JEE MAIN ONLINE PAPER 2021

Held on July 25, 2021 (Morning)

Instructions

1. This test will be a 3 hours Test.
2. This test consists of Physics, Chemistry and Mathematics questions with equal weightage of 100 marks.
3. Each question is of 4 marks.
4. In the question paper consisting of Physics (Q.no. 1 to 30), Chemistry (Q.no. 31 to 60) and Mathematics (Q.no. 61 to 90). There are two sections for each subject (Section-A : MCQ Type & Section-B : Numerical Response Type). Section-A consists of 20 multiple choice questions & Section-B consists of 10 Numerical Value type Questions. **Candidates have a choice to Answer 5 out of the 10 numerical value answer based questions per section.**
5. There will be only one correct choice in the given four choices in Section-A. For each question 4 marks will be awarded for correct choice, 1 mark will be deducted for incorrect choice and zero mark will be awarded for not attempted question. For Section-B questions 4 marks will be awarded for correct answer and zero for unattempted and incorrect answer.
6. Any textual, printed or written material, mobile phones, calculator etc. is not allowed for the students appearing for the test.
7. All calculations/written work should be done in the rough sheet provided.

PHYSICS

Section -A

Q.1 For a gas $C_P - C_V = R$ in a state P and $C_P - C_V = 1.10\ R$ in a state Q, T_P and T_Q are the temperatures in two different states P and Q respectively. Then

(1) $T_P = T_Q$

(2) $T_P < T_Q$

(3) $T_P = 0.9\ T_Q$

(4) $T_P > T_Q$

Q.2 Given below are two statements : one is labelled as **Assertion A** and the other is labelled as **Reason R**.

Assertion A : Moment of inertia of a circular disc of mass 'M' and radius 'R' about X, Y axes (passing through its plane) and Z-axis which is perpendicular to its plane were found to be I_x, I_y and I_z respectively. The respective radii of gyration about all the three axes will be the same.

Reason R : A rigid body making rotational motion has fixed mass and shape. In the light of the above statements, choose the most appropriate answer from the options given below :

(1) Both **A** and **R** are correct but **R** is NOT the correct explanation of **A**.

(2) **A** is not correct but **R** is correct.

(3) **A** is correct but **R** is not correct.

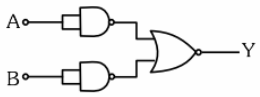
(4) Both **A** and **R** are correct and **R** is the correct explanation of A.

Q.3 What should be the order of arrangement of de-Broglie wavelength of electron (λ_e), an α-particle (λ_α) and proton (λ_p) given that all have the same kinetic energy ?

(1) $\lambda_e = \lambda_p = \lambda_\alpha$ (2) $\lambda_e < \lambda_p < \lambda_\alpha$

(3) $\lambda_e > \lambda_p > \lambda_\alpha$ (4) $\lambda_e = \lambda_p > \lambda_\alpha$

Q.4 Identify the logic operation carried out.



(1) OR (2) AND

(3) NOR (4) NAND

Q.5 A particle of mass 4M at rest disintegrates into two particles of mass M and 3M respectively having non zero velocities. The ratio of de-Broglie wavelength of particle of mass M to that of mass 3M will be :

(1) 1 : 3 (2) 3 : 1

(3) 1 : $\sqrt{3}$ (4) 1 : 1

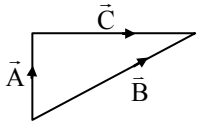
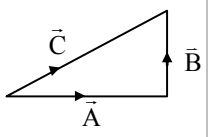
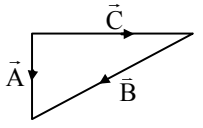
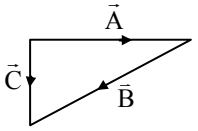
Q.6 Some nuclei of a radioactive material are undergoing radioactive decay. The time gap between the instances when a quarter of the nuclei have decayed and when half of the nuclei have decayed is given as :

(where λ is the decay constant)

(1) $\frac{1}{2}\frac{\ell n2}{\lambda}$ (2) $\frac{\ell n2}{\lambda}$

(3) $\frac{2\ell n2}{\lambda}$ (4) $\frac{\ell n\frac{3}{2}}{\lambda}$

Q.7 Match **List I** with **List II**.

	List-I		**List-II**
(a)	$\vec{C}-\vec{A}-\vec{B}=0$	(i)	
(b)	$\vec{A}-\vec{C}-\vec{B}=0$	(ii)	
(c)	$\vec{B}-\vec{A}-\vec{C}=0$	(iii)	
(d)	$\vec{A}+\vec{B}=-\vec{C}$	(iv)	

Choose the correct answer from the options given below :

(1) (a) → (iv), (b) → (i) , (c) → (iii), (d) → (ii)

(2) (a) → (iv), (b) → (iii) , (c) → (i), (d) → (ii)

(3) (a) → (iii), (b) → (ii) , (c) →(iv), (d) → (i)

(4) (a) → (i), (b) → (iv) , (c) → (ii), (d) → (iii)

Q.8 A parallel plate capacitor with plate area 'A' and distance of separation 'd' is filled with a dielectric. What is the capacity of the capacitor when permittivity of the dielectric varies as :

$\varepsilon(x) = \varepsilon_0 + kx$, for $\left(0 < x \le \frac{d}{2}\right)$

$\varepsilon(x) = \varepsilon_0 + k\,(d-x)$, for $\left(\frac{d}{2} \le x \le d\right)$

(1) $\left(\varepsilon_0 + \frac{kd}{2}\right)^{2/kA}$ (2) $\frac{kA}{2\ln\left(\frac{2\varepsilon_0+kd}{2\varepsilon_0}\right)}$

(3) 0 (4) $\frac{kA}{2}\ln\left(\frac{2\varepsilon_0}{2\varepsilon_0-kd}\right)$

Q.9 A monoatomic ideal gas, initially at temperature T_1 is enclosed in a cylinder fitted with a frictionless piston. The gas is allowed to expand adiabatically to a temperature T_2 by releasing the piston suddenly. If l_1 and l_2 are the lengths of the gas column, before and after the expansion respectively, then the value of $\frac{T_1}{T_2}$ will be :

(1) $\left(\frac{\ell_1}{\ell_2}\right)^{\frac{2}{3}}$ (2) $\left(\frac{\ell_2}{\ell_1}\right)^{\frac{2}{3}}$

(3) $\frac{\ell_2}{\ell_1}$ (4) $\frac{\ell_1}{\ell_2}$

Q.10 A ray of laser of a wavelength 630 nm is incident at an angle of 30° at the diamond-air interface. It is going from diamond to air. The refractive index of diamond is 2.42 and that of air is 1. Choose the correct option.

(1) angle of refraction is 24.41°

(2) angle of refraction is 30°

(3) refraction is not possible

(4) angle of refraction is 53.4°

Q.11 Two wires of same length and radius are joined end to end and loaded. The Young's modulii of the materials of the two wires are Y_1 and Y_2. The combination behaves as a single wire then its Young's modulus is :

(1) $Y = \frac{2Y_1Y_2}{3(Y_1+Y_2)}$ (2) $Y = \frac{2Y_1Y_2}{Y_1+Y_2}$

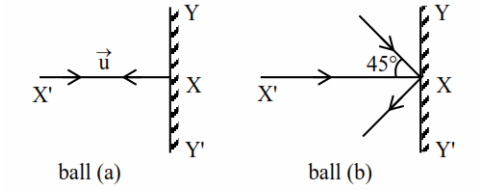
(3) $Y = \frac{Y_1Y_2}{2(Y_1+Y_2)}$ (4) $Y = \frac{Y_1Y_2}{Y_1+Y_2}$

Q.12 The half-life of ^{198}Au is 3 days. If atomic weight of ^{198}Au is 198 g/mol then the activity of 2 mg of ^{198}Au is [in disintegration/second] :

(1) 2.67×10^{12} (2) 6.06×10^{18}

(3) 32.36×10^{12} (4) 16.18×10^{12}

Q.13 Two billiard balls of equal mass 30 g strike a rigid wall with same speed of 108 kmph (as shown) but at different angles. If the balls get reflected with the same speed then the ratio of the magnitude of impulses imparted to ball 'a' and ball 'b' by the wall along 'X' direction is :



(1) 1 : 1 (2) $\sqrt{2}$: 1

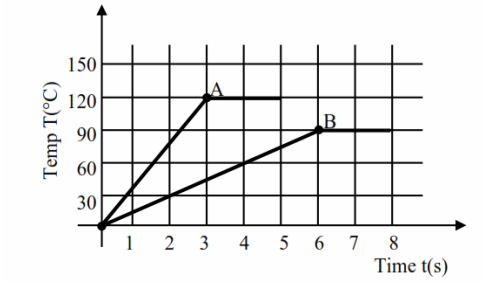
(3) 2 : 1 (4) 1 : $\sqrt{2}$

Q.14 In the Young's double slit experiment, the distance between the slits varies in time as $d(t) = d_0 + a_0 \sin\omega t$; where d_0, ω and a_0 are constants. The difference between the largest fringe width and the smallest fringe width obtained over time is given as :

(1) $\frac{2\lambda D(d_0)}{(d_0^2 - a_0^2)}$ (2) $\frac{2\lambda D a_0}{(d_0^2 - a_0^2)}$

(3) $\frac{\lambda D}{d_0^2} a_0$ (4) $\frac{\lambda D}{d_0 + a_0}$

Q.15 Two different metal bodies A and B of equal mass are heated at a uniform rate under similar conditions. The variation of temperature of the bodies is graphically represented as shown in the figure. The ratio of specific heat capacities is :



(1) $\frac{8}{3}$ (2) $\frac{3}{8}$ (3) $\frac{3}{4}$ (4) $\frac{4}{3}$

Q.16 A linearly polarized electromagnetic wave in vacuum is

$E = 3.1\cos[(1.8)z - (5.4 \times 10^6)t]\,\hat{i}$ N/C

is incident normally on a perfectly reflecting wall at z = a. Choose the correct option

(1) The wavelength is 5.4 m

(2) The frequency of electromagnetic wave is 54×10^4 Hz.

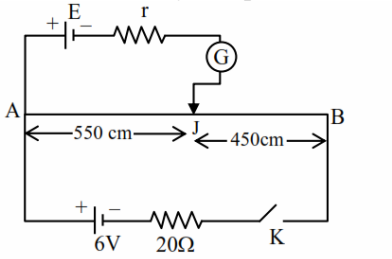
(3) The transmitted wave will be

$3.1\cos[(1.8)z - (5.4 \times 10^6)t]\,\hat{i}$ N/C

(4) The reflected wave will be

$3.1\cos[(1.8)z + (5.4 \times 10^6)t]\,\hat{i}$ N/C

Q.17 In the given figure, there is a circuit of potentiometer of length AB = 10 m. The resistance per unit length is 0.1 Ω per cm. Across AB, a battery of emf E and internal resistance 'r' is connected. The maximum value of emf measured by this potentiometer is :



(1) 5 V (2) 2.25 V

(3) 6 V (4) 2.75 V

Q.18 In amplitude modulation, the message signal $V_{m(t)} = 10 \sin(2\pi \times 10^5 t)$ volts and Carrier signal $V_{c(t)} = 20 \sin(2\pi \times 10^7 t)$ volts

The modulated signal now contains the message signal with lower side band and upper side band frequency, therefore the bandwidth of modulated signal is α kHz. The value of α is :

(1) 200 kHz (2) 50 kHz

(3) 100 kHz (4) 0

Q.19 Water droplets are coming from an open tap at a particular rate. The spacing between a droplet observed at 4^{th} second after its fall to the next droplet is 34.3 m. At what rate the droplets are coming from the tap ? (Take g = 9.8 m/s^2)

(1) 3 drops / 2 seconds (2) 2 drops / second

(3) 1 drop / second (4) 1 drop / 7 seconds

Q.20 The minimum and maximum distances of a planet revolving around the Sun are x_1 and x_2. If the minimum speed of the planet on its trajectory is v_0 then its maximum speed will be :

(1) $\frac{v_0 x_1^2}{x_2}$ (2) $\frac{v_0 x_2^2}{x_1^2}$

(3) $\frac{v_0 x_1}{x_2}$ (4) $\frac{v_0 x_2}{x_1}$

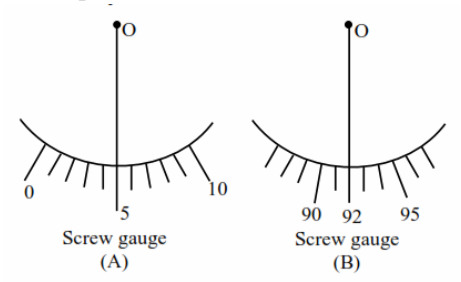
Section -B

Q.21 A body of mass 2 kg moving with a speed of 4 m/s. makes an elastic collision with another body at rest and continues to move in the original direction but with one fourth of its initial speed. The speed of the two body centre of mass is $\frac{x}{10}$ m/s. Then the value of x is_____.

Q.22 Student A and Student B used two screw gauges of equal pitch and 100 equal circular divisions to measure the radius of a given wire. The actual value of the radius of the wire is 0.322 cm. The absolute value of the difference between the final circular scale readings observed by the students A and B is _____.

[Figure shows position of reference 'O' when jaws of screw gauge are closed]

Given pitch = 0.1 cm.



Q.23 An inductor of 10 mH is connected to a 20 V battery through a resistor of 10 kΩ and a switch. After a long time, when maximum current is set up in the circuit, the current is switched off. The current in the circuit after 1 μs is $\frac{x}{100}$ mA. Then x is equal to _____.

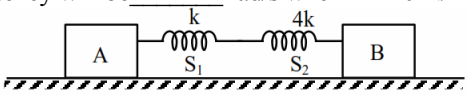
(Take $e^{-1} = 0.37$)

Q.24 A circular conducting coil of radius 1 m is being heated by the change of magnetic field B passing perpendicular to the plane in which the coil is laid. The resistance of the coil is 2 μΩ. The magnetic field is slowly switched off such that its magnitude changes in time as

$B = \frac{4}{\pi} \times 10^{-3}\ T \left(1 - \frac{t}{100}\right)$

The energy dissipated by the coil before the magnetic field is switched off completely is E = _____ mJ.

Q.25 In the reported figure, two bodies A and B of masses 200 g and 800 g are attached with the system of springs. Springs are kept in a stretched position with some extension when the system is released. The horizontal surface is assumed to be frictionless. The angular frequency will be_____ rad/s when k = 20 N/m.



Q.26 The value of aluminium susceptibility is 2.2×10^{-5}. The percentage increase in the magnetic field if space within a current carrying toroid is filled with aluminium is $\frac{x}{10^4}$. Then the value of x is _____.

Q.27 A particle of mass 1 mg and charge q is lying at the mid-point of two stationary particles kept at distance '2 m' when each is carrying same charge 'q'. If the free charged particle is displaced from its equilibrium position through distance 'x' (x << 1 m). The particle executes SHM. Its angular frequency of oscillation will be _____ $\times 10^5$ rad/s if $q^2 = 10C^2$

Q.28 An electric bulb rated as 200 W at 100 V is used in a circuit having 200 V supply. The resistance 'R'that must be put in series with the bulb so that the bulb delivers the same power is _____ Ω.

Q.29 A pendulum bob has a speed of 3 m/s at its lowest position. The pendulum is 50 cm long. The speed of bob, when the length makes an angle of 60° to the vertical will be (g = 10 m/s) _____ m/s.

Q.30 A particle of mass 'm' is moving in time 't' on a trajectory given by

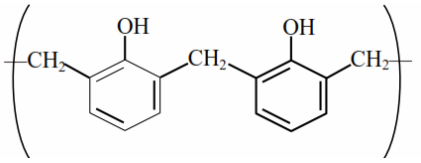
$\vec{r} = 10\,\alpha t^2\,\hat{i} + 5\beta\,(t-5)\,\hat{j}$

Where α and β are dimensional constants.
The angular momentum of the particle becomes the same as it was for t = 0 at time
t = _____seconds.

CHEMISTRY

Section -A

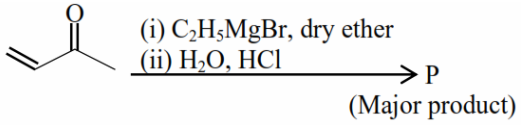
Q.31



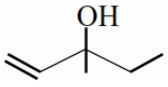
is a repeating unit for :
(1) Novolac (2) Buna–N
(3) Acrilan (4) Neoprene

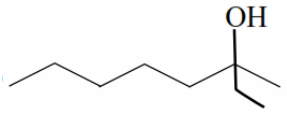
Q.32 Which one of the following species responds to an external magnetic field?
(1) $[Fe(H_2O)_6]^{3+}$ (2) $[Ni(CN)_4]^{2-}$
(3) $[Co(CN)_6]^{3-}$ (4) $[Ni(CO)_4]$

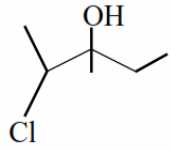
Q.33

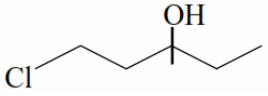


Consider the above reaction, the major product 'P' is:

(1) 

(2) 

(3) 

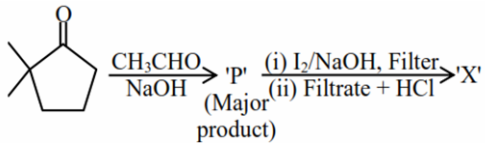
(4) 

Q.34 Sodium stearate $CH_3(CH_2)_{16}COO^-Na^+$ is an anionic surfactant which forms micelles in oil. Choose the **correct** statement for it from the following :
(1) It forms spherical micelles with $CH_3(CH_2)_{16}$ –group pointing towards the centre of sphere.
(2) It forms non–spherical micelles with $-COO^-$ group pointing outwards on the surface.
(3) It forms spherical micelles with $CH_3(CH_2)_{16}$– group pointing outwards on the
(4) It forms non–spherical micelles with $CH_3(CH_2)_{16}$ –group pointing towards the centre.

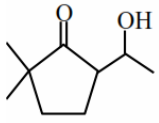
Q.35 The water soluble protein is :
(1) Fibrin (2) Albumin
(3) Myosin (4) Collagen

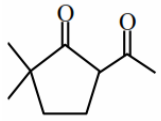
Q.36 At 298.2 K the relationship between enthalpy of bond dissociation (in kJ mol^{-1}) for hydrogen (E_H) and its isotope, deuterium (E_D), is best described by :
(1) $E_H = \frac{1}{2}E_D$ (2) $E_H = E_D$
(3) $E_H \simeq E_D - 7.5$ (4) $E_H = 2E_D$

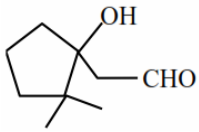
Q.37

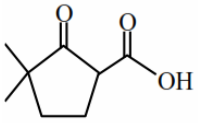


Consider the given reaction, the product 'X' is:

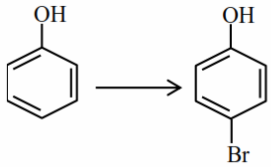
(1) 

(2) 

(3) 

(4) 

Q.38



(Major product)

The given reaction can occur in the presence of :
(a) Bromine water
(b) Br_2 in CS_2, 273 K
(c) $Br_2/FeBr_3$
(d) Br_2 in $CHCl_3$, 273 K
(1) (b) and (d) only (2) (a) and (c) only
(3) (b), (c) and (d) only (4) (a), (b) and (d) only

Q.39 Given below are two statements, one is labelled as **Assertion (A)** and other is labelled as **Reason (R)**.

Assertion (A) : Gabriel phthalimide synthesis cannot be used to prepare aromatic primary amines.

Reason (R) : Aryl halides do not undergo nucleophilic substitution reaction.

In the light of the above statements, choose the **correct** answer from the options given below :

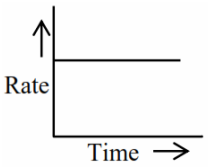
(1) Both **(A)** and **(R)** true but **(R)** is not the correct explanation of **(A)**.

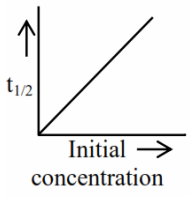
(2) **(A)** is false but **(R)** is true.

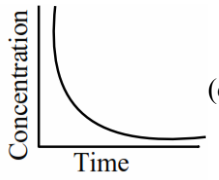
(3) Both **(A)** and **(R)** true and **(R)** is correct explanation of **(A)**.

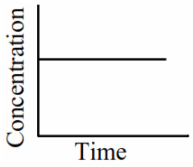
(4) **(A)** is true but **(R)** is false.

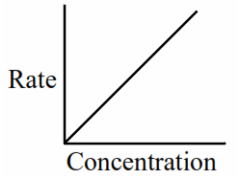
Q.40 For the following graphs,

(a) 

(b) 

(c) 

(d) 

(e) 

Choose from the options given below, the **correct** one regarding order of reaction is :

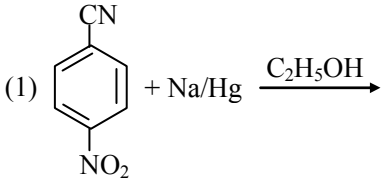
(1) (b) zero order (c) and (e) First order

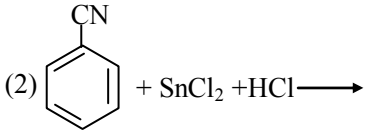
(2) (a) and (b) Zero order (e) First order

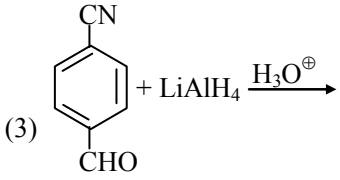
(3) (b) and (d) Zero order (e) First order

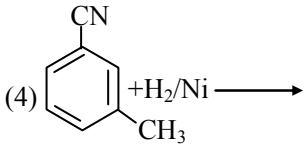
(4) (a) and (b) Zero order (c) and (e) First order

Q.41 Which one of the products of the following reactions **does not** react with Hinsberg reagent to form sulphonamide?

(1) 

(2) 

(3) 

(4) 

Q.42 The ionic radii of K^+, Na^+, Al^{3+} and Mg^{2+} are in the order :

(1) $Na^+ < K^+ < Mg^{2+} < Al^{3+}$

(2) $Al^{3+} < Mg^{2+} < K^+ < Na^+$

(3) $Al^{3+} < Mg^{2+} < Na^+ < K^+$

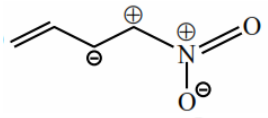
(4) $K^+ < Al^{3+} < Mg^{2+} < Na^+$

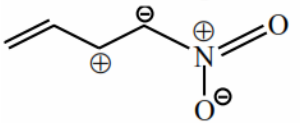
Q.43 Which one of the following compounds of Group–14 elements is **not** known?

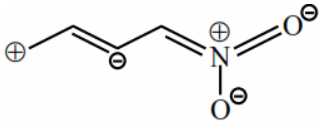
(1) $[GeCl_6]^{2-}$ (2) $[Sn(OH)_6]^{2-}$

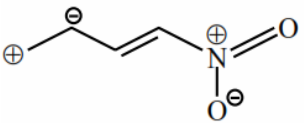
(3) $[SiCl_6]^{2-}$ (4) $[FiF_6]^{2-}$

Q.44 Which one among the following resonating structures is **not** correct?

(1) 

(2) 

(3) 

(4) 

Q.45 Given below are two statements :
Statement I **:** None of the alkaline earth metal hydroxides dissolve in alkali.
Srtatement II : Solubility of alkaline earth metal hydroxides in water increases down the group. In the light of the above statements, choose the **most appropriate** answer from the options given below :
(1) **Statement I** is correct but **Statement II** is incorrect.
(2) **Statement I** is incorrect but **Statement II** is correct.
(3) **Statement I** and **Statement II** both are incorrect.
(4) **Statement I** and **Statement II** both are correct.

Q.46 The correct order of following 3d metal oxides, according to their oxidation numbers is :
(a) CrO_3 (b) Fe_2O_3 (c) MnO_2 (d) V_2O_5 (e) Cu_2O
(1) (d) > (a) > (b) > (c) > (e)
(2) (a) > (c) > (d) > (b) > (e)
(3) (a) > (d) > (c) > (b) > (e)
(4) (c) > (a) > (d) > (e) > (b)

Q.47 Which one of the following chemical agent is **not** being used for dry–cleaning of clothes?
(1) H_2O_2 (2) CCl_4
(3) Liquid CO_2 (4) $Cl_2C = CCl_2$

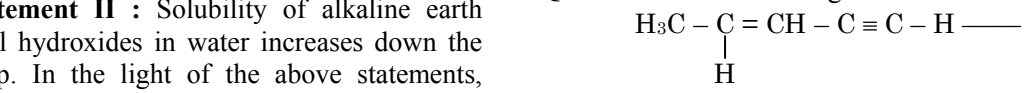
Q.48 Which one of the following compounds will liberate CO_2, when treated with $NaHCO_3$?
(1) $(CH_3)_3\overset{\oplus}{N}H\overset{\ominus}{Cl}$ (2) $(CH_3)_4\overset{\oplus}{N}\overset{\ominus}{O}H$
(3) $CH_3-\overset{\overset{O}{\|}}{C}-NH_2$ (4) CH_3NH_2

Q.49 In the leaching of alumina from bauxite, the ore expected to leach out in the process by reacting with NaOH is :
(1) TiO_2 (2) Fe_2O_3
(3) ZnO (4) SiO_2

Q.50 An organic compound 'A' C_4H_8 on treatment with $KMnO_4/H^+$ yields compound 'B' C_3HO. Compound 'A' also yields compound 'B' an ozonolysis. Compound 'A' is :
(1) 2–Methylpropene
(2) 1–Methylcyclopropane
(3) But–2–ene
(4) Cyclobutane

Section -B

Q.51 The number of sigma bonds in



Q.52 Three moles of AgCl get precipitated when one mole of an octahedral co-ordination compound with empirical formula $CrCl_3.3NH_3.3H_2O$ reacts with excess of silver nitrate. The number of chloride ions satisfying the secondary valency of the metal ion is_____ .

Q.53 A source of monochromatic radiation of wavelength 400 nm provides 1000 J of energy in 10 seconds. When this radiation falls on the surface of sodium, $x \times 10^{20}$ electrons are ejected per second. Assume that wavelength 400 nm is sufficient for ejection of electron from the surface of sodium metal. The value of x is _____ .
(Nearest integer)
($h = 6.626 \times 10^{-34}$ Js)

Q.54 CO_2 gas is bubbled through water during a soft drink manufacturing process at 298 K. If CO_2 exerts a partial pressure of 0.835 bar then x m mol of CO_2 would dissolve in 0.9 L of water. The value of x is _____ . (Nearest integer)
(Henry' s law constant for CO_2 at 298 K is 1.67×10^3 bar)

Q.55 For the reaction A + B $\rightleftharpoons$ 2C the value of equilibrium constant is 100 at 298 K. If the initial concentration of all the three species is 1 M each, then the equilibrium concentration of Cis $x \times 10^{-1}$ M. The value of x is _____ .
(Nearest integer)

Q.56 Consider the cell at 25°C
$Zn \mid Zn^{2+}(aq), (1\ M) \parallel Fe^{3+}(aq), Fe^{2+}(aq) \mid Pt(s)$
The fraction of total iron present as Fe^{3+} ion at the cell potential of 1.500 V is $x \times 10^{-2}$.
The value of x is _____ . (Nearest integer)
(Given := $E^0_{Fe^{3+}/Fe^{2+}} = 0.77V$, $E^0_{Zn^{2+}/Zn} = -0.76V$

Q.57 At 298 K, the enthalpy of fusion of a solid (X) is 2.8 kJ mol–1 and the enthalpy of vaporisation of the liquid (X) is 98.2 kJ mol^{-1}. The enthalpy of sublimation of the substance (X) in kJ mol^{-1} is_____ . (in nearest integer)

Q.58 A home owner uses 4.00×10^3 m^3 of methane (CH_4) gas, (assume CH_4 is an ideal gas) in a year to heat his home. Under the pressure of 1.0 atm and 300 K, mass of gas used is $x \times 10^5$ g. The value of x is _____ . (Nearest integer)
(Given R = 0.083 L atm $K^{-1}mol^{-1}$)

Q.59 When 10 mL of an aqueous solution of Fe^{2+} ions was titrated in the presence of dil H_2SO using diphenylamine indicator, 15 mL of .02 M solution of K_2Cr_2O was required to get the end point. The molarity of the solution containing Fe^{2+} ions is $x \times 10^{-2}$ M. The value of x is _____.(Nearest integer)

Q.60 Consider the complete combustion of butane, the amount of butane utilized to produce 72.0 g of water is _____ $\times 10^{-1}$ g. (in nearest integer)

MATHEMATICS

Section -A

Q.61 A spherical gas balloon of radius 16 meter subtends an angle 60° at the eye of the observer A while the angle of elevation of its center from the eye of A is 75°. Then the height (in meter) of the top most point of the balloon from the level of the observer's eye is :

(1) $8(2+2\sqrt{3}+\sqrt{2})$

(2) $8(\sqrt{6}+\sqrt{2}+2)$

(3) $8(\sqrt{2}+2+\sqrt{3})$

(4) $8(\sqrt{6}-\sqrt{2}+2)$

Q.62 Let $f(x) = 3\sin^4 x + 10\sin^3 x + 6\sin^2 x - 3$, $x \in \left[\frac{\pi}{6}, \frac{\pi}{2}\right]$. Then , f is :

(1) increasing in $\left[-\frac{\pi}{6}, \frac{\pi}{2}\right]$

(2) decreasing in $\left[0, \frac{\pi}{2}\right]$

(3) increasing in $\left(-\frac{\pi}{6}, 0\right)$

(4) decreasing in $\left(-\frac{\pi}{6}, 0\right)$

Q.63 Let S_n be the sum of the first n terms of an arithmetic progression. If $S_{3n} = 3S_{2n}$, then the value of $\frac{S_{4n}}{S_{2n}}$ is :

(1) 6 (2) 4 (3) 2 (4) 8

Q.64 The locus of the centroid of the triangle formed by any point P on the hyperbola $16x^2 - 9y^2 + 32x + 36y - 164 = 0$, and its foci is :

(1) $16x^2 - 9y^2 + 32x + 36y - 36 = 0$
(2) $9x^2 - 16y^2 + 36x + 32y - 144 = 0$
(3) $16x^2 - 9y^2 + 32x + 36y - 144 = 0$
(4) $9x^2 - 16y^2 + 36x + 32y - 36 = 0$

Q.65 Let the vectors

$(2 + a + b)\hat{i} + (a + 2b + c)\hat{j} - (b + c)\hat{k}$, $(1 + b)\hat{i} + 2b\hat{j} - b\hat{k}$ and $(2 + b)\hat{i} + 2b\hat{j} + (1 - b)\hat{k}$ a, b, c $\in$ R

be co-planar. Then which of the following is true?

(1) 2b = a + c (2) 3c = a + b
(3) a = b + 2c (4) 2a = b + c

Q.66 Let f : R $\to$ R be defined as

$$f(x) = \begin{cases} \frac{\lambda|x^2 - 5x + 6|}{\mu(5x - x^2 - 6)}, & x < 2 \\ e^{\frac{\tan(x-2)}{x-[x]}}, & x > 2 \\ \mu, & x = 2 \end{cases}$$

where [x] is the greatest integer less than or equal to x. If f is continuous at x = 2, then $\lambda + \mu$ is equal to :

(1) e(–e + 1) (2) e(e – 2)
(3) 1 (4) 2e–1

Q.67 The value of the definite integral $\int_{\pi/24}^{5\pi/24} \frac{dx}{1+\sqrt[3]{\tan 2x}}$ is :

(1) $\frac{\pi}{3}$ (2) $\frac{\pi}{6}$ (3) $\frac{\pi}{12}$ (4) $\frac{\pi}{18}$

Q.68 If b is very small as compared to the value of a, so that the cube and other higher powers of $\frac{b}{a}$ an be neglected in the identity

$\frac{1}{a-b} + \frac{1}{a-2b} + \frac{1}{a-3b} + ... + \frac{1}{a-nb}$
$= \alpha n + \beta n^2 + \gamma n^3$,
Then the value of γ is :

(1) $\frac{a^2+b}{3a^3}$ (2) $\frac{a+b}{3a^2}$

(3) $\frac{b^2}{3a^3}$ (4) $\frac{a+b^2}{3a^3}$

Q.69 Let y = y(x) be the solution of the differential equation $\frac{dy}{dx} = 1 + xe^{y-x}, -\sqrt{2} < x < \sqrt{2}$, $y(0) = 0$ then, the minimum value of y(x), $x \in \left(-\sqrt{2}, \sqrt{2}\right)$ is equal to :

(1) $\left(2-\sqrt{3}\right) - \log_e 2$

(2) $\left(2+\sqrt{3}\right) + \log_e 2$

(3) $\left(1+\sqrt{3}\right) - \log_e \left(\sqrt{3}-1\right)$

(4) $\left(1-\sqrt{3}\right) - \log_e \left(\sqrt{3}-1\right)$

Q.70 The Boolean expression $(p \Rightarrow q) \wedge (q \Rightarrow \sim p)$ is equivalent to :

(1) ~q (2) q (3) p (4) ~p

Q.71 The area (in sq. units) of the region, given by the set $\{(x, y) \in \mathbf{R} \times \mathbf{R} \mid x \geq 0, 2x^2 \leq y \leq 4 - 2x\}$ is :

(1) $\frac{8}{3}$ (2) $\frac{17}{3}$ (3) $\frac{13}{3}$ (4) $\frac{7}{3}$

Q.72 The sum of all values of x in $[0, 2\pi]$, for which $\sin x + \sin 2x + \sin 3x + \sin 4x = 0$, is equal to :

(1) 8π (2) 11π (3) 12π (4) 9π

Q.73 Let $g : \mathbf{N} \to \mathbf{N}$ be defined as
$g(3n + 1) = 3n + 2$,
$g(3n + 2) = 3n + 3$,
$g(3n + 3) = 3n + 1$, for all $n \geq 0$.
Then which of the following statements is true ?

(1) There exists an onto function $f : \mathbf{N} \to \mathbf{N}$ such that fog = f

(2) There exists a one–one function $f: \mathbf{N} \to \mathbf{N}$ such that fog = f

(3) gogog = g

(4) There exists a function $f : \mathbf{N} \to \mathbf{N}$ such that gof = f

Q.74 Let $f : [0, \infty) \to [0, \infty)$ be defined as

$f(x) = \int_0^x [y]dy$

where [x] is the greatest integer less than or equal to x. Which of the following is true?

(1) f is continuous at every point in $[0,\infty)$ and differentiable except at the integer points.

(2) f is both continuous and differentiable except at the integer points in $[0, \infty)$.

(3) f is continuous everywhere except at the integer points in $[0, \infty)$.

(4) f is differentiable at every point in $[0, \infty)$

Q.75 The values of a and b, for which the system of equations
$2x + 3y + 6z = 8$
$x + 2y + az = 5$
$3x + 5y + 9z = b$
has no solution, are :

(1) $a = 3, b \neq 13$ (2) $a \neq 3, b \neq 13$

(3) $a \neq 3, b = 3$ (4) $a = 3, b = 13$

Q.76 Let 9 distinct balls be distributed among 4 boxes, B_1, B_2, B_3 and B_4. If the probability of B_3 contains exactly 3 balls is $k\left(\frac{3}{4}\right)^9$ then k lies in the set :

(1) $\{x \in \mathbf{R} : |x - 3| < 1\}$

(2) $\{x \in \mathbf{R} : |x - 2| \leq 1\}$

(3) (3) $\{x \in \mathbf{R} : |x - 1| < 1\}$

(4) (4) $\{x \in \mathbf{R} : |x - 5| \leq 1\}$

Q.77 Let a parabola P be such that its vertex and focus lie on the positive x–axis at a distance 2 and 4 units from the origin, respectively. If tangents are drawn from O(0, 0) to the parabola P which meet P at S and R, then the area (in sq. units) of ΔSOR is equal to :

(1) $16\sqrt{2}$ (2) 16 (3) 32 (4) $8\sqrt{2}$

Q.78 The number of real roots of the equation $e^{6x} - e^{4x} - 2e^{3x} - 12e^{2x} + e^x + 1 = 0$ is :

(1) 1 (2) 4 (3) 6 (4) 1

Q.79 Let an ellipse E : $\frac{x^2}{a^2}+\frac{y^2}{b^2} = 1$ $a^2 > b^2$, passes through $\left(\sqrt{\frac{3}{2}},1\right)$ and has eccentricity $\frac{1}{\sqrt{3}}$. If a circle, centered at focus F(α, 0), $\alpha > 0$, of E and radius $\frac{2}{\sqrt{3}}$, intersects E at two points P and Q, then PQ^2 is equal to :

(1) $\frac{8}{3}$ (2) $\frac{4}{3}$ (3) $\frac{16}{3}$ (4) 3

Q.80 Let the foot of perpendicular from a point P(1, 2, –1) to the straight line L : $\frac{x}{1} = \frac{y}{0} = \frac{z}{-1}$ Be N. Let a line be drawn from P parallel to the plane x + y + 2z = 0 which meets L at point Q. If α is the acute angle between the lines PN and PQ, then cosα is equal to _____.

(1) $\frac{1}{\sqrt{5}}$ (2) $\frac{\sqrt{3}}{2}$ (3) $\frac{1}{\sqrt{3}}$ (4) $\frac{1}{2\sqrt{3}}$

Section -B

Q.81 Let y = y(x) be solution of the following differential equation

$e^y \frac{dy}{dx} - 2e^y \sin x + \sin x \cos^2 x = 0$, $y\left(\frac{\pi}{2}\right) = 0$

If $y(0) = \log_e(\alpha + \beta e^{-2})$, then $4(\alpha + \beta)$ is equal to_____.

Q.82 If the value of

$$\left(1+\frac{2}{3}+\frac{6}{3^2}+\frac{10}{3^3}+.....\text{upto } \infty\right)^{\log(0.25)\left(\frac{1}{3}+\frac{1}{3^2}+\frac{1}{3^3}+\text{upto } \infty\right)}$$

is l, then l^2 is equal to _____.

Q.83 Consider the following frequency distribution

class :	10–20	20–30	30–40	40–50	50–60
Frequency :	α	110	54	30	β

If the sum of all frequencies is 584 and median is 45, then $|\alpha - \beta|$ is equal to_____.

Q.84 Let $\vec{p} = 2\hat{i} + 3\hat{j} + \hat{k}$ and $\vec{p} = \hat{i} + 2\hat{j} + \hat{k}$ be two vectors. If a vector $\vec{r} = (\alpha\hat{i} + \beta\hat{j} + \gamma\hat{k})$ is perpendicular to each of the vectors $(\vec{p} + \vec{q})$ and $(\vec{p} - \vec{q})$, and $|\vec{r}| = \sqrt{3}$, then $|\alpha| + |\beta| + |\gamma|$ is equal to ——

Q.85 The ratio of the coefficient of the middle term in the expansion of $(1 + x)^{20}$ and the sum of the coefficients of two middle terms in expansion of $(1 + x)^{19}$ is _____.

Q.86 Let $M = \left\{A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} : a, b, c, d \in \{\pm 3, \pm 2 \pm 1, 0\}\right\}$

Define $f : M \to \mathbf{Z}$, as f(A) = det(A), for all $A \in M$, where **Z** is set of all integers. Then the number of $A \in M$ such that f(A) = 15 is equal to _____.

Q.87 There are 5 students in class 10, 6 students in class 11 and 8 students in class 12. If the number of ways, in which 10 students can be selected from them so as to include at least 2 students from each class and at most 5 students from the total 11 students of class 10 and 11 is 100 k, then k is equal to _____.

Q.88 If α, β are roots of the equation $x^2 + (5\sqrt{2})x + 10 = 0$, $\alpha > \beta$ and $P_n = \alpha^n - \beta^n$ for each positive integer n, then the value of $\left(\frac{P_{17}P_{20} + 5\sqrt{2}P_{17}P_{19}}{P_{18}P_{19} + 5\sqrt{2}P_{18}^2}\right)$ is equal to _____.

Q.89 The term independent of 'x' in the expansion of $\left(\frac{x+1}{x^{2/3} - x^{1/3} + 1} - \frac{x-1}{x - x^{1/2}}\right)^{10}$, where $x \neq 0, 1$ is equal to ____.

Q.90 Let

$$S = \left\{n \in N : \begin{pmatrix} 0 & i \\ 1 & 0 \end{pmatrix}^n \begin{pmatrix} a & b \\ c & d \end{pmatrix}^n = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \forall a, b, c, d \in R\right\},$$

where $i\sqrt{-1}$. Then the number of 2–digit numbers in the set S is _____.

JEE MAIN ONLINE PAPER 2021

Held on JULY 25, 2021 (Morning)

Hints & Solutions

PHYSICS

Section -A

1.[4] $C_P - C_V = R$ for ideal gas and gas behaves as ideal gas at high temperature
so $T_P > T_Q$

2.[2] $I_z = I_x + I_y$ (using perpendicular axis theorem)
& $I = mk^2$ (K : radius of gyration)

So $mK_z^2 = mK_x^2 + mK_y^2$

$K_z^2 = K_x^2 + K_y^2$

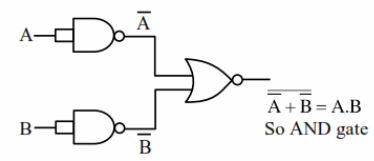
so radius of gyration about axes x, y & z won't be same hence assertion A is not correct. Reason R is correct statement (property of a rigid body)

3.[3] $\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}} \propto \frac{1}{\sqrt{m}}$

$m_\alpha > m_p > m_e$

so $\lambda_e > \lambda_p > \lambda_\alpha$

4.[2]



5.[4] $\lambda = \frac{h}{p}$

both the particles will move with momentum same in magnitude & opposite in direction.
So De-Broglie wavelength of both will be same i.e. ratio 1 : 1

6.[4] $\frac{3N_0}{4} = N_0 e^{-\lambda t_1}$

$\frac{N_0}{2} = N_0 e^{-\lambda t_2}$

$\ln(3/4) = -\lambda t_1$ ....(i)

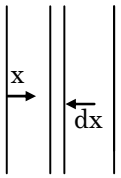
$\ln(1/2) = -\lambda t_2$ .... (i)

$\ln(3/4) - \ln(1/2) = \lambda (t_2 - t_1)$ .... (i)

$\Delta t = \frac{\ln(3/2)}{\lambda}$

7.[2] (a) $\vec{C} = \vec{A} + \vec{B}$
Option (iv)
(b) $\vec{A} = \vec{B} + \vec{C} = \vec{C} + \vec{B}$
Option (iii)
(c) $\vec{B} = \vec{A} + \vec{C}$
Option (i)
(d) $\vec{A} + \vec{B} + \vec{C} = 0$
Option (ii)

8.[2]



Taking an element of width dx at a distance x ($x < d/2$) from left plate

$dc = \frac{(\varepsilon_0 + kx)A}{dx}$

Capacitance of half of the capacitor

$\frac{1}{C} = \int_0^{d/2} \frac{1}{dc} = \frac{1}{A} \int_0^{d/2} \frac{dx}{\varepsilon_0 + kx}$

$\frac{1}{C} = \frac{1}{kA} \ln\left(\frac{\varepsilon_0 + kd/2}{\varepsilon_0}\right)$

Capacitance of second half will be same

$C_{eq} = \frac{C}{2} = \frac{kA}{2\ell n\left(\frac{2\varepsilon_0 + kd}{2\varepsilon_0}\right)}$

9.[2] $PV^r = \text{const.}$

$TV^{r-1} = \text{const.}$

$T(\ell)^{\frac{5}{3}-1} = \text{const.}$

$\frac{T_1}{T_2} = \left(\frac{\ell_2}{\ell_1}\right)^{\frac{2}{3}}$

10.[3] $\sin\theta_C = \frac{1}{\mu} = \frac{1}{2\mu_2} < \sin\theta_C$

$\sin\theta > \sin\theta_C$

$\theta > \theta_C$

Total internal reflection will happen

11.[2] In series combination $\Delta\ell = \ell_1 + \ell_2$

$$Y = \frac{F/A}{\Delta\ell/\ell} \Rightarrow \Delta\ell = \frac{F\ell}{AY}$$

$$\Rightarrow \Delta\ell \propto \frac{\ell}{Y}$$

Equivalent length of rod after joing is = 2ℓ

As, lengths are same and force is also same in series

$$\Delta\ell = \Delta\ell_1 + \Delta\ell_2$$

$$\frac{\ell_{eq}}{Y_{eq}} = \frac{\ell}{Y_1} + \frac{\ell}{Y_2} \Rightarrow \frac{2\ell}{Y} = \frac{\ell}{Y_1} + \frac{\ell}{Y_2}$$

$$\therefore Y = \frac{2Y_1Y_2}{Y_1 + Y_2}$$

12.[4] $A = \lambda N$

$$\lambda = \frac{\ell n2}{t_{1/2}} = \frac{\ell n2}{3\times24\times60\times60}\,sec^{-1} = 2.67\times10^{-6}\,sec^{-1}$$

N = Number of atoms in 2 mg Au

$$= \frac{2\times10^{-3}}{198} \times 6\times10^{23} = 6.06\times10^{15}$$

$A = \lambda N = 1.618 \times 10^{13} = 16.18 \times 10^{12}$ dps

13.[2] Impulse = change in momentum

Ball (a) $\left|\overrightarrow{\Delta p}\right| = 2mu = J_1$

Ball (b) $\left|\overrightarrow{\Delta p}\right| = 2\,mu\cos45^\circ = J_2$

$$\frac{J_1}{J_2} = \frac{1}{\cos45^\circ} = \sqrt{2}$$

14.[2] Fringe Width , $\beta = \frac{\lambda D}{d}$

$\beta_{max} \Rightarrow d_{min}$ and $\beta_{min} \Rightarrow d_{max}$

$d = d_0 + a_0 \sin\omega t$

$d_{max} = d_0 + a_0$ and $d_{min} = d_0 - a_0$

$$\therefore \beta_{min} = \frac{\lambda D}{d_0 + a_0} \text{ and } \beta_{max} = \frac{\lambda D}{d_0 - a_0}$$

$$\therefore \beta_{max} - \beta_{min} = \frac{\lambda D}{d_0 - a_0} - \frac{\lambda D}{d_0 + a_0} = \frac{2\lambda D a_0}{d_0^2 - a_0^2}$$

15.[2] $\left(\frac{\Delta Q}{\Delta t}\right)_A = \left(\frac{\Delta Q}{\Delta t}\right)_B$

$$mS_A\left(\frac{\Delta T}{\Delta t}\right)_A = mS_B\left(\frac{\Delta T}{\Delta t}\right)_B$$

$$\frac{S_A}{S_B} = \frac{\left(\frac{\Delta T}{\Delta t}\right)_A}{\left(\frac{\Delta T}{\Delta t}\right)_B} = \frac{90/6}{120/3} = \frac{15}{40} = \frac{3}{8}$$

16.[4] Reflected wave will have direction opposite to incident wave.

17.[1] Max. voltage that can be measured by this potentiometer will be equal to potential drop across AB

$R_{AB} = 10 \times 0.1 \times 100 = 100$ ohm.

$$\therefore V_{AB} = \frac{6}{20+100} \times 100 = 6 \times \frac{100}{120} = 5V$$

18.[1] Bandwidth = $2 \times f_m$

$= 2 \times 10^5$ HZ = 200 KHZ

19.[3] In 4 sec. 1st drop will travel

$$\Rightarrow \frac{1}{2} \times (9.8) \times (4)^2 = 78.4 \text{ m}$$

$\therefore$ 2nd drop would have travelled

$\Rightarrow 78.4 - 34.3 = 44.1$ m.

Time for 2nd drop

$$\frac{1}{2}(9.8)t^2 = 44.1$$

$\boxed{t = 3 \text{ sec}}$

$\therefore$ each drop have time gap of 1 sec

$\therefore$ 1 drop per sec

20.[4] Angular momentum conservation equation

$$v_0x_2 = v_1x_1$$

$$v_1 = \frac{v_0x_2}{x_1}$$

Section -B

21.[25] $p_i = p_f$

$2 \times 4 = 2 \times 1 + m_2 \times v_2$

$m_2v_2 = 6$ .....(i)

by coefficient of restitution

$$1 = \frac{v_2 - 1}{4} \Rightarrow v_2 = 5\text{m/s}$$

by (i)

$m_2 \times 5 = 6$

$m_2 = 1.2$ kg

$$v_{cm} = \frac{m_1v_1 + m_2v_2}{m_1 + m_2}$$

$$v_{cm} = \frac{2\times1 + 1.2\times5}{2 + 1.2} = \frac{8}{3.2} = \frac{25}{10}$$

$\boxed{x = 25}$

22.[13] For (A)

Reading = MSR + CSR + Error

$0.322 = 0.300 + CSR + 5 \times LC$

$0.322 = 0.300 + CSR + 0.005$

CSR = 0.017
For B
Reading = MSR + CSR + Error
0.322 = 0.200 + CSR + 0.092
CSR = 0.030
Difference = 0.030 – 0.017 = 0.013 cm

Division on circular scale = $\frac{0.013}{0.001} = 13$

23.[74] $I_{max} = \frac{V}{R} = \frac{20V}{10K\Omega} = 2mA$

For LR – decay circuit

$I = I_{max}\, e^{-Rt/L}$

$I = 2mA\, e^{\frac{-10\times10^{3}\times1\times10^{-6}}{10\times10^{-3}}}$

$I = 2mA\, e^{-1}$

$I = 2 \times 0.37$ mA

$I = \frac{74}{100}$ mA

$\boxed{x = 74}$

24.[80] $\phi = \vec{B}.\vec{S}$

$\phi = \frac{4}{\pi} \times 10^{-3}\left(1 - \frac{t}{100}\right).\pi R^2$

$\phi = 4 \times 10^{-3} \times (1)^2\left(1 - \frac{t}{100}\right)$

$\varepsilon = \frac{-d\phi}{dt}$

$\varepsilon = \frac{-d}{dt}\left(4\times10^{-3}\left(1 - \frac{t}{100}\right)\right)$

$\varepsilon = 4 \times 10^{-3}\left(\frac{1}{100}\right) = 4 \times 10^{-5}V$

When B = 0

$1 - \frac{t}{100} = 0$

$t = 100$ sec

$\text{Heat} = \frac{\varepsilon^2}{R}\, t$

$\text{Heat} = \frac{(4\times10^{-5})^2}{2\times10^{-6}} \times 100J$

$\text{Heat} = \frac{16\times10^{-10}\times100}{2\times10^{-6}}$ J

Heat = 0.08 J

Heat = 80 mJ

25.[10] $\omega = \sqrt{\frac{k_{eq}}{\mu}}$

μ = reduced mass

springs are in series connection

$k_{eq} = \frac{k_1k_2}{k_1 + k_2}$

$k_{eq} = \frac{k\times4k}{5k} = \frac{4k}{5}$

$k_{eq} = \frac{4\times20}{5}$ N /m = 16N/m

$\mu = \frac{m_1m_2}{m_1 + m_2} = \frac{0.2\times0.8}{0.2 + 0.8} = 0.16$ kg

$\omega = \sqrt{\frac{16}{0.16}} = \sqrt{100} = 10$

26.[22] $B = \mu.(H+1)$

$B = \mu.H\left(1 + \frac{1}{H}\right)$

$B = B_0(1+x)$

$B - B_0 = B_0x$

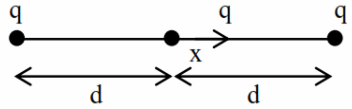
$B - B_0 = B_0x$

$\frac{B - B_0}{B_0} = x$

$\frac{B - B_0}{B_0} \times 100 = 100x$

$= 2.2 \times 10^{-3} = \frac{22}{10^4}$

27.[6]



Net force on free charged particle

$F = \frac{kq^2}{(d+x)^2} - \frac{kq^2}{(d-x)^2}$

$F = -kq^2\left[\frac{4dx}{(d^2 - x^2)^2}\right]$

$a = -\frac{4kq^2d}{m}\left(\frac{x}{d^4}\right)$

$a = -\left(\frac{4kq^2}{md^3}\right)x$

So, angular frequency

$\omega = \sqrt{\frac{4kq^2}{md^3}}$

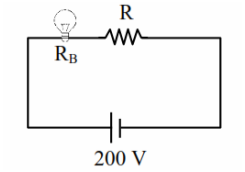
$\omega = \sqrt{\frac{4\times9\times10^{9}\times10}{1\times10^{-6}\times1^3}}$

$\omega = 6 \times 10^8$ rad/sec

28.[50] Power, $P = \frac{V^2}{R_B}$

$$R_B = \frac{V^2}{P} = \frac{100\times100}{200}$$

$R_B = 50\Omega$

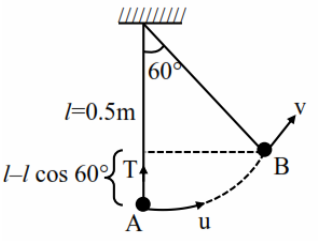


To produce same power, same voltage (i.e. 100 V) should be across the bulb

Hence, $R = R_B$

$R = 50\ \Omega$

29.[2]



Applying work energy theorem :

$W_g + W_T = \Delta K$

$-mgl\,(1-\cos60^\circ) = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$

$v^2 = u^2 - 2gl\,(1-\cos60^\circ)$

$v^2 = 9 - 2\times10\times0.5\left(\frac{1}{2}\right)$

$v^2 = 4$

$v = 2m/s$

30.[10] $\vec{r} = 10\alpha t^2\,\hat{i} + 5\beta(t-5)\,\hat{j}$

$\vec{v} = 20\alpha t\,\hat{i} + 5\beta\,\hat{j}$

$\hat{L} = m(\vec{r}\times\vec{v})$

$= m[10\alpha t^2\,\hat{i} + 5\beta(t-5)\,\hat{j}] \times [20\alpha t\,\hat{i} + 5\beta\,\hat{j}]$

$\hat{L} = m[50\alpha\beta t^2\,\vec{k} - 100\alpha\beta(t^2-5t)\,\vec{k}]$

At $t = 0$, $\hat{L} = \vec{0}$

$50\alpha\beta t^2 - 100\alpha\beta(t^2-5t) = 0$

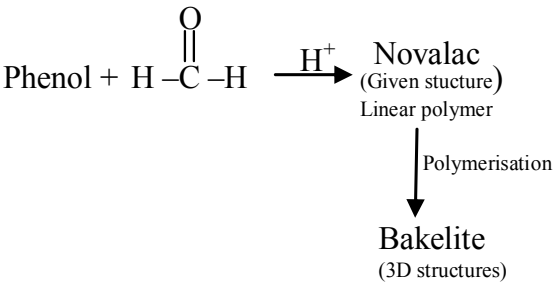
$t - 2(t-5) = 0$

$t = 10$ sec

CHEMISTRY

Section -A

31.[1]



32.[1] $[\overset{+3}{Fe}(H_2O)_6]^{3+}$

Fe^{3+} : $[Ar]3d^5$

Hybridisation : sp^3d^2

Magnetic nature : Paramagnetic (so this complex response to external magnetic field)

2. $[Ni(CN)_4]^{2-}$

Ni^{2+} : $[Ar]3d^8$

Hybridisation : dsp^2

Magnetic nature : diamagnetic

3. $[Co(CN)_6]^{3-}$

CO^{3+} : $[Ar]3d^6$

Hybridisation : d^2sp^3

Magnetic nature : diamagnetic

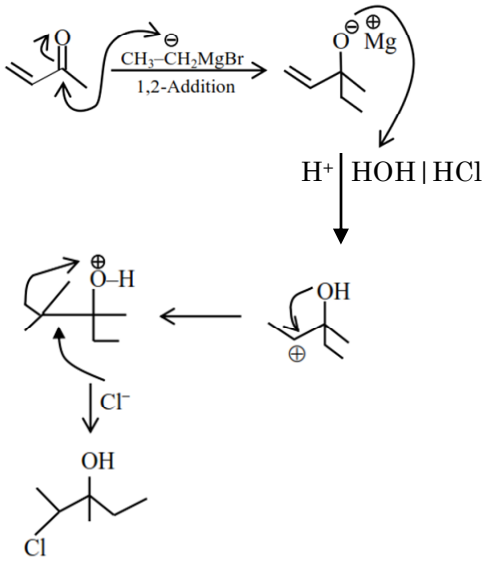
4. $[Ni(CO)_4]$

Ni [Ar] $3d^84s^2$

Hybridisation : sp^3

Magnetic nature : diamagnetic

33.[3]

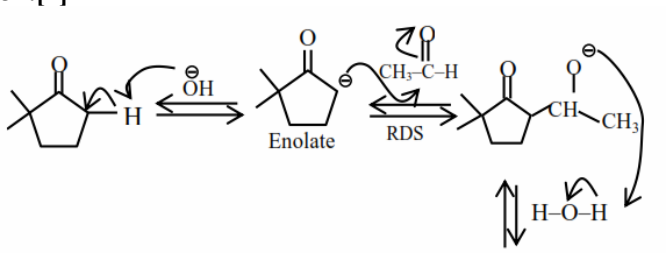


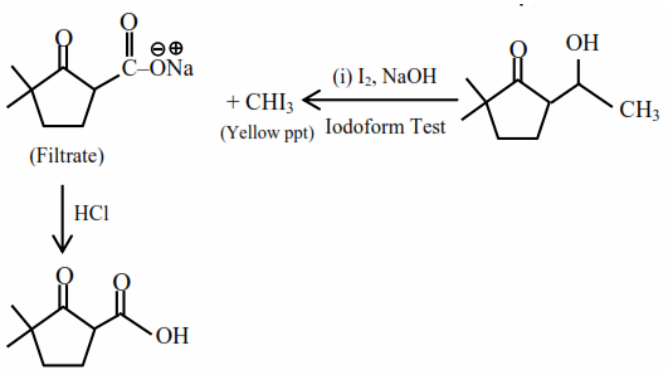
34.[1] Forms spherical micelles with $CH_3(CH_2)_{16}$ group pointing towards the centre of sphere

35.[2] Albumin is water soluble.

36.[3] Enthalpy of bond dissociation (kJ/mole) at 298.2K
For , hydrogen = 435.88
For , Deuterium = 443.35
$\therefore E_H \simeq E_D - 7.5$

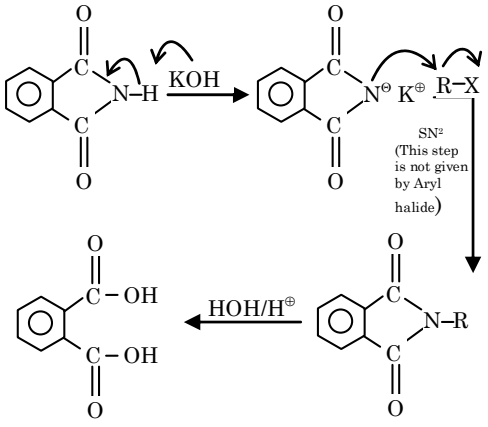
37.[4]





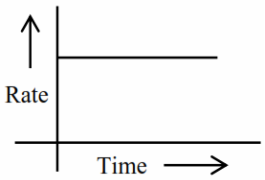
38.[3] Bromine water gives tribromo products, other gives monobromo products in which para is major product.

39.[3] Gabriel pthalamide synthesis

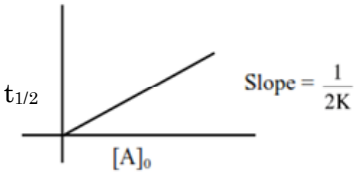


+ R – NH_2
(primary aliphatic amine)

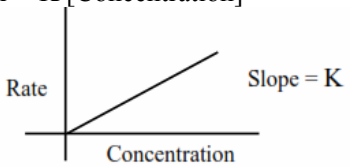
40.[2] **Official Ans by NTA (1)**
For zero order reaction's
rate = K [Reactant]^0
$\Rightarrow$ r = k

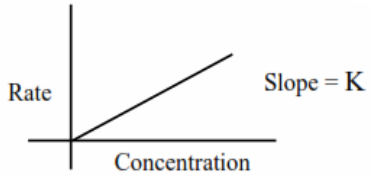


$t_{1/2} = \frac{[A]_0}{2K}$ for zero order

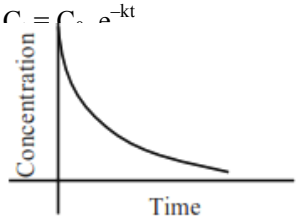


For first order reaction $\rightarrow$
r = K [Concentration]

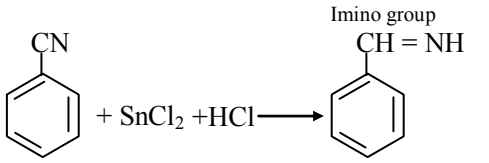




Reactant concentration after time t $\rightarrow$
$C = C_0 e^{-kt}$



41.[2]

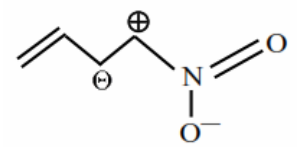


Imino group does not react with Hinsberg reagent

42.[3] Al^{3+}, Mg^{2+} and Na+ are isoelectronic ionic species. For monoatomic ionic isoelectronic species as positive charge increases ionic size decreases.
The order of size of Na^+ & K^+ is $Na^+ < K^+$
$\therefore$ order of ionic radii is : $Al^{3+} < Mg^{2+} < Na^+ < K^+$

43.[3] $[SiCl_6]^{2-}$ does not exist due to steric crowding of surrounding atoms.

44.[1]



It is unstable RS (due to similar charge on adjacent atom)

45.[2] Statement-I is incorrect
$Be(OH)_2$ dissolve in alkali due to it's amphoteric nature.
Statement-II is correct
Solubility of alkaline earth metal hydroxide in water increases down the group due to rapid decreases in lattice energy as compared to hydration energy.

46.[3] (a) $\overset{+6}{Cr}O_3$ (d) $\overset{+5}{V_2}O_5$
(b) $\overset{+3}{Fe}O_3$ (e) $\overset{+1}{Cu_2}O$
(e) $\overset{+4}{Mn}O_2$
So order of oxidation state
a > d > c > b > e

47.[1] **Official Ans by NTA (2)**
CO_2, CCl_4 and $Cl_2C = CCl_2$ are used as dry cleaning agents for clothes. H_2O_2 is used as bleaching agent in laundry.

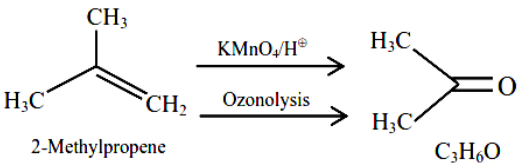
48.[1]

$$(CH_3)_3NHCl + NaHCO_3 \rightarrow H_2CO_3 + (CH_3)_3N + NaCl$$
$$\downarrow$$
$$CO_2 + H_2O$$

49.[4] In bauxite impurities of Fe_2O_3, TiO_2 and SiO_2 are present , Fe_2O_3 and TiO_2 are basic oxides therefore does not reacts with or dissolve in NaOH whereas SiO_2 is acidic oxide it gets dissolve in NaOH, hence leach out

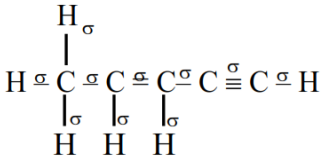
$$SiO_2 + 2NaOH \rightarrow Na_2SiO_3(aq.) + H_2O$$

50.[1]



Section -B

51.[10]



numbers of σ bonds = 10

52.[0] Mole of AgCl precipitated is equal the mole of Cl^- present in ionization sphere.

$$[Cr(H_2O)_3(NH_3)_3]Cl_3 \rightarrow [Cr(H_2O)_3(NH_3)_3]^{3+} + 3Cl^-$$
1 mole 1 mole 3 mole

$AgNO_3$ (Excess) ↓ AgCl (3mole)

Since none of Cl^- is present in the co-ordination sphere. Therefore answer is zero.

53.[2] Total energy provided by
Source per second $= \frac{1000}{10} = 100J$

Energy required to eject electron $= \frac{hc}{\lambda}$

$$= \frac{6.626\times10^{-34}}{400\times10^{-9}} \times 3 \times 10^8$$

Number of electrons ejected

$$= \frac{100}{\frac{6.626\times10^{-34}\times3\times10^8}{400\times10^{-9}}}$$

$$= \frac{400\times10^{-7}\times10^{26}}{6.626\times3}$$

$$= \frac{40\times10^{-20}}{6.626\times3}$$

$$= 2.01 \times 10^{20}$$

54.[25] From Henry's law

$P_{gas} = K_H.X_{gas}$

$$0.835 = 1.67 \times 10^3 \frac{n(CO_2)}{\frac{0.9\times1000}{18}}$$

$n(CO_2) = 0.025$

Millimoles of $CO_2 = 0.025 \times 1000 = 25$

55.[25]

A	+	B $\rightleftharpoons$	2C
1		1	1
–x		–x	2x
1–x		1–x	1+ 2x

$$K = \frac{[C]_{eq}^2}{[A]_{eq}[B]_{eq}} = \frac{(1+2x)^2}{(1-x)(1-x)}$$

$$100 = \left(\frac{1+2x}{1-x}\right)^2$$

$$\left(\frac{1+2x}{1-x}\right) = 10$$

$$x = \frac{3}{4}$$

$[C]e_{q.} = 1 + 2x$

$= 1 + 2\left(\frac{3}{4}\right)$

$= 2.5$ M

25×10^{-1} M

56.[24]

$$Zn \longrightarrow Zn^{2+} + 2e^-$$
$$2Fe^{3+} \longrightarrow 2e^- + 2e^{2+}$$

$$Zn + 2Fe^{3+} \longrightarrow Zn^{2+} + 2Fe^{2+}$$

$$E^0_{cell} = 0.77 - (0.76)$$
$$= 1.53 \text{ V}$$

$$1.50 = 1.53 - \frac{0.06}{2}\log\left(\frac{Fe^{2+}}{Fe^{3+}}\right)^2$$

$$\log\left(\frac{Fe^{2+}}{Fe^{3+}}\right) = \frac{0.03}{0.06} = \frac{1}{2}$$

$$\frac{[Fe^{2+}]}{[Fe^{3+}]} = 10^{1/2} = \sqrt{10}$$

$$\frac{[Fe^{3+}]}{[Fe^{2+}]} = \frac{1}{\sqrt{10}}$$

$$\frac{[Fe^{3+}]}{[Fe^{2+}]+[Fe^{3+}]} = \frac{1}{1+\sqrt{10}} = \frac{1}{4.16}$$

$= 0.2402$

$= 24 \times 10^{-2}$

57.[101] $\Delta H_{sub} = \Delta H_{fus} + \Delta H_{vap}$

$= 2.8 + 98.2$

$= 101$ kJ/mol

58.[26] $n(CH_4) = \frac{PV}{RT}$

$$= \frac{1\times 4\times 10^3\times 1000}{0.083\times 300}$$

Weight of CH_4

$$= \frac{40\times 16\times 10^5}{0.083\times 300}\text{ gm}$$

$= 25.7 \times 10^5$ gm

59.[18] milli-equivalents of Fe^{2+} = milli-equivalents of $K_2Cr_2O_7$

$M \times 10 \times 1 = 0.02 \times 15 \times 6$

$M = 0.18 = 18 \times 10^{-2}$ M

60.[464]

$$C_4H_{10} + \frac{13}{2}O_2 \rightarrow 4CO_2 + 5H_2O$$

moles of $H_2O = \frac{72}{18} = 4$

Moles of C_4H_{10} used $= \frac{1}{5}\times 4$

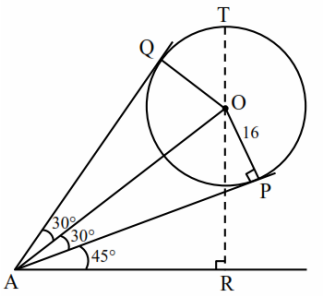
Weight of C_4H_{10} used $= \frac{4}{5}\times 58$

$= 46.4$gm

MATHEMATICS

Section -A

61.[2]



$O \rightarrow$ centre of sphere

$P,Q \rightarrow$ point of contact of tangents from A

Let T be top most point of balloon & R be foot of perpendicular from O to ground.

From triangle OAP, $OA = 16\cos ec30° = 32$

From triangle ABO, $OR = OA \sin 75°$

$$= 32\frac{\left(\sqrt{3}+1\right)}{2\sqrt{2}}$$

So level of top most point = OR + OT

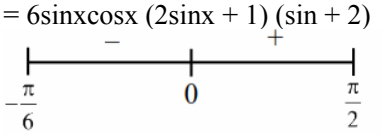
$= 8\left(\sqrt{6}+\sqrt{2}+2\right)$

62.[4] $f(x) = 3\sin^4 x + 10\sin^3 x + 6\sin^2 x - 3,\ x \in \left[\frac{\pi}{6}, \frac{\pi}{2}\right]$

$f(x) = 12\sin^3 x\cos x + 30\sin^2 x\cos x + 12\sin x\cos x$

$= 6\sin x\cos x\,(2\sin^2 x + 5\sin x + 2)$

$= 6\sin x\cos x\,(2\sin x + 1)(\sin x + 2)$



Decreasing in $\left(-\frac{\pi}{6}, 0\right)$

63.[1] Let a be first term and d be common diff. of this A.P.

Given $S_{3n} = 3S_{2n}$

$\Rightarrow \frac{3n}{2}[2a + (3n-1)d] = 3\,\frac{2n}{2}[2a + (2n-1)d]$

$\Rightarrow 2a + (3n-1)d = 4a + (4n-2)d$

$\Rightarrow 2a + (n-1)d = 0$

Now $\frac{S_{4n}}{S_{2n}} = \frac{\frac{4n}{2}[2a + (4a-1)d]}{\frac{2n}{2}[2a + (2n-1)d]}$

$= \frac{2\left[\underbrace{2a + (n-1)d}_{=0} + 3nd\right]}{\left[\underbrace{2a + (n-1)d}_{=0} + nd\right]}$

$= \frac{6nd}{nd} = 6$

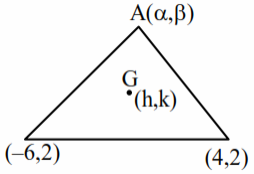
64.[1] Given hyperbola is

$16(x+1)^2 - 9(y-2)^2 = 164 + 16 - 36 = 144$

$\frac{(x+1)^2}{9} - \frac{(y-2)^2}{16} = 1$

Eccentricity, $e = \sqrt{1 + \frac{16}{9}} = \frac{5}{3}$

$\Rightarrow$ foci are (4, 2) and (–6, 2)



Let the centroid be (h, k)

& A(α, β) be point on hyperbola

So $h = \frac{\alpha - 6 + 4}{3}$, $K = \frac{\beta + 2 + 2}{3}$

$\Rightarrow \alpha = 3h + 2,\ \beta = 3k - 4$

(α, β) lies on hyperbola so

$16(3h + 2 + 1)^2 - 9(3k - 4 - 2)^2 = 144$

$\Rightarrow 144(h+1)^2 - 81(k-2)^2 = 144$

$\Rightarrow 16(h^2 + 2h + 1) - 9(k^2 - 4k + 4) = 16$

$\Rightarrow 16x^2 - 9y^2 + 32x + 36y - 36 = 0$

65.[1] If the vectors are co-planar,

$$\begin{vmatrix} a+b+2 & a+2b+c & -b-c \\ b+1 & 2b & -b \\ b+2 & 2b & 1-b \end{vmatrix} = 0$$

Now $R_3 \to R_3 - R_2$, $R_1 \to R_1 - R_2$

So $\begin{vmatrix} a+1 & a+c & -c \\ b+1 & 2b & -b \\ 1 & 0 & 1 \end{vmatrix} = 0$

$= (a+1)2b - (a+c)(2b+1) - c(-2b)$

$= 2ab + 2b - 2ab - a - 2bc - c + 2bc$

$= 2b - a - c = 0$

66.[1] $\lim\limits_{x\to 2+} f(x) = \lim\limits_{x\to 2+} e^{\frac{\tan(x-2)}{x-2}} = e^1$

$\lim\limits_{x\to 2-} f(x) = \lim\limits_{x\to 2-} \frac{-\lambda(x-2)(x-3)}{\mu(x-2)(x-3)} = -\frac{\lambda}{\mu}$

For continuity $\mu = e = -\frac{\lambda}{\mu} \Rightarrow \mu = e,\ \lambda = -e^2$

$\lambda + \mu = e(-e + 1)$

67.[3] Let $I = \int_{\pi/24}^{5\pi/24} \frac{(\cos 2x)^{1/3}}{(\cos 2x)^{1/3} + (\sin 2x)^{1/3}}\,dx$ ...(i)

$\Rightarrow I = \int_{\pi/24}^{5\pi/24} \frac{\left(\cos\left\{2\left(\frac{\pi}{4} - x\right)\right\}\right)^{\frac{1}{3}}}{\left(\cos\left\{2\left(\frac{\pi}{4} - x\right)\right\}\right)^{\frac{1}{3}} + \left(\sin\left\{2\left(\frac{\pi}{4} - x\right)\right\}\right)^{\frac{1}{3}}}\,dx$

$\left\{\int_a^b f(x)dx = \int_a^b f(a+b-x)dx\right\}$

So $I \int_{\pi/24}^{5\pi/24} \frac{(\sin 2x)^{1/3}}{(\sin 2x)^{1/3} + (\cos 2x)^{1/3}}\,dx$ ....(ii)

Hence $2I \int_{\pi/24}^{5\pi/24} dx$ [(i) + (ii)]

$\Rightarrow 2I = \frac{4\pi}{24} \Rightarrow \boxed{I = \frac{\pi}{12}}$

68.[3] $(a-b)^{-1} + (a-2b)^{-1} + \ldots + (a-nb)^{-1}$

$= \frac{1}{a}\sum_{r=1}^{n}\left(1 - \frac{rb}{a}\right)^{-1}$

$$= \frac{1}{a}\sum_{r=1}^{n}\left\{\left(1+\frac{rb}{a}+\frac{r^2b^2}{a^2}\right)+(\text{terms to be neglected})\right\}$$

$$= \frac{1}{a}\left[n+\frac{n(n+1)}{2}\cdot\frac{b}{a}+\frac{n(n+1)(2n+1)}{6}\cdot\frac{b^2}{a^2}\right]$$

$$= \frac{1}{a}\left[n^3\left(\frac{b^2}{3a^2}\right)+\ldots\right]$$

So $\gamma = \frac{b^2}{3a^3}$

69.[4] $\frac{dy-dx}{e^{y-x}} = x\,dx$

$\Rightarrow \frac{dy-dx}{e^{y-x}} = x\,dx$

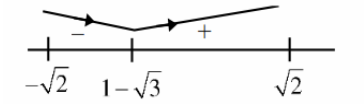
$\Rightarrow -e^{x-y} = \frac{x^2}{2} + c$

At $x = 0, y = 0 \Rightarrow c = -1$

$\Rightarrow e^{x-y} = \frac{2-x^2}{2}$

$\Rightarrow y = x - \ell n\left(\frac{2-x^2}{2}\right)$

$\Rightarrow \frac{dy}{dx} = 1 + \frac{2x}{2-x^2} = \frac{2+2x-x^2}{2-x^2}$



So minimum value occurs at $x = 1-\sqrt{3}$

$$y(1-\sqrt{3}) = (1-\sqrt{3}) - \ell n\left(\frac{2-(4-2\sqrt{3})}{2}\right)$$

$$= (1-\sqrt{3}) - \ell n(1-\sqrt{3})$$

70.[4] $(p \to q) \wedge (q \to \sim p)$

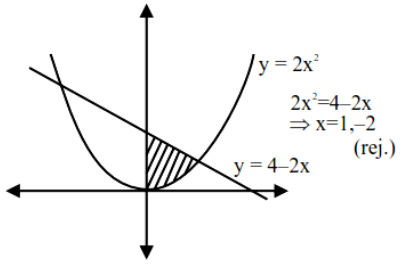
$\equiv (\sim p \vee q) \wedge (\sim q \vee \sim p)$ $\{p \to q \equiv \sim p \vee q\}$

$\equiv (\sim p \vee q) \wedge (\sim p \vee \sim q)$ {commutative property}

$\equiv \sim p \vee (q \wedge \sim q)$ {distributive property}

$\equiv \sim p$

71.[4]



Required area $= \int_0^1 (4-2x-2x^2)\,dx$

$$= 4x - x^2 - \frac{2x^3}{3}\Big|_0^1$$

$$= 4 - 1 - \frac{2}{3} = \frac{7}{3}$$

72.[4] $(\sin x + \sin 4x) + (\sin 2x + \sin 3x) = 0$

$\Rightarrow 2\sin\frac{5x}{2}\left\{\cos\frac{3x}{2}+\cos\frac{x}{2}\right\} = 0$

$\Rightarrow 2\sin\frac{5x}{2}\left\{2\cos x\cos\frac{x}{2}\right\} = 0$

$2\sin\frac{5x}{2} = 0 \Rightarrow \frac{5x}{2} = 0, \pi, 2\pi, 3\pi, 4\pi, 5\pi$

$\Rightarrow x = 0, \frac{2\pi}{5}, \frac{4\pi}{5}, \frac{6\pi}{5}, \frac{8\pi}{5}, 2\pi$

$\cos\frac{x}{2} = 0 \Rightarrow \frac{x}{2} = \frac{\pi}{2} \Rightarrow x = \pi$

$\cos x = 0 \Rightarrow x = \frac{\pi}{2}, \frac{3\pi}{2}$

So sum $= 6\pi + \pi + 2\pi = 9\pi$

73.[1] $g : N \to N$ $g(3n+1) = 3n+2$

$g(3n+2) = 3n+3$

$g(3n+3) = 3n+1$

$$g(x) = \begin{cases} x+1 & x = 3k+1 \\ x+1 & x = 3k+2 \\ x-2 & x = 3k+3 \end{cases}$$

$$g(g(x)) = \begin{cases} x+2 & x = 3k+1 \\ x-1 & x = 3k+2 \\ x-1 & x = 3k+3 \end{cases}$$

$$g(g(g(x))) = \begin{cases} x & x = 3k+1 \\ x & x = 3k+2 \\ x & x = 3k+3 \end{cases}$$

If $f : N \to N$, f is a one-one function such that $f(g(x)) = f(x) \Rightarrow g(x) = x$, which is not the case

If $f : N \to N$ is an onto function such that $f(g(x)) = f(x)$, one possibility is

$$f(x) = \begin{cases} n & x = 3k+1 \\ n & x = 3k+2 \\ n & x = 3k+3 \end{cases} \quad n \in N_0$$

Here $f(x)$ is onto, also $f(g(x)) = f(x)\ \forall\ x \in N$

74.[1] $f : [0, \infty) \to [0, \infty), f(x) = \int_0^x [y]dy$

Let $x = n + f, f \in (0, 1)$

So $f(x) = 0 + 1 + 2 + \ldots + (n-1) + \int_n^{n+f} n\, dy$

$$f(x) = \frac{n(n-1)}{2} + nf$$

$$\frac{[x]([x]-1)}{2} + [x]\{x\}$$

Not $\lim\limits_{x \to n^-} f(x) = \frac{n(n-1)}{2}$, $\lim\limits_{x \to n^-} f(x)$

$$= \frac{(n-1)(n-2)}{2} + (n-1)$$

$$= \frac{n(n-1)}{2} \; (n \in N_0)$$

so $f(x)$ is cont. $\forall$ $x \geq 0$ and diff. except at integer points

75.[1]

$$D = \begin{vmatrix} 2 & 3 & 6 \\ 1 & 2 & a \\ 3 & 5 & 9 \end{vmatrix} = 3 - a$$

$$D = \begin{vmatrix} 2 & 3 & 8 \\ 1 & 2 & 5 \\ 3 & 5 & b \end{vmatrix} = b - 13$$

If $a = 3$, $b \neq 13$, no solution.

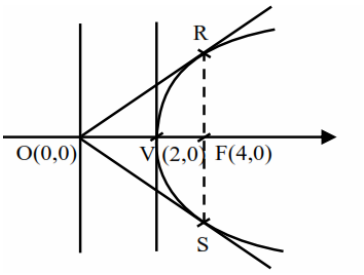
76.[1] required probability $= \frac{{}^9C_3 \cdot 3^6}{4^9}$

$$= \frac{{}^9C_3}{27} \cdot \left(\frac{3}{4}\right)^9$$

$$= \frac{28}{9} \cdot \left(\frac{3}{4}\right)^9 \Rightarrow k = \frac{28}{9}$$

Which satisfies $|x - 3| < 1$

77.[2]



Clearly RS is latus-rectum

$\because$ VF = 2 = a

$\therefore$ RS = 4a = 8

Now OF = 2a = 4

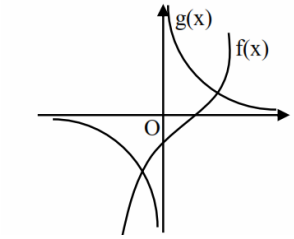
$\Rightarrow$ Area of triangle ORS = 16

78.[1] $e^{6x} - e^{4x} - 2e^{3x} - 12e^{2x} + e^x + 1 = 0$

$$\Rightarrow \left(e^{3x} - 1\right)^2 - e^x\left(e^{3x} - 1\right) = 12e^{2x}$$

$$\left(e^{3x} - 1\right)^2\left(e^x - e^{-x} - e^{-2x}\right) = 12$$

$$\Rightarrow \underbrace{e^x - e^{-x} - e^{-2x}}_{\text{increasing (let } f(x))} = \underbrace{\frac{12}{e^{3x} - 1}}_{\text{decreasing (let } g(x))}$$



$\Rightarrow$ No. of real roots = 2

79.[3] $\frac{3}{2a^2} + \frac{1}{b^2} = 1$ and $1 - \frac{b^2}{a^2} = \frac{1}{3}$

$\Rightarrow a^2 = 3$ $b^2 = 3$

$$\Rightarrow \frac{x^2}{3} + \frac{y^2}{2} = 1 \qquad \ldots(i)$$

Its focus is (1,0)

Now, eqn of circle is

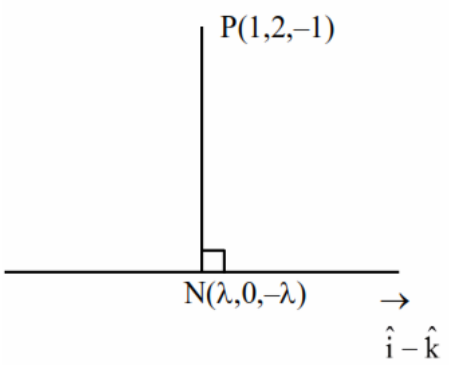
$$(x-1)^2 + y^2 = \frac{4}{3}. \qquad \ldots(ii)$$

Solving (i) and (ii) we get

$$y = \pm\frac{2}{\sqrt{3}}, \; x = 1$$

$$PQ^2 = \left(\frac{4}{\sqrt{3}}\right)^2 = \frac{16}{3}$$

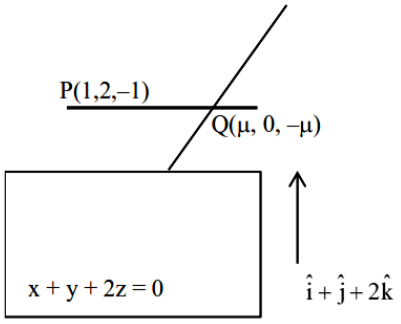
80.[3]



$\overrightarrow{PN}.(\hat{i} - \hat{k}) = 0$

$\Rightarrow$ N (1, 0, –1)

Now,



$\overrightarrow{PQ}.(\hat{i}+\hat{j}+2\hat{k})=0$

$\Rightarrow \mu = -1$

$\Rightarrow Q(-1, 0, 1)$

$\overrightarrow{PN}=2\hat{j}$ and $\overrightarrow{PQ}=2\hat{i}+2\hat{j}-2\hat{k}$

$\Rightarrow \cos\alpha = \frac{1}{\sqrt{3}}$

Section -B

81.[4] Let $e^y = t$

$\Rightarrow \frac{dt}{dx} - (2\sin x)t = -\sin x\cos^2 x$

I.F. $= e^{2\cos x}$

$\Rightarrow t.\, e^{2\cos x} \int e^{2\cos x}.(-\sin x\cos^2 x)dx$

$\Rightarrow e^y.e^{2\cos x} \int e^{2z}.z^2 dz,\ z = e^{2\cos x}$

$\Rightarrow e^y.e^{2\cos x}\frac{1}{2}.\cos^2 x.e^{2\cos x} - \frac{1}{2}\cos x.e^{2\cos x} + \frac{e^{2\cos x}}{4} + C$

at $x = \frac{\pi}{2}, y = 0 \Rightarrow C = \frac{3}{4}$

$\Rightarrow e^y = \frac{1}{2}\cos^2 x - \frac{1}{2}\cos x + \frac{1}{4} + \frac{3}{4}.e^{-2\cos x}$

$\Rightarrow y = \log\left[\frac{\cos^2 x}{2} - \frac{\cos x}{2} + \frac{1}{4} + \frac{3}{4}e^{-2\cos x}\right]$

put $x = 0$

$\Rightarrow y = \log\left[\frac{1}{4} + \frac{3}{4}e^{-2}\right] \Rightarrow \alpha = \frac{1}{4}, \beta = \frac{3}{4}$

82.[3] $\ell = \left(\underbrace{1+\frac{2}{3}+\frac{6}{3^2}+\frac{10}{3^3}+....}_{S}\right)^{\log_{0.25}\left(\frac{1}{3}+\frac{1}{3^2}+....\right)}$

$S = 1 + \frac{2}{3}+\frac{6}{3^2}+\frac{10}{3^3}.....$

$\frac{S}{3} = \frac{1}{3}+\frac{2}{3^2}+\frac{6}{3^3}+...$

$- \quad - \quad - \quad -$

$\frac{2S}{3} = 1 + \frac{1}{3} + \frac{4}{3^2} + \frac{4}{3^3}....$

$\frac{2S}{3} = \frac{4}{3}+\frac{4}{3^2}+\frac{4}{3^3}+....$

$S = \frac{3}{2}\left(\frac{4/3}{1-1/3}\right) = 3$

Now $\ell = (3)^{\log_{0.25}\left(\frac{1/3}{1-1/3}\right)}$

$\ell = 3^{\log_{((1/4))}\left(\frac{1}{2}\right)} = 3^{1/2} = \sqrt{3}$

$\Rightarrow \ell^2 = 3$

83.[164]

$\because$ Sum of frequencies = 584

$\Rightarrow \alpha + \beta = 390$

Now, Median is at $\frac{584}{2} = 292^{th}$

$\because$ Median = 45 (lies in class 40 – 50)

$\Rightarrow \alpha + 110 + 54 + 15 = 292$

$\Rightarrow \alpha = 113, \beta = 277$

$\Rightarrow |\alpha - \beta| = 164$

84.[3] $\vec{p} = 2\hat{i}+3\hat{j}+\hat{k}$ (given)

$\vec{q} = \hat{i}+2\hat{j}+\hat{k}$

Now $(\vec{p}+\hat{q}) \times (\vec{p}-\vec{q}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 5 & 2 \\ 1 & 1 & 0 \end{vmatrix}$

$= -2\hat{i}-2\hat{j}-2\hat{k}$

$\Rightarrow \vec{r} = \pm\sqrt{3}\frac{((\vec{p}+\vec{q})\times(\vec{p}-\vec{q}))}{[(\vec{p}+\vec{q})\times(\vec{p}-\vec{q})]}$

$= \pm\frac{\sqrt{3}(-2\hat{i}-2\hat{j}-2\hat{k})}{\sqrt{2^2+2^2+2^2}}$

$\vec{r} = \pm = \left(-\hat{i}-\hat{j}-\hat{k}\right)$

According to question

$\vec{r} = \alpha\hat{i}+\beta\hat{j}+\gamma\hat{k}$

So $|\alpha| = 1, |\beta| = 1, |\gamma| = 1$

$\Rightarrow |\alpha| + |\beta| + |\gamma| = 3$

85.[1] Coeff. of middle term in $(1 + x)^{20} = {}^{20}C_{10}$ & Sum of Coeff. of two middle terms in $(1 + x)^{19} = {}^{19}C_9 + {}^{19}C_{10}$

So required ratio $= \frac{{}^{20}C_{10}}{{}^{19}C_9 + {}^{19}C_{10}} = \frac{{}^{20}C_{10}}{{}^{20}C_{10}} = 1$

86.[16] $|A| = ad - bc = 15$

Where $a,b,c,d \in \{\pm3, \pm2, \pm1, 0\}$

Case I $ad = 9$ & $bc = -6$

for ad possible pairs are (3,3), (–3, –3)

For bc possible pairs are (3,–2), (–3,2), (–2,3),(2,–3)

So total matrix = $2 \times 4 = 8$

Case II $ad = 6$ & $bc = -9$

Similarly total matrix = $2 \times 4 = 8$

$\Rightarrow$ Total such matrices are = 16

87.[238]

Class	10^{th}	11^{th}	12^{th}	
Totalstudent	5	6	8	
Number of selection	2	3	5	$\Rightarrow {}^5C_2 \times {}^6C_3 \times {}^8C_5$
	2	2	6	$\Rightarrow {}^5C_2 \times {}^6C_2 \times {}^8C_6$
	3	2	5	$\Rightarrow {}^5C_3 \times {}^6C_2 \times {}^8C_5$

$\Rightarrow$ Total number of ways = 23800

According to question

100 K = 23800

$\Rightarrow$ K = 238

88.[1] $x^2 + 5\sqrt{2}x + 10 = 0$

& $P_n = \alpha^n - \beta^n$ (Given)

Now $\dfrac{P_{17}P_{20} + 5\sqrt{2}P_{17}P_{19}}{P_{18}P_{19} + 5\sqrt{2}P_{18}^2} = \dfrac{P_{17}(P_{20} + 5\sqrt{2}P_{19})}{P_{18}(P_{19} + 5\sqrt{2}P_{18})}$

$\dfrac{P_{17}(\alpha^{20} - \beta^{20} + 5\sqrt{2}(\alpha^{19} - \beta^{19})}{P_{18}(\alpha^{19} - \beta^{19} + 5\sqrt{2}(\alpha^{18} - \beta^{18})}$

$\dfrac{P_{17}(\alpha^{19}(\alpha + 5\sqrt{2}) - \beta^{19}(\beta + 5\sqrt{2}))}{P_{18}(\alpha^{18}(\alpha + 5\sqrt{2}) - \beta^{18}(\beta + 5\sqrt{2}))}$

Since $\alpha + 5\sqrt{2} = -10/\alpha$ .....(1)

& $\beta + 5\sqrt{2} = -10/\beta$ ....(2)

Now put there values in above expression

$= \dfrac{-10P_{17}P_{18}}{-10P_{18}P_{17}} = 1$

89.[210] $\left(\left(x^{1/3} + 1\right) - \left(\dfrac{x^{1/2} + 1}{x^{1/2}}\right)\right)^{10}$

$\left(x^{1/3} - \dfrac{1}{x^{1/2}}\right)^{10}$

Now General Term

$T_{r+1} = {}^{10}C_r \left(x^{1/3}\right)^{10-r}\left(-\dfrac{1}{x^{1/2}}\right)^r$

For independent term

$\dfrac{10-r}{3} - \dfrac{r}{2} = 0 \Rightarrow \rho = 4$

$\Rightarrow T_5 = {}^{10}C_4 = 210$

90.[11]

Let $X = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ & $A = \begin{pmatrix} O & i \\ 1 & 0 \end{pmatrix}^n$

$\Rightarrow AX = IX$

$\Rightarrow A = I$

$\begin{pmatrix} 0 & i \\ 1 & 0 \end{pmatrix}^n = I$

$\Rightarrow A^8 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow$ n is multiple of 8

So number of 2 digit numbers in the set

S = 11 (16, 24, 32, ...,96)