}\right) = \pm 6$

$|\vec{a}.\vec{b}| = 6$

73.[1] $|x|^2 - |x| - 12 = 0$

$(|x| + 3)(|x| - 4) = 0$

$|x| = 4 \Rightarrow x = \pm 2$

74.[3] $\therefore$ (gof) exist $\Rightarrow$ gof is bijective

$\Rightarrow$ 'f' must be one-one and 'g' must be ONTO

75.[1] $P = \begin{bmatrix} 1 & 0 \\ \frac{1}{2} & 1 \end{bmatrix}$

$$P^2 = \begin{bmatrix} 1 & 0 \\ \frac{1}{2} & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ \frac{1}{2} & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$$

$$P_3 = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ \frac{1}{2} & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \frac{3}{2} & 1 \end{bmatrix}$$

$$P^4 = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$$

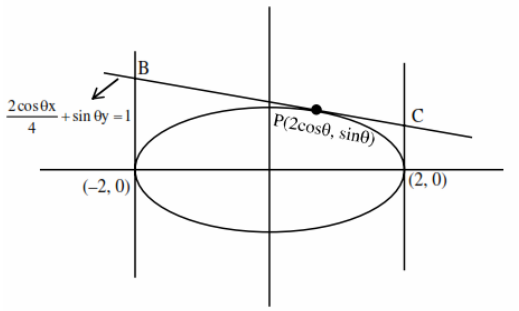
$$\therefore P^{50} = \begin{bmatrix} 1 & 0 \\ 25 & 1 \end{bmatrix}$$

76.[2] $\text{mean} = \sum x_i p_i = \sum_{r=0}^{\infty} r.\frac{1}{3^r} = \frac{3}{4}$

$P(x \text{ is even}) = \frac{1}{3^2} + \frac{1}{3^4} + ... \infty$

$$= \frac{\frac{1}{9}}{1-\frac{1}{9}} = \frac{1/9}{8/9} = \frac{1}{8}$$

77.[1]



$\frac{x^2}{4} + \frac{y^2}{1} = 1$ Equation of tangent is $(\cos\theta)x + 2\sin\theta y = 2$

$B\left(-2, \frac{1+\cos\theta}{\sin\theta}\right)$, $C\left(2, \frac{1-\cos\theta}{\sin\theta}\right)$

$B\left(-2\cot\frac{\theta}{2}\right)$ $C\left(2, \tan\frac{\theta}{2}\right)$

Equation of circle is

$$(x+2)(x-2) + \left(y - \cot\frac{\theta}{2}\right)\left(y - \tan\frac{\theta}{2}\right) = 0$$

$$x^2 - 4 + y^2 - \left(\tan\frac{\theta}{2} + \cot\frac{\theta}{2}\right)y + 1 = 0$$

so, $(\sqrt{3}, 0)$ satisfying option (1)

78.[3] $\frac{x^2 - y^2}{1-(-5)} = \frac{xy}{-2}$

$$\frac{x^2 - y^2}{6} = \frac{xy}{-2}$$

$\Rightarrow x^2 - y^2 = -3xy$

$\Rightarrow x^2 + 3xy - y^2 = 0$

79.[3] ${}^nP_r = {}^nP_{r+1} \Rightarrow \frac{n!}{(n-r)!} = \frac{n!}{(n-r-1)!}$

$\Rightarrow (n-r) = 1$ ....(1)

${}^nC_r = {}^nC_{r-1}$

$$\Rightarrow \frac{n!}{r!(n-r)!} = \frac{n!}{(n-r)!(n-r+1)!}$$

$$\Rightarrow \frac{1}{r(n-r)!} = \frac{1}{r(n-r+1)(n-r)!}$$

$\Rightarrow n - r + 1 = r$

$\Rightarrow n + 1 = 2r$ ....(2)

(1) $\Rightarrow 2r - 1 - r = 1 \Rightarrow r = 2$

80.[1] $xdy = (y + x^3\cos x)dx$

$xdy = ydx + x^3\cos x dx$

$$\frac{xdy - ydx}{x^2} = \frac{x^3\cos x dx}{x^2}$$

$$\frac{d}{dx}\left(\frac{y}{x}\right) = \int x\cos x\,dx$$

$\frac{y}{x} = x\sin x + \cos x + C$

$\Rightarrow 0 = -1 + C \Rightarrow C = 1, x = \pi, y = 0$

so $\frac{y}{x} = = x\sin x + \cos x + 1$

$y = x\sin x + \cos x + x$ $\quad x = \frac{\pi}{2}$

$$y = \left(\frac{\pi}{2}\right) = \frac{\pi^2}{4} + \frac{\pi}{2}$$

Section -B

81.[98] 1. ${}^nC_0$, +3. ${}^nC_1$, 5. ${}^nC_2$, +.....+(2n+1) . ${}^nC_n$

$T_r = (2r+1){}^nC_r$

$S = \Sigma T_r$

$S = \Sigma(2r+1){}^nCr = \Sigma 2r{}^nC_r + \Sigma{}^nC_r$

$S = 2\,(n.2^{n-1}) + 2^n = 2_n\,(n+1)$

$2^n(n+1) = 2^{100}\,.\,101 \Rightarrow n = 100$

$$2\left[\frac{n-1}{2}\right] = 2\left[\frac{99}{2}\right] = 98$$

82.[39] $f(x) = \begin{cases} \dfrac{P(x)}{\sin(x-2)}, & x \neq 2 \\ 7, & x = 2 \end{cases}$

P"(x) = const. $\Rightarrow$ P(x) is a 2 degree polynomial

$f(x)$ is cont. at x = 2

$f(2^+) = f(2^-)$

$\lim\limits_{x\to 2^+} \dfrac{P(x)}{\sin(x-2)} = 7$

$\lim\limits_{x\to 2^+} \dfrac{(x-2)(ax+b)}{\sin(x-2)} = 7 \Rightarrow \boxed{2a + b = 7}$

$P(x) = (x - 2)(ax + b)$

$P(3) = (3 - 2)(3a + b) = 9 \Rightarrow \boxed{3a + b = 9}$

$\boxed{a = 2, b = 3}$

$P(5) = (5 - 2)(2.5 + 3) = 3.13 = 39$

83.[1] Equation of circle is $(x^2 - y^2) + 2y^2 + 2x = 0$

$x^2 + y^2 + 2x = 0$

Centre : (–1, 0)

Parabola : $x^2 - 6x - y + 13 = 0$

$(x - 3)^2 = y - 4$

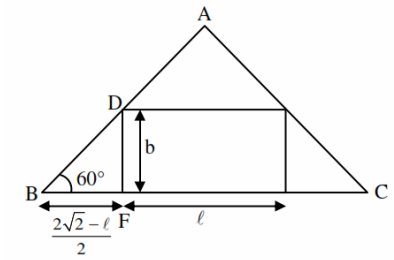
Vertex : (3, 4)

Equation of line $\equiv y - 0 = \dfrac{4-0}{3+1}(x + 1)$

$y = x + 1$

y-intercept = 1

84.[3]



In ΔDBF

$\tan 60^\circ = \dfrac{2b}{2\sqrt{2}-\ell} \Rightarrow \dfrac{\sqrt{3}(2\sqrt{2}-\ell)}{2}$

A = Area of rectangle = $\ell \times b$

$A = \ell \times \dfrac{\sqrt{3}}{2}(2\sqrt{2} - \ell)$

$\dfrac{dA}{d\ell} = \dfrac{\sqrt{3}}{2}(2\sqrt{2} - \ell)\dfrac{\ell\sqrt{3}}{2} = 0$

$\boxed{\ell = \sqrt{2}}$

$A = \ell \times b = \sqrt{2} \times \dfrac{\sqrt{3}}{2}(\sqrt{2}) = \sqrt{3}$

$\Rightarrow \boxed{A^2 = 3}$

85.[60] $(\vec{a} + 3\vec{b}) \perp (7\vec{a} - 5\vec{b})$

$(\vec{a} + 3\vec{b})(7\vec{a} - 5\vec{b}) = 0$

$7|\vec{a}|^2 - 15|\vec{b}|^2 + 16\vec{a}.\vec{b} = 0....(1)$

$(\vec{a} - 4\vec{b}).(7\vec{a} - 2\vec{b}) = 0$

$7|\vec{a}|^2 + 8|\vec{b}|^2 - 30\vec{a}.\vec{b} = 0....(2)$

from (1) & (2)

$|\vec{a}| = |\vec{b}|$

$\cos\theta = \dfrac{|\vec{b}|}{2|\vec{a}|} \therefore \theta = 60^\circ$

86.[1] $y' \dfrac{2y}{x\ell nx}$

$\Rightarrow \dfrac{dy}{y} = \dfrac{2dx}{x\ell nx}$

$\Rightarrow \ell n|y| = 2\ell n|\ell nx| + C$

put x = 2, $y = (\ell n2)^2$

$\Rightarrow c = 0$

$\Rightarrow y = (\ell nx)^2$

$\Rightarrow f(e) = 1$

87.[13] $a^2 + b^2 + c^2 = (a + b + c)^2 - 2\Sigma ab = -3$

$(ab + bc + ca)^2 = \Sigma(ab)^2 + 2abc\Sigma a$

$\Rightarrow \Sigma(ab)^2 = -2$

$a^4 + b^4 + c^4 = (a^2 + b^2 + c^2)^2 - 2\Sigma(ab)^2$

$= 9 - 2(-2) = 13$

88.[4] P (Head) = $\dfrac{1}{2}$

$1 - P(\text{All tail}) \geq 0.9$

$\Rightarrow 1 - \left(\dfrac{1}{2}\right)^n \geq 0.9$

$\Rightarrow \left(\dfrac{1}{2}\right)^n \leq \dfrac{1}{10}$

$\Rightarrow n_{min} = 4$

89.[55] ${}^nC_7\, 2^{n-7} = \dfrac{1}{3^7} = {}^nC_8\, 2^{n-8}\, \dfrac{1}{3^8}$

$\Rightarrow n - 7 = 48 \Rightarrow n = 55$

90.[1] $\begin{vmatrix} k+1 & 4 & 6 \\ 1 & 2 & 3 \\ 3 & 2 & 1 \end{vmatrix} = 0$

$(k + 1)[2 - 6] - 4[1 - 9] + 6[2 - 6] = 0$

$k = 1$