JEE MAIN ONLINE PAPER 2021

Held on July, 22, 2021 (Evening)

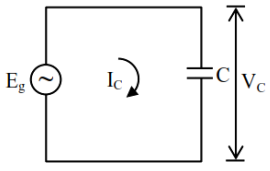
Instructions

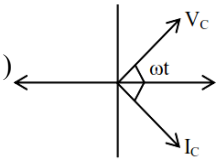
1. This test will be a 3 hours Test.
2. This test consists of Physics, Chemistry and Mathematics questions with equal weightage of 100 marks.
3. Each question is of 4 marks.
4. In the question paper consisting of Physics (Q.no. 1 to 30), Chemistry (Q.no. 31 to 60) and Mathematics (Q.no. 61 to 90). There are two sections for each subject (Section-A : MCQ Type & Section-B : Numerical Response Type). Section-A consists of 20 multiple choice questions & Section-B consists of 10 Numerical Value type Questions. **Candidates have a choice to Answer 5 out of the 10 numerical value answer based questions per section.**
5. There will be only one correct choice in the given four choices in Section-A. For each question 4 marks will be awarded for correct choice, 1 mark will be deducted for incorrect choice and zero mark will be awarded for not attempted question. For Section-B questions 4 marks will be awarded for correct answer and zero for unattempted and incorrect answer.
6. Any textual, printed or written material, mobile phones, calculator etc. is not allowed for the students appearing for the test.
7. All calculations/written work should be done in the rough sheet provided.

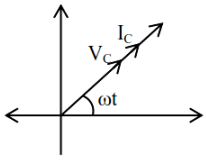
PHYSICS

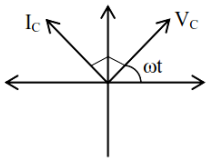
Section -A

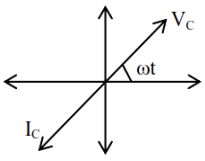
Q.1 In a circuit consisting of a capacitance and a generator with alternating emf $E_g = E_{g0} \sin \omega t$, V_C and I_C are the voltage and current. Correct phasor diagram for such circuit is :



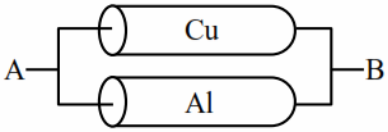
(1) 

(2) 

(3) 

(4) 

Q.2 A Copper (Cu) rod of length 25 cm and crosssectional area 3 mm^2 is joined with a similar Aluminium (Al) rod as shown in figure. Find the resistance of the combination between the ends A and B.
(Take Resistivity of Copper = $1.7 \times 1 \times 10^{-8}$ Ωm
Resistivity of Aluminium = 2.6×10^{-8} Ωm)



(1) 2.170 mΩ (2) 1.420 mΩ
(3) 0.0858 mΩ (4) 0.858 mΩ

Q.3 What will be the projection of vector $\vec{A} = \hat{i} + \hat{j} + \hat{k}$ on vector $\vec{B} = \hat{i} + \hat{j}$?

(1) $\sqrt{2}(\hat{i} + \hat{j} + \hat{k})$ (2) $2(\hat{i} + \hat{j} + \hat{k})$

(3) $\sqrt{2}(\hat{i} + \hat{j})$ (4) $(\hat{i} + \hat{j})$

Q.4 A porter lifts a heavy suitcase of mass 80 kg and at the destination lowers it down by a distance of 80 cm with a constant velocity. Calculate the workdone by the porter in lowering the suitcase.
(take g = 9.8 ms^{-2})

(1) –62720.0 J (2) –627.2 J

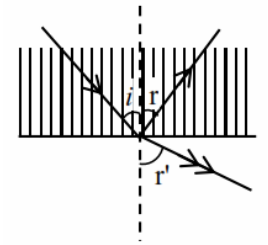
(3) +627.2 J (4) 784.0 J

Q.5 T_0 is the time period of a simple pendulum at a place. If the length of the pendulum is reduced to $\left(\frac{1}{16}\right)$ times of its initial value, the modified time period is :

(1) T_0 (2) $8\pi T_0$

(3) $4T_0$ (4) $\frac{1}{4}T_0$

Q.6 A ray of light passes from a denser medium to a rarer medium at an angle of incidence *i*. The reflected and refracted rays make an angle of 90° with each other. The angle of reflection and refraction are respectively r and r'. The critical angle is given by :



(1) $\sin^{-1}(\cot r)$ (2) $\tan^{-1}(\sin i)$

(3) $\sin^{-1}(\tan r')$ (4) $\sin^{-1}(\tan r)$

Q.7 **Statement I :** The ferromagnetic property depends on temperature. At high temperature, ferromagnet becomes paramagnet.

Statement II : At high temperature, the domain wall area of a ferromagnetic substance increases. In the light of the above statements, choose the **most appropriate** answer from the options given below :

(1) **Statement I** is true but **Statement II** is false

(2) Both **Statement I** and **Statement II** are true

(3) Both **Statement I** and **Statement II** are false

(4) **Statement I** is false but **Statement II** is true

Q.8 A bullet of '4g' mass is fired from a gun of mass 4 kg. If the bullet moves with the muzzle speed of 50 ms^{-1}, the impulse imparted to the gun and velocity of recoil of gun are :

(1) 0.4 kg ms^{-1}, 0.1^{-1}

(2) 0.2 kg ms^{-1}, 0.05 ms^{-1}

(3) 0.2 kg ms^{-1}, 0.1ms^{-1}

(4) 0.4 kg ms^{-1}, 0.05ms^{-1}

Q.9 Choose the correct option :

(1) True dip is not mathematically related to apparent dip.

(2) True dip is less than apparent dip.

(3) True dip is always greater than the apparent dip.

(4) True dip is always equal to apparent dip.

Q.10 Consider a situation in which a ring, a solid cylinder and a solid sphere roll down on the same inclined plane without slipping. Assume that they start rolling from rest and having identical diameter.

The **correct** statement for this situation is:-

(1) The sphere has the greatest and the ring has the least velocity of the centre of mass at the bottom of the inclined plane.

(2) The ring has the greatest and the cylinder has the least velocity of the centre of mass at the bottom of the inclined plane.

(3) All of them will have same velocity.

(4) The cylinder has the greatest and the sphere has the least velocity of the centre of mass at the bottom of the inclined plane.

Q.11 Consider a situation in which reverse biased current of a particular P-N junction increases when it is exposed to a light of wavelength $\leq$ 621 nm. During this process, enhancement in carrier concentration takes place due to generation of hole-electron pairs. The value of band gap is nearly.

(1) 2 eV (2) 4 eV

(3) 1 eV (4) 0.5 eV

Q.12 A nucleus with mass number 184 initially at rest emits an α-particle. If the Q value of the reaction is 5.5 MeV, calculate the kinetic energy of the α- particle.

(1) 5.0 MeV (2) 5.5 MeV
(3) 0.12 MeV (4) 5.38 MeV

Q.13 An electron of mass m_e and a proton of mass m_p are accelerated through the same potentialdifference. The ratio of the de-Broglie wavelengthassociated with the electron to that with the proton is :-

(1) $\frac{m_p}{m_e}$ (2) 1

(3) $\sqrt{\frac{m_p}{m_e}}$ (4) $\frac{m_e}{m_p}$

Q.14 Match **List-I** with **List-II** :

	List-I		**List-II**
(a)	$\omega L > \frac{1}{\omega C}$	(i)	Current is in phase with emf
(b)	$\omega L = \frac{1}{\omega C}$	(ii)	Current lags behind the applied emf
(c)	$\omega L < \frac{1}{\omega C}$	(iii)	Maximum current occurs
(d)	Resonant frequency	(iv)	Current leads the emf

Choose the **correct** answer from the options given below :

(1) (a) – (ii) ; (b) – (i) ; (c) - (iv) ; (d) – (iii)
(2) (a) – (ii) ; (b) – (i) ; (c) - (iii) ; (d) – (iv)
(3) (a) – (iii) ; (b) – (i) ; (c) - (iv) ; (d) – (ii)
(4) (a) – (iv) ; (b) – (iii) ; (c) - (ii) ; (d) – (i)

Q.15 What should be the height of transmitting antenna and the population covered if the television telecast is to cover a radius of 150 km ? The average population density around the tower is 2000/km and the value of Re = 6.5×10^6 m.

(1) Height = 1731 m
Population Covered = 1413×10^5
(2) Height = 1241 m
Population Covered = 7×10^5
(3) Height = 1600 m
Population Covered = 2×10^5
(4) Height = 1800 m
Population Covered = 1413×10^8

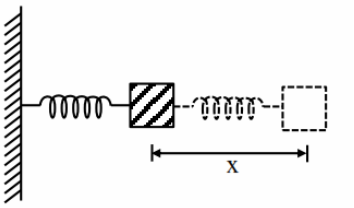
Q.16 What will be the average value of energy for a monoatomic gas in thermal equilibrium at temperature T ?

(1) $\frac{2}{3}k_BT$ (2) k_BT (3) $\frac{3}{2}k_BT$ (4) $\frac{1}{2}k_BT$

Q.17 at Intensity of sunlight is observed as 0.092 Wm^{-2} at a point in free space. What will be the peak value of magnetic field at that point ?
($\varepsilon_0 = 8.85 \times 10^{-12} C^2N^{-1}m^{-2}$)

(1) 2.77×10^{-8} T (2) 1.96×10^{-8}T
(3) 8.31T (4) 5.88T

Q.18 The motion of a mass on a spring, with spring constant K is as shown in figure.



The equation of motion is given by x(t) = $A\sin\omega t + B\cos\omega t$ with $\omega = \sqrt{\frac{K}{m}}$

Suppose that at time t = 0, the position of mass is x(0) and velocity v (0), then its displacement can also be represented as $x(t) = C\cos(\omega t - \phi)$, where C and ϕ are :

(1) $C = \sqrt{\frac{2v(0)^2}{\omega^2} + x(0)^2}$, $\phi = \tan^{-1}\left(\frac{v(0)}{x(0)\omega}\right)$

(2) $C = \sqrt{\frac{2v(0)^2}{\omega^2} + x(0)^2}$, $\phi = \tan^{-1}\left(\frac{x(0)\omega}{2v(0)}\right)$

(3) $C = \sqrt{\frac{v(0)^2}{\omega^2} + x(0)^2}$, $\phi = \tan^{-1}\left(\frac{x(0)\omega}{v(0)}\right)$

(4) $C = \sqrt{\frac{v(0)^2}{\omega^2} + x(0)^2}$, $\phi = \tan^{-1}\left(\frac{x(0)\omega}{v(0)}\right)$

Q.19 An electric dipole is placed on x-axis in proximity to a line charge of linear charge density 3.0×10^{-6} C/m. Line charge is placed on z-axis and positive and negative charge of dipole is at a distance of 10 mm and 12 mm from the origin respectively. If total force of 4 N is exerted on the dipole, find out the amount of positive or negative charge of the dipole.

(1) 815.1 nC (2) 8.8 μC
(3) 0.485 mC (4) 4.44 μC

Q.20 A body is projected vertically upwards from the surface of earth with a velocity sufficient enough to carry it to infinity. The time taken by it to reach height h is _____ S.

(1) $\sqrt{\frac{R_e}{2g}}\left[\left(1+\frac{h}{R_e}\right)^{3/2}-1\right]$

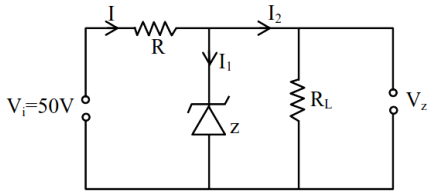
(2) $\sqrt{\frac{R_e}{g}}\left[\left(1+\frac{h}{R_e}\right)^{3/2}-1\right]$

(3) $\frac{1}{3}\sqrt{\frac{R_e}{2g}}\left[\left(1+\frac{h}{R_e}\right)^{3/2}-1\right]$

(4) $\frac{1}{3}\sqrt{\frac{2R_e}{g}}\left[\left(1+\frac{h}{R_e}\right)^{3/2}-1\right]$

Section -B

Q.21 In a given circuit diagram, a 5 V zener diode along V with a series resistance is connected across a 50V power supply. The minimum value of the resistance required, if the maximum zener current is 90 mA will be _____ Ω.



Q.22 The position of the centre of mass of a uniform semi-circular wire of radius 'R' placed in x-y plane with its centre at the origin and the line joining its ends as x-axis is given by $\left(0,\frac{xR}{\pi}\right)$

Then, the value of |x| is _____.

Q.23 In an electric circuit, a call of certain emf provides a potential difference of 1.25 V across a load resistance of 5 Ω. However, it provides a potential difference of 1 V across a load resistance of 2Ω. The emf of the cell is given by $\frac{x}{10}$ V. Then the value of x is ——

Q.24 The total charge enclosed in an incremental volume of 2×10^{-9} m^3 located at the origin is _____ nC, if electric flux density of its field is found as $D = e^{-x}\sin y\,\hat{i} - e^{-x}\cos y\,\hat{j} + 2z\hat{k}$ C/m^2

Q.25 Three particles P, Q and R are moving along the vectors $\vec{A}=\hat{i}+\hat{j}$, $\vec{B}=\hat{j}+\hat{k}$ and $\vec{C}=-\hat{i}+\hat{j}$ respectively. They strike on a point and start to move in different directions. Now particle P is moving normal to the plane which contains vector $\vec{A}$ and $\vec{B}$. Similarly particle Q is moving normal to the plane which contains vector $\vec{A}$ and $\vec{C}$. The angle between the direction of motion of P and Q is $\cos^{-1}\left(\frac{1}{\sqrt{x}}\right)$.

Then the value of x is ___

Q.26 The centre of a wheel rolling on a plane surface moves with a speed *v*. A particle on the rim of the wheel at the same level as the centre will be moving at a speed $\sqrt{x}v_0$. Then the value of x is _____.

Q.27 A ray of light passing through a prism ($\mu=\sqrt{3}$)suffers minimum deviation. It is found that the angle of incidence is double the angle of refraction within the prism. Then, the angle of prism is _____ (in degrees)

Q.28 The area of cross-section of a railway track is 0.01 m^2. The temperature variation is 10°C. Coefficient of linear expansion of material of track is 10^{-5}/°C. The energy stored per meter in the track is ___ J/m.
(Young's modulus of material of track is $10^{11}Nm^{-2}$)

Q.29 Three students S_1, S_2 and S_3 perform an experiment for determining the acceleration due to gravity (g) using a simple pendulum. They use different lengths of pendulum and record time for different number of oscillations. The observations are as shown in the table.

Student No.	Length of pendulum (cm)	No. of oscillations (n)	Total time for n oscillations	Time period (s)
1.	64.0	8	128.0	16.0
2.	64.0	4	64.0	16.0
3.	20.0	4	36.0	9.0

(Least count of length = 0.1 m
least count for time = 0.1 s)
If E_1, E_2 and E are the percentage errors in 'g' for students 1, 2 and 3 respectively, then the minimum percentage error is obtained by student no._____.

Q.30 In 5 minutes, a body cools from 75°C to 65°C at room temperature of 25°C. The temperature of body at the end of next 5 minutes is _____°C.

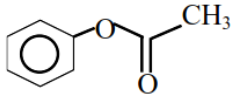
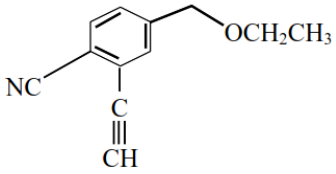
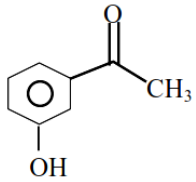
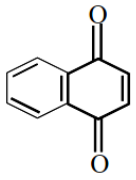
CHEMISTRY

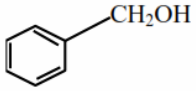
Section -A

Q.31 The water having more dissolved O_2 is :
(1) boiling water (2) water at 80°C
(3) polluted water (4) water at 4°C

Q.32 Which one of the following statements for D.I. Mendeleeff, is **incorrect**?
(1) He authored the textbook – Principles of Chemistry.
(2) At the time, he proposed Periodic Table of elements structure of atom was known.
(3) Element with atomic number 101 is named after him.
(4) He invented accurate barometer.

Q.33 Which purification technique is used for high boiling organic liquid compound decomposes near its boiling point)?
(1) Simple distillation
(2) Steam distillation
(3) Fractional distillation
(4) Reduced pressure distillation

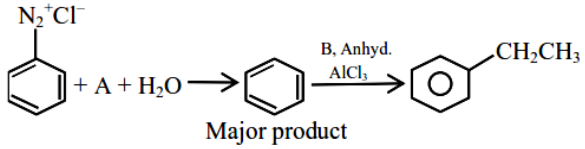
Q.34 Which of the following compounds will provide a tertiary alcohol on reaction with excess of CH3MgBr followed by hydrolysis?
(1) 
(2) 
(3) 
(4) 

Q.35 Which of the following compounds does not exhibit resonance?
(1) $CH_3CH_2OCH = CH_2$
(2) 
(3) $CH_3CH_2CH_2CONH_2$
(4) $CH_3CH_2CH = CH–CHCH_2NH_2$

Q.36 Match **List-I** with **List-II**

	List-I (Elements)		**List-II (Properties)**
(a)	Ba	(i)	Organic solvent soluble compounds
(b)	Ca	(ii)	Outer electronic configuration $6s^2$
(c)	Li	(iii)	Oxalate insoluble in water
(d)	Na	(iv)	Formation of very strong monoacidic base

Choose the **correct** answer from the options given below :
(1) (a)-(ii), (b)-(iii), (c)-(i) and (d)-(iv)
(2) (a)-(iv), (b)-(i), (c)-(ii) and (d)-(iii)
(3) (a)-(iii), (b)-(ii), (c)-(iv) and (d)-(i)
(4) (a)-(i), (b)-(iv), (c)-(ii) and (d)-(iii)

Q.37


In the chemical reactions given above A and B respectively are :
(1) H_3PO_2 and CH_3CH_2Cl
(2) CH_3CH_2OH and H_3PO_2
(3) H_3PO_2 and $CH_3CH_2PO_2$
(4) CH_3CH_2Cl and H_3PO_2

Q.38 Isotope(s) of hydrogen which emits low energy β–particles with $t_{½}$ value > 12 years is / are
(1) Protium
(2) Tritium
(3) Deuterium
(4) Deuterium and Tritium

Q.39 Match **List-I** with **List-II**

	List-I (Species)		**List-II (Hybrid Orbitals)**
(a)	SF_4	(i)	sp^3d^2
(b)	IF_5	(ii)	d^2sp^3
(c)	NO_2^+	(iii) (iv)	sp^3d sp^3
(d)	NO_4^+	(v)	sp

Choose the **correct** answer from the options given below :
(1) (a)-(i), (b)-(ii), (c)-(v) and (d)-(iii)
(2) (a)-(ii), (b)-(i), (c)-(iv) and (d)-(v)
(3) (a)-(iii), (b)-(i), (c)-(v) and (d)-(iv)
(4) (a)-(iv), (b)-(iii), (c)-(ii) and (d)-(v)

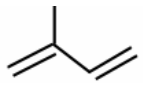
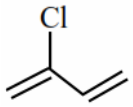
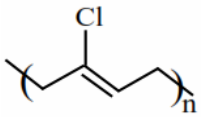
Q.40 When silver nitrate solution is added to potassium iodide solution then the sol produced is :
(1) AgI/I^- (2) AgI/Ag^+
(3) KI/NO_3^- (4) $AgNO_3/NO_3^-$

Q.41 Which of the following molecules does not show stereo isomerism ?
(1) 3,4-Dimethylhex-3-ene
(2) 3-Methylhex-1-ene
(3) 3-Ethylhex-3-ene
(4) 4-Methylhex-1-ene

Q.42 Given below are the statements about diborane
(a)Diborane is prepared by the oxidation of $NaBH_4$ with I_2
(b) Each boron atom is in sp^2 hybridized state
(c) Diborane has one bridged 3 centre-2-electron bond
(d) Diborane is a planar molecule
The option with **correct** statement(s) is –
(1) (c) and (d) only (2) (a) only
(3) (c) only (4) (a) and (b) only

Q.43 Which one of the following group-15 hydride is the strongest reducing agent ?
(1) AsH_3 (2) BiH_3
(3) PH_3 (4) SbH_3

Q.44 Match **List-I** with **List-II :**

	List-I		**List-II**
(a)	Chloroprene	(i)	
(b)	Neoprene	(ii)	
(c)	Acrylonitrile	(iii)	
(d)	Isoprene	(iv)	$CH_2 = CH–CN$

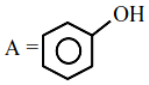
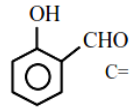
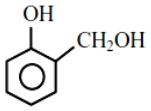
Choose the **correct** answer from the options given below :
(1) (a) - (iii), (b)-(iv), (c) -(ii), (d) -(i)
(2) (a) - (ii), (b)-(iii), (c) -(iv), (d) -(i)
(3) (a) - (ii), (b)-(i), (c) -(iv), (d) -(iii)
(4) (a) - (iii), (b)-(i), (c) -(iv), (d) -(ii)

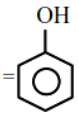
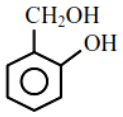
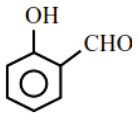
Q.45 The set having ions which are coloured and paramagnetic both is -
(1) Cu^{2+}, Cr^{3+}, Sc^+ (2) Cu^{2+}, Zn^{2+}, Mn^{4+}
(3) Sc^{3+} , V^{5+}, Ti^{4+} (4) Ni^{2+} , Mn^{7+}, Hg^{2+}

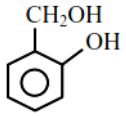
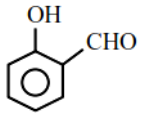
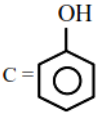
Q.46 Thiamine and pyridoxine are also known respectively as :
(1) Vitamin B_2 and Vitamin E
(2) Vitamin E and Vitamin B_2
(3) Vitamin B_6 and Vitamin B_2
(4) Vitamin B1 and Vitamin B_6

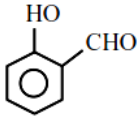
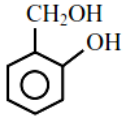
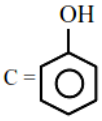
Q.47 Sulphide ion is soft base and its ores are commonfor metals.
(a) Pb (b) Al
(c) Ag (d) Mg
Choose the **correct** answer from the options given below :
(1) (a) and (c) only (2) (a) and (d) only
(3) (a) and (b) only (4) (c) and (d) only

Q.48 An organic compound A ($C6H_6O$) gives dark green colouration with ferric chloride. On treatment with $CHCl_3$ and KOH, followed by acidification gives compound B. Compound B can also be obtained from compound C on reaction with pyridinium chlorochromate (PCC). Identify A, B and C.

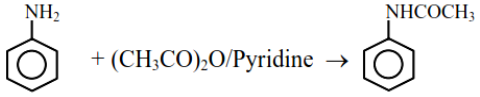
(1) A =  B =  C= 

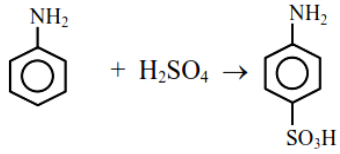
(2) A =  B =  C = 

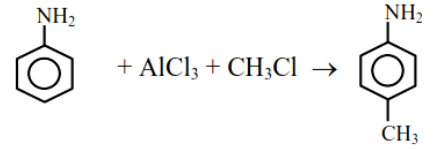
(3) A =  B =  C = 

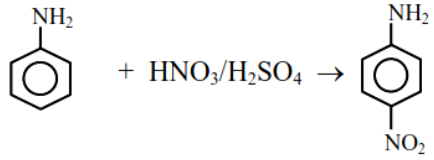
(4) A =  B=  C = 

Q.49 Which one of the following reactions does not occur ?

(1) 

(2) 

(3) 

(4) 

Q.50 Which one of the following 0.06 M aqueous solutions has lowest freezing point ?

(1) $Al_2(SO_4)_3$ (2) $C_6H_{12}O_6$

(3) KI (4) K_2SO_4

Section -B

Q.51 The total number of unpaired electrons present in $[Co(NH_3)_6]Cl_2$ and $[Co(NH_3)_6]Cl_3$ is

Q.52 Methylation of 10 g of benzene gave 9.2 g of toluene. Calculate the percentage yield of toluene_____. (Nearest integer)

Q.53 The number of acyclic structural isomers (including geometrical isomers) for pentene are ____

Q.54 Assume a cell with the following reaction

$Cu_{(s)} + 2Ag^+ (1 \times 10^{-3}\ M) \rightarrow Cu^{2+} (0.250M) + 2Ag_{(s)}$ $E^{\ominus}_{cell} = 2.97V$

Ecell for the above reaction is _____V. (Nearest integer)

[Given : log 2.5 = 0.3979, T = 298 K]

Q.55 Value of K_p for the equilibrium reaction $N_2O_4(g) \rightleftharpoons 2NO_{2(g)}$ at 288 K is 47.9 . The K_C for this reaction at same temperature is ____ (Nearest integer)

($R = 0.083$ L bar K^{-1} mol^{-1})

Q.56 If the standard molar enthalpy change for combustion of graphite powder is -2.48×10^2 kJ mol^{-1}, the amount of heat generated on combustion of 1 g of graphite powder is ——— kJ. (Nearest integer)

Q.57 A copper complex crystallising in a CCP lattice with a cell edge of 0.4518 nm has been revealed by employing X-ray diffraction studies. The density of a copper complex is found to be 7.62 g cm^{-3} The molar mass of copper complex is —— g mol^{-1}

(Nearest integer)

[Given : $N_A = 6.022 \times 10^{23} mol^{-1}$]

Q.58 Number of electrons that Vanadium (Z = 23) has in p-orbitals is equal to ——

Q.59 $N_2O_{5(g)} \rightarrow 2NO_{2(g)} + \frac{1}{2} O_{2(g)}$

In the above first order reaction the initial concentration of N_2O_5 is 2.40×10^{-2} mol L^{-1} at 318 K . The concentration of N_2O_5 after 1 hour was 1.60×10^{-2} mol L^{-1} . The rate constant of the reaction at 318 K is ____ $\times 10^{-3}$ min^{-1} . (Nearest integer)

[Given : log 3 = 0.477, log 5 = 0.699]

Q.60 If the concentration of glucose ($C_6H_{12}O_6$) in blood is 0.72 g L^{-1} , the molarity of glucose in blood is ——— $\times 10^{-3}$M. (Nearest integer)

[Given : Atomic mass of C = 12, H = 1, O = 16 u]

MATHEMATICS

Section -A

Q.61 Let L be the line of intersection of planes $\vec{r}\cdot(\hat{i}-\hat{j}+2\hat{k})=2$ and $\vec{r}\cdot(2\hat{i}+\hat{j}-\hat{k})=2$. If $P(\alpha, \beta, \gamma)$ is the foot of perpendicular on L from the point (1, 2, 0), then the value of $35(\alpha+\beta+\gamma)$ is equal to :
(1) 101 (2) 119 (3) 143 (4) 134

Q.62 Let S_n denote the sum of first n-terms of an arithmetic progression. If $S_{10}=530$, $S_5=140$, then $S_{20}-S_6$ is equal to :
(1) 1862 (2) 1842 (3) 1852 (4) 1872

Q.63 let $f : \mathbf{R} \to \mathbf{R}$ be defined as

$$f(x)=\begin{cases}-\frac{4}{3}x^3+2x^2+3x, & x>0\\ 3xe^x, & x\le 0\end{cases}$$

Then f is increasing function in the interval
(1) $\left(-\frac{1}{2},2\right)$ (2) (0, 2)
(3) $\left(-1,\frac{3}{2}\right)$ (4) (–3, –1)

Q.64 Let y = y(x) be the solution of the differential equation $\text{cosec}^2 x dy + 2dx = (1 + y\cos 2x)\text{cosec}^2 x dx$, with $y\left(\frac{\pi}{4}\right)=0$ Then, the value of $(y(0)+1)^2$ is equal to :
(1) $e^{1/2}$ (2) $e^{-1/2}$ (3) e^{-1} (4) e

Q.65 Four dice are thrown simultaneously and the numbers shown on these dice are recorded in 2 × 2 matrices. The probability that such formed matrices have all different entries and are non-singular, is :
(1) $\frac{45}{162}$ (2) $\frac{23}{81}$ (3) $\frac{22}{81}$ (4) $\frac{43}{162}$

Q.66 Let a vector $\vec{a}$ be coplanar with vectors $\vec{b}=2\hat{i}+\hat{j}+\hat{k}$ and $\vec{c}=\hat{i}-\hat{j}+\hat{k}$. If $\vec{a}$ is perpendicular to $\vec{d}=3\hat{i}+2\hat{j}+6\hat{k}$, and $|\vec{a}|=\sqrt{10}$. Then a possible value of $[\vec{a}\ \vec{b}\ \vec{c}]+[\vec{a}\ \vec{b}\ \vec{d}]+[\vec{a}\ \vec{c}\ \vec{d}]$ is equal to -
(1) –42 (2) –40 (3) –29 (4) –38

Q.67 If $\int_0^{100\pi}\frac{\sin^2 x}{e^{\left(\frac{x}{\pi}-\left[\frac{x}{\pi}\right]\right)}}dx=\frac{\alpha\pi^3}{1+4\pi^2}$, $\alpha \in \mathbf{R}$ where [x] is the greatest integer less than or equal to x, then the value of α is -
(1) $200(1-e^{-1})$ (2) $100(1-e)$
(3) $50(e-1)$ (4) $150(e^{-1}-1)$

Q.68 Let three vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ be such that $\vec{a}\times\vec{b}=\vec{c}, \vec{b}\times\vec{c}=\vec{a}$ and $|\vec{a}|=2$. Then which one of the following is **not** true ?
(1) $\vec{a}\times((\vec{b}+\vec{c})\times(\vec{b}-\vec{c}))=\vec{0}$
(2) Projection of $\vec{a}$ on $(\vec{b}+\vec{c})$ is 2
(3) $[\vec{a}\ \vec{b}\ \vec{c}]+[\vec{c}\ \vec{a}\ \vec{b}]=8$
(4) $|\vec{3a}+\vec{b}-2\vec{c}|=51$

Q.69 The values of λ and μ such that the system of equations x + y + z = 6, 3x + 5y + 5z = 26, x + 2y + λz = μ has no solution, are :
(1) $\lambda=3$, $\mu=5$ (2) $\lambda=3$, $\mu\neq 10$
(3) $\lambda\neq 2$, $\mu=10$ (4) $\lambda=2$, $\mu\neq 10$

Q.70 If the shortest distance between the straight lines $3(x-1)=6(y-2)=2(z-1)$ and $4(x-2)=2(y-\lambda)=(z-3)$, $\lambda \in \mathbf{R}$ is $\frac{1}{\sqrt{38}}$, then the integral value of λ is equal to :
(1) 3 (2) 2 (3) 5 (4) –1

Q.71 Which of the following Boolean expressions is **not** a tautology ?
(1) $(p \Rightarrow q) \vee (\sim q \Rightarrow p)$
(2) $(q \Rightarrow p) \vee (\sim q \Rightarrow p)$
(3) $(p \Rightarrow \sim q) \vee (\sim q \Rightarrow p)$
(4) $(\sim p \Rightarrow q) \vee (\sim q \Rightarrow p)$

Q.72 Let $A = [a_{ij}]$ be a real matrix of order 3 × 3, such that $a_{i1} + a_{i2} + a_{i3} = 1$, for i = 1, 2, 3. Then, the sum of all the entries of the matrix A^3 is equal to :
(1) 2 (2) 1
(3) 3 (4) 9

Q.73 Let [x] denote the greatest integer less than or equal to x. Then, the values of $x \in \mathbf{R}$ satisfying the equation $[e^x]^2 + [e^x + 1] - 3 = 0$ lie in the interval :

(1) $\left[0, \frac{1}{e}\right)$ (2) $[\log_e 2, \log_e 3)$

(3) $[1, e)$ (4) $[0, \log_e 2]$

Q.74 Let the circle
$S : 36x^2 + 36y^2 - 108x + 120y + C = 0$ be such that it neither intersects nor touches the co-ordinate axes. If the point of intersection of the lines, $x - 2y = 4$ and $2x - y = 5$ lies inside the circle S, then :

(1) $\frac{25}{9} < C < \frac{13}{3}$ (2) $100 < C < 165$

(3) $81 < C < 156$ (4) $100 < C < 156$

Q.75 Let n denote the number of solutions of the equation $z^2 + 3\bar{z} = 0$, where z is a complex number. Then the value of $\sum_{k=0}^{\infty} \frac{1}{n^k}$ is equal to

(1) 1 (2) $\frac{4}{3}$ (3) $\frac{3}{2}$ (4) 2

Q.76 The number of solutions of $\sin^7 x + \cos^7 x = 1$, $x \in [0, 4\pi]$ is equal to

(1) 11 (2) 7

(3) 5 (4) 9

Q.77 If the domain of the function $f(x) = \frac{\cos^{-1}\sqrt{x^2 - x + 1}}{\sqrt{\sin^{-1}\left(\frac{2x-1}{2}\right)}}$ is the interval $(\alpha, \beta]$, then $\alpha + \beta$ is equal to -

(1) $\frac{3}{2}$ (2) 2 (3) $\frac{1}{2}$ (4) 1

Q.78 Let $f : \mathbf{R} \to \mathbf{R}$ be defined as

$$f(x) = \begin{cases} \frac{x^3}{(1-\cos 2x)^2} \log_e\left(\frac{1+2xe^{-2x}}{(1-xe^{-x})^2}\right) & , x \neq 0 \\ \alpha & , x = 0 \end{cases}$$

If f is continuous at x = 0, then α is equal to :

(1) 1 (2) 3 (3) 0 (4) 2

Q.79 Let a line $L : 2x + y = k$, $k > 0$ be a tangent to the hyperbola $x^2 - y^2 = 3$. If L is also a tangent to the parabola $y^2 = \alpha x$, then α is equal to :

(1) 12 (2) –12

(3) 24 (4) –24

Q.80 Let $E_1 : \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, $a > b$. Let E_2 be another ellipse such that it touches the end points of major axis of E_1 and the foci of E_2 are the end points of minor axis of E_1. If E_1 and E_2 have same eccentricities, then its value is -

(1) $\frac{-1+\sqrt{5}}{2}$ (2) $\frac{-1+\sqrt{8}}{2}$

(3) $\frac{-1+\sqrt{3}}{2}$ (4) $\frac{-1+\sqrt{6}}{2}$

Section -B

Q.81 Let A = {0, 1, 2, 3, 4, 5, 6, 7}. Then the number of bijective functions $f : A \to A$ such that $f(1) + f(2) = 3 - f(3)$ is equal to

Q.82 If the digits are not allowed to repeat in any number formed by using the digits 0, 2, 4, 6, 8, then the number of all numbers greater than 10,000 is equal to _____.

Q.83 Let $A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$. Then the number of 3×3 matrices B with entries from the set {1, 2, 3, 4, 5} and satisfying AB = BA is _____ .

Q.84 Consider the following frequency distribution

Class :	0-6	6-12	12-18	18-24	24-30
Frequency :	a	b	12	9	5

If mean = $\frac{309}{22}$ and median = 14, then the value $(a - b)^2$ is equal to _____ .

Q.85 The sum of all the elements in the set {n ∈ {1, 2, ….., 100} | H.C.F. of n and 2040 is 1} is equal to _____.

Q.86 The area (in sq. units) of the region bounded by the curves $x^2 + 2y - 1 = 0$, $y^2 + 4x - 4 = 0$ and $y^2 - 4x - 4 = 0$, in the upper half plane is _____.

Q.87 Let $f : \mathbf{R} \to \mathbf{R}$ be a function defined as

$$f(x)\begin{cases} 3\left(1-\frac{|x|}{2}\right) & \text{if } |x| \le 2 \\ 0 & \text{if } |x| > 2 \end{cases}$$

Let $g : \mathbf{R} \to \mathbf{R}$ be given by
$g(x) = f(x + 2) - f(x - 2)$
If n and m denote the number of point in R where g is not continuous and not differentiable respectively, then n + m is equal to ____ .

Q.88 If the constant term, in binomial expansion of $\left(2x^r + \frac{1}{x^2}\right)^{10}$ is 180, then r is equal to _____ .

Q.89 Let y = y(x) be the solution of the differential equation

$$\left((x+2)e^{\left(\frac{y+1}{x+2}\right)} + (y+1)\right)dx = (x+2)dy,\ y(1) =$$

1. If the domain of y = y(x) is an open interval (α, β), then $|\alpha + \beta|$ is equal to _____ .

Q.90 The number of elements in the set $\{n \in \{1, 2, 3, \ldots\ldots, 100\} \mid (11)^n > (10)^n + (9)^n\}$ is _____ .

JEE MAIN ONLINE PAPER 2021

Held on JULY 22, 2021 (Evening)

Hints & Solutions

PHYSICS

Section -A

1.[3] In capacitor, current lead voltage by $\frac{\pi}{2}$

2.[4] $R = \frac{R_1R_2}{R_1+R_2} = \frac{\ell}{A}.\frac{\rho_1\rho_2}{\rho_1+\rho_2}$

$R = \frac{25\times10^{-2}}{3\times10^{-6}} \times \frac{1.7\times2.6\times10^{-16}}{4.3\times10^{-8}}$

$R = 0.858m\Omega$

3.[4] $(A\cos\theta)\ \vec{B} = A\left(\frac{\vec{A}.\vec{B}}{AB}\right)\hat{B} = \frac{\vec{A}.\vec{B}}{B}.\hat{B}$

$= \frac{2}{\sqrt{2}}\left(\frac{\hat{i}+\hat{j}}{\sqrt{2}}\right) = \hat{i}+\hat{j}$

4.[2] $W_{Porter} + W_{mg} = \Delta K.E. = 0$

$W_{Porter} = -W = -mgh$

$= -80 \times 9.8 \times 0.8 = -627.2$ J

5.[4] $T_0 = 2\pi\sqrt{\frac{\ell}{g}}$

New time period $T = 2\pi\sqrt{\frac{\ell/16}{g}} = \frac{2\pi}{4}\sqrt{\frac{\ell}{g}}$

$T = \frac{T_0}{4}$

6.[4] $r + r' + 90° = 180° \Rightarrow r' = 90 - r = 90 - i$

$n_1 \sin i = n_2 \sin r' = n_2 \sin(90-i)$

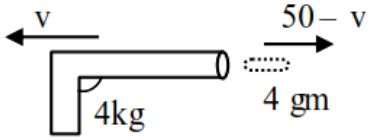
$n_1 \sin i = n_2 \cos i \Rightarrow \tan i = \frac{n_2}{n_1}$

Now $\sin C = \frac{n_2}{n_1} = \tan i$

$\Rightarrow C = \sin^{-1}(\tan i) = \sin^{-1}(\tan r)$

7.[1] As temperature increases, domains disintegrate so ferromagnetism decreases and above curie temperature it become paramagnet.

8.[2]



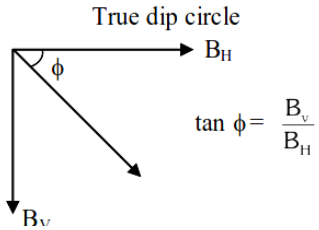
By momentum conservation

$4 \times 10^{-3}(50 - v) - 4v = 0$

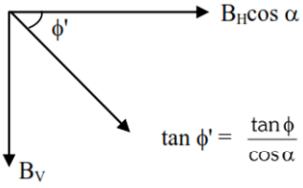
$v = \frac{4\times10^{-3}\times50}{4+4\times10^{-3}} = \approx 0.05 ms^{-1}$

Impulse $J = mv = 4 \times .05 = 0.2$ kgms^{-1}

9.[2] If apparent dip circle is at an angle α with true dip circle then



Apparent dip circle



As $\cos\alpha < 1$

Hence true dip (ϕ) is less than apparent dip (ϕ')

10.[1] $a = \frac{g\sin\theta}{1+\frac{I}{mR^2}}$

$I_{ring} > I_{solid\ cylinder} > I_{solid\ sphere}$

$\Rightarrow a_{ring} < a_{solid\ cylinder} < a_{solid\ sphere}$

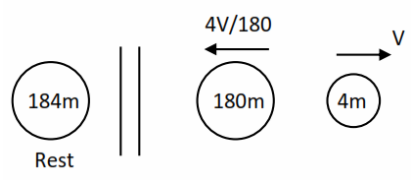
$\Rightarrow v_{ring} < v_{solid\ cylinder} < v_{solid\ sphere}$

11.[1] Band gap $= \frac{hc}{\lambda_0}$

λ_0 ; threshold wavelength

Band gap $= \frac{1242ev-nm}{621nm} = 2eV$

12.[4]



$$\frac{1}{2}(4m)v^2+\frac{1}{2}(180m)\left(\frac{4}{180}\right)^2 = 5.5MeV$$

$$\Rightarrow \frac{1}{2}4mv^2\left[1+45\left(\frac{4}{180}\right)^2\right] = 5.5MeV$$

$$\Rightarrow K.E\alpha = \frac{5.5}{1+45.\left(\frac{4}{180}\right)^2}MeV$$

$KE_\alpha = 5.38MeV$

13.[3] $KE = e\Delta V$

$$\lambda_e = \frac{h}{\sqrt{2m_e(e\Delta V)}}$$

$$\lambda_p = \frac{h}{\sqrt{2m_p(e\Delta V)}}$$

$$\Rightarrow \frac{\lambda_e}{\lambda_p} = \sqrt{\frac{m_p}{m_e}}$$

14.[1] For $x_L > x_C$, voltage leads the current (ii)
(b) For $x_L = x_C$, voltage & current are in same phase (i)
(c) For $x_L < x_C$, current leads the voltage (iv)
(d) For resonant frequency $x_L = x_C$, current is maximum (iii)

15.[1] Radius covered $r = \sqrt{2RH_T}$

$150km = \sqrt{2\times(6.5\times10^6 m)H_T}$

$(150\ km\times 10^3)^2 = 2\times 6.5\times 10^6\ H_T$

$H_T = 1731m$

Population covered $= (\pi r^2)(2000/km^2)$

$= 3.14\times\ (150)^2\times 2000 = 1413\times 10^5$

16.[3] As per Equi-partition law :
Each degree of freedom contributes

$\frac{1}{2}k_BT$ Average Energy

In monoatomic gas D.O.F. = 3

$$\Rightarrow \text{Average energy} = 3\times\frac{1}{2}k_BT = \frac{3}{2}k_BT$$

17.[1] $I_{avg} = \frac{B_0^2C}{2\mu_0}$ & $\frac{1}{\mu_0} = \epsilon_0 C^2$

$$I = \frac{B_0^2C}{2}\epsilon_0 C^3$$

$$B_0 = \sqrt{\frac{2I}{\epsilon_0 C^3}}$$

$B_0 = 2.77\times 10^{-8}T$

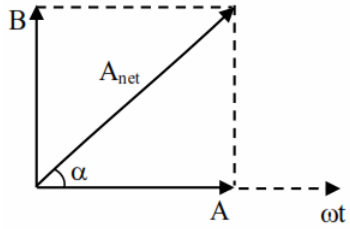
18.[4] $x = A\sin\omega t + B\cos\omega t$

$v = \frac{dx}{dt} = A\omega\cos\omega t - B\omega\sin\omega t$

At $t = 0$, $x(0) = B$

$v(0) = A\omega$

$x = A\sin\omega t + B\sin(\omega t + 90°)$



$A_{net} = \sqrt{A^2+B^2}$

$\tan\alpha = \frac{B}{A} \Rightarrow \cot\alpha = \frac{A}{B}$

$\Rightarrow x = \sqrt{A^2+B^2}\sin(\omega t+\alpha)$

$\Rightarrow x = \sqrt{A^2+B^2}\cos(\omega t-(90-\alpha))$

$x = C\cos(\omega t-\phi)$

$\Rightarrow C = \sqrt{A^2+B^2}$

$$C = \sqrt{\frac{[v(0)]^2}{\omega^2}+[x(0)]^2}$$

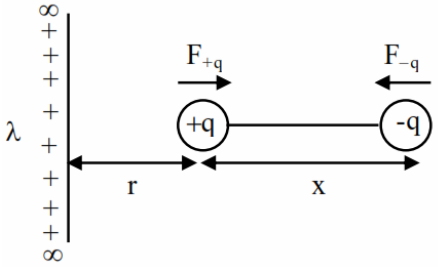
$\phi = 90-\alpha$

$\tan\alpha = \cos\alpha\ \frac{A}{B}$

$$\Rightarrow \tan\alpha = \frac{v(0)}{x(0).\omega}$$

$$\phi = \tan^{-1}\left(\frac{v(0)}{x(0).\omega}\right)$$

19.[4]



r = 10 mm, x = 2,,

$$\left|\vec{F}_q\right| = \frac{2k\lambda}{r} \cdot q$$

$$\left|\vec{F}_{-q}\right| = \frac{2k\lambda}{r+x}.q$$

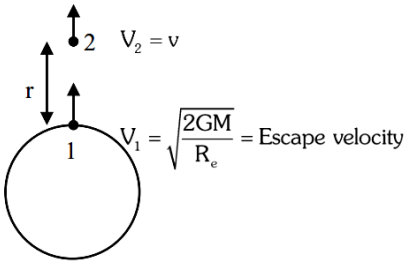
$$\Rightarrow \left|\vec{F}_{net}\right| = \frac{2k\lambda}{r} - \frac{2k\lambda q}{r+x}$$

$$\left|\vec{F}_{net}\right| = \frac{2k\lambda q.x}{r(r+x)}$$

$$4 = \frac{2\times 9\times 10^9 \times 3\times 10^{-6}\times q\times 2mm}{10mm.12mm}$$

$$\Rightarrow q = 4.44\mu C$$

20.[4]



Applying energy conservation from (1) to (2)

$$\frac{1}{2}m.\left(\frac{2GM}{R_e}\right) - \frac{2GMm}{R_e} = \frac{1}{2}mv^2 - \frac{GMm}{R+r}$$

$$\Rightarrow \frac{1}{2}mv^2 = \frac{2GMm}{R+r}$$

$$\Rightarrow v = \sqrt{\frac{2GM}{R+r}} = \frac{dr}{dt}$$

$$\sqrt{2GM}\int_0^t dt = \int_{R_e}^{R_e+h}\left(\sqrt{R+r}\right)dr$$

$$\sqrt{2GM}\,.\,t = \frac{2}{3}\left[(R+r)^{3/2}\right]_{R_e}^{Re+h}$$

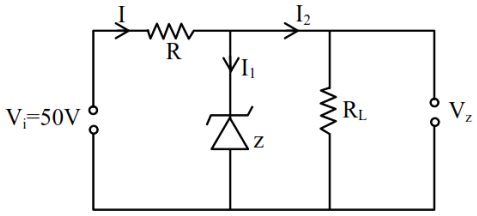
$$t = \frac{2}{3}\sqrt{\frac{R_e^3}{2GM}}\left[\left(1+\frac{h}{R_e}\right)^{3/2} - 1\right]$$

$$\frac{GM}{R_e^2} = g$$

$$t = \frac{1}{3}\sqrt{\frac{2R_e}{g}}\left[\left(1+\frac{h}{R_e}\right)^{3/2} - 1\right]$$

Section -B

21.[500]



Voltage across R_L = 5V

$$\Rightarrow i_2 = \frac{5}{R_L}$$

Also voltage across R = 50 – 5 = 45 volt

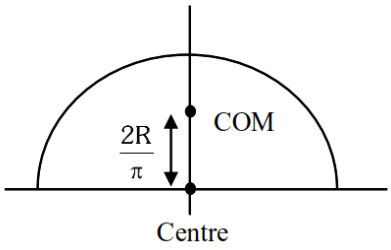
$$\text{By } v = iR \Rightarrow P = \frac{v}{i} = \frac{45}{i_1 + i_2}$$

$$R = \frac{45}{90mA + \frac{5}{R_L}}$$

Current in zener diode is maximum when $R_L \to \infty$ ($i_2 \to 0$) and $i_1 = i$)

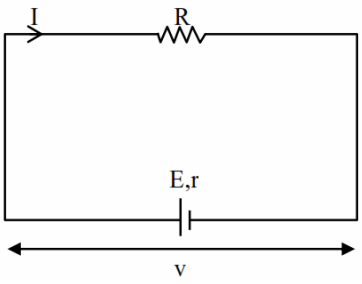
$$\text{So } R = \frac{45}{90mA} = 500\Omega$$

22.[2] COM of semi-circular ring is at $\frac{2R}{\pi}$



Distance from centre $\Rightarrow$ x = 2

23.[15]



$$\text{Terminal voltage } v = iR = \frac{ER}{R+r}$$

$$1^{st} \to 1.25 = \frac{E(5)}{5+r} \ldots.(i)$$

$2^{nd} \rightarrow 1 = \frac{E(2)}{2+r}$ ....(ii)

By (i) and (ii)

$r = 1\Omega$, $E = \frac{3}{2} V = \frac{15}{10}$ volt

$\Rightarrow x = 15$

24.[4] Electric flux density

$(\vec{D}) = \frac{\text{charge}}{\text{Area}} \times \hat{r} = \frac{Q}{4\pi r^2}\hat{r} = \epsilon_0 \left(\frac{Q}{4\pi\epsilon_0 r^2}\hat{r}\right)$

$\vec{E} = \frac{\vec{D}}{\epsilon_0} = \frac{e^{-x}\sin y\hat{i} - e^{-x}\cos y\hat{j} + 2z\hat{k}}{\epsilon_0}$

Also by Gauss's law

$\frac{\rho}{\epsilon_0} = \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right).\vec{E}$

$= \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right)\frac{\vec{D}}{\epsilon_0}$

$\Rightarrow \rho = \frac{\partial}{\partial x}(e^{-x}\sin y) + \frac{\partial}{\partial y}(-e^{-x}\cos y) + \frac{\partial}{\partial z}(2z)$

$\rho = -e^{-x}\sin y + e^{-x}\sin y + 2$

At origin $\rho = -e^{-0}\sin 0 + e^{-0}\sin 0 + 2$

$\rho = 2C/m^3$

$\text{Charge} = \rho \times \text{volume} = 2 \times 2 \times 10^{-9} = 4 \times 10^{-9} = 4\,nC$

25.[3] Direction of P, $\hat{V}_1 = \pm \frac{\vec{A}\times\vec{B}}{|\vec{A}\times\vec{B}|} = \pm \frac{\vec{i} - \vec{j} + \vec{k}}{\sqrt{3}}$

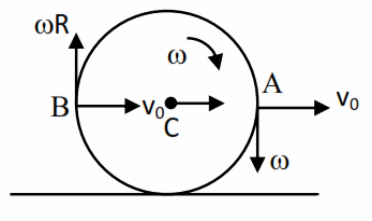
Direction of Q, $\hat{V}_2 = \pm \frac{\vec{A}\times\vec{C}}{|\vec{A}\times\vec{C}|} = \pm \frac{2\hat{k}}{2} = \pm \hat{k}$

Angle between $\hat{V}_1$ and $\hat{V}_2$

$\frac{\hat{V}_1.\hat{V}_1}{|\hat{V}_1||\hat{V}_2|} = \frac{\pm 1/\sqrt{3}}{(1)(1)} = \pm \frac{1}{\sqrt{3}}$

$\Rightarrow x = 3$

26.[2]



For no slipping $V_0 = \omega R$

Now $V_A = V_B = \sqrt{V_0^2 + (\omega R)^2}$

$= \sqrt{2}\, V_0$

$\Rightarrow x = 2$

27.[60] At minimum deviation $r_1 = r_2 = \frac{A}{2}$

Also given $i = 2r_1 = A$

Now $1.\sin i = \sqrt{3}\sin r_1$

$1 \sin A = \sqrt{3}\sin\frac{A}{2}$

$\Rightarrow 2\sin\frac{A}{2}\cos\frac{A}{2} = \sqrt{3}\sin\frac{A}{2}$

$\Rightarrow \cos\frac{A}{2} = \frac{\sqrt{3}}{2} \Rightarrow \frac{A}{2} = 30^\circ$

$\Rightarrow A = 60^\circ$

28.[5] Elastic energy $= \frac{Y}{2}(\text{strain})^2 \times \text{Area}$

$(\text{strain} = \frac{\Delta\ell}{\ell} = \alpha\,\Delta T = 10^{-5} \times 10 = 10^{-4})$

$= \frac{10^{11}}{2} \times (10^{-4})^2 \times 10^{-2} = 5 J/m$

29.[1] $T = 2\pi\sqrt{\frac{\ell}{g}} \Rightarrow g\ \frac{4\pi^2\ell}{T^2}$

$\frac{\Delta g}{g} = \frac{\Delta\ell}{\ell} + \frac{2\Delta T}{T}$

$\Delta T = \frac{\text{least count of time } (\Delta T_0)}{\text{number of oscillations}(n)}$

$\frac{\Delta g}{g} = \frac{\Delta\ell}{\ell} + \frac{2\Delta T}{T}$

As $\Delta\ell$ and ΔT_0 are same for all observations so $\frac{\Delta g}{g}$ is minimum for highest value of ℓ, n ant T

$\Rightarrow$ Minimum percentage error in g is for student number-1

30.[57] By newton's law of cooling (with approximation)

$\frac{\Delta T}{\Delta t} = -C\,(T_{avg} - T_s)$

$1^{st}\ \frac{-10^\circ C}{5\,min} = -C\,(70^\circ C - 25^\circ C)$

$\Rightarrow C = \frac{2}{45}\ min^{-1}$

$2^{nd}\ \frac{T-65}{5\,min} = -C\left(\frac{T+65}{2} - 25\right) = -\left(\frac{2}{45}\right)\left(\frac{T+15}{2}\right)$

$\Rightarrow 9(T-65) = -(T+15)$

$\Rightarrow 10T = 570$

$\Rightarrow T = 57^\circ C$

Alternate solution :

Newton's law of cooling (without approximation)

$T_P - T_S = (T_i - T_S)\, e^{-ct}$

1^{st} $\quad 65 - 25 = (75 - 25)\, e^{-5C} \Rightarrow e^{-5C} = \frac{4}{5}$

2^{nd} $\quad T - 25 = (65 - 25)e^{-5C} = 40 \times \frac{4}{5} = 32$

$T = 57^{\circ}C$

CHEMISTRY

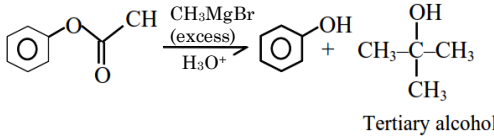
Section -A

31.[4] On heating concentration of O_2 in water decreases. So boiling water and water at 80°C having less O_2 concentration. Polluted water also having less O_2 concentration. So water at 4°C having maximum O_2 concentration.

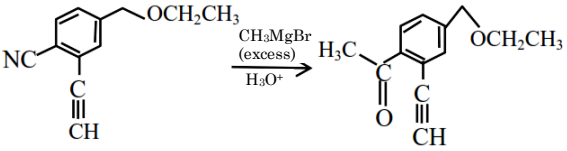
32.[2] At the time, he proposed the periodic table but structure of atom was unknown.

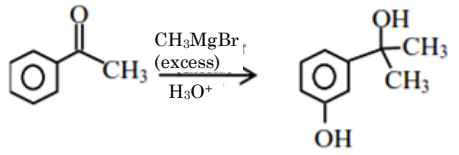
33.[4] Reduced pressure distillation or vacuum distillation is used for the purification of high boiling organic liquids which decomposes at or below their boiling point.

34.[1]

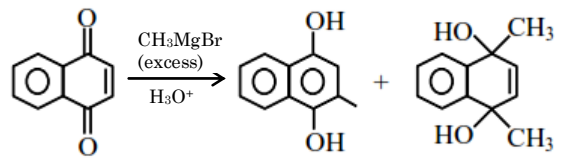


Tertiary alcohol





Phenolic –OH group and tertiary alcohol is present thus two functional groups are present in the product



Two 3° alcohol

Since the given question is single correct choice the best appropriate option is (A)

35.[4] $CH_3 - CH_2 - CH = CH - CH_2 - NH_2$

No conjugation thus resonance is not possible.

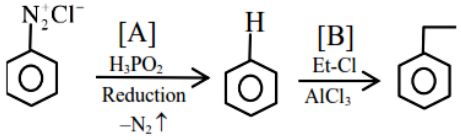
36.[1] (a) 'Ba' having outer electronic configuration $6s^2$

(b) CaC_2O_4 is water insoluble

(c) 'Li' is soluble in organic solvents

(d) NaOH is strong Monoacidic base among given.

37.[1]



38.[2] ${}_1^1H$ and ${}_1^2H$ are stable while ${}_1^3H$ is radioactive.

39.[3] (a) SF_4 – SP^3d hybridisation

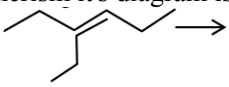
(b) IF_5 – sp^3d^2 hybridisation

(c) NO_2^+ – sp hybridisation

(d) NO_4^+ – sp^3 hybridisation

40.[1] $\underset{\text{(drop by drop)}}{AgNO_3(aq.)} + \underset{\text{excess}}{KI(aq.)} \longrightarrow \underset{\text{Sol}}{AgI / I^-}$

41.[3] 3-Ethylhex-3-ene will not show stereo isomerism it's diagram is.



(1) Not show geometrical isomerism

(2) Not show optical isomerism

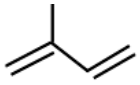
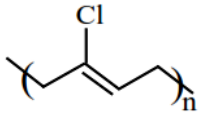
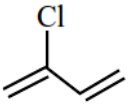
42.[2] Diborane is prepared by the reaction of $NaBH_4$ with I_2

$2NaBH_4 + I_2 \rightarrow B_2H_6 + 2NaI + H_2$

In diborane, 'B' is sp^3 hybrid, it is Non-planar and two 3c–2e bonds are present.

43.[2] Among 15^{th} group hydrides, BiH_3 is strongest reducing agent.

44.[2]

	List-I		List-II
(a)	Chloroprene	(i)	
(b)	Neoprene	(ii)	
(c)	Acrylonitrile	(iii)	$CH_2 = CH - CN$
(d)	Isoprene	(iv)	

45.[1]

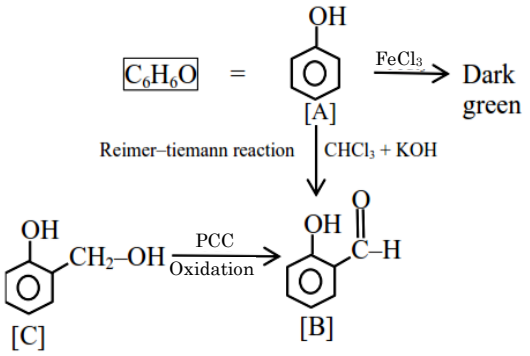
Cu^{2+} : $[Ar]3d^9 4s^0$
Cr^{3+} : $[Ar]3d^3 4s^0$
Sc^{+} : $[Ar]3d^1 4s^1$

All are coloured and paramagnetic due to presence of unpaired electrons

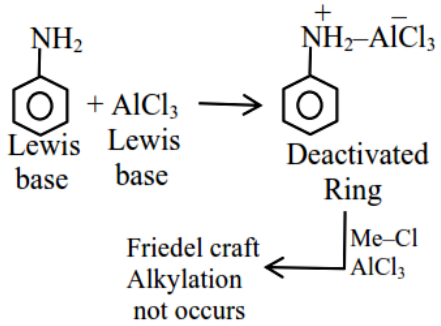
46.[4] Vitamine-B_1 is also known as Thiamine while vitamin B-6 is known as Pyridoxine

47.[1] Pb and Ag commonly exist in the form of sulphide ore like PbS (galena) and Ag_2S (Argentite) 'Al' is mainly found in the form of oxide ore whereas 'Mg' is found in the form of halide ore.

48.[1]



49.[3]



(1) Aniline is lewis base give acid base reaction with $AlCl_3$ and form Anilinium ion

(2) Anilinium ion has strongest deactivated ring so further Friedel craft Alkylation not occurs.

50.[1] $T_f - T_f = i\,K_f.m$

For minimum T_f

'i' should be maximum.

$Al_2(SO_4)_3$ i = 5
$C_6H_{12}O_6$ i = 1
KI i = 2
K_2SO_4 i = 3

Section -B

51.[3] **Ans by NTA is [1]**

$[Co(NH_3)_6]Cl_2$

Co^{2+} ; $[Ar]\ 3d^7 4s^0 4p^0$

For this complex Δ_0 < P.E., so pairing of electrons does not take place.

sp^3d^2 hybridisation

Total 3 unpaired electrons are present.

$[Co(NH_3)_6]Cl_3$

Co^{3+}: $[Ar]3d^6\ 4s^0 4p^0$

d^2sp^3 hybridisation

NH_3 acts as SFL because Δ_0 > P.E.

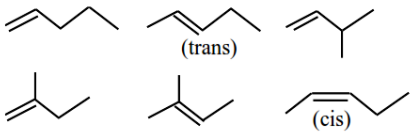
So here all electrons becomes paired.

52.[78] $C_6H_6 + CH_3Cl \rightarrow C_6H_5CH_3 + HCl$

$\frac{10}{78}$ $\left(\frac{10}{78}\times 92\right)$ gm $\Rightarrow$

$\frac{A_y}{T_y}$ = % yield = $\frac{9.2}{920} \times 78 \times 100 \Rightarrow 78\%$

53.[6]



54.[3] $E = E^o - \frac{0.059}{2}\log\frac{[Cu^{+2}]}{[Ag^+]^2}$

$= 2.97 - \frac{0.059}{2}\log\frac{0.25}{(10^{-3})^2} = 2.81V$

55.[2] $K_C = \frac{K_p}{RT} = \frac{47.9}{0.083\times 288} = 2$

56.[21] 1 mol graphite = 12 gm C

Ans. = $\frac{248}{12}$ = 20.67kJ / gm heat evolved

57.[106] $d\left(\frac{gm}{cc}\right) = \frac{4\times\frac{M}{N_A}}{(a\ cm)^3}$

$$7.62 = \frac{4\times M/6.022\times 10^{23}}{(0.1518\times 10^{-7}\,cm)^3}$$

$\Rightarrow$ M = 105.8 g/mol

58.[12] $_{23}V$: $1s^2\ 2s^2\ 2p^6 3s^2 3p^6 3d^3\ 4s^2$

Number of electrons in p-orbitals is equal to 12.00

59.[7] $K=\frac{2.303}{t}\log\frac{[N_2O_5]_0}{[N_2O_5]_t}$

$=\frac{2.303}{60}\log\frac{2.4}{1.6}=6.76\times10^{-3}\ min^{-1}\approx7\times10^{-3}\ min^{-1}$

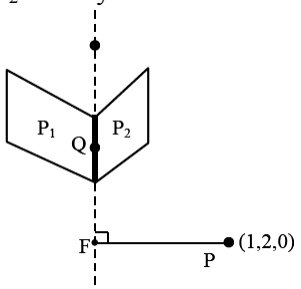
60.[4] $[\text{Glu cose}]=\frac{C(gm/\ell)}{M(gm/mol)}=\frac{0.72}{180}=4\times10^{-3}M$

MATHEMATICS

Section -A

61.[2] $P_1: x-y+2z=2$

$P_2: 2x-y-3=2$



Let line of Intersection of planes P_1 and P_2 cuts xy plane in point Q.

$\Rightarrow$ z-coordinate of point Q is zero

$\Rightarrow$ and $\left.\begin{matrix}x-y=2\\2x+y=2\end{matrix}\right\}\Rightarrow x=\frac{4}{3},y=\frac{-2}{3}$

$\Rightarrow Q\left(\frac{4}{3},\frac{-2}{3},0\right)$

Vector parallel to the line of intersection

$$\vec{a}=\begin{vmatrix}i&j&k\\1&-1&2\\2&1&-1\end{vmatrix}=-\hat{i}+5\hat{j}+3\hat{k}$$

Equation of Line of intersection

$$\frac{x-\frac{4}{3}}{-1}=\frac{y+\frac{2}{3}}{5}=\frac{z-0}{3}=\lambda\ \text{(say)}$$

Let coordinates of foot of perpendicular be

$$F\left(-\lambda+\frac{4}{3},5\lambda-\frac{2}{3},3\lambda\right)$$

$$\overrightarrow{PF}=\left(-\lambda+\frac{1}{3}\right)\hat{i}+\left(5\lambda-\frac{8}{3}\right)\hat{j}+(3\lambda)\hat{k}$$

$\overrightarrow{PF}.\vec{a}=0$

$$\Rightarrow\lambda-\frac{1}{3}+25\lambda-\frac{40}{3}+9\lambda=0$$

$$\Rightarrow35\lambda=\frac{41}{3}\Rightarrow\lambda=\frac{41}{105}$$

Now, $\alpha=-\lambda+\frac{4}{3},\ \beta=5\lambda-\frac{2}{3},\ \gamma=3\lambda$

$$\Rightarrow\alpha+\beta+\gamma=7\lambda+\frac{2}{3}$$

$$=7\left(\frac{41}{105}\right)+\frac{2}{3}$$

$$=\frac{51}{15}$$

$$\Rightarrow35(\alpha+\beta+\gamma)=\frac{51}{15}\times35=119$$

62.[1] $S_{10}=530\Rightarrow\frac{10}{2}\{2a+9d\}=530$

$\Rightarrow 2a+9d=106$ ....(1)

and $S_5=140\ \Rightarrow\frac{5}{2}\{2a+4d\}=140$

$\Rightarrow 2a+4d=56$ ..... (2)

$\Rightarrow 5d=50\Rightarrow \boxed{d=10}\Rightarrow \boxed{a=8}$

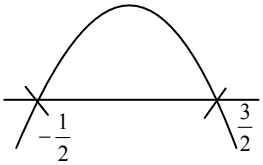
Now, $S_{20}-S_6=\frac{20}{2}\{2a+19d\}-\frac{6}{2}\{2a+5d\}$

$= 14a + 145d$

$= (14 \times 8) + (175 \times 10)$

$= 1862$

63.[3] $f'(x)\begin{cases}-4x^2+4x+3 & x>0\\ 3e^x(1+x) & x\le0\end{cases}$



For $x > 0$, $f'(x)=-4x^2+4x+3$

f(x) is increasing in $\left(-\frac{1}{2},\frac{3}{2}\right)$

For $x \le 0$, $f'(x) = 33e^x(1+x)$

$f'(x) > 0\ \forall\ x\in(-1, 0)$

$\Rightarrow$ f(x) is increasing in (–1, 0)

So, in complete domain, f(x) is increasing in $\left(-1,\frac{3}{2}\right)$

64.[3] $\frac{dy}{dx}+2\sin^2x=1+y\cos2x$

$$\Rightarrow\frac{dy}{dx}+(-\cos2x)y=\cos2x$$

I.F. $=e^{\int-\cos2xdx}=e^{\frac{\sin2x}{2}}$

Solution of D.E.

$$y\left(e^{-\frac{\sin 2x}{2}}\right)=\int(\cos 2x)\left(e^{-\frac{\sin 2x}{2}}\right)dx+c$$

$$\Rightarrow y\left(e^{-\frac{\sin 2x}{2}}\right)=-e^{-\frac{\sin 2x}{2}}+c$$

Given $y\left(\frac{\pi}{4}\right)=0$

$$\Rightarrow 0=-e^{-\frac{1}{2}}+c \Rightarrow c=e^{-\frac{1}{2}}$$

$$\Rightarrow y\left(e^{-\frac{\sin 2x}{2}}\right)=-e^{-\frac{\sin 2x}{2}}+e^{-\frac{1}{2}}$$

at x = 0

$$y=-1+e^{-\frac{1}{2}}$$

$$\Rightarrow y(0)=-1+e^{-\frac{1}{2}} \Rightarrow (y(0)+1)^2=e^{-1}$$

65.[4] $A=\begin{vmatrix}a & b\\ c & d\end{vmatrix}$ $|A| = ad - bc$

Total case = 6^4

For non-singular matrix $|A| \neq 0 \Rightarrow ad - bc \neq 0$

$\Rightarrow ad \neq bc$

And a, b, c, d are all different numbers in the set {1, 2, 3, 4, 5, 6}

Now for ad = bc

(i) $6 \times 1 = 2 \times 3$

$\Rightarrow$ $\left.\begin{matrix}a=6,\ b=2,\ c=3,\ d=1\\ \text{or } a=1,\ b=2,\ c=3,\ d=6\\ \vdots\end{matrix}\right\}$ 8 such cases

(ii) $6 \times 2 = 3 \times 4$

$\Rightarrow$ $\left.\begin{matrix}a=6,\ b=3,\ c=4,\ d=2\\ \text{or } a=2,\ b=3,\ c=4,\ d=6\\ \vdots\end{matrix}\right\}$ 8 such cases

Favourable cases

$= {}^6C_4 \lfloor 4 - 16$

required probability

$$=\frac{{}^6C_4\lfloor 4-16}{6^4}=\frac{43}{162}$$

66.[1] $\vec{a}=\lambda\vec{b}+\mu\vec{c}=\hat{i}(2\lambda+\mu)+\hat{j}(\lambda-\mu)+\hat{k}(\lambda+\mu)$

$\vec{a}\cdot\vec{d}=0=3(2\lambda+\mu)+2(\lambda-\mu)+6(\lambda+\mu)$

$\Rightarrow 14\lambda + 7\mu = 0 \Rightarrow \mu = -2\lambda$

$\Rightarrow \vec{a}=(0)\hat{i}-3\lambda\hat{j}+(-\lambda)\hat{k}$

$\Rightarrow |\vec{a}|=\sqrt{10}\,|\lambda|=\sqrt{10} \Rightarrow |\lambda|=1$

$\Rightarrow \lambda = 1$ or -1

$[\vec{a}\ \vec{b}\ \vec{c}]=0$

$[\vec{a}\ \vec{b}\ \vec{c}]+[\vec{a}\ \vec{b}\ \vec{d}]+[\vec{a}\ \vec{c}\ \vec{d}]=[\vec{a}\ \vec{b}+\vec{c}\ \vec{d}]$

$$=\begin{vmatrix}0 & -3\lambda & \lambda\\ 3 & 0 & 2\\ 3 & 2 & 6\end{vmatrix}$$

$= 3\lambda(12) + \lambda(6) = 42\lambda = -42$

67.[1] $$\int_0^{100\pi}\frac{\sin^2 x}{e^{\left\{\frac{x}{\pi}\right\}}}dx=100\int_0^{\pi}\frac{\sin^2 x}{e^{\frac{x}{\pi}}}dx$$

$$\int_0^{100\pi} e^{\frac{-x}{\pi}}\frac{(1-\cos 2x)}{2}dx$$

$$=50\left\{\int_0^{\pi} e^{\frac{-x}{\pi}}dx-\int_0^{\pi} e^{\frac{-x}{\pi}}\cos 2x dx\right\}$$

$$I_1=\int_0^{\pi} e^{\frac{-x}{\pi}}dx=[-\pi e^{\frac{-x}{\pi}}]_0^{\pi}=\pi(1-e^{-1})$$

$$I_2=\int_0^{\pi} e^{\frac{-x}{\pi}}\cos 2x dx$$

$$=-\pi e^{\frac{-x}{\pi}}\cos 2x]_0^{\pi}-\int-\pi e^{\frac{-x}{\pi}}(-2\sin 2x)dx$$

$$=\pi(1-e^{-1})-2\pi\int_0^{\pi} e^{\frac{-x}{\pi}}\sin 2x dx$$

$$=\pi(1-e^{-1})-2\pi\left\{-\pi e^{\frac{-x}{n}}\sin 2x]_0^{\pi}-\int_0^{\pi}-\pi e^{\frac{-x}{\pi}}2\cos 2x dx\right\}$$

$$\pi(1-e^{-1})-4\pi^2 I_2$$

$$\Rightarrow I_2=\frac{\pi(1-e^{-1})}{1+4\pi^2}$$

$$\therefore\ I=50\left\{\pi(1-e^{-1})-\frac{\pi(1-e^{-1})}{1+4\pi^2}\right\}$$

$$=\frac{200(1-e^{-1})\pi^3}{1+4\pi^2}$$

68.[4] (1) $\vec{a}\times((\vec{b}+\vec{c})\times(\vec{b}-\vec{c}))$

$= \vec{a}(-\vec{b}\times\vec{c}+\vec{c}\times\vec{b})=2(\vec{a}\times(\vec{b}\times\vec{c}))$

$= -2(\vec{a}\times\vec{a}) = \vec{0}$

(2) Projection of $\vec{a}$ on $\vec{b}\times\vec{c}$

$= \dfrac{\vec{a}\cdot(\vec{b}\times\vec{c})}{|\vec{b}\times\vec{c}|} = \dfrac{\vec{a}\cdot\vec{a}}{|\vec{a}|} = |\vec{a}| = 2$

(3) $[\vec{a}\,\vec{b}\,\vec{c}]+[\vec{c}\,\vec{a}\,\vec{b}] = 2[\vec{a}\,\vec{b}\,\vec{c}] = 2\vec{a}\cdot(\vec{b}\times\vec{c})$

$= 2\vec{a}\cdot\vec{a} = 2|\vec{a}|^2 = 8$

(4) $\vec{a}\times\vec{b} = \vec{c}$ and $\vec{b}\times\vec{c} = \vec{a}$

$\Rightarrow \vec{a}, \vec{b}, \vec{c}$ are mutually $\perp$ vectors

$\therefore |\vec{a}\times\vec{b}| = |\vec{c}| \Rightarrow |\vec{a}||\vec{b}| = |\vec{c}| \Rightarrow |\vec{b}| = \dfrac{|\vec{C}|}{2}$

Also, $|\vec{b}\times\vec{c}| = |\vec{a}| \Rightarrow |\vec{b}||\vec{c}| = 2 \Rightarrow |\vec{c}| = 2$ &

$|\vec{b}| = 1$

$|3\vec{a}+\vec{b}-2\vec{c}|^2 = (3\vec{a}+\vec{b}-2\vec{c})\cdot(3\vec{a}+\vec{b}-2\vec{c})$

$= 9|\vec{a}|^2 + |\vec{b}|^2 + 4|\vec{c}|^2$

$= (9\times4) + 1 + (4\times4)$

$= 36 + 1 + 16 = 53$

69.[4] $x + y + z = 6$ ... (i)

$3x + 5y + 5z = 26$ ... (ii)

$x + 2y + \lambda z = \mu$ ...(iii)

$5\times$ (i) – (ii) $\Rightarrow 2x = 4 \Rightarrow x = 2$

$\therefore$ from (i) and (iii)

$y + z = 4$ ...(iv)

$2y + \lambda z = \mu - 2$ ...(v)

(v) – 2 × (iv)

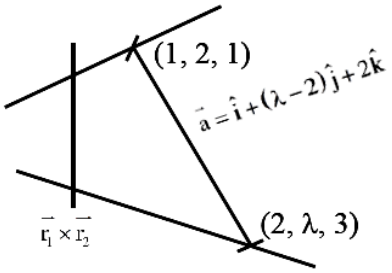
$\Rightarrow (\lambda - 2)z = \mu - 10$

$\Rightarrow z = \dfrac{\mu-10}{\lambda-2}$ & $y = 4 - \dfrac{\mu-10}{\lambda-2}$

$\therefore$ For no solution $\lambda = 2$ and $m \neq 10$

70.[1] $L_1 : \dfrac{(x-1)}{2} = \dfrac{(y-2)}{1} = \dfrac{(z-1)}{3}$ $\vec{r}_1 = 2\hat{i}+\hat{j}+3\hat{k}$

$L_2 : \dfrac{(x-2)}{1} = \dfrac{(y-\lambda)}{2} = \dfrac{(z-3)}{4}$ $\vec{r}_2 = \hat{i}+2\hat{j}+4\hat{k}$



Shortest distance = Projection of $\vec{a}$ on $\vec{r}_1\times\vec{r}_2 = \dfrac{|\vec{a}.(\vec{r}_1\times\vec{r}_2)|}{|\vec{r}_1\times\vec{r}_2|}$

$|\vec{a}.(\vec{r}_1\times\vec{r}_2)| = \begin{vmatrix} 1 & \lambda-2 & 2 \\ 2 & 1 & 3 \\ 1 & 2 & 4 \end{vmatrix} = |14-5\lambda|$

$|\vec{r}_1\times\vec{r}_2| = \sqrt{38}$

$\therefore \dfrac{1}{\sqrt{38}} = \dfrac{|14-5\lambda|}{\sqrt{38}}$

$\Rightarrow |14-5\lambda| = 1$

$\Rightarrow 14 - 5\lambda = 1$ or $14 - 5\lambda = -1$

$\Rightarrow \lambda = \dfrac{13}{5}$ or 3

$\therefore$ Integral value of $\lambda = 3$

71.[4] (1) $(p \to q) \vee (\sim q \to p)$

$= (\sim p \to q) \vee (q \vee p)$

$= (\sim p \vee p) \vee q$

$= t \vee q = t$

(2) $(q \to p) \vee (\sim q \to p)$

$= (\sim q \vee p) \vee (q \vee p)$

$= (\sim q \vee q) \vee p$

$= t \vee p = t$

(3) $(p \to \sim q) \vee (\sim q \to p)$

$= (\sim p \vee \sim q) \vee (q \vee p)$

$= (\sim p \vee p) \vee (\sim q \vee q)$

$= t \vee t = t$

(4) $(\sim q \to q) \vee (\sim q \to p)$

$= (p \vee q) \vee (q \vee p)$

$= (p \vee p) \vee (\theta \vee p)$

$= p \vee q$

Which is not a tautology

72.[3] $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$

Let $x = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

$AX = \begin{bmatrix} a_{11}+a_{12}+a_{13} \\ a_{21}+a_{22}+a_{23} \\ a_{31}+a_{32}+a_{33} \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

$\Rightarrow AX = X$

Replace X by AX

$A^2X = AX = X$

Replace X by AX

$A^3X = AX = X$

Let $A^3 = \begin{bmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ z_1 & z_2 & z_3 \end{bmatrix}$

$$A^3\begin{bmatrix}1\\1\\1\end{bmatrix} = \begin{bmatrix} x_1+x_2+x_3 \\ y_1+y_2+y_3 \\ z_1+z_2+z_3 \end{bmatrix} = \begin{bmatrix}1\\1\\1\end{bmatrix}$$

Sum of all the element = 3

73.[4] $\left[e^x\right]^2 + [e^x + 1] - 3 = 0$

$\Rightarrow [e^x]^2 + [e^x] + 1 - 3 = 0$

Let $[e^x] = t$

$\Rightarrow t^2 + t - 2 = 0$

$\Rightarrow t = -2, 1$

$[e^x] = -2$ (Not possible)

or $[e^x] = 1 \quad \therefore 1 \le e^x < 2$

$\Rightarrow \ln(1) \le x < \ln(2)$

$\Rightarrow 0 \le x < \ln(2)$

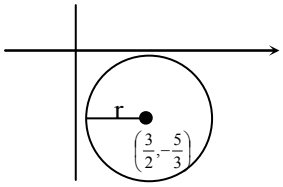
$\Rightarrow x \in [0, \ln 2)$

74.[4] $S : 36x^2 + 36y^2 - 108x + 120y + C = 0$

$$x^2 + y^2 - 3x + \frac{10}{3}y + \frac{C}{36} = 0$$

$$\text{Centre} \equiv (-g,-f) \equiv \left(\frac{3}{2}, \frac{-10}{6}\right)$$

$$\text{radius} = r = \sqrt{\frac{9}{4} + \frac{100}{36} - \frac{C}{36}}$$



Now,

$$\Rightarrow r < \frac{3}{2}$$

$$\Rightarrow \frac{9}{4} + \frac{100}{36} - \frac{C}{36} < \frac{9}{4}$$

$\Rightarrow C > 100$ ... (i)

Now point of intersection of $x - 2y = 4$ and $2x - y = 5$ is $(2, -1)$, which lies inside the circle S.

$\therefore S(2, -1) < 0$

$$\Rightarrow (2)^2 + (-1)^2 - 3(2) + \frac{10}{3}(-1) + \frac{C}{36} < 0$$

$$\Rightarrow 4 + 1 - 6 - \frac{10}{3} + \frac{C}{36} < 0$$

$\boxed{C < 156}$ ....(ii)

From (1) & (2)

$\boxed{100 < C < 156}$ Ans

75.[2] $z^2 + 3\bar{z} = 0$

Put $z = x + iy$

$\Rightarrow x^2 - y^2 + 2ixy + 3(x - iy) = 0$

$\Rightarrow (x^2 - y^2 + 3x) + i(2xy - 3y) = 0 + i0$

$\therefore x^2 - y^2 + 3x = 0$ ..... (i)

$2xy - 3y = 0$ .... (ii)

$$x = \frac{3}{2},\ y = 0$$

Put $x = \frac{3}{2}$ in equation (1)

$$\frac{9}{4} - y^2 + \frac{9}{2} = 0$$

$$y^2 = \frac{27}{4} \Rightarrow y = \pm\frac{3\sqrt{3}}{2}$$

$$\therefore (x, y) = \left(\frac{3}{2}, \frac{3\sqrt{3}}{2}\right), \left(\frac{3}{2}, \frac{-3\sqrt{3}}{2}\right)$$

Put $y = 0 \Rightarrow x^2 - 0 + 3x = 0$

$x = 0, -3$

$\therefore (x, y) = (0, 0), (-3, 0)$

$\therefore$ No of solutions = n = 4

$$\sum_{K=0}^{\infty}\left(\frac{1}{n^k}\right) = \sum_{K=0}^{\infty}\left(\frac{1}{4^k}\right)$$

$$= \frac{1}{1} + \frac{1}{4} + \frac{1}{16} + \frac{1}{64} + ...$$

$$= \frac{1}{1 - \frac{1}{4}} = \frac{4}{3}$$

76.[3] $\sin^7 x \le \sin^2 x \le 1$ ...(1)

and $\cos^7 x \le \cos^2 x \le 1$ ...(2)

also $\sin^2 x + \cos^2 x = 1$

$\Rightarrow$ equality must hold for (1) & (2)

$\Rightarrow \sin^7 x = \sin^2 x$ & $\cos^7 x = \cos^2 x$

$\Rightarrow \sin x = 0$ & $\cos x = 1$

or

$\cos x = 0$ & $\sin x = 1$

$$\Rightarrow x = 0, 2\pi, 4\pi, \frac{\pi}{2}, \frac{5\pi}{2}$$

$\Rightarrow 5$ solutions

77.[1] $0 \le x^2 - x + 1 \le 1$

$\Rightarrow x^2 - x \le 0$

$\Rightarrow x \in [0, 1]$

Also, $0 < \sin^{-1}\left(\frac{2x-1}{2}\right) \le \frac{\pi}{2}$

$\Rightarrow 0 < \frac{2x-1}{2} \le 1$

$\Rightarrow 0 < 2x - 1 \le 2$

$1 < 2x \le 3$

$\frac{1}{2} < x \le \frac{3}{2}$

Taking intersection

$x \in \left(\frac{1}{2}, 1\right]$

$\Rightarrow \alpha = \frac{1}{2}, \beta = 1$

$\Rightarrow \alpha + \beta = \frac{3}{2}$

78.[1] For continuity

$\lim\limits_{x\to 0} \frac{x^3}{4\sin^4 x}(\ell n(1+2xe)^{-2x}) - 2\ell n(1 - xe^{-x}) = \alpha$

$\lim\limits_{x\to 0} \frac{1}{4x}[2xe^{-2x} + 2xe - x] = \alpha$

$= \frac{1}{4}(4) = \alpha = 1$

79.[4] Tangent to hyperbola of

Slope m = –2(given)

$y = -2x \pm \sqrt{3(3)}$

$(y = mx \pm \sqrt{a^2m^2 - b^2})$

$\Rightarrow y + 2x = \pm 3 \Rightarrow 2x + y = 3 (k > 0)$

For parabola $y^2 = \alpha x$

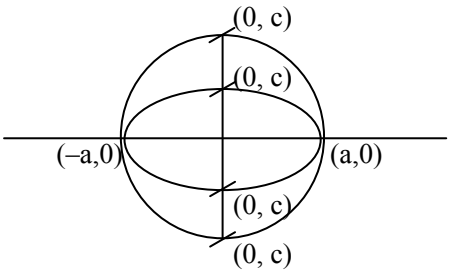
$y = mx + \frac{\alpha}{4m}$

$\Rightarrow y = -2x + \frac{\alpha}{-8}$

$\Rightarrow \frac{\alpha}{-8} = 3$

$\Rightarrow \alpha = -24$

80.[1]



$e^2 = 1 - \frac{b^2}{a^2}$

$e^2 = 1 - \frac{a^2}{c^2}$

$\Rightarrow \frac{b^2}{a^2} = \frac{a^2}{c^2}$

$\Rightarrow c^2 = \frac{a^4}{b^2} \Rightarrow c = \frac{a^2}{b}$

Also b = ce

$\Rightarrow c = \frac{b}{e}$

$\frac{b}{e} = \frac{a^2}{b}$

$\Rightarrow e = \frac{b^2}{a^2} = 1 - e^2$

$\Rightarrow e^2 + e - 1 = 0$

$\Rightarrow e = \frac{-1 + \sqrt{5}}{2}$

Section -B

81.[720] f(1) + f(2) = 3 – f(3)

$\Rightarrow$ f(1) + f(2) = 3 + f(3) = 3

The only possibility is : 0 + 1 + 2 = 3

$\Rightarrow$ Elements 1, 2, 3 in the domain can be mapped with 0, 1, 2 only.

So number of bijective functions.

$= \lfloor 3 \times \lfloor 5 = 720$

82.[96]

2, 4, 6, 8				

$= 4 \times 4 \times 3 \times 3 \times 2 = 96$

83.[3125]

Let matrix $B = \begin{bmatrix} a & b & c \\ d & e & f \\ g & n & i \end{bmatrix}$

$\because$ AB = BA

$$\begin{bmatrix}0&1&0\\1&0&0\\0&0&1\end{bmatrix}\begin{bmatrix}a&b&c\\d&e&F\\g&h&i\end{bmatrix}=\begin{bmatrix}a&b&c\\d&e&f\\g&h&i\end{bmatrix}\begin{bmatrix}0&1&0\\1&0&0\\0&0&1\end{bmatrix}$$

$$\begin{bmatrix}d&e&f\\a&b&c\\g&h&i\end{bmatrix}=\begin{bmatrix}b&a&c\\e&d&f\\h&g&i\end{bmatrix}$$

$\Rightarrow d = b, e = a, f = c, g = h$

$\therefore$ Matrix $B = \begin{bmatrix}a&b&c\\b&a&c\\g&g&i\end{bmatrix}$

No. of ways of selecting a, b, c, g, i

$= 5 \times 5 \times 5 \times 5 \times 5$

$= 5^5 = 3125$

$\therefore$ No. of matrices B = 3125

84.[4]

Class	Frequency	xi	fi xi
0-6	a	3	3a
6-12	b	9	9b
12-18	12	15	180
18-24	9	21	189
24-30	5	27	135
	N=(26+a+b)		(504 +3a+9b)

$$\text{Mean} = \frac{3a+9b+180+189+135}{a+b+26} = \frac{309}{22}$$

$\Rightarrow 66a + 198b + 11088 = 309a + 308b + 8034$

$\Rightarrow 243a + 111b = 3054$

$\Rightarrow \boxed{81a + 37b = 1018} \rightarrow (1)$

$$\text{Now, Median} = 12 + \frac{\frac{a+b+26}{2} - (a+b)}{12} \times 6 = 14$$

$$\Rightarrow \frac{13}{2} - \left(\frac{a+b}{4}\right) = 2$$

$$\Rightarrow \frac{a+b}{4} = \frac{9}{2}$$

$\Rightarrow a + b = 18 \rightarrow (2)$

From equation (1) & (2)

$a = 8, b = 10$

$\therefore (a - b)^2 = (8 - 10)^2$

85.[1251]

$2040 = 2^3 \times 3 \times 5 \times 17$

n should not be multiple of 2, 3, 5 and 17

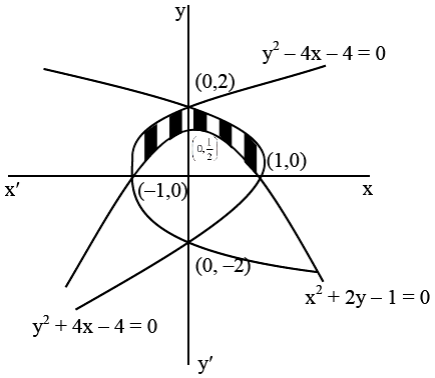
Sum of all n = (1 + 3 + 5 ..... + 99) – (3 + 9 + 15 + 21 + ... + 99) – (5 + 25 + 35 + 55 + 65 + 85 + 95) – (17)

$$= 2500 - \frac{17}{2}(3+99) - 365 - 17$$

$= 2500 – 867 – 365 – 17$

$= 1251$

86.[2]



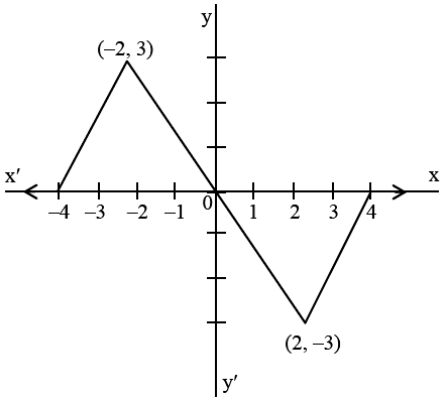
Required Area (shaded)

$$= 2\left[\int_0^2\left(\frac{4-y^2}{4}\right)dy - \int_0^1\left(\frac{1-x^2}{2}\right)dx\right]$$

$$= 2\left[\frac{4}{3} - \frac{1}{3}\right] = (2)$$

87.[4] $f(x-2)\begin{cases}\frac{3x}{2} & -4 \le x \le -2\\ -\frac{3x}{2} & -2 < x \le 0\\ 0 & x \in (-\infty,-4) \cup (0,+\infty)\end{cases}$

$$g(x) = f(x+2) - f(x-2)\begin{cases}\frac{3x}{2}+6 & -4 \le x \le -2\\ -\frac{3x}{2} & -2 < x < 2\\ \frac{3x}{2}-6 & 2 \le x \le 4\\ 0 & x \in (-\infty,-4) \cup (4,\infty)\end{cases}$$



n = 0

m = 4 $\Rightarrow$ (n + m = 4)

88.[8] $\left(2x^r + \frac{1}{x^2}\right)^{10}$

General term = ${}^{10}C_R(2x^2)^{10-R}X^{-2R}$

$\Rightarrow 2^{10-R}\,{}^{10}C_R = 180$ ..... (1)

& (10–R) r – 2R = 0

$r = \frac{2R}{10-R}$

$r = \frac{2(R-10)}{10-R} + \frac{20}{10-R}$

$\Rightarrow r = -2 + \frac{20}{10-R}$ ..... (2)

R = 8 or 5 reject equation (1) not satisfied

At R = 8

$2^{10-R}\,{}^{10}C_R = 180 \Rightarrow \boxed{r = 8}$

89.[4] y + 1 = Y $\Rightarrow$ dy = dY

x + 2 = X $\Rightarrow$ dx = dX

$\Rightarrow \left(Xe^{\frac{Y}{x}} + Y\right)dX = XdY$

$\Rightarrow XdY - YdX = Xe^{\frac{Y}{x}}dX$

$\Rightarrow d\left(\frac{Y}{X}\right)e^{-\frac{Y}{X}} = \frac{dX}{X}$

$-e^{-\frac{Y}{X}} = \ell|X| + c$

$(3, 2) \Rightarrow -e^{\frac{-2}{3}} = \ell|3| + c$

$-e^{-\frac{Y}{X}} = \ell n|X| - e^{-\frac{2}{3}} - \ell n3$

$e^{-\frac{Y}{X}} = e^{2/3} + \ell n3 - \ell n|X| > 0$

$\ell n|X| < (e^{2/3} + \ell n3)$

Let $\lambda = (e^{2/3} + \ell n3)$

$|x + 2| < e^{\lambda}$

$-e^{\lambda} < x + 2 < e^{\lambda}$

$-e^{\lambda} - 2 < x < e^{\lambda} - 2$

$\alpha \qquad \beta$

$\alpha + \beta = -4 \Rightarrow |\alpha + \beta| = 4$

Although x = –2 should be excluded from domain but according to the given problem it will the most appropriate solution.

90.[96] $11^n > 10^n + 9^n$

$\Rightarrow 11^n - 9^n > 10^n$

$\Rightarrow (10 + 1)^n - (10 - 1)^n > 10^n$

$\Rightarrow \{{}^nC_1 \cdot 10^{n-1} + {}^nC_3 10^{n-0} + {}^nC_5 10^{n-5} + ...\} > 10^n$

$\Rightarrow 2n \cdot 10^{n-1} + 2\{{}^nC_3 10n^{-3} + {}^nC_5 10^{n-5} + ...\} > 10^n$ .....(1)

For n = 5

$10^5 + 2\{{}^5C_n\, 10^2 + {}^5C_5\} > 10^5$ (true)

For n = 6, 7, 8, ... 100

$2n\,10^{n-1} > 10^n$

$\Rightarrow 2n10^{n-1} + 2\{{}^nC_3\, 10^{n-3} + {}^nC_5\, 10^{n-5} + ...\} > 10^n$

$\Rightarrow 11^n - 9^n > 10^n$ For n = 5, 6, 7, ... 100

for n = 4, inequality (1) is not satisfied

$\Rightarrow$ Inequality does not hold good for

N = 1. 2, 3, 4

So, required number of elements = 96