JEE MAIN ONLINE PAPER 2021

Held on JULY 20, 2021 (Morning)

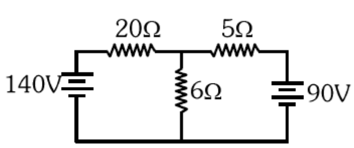
Instructions

1. This test will be a 3 hours Test.
2. This test consists of Physics, Chemistry and Mathematics questions with equal weightage of 100 marks.
3. Each question is of 4 marks.
4. In the question paper consisting of Physics (Q.no. 1 to 30), Chemistry (Q.no. 31 to 60) and Mathematics (Q.no. 61 to 90). There are two sections for each subject (Section-A : MCQ Type & Section-B : Numerical Response Type). Section-A consists of 20 multiple choice questions & Section-B consists of 10 Numerical Value type Questions. **Candidates have a choice to Answer 5 out of the 10 numerical value answer based questions per section.**
5. There will be only one correct choice in the given four choices in Section-A. For each question 4 marks will be awarded for correct choice, 1 mark will be deducted for incorrect choice and zero mark will be awarded for not attempted question. For Section-B questions 4 marks will be awarded for correct answer and zero for unattempted and incorrect answer.
6. Any textual, printed or written material, mobile phones, calculator etc. is not allowed for the students appearing for the test.
7. All calculations/written work should be done in the rough sheet provided.

PHYSICS

Section -A

Q.1



The value of current in the 6Ω resistance is:

(1) 4A (2) 8A
(3) 10A (4) 6A

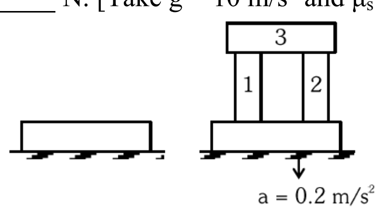
Q.2 The normal reaction 'N' for a vehicle of 800 kg mass, negotiating a turn on a 30° banked road at maximum possible speed without skidding is_______ $\times 10^3$ kg m/s^2.

(1) 10.2 (2) 7.2
(3) 12.4 (4) 6.96

Q.3 A radioactive material decays by simultaneousemissions of two particles with half lives of 1400 years and 700 years respectively. What will be the time after the which one third of the material remains ? (Take ln 3 = 1.1)

(1) 1110 years (2) 700 years
(3) 340 year (4) 740 year

Q.4 A steel block of 10 kg rests on a horizontal floor as shown. When three iron cylinders are placed on it as shown, the block and cylinders go down with an acceleration 0.2 m/s^2. The normal reaction R' by the floor if mass of the iron cylinders are equal and of 20 kg each, is ______ N. [Take g = 10 m/s^2 and μ_s = 0.2]



(1) 716 (2) 686
(3) 714 (4) 684

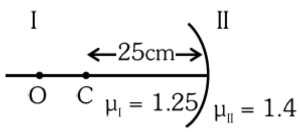
Q.5 AC voltage $V(t) = 20 \sin\omega t$ of frequency 50 Hz is applied to a parallel plate capacitor. The separation between the plates is 2 mm and the area is 1 m^2. The amplitude of the oscillating displacement current for the applied AC voltage is _______.

[Take $\varepsilon_0 = 8.85 \times 10^{-12}$ F/m]

(1) 21.14 μA (2) 83.37 μA

(3) 27.79 μA (4) 55.58 μA

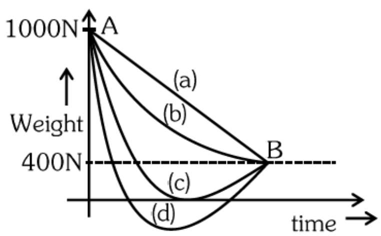
Q.6 Region I and II are separated by a spherical surface of radius 25 cm. An object is kept in region I at a distance of 40 cm from the surface. The distance of the image from the surface is :



(1) 55.44 cm (2) 9.52 cm

(3) 18.23 cm (4) 37.58 cm

Q.7 A person whose mass is 100 kg travels from Earth to Mars in a spaceship. Neglect all other objects in sky and take acceleration due to gravity on the surface of the Earth and Mars as 10 m/s^2 and 4 m/s^2 respectively. Identify from the below figures, the curve that fits best for the weight of the passenger as a function of time.



(1) (c) (2) (a)

(3) (d) (4) (b)

Q.8 The amount of heat needed to raise the temperature of 4 moles of a rigid diatomic gas from 0°C to 50°C when no work is done is _______. (R is the universal gas constant)

(1) 250 R (2) 750 R

(3) 175 R (4) 500 R

Q.9 If $\vec{A}$ and $\vec{B}$ are two vectors satisfying the relation. $\vec{A}\cdot\vec{B} = |\vec{A}\times\vec{B}|$. Then the value of $|\vec{A}-\vec{B}|$ will be :

(1) $\sqrt{A^2+B^2}$ (2) $\sqrt{A^2+B^2+\sqrt{2}AB}$

(3) $\sqrt{A^2+B^2+2AB}$ (4) $\sqrt{A^2+B^2-\sqrt{2}AB}$

Q.10 A deuteron and an alpha particle having equal kinetic energy enter perpendicular into a magnetic field. Let r_d and r_α be their respective radii of circular path. The value of $\frac{r_d}{r_\alpha}$ is equal to :

(1) $\frac{1}{\sqrt{2}}$ (2) $\sqrt{2}$

(3) 1 (4) 2

Q.11 A nucleus of mass M emits γ-ray photon of frequency 'ν'. The loss of internal energy by the nucleus is : [Take 'c' as the speed of electromagnetic wave]

(1) hν (2) 0

(3) $h\nu\left[1-\frac{h\nu}{2Mc^2}\right]$ (4) $h\nu\left[1+\frac{h\nu}{2Mc^2}\right]$

Q.12 A certain charge Q is divided into two parts q and (Q – q). How should the charges Q and q be divided so that q and (Q – q) placed at a certain distance apart experience maximum electrostatic repulsion ?

(1) $Q = \frac{q}{2}$ (2) Q = 2q

(3) Q = 4q (4) Q = 3q

Q.13 A current of 5 A is passing through a non-linear magnesium wire of cross-section 0.04 m^2. At every point the direction of current density is at an angle of 60° with the unit vector of area of cross-section. The magnitude of electric field at every point of the conductor is :

(Resistivity of magnesium $\rho = 44 \times 10^{-8}$ Ωm)

(1) 11×10^{-2} V/m (2) 11×10^{-7} V/m

(3) 11×10^{-5} V/m (4) 11×10^{-3} V/m

Q.14 Consider a mixture of gas molecule of types A, B and C having masses $m_A < m_B < m_C$. The ratio of their root mean square speeds at normal temperature and pressure is :

(1) $v_A = v_B = v_C = 0$ (2) $\frac{1}{v_A} > \frac{1}{v_B} > \frac{1}{v_C}$

(3) $v_A = v_B \neq v_C$ (4) $\frac{1}{v_A} < \frac{1}{v_B} < \frac{1}{v_C}$

Q.15 A butterfly is flying with a velocity $4\sqrt{2}$ m/s in North-East direction. Wind is slowly blowing at 1 m/s from North to South. The resultant displacement of the butterfly in 3 seconds is :

(1) 3m (2) 20m

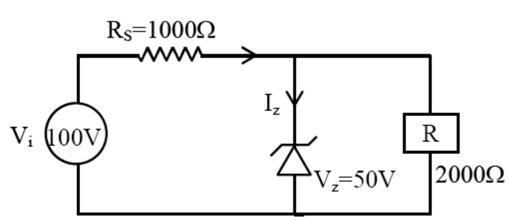
(3) $12\sqrt{2}$ m (4) 15m

Q.16 The value of tension in a long thin metal wire has been changed from T_1 to T_2. The lengths of the metal wire at two different values of tension T_1 and T_2 are ℓ_1 and ℓ_2 respectively. The actual length of the metal wire is :

(1) $\frac{T_1\ell_2 - T_2\ell_1}{T_1 - T_2}$ (2) $\frac{T_1\ell_1 - T_2\ell_2}{T_1 - T_2}$

(3) $\frac{\ell_1 + \ell_2}{2}$ (4) $\sqrt{T_1T_2\,\ell_1 + \ell_2}$

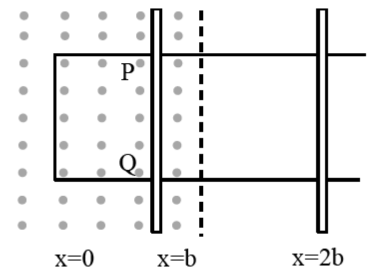
Q.17 For the circuit shown below, calculate the value of I_z :

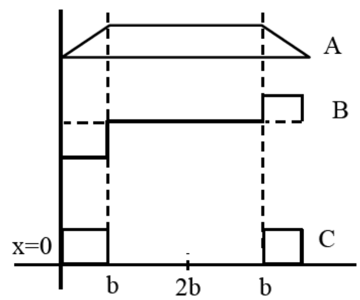


(1) 25 mA (2) 0.15 A

(3) 0.1A (4) 0.05A

Q.18 The arm PQ of a rectangular conductor is moving from x = 0 to x = 2b outwards and then inwards from x = 2b to x = 0 as shown in the figure. A uniform magnetic field perpendicular to the plane is acting from x = 0 to x = b. Identify the graph showing the variation of different quantities with distance





(1) A-Flux, B-Power dissipated, C-EMF

(2) A-Power dissipated, B-Flux, C-EMF

(3) A-Flux, B-EMF, C-Power dissipated

(4) A-EMF, B-Power dissipated, C-Flux

Q.19 The entropy of any system is given by

$$S = \alpha^2\beta\ell n\left[\frac{\mu kR}{J\beta^2} + 3\right]$$

where α and β are the constants. μ, J, k and R are no. of moles, mechanical equivalent of heat, Boltzmann constant and gas constant respectively.

$$\left[\text{Take}\, S = \frac{dQ}{T}\right]$$

Choose the incorrect option from the following :

(1) α and J have the same dimensions.

(2) S, β, k and μR have the same dimensions.

(3) S and α have different dimensions.

(4) α and k have the same dimensions.

Q.20 The radiation corresponding to $3 \rightarrow 2$ transition of a hydrogen atom falls on a gold surface to generate photoelectrons. These electrons are passed through a magnetic field of 5×10^{-4} T. Assume that the radius of the largest circular path followed by these electrons is 7 mm, the work function of the metal is :

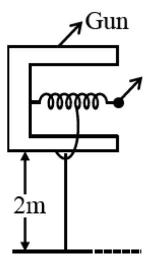
(Mass of electron = 9.1×10^{-31} kg)

(1) 1.36 eV (2) 1.88 eV

(3) 0.16 ev (4) 0.82 eV

Section -B

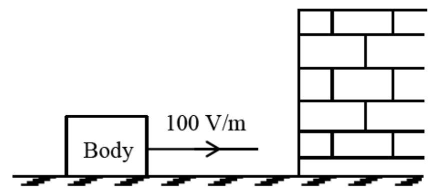
Q.21 In a spring gun having spring constant 100 N/m a small ball 'B' of mass 100 g is put in its barrel (as shown in figure) by compressing the spring through 0.05 m. There should be a box placed at a distance 'd' on the ground so that the ball falls in it. If the ball leaves the gun horizontally at a height of 2 m above the ground. The value of d is ____ m. ($g = 10\ m/s^2$)



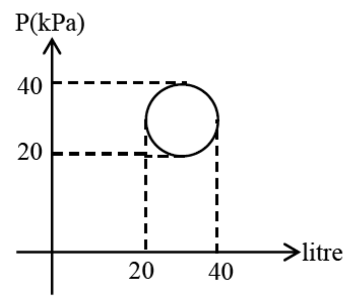
Q.22 In an LCR series circuit, an inductor 30 mH and a resistor 1Ω are connected to an AC source of angular frequency 300 rad/s. The value of capacitance for which, the current leads the voltage by 45° is $\frac{1}{x} \times 10^{-3}$ F. Then the value of x is ____.1x

Q.23 The amplitude of wave disturbance propagating in the positive x-direction is given by $y = \frac{1}{(1+x)^2}$ at time t = 0 and $y = \frac{1}{1+(x-2)^2}$ at t = 1s, where x and y are in meres. The shape of wave does not change during the propagation. The velocity of the wave will be ____m/s.

Q.24 A body having specific charge 8 μC/g is resting on a frictionless plane at a distance 10 cm from the wall (as shown in the figure). It starts moving towards the wall when a uniform electric field of100 V/m is applied horizontally towards the wall. If the collision of the body with the wall is perfectly elastic, then the time period of the motion will be ______ s.



Q.25 In the reported figure, heat energy absorbed by a system in going through a cyclic process is ______ πJ.



Q.26 A circular disc reaches from top to bottom of an inclined plane of length 'L'. When it slips down the plane, it takes time 't_1'. When it rolls down the plane, it takes time t_2. The value of $\frac{t_2}{t_1}$ is $\sqrt{\frac{3}{x}}$. The value of x will be ______.

Q.27 A rod of mass M and length L is lying on a horizontal frictionless surface. A particle of mass 'm' travelling along the surface hits at one end of the rod with a velocity 'u' in a direction perpendicular to the rod. The collision is completely elastic. After collision, particle comes to rest. The ratio of masses $\left(\frac{m}{M}\right)$ is $\frac{1}{x}$.

The value of 'x' will be ______.

Q.28 An object viewed from a near point distance of 25 cm, using a microscopic lens with magnification '6', gives an unresolved image. A resolved image is observed at infinite distance with a total magnification double the earlier using an eyepiece along with the given lens and a tube of length 0.6 m, if the focal length of the eyepiece is equal to ______ cm.

Q.29 The frequency of a car horn encountered a change from 400 Hz to 500 Hz. When the car approaches a vertical wall. If the speed of sound is 330 m/s. Then the speed of car is ______ km/h.

Q.30 A carrier wave VC(t) = 160 sin $(2\pi \times 10^6 t)$ volts is made to vary between V_{max} = 200 V and V_{min} = 120 V by a message signal $V_m(t)$ = Am $\sin(2\pi \times 10^3 t)$ volts. The peak voltage A_m of the modulating signal is _______.

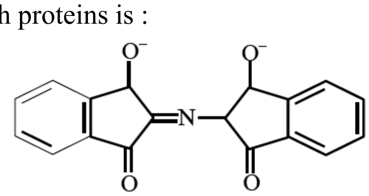
CHEMISTRY

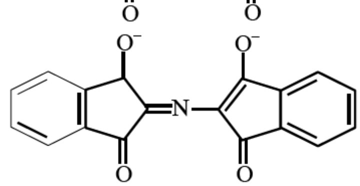
Section -A

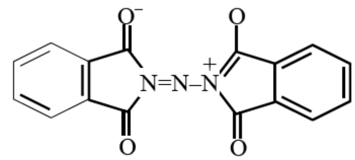
Q.31 According to the valence bond theory the hybridization of central metal atom is dsp^2 for which one of the following compounds?

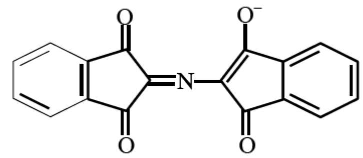
(1) $NiCl_2.6H_2O$ (2) $K_2[Ni(CN)_4]$
(3) $[Ni(CO)_4]$ (4) $Na_2[NiCl_4]$

Q.32 The correct structure of Rhumann's Purple, the compound formed in the reaction of ninhydrin with proteins is :

(1) 

(2) 

(3) 

(4) 

Q.33 Green chemistry in day–to–day life is in the use of:

(1) Chlorine for bleaching of paper
(2) Large amount of water alone for washing clothes
(3) Tetrachloroethene for laundry
(4) Liquified CO_2 for dry cleaning of clothes

Q.34 The correct order of intensity of colors of the compounds is :

(1) $[Ni(CN)_4]^{2-} > [NiCl_4]^{2-} > [Ni(H_2O)_6]^{2+}$
(2) $[Ni(H_2O)_6]^{2+} > [NiCl_4]^{2-} > [Ni(CN)_4]^{2-}$
(3) $[NiCl_4]^{2-} > [Ni(H_2O)_6]^{2+} > [Ni(CN)_4]^{2-}$
(4) $[NiCl_4]^{2-} > [Ni(CN)_4]^{2-} > [Ni(H_2O)_6]^{2+}$

Q.35 The set in which compounds have different nature is :

(1) $B(OH)_3$ and H_3PO_3
(2) $B(OH)_3$ and $Al(OH)_3$
(3) NaOH and $Ca(OH)_2$
(4) $Be(OH)_2$ and $Al(OH)_3$

Q.36 The species given below that does NOT show disproportionation reaction is :

(1) BrO_4^- (2) BrO^- (3) BrO_2^- (4) BrO_3^-

Q.37 A Given below are two statements. One is labelled as **Assertion A** and the other is labelled as **Reason R**.

Assertion A : Sharp glass edge becomes smooth on heating it upto its melting point.

Reason R : The viscosity of glass decreases on melting.

Choose the most appropriate answer from the options given below.

(1) **A** is true but **R** is false

(2) Both **A** and **R** are true but **R** is NOT the correct explanation of **A**.

(3) **A** is false but **R** is true.

(4) Both **A** and **R** are true and **R** is the correct explanation of **A**.

Q.38 Orlon fibres are made up of :

(1) Polyacrylonitrile (2) Polyesters

(3) Polyamide (4) Cellulose

Q.39 Given below are two statements : One is labelled as **Assertion A** and other is labelled as **Reason R**.

Assertion A : The dihedral angles in H_2O_2 in gaseous phase is 90.2° and in solid phase is 111.5°.

Reason R : The change in dihedral angle in solid and gaseous phase is due to the difference in the intermolecular forces.

Choose the most appropriate answer from the options given below for **A** and **R**.

(1) **A** is correct but **R** is not correct.

(2) Both **A** and **R** are correct but R is not the correct explanation of A.

(3) Both **A** and **R** are correct and **R** is the correct explanation of **A**.

(4) **A** is not correct but **R** is correct.

Q.40 Chemical nature of the nitrogen oxide compound obtained from a reaction of concentrated nitric acid and P_4O_{10} (in 4 : 1 ratio) is :

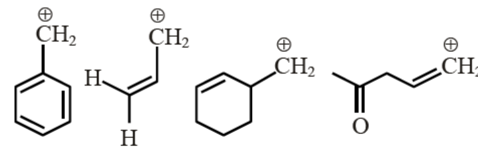
(1) acidic (2) basic

(3) amphoteric (4) neutral

Q.41 An inorganic Compound 'X' on treatment with concentrated H_2SO_4 produces brown fumes and gives dark brown ring with $FeSO_4$ in presence of concentrated H_2SO_4. Also Compound 'X' gives precipitate 'Y', when its solution in dilute HCl is treated with H_2S gas. The precipitate 'Y' on treatment with concentrated HNO_3 followed by excess of NH_4OH further gives deep blue coloured solution, Compound 'X' is:

(1) $Co(NO_3)_2$ (2) $Pb(NO_2)_2$

(3) $Cu(NO_3)_2$ (4) $Pb(NO_3)_2$

Q.42 

(A) (B) (C) (D)

Among the given species the Resonance stabilised carbocations are:

(1) (C) and (D) only

(2) (A), (B) and (D) only

(3) (A) and (B) only

(4) (A), (B) and (C) only

Q.43 A s-block element (M) reacts with oxygen to form an oxide of the formula MO_2. The oxide is pale yellow in colour and paramagnetic. The element (M) is:

(1) Mg (2) Na

(3) Ca (4) K

Q.44 In the given reaction 3-Bromo-2, 2-dimethyl butane $\xrightarrow{C_2H_5OH}$ 'A' (Major Product) Product A is -

(1) 2-Ethoxy-3, 3-dimethyl butane

(2) 1-Ethoxy-3, 3-dimethyl butane

(3) 2-Ethoxy-2, 3-dimethyl butane

(4) 2-Hydroxy-3, 3-dimethyl butane

Q.45 The metal that can be purified economically by fractional distillation method is:

(1) Fe (2) Zn (3) Cu (4) Ni

Q.46 Compound A is converted to B on reaction with CHCl3 and KOH. The compound B is toxic and can be decomposed by C. A, B and C respectively are :

(1) primary amine, nitrile compound, conc. HCl

(2) secondary amine, isonitrile compound, conc. NaOH

(3) primary amine, isonitrile compound, conc. HCl

(4) secondary amine, nitrile compound, conc. NaOH

Q.47 The conditions given below are in the context of observing Tyndall effect in colloidal solutions:

(A) The diameter of the colloidal particles is comparable to the wavelength of light used.

(B) The diameter of the colloidal particles is much smaller than the wavelength of light used.

(C) The diameter of the colloidal particles is much larger than the wavelength of light used.

(D) The refractive indices of the dispersed phase and the dispersion medium are comparable.

(E) The dispersed phase has a very different refractive index from the dispersion medium.

Choose the most appropriate conditions from the options given below:

(1) (A) and (E) only (2) (C) and (D) only

(3) (A) and (D) only (4) (B) and (E) only

Q.48 Identify the incorrect statement from the following

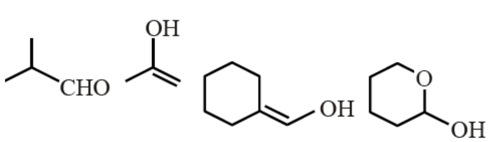
(1) Amylose is a branched chain polymer of glucose

(2) Starch is a polymer of α-D glucose

(3) β-Glycosidic linkage makes cellulose polymer

(4) Glycogen is called as animal starch

Q.49



(I) (II) (III) (IV)

Which among the above compound/s does/do not form Silver mirror when treated with Tollen's reagent?

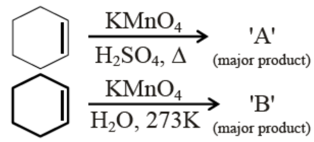
(1) (I), (III) and (IV) only

(2) Only (IV)

(3) Only (II)

(4) (III) and (IV) only

Q.50



For above chemical reactions, identify the correct statement from the following:

(1) Both compound 'A' and compound 'B' are dicarboxylic acids

(2) Both compound 'A' and compound 'B' are diols

(3) Compound 'A' is diol and compound 'B' is dicarboxylic acid

(4) Compound 'A' is dicarboxylic acid and compound 'B' is diol

Section -B

Q.51 The number of lone pairs of electrons on the central I atom in I_3^- is _________.

Q.52 250 mL of 0.5 M NaOH was added to 500 mL of 1 M HCl. The number of unreacted HCl molecules in the solution after complete reaction is _______ $\times 10^{21}$. (Nearest integer)
(NA = 6.022×10^{23})

Q.53 The Azimuthal quantum number for the valence electrons of Ga^+ ion is _______.
(Atomic number of Ga = 31)

Q.54 The spin-only magnetic moment value for the complex $[Co(CN)_6]^{4-}$ is ________ BM.
[At. no. of Co = 27]

Q.55 $2SO_2(g) + O_2(g) \rightleftharpoons 2SO_3(g)$

In an equilibrium mixture, the partial pressures are

$P_{SO_3} = 43 kPa$; $P_{O_2} = 530 Pa$ and

$P_{SO_2} = 45$ kPa.

The equation constant k_P = ____ $\times$ 10^{-2}. (Nearest integer)

Q.56 The number of nitrogen atoms in a semicarbazone molecule of acetone is _________.

Q.57 To synthesise 1.0 mole of 2-methylpropan-2-ol from Ethylethanoate ________ equivalents of CH_3MgBr reagent will be required. (Integer value)

Q.58 The inactivation rate of a viral preparation is proportional to the amount of virus. In the first minute after preparation, 10% of the virus is inactivated. The rate constant for viral inactivation is __________ $\times 10^{-3}$ min^{-1}. (Nearest integer)

[Use : ln 10 = 2.303 ; $\log_{10} 3 = 0.477$; property of logarithm : $\log x^y = y \log x$]

Q.59 An average person needs about 10000 kJ energy per day. The amount of glucose (molar mass = 180.0 g mol^{-1}) needed to meet this energy requirement is _________ g.

(Use : $\Delta_C H$(glucose) = –2700 kJ mol^{-1})

Q.60 At 20°C, the vapour pressure of benzene is 70 torr and that of methyl benzene is 20 torr. The mole fraction of benzene in the vapour phase at 20°C above an equimolar mixture of benzene and methyl benzene is ________ $\times 10^{-2}$. (Nearest integer)

MATHEMATICS

Section -A

Q.61 The Boolean expression

$(P \wedge \sim q) \Rightarrow (q \vee \sim p)$ is equivalent to

(1) $q \Rightarrow p$ (2) $p \Rightarrow q$

(3) $\sim q \Rightarrow p$ (4) $p \Rightarrow \sim q$

Q.62 Let a be a positive real number such that $\int_0^a e^{x-[x]}dx = 10e - 9$, where [x] is the greatest integer less than or equal to x. Then a is equal to :

(1) $10 - \log_e(1 + e)$ (2) $10 + \log_e 2$

(3) $10 + \log_e 3$ (4) $10 + \log_e(1+ e)$

Q.63 The mean of 6 distinct observations is 6.5 and their variance is 10.25. If 4 out of 6 observations are 2, 4, 5 and 7, then the remaining two observations are

(1) 10, 11 (2) 3, 18

(3) 8, 13 (4) 1, 20

Q.64 The value of the integral $\int_{-1}^{1} \log_e(\sqrt{1-x} + \sqrt{1+x})dx$ is equal to

(1) $\frac{1}{2}\log_e 2 + \frac{\pi}{4} - \frac{3}{2}$ (2) $2\log_e 2 + \frac{\pi}{4} - 1$

(3) $\log_2 2 + \frac{\pi}{4} - 1$ (4) $2\log_e 2 + \frac{\pi}{4} - \frac{1}{2}$

Q.65 If α and β are the distinct roots of the equation $x^2 + (3)^{1/4}x + 3^{1/2} = 0$, then the value of $\alpha^{96}(\alpha^{12} - 1) + \beta^{96}(\beta^{12} - 1)$ is equal to

(1) 56×3^{25} (2) 56×3^{24}

(3) 52×3^{24} (4) 28×3^{25}

Q.66 Let $A = \begin{bmatrix} 2 & 3 \\ a & 0 \end{bmatrix}$, $a \in \mathbf{R}$ be written as P + Q where P is symmdetric matrix and Q is skew symmetric matrix. If det (Q) = 9, then the modulus of the sum of all possible values of determinant of P is equal to -

(1) 36 (2) 24

(3) 45 (4) 18

Q.67 If z and ω are two complex numbers such that $|z\omega| = 1$ and $\arg(z) - \arg(\omega) = \frac{3\pi}{2}$, then $\arg\left(\frac{1-2\bar{z}\omega}{1+3\bar{z}\omega}\right)$ is :

(Here arg(z) denotes the principal argument of complex number z)

(1) $\frac{\pi}{4}$ (2) $-\frac{3\pi}{4}$

(3) $-\frac{\pi}{4}$ (4) $\frac{3\pi}{4}$

Q.68 If in triangle ABC, AB = 5 units, $\angle B = \cos^{-1}\left(\frac{3}{5}\right)$ and radius of circumcircle of ΔABC is 5 units, then the area (in sq. units) of ΔABC is

(1) $10+6\sqrt{2}$ (2) $8+2\sqrt{2}$

(3) $6+8\sqrt{3}$ (4) $4+2\sqrt{3}$

Q.69 Let [x] denote the greatest integer $\le$ x, where x $\in$ **R**. If the domain of the real valued function $f(x)=\sqrt{\frac{|[x]|-2}{|[x]|-3}}$ is $(-\infty, a) \cup [b, c) \cup [4, \infty)$, $a < b < c$, then the value of a + b + c is

(1) 8 (2) 1 (3) –2 (4) –3

Q.70 Let y = y(x) be the solution of the differential equation $x\tan\left(\frac{y}{x}\right)dy = \left(y\tan\left(\frac{y}{x}\right) - x\right)dx$, $-1 \le x \le 1$, $y\left(\frac{1}{2}\right)=\frac{\pi}{6}$. Then the area of the region bounded by the curves x = 0, $x=\frac{1}{\sqrt{2}}$ and y = y(x) in the upper half plane is -

(1) $\frac{1}{8}(\pi-1)$ (2) $\frac{1}{12}(\pi-3)$

(3) $\frac{1}{4}(\pi-2)$ (4) $\frac{1}{6}(\pi-1)$

Q.71 The coefficient of x^{256} in the expansion of $(1-x)^{101}(x^2+x+1)^{100}$ is

(1) ${}^{100}C_{16}$ (2) ${}^{100}C_{15}$

(3) $-{}^{100}C_{16}$ (4) $-{}^{100}C_{15}$

Q.72 Let A = $[a_{ij}]$ be a 3×3 matrix, where

$$a_{ij} = \begin{cases} 1 & , \text{ if } i=j \\ -x & , \text{ if } |i-j|=1 \\ 2x+1 & , \text{ otherwise} \end{cases}$$

Let a function f : **R** → **R** be defined as f(x) = det(A). Then the sum of maximum and minimum values of f on **R** is equal to -

(1) $-\frac{20}{27}$ (2) $\frac{88}{27}$ (3) $\frac{20}{27}$ (4) $-\frac{88}{27}$

Q.73 Let $\vec{a}=2\hat{i}+\hat{j}-2\hat{k}$ and $\vec{b}=\hat{i}+\hat{j}$. If $\vec{c}$ is a vector such that $\vec{a}\cdot\vec{c}=|\vec{c}|$, $|\vec{c}-\vec{a}|=2\sqrt{2}$ and the angle between $(\vec{a}\times\vec{b})$ and $\vec{c}$ is $\frac{\pi}{6}$, then the value of $|(\vec{a}\times\vec{b})\times\vec{c}|$ is -

(1) $\frac{2}{3}$ (2) 4 (3) 3 (4) $\frac{3}{2}$

Q.74 The number of real roots of the equation $\tan^{-1}\sqrt{x(x+1)}+\sin^{-1}\sqrt{x^2+x+1}=\frac{\pi}{4}$ is

(1) 1 (2) 2 (3) 4 (4) 0

Q.75 Let y = y(x) be the solution of the differential equation $e^x\sqrt{1-y^2}dx+\left(\frac{y}{x}\right)dy=0$, y(1) = –1. The the value of $(y(3))^2$ is equal to -

(1) $1-4e^3$ (2) $1-4e^6$

(3) $1+4e^3$ (4) $1+4e^6$

Q.76 Let 'a' be a real number such that the function $f(x) = ax^2 + 6x - 15$, x $\in$ **R** is increasing in $\left(-\infty, \frac{3}{4}\right)$ and decreasing in $\left(\frac{3}{4}, \infty\right)$. Then the function $g(x) = ax^2 - 6x + 15$, x $\in$ **R** has a -

(1) local maximum at $x=-\frac{3}{4}$

(2) local minimum at $x=-\frac{3}{4}$

(3) local maximum at $x=\frac{3}{4}$

(4) local minimum at $x=\frac{3}{4}$

Q.77 Let a function f : R → R be defined as

$$f(x) = \begin{cases} \sin x - e^x & \text{if } x \le 0 \\ a+[-x] & \text{if } 0<x<1 \\ 2x-b & \text{if } x \ge 1 \end{cases}$$

Where [x] is the greatest integer less than or equal to x. If f is continuous on **R**, then (a + b) is equal to

(1) 4 (2) 3 (3) 2 (4) 5

Q.78 Words with or without meaning are to be formed using all the letters of the word EXAMINATION. The probability that the letter M appears at the fourth position in any such word is

(1) $\frac{1}{66}$ (2) $\frac{1}{11}$ (3) $\frac{1}{9}$ (4) $\frac{2}{11}$

Q.79 The probability of selecting integers $a \in [-5, 30]$ such that $x^2 + 2(a + 4)x - 5a + 64 > 0$, for all $x \in \mathbf{R}$ is

(1) $\frac{7}{36}$ (2) $\frac{2}{9}$ (3) $\frac{1}{6}$ (4) $\frac{1}{4}$

Q.80 Let the tangent to the parabola $S : y^2 = 2x$ at the point P(2, 2) meet the x-axis at Q and normal at it meet the parabola S at the point R. Then the area (in sq. units) of the triangle PQR is equal to

(1) $\frac{25}{2}$ (2) $\frac{35}{2}$ (3) $\frac{15}{2}$ (4) 25

Section -B

Q.81 Let $\vec{a}, \vec{b}, \vec{c}$ be three mutually perpendicular vectors of the same magnitude and equally inclined at an angle θ, with the vector a + b + c. Then 36 cos2θ is equal to _____ .

Q.82 Let $A = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix}$ and $B = 7A^{20} - 20A^7 + 2I$,

Where I is an identity matrix of order 3× 3. If $B = [b_{ij}]$, then b_{13} is equal to ______.

Q.83 Let P be a plane passing through the points (1, 0, 1), (1, –2, 1) and (0, 1, –2). let a vector $\vec{a} = \alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}$ be such that $\vec{a}$ is parallel to the plane P, perpendicular to $(\hat{i} + 2\hat{j} + 3\hat{k})$ and $\vec{a}(\hat{i} + \hat{j} + 2\hat{k}) = 2$, then $(\alpha - \beta + \gamma)^2$ equals _____ .

Q.84 The number of rational terms in the binomial expansion of $(4^{\frac{1}{4}} + 5^{\frac{1}{6}})^{120}$ is ______ .

Q.85 If the shortest distance between the lines $\vec{r_1} = \alpha\hat{i} + 2\hat{j} + 2\hat{k} + \lambda(\hat{i} - 2\hat{j} + 2\hat{k})$, $\lambda \in \mathbf{R}$, $\alpha > 0$ and $\vec{r_2} = -4\hat{i} - \hat{k} + \mu(3\hat{i} - 2\hat{j} - 2\hat{k})$, $\mu \in \mathbf{R}$ is 9, then α is equal to ______ .

Q.86 Let T be the tangent to the ellipse $E : x^2 + 4y^2 = 5$ at the point P(1, 1). If the area of the region bounded by the tangent T, ellipse E, lines x = 1 and $x = \sqrt{5}$ is $\alpha = \sqrt{5} + \beta + \gamma \cos^{-1}\left(\frac{1}{\sqrt{5}}\right)$, then $|\alpha + \beta + \gamma|$ is equal to _______.

Q.87 Let a, b, c, d be in arithmetic progression with common difference λ. If

$$\begin{vmatrix} x+a-c & x+b & x+a \\ x-1 & x+c & x+b \\ x-b+d & x+d & x+c \end{vmatrix} = 2,$$ then value of λ^2 is equal to ______ .

Q.88 There are 15 players in a cricket team, out of which 6 are bowlers, 7 are batsmen and 2 are wicket keepers. The number of ways, a team of 11 players be selected from them so as to include at least 4 bowlers, 5 batsmen and 1 wicketkeeper, is__________.

Q.89 Let y = mx + c, m > 0 be the focal chord of $y^2 = -64x$, which is tangent to $(x + 10)^2 + y^2 = 4$. Then, the value of $4\sqrt{2}(m + c)$ is equal to ______.

Q.90 If the value of $\lim_{x \to 0}(2 - \cos x\sqrt{\cos 2x})^{\left(\frac{x+2}{x^2}\right)}$ is equal to e^a, then a is equal to _____ .

JEE MAIN ONLINE PAPER 2021

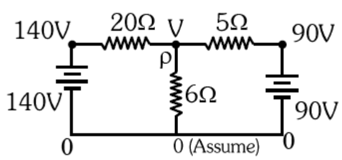
Held on JULY 20, 2021 (Morning)

Hints & Solutions

PHYSICS

Section -A

1.[3]



Applying KCL at point P,

$$\frac{V-0}{6}+\frac{V-90}{5}+\frac{v-140}{20}=0$$

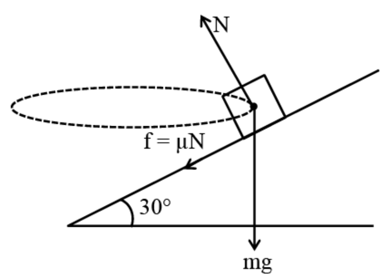
$\Rightarrow$ 10V + 12V – 1080 + 3v–420 = 0

$\Rightarrow$ V= 60

$\therefore$ current in $6\Omega=\frac{V-0}{6}=10A$

hence option 3.

2.[1]



At V_{max}, f will be limiting in nature.

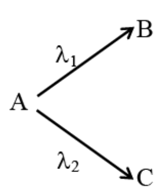
$\therefore$Balancing force in vertical direction,

Ncos30° – mg – μNcos60° = 0

$\Rightarrow$ N [cos30° – μcos60°] = mg

$$\therefore\ N=\frac{800\times10}{(0.87-0.1)}\approx10.2\times10^{3}kgm/s^{2}$$

3.[4]



Given,

$$\lambda_1=\frac{\ell n2}{700}/\text{year},\ \lambda_2=\frac{\ell n2}{1400}/\text{year}$$

$$\therefore\ \lambda_{net}=\lambda_1+\lambda_2=\ell n2\left[\frac{1}{700}+\frac{1}{1400}\right]$$

$$=\frac{3\ell n2}{1400}/\text{year}$$

Now, Let initial no. of radioactive nuclei be No.

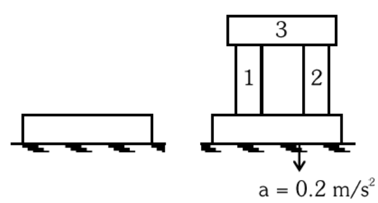
$$\therefore\ \frac{N_0}{3}=N_0e^{-\lambda_{net}t}$$

$$\Rightarrow\ \ell n\frac{1}{3}=-\lambda_{net}t$$

$$\Rightarrow\ 1.1=\frac{3\times0.693}{1400}t\Rightarrow t\approx740\ \text{year}$$

Hence option 4

4.[2]



Writing force equation in vertical direction

Mg – N = Ma

$\Rightarrow$ 70g – N = 70 × 0.2

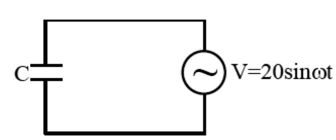
$\Rightarrow$ N = 70 [g – 0.2] = 70 × 9.8

$\therefore$ N = 686 Newton

Note : Since there is no compressive normal from the sides, hence friction will not act.

Hence option 2.

5.[3]



From the given information,

$$C=\frac{\epsilon_0 A}{d}=\frac{\epsilon_0\times1}{2\times10^{-3}}F$$

$$\therefore\ X_C = \frac{1}{\omega C} = \frac{2\times10^{-3}}{2\times50\pi\times\epsilon_0} = \frac{2\times10^{-3}}{25\times4\pi\epsilon_0}\Omega$$

$$\therefore\ X_C = \frac{2\times10^{-3}}{25}\times9\times10^{9} = \frac{18}{25}\times10^{6}\Omega$$

$$\therefore\ i_0 = \frac{V_0}{X_C} = \frac{20\times25}{18}\times10^{-6}A = 27.47\mu A$$

The value of amplitude of displacement current will be same as value of amplitude of conventional current.

Hence option 3.

6.[4] $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2-\mu_1}{R}$

$$\frac{1.4}{v} - \frac{1.25}{-40} = \frac{1.4-1.25}{-25}$$

$$\frac{1.4}{v} - \frac{0.15}{25} = \frac{1.25}{40}$$

v = –37.58 cm

Hence option (4)

7.[1] At neutral point g = 0 so graph (C) is correct

Hence option (1).

8.[4] $\Delta Q = \Delta U + \Delta W$

Here $\Delta W = 0$

$\Delta Q = \Delta U = nC_V\Delta T$

$$\Delta Q = 4\times\frac{5R}{2}(50) = 500R$$

Hence option (4)

9.[4] $\vec{A}\cdot\vec{B} = |\vec{A}\times\vec{B}|$

$AB\cos\theta = AB\sin\theta \Rightarrow \theta = 45°$

$$|\vec{A}\times\vec{B}| = \sqrt{A^2+B^2-2AB\cos45°}$$

$$= \sqrt{A^2+B^2-\sqrt{2}AB}$$

Hence option (4)

10.[2] $r = \frac{mv}{qB} = \frac{\sqrt{2mk}}{qB}$

$$\frac{r_d}{r_\alpha} = \sqrt{\frac{m_d}{m_\alpha}}\frac{q_\alpha}{q_d} = \sqrt{\frac{2}{4}}\left(\frac{2}{1}\right) = \sqrt{2}$$

Hence option (2)

11.[4] Energy of γ ray $[E_\gamma] = h\nu$

Momentum of γ ray $[P_\gamma] = \frac{h}{\lambda} = \frac{h\nu}{C_v}$

Total momentum is conserved

$$\vec{P}_\gamma + \vec{P}_{Nu} = 0$$

where $\vec{P}_{Nu}$ = Momentum of decayed nuclei

$\Rightarrow P_\gamma = P_{Nu}$

$\Rightarrow \frac{h\nu}{C} = P_{Nu}$

$\Rightarrow$ K.E. of nuclei

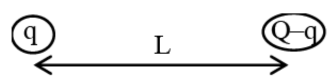
$$= \frac{1}{2}Mv^2 = \frac{(P_{Nu})^2}{2M} = \frac{1}{2M}\left[\frac{h\nu}{C}\right]^2$$

Loss in internal energy = E_γ + $K.E_{Nu}$

$$= h\nu + \frac{1}{2M}\left[\frac{h\nu}{C}\right]^2$$

$$= h\nu\left[1+\frac{h\nu}{2MC^2}\right]$$

12.[2]



$$F_q = \frac{kq(Q-q)}{L^2} = \frac{k}{L^2}(qQ-q^2)$$

$\frac{dF}{dq} = 0$ when force is maximum

$$\frac{dF}{dq} = \frac{k}{L^2}[Q-2q] = 0$$

$\Rightarrow Q - 2q = 0 \Rightarrow Q = 2q$

13.[3] $I = \vec{J}\cdot\vec{A} = JA\cos(\theta)$

$$5 = J\left(\frac{4}{100}\right)\times\cos(60)$$

J = 5 × 50 = 250 A/m^2

Now, $\vec{E} = \rho\cdot\vec{J}$

= 44 × 10^{-8} × 250 = 11 × 10^{-5} V/m

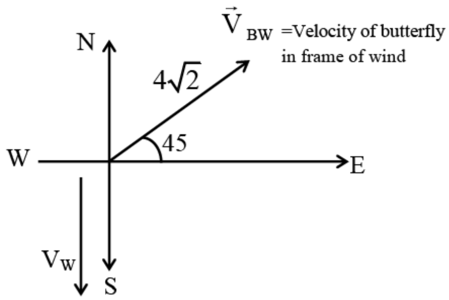
14.[4] $V_{RMS} = \sqrt{\frac{3RT}{M}}$

$m_A < m_B < m_C$

$\Rightarrow V_A > V_B > V_C$

$\Rightarrow \frac{1}{V_A} < \frac{1}{V_B} < \frac{1}{V_C}$

15.[4]



$\vec{V}_{BW} = 4\sqrt{2}\cos 45\hat{i} + 4\sqrt{2}\sin 45\hat{j} = 4\hat{i} + 4\hat{j}$

$\vec{V}_w = -\hat{j}$

$\vec{V}_B = \vec{V}_{BW} + \vec{V}_W = 4\hat{i} + 3\hat{j}$

$\vec{S}_B = \vec{V}_B \times t = (4\hat{i} + 3\hat{j}) \times 3 = 12\hat{i} + 9\hat{j}$

$|\vec{S}_B| = \sqrt{(12)^2 + (9)^2} = 15m$

16.[1] $Y = \frac{FL}{A\Delta L}$

$\Rightarrow Y = \frac{T_1\ell_0}{A(\ell_1 - \ell_0)} = \frac{T_2\ell_0}{A(\ell_2 - \ell_0)}$

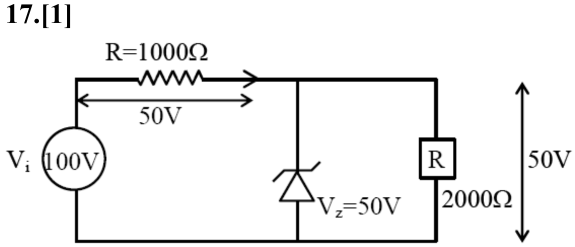
$1 = \frac{T_1(\ell_2 - \ell_0)}{T_2(\ell_1 - \ell_0)}$

$T_2\ell_1 - T_2\ell_0 = T_1\ell_2 - T_1\ell_0$

$(T_1 - T_2)\ \ell_0 = T_1\ell_2 - T_2\ell_1$

$\ell_0 = \left(\frac{T_1\ell_2 - T_2\ell_1}{T_1 - T_2}\right)$

17.[1]



$I = \frac{50}{2000} = 25mA$

$Iz = I_{1000} - I_{2000}$

$I = \frac{50}{2000} = 25mA$

$I_z = I_{1000} - I_{2000}$

$= 50 - 25 = 25mA$

18.[3] As rod moves in field area increases upto x = b then field is absent and again flux is generated on return journey from x = b to x = 0. Thus plot A for flux.

$\Rightarrow e = -\frac{d\phi}{dt}$ curve B for emf

$\Rightarrow$ Power dissipated = vi

$\Rightarrow$ curve C for power dissipated

19.[4] $S = \alpha^2\beta\ell_n\left(\frac{\mu KR}{J\beta^2} + 3\right)$

$S = \frac{Q}{T} = \text{joulek/k}$

$[\alpha^2 \cdot \beta]$ joule/k

$PV = nRT \quad \left[\frac{\mu KR}{J\beta^2}\right] = 1$

$R = \frac{\text{Joule}}{K}$

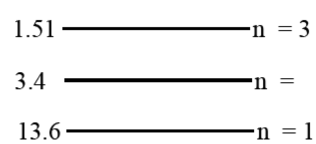
$\Rightarrow R = \frac{\text{Joule}}{K}, K = \frac{\text{Joule}}{R}$

$\Rightarrow \beta = \left(\frac{\text{Joule}}{K}\right)$

$\alpha^2\beta = \left(\frac{\text{Joule}}{K}\right)$

$\Rightarrow \alpha$ = dimesnsionless

20.[4]



$3 \rightarrow 2 \Rightarrow 1.89$ eV

$5 \times 10^{-4}T \quad r = 7mm$

$r = \frac{mv}{qB} \Rightarrow mv = qrB$

$\Rightarrow E = \frac{P^2}{2m} = \frac{(qRB)^2}{2m}$

$$= \frac{(1.6\times10^{-19}\times7\times10^{-3}\times5\times10^{-4})^2}{2\times9.1\times10^{-31}\text{ joule}}$$

$$= \frac{3136\times10^{-52}}{18.2\times10^{-31}\times1.6\times10^{-19}}\text{eV}$$

= 1.077 eV

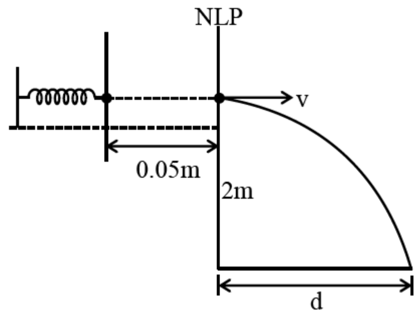
We know work function

= energy incident – (KE)electron

$\phi = 1.89 - 1.077 = 0.813$ eV

Section -B

21.[1]



$$\frac{1}{2}kx^2 = \frac{1}{2}mv^2$$

$Kx^2 = mv^2$

$$v = x\sqrt{\frac{k}{m}} = 0.05\sqrt{\frac{100}{0.1}} = 0.05\times10\sqrt{10}$$

$v = 0.5\sqrt{10}$

From $h = \frac{1}{2}gt^2$

$$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2\times2}{10}} = -\frac{2}{\sqrt{10}}$$

$$\therefore\ d = vt = 0.5\sqrt{10}\times\frac{2}{\sqrt{10}} = 1\text{m}$$

22.[3] $\tan\phi = \frac{X_C - X_L}{R}$

$$\tan 45 = \frac{X_C - X_L}{R}$$

$X_C - X_L = R$

$$\frac{1}{\omega C} - \omega L = R$$

$$\frac{1}{\omega C} - 300\times0.03 = 1$$

$$\frac{1}{\omega C} = 10$$

$$C = \frac{1}{10\omega} = \frac{1}{10\times300}$$

$$C = \frac{1}{3}\times10^{-3}$$

X = 3

23.[2] At t = 0, $y = \frac{1}{1+x^2}$

at time t = t, $y = \frac{1}{1+(x-vt)^2}$

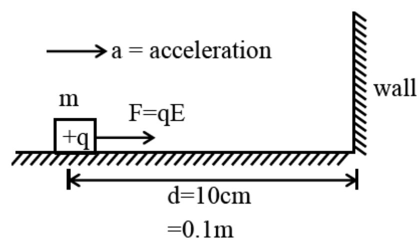
at t = 1, $y = \frac{1}{1+(x-v)^2}$ .... (i)

at t = 1, $y = \frac{1}{1+(x-2)^2}$ ....(ii)

comparing (1) & (ii)

v = 2m/s

24.[1]



F = ma

qE = ma

$$a = \frac{qE}{m}$$

Now $d = \frac{1}{2}at^2$

$$t = \sqrt{\frac{2d}{a}}$$

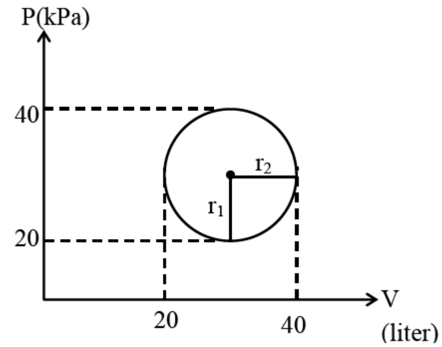
$$t = \sqrt{\frac{2d}{\left(\frac{qE}{m}\right)}}$$

$$t = \sqrt{\frac{2\times0.1}{\left(\frac{8\times10^{-6}}{10^{-3}}\right)\times100}} = \frac{1}{2}$$

$\therefore$ Time period = 2t = 1sec

Ans = 1.00

25.[100]



For complete cyclic process

$\Delta U = 0$

$\therefore$ from $\Delta Q = \Delta U + W$

$= 0 + W$

$\Delta Q = W$

= Area

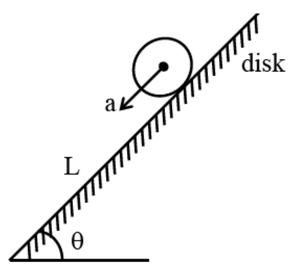
$= \pi r_1 \cdot r_2$

$= \pi \times (10 \times 10^3) \times (10 \times 10^{-3})$

$\Delta Q = 100\ \pi$

$\therefore$ Ans. = 100

26.[2]



If disk slips on inclined plane, then it's acceleration

$a_1 = g\sin\theta$

$$L = \frac{1}{2}a_1 t_1^2$$

$$\Rightarrow t_1 = \sqrt{\frac{2L}{a_1}} \quad \text{..... (i)}$$

If disk rolls on inclined plane, its acceleration,

$$a_2 = \frac{g\sin\theta}{1 + \frac{I}{mR^2}}$$

$$a_2 = \frac{g\sin\theta}{1 + \frac{MR^2}{2mR^2}}$$

$$a_2 = \frac{2}{3}g\sin\theta$$

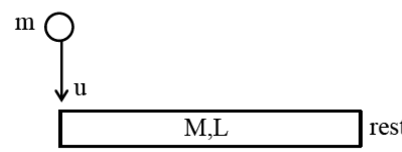
Now, $L = \frac{1}{2}a_2 \cdot t_2^2$

$$\Rightarrow t_2 = \sqrt{\frac{2L}{a_2}} \quad \text{.....(ii)}$$

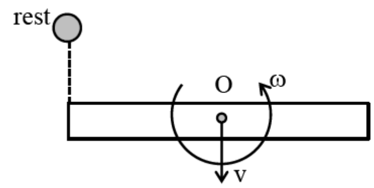
Now $\frac{t_2}{t_1} = \sqrt{\frac{a_1}{a_2}} = \sqrt{\frac{3}{2}}$

$\Rightarrow x = 2$

27.[4]



Just before collision



Just after collision

From momentum conservation, $P_1^0 = P_f$

$mu = Mv$ …. (i)

From angular momentum conservation about O,

$$mu \cdot \frac{L}{2} = \frac{ML^2}{12}\omega$$

$$\Rightarrow \omega = \frac{6mu}{ML} \quad \text{.... (ii)}$$

From $e = \frac{\text{R.V.S.}}{\text{R.V.A.}}$

$$1 = \frac{v + \frac{\omega L}{2}}{u}$$

$$v + \frac{\omega L}{2} = u$$

$$v + \frac{3mu}{M} = u$$

$$\frac{mu}{M} + \frac{3mu}{M} = u$$

$$\frac{4mu}{M} = u$$

$$\frac{m}{M} = \frac{1}{4}$$

X = 4

28.[25] For simple microscope

$$m = 1 + \frac{D}{f_0}$$

$$6 = 1 + \frac{D}{f_0}$$

$$5 = \frac{25}{f_0}$$

$f_0 = 5cm$

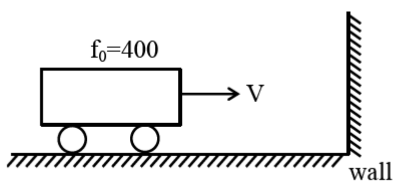
for compound microscope

$$m = \frac{\ell . D}{f_0 \cdot f_e}$$

$$12 = \frac{60 \times 25}{5 \cdot f_e}$$

$f_e = 25cm$

29.[132]



Wall as an observer

Frequency received by wall

$$f_1 = f_0\left(\frac{C}{C-V}\right)$$

Again wall as a source

Frequency received by observer on car

$$f_2 = f_1\left(\frac{C+V}{C}\right)$$

$$f_2 = f_0\left(\frac{C+V}{C-V}\right)$$

$$500 = 400\left(\frac{C+V}{C-V}\right)$$

$$\frac{5}{4} = \frac{C+V}{C-V}$$

$C = 9V$

$$V = \frac{C}{9} = \frac{330}{9} m/s$$

$$V = \frac{330}{9} \times \frac{18}{5} = 132 km/hr$$

30.[40] Maximum amplitude

$A_{max} = A_m + A_C$

$\Rightarrow V_{max} = V_m + V_C$

$200 = V_m + 160$

$V_m = 40$

$\therefore$ Peak voltage $A_m = 40$

Ans. 40

CHEMISTRY

Section -A

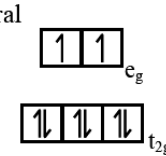
31.[2]

1) $NiCl_2.6H_2O$

$Ni^{+2} \rightarrow [Ar]_{18}3d^84s^0$

C.N. = 6 octahedral

splitting



Hybridisation sp^3d^2

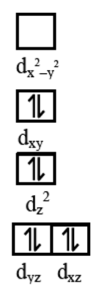
2) $K_2[Ni(CN)_4]$

C.N. 4 $\quad Ni^{+2} \rightarrow [Ar]_{18}3d^84s^0$

$CN^- \rightarrow$ Strong field ligand

Hybridisation dsp^2

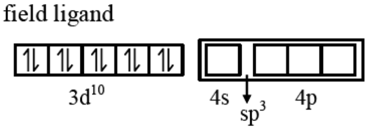
Square planar splitting



3) $Ni(CO)_4$

CO - Strong field ligand

$Ni \rightarrow [Ar]_{18}$

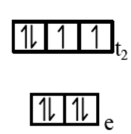


4) $Na_2[NiCl_4]$ $\quad Ni \rightarrow [Ar]_{18}3d^84s^0$

$Cl^{\ominus} \rightarrow$ weak field ligand

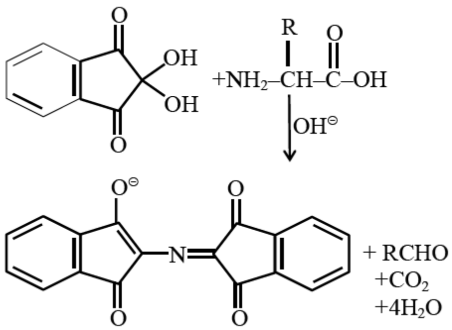
C.N. 4

tetrahedral splitting



Hybridisation sp^3

32.[4]



Ninhydrin Test

33.[4] Chlorine gas was used earlier for bleaching paper. These days, hydrogen peroxide (H_2O_2) with suitable catalyst.

Tetra chlroroethene ($Cl_2C{=}CCl_2$) was earlier used as solvent for dry cleaning. The compound contaminates the ground water and is also a suspected carcinogen. Replacement of halogenated solvent by liquid CO_2 will result in less harm to groundwater.

Hence given statement (4) is correct.

34.[3]

$[NiCl_4]^{2-} > [Ni(H_2O)_6]^{2+} > [Ni(CN)_4]^{2-}$

Splitting energy order	Δ_t	$<$	Δ_0	$<$	Δ_{sq}

absorbed energy order $[NiCl_4]^{2-} < [Ni(H_2O)_6]^{2+} < [Ni(CN)_4]^{2-}$

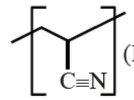
intensity of colour of compound $[NiCl_4]^{2-} > [Ni(H_2O)_6]^{2+} > [Ni(CN)_4]^{2-}$

35.[2]
(1) $B(OH)_3$ acidic and H_3PO_3 acidic
(2) $B(OH)_3$ acidic and $Al(OH)_3$ amphoteric
(3) NaOH basic and $Ca(OH)_2$ basic
(4) $Be(OH)_2$ amphoteric and $Al(OH)_3$ amphoteric

36.[1] in BrO_4^-, Br is in highest oxidation state (+7), So it cannot oxidise further hence it cannot show disproportionation reaction.

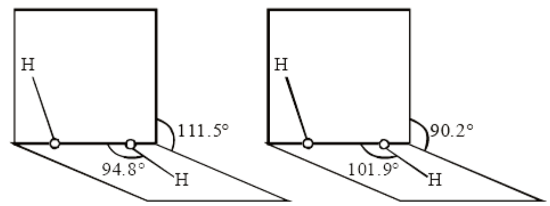
37.[2] Hence given statement (A) is not correct
But statement (B) is correct

38.[1] $\rightarrow$ orlon fibers are made up of Polyacrylonitrile



(PAN)

39.[4]



(a) Gas phase (a) Solid phase

(a) H_2O_2 structure in gas phase, dihedral angle is 111.5°. (b) H_2O_2 structure in solid phase at 110K, dihedral angle is 90.2°.

Hence given statement (A) is not correct But statement (B) is correct.

40.[1] $4HNO_3 + P_4O_{10}$

$\downarrow$

$2N_2O_5 + (HPO_3)_4$

Ans. N_2O_5 is acidic in nature.

41.[3]

$\underset{\substack{X \\ \text{(Anion)}}}{NO_3^-} + \underset{\text{(conc.)}}{H_2SO_4} \rightarrow \underset{\text{Brown fumes}}{NO_2\uparrow} + H_2O$

$\underset{\text{Sol}}{FeSO_4} + \underset{\text{conc}}{H_2SO_4} + \underset{X}{NO_3^-}$

$\downarrow$

$\underset{\text{(Dark Brown ring)}}{[Fe(H_2O)_5(NO)SO_4]}$

$\underset{\substack{X \\ \text{(cation)}}}{Cu^{2+}} + \underset{\text{(Group–II reagent)}}{(\text{dil HCl} + H_2S)}$

$\downarrow$

$CuS\downarrow$ (Black ppt) (Y)

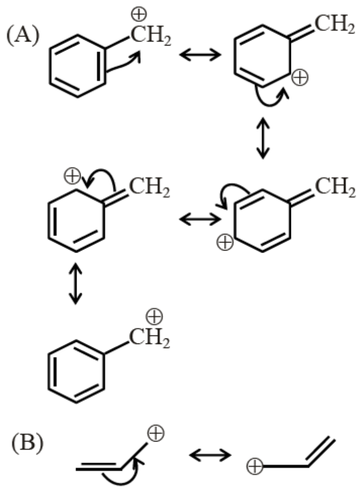
$\underset{(Y)}{CuS} \xrightarrow[HNO_3]{Conc^n} \overset{\text{Soluble}}{Cu(NO_3)_2} + NO_2 + S + H_2O$

$\downarrow$ Excess NH_4OH Sol^n

$[Cu(NH_3)_4]^{2+}$
Deep blue colour solution.

$\therefore X \rightarrow Cu(NO_3)_2$

42.[3] (A) and (B) only in Resonance



43.[4] (A) $2Mg + O_2 \rightarrow 2MgO$ (Diamagnetic)

(B) $2Na + O_2 \rightarrow Na_2O$ (Diamagnetic)

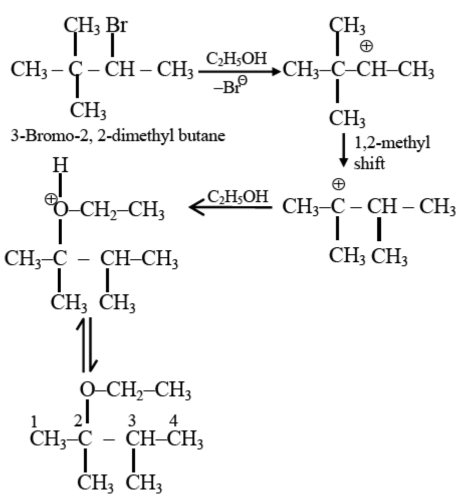
$2Na + \underset{(excess)}{O_2} \rightarrow Na_2O_2$ (Diamagnetic)

(C) $2Ca + O_2 \rightarrow 2CaO$ (Diamagnetic)

$Ca + O_2 \rightarrow CaO_2$ (Diamagnetic)

(D) $K + \underset{(excess)}{O_2} \rightarrow KO_2$ (Paramagnetic)

44.[3]



2 Ethoxy –2,3–dimethyl butane

45.[2] Zinc can be purified economically by fractional distillation.

46.[3]

$$\underset{\substack{1^\circ \text{ amine} \\ (A)}}{R\text{–}NH_2} \xrightarrow{CHCl_3} \underset{\substack{(B) \\ \text{(Isonitrile)}}}{R\text{–}N{\equiv}C} \xrightarrow[(HCl)]{H_3O^{\oplus}} \underset{(C)}{R\text{-}NH_2} + HCOOH$$

47.[1] The phenomenon of scattering of light by colloidal particles as a result of which the path of the beam becomes visible is called a tyndall effect. smaller the diameter and similar the magnitude of refractive indices, lesser is the scattering and hence the tyndall effect and viced-versa.

The diameter of the dispersed phase particle should not be smaller than the wavelength of light used because they won't be able to scatter the light so, therefore, the diameter of the dispersed particles should be equal or not much smaller than the wavelength of the light used.

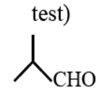
2. The refractive indies (i.e. the ratio of the velocity of light in vacuum to the velocity of light in any medium) of the dispersed phase and the dispersion medium should differ greatly in magnitude than only the particles will be able to scatter the light and tyndall effect will be obersved. On the other hand, if the refractive indices of the dispersed phase and dispersion medium are almost similar in magnitude, then there will be no scattering of light and hence, therefore, no tyndall effect effect is observed.

Hence answer A and E are correct.

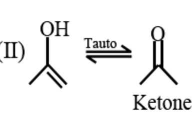
48.[1] Amylose is a linear chain polymer of α-D-glucose while amylopectine is branched chain polymer of α-D-glucose.

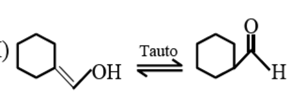
49.[3] Aldehydes give ⊕ve Tollen's test (silver mirror test)

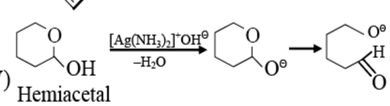
test) Tollen's test

(I)  Positive

Negative

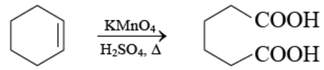
(II) 

(III)  Positive

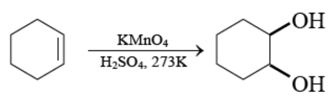
(IV)  Positive

Hemiacetal

50.[4]



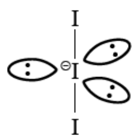
dicarboxylic acid
(A)



Diol
(B)

Section -B

51.[3] I_3^- :



The number of lone pairs of electron on the central atom is 3.

52.[226]

We known that no. of moles = V_{litre} × Molarity
& No. of millimoles = V_{ml} × Molarity
so millimoles of NaOH = 250 × 0.5
= 125
Millimoles of HCl = 500 × 1 = 500
Now reaction is

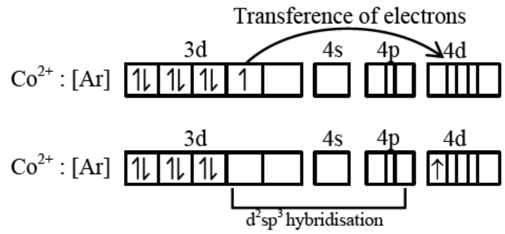
	NaOH	+ HCl	→ NaCl	+ H_2O
t = 0	125	500	0	0
t = t	0	375	125	125

so millimoles of HCl left = 375
Moles of HCl = 375×10^{-3}
No. of HCl molecules
$= 6.022 \times 10^{23} \times 375 \times 10^{-3}$
$= 225.8 \times 10^{21}$
$\simeq 226 \times 10^{21} = 226$

53.[0] Ga^+ : $1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^6\ 3d^{10}\ 4s^2$
The azimuthal quantum number for the valence electrons (4s-subshell) of Ga^+ ion is zero(0).

54.[2] $[Co(CN)_6]^{4-}$
$x + 6 \times (-1) = -4$
$\boxed{x = \pm 2}$

Co^{2+} : $[Ar]3d^7$
and CN^- is a strong field ligand which can pair electron of central atom.



It has one unpaired electron (n) in 4d-subshell.
So spin only magnetic moment (μ) = $\sqrt{n(n+2)}$ BM
where n = number of unpaired electrons.
$\mu = \sqrt{3}$ B.M.
$\boxed{\mu = 1.73\,BM}$

55.[17228]

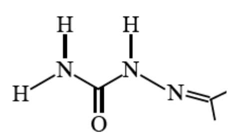
Official Ans. by NTA (172)
$2SO_{2(g)} + O_{2(g)} = 2SO_{3(g)}$

$$K_P = \frac{(pSO_{3(g)})^2}{pSO_2(g)} \times pO_{2(g)}$$

$$= \frac{43 \times 43}{45 \times 45} \times 530\ Pa^{-1}$$

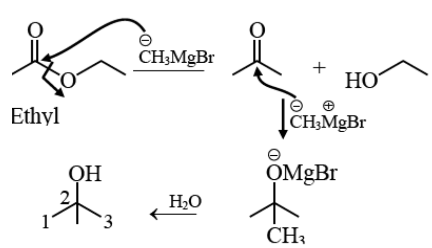
$= 172.28 \times 10^{-5}\ Pa^{-1}$
= 172.28 atm
= 17228 × 10–2 atm
Ans is 17228

56.[3]



Semicarbazone molecule of acetone

57.[2]



2-Methylpropan-2-ol

58.[106] As the unit of rate constant is min^{-1} so it must be a first order reaction

$K \times t = 2.303 \log A_0/At$

in 1 min 10% is in activated so tabing

$A_0 = 100$ $A_t = 90$ in 1 min

So $K \times 1 = 2.303 \times \log\frac{100}{90}$

$= 2.303 \times (\log 10 - 2\log 3)$

$= 2.303 \times (1 - 2 \times 0.477)$

$= 0.10593$

$= 105.93 \times 10^{-3}$

≈ 106

59.[667] 1 mole glucose give 2700 kJ energy so mole of glucose needed for 10^5 kJ energy

$= \frac{10000}{2700} = 370$ moles

wt. of glucose = 3.10 × 180 = 666.666

≈ 667 gm

$\frac{Y_{Benzene}}{Y_{MB}} = \frac{P_B^0 X_B}{P_{MB}^0 X_{MB}} = \frac{70\times 1}{20\times 1} = \frac{7}{2}$

$Y_{Benzene} = \frac{7}{9} = 77.77\times 10^{-2}$

$= 78\times 10^{-12}$

60.[78] $P_B^0 = 40$ $P_T^0 = 20$ $K_B = 0.5 = K_T$

Now, $y_B = \frac{K_B P_B^0}{K_B P_B^0 + K_T P_T^0}$

$= \frac{70\times 0.5}{70\times 0.5 + 20\times 0.5}$

MATHEMATICS

Section -A

61.[2]

p	q	~p	~q	p∧~q	q∨~p	(p∧~q) ⇒ (p∨~p)	p⇒q
T	F	F	T	T	F	F	F
F	T	T	F	F	T	T	T
T	T	F	F	F	T	T	T
F	F	T	T	F	T	T	T

∴ (p∧~q) ⇒ (q∨~p)

≡ p ⇒ q

So, option (2) is correct

62.[2] a > 0

Let $n \le a < n + 1$, $n \in W$

∴ a = [a] + {a}

⇓ ⇓

GIF Fractional part

Here [a] = n

Now, $\int_0^a e^{x-[x]}dx = 10e - 9$

$\Rightarrow \int_0^a e^{[x]}dx + \int_0^a e^{x-[x]}dx = 10e - 9$

$\therefore\ n\int_0^1 e^x dx + \int_n^a e^{x-n}dx = 10e - 9$

$\Rightarrow n(e - 1) + (e^{a-n} - 1) = 10e - 9$

$\therefore$ n = 0 and {a} = $\log_e 2$

So, a = [a] + {a} = $(10 + \log_e 2)$

⇒ option (2) is correct

63.[1] Let other two numbers be a, (21 – a)

Now,

$10.25 = \frac{(4+16+25+49+a^2+(21-a)^2)}{6} - (6.5)^2$

(Using formula for variance)

$6(10.25) + 6(6.5)^2 = 94 + a^2 + (21 - a)^2$

$a^2 + (21 - a)^2 = 221$

a = 10 and (21 – a) = 21 – 10 = 11

So, remaining two observations are 10, 11.

⇒ Option (1) is correct.

64.[3] **Official Ans. by NTA (2)**

Let $I = 2\int_0^1 \underbrace{\ln(\sqrt{1-x}+\sqrt{1+x})}_{(I)}\ \underbrace{1}_{(II)}\ dx$

(I.B.P.)

$\therefore\ I = 2\Big[(x.\ln(\sqrt{1-x}+\sqrt{1-x}))_0^1$

$-\int_0^1 x\left(\frac{1}{\sqrt{1-x}+\sqrt{1+x}}\right)\cdot\left(\frac{1}{2\sqrt{1+x}} - \frac{1}{2\sqrt{1-x}}\right)dx\Big]$

$2(\ln\sqrt{2} - 0) - \frac{2}{2}\int_0^1 \frac{x\sqrt{1-x} - \sqrt{1+x}dx}{(\sqrt{1-x}+\sqrt{1+x})\sqrt{1-x^2}}$

$= (\log_e 2) - \int_0^1 \frac{x(2-2\sqrt{1-x^2})}{-2x\sqrt{1-x^2}}dx$

(After rationalisation)

$= (\log_e 2) + \int_0^1 \left(\frac{1-\sqrt{1-x^2}}{\sqrt{1-x^2}} \right) dx$

$= (\log_e 2) + (\sin^{-1} x)_0^1 - 1$

$= \log_e 2 + \left(\frac{\pi}{2} - 0 \right) - 1$

$\therefore\ I = (\log_e 2) + \frac{\pi}{2} - 1$

$\Rightarrow$ option (3) is correct

65.[3] As, $(\alpha^2 + \sqrt{3}) = -(3)^{1/4} \cdot \alpha$

$\Rightarrow\ (\alpha^4 + 2\sqrt{3}\alpha^2 + 3) = \sqrt{3}\alpha^2$

(On squaring)

$\therefore\ (\alpha^4 + 3) = (-)\sqrt{3}\alpha^2$

$\Rightarrow \alpha^8 + 6\alpha^4 + 9 = 3\alpha^4$ (Again squaring)

$\therefore\ \alpha^8 + 3\alpha^4 + 9 = 0$

$\Rightarrow \alpha^8 = -9 - 3\alpha^4$

(multiply by α^4)

So, $\alpha^{12} = -9\alpha^4 - 3\alpha^8$

$\therefore\ \alpha^{12} = -9\alpha^4 - 3(-9-3\alpha^4)$

$\Rightarrow \alpha^{12} = \cancel{-9\alpha^4} + 27 + \cancel{9\alpha^4}$

Hence, $\boxed{a^{12} = (27)^2}$

$\Rightarrow (\alpha^{12})^8 = (27)^8$

$\Rightarrow \alpha^{96} = (3)^{24}$

similarly $\beta^{96} = (3)^{24}$

$\therefore\ \alpha^{96}(\alpha^{12}-1) + \beta^{96}(\beta^{12}-1) = (3)^{24} \times 52$

$\Rightarrow$ Option (3) is correct

66.[1] $A = \begin{bmatrix} 2 & 3 \\ a & 0 \end{bmatrix}, a \in \mathbf{R}$

and $P = \frac{A + A^T}{2} = \begin{bmatrix} 2 & \frac{3+a}{2} \\ \frac{a+3}{2} & 0 \end{bmatrix}$

and $Q = \frac{A - A^T}{2} = \begin{bmatrix} 0 & \frac{3-a}{2} \\ \frac{a-3}{2} & 0 \end{bmatrix}$

As, del (Q) = 9

$\Rightarrow (a-3)^2 = 36$

$\Rightarrow a = 3 \pm 6$

$\therefore\ \boxed{a = 9, -3}$

$\therefore\ \det.(P) = \begin{vmatrix} 2 & \frac{3+a}{2} \\ \frac{a+3}{2} & 0 \end{vmatrix}$

$= 0 - \frac{(a-3)^2}{4} = 0$ for $a = -3$

$= 0 - \frac{(a-3)^2}{4} = -\frac{1}{4}(12)(12)$, for $a = 9$

$\therefore$ Modulus of the sum of all possible values of det. (P) = |–36| + |0| = 36 Ans.

$\Rightarrow$ Option (1) is correct

67.[2] Official Ans. by NTA (3)

As $|z\omega| = 1$

$\Rightarrow$ if $|z| = r$, then $|\omega| = \frac{1}{r}$

Let arg (z) = θ

$\therefore\ \arg(\omega) = \left(\theta - \frac{3\pi}{2} \right)$

So, $z = re^{i\theta}$

$\Rightarrow\ \bar{z} = re^{i(-\theta)}$

$\omega = \frac{1}{r} e^{i\left(\theta - \frac{3\pi}{2}\right)}$

Now, consider

$\frac{1 - 2\bar{z}w}{1 + 3\bar{z}\omega} = \frac{1 - 2e^{i\left(-\frac{3\pi}{2}\right)}}{1 + 3e^{i\left(-\frac{3\pi}{2}\right)}} = \left(\frac{1-2i}{1+3i} \right)$

$= \frac{(1-2i)(1-3i)}{(1+3i)(1-3i)} = -\frac{1}{2}(1+i)$

$\therefore$ prin arg $\left(\frac{1 - 2\bar{z}\omega}{1 + 3\bar{z}\omega} \right)$

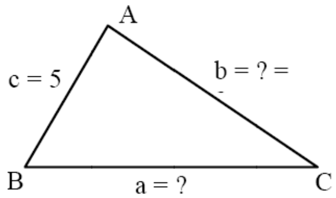
$=$ prin arg $\left(\frac{1 - 2\bar{z}\omega}{1 + 3\bar{z}\omega} \right)$

$= \left(-\frac{1}{2}(1+i) \right)$

$= -\left(\pi - \frac{\pi}{4} \right) = \frac{-3\pi}{4}$

So, option (2) is correct

68.[3]



As, $\cos B = \frac{3}{5} \Rightarrow \boxed{B = 53°}$

As, $R = 5 \Rightarrow \frac{c}{\sin c} = 2R$

$\Rightarrow \frac{5}{10} = \sin c \Rightarrow \boxed{C = 30°}$

Now, $\frac{b}{\sin B} = 2R \Rightarrow \boxed{b = 2(5)\left(\frac{4}{5}\right) = 8}$

Now, by cosine formula

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

$$\Rightarrow \frac{3}{5} = \frac{a^2 + 25 - 64}{2(5)a}$$

$$\Rightarrow a^2 - 6a - 3g = 0$$

$$\therefore\ a = \frac{6 \pm \sqrt{192}}{2} = \frac{6 \pm 8\sqrt{3}}{2}$$

$\Rightarrow \boxed{3 + 4\sqrt{3}}$ (Reject $a = 3 - 4\sqrt{3}$)

Now, $\Delta = \frac{abc}{4R} = \frac{(3+4\sqrt{3})(8)(5)}{4(5)} = 2(3 + 4\sqrt{3})$

$\Rightarrow \Delta = (6 + 8\sqrt{3})$

$\Rightarrow$ Option (3) is correct

69.[3] For domain,

$$\frac{|[x]| - 2}{|[x]| - 3} \geq 0$$

Case : I When $|[x]| - 2 \geq 0$ and $|[x]| - 3 > 0$

$\therefore x \in (-\infty, -3) \cup [4, \infty)$ ...(1)

Case II : When $|[x]| - 2 \leq 0$ and $|[x]| - 3 < 0$

$\therefore x \in [-2, 3)$ ...(2)

So, from (1) and (2)

We get

Domain of function

$= (-\infty, -3) \cup [-2, 3) \cup [4, \infty)$

$\therefore (a + b + c) = -3 + (-2) + 3 = -2\ (a < b < c)$

$\Rightarrow$ Option (3) is correct

70.[1] We have

$$\frac{dy}{dx} = \frac{x\left(\frac{y}{x}.\tan\frac{y}{x} - 1\right)}{x \tan\frac{y}{x}}$$

$$\therefore\ \frac{dy}{dx} = \frac{y}{x} - \cot\left(\frac{y}{x}\right)$$

Put $\frac{y}{x} = v$

$\Rightarrow y = vn$

$$\therefore\ \frac{dy}{dx} = v + \frac{ndv}{dx}$$

Now, we get

$$v + n\frac{dv}{dx} = v - \cot(v)$$

$$\Rightarrow \int (\tan)dv = -\int \frac{dx}{x}$$

$$\therefore\ \ell n\left|\sec\left(\frac{y}{x}\right)\right| = -\ell n\,|x| + c$$

As $\left(\frac{1}{2}\right) = \left(\frac{y}{x}\right) \Rightarrow \boxed{C = 0}$

$$\therefore\ \sec\left(\frac{y}{x}\right) = \frac{1}{x}$$

$$\Rightarrow \cos\left(\frac{y}{x}\right) = x$$

$\therefore y = x\cos^{-1}(x)$

So, required bounded area

$$= \int_0^{1/\sqrt{2}} \underset{(II)}{x}\ (\underset{(I)}{\cos^{-1} x})\,dx = \left(\frac{\pi - 1}{8}\right)$$

(I.B.P.)

$\therefore$ option (1) is correct

71.[2] $(1-x)^{100} \cdot (x^2 + x + 1)^{100} \cdot (1-x)$

$= ((1-x)(x^2 + x + 1))^{100}(1-x)$

$= ((1^3 - x^3)(x^2 + x + 1))^{100}(1-x)$

$= (1^3 - x^3)^{100}(1 - x)$

$= (1 - x^3)^{100}(1 - x)$

$$= \underbrace{(1 - x^3)^{100}}_{\text{No term of } x^{256}} - \underbrace{x(1 - x^3)^{100}}_{\text{We find cofficient of } x^{255}}$$

Required coefficient $(-1) \times (-{}^{100}C_{85})$

$= {}^{100}C_{85} = {}^{100}C_{15}$

72.[4] $A = \begin{bmatrix} 1 & -x & 2x+1 \\ -x & 1 & -x \\ 2x+1 & -x & 1 \end{bmatrix}$

$|A| = 4x^3 - 4x^2 - 4x = f(x)$

$f(x) = 4(3x^2 - 2x - 1) = 0$

$\Rightarrow x = 1;\ x = \frac{-1}{3}$

$\therefore \underbrace{f(1) = -4}_{\text{min}}\ ;\ \underbrace{f\left(-\frac{1}{3}\right) = \frac{20}{27}}_{\text{max.}}$

Sum $= -4 + \frac{20}{27} = -\frac{88}{27}$

73.[4] $|\vec{a}| = 3 = a\ ;\ \vec{a} \times \vec{c} = c$

Now $|\vec{c} - \vec{a}| = 2\sqrt{2}$

$\Rightarrow c^2 + a^2 - 2\vec{c} \cdot \vec{a} = 8$

$\Rightarrow c^2 + 9 - 2(c) = 8$

$\Rightarrow c^2 - 2c + 1 = 0 \Rightarrow c = 1 = |\vec{c}|$

Also, $\vec{a} \times \vec{b} = 2\hat{i} - 2\hat{j} + \hat{k}$

Given $(\vec{a} \times \vec{b}) = |\vec{a} \times \vec{b}||\vec{c}|\sin\frac{\pi}{6}$

$= (3)(1)(1/2)$

$= 3/2$

74.[4] $\tan^{-1}\sqrt{x^2 + x)} + \sin^{-1}\sqrt{x^2 + x + 1} = \frac{\pi}{4}$

For equation to be defined

$x^2 + x \geq 0$

$\Rightarrow x^2 + x + 1 \geq 1$

$\therefore$ only possibility that the equation is defined

$x^2 + x = 0 \Rightarrow x = 0, x = -1$

None of these value satisfy

$\therefore$ No of roots = 0

75.[2] $e^x\sqrt{1-y^2}\,dx + \frac{y}{x}dy = 0$

$\Rightarrow e^x\sqrt{1-y^2}\,dx + \frac{-y}{x}dy$

$\int \frac{-y}{\sqrt{1-y^2}}dy = \int \underset{\text{II}}{e^x}\ \underset{1}{x}\,dx$

$\Rightarrow \sqrt{1-y^2} = e^x(x-1) + c$

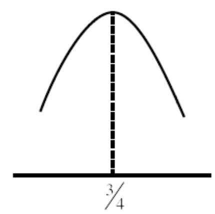
Given : At $x = 1, y = -1$

$\Rightarrow 0 = 0 + c \Rightarrow c = 0$

$\therefore \sqrt{1-y^2} = e^x(x-1)$

At $x = 3\ \ 1 - y^2 = (e^3\ \ 2)^2 \Rightarrow y^2 = 1 - 4e^6$

76.[1]



$\frac{-B}{2A} = \frac{3}{4}$

$\Rightarrow \frac{-(6)}{2a} = \frac{3}{4}$

$\Rightarrow a = \frac{-6 \times 4}{6} \Rightarrow a = -4$

$\therefore g(x) = 4x^2 - 6x + 15$

Local max. at $x = \frac{-B}{2A} = -\frac{(-6)}{2(-4)}$

$= \frac{-3}{4}$

77.[2] Continuous at x = 0

$f(0^+) = f(0^-) \Rightarrow a - 1 = 0 - e^0$

$\Rightarrow a = 0$

Continuous at x = 1

$f(1^+) = f(1^-)$

$\Rightarrow 2(1) - b = a + (-1)$

$\Rightarrow b = 2 - a + 1 \Rightarrow b = 3$

$\therefore a + b = 3$

78.[2] AAEIIMNNOTX

---------M --------

Total words with M at fourth place $= \frac{10!}{2!2!2!}$

Total words $= \frac{11!}{2!2!2!}$

Required probability $= \frac{10!}{11!} = \frac{1}{11}$

79.[2] $D < 0$

$\Rightarrow 4(a+4)^2 - 4(-5a+64) < 0$

$\Rightarrow a^2 + 16 + 8a + 5a - 64 < 0$

$\Rightarrow a^2 + 13a - 48 < 0$

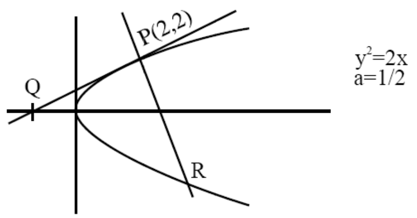
$\Rightarrow (a+16)(a-3) < 0$

$\Rightarrow a \in (-16, 3)$

$\therefore$ Possible a : $\{-5, -4, \ldots, 3\}$

$\therefore$ Required probability $= \frac{8}{36} = \frac{2}{9}$

80.[1]



Tangent at P: $y(2) = 2(1/2)(x+2)$

$\Rightarrow 2y = x + 2$

$\therefore Q = (-2, 0)$

Normal at P : $y - 2 = -\frac{(2)}{2\frac{1}{2}}(x-2)$

$\Rightarrow y - 2 = -2(x-2)$

$\Rightarrow y = 6 - 2x$

$\therefore$ Solving with $y^2 = 2x \Rightarrow R\left(\frac{9}{2} - 3\right)$

$\therefore$ Ar (ΔPQR) $= \frac{1}{2}\begin{vmatrix} 2 & 2 & 1 \\ -2 & 1 & 1 \\ \frac{9}{2} & -3 & 1 \end{vmatrix}$

$= \frac{25}{2}$ sq. units

Section -B

81.[4] $|\vec{a}+\vec{b}+\vec{c}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a}\cdot\vec{b} + \vec{a}\cdot\vec{c} + \vec{b}\cdot\vec{c}) = 3$

$|\vec{a}+\vec{b}+\vec{c}| = \sqrt{3}$

$\vec{a}\cdot(\vec{a}+\vec{b}+\vec{c})\,|\vec{a}| + |\vec{a}+\vec{b}+\vec{c}|\cos\theta$

$\Rightarrow I = \sqrt{3}\cos\theta$

$\Rightarrow \cos 2\theta = -\frac{1}{3}$

$\Rightarrow 36\cos 2\theta = \boxed{4}$

82.[910] Let $A = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix} = I + C$

Where $I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$, $C = \begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \end{pmatrix}$

$C^2 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$, $C^3 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

$= C^4 = C^5 = \ldots$

$B = 7A^{20} - 20A^7 + 2I$

$= 7(I+C)^{20} - 20(I+C)^7 + 2I$

$= 7(I + 20C + {}^{20}C_2 C^2) - 20(I + 7C + {}^7C_2 C^3) + 2I$

So

$B_{13} = 7 \times {}^{20}C_2 - 20 \times {}^7C_2 = 910$

83.[81] Equation of plane :

$\begin{vmatrix} x-1 & y-0 & z-1 \\ 1-1 & 2 & 1-1 \\ 1-0 & 0-1 & 1+2 \end{vmatrix} = 0$

$\Rightarrow 3x - z - 2 = 0$

$\vec{a} = \alpha\hat{i} + \beta\hat{j} + \gamma\hat{k} \parallel$ to $3x - z - 2 = 0$

$\Rightarrow 3\alpha - \gamma = 0$ ..... (1)

$\vec{a} \perp i + 2\hat{j} + 3\hat{k}$

$\Rightarrow \alpha + 2\beta + 3\gamma = 0$ ..... (2)

$\vec{a}(\hat{i} + \hat{j} + 3\hat{k}) = 0$

$\Rightarrow \alpha + \beta + 2\gamma = 2$ .....(3)

on solving 1, 2 & 3

$\alpha = 1, \beta = -5, \gamma = 3$

So $(\alpha - \beta + \gamma) = \boxed{81}$

84.[21] $(4^{\frac{1}{4}} + 5^{\frac{1}{6}})^{120}$

$T_{r+1} = {}^{120}C_r(2^{1/2})^{120-r}(5)^{r/6}$

for rational terms $r = 6\lambda$ $0 \le r \le 120$

so total no. of forms are 21.

85.[6] If $\vec{r} = \vec{a} + \lambda\vec{b}$ and $\vec{r} = \vec{c} + \lambda\vec{d}$

then shortest distance between two lines is

$L = \frac{(\vec{a} - \vec{c})\cdot(\vec{b} \times \vec{d})}{|b \times d|}$

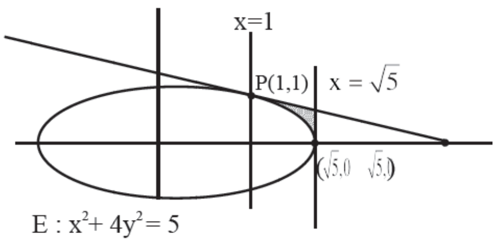
$\therefore\ \vec{a}-\vec{c}=((\alpha+4)\hat{i}+2\hat{j}+3\hat{k})$

$\dfrac{\vec{b}\times\vec{d}}{|b\times d|}=\dfrac{(2\hat{i}+2\hat{j}+\hat{k})}{3}$

$\therefore\ ((\alpha+4)\hat{i}+2\hat{j}+3\hat{k})\cdot\dfrac{(2\hat{i}+2\hat{j}+\hat{k})}{3}=9$

or $\alpha = 6$

86.[1]



E : $x^2+4y^2=5$

Tangent at P : $x + 4y = 5$

Required area

$$=\int_{12}^{\sqrt{5}}\left(\frac{5-x}{4}-\frac{\sqrt{5-x^2}}{2}\right)dx$$

$$=\left[\frac{5x}{4}-\frac{x^2}{8}-\frac{x}{4}\sqrt{5-x^2}-\frac{5}{2}\sin^{-1}\frac{x}{\sqrt{5}}\right]_1^{\sqrt{5}}$$

$$=\frac{5}{4}\sqrt{5}-\frac{5}{4}-\frac{5}{4}\cos^{-1}\left(\frac{1}{\sqrt{5}}\right)$$

It we assume $\alpha, \beta, \gamma, \in Q$ (Not given in question) then $\alpha=\frac{5}{4}$, $\beta=-\frac{5}{4}$ & $\gamma=-\frac{5}{4}$

$|\alpha + \beta + \gamma| = 1.25$

87.[1] $\begin{vmatrix} x+a-c & x+b & x+a \\ x-1 & x+c & x+b \\ x-b+d & x+d & x+c \end{vmatrix}=2$

$C_2 \to C_2 - C_3$

$\Rightarrow\begin{vmatrix} x-2\lambda & \lambda & x+a \\ x-1 & \lambda & x+b \\ x+2\lambda & \lambda & x+c \end{vmatrix}=2$

$R_2 \to R_2 - R_1 \quad R_3 \to R_3 - R_1$

$\Rightarrow\lambda\begin{vmatrix} x-2\lambda & 1 & x+a \\ 2\lambda-1 & 0 & \lambda \\ 4\lambda & 0 & 2\lambda \end{vmatrix}=2$

$\Rightarrow 1(4\lambda^2-4\lambda^2+2\lambda)=2$

$\Rightarrow \lambda^2 = 1$

88.[777] 15 : Players

6 : Bowlers

7 : Batsman

2 : Wicket keepers

Total number of ways for at least 4 bowlers, 5 batsman & 1 wicket keeper

$= {}^6C_4({}^7C_6\times{}^2C_1\times{}^7C_5\times{}^2C_2)+{}^6C_5\times{}^7C_5\times{}^2C_1$

$= 777$

89.[34] $y^2 = 64x$

focus : (–16, 0)

y = mx + c is focal chord

$\Rightarrow \chi = 16\text{ m}$ ...... (1)

y = mx + c is tangent to $(x + 10)^2 + y^2 = 4$

$\Rightarrow y = m(x+10)\pm 2\sqrt{1+m^2}$

$\Rightarrow c = 10m \pm 2\sqrt{1+m^2}$

$16m = 10m \pm 2\sqrt{1+m^2}$

$\Rightarrow 6m = 2\sqrt{1+m^2}\ \ (m > 0)$

$\Rightarrow 9m^2 = 1 + m^2$

$m=\dfrac{1}{2\sqrt{2}}$ & $c=\dfrac{8}{\sqrt{2}}$

$4\sqrt{2}(m+c)=4\sqrt{2}\left(\dfrac{17}{2\sqrt{2}}\right)=\boxed{34}$

90.[3] $\lim\limits_{x\to0}(2-\cos x\sqrt{\cos x})^{\frac{x+2}{x^2}}$

form : 1^∞

$= e^{\lim\limits_{x\to0}\left(\frac{1-\cos x\sqrt{\cos 2x}}{x^2}\right)\times(x+2)}$

Now $\lim\limits_{x\to0}\dfrac{1-\cos x\sqrt{\cos 2x}}{x^2}$

$$=\lim_{x\to0}\frac{\sin x\sqrt{\cos 2x}-\cos x\times\frac{1}{2\sqrt{\cos 2x}}\times(-2\sin 2x)}{2x}$$

(by L Hospital Rules)

$\lim\limits_{x\to0}\dfrac{\sin x\cos 2x+\sin 2x\cdot\cos x}{2x}$

$=\dfrac{1}{2}+1=\dfrac{3}{2}$

So, $e^{\lim\limits_{x\to0}\left(\frac{1-\cos x\sqrt{\cos 2x}}{x^2}\right)(x+2)}$

$= e^{\frac{3}{2}\times2}=e^3$

$\Rightarrow \boxed{a = 3}$