JEE MAIN ONLINE PAPER 2021

Held on March 18, 2021 (Evening)

Instructions

1. This test will be a 3 hours Test.
2. This test consists of Physics, Chemistry and Mathematics questions with equal weightage of 100 marks.
3. Each question is of 4 marks.
4. In the question paper consisting of Physics (Q.no. 1 to 30), Chemistry (Q.no. 31 to 60) and Mathematics (Q.no. 61 to 90). There are two sections for each subject (Section-A : MCQ Type & Section-B : Numerical Response Type). Section-A consists of 20 multiple choice questions & Section-B consists of 10 Numerical Value type Questions. **Candidates have a choice to Answer 5 out of the 10 numerical value answer based questions per section.**
5. There will be only one correct choice in the given four choices in Section-A. For each question 4 marks will be awarded for correct choice, 1 mark will be deducted for incorrect choice and zero mark will be awarded for not attempted question. For Section-B questions 4 marks will be awarded for correct answer and zero for unattempted and incorrect answer.
6. Any textual, printed or written material, mobile phones, calculator etc. is not allowed for the students appearing for the test.
7. All calculations/written work should be done in the rough sheet provided.

PHYSICS

Section-A

Q.1 Which of the following statements are correct?

(A) Electric monopoles do not exist whereas magnetic monopoles exist.

(B) Magnetic field lines due to a solenoid at its ends and outside cannot be completely straight and confined.

(C) Magnetic field lines are completely confined within a toroid.

(D) Magnetic field lines inside a bar magnet are not parallel.

(E) $\chi = -1$ is the condition for a perfect diamagnetic material, where χ is its magnetic susceptibility.

Choose the correct answer from the options given below :

(1) (C) and (E) only

(2) (B) and (D) only

(3) (A) and (B) only

(4) (B) and (C) only

Q.2 An object of mass m_1 collides with another object of mass m_2, which is at rest. After the collision the objects move with equal speeds in opposite direction. The ratio of the masses $m_2 : m_1$ is :

(1) 3 : 1　　(2) 2 : 1

(3) 1 : 2　　(4) 1 : 1

Q.3 For an adiabatic expansion of an ideal gas, the fractional change in its pressure is equal to (where γ is the ratio of specific heats):

(1) $-\gamma\frac{dV}{V}$　　(2) $-\gamma\frac{V}{dV}$

(3) $-\frac{1}{\gamma}\frac{dV}{V}$　　(4) $\frac{dV}{V}$

Q.4 A proton and an α-particle, having kinetic energies K_p and K_α, respectively, enter into a magnetic field at right angles. The ratio of the radii of trajectory of proton to that of α-particle is 2 : 1. The ratio of $K_p : K_\alpha$ is :

(1) 1 : 8　　(2) 8 : 1

(3) 1 : 4　　(4) 4 : 1

Q.5 A plane electromagnetic wave propagating along y-direction can have the following pair of electric field ($\vec{E}$) and magnetic field ($\vec{B}$) components.

(1) E_y, B_y or E_z, B_z (2) E_y, B_x or E_x, B_y

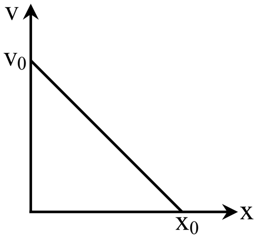
(3) E_x, B_z or E_z, B_x (4) E_x, B_y or E_y, B_x

Q.6 Consider a uniform wire of mass M and length L. It is bent into a semicircle. Its moment of inertia about a line perpendicular to the plane of the wire passing through the centre is :

(1) $\frac{1}{4}\frac{ML^2}{\pi^2}$ (2) $\frac{2}{5}\frac{ML^2}{\pi^2}$

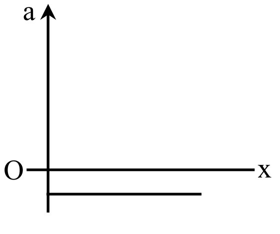
(3) $\frac{ML^2}{\pi^2}$ (4) $\frac{1}{2}\frac{ML^2}{\pi^2}$

Q.7 The velocity-displacement graph of a particle is shown in the figure.



The acceleration-displacement graph of the same particle is represented by :

(1)



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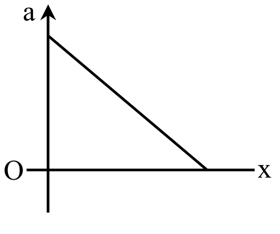

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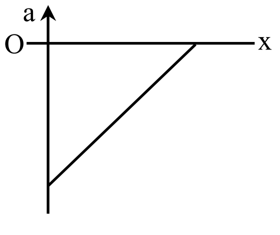

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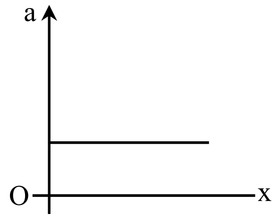
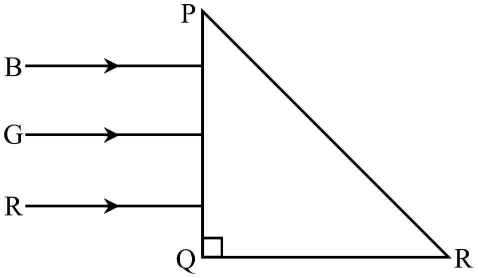


Q.8 The correct relation between α (ratio of collector current to emitter current) and β (ratio of collector current to base current) of a transistor is :

(1) $\beta = \frac{\alpha}{1+\alpha}$ (2) $\alpha = \frac{\beta}{1-\alpha}$

(3) $\beta = \frac{1}{1-\alpha}$ (4) $\alpha = \frac{\beta}{1+\beta}$

Q.9 Three rays of light, namely red (R), green (G) and blue (B) are incident on the face PQ of a right angled prism PQR as shown in figure.



The refractive indices of the material of the prism for red, green and blue wavelength are 1.27, 1.42 and 1.49 respectively. The colour of the ray(s) emerging out of the face PR is :

(1) green (2) red

(3) blue and green (4) blue

Q.10 If the angular velocity of earth's spin is increased such that the bodies at the equator start floating, the duration of the day would be approximately :

(Take : g = 10 ms^{-2}, the radius of earth, R = 6400 × 10^3 m, Take π = 3.14)

(1) 60 minutes (2) does not change

(3) 1200 minutes (4) 84 minutes

Q.11 The decay of a proton to neutron is :

(1) not possible as proton mass is less than the neutron mass
(2) possible only inside the nucleus
(3) not possible but neutron to proton conversion is possible
(4) always possible as it is associated only with β+ decay

Q.12 In a series LCR circuit, the inductive reactance (X_L) is 10 Ω and the capacitive reactance (X_C) is 4 Ω. The resistance (R) in the circuit is 6 Ω. The power factor of the circuit is :

(1) $\frac{1}{2}$ (2) $\frac{1}{2\sqrt{2}}$

(3) $\frac{1}{\sqrt{2}}$ (4) $\frac{\sqrt{3}}{2}$

Q.13 The angular momentum of a planet of mass M moving around the sun in an elliptical orbit is $\vec{L}$. The magnitude of the areal velocity of the planet is :

(1) $\frac{4L}{M}$ (2) $\frac{L}{M}$

(3) $\frac{2L}{M}$ (4) $\frac{L}{2M}$

Q.14 The function of time representing a simple harmonic motion with a period of $\frac{\pi}{\omega}$ is :

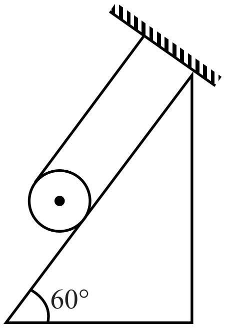
(1) $\sin(\omega t) + \cos(\omega t)$

(2) $\cos(\omega t) + \cos(2\omega t) + \cos(3\omega t)$

(3) $\sin^2(\omega t)$

(4) $3\cos\left(\frac{\pi}{4} - 2\omega t\right)$

Q.15 A solid cylinder of mass m is wrapped with an inextensible light string and, is placed on a rough inclined plane as shown in the figure. The frictional force acting between the cylinder and the inclined plane is :



[The coefficient of static friction, μ_s, is 0.4]

(1) $\frac{7}{2}$mg (2) 5 mg

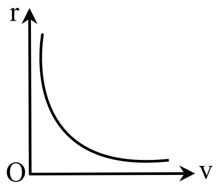
(3) $\frac{mg}{5}$ (4) 0

Q.16 The time taken for the magnetic energy to reach 25% of its maximum value, when a solenoid of resistance R, inductance L is connected to a battery, is :

(1) $\frac{L}{R}\ell n5$ (2) infinite

(3) $\frac{L}{R}\ell n2$ (4) $\frac{L}{R}\ell n10$

Q.17 A particle of mass m moves in a circular orbit under the central potential field, $U(r) = \frac{-C}{r}$, where C is a positive constant. The correct radius – velocity graph of the particle's motion is :

(1)


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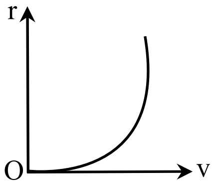

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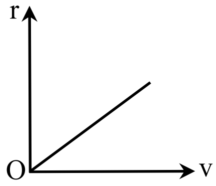

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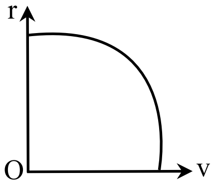


Q.18 An ideal gas in a cylinder is separated by a piston in such a way that the entropy of one part is S_1 and that of the other part is S_2. Given that $S_1 > S_2$. If the piston is removed then the total entropy of the system will be :

(1) $S_1 \times S_2$ (2) $S_1 - S_2$

(3) $\frac{S_1}{S_2}$ (4) $S_1 + S_2$

Q.19 Consider a sample of oxygen behaving like an ideal gas. At 300 K, the ratio of root mean square (rms) velocity to the average velocity of gas molecule would be :
(Molecular weight of oxygen is 32 g/mol ; $R = 8.3\ J\ K^{-1}\ mol^{-1}$)

(1) $\sqrt{\frac{3}{3}}$ (2) $\sqrt{\frac{8}{3}}$

(3) $\sqrt{\frac{3\pi}{8}}$ (4) $\sqrt{\frac{8\pi}{3}}$

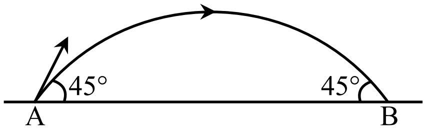
Q.20 The speed of electrons in a scanning electron microscope is 1×10^7 ms^{-1}. If the protons having the same speed are used instead of electrons, then the resolving power of scanning proton microscope will be changed by a factor of :

(1) 1837 (2) $\frac{1}{1837}$

(3) $\sqrt{1837}$ (4) $\frac{1}{\sqrt{1837}}$

Section-B

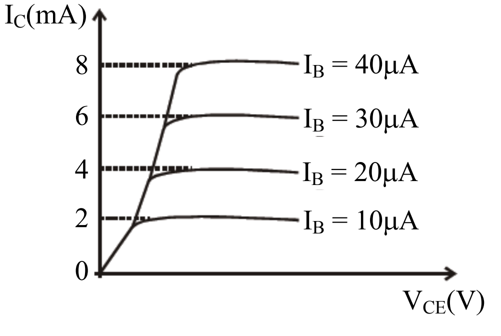
Q.21 The projectile motion of a particle of mass 5 g is shown in the figure.



The initial velocity of the particle is $5\sqrt{2}$ ms^{-1} and the air resistance is assumed to be negligible. The magnitude of the change in momentum between the points A and B is $x \times 10^{-2}$ kgms^{-1}. The value of x, to the nearest integer, is _______.

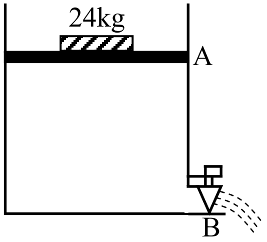
Q.22 A ball of mass 4 kg, moving with a velocity of 10 ms^{-1}, collides with a spring of length 8 m and force constant 100 Nm^{-1}. The length of the compressed spring is x m. The value of x, to the nearest integer, is _____.

Q.23 The typical output characteristics curve for a transistor working in the common-emitter configuration is shown in the figure.



The estimated current gain from the figure is.

Q.24 Consider a water tank as shown in the figure. It's cross-sectional area is 0.4 m^2. The tank has an opening B near the bottom whose cross-section area is 1 cm^2. A load of 24 kg is applied on the water at the top when the height of the water level is 40 cm above the bottom, the velocity of water coming out the opening B is v ms^{-1}. The value of v, to the nearest integer, is _______ . [Take value of g to be 10 ms^{-2}]



Q.25 A TV transmission tower antenna is at a height of 20 m. Suppose that the receiving antenna is at.

(i) ground level

(ii) a height of 5 m.

The increase in antenna range in case (ii) relative to case (i) is n%.

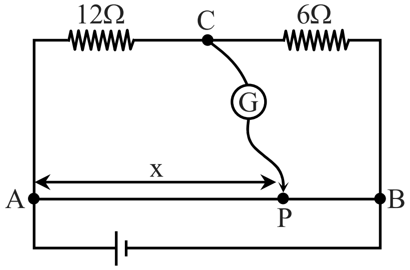
The value of n, to the nearest integer, is .

Q.26 The radius of a sphere is measured to be (7.50 ± 0.85) cm. Suppose the percentage error in its volume is x. The value of x, to the nearest x, is _______.

Q.27 An infinite number of point charges, each carrying 1 μC charge, are placed along the y-axis at y = 1 m, 2 m, 4 m, 8 m................. The total force on a 1 C point charge, placed at the origin, is $x \times 10^3$ N. The value of x, to the nearest integer, is ________.

[Take $\frac{1}{4\pi\epsilon_0} = 9 \times 10^9$ Nm^2/C^2]

Q.28 Consider a 72 cm long wire AB as shown in the figure. The galvanometer jockey is placed at P on AB at a distance x cm from A. The galvanometer shows zero deflection.



The value of x, to the nearest integer, is

Q.29 Two wires of same length and thickness having specific resistances 6Ω cm and 3Ω cm respectively are connected in parallel. The effective resistivity is ρ Ω cm. The value of ρ, to the nearest integer, is_____.

Q.30 A galaxy is moving away from the earth at a speed of 286 kms^{-1}. The shift in the wavelength of a red line at 630 nm is $x \times 10^{-10}$ m. The value of x, to the nearest integer, is_____.
[Take the value of speed of light c, as $3 \times 10^8\ ms^{-1}$]

CHEMISTRY

Section-A

Q.31 The oxidation states of nitrogen in NO, NO_2, N_2O and NO_3^- are in the order of :

(1) $NO_3^- > NO_2 > NO > N_2O$

(2) $NO_2 > NO_3^- > NO > N_2O$

(3) $N_2O > NO_2 > NO > NO_3^-$

(4) $NO > NO_2 > N_2O > NO_3^-$

Q.32 In basic medium, H_2O_2 exhibits which of the following reactions ?
(A) $Mn^{2+} \rightarrow Mn^{4+}$
(B) $I_2 \rightarrow I^-$
(C) $PbS \rightarrow PbSO_4$
Choose the most appropriate answer from the options given below :
(1) (A), (C) only
(2) (A) only
(3) (B) only
(4) (A), (B) only

Q.33 In the reaction of hypobromite with amide, the carbonyl carbon is lost as :

(1) CO_3^{2-}

(2) HCO_3^-

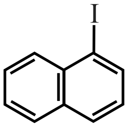
(3) CO_2

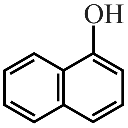
(4) CO

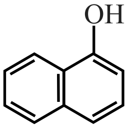
Q.34 The oxide that shows magnetic property is :

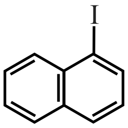
(1) SiO_2	(2) Mn_3O_4
(3) Na_2O	(4) MgO

Q.35 Main Products formed during a reaction of 1-methoxy naphthalene with hydroiodic acid are :

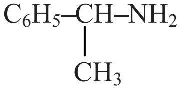
(1)  and CH_3OH

(2)  and CH_3I

(3)  and CH_3OH

(4)  and CH_3I

Q.36 Deficiency of vitamin K causes :
(1) Increase in blood clotting time
(2) Increase in fragility of RBC's
(3) Cheilosis
(4) Decrease in blood clotting time

Q.37 An organic compound "A" on treatment with benzene sulphonyl chloride gives compound B. B is soluble in dil. NaOH solution. Compound A is :
(A) $C_6H_5-N-(CH_3)_2$
(B) $C_6H_5-NHCH_2CH_3$
(C) $C_6H_5-CH_2NHCH_3$
(D) 
$C_6H_5-\underset{\displaystyle CH_3}{\underset{|}{CH}}-NH_2$

Q.38 The first ionization energy of magnesium is smaller as compared to that of elements X and Y, but higher than that of Z. the elements X, Y and Z, respectively, are :
(1) chlorine, lithium and sodium
(2) argon, lithium and sodium
(3) argon, chlorine and sodium
(4) neon, sodium and chlorine

Q.39 The secondary valency and the number of hydrogen bonded water molecule(s) in $CuSO_4 \cdot 5H_2O$, respectively, are :

(1) 6 and 4	(2) 4 and 1
(3) 6 and 5	(4) 5 and 1

Q.40 Given below are two statements :

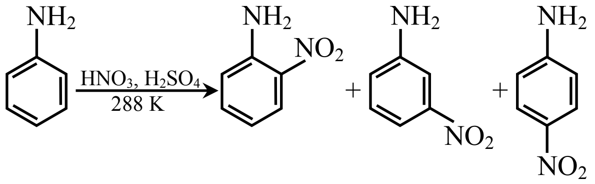
Statement I : Bohr's theory accounts for the stability and line spectrum of Li+ ion.

Statement II : Bohr's theory was unable to explain the splitting of spectral lines in the presence of a magnetic field.

In the light of the above statements, choose the most appropriate answer from the options given below :

(1) Both statement I and statement II are true.
(2) Statement I is false but statement II is true.
(3) Both statement I and statement II are false.
(4) Statement I is true but statement II is false.

Q.41



(A) **(B)** **(C)**

Consider the given reaction, percentage yield of :

(1) C > A > B (2) B > C > A
(3) A > C > B (4) C > B > A

Q.42 The charges on the colloidal CdS sol and TiO_2 sol are, respectively :

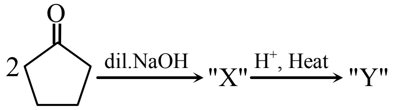
(1) positive and positive
(2) positive and negative
(3) negative and negative
(4) negative and positive

Q.43 Match List-I with List-II :

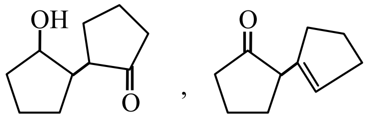
List - I	**List - II**
(Class of Chemicals)	(Example)
(A) Antifertility drug	(i) Meprobamate
(B) Antibiotic	(ii) Alitame
(C) Tranquilizer	(iii) Norethindrone
(D) Artificial Sweetener	(iv) Salvarsan

(1) A-ii, B-iii, C-iv, D-i
(2) A-iv, B-iii, C-ii, D-i
(3) A-iii, B-iv, C-i, D-ii
(4) A-ii, B-iv, C-i, D-iii

Q.44



Consider the above reaction, the product 'X' and 'Y' respectively are :

(1)


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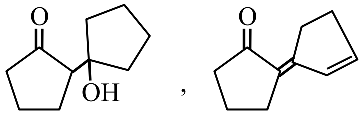

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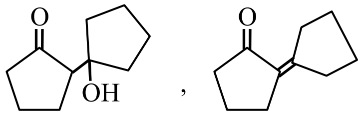

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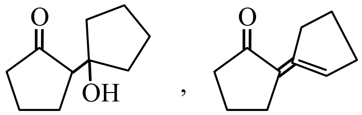


Q.45 Match List-I with List-II :

List-I	**List-II**
(A) Be	(i) Treatment of cancer
(B) Mg	(ii) Extraction of metals
(C) Ca	(iii) Incendiary bombs and signals
(D) Ra	(iv) Windows of X-ray tubes
	(v) Bearings for motor engines.

Choose the most appropriate answer the option given below :

(1) A-iv, B-iii, C-i, D-ii
(2) A-iv, B-iii, C-ii, D-i
(3) A-iii, B-iv, C-v, D-ii
(4) A-iii, B-iv, C-ii, D-v

Q.46 Given below are two statements :

Statement-I : C_2H_5OH and AgCN both can generate nucleophile.

Statement-II : KCN and AgCN both will generate nitrile nucleophile with all reaction conditions.

Choose the most appropriate option :

(1) Statement I is true but statement II is false
(2) Both statement I and statement II are true
(3) Statement I is false but statement II is true
(4) Both statement I and statement II are false

Q.47 Given below are two statements :

Statement-I : Non-biodegradable wastes are generated by the thermal power plants.

Statement-II : Bio-degradable detergents leads to eutrophication.

In the light of the above statements, choose the most appropriate answer from the option given below :

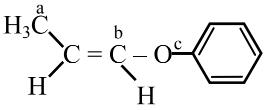
(1) Both statement I and statement II are false
(2) Statement I is true but statement II is false
(3) Statement I is false but statement II is true
(4) Both statement I and statement II are true.

Q.48 Match list-I with list-II :

List-I	**List-II**
(A) Mercury	(i) Vapour phase refining
(B) Copper	(ii) Distillation refining
(C) Silicon	(iii) Electrolytic refining
(D) Nickel	(iv) Zone refining

Choose the most appropriate answer from the option given below :
(1) A-i, B-iv, C-ii, D-iii
(2) A-ii, B-iii, C-i, D-iv
(3) A-ii, B-iii, C-iv, D-i
(4) A-ii, B-iv, C-iii, D-i

Q.49 In the following molecules,



Hybridisation of carbon a, b and c respectively are :
(1) sp^3, sp, sp (2) sp^3, sp^2, sp
(3) sp^3, sp^2, sp^2 (4) sp^3, sp, sp^2

Q.50 A hard substance melts at high temperature and is an insulator in both solid and in molten state. This solid is most likely to be a / an :
(1) Ionic solid (2) Molecular solid
(3) Metallic solid (4) Covalent solid

Section-B

Q.51 A reaction has a half life of 1 min. The time required for 99.9% completion of the reaction is ______ min. (Round off to the Nearest integer) [Use : ln 2= 0.69, ln 10 = 2.3]

Q.52 The molar conductivities at infinite dilution of barium chloride, sulphuric acid and hydrochloric acid are 280, 860 and 426 Scm^2 mol^{-1} respectively. The molar conductivity at infinite dilution of barium sulphate is ________S cm^2 mol^{-1} (Round off to the Nearest Integer).

Q.53 The number of species below that have two lone pairs of electrons in their central atom is ____ (Round off to the Nearest integer)

SF_4, BF_4^-, ClF_3, AsF_3, PCl_5, BrF_5, XeF_4, SF_6

Q.54 A xenon compound 'A' upon partial hydrolysis gives XeO_2F_2. The number of lone pair of electrons present in compound A is ______ (Round off to the Nearest integer)

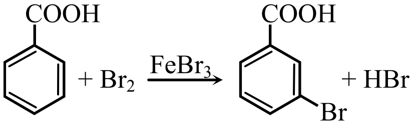
Q.55 The gas phase reaction $2A(g) \rightleftharpoons A_2(g)$ at 400 K has $\Delta G^o = + 25.2$ kJ mol^{-1}. The equilibrium constant K_C for this reaction is ______× 10^{-2}. (Round off to the Nearest integer)
[Use : R = 8.3 J $mol^{-1}K^{-1}$, ln 10 = 2.3
$\log_{10} 2 = 0.30$, 1 atm = 1 bar]
[antilog (–0.3) = 0.501]

Q.56 In Tollen's test for aldehyde, the overall number of electron(s) transferred to the Tollen's reagent formula $[Ag(NH_3)_2]^+$ per aldehyde group to form silver mirror is_______.(Round off to the Nearest integer)

Q.57 The solubility of $CdSO_4$ in water is 8.0×10^{-4} mol L^{-1}. Its solubility in 0.01 M H_2SO_4 solution is ______× 10^{-6} mol L^{-1}. (Round off to the Nearest integer) (Assume that solubility is much less than 0.01 M)

Q.58 A solute a dimerizes in water. The boiling point of a 2 molar solution of A is 100.52ºC. The percentage association of A is.____.
(Round off to the Nearest integer)
[Use : K_b for water = 0.52 K kg mol^{-1}
Boiling point of water = 100°C]

Q.59 10.0 ml of Na_2CO_3 solution is titrated against 0.2 M HCl solution. The following titre values were obtained in 5 readings. 4.8 ml, 4.9 ml, 5.0 ml, 5.0 ml and 5.0 ml Based on these readings, and convention of titrimetric estimation of concentration of Na_2CO_3 solution is ________ mM. (Round off to the Nearest integer)

Q.60 

Consider the above reaction where 6.1 g of benzoic acid is used to get 7.8 g of m-bromo benzoic acid. The percentage yield of the product is_____.

(Round off to the Nearest integer)

[Given : Atomic masses : C = 12.0u, H : 1.0u, O : 16.0u, Br = 80.0 u]

MATHEMATICS

Section-A

Q.61 Let y = y(x) be the solution of the differential equation $\frac{dy}{dx} = (y+1)\left((y+1)e^{x^2/2} - x\right), 0 < x < 2.1,$ with y(2) = 0. Then the value of $\frac{dy}{dx}$ at x = 1 is equal to :

(1) $\frac{-e^{3/2}}{(e^2+1)^2}$ (2) $-\frac{2e^2}{(1+e^2)^2}$

(3) $\frac{e^{5/2}}{(1+e^2)^2}$ (4) $\frac{5e^{1/2}}{(e^2+1)^2}$

Q.62 In a triangle ABC, if $|\overrightarrow{BC}| = 8, |\overrightarrow{CA}| = 7,$ $|\overrightarrow{AB}| = 10,$ then the projection of the vector $\overrightarrow{AB}$ on $\overrightarrow{AC}$ is equal to :

(1) $\frac{25}{4}$ (2) $\frac{85}{14}$

(3) $\frac{127}{20}$ (4) $\frac{115}{16}$

Q.63 Let the system of linear equations

$4x + \lambda y + 2z = 0$

$2x - y + z = 0$

$\mu x + 2y + 3z = 0, \lambda, \mu \in R.$

has a non-trivial solution. Then which of the following is true ?

(1) $\mu = 6, \lambda \in R$ (2) $\lambda = 2, \mu \in R$

(3) $\lambda = 3, \mu \in R$ (4) $\mu = -6, \lambda \in R$

Q.64 Let f : R – {3} → R – {1} be defined by $f(x) = \frac{x-2}{x-3}$. Let g : R → R be given as g(x) = 2x – 3. Then, the sum of all the values of x for which $f^{-1}(x) + g^{-1}(x) = \frac{13}{2}$ is equal to

(1) 7 (2) 2 (3) 5 (4) 3

Q.65 Let the centroid of an equilateral triangle ABC be at the origin. Let one of the sides of the equilateral triangle be along the straight line x + y = 3. If R and r be the radius of circumcircle and incircle respectively of ΔABC, then (R + r) is equal to :

(1) $\frac{9}{\sqrt{2}}$ (2) $7\sqrt{2}$

(3) $2\sqrt{2}$ (4) $3\sqrt{2}$

Q.66 Consider a hyperbola H : $x^2 - 2y^2 = 4$. Let the tangent at a point $P(4, \sqrt{6})$ meet the x-axis at Q and latus rectum at $R(x_1, y_1)$, $x_1 > 0$. If F is a focus of H which is nearer to the point P, then the area of ΔQFR is equal to

(1) $4\sqrt{6}$ (2) $\sqrt{6} - 1$

(3) $\frac{7}{\sqrt{6}} - 2$ (4) $4\sqrt{6} - 1$

Q.67 If P and Q are two statements, then which of the following compound statement is a tautology ?

(1) $((P \Rightarrow Q) \wedge \sim Q) \Rightarrow Q$

(2) $((P \Rightarrow Q) \wedge \sim Q) \Rightarrow \sim P$

(3) $((P \Rightarrow Q) \wedge \sim Q) \Rightarrow P$

(4) $((P \Rightarrow Q) \wedge \sim Q) \Rightarrow (P \wedge Q)$

Q.68 Let $g(x) = \int_0^x f(t)\,dt$, where f is continuous function in [0, 3] such that $\frac{1}{3} \le f(t) \le 1$ for all t ∈ [0, 1] and $0 \le f(t) \le \frac{1}{2}$ for all t ∈ (1, 3]. The largest possible interval in which g(3) lies is :

(1) $\left[-1, -\frac{1}{2}\right]$ (2) $\left[-\frac{3}{2}, -1\right]$

(3) $\left[\frac{1}{3}, 2\right]$ (4) [1, 3]

Q.69 Let S_1 be the sum of first 2n terms of an arithmetic progression. Let S_2 be the sum of first 4n terms of the same arithmetic progression. If $(S_2 - S_1)$ is 1000, then the sum of the first 6n terms of the arithmetic progression is equal to:

(1) 1000 (2) 7000

(3) 5000 (4) 3000

Q.70 Let a complex number be $w = 1-\sqrt{3}i$. Let another complex number z be such that $|zw| = 1$ and $\arg(z) - \arg(w) = \frac{\pi}{2}$. Then the area of the triangle with vertices origin, z and w is equal to :

(1) 4 (2) $\frac{1}{2}$ (3) $\frac{1}{4}$ (4) 2

Q.71 Let in a series of 2n observations, half of them are equal to a and remaining half are equal to –a. Also by adding a constant b in each of these observations, the mean and standard deviation of new set become 5 and 20, respectively. Then the value of $a^2 + b^2$ is equal to :

(1) 425 (2) 650

(3) 250 (4) 925

Q.72 Let $S_1 : x^2 + y^2 = 9$ and $S_2 : (x-2)^2 + y^2 = 1$. Then the locus of center of a variable circle S which touches S_1 internally and S_2 externally always passes through the points :

(1) $(0, \pm\sqrt{3})$ (2) $\left(\frac{1}{2}, \pm\frac{\sqrt{5}}{2}\right)$

(3) $\left(2, \pm\frac{3}{2}\right)$ (4) $(1, \pm 2)$

Q.73 Let $\vec{a}$ and $\vec{b}$ be two non-zero vector perpendicular to each other and $|\vec{a}| = |\vec{b}|$. If $|\vec{a}\times\vec{b}| = |\vec{a}|$, then the angle between the vectors $(\vec{a}+\vec{b}+(\vec{a}\times\vec{b}))$ and $\vec{a}$ is equal to :

(1) $\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$ (2) $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$

(3) $\cos^{-1}\left(\frac{1}{\sqrt{2}}\right)$ (4) $\sin^{-1}\left(\frac{1}{\sqrt{6}}\right)$

Q.74 Let in a Binomial distribution, consisting of 5 independent trials, probabilities of exactly 1 and 2 successes be 0.4096 and 0.2048 respectively. Then the probability of getting exactly 3 successes is equal to :

(1) $\frac{32}{625}$ (2) $\frac{80}{243}$

(3) $\frac{40}{243}$ (4) $\frac{128}{625}$

Q.75 Let a tangent be drawn to the ellipse $\frac{x^2}{27} + y^2 = 1$ at $(3\sqrt{3}\cos\theta, \sin\theta)$ where $\theta \in \left(0, \frac{\pi}{2}\right)$. Then the value of θ such that the sum of intercepts on axes made by this tangent is minimum is equal to :

(1) $\frac{\pi}{8}$ (2) $\frac{\pi}{4}$ (3) $\frac{\pi}{6}$ (4) $\frac{\pi}{3}$

Q.76 Define a relation R over a class of n × n real matrices A and B as "ARB iff there exists a non-singular matrix P such that $PAP^{-1} = B$". Then which of the following is true ?

(1) R is symmetric, transitive but not reflexive,

(2) R is reflexive, symmetric but not transitive

(3) R is an equivalence relation

(4) R is reflexive, transitive but not symmetric

Q.77 A pole stands vertically inside a triangular park ABC. Let the angle of elevation of the top of the pole from each corner of the park be $\frac{\pi}{3}$. If the radius of the circumcircle ot ΔABC is 2, then the height of the pole is equal to :

(1) $\frac{2\sqrt{3}}{3}$ (2) $2\sqrt{3}$

(3) $\sqrt{3}$ (4) $\frac{1}{\sqrt{3}}$

Q.78 If $15\sin^4\alpha + 10\cos^4\alpha = 6$, for some a ∈ R, then the value of $27\sec^6\alpha + 8\text{cosec}^6\alpha$ is equal to :

(1) 350 (2) 500 (3) 400 (4) 250

Q.79 The area bounded by the curve

$4y^2 = x^2(4 - x)(x - 2)$ is equal to :

(1) $\frac{\pi}{8}$ (2) $\frac{3\pi}{8}$ (3) $\frac{3\pi}{2}$ (4) $\frac{\pi}{16}$

Q.80 Let $f : R \to R$ be a function defined as

$$f(x) = \begin{cases} \dfrac{\sin(a+1)x + \sin 2x}{2x} & , \text{ if } x < 0 \\ b & , \text{ if } x = 0 \\ \dfrac{\sqrt{x+bx^3} - \sqrt{x}}{bx^{5/2}} & , \text{ if } x > 0 \end{cases}$$

If f is continuous at x = 0, then the value of a + b is equal to :

(1) $-\frac{5}{2}$ (2) –2

(3) –3 (4) $-\frac{3}{2}$

Section-B

Q.81 If $f(x)$ and $g(x)$ are two polynomials such that the polynomial $P(x) = f(x^3) + xg(x^3)$ is divisible by $x^2 + x + 1$, then $P(l)$ is equal to ______.

Q.82 Let I be an identity matrix of order 2 × 2 and $P = \begin{bmatrix} 2 & -1 \\ 5 & -3 \end{bmatrix}$. Then the value of $n \in N$ for which $P^n = 51 - 8P$ is equal to ________.

Q.83 If $\sum_{r=1}^{10} r!(r^3 + 6r^2 + 2r + 5) = \alpha(11!)$, then the value of α is equal to ________.

Q.84 The term independent of x in the expansion of $\left[\frac{x+1}{x^{2/3} - x^{1/3} + 1} - \frac{x-1}{x - x^{1/2}}\right]^{10}$, $x \neq 1$, is equal to ________.

Q.85 Let P(x) be a real polynomial of degree 3 which vanishes at x = –3. Let P(x) have local minima at x = 1, local maxima at x = –1 and $\int_{-1}^{1} P(x)\,dx = 18$, then the sum of all the coefficients of the polynomial P(x) is equal to ________.

Q.86 Let the mirror image of the point (1, 3, a) with respect to the plane $\vec{r} \cdot (2\hat{i} - \hat{j} + \hat{k}) - b = 0$ be (–3, 5, 2). Then the value of $|a + b|$ is equal to _________.

Q.87 Let $f : R \to R$ satisfy the equation $f(x + y) = f(x). f(y)$ for all $x, y \in R$ and $f(x) \neq 0$ for any $x \in R$. If the function f is differentiable at x = 0 and $f'(0) = 3$, then $\lim_{h\to 0} \frac{1}{h}(f(h) - 1)$ is equal to ____.

Q.88 Let ${}^nC_r$ denote the binomial coefficient of x^r in the expansion of $(1 + x)^n$.

If $\sum_{k=0}^{10} (2^2 + 3k)\ {}^nC_k = \alpha.3^{10} + \beta.2^{10}$, $\alpha, \beta \in R$, then $\alpha + \beta$ is equal to _______.

Q.89 Let P be a plane containing the line $\frac{x-1}{3} = \frac{y+6}{4} = \frac{z+5}{2}$ and parallel to the line $\frac{x-3}{4} = \frac{y-2}{-3} = \frac{z+5}{7}$. If the point $(1, -1, \alpha)$ lies on the plane P, then the value of $|5\alpha|$ is equal to ________.

Q.90 Let y = y(x) be the solution of the differential equation $xdy - ydx = \sqrt{(x^2 - y^2)}\,dx$, $x \geq 1$, with $y(1) = 0$. If the area bounded by the line x = 1, $x = e^{\pi}$, y = 0 and y = y(x) is $\alpha e^{2\pi} + \beta$, then the value of $10(\alpha + \beta)$ is equal to ______.

JEE MAIN ONLINE PAPER 2021

Held on March 18, 2021 (Evening)

Hints & Solutions

PHYSICS

SECTION-A

1.[1] Statement (C) is correct because, the magnetic field outside the toroid is zero and they form closed loops inside the toroid itself.

Statement (E) is correct because we know that super conductors are materials inside which the net magnetic field is always zero and they are perfect diamagnetic.

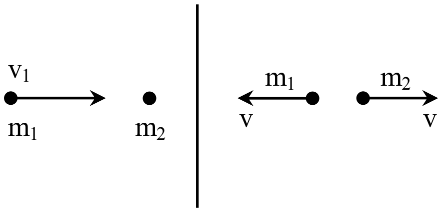
$\mu_r = 1 + \chi$

$\chi = -1$

$\mu_r = 0$

For superconductors.

2.[1]



$m_1 v_1 = -m_1 v + m_2 v$

$$v_1 = -v + \frac{m_2}{m_1} v$$

$$\frac{(v_1 + v)}{v} = \frac{m_2}{m_1}$$

$$e = \frac{2v}{v_1} = 1$$

$$v = \frac{v_1}{2}$$

$$\frac{v_1 + v_1/2}{v_1/2} = \frac{m_2}{m_1}$$

$$3 = \frac{m_2}{m_1}$$

3.[1] PV^γ = constant

Differentiating

$$\frac{dP}{dV} = -\frac{\gamma P}{V}$$

$$\frac{dP}{P} = -\frac{\gamma dV}{V}$$

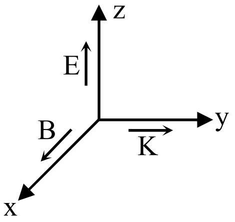
4.[4] $r = \frac{mv}{qB} = \frac{p}{qB}$ $\qquad$ $\frac{m_\alpha}{m_p} = 4$

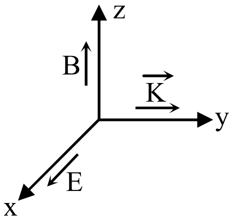
$$\frac{r_p}{r_\alpha} = \frac{p_p}{q_p}\frac{q_\alpha}{p_\alpha} = \frac{2}{1}$$

$$\frac{p_p}{p_\alpha} = \frac{2q_p}{q_\alpha} = 2\left(\frac{1}{2}\right) \Rightarrow \frac{p_p}{p_\alpha} = 1$$

$$\frac{K_p}{K_\alpha} = \frac{p_p^2}{p_\alpha^2}\frac{m_\alpha}{m_p} = 1 \times 4$$

5.[3]





6.[3] $\pi r = L \Rightarrow r = \frac{L}{\pi}$

$$I = Mr^2 = \frac{ML^2}{\pi^2}$$

7.[3] $v = -\left(\frac{v_0}{x_0}\right)x + v_0$

$$a = \frac{vdv}{dx}$$

$$a = \left[-\left(\frac{v_0}{x_0}\right)x + v_0\right]\left[-\frac{v_0}{x_0}\right]$$

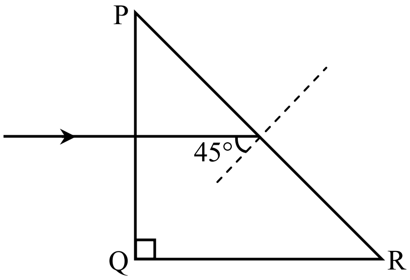
$$a = \left(\frac{v_0}{x_0}\right)^2 x - \frac{v_0^2}{x_0}$$

8.[4] $\alpha = \frac{I_C}{I_E}, \beta = \frac{I_C}{I_B} \Rightarrow I_E = I_B + I_C$

$$\alpha = \frac{I_C}{I_B + I_C} = \frac{1}{\frac{I_B}{I_C} + 1}$$

$$\alpha = \frac{1}{\frac{1}{\beta}+1} \Rightarrow \alpha = \frac{\beta}{1+\beta}$$

9.[2]



Assuming that the right angled prism is an isoceles prism, so the other angles will be 45° each.

⇒ Each incident ray will make an angle of 45° with the normal at face PR.

⇒ The wavelength corresponding to which the incidence angle is less than the critical angle, will pass through PR.

⇒ θ_C = critical angle

$$\Rightarrow \quad \theta_C = \sin^{-1}\left(\frac{1}{\mu}\right)$$

⇒ If $\theta_C \geq 45°$

the light ray will pass

$$\Rightarrow \quad (\theta_C)_{Red} = \sin^{-1}\left(\frac{1}{1.27}\right) = 51.94°$$

Red will pass.

$$\Rightarrow \quad (\theta_C)_{Green} = \sin^{-1}\left(\frac{1}{1.42}\right) = 44.76°$$

Green will not pass.

$$\Rightarrow \quad (\theta_C)_{Blue} = \sin^{-1}\left(\frac{1}{1.49}\right) = 42.15°$$

Blue will not pass.

⇒ So only red will pass through PR.

10.[4] For objects to float

$mg = m\omega^2 R$

ω = angular velocity of earth.

R = Radius of earth

$$\omega = \sqrt{\frac{g}{R}} \qquad \text{... (1)}$$

Duration of day = T

$$T = \frac{2\pi}{\omega} \qquad \text{... (2)}$$

$$\Rightarrow T = 2\pi\sqrt{\frac{R}{g}}$$

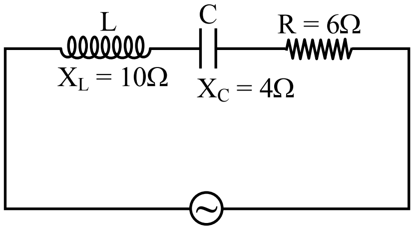
$$= 2\pi\sqrt{\frac{6400 \times 10^3}{10}}$$

$$\Rightarrow \frac{T}{60} = 83.775 \text{ minutes}$$

$$\approx 84 \text{ minutes}$$

11.[2] It is possible only inside the nucleus and not otherwise.

12.[3]

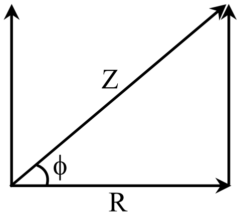


We know that power factor is cos ϕ,

$$\cos\phi = \frac{R}{Z} \qquad \text{... (1)}$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2} \qquad \text{... (2)}$$

$(\omega L - 1/\omega C)$

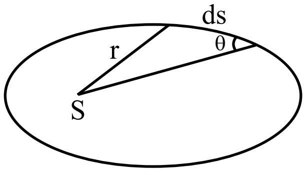


$$\Rightarrow Z = \sqrt{6^2 + (10-4)^2}$$

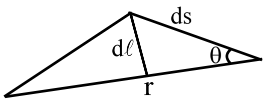
$$\Rightarrow Z = 6\sqrt{2} \mid \cos\phi = \frac{6}{6\sqrt{2}}$$

$$\cos\phi = \frac{1}{\sqrt{2}}$$

13.[4]



For small displacement ds of the planet its area can be written as



$$dA = \frac{1}{2} r d\ell$$

$$= \frac{1}{2} r ds \sin\theta$$

$$\text{A.vel} = \frac{dA}{dt} = \frac{1}{2} r \sin\theta \frac{ds}{dt} = \frac{Vr\sin\theta}{2}$$

$$= \frac{dA}{dt} = \frac{1}{2}\frac{mVr\sin\theta}{m} = \frac{L}{2m}$$

14.[4] Time period $T = \frac{2\pi}{\omega'}$

$$\frac{\pi}{\omega} = \frac{2\pi}{\omega'}$$

$\omega' = 2\omega \rightarrow$ Angular frequency of SHM

Option (3)

$$\sin^2 \omega t = \frac{1}{2}(2\sin^2 \omega t) = \frac{1}{2}(1 - \cos 2\omega t)$$

Angular frequency of $\left(\frac{1}{2} - \frac{1}{2}\cos 2\omega t\right)$ is 2ω

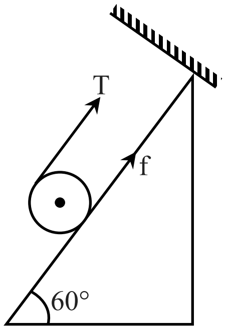
Option (4)

Angular frequency of SHM

$3\cos\left(\frac{\pi}{4} - 2\omega t\right)$ is 2ω.

So option (3) & (4) both have angular frequency 2ω but option (4) is direct answer.

15.[3]



Let's take solid cylinder is in equilibrium

$T + f = mg \sin 60$ ... (i)

$TR - fR = 0$ ... (ii)

Solving we get

$$T = f_{red} = \frac{mg\sin\theta}{2}$$

But limiting friction < required friction

$$\mu mg\cos 60° < \frac{mg\sin 60°}{2}$$

$\therefore$ Hence cylinder will not remain in equilibrium

Hence f = kinetic

$= \mu_k N = \mu_k mg\cos 60°$

$$= \frac{mg}{5}$$

16.[3] Magnetic energy $= \frac{1}{2}Li^2 = 25\%$

$$\text{ME} \Rightarrow 25\% \Rightarrow i = \frac{i_0}{2}$$

$i = i_0 (1 - R^{-Rt/L})$ for charging

$$t = \frac{L}{R}\ell n2$$

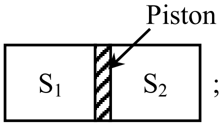
17.[1] $U(r) = \frac{-C}{r}$

$$F = -\frac{dU}{dr} = -\frac{C}{r^2}$$

$$|F| = \frac{mv^2}{r}$$

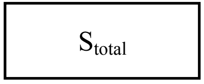
$$\frac{C}{r^2} = \frac{mv^2}{r} \Rightarrow v^2 \propto \frac{1}{r}$$

18.[4]



; $S_1 > S_2$

After piston is removed



; $S_{total} = S_1 + S_2$

19.[3] $v_{rms} = \sqrt{\frac{3RT}{M}}$

$$v_{avg} = \sqrt{\frac{8}{\pi}\frac{RT}{M}}$$

$$\frac{v_{rms}}{v_{avg}} = \sqrt{\frac{3\pi}{8}}$$

20.[1] Resolving power (RP) $\propto \frac{1}{\lambda}$

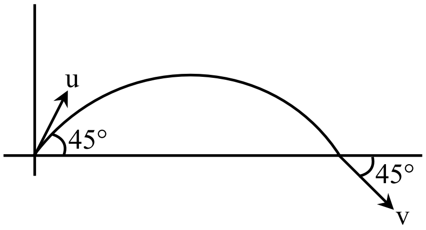
$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

$$\text{So (RP)} \propto \frac{mv}{h}$$

RP $\propto$ P $\Rightarrow$ RP $\propto$ mv $\Rightarrow$ RP $\propto$ m

SECTION-B

21.[5]



$|\vec{u}| = |\vec{v}|$ ... (1)

$$\vec{u} = u\cos 45\hat{i} + u\sin 45\hat{j} \qquad \text{... (2)}$$

$$\vec{v} = v\cos 45\hat{i} + v\sin 45\hat{j} \qquad \text{... (3)}$$

$$|\overrightarrow{\Delta P}| = |m(\vec{v} - \vec{u})| \qquad \text{... (4)}$$

$\Delta P = 2mu \sin 45°$

$$= 2 \times 5 \times 10^{-3} \times 5\sqrt{2} \times \frac{1}{\sqrt{2}}$$

$= 50 \times 10^{-3} = 5 \times 10^{-2}$

22.[6] Let's say the compression in the spring by : y. So, by work energy theorem we have

$$\Rightarrow \frac{1}{2}mv^2 = \frac{1}{2}ky^2$$

$$\Rightarrow y = \sqrt{\frac{m}{k}} \cdot v$$

$$\Rightarrow y = \sqrt{\frac{4}{100}} \times 10$$

$\Rightarrow y = 2m$

$\Rightarrow$ final length of spring = 8 – 2 = 6m

23.[200]

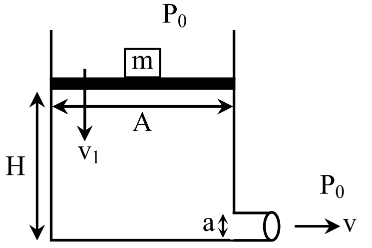
$$\beta = \frac{\Delta I_C}{\Delta I_B} = \frac{2 \times 10^{-3}}{10 \times 10^{-6}}$$

$$\beta = \frac{1}{5} \times 10^3$$

$\beta = 2 \times 10^2$

$\beta = 200$

24.[3]



m = 24 kg

A' = 0.4 m^2

a = 1 cm^2

H = 40 cm

Using Bernoulli's equation

$$\Rightarrow \left(P_0 + \frac{mg}{A}\right) + \rho g H + \cancelto{0}{\frac{1}{2}\rho v_1^2}$$

$$= P_0 + 0 + \frac{1}{2}\rho v^2 \qquad \text{... (1)}$$

$\Rightarrow$ Neglecting v_1

$$\Rightarrow v = \sqrt{2gH + \frac{2mg}{A\rho}}$$

$$\Rightarrow v = \sqrt{8 + 1.2}$$

$\Rightarrow v = 3.033$ m/s

$\Rightarrow v \simeq 3$ m/s

25.[50] Range $= \sqrt{2Rh}$

Range (i) $= \sqrt{2Rh}$

Range (ii) $= \sqrt{2Rh} + \sqrt{2Rh'}$

where h = 20m & h' = 5m

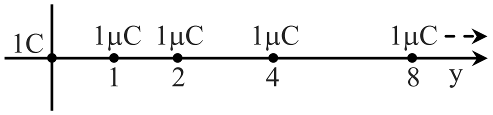
$$\text{Ans} = \frac{\sqrt{2Rh'}}{\sqrt{2Rh}} \times 100\% = \frac{\sqrt{5}}{\sqrt{20}} \times 100\% = 50\%$$

26.[34] $\because\ v = \frac{4}{3}\pi r^3$

taking log & then differentiate

$$\frac{dV}{V} = 3\frac{dr}{r} = \frac{3 \times 0.85}{7.5} \times 100 = 34\%$$

27.[12]



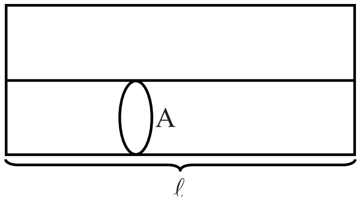
$$F = k(1C)\,(1\mu C)\left[1 + \frac{1}{2^2} + \frac{1}{4^2} + \frac{1}{8^2} + ...\right]$$

$$= 9 \times 10^3 \left[\frac{1}{1 - \frac{1}{4}}\right] = 12 \times 10^3 \text{N}$$

28.[48] In Balanced conditions

$$\frac{12}{6} = \frac{x}{72 - x} \Rightarrow x = 48 \text{ cm}$$

29.[4] $\because$ in parallel



$$R_{net} = \frac{R_1 R_2}{R_1 + R_2}$$

$$\frac{\rho\ell}{2A} = \frac{\rho_1\frac{\ell}{A} \times \rho_2\frac{\ell}{A}}{\rho_1\frac{\ell}{A} + \rho_2\frac{\ell}{A}}$$

$$\frac{\rho}{2}=\frac{6\times3}{6+3}=2\Rightarrow\rho=4$$

30.[6] $\frac{\Delta\lambda}{\lambda}c=v$

$$\Delta\lambda=\frac{v}{c}\times\lambda=\frac{286}{3\times10^5}\times630\times10^{-9}=6\times10^{-10}$$

CHEMISTRY

SECTION-A

31.[1] The oxidation states of Nitrogen in following molecules are as follows

$NO_3^- \rightarrow +5$

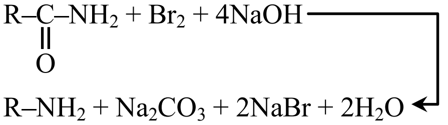
$NO_2 \rightarrow +4$

$NO \rightarrow +2$

$N_2O \rightarrow +1$

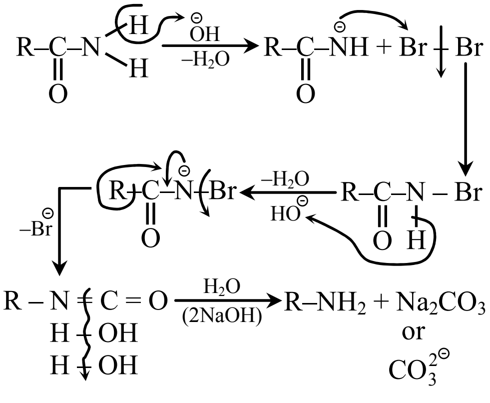
32.[4] In basic medium, oxidising action of H_2O_2.
$Mn^{2+} + H_2O_2 \rightarrow Mn^{+4} + 2OH^-$
In basic medium, reducing action of H_2O_2
$I_2 + H_2O_2 + 2OH^- \rightarrow 2I^- + 2H_2O + O_2$
In acidic medium, oxidising action of H_2O_2.
$PbS(s) + 4H_2O_2(aq) \rightarrow PbSO_4(s) + 4H_2O(\ell)$

33.[1]



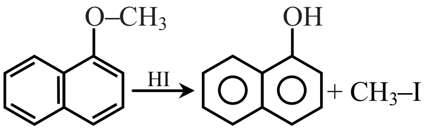
$R\text{–}\underset{\underset{O}{\|}}{C}\text{–}NH_2 + Br_2 + 4NaOH \rightarrow R\text{–}NH_2 + Na_2CO_3 + 2NaBr + 2H_2O$

Mechanism

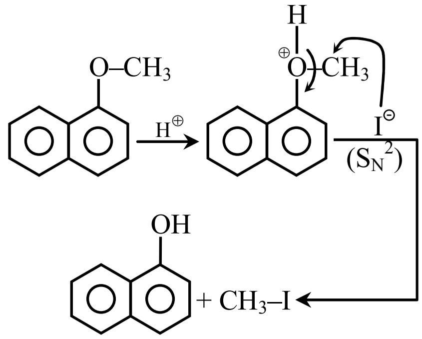


34.[2] Mn_3O_4 shows magnetic properties.

35.[2]

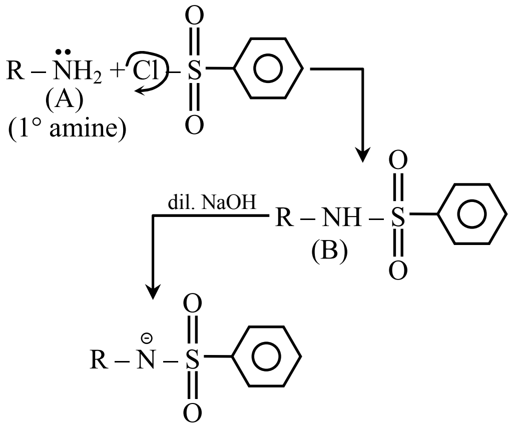


Mechanism



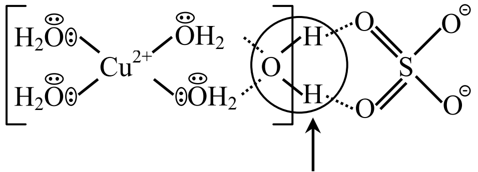
36.[1] Due to deficiency of Vitmain K causes increases in blood clotting time.
Note : Vitamin K related to blood factor.

37.[4] Hinsberg reagent (Benzene sulphonyl chloride) gives reaction product with 1° amine and it is soluble in dil. NaOH.



38.[3] The 1st IE order of 3rd period is
Na < Al < Mg < Si < S < P < Cl < Ar
X & Y are Ar & Cl
Z is sodium (Na).

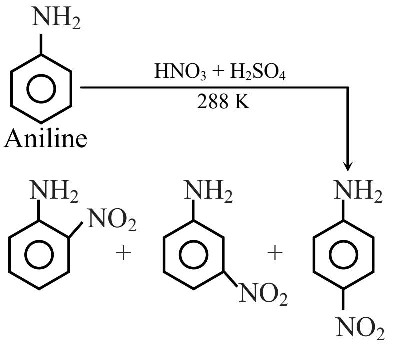
39.[2]



Hydrogen bonded water molecule = 1
Secondary valency = 4

40.[2] Statement-I is false since Bohr's theory accounts for the stability and spectrum of single electronic species (eg : He^+, Li^{2+} etc)
Statement II is true.

41.[4]



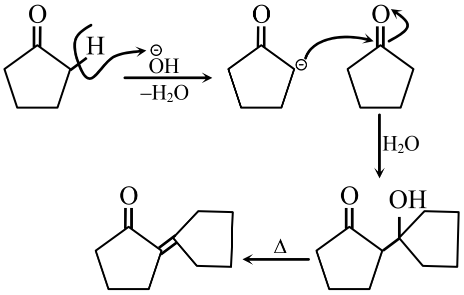
(A) (B) (C)
(2%) (47%) (51%)

% yield order ⇒ C > B > A

42.[4] CdS sol → –ve sol
TiO_2 sol → +ve sol

43.[3] (A) Antifertility drug → (iii) Nor ethindrone
(B) Antibiotic → (iv) Salvarsan
(C) Tranquilizer → (i) Meprobamate
(D) Artificial sweetener → (ii) Alitame
Ans : A - iii ; B - iv ; C - i ; D - ii

44.[3]



(Y) (X)

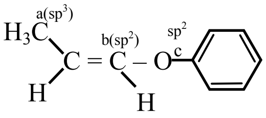
45.[2]
(a) Be → it is used in the Windows of X-ray tubes
(b) Mg → it is used in the Incendiary bombs and signals
(c) Ca → it is used in the Extraction of metals
(d) Ra → it is used in the Treatment of cancer

46.[1]

47.[4] Non-biodegradable wastes are generated by the thermal power plants which produces fly ash. Detergents which are biodegradable causes problem called eutrophication which kills animal life by deprieving it of oxygen.

48.[3] (a) Mercury → Distillation refining
(b) Copper → Electrolytic refining
(c) Silicon → Zone refining
(d) Nickel → Vapour phase refining

49.[3]



50.[4] Covalent or network solid have very high melting point and they are insulators in their solid and molten form.

SECTION-B

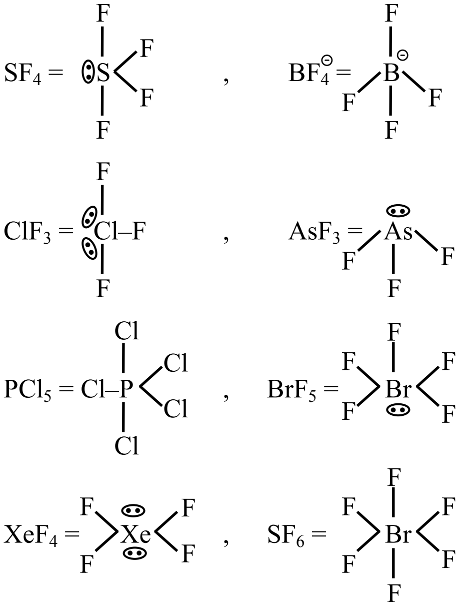
51.[10]
$$\frac{t_{99.9\%}}{t_{50\%}} = \frac{\frac{1}{K}\text{In}\frac{100}{0.1}}{\frac{1}{K}\text{In}2}$$
$$= \frac{\text{In}1000}{\text{In}2} \times t_{50\%} = \frac{3\text{In}10}{\text{In}2} \times 1$$
$$= \frac{3 \times 2.3}{0.69} = 10$$

52.[288]

From Kohlrausch's law

$$\Lambda_m^\infty(BaSO_4) = \Lambda_m^\infty(Ba^{2+}) + \Lambda_m^\infty(SO_4^{2-})$$
$$\Lambda_m^\infty(BaSO_4) = \Lambda_m^\infty(BaCl_2) + \Lambda_m^\infty(H_2SO_4) - 2\,\Lambda_m^\infty(HCl)$$
$$= 280 + 860 - 2(426) = 288\ \text{Scm}^2\ \text{mol}^{-1}$$

53.[2]

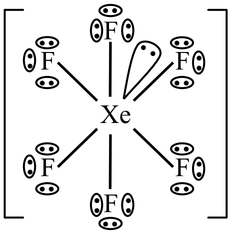


Two l.p. on central atom is = ClF_3, XeF_4

54.[19] $XeF_6 + 2H_2O \rightarrow XeO_2F_2 + 4HF$

(A) (Limited water)

Structure of 'A'



Total l.p. on (A) = 19

55.[2] Using formula

$\Delta_rG^0 = -RT\ln K_p$

$25200 = -2.3 \times 8.3 \times 400 \log(K_p)$

$K_p = 10^{-3.3} = 10^{-3} \times 0.501$

$= 5.01 \times 10^{-4}\ Bar^{-1}$

$= 5.01 \times 10^{-9}\ Pa^{-1}$

$= \dfrac{K_C}{8.3 \times 400}$

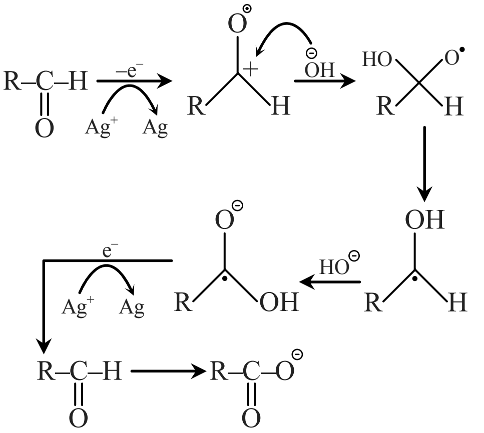
$K_C = 1.66 \times 10^{-5}\ m^3/mole$

$= 1.66 \times 10^{-2}\ L/mol$

56.[2] $AgNO_3 + NaOH \rightarrow AgOH + NaNO_3$

$2AgOH \rightarrow Ag_2O + H_2O$

$Ag_2O + 4NH_3 + H_2O \rightarrow 2Ag(NH_3)_2^+ + 2\overset{\ominus}{O}H$



Total $2e^-$ transfer to Tollen's reagnet

57.[64] In pure water,

$K_{sp} = S^2 = (8 \times 10^{-4})^2$

$= 64 \times 10^{-8}$

In 0.01 M H_2SO_4

$$H_2SO_{4(sq)} \rightarrow \underset{0.02}{2H^+_{(aq)}} + \underset{0.01}{SO_4^{2-}(aq.)}$$

$$\underset{x}{BaSO_{4(s)}} \rightleftharpoons \underset{x}{Ba^{2+}_{(aq.)}} + \underset{(x+0.01)}{SO^{2-}_{4(aq)}}$$

$K_{sp} = x\,(x + 0.01)$

$= 64 \times 10^{-8}$

$x + 0.01 \cong 0.01$ M

So, $x\,(0.01) = 64 \times 10^{-8}$

$x = 64 \times 10^{-6}$ M

58.[100]

$\Delta T_b = T_b - T_b^0$

$100.52 - 100 = 0.52°C$

$i = \left(1 - \dfrac{\alpha}{2}\right)$

$\because = \Delta T_b = i\,K_b \times m$

$0.52 = \left(1 - \dfrac{\alpha}{2}\right) \times 0.52 \times 2$

$\alpha = 1$

So, percentage association = 100%

59.[50] Most precise volume of HCl = 5 ml at equivalence point

Meq. of Na_2CO_3 = meq. of HCl.

Let molarity of Na_2CO_3 solution = M, then

$M \times 10 \times 2 = 0.2 \times 5 \times 1$

$M = 0.05$ mol / L

$= 0.05 \times 1000$

$= 50$ mM

60.[78] Moles of Benzoic acid $= \dfrac{6.1}{122}$

= moles of m-bromobenzoic acid

So, weight of m-bromobenzoic acid

$= \dfrac{6.1}{122} \times 201\text{gm}$

$= 10.05$ gm

$\%\text{yield} = \dfrac{\text{Actual weight}}{\text{Theoretical weight}} \times 100$

$= \dfrac{7.8}{10.05} \times 100$

$= 77.61\%$

MATHEMATICS

SECTION-A

61.[1] Let y + 1 = Y

$\therefore\ \dfrac{dy}{dx} = Y^2 e^{\frac{x^2}{2}} - xY$

Put $-\frac{1}{Y}=k$

$\Rightarrow \frac{dK}{dx}+k(-x)=e^{\frac{x^2}{2}}$

I.F. = $e^{-\frac{x^2}{2}}$

$\therefore k=(x+c)\,e^{x^2/2}$

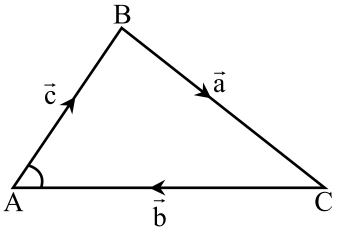
Put $k=-\frac{1}{y+1}$

$\therefore\ y+1=-\frac{1}{(x+c)e^{x^2/2}}$ ...(i)

when x = 2, y = 0, then $c=-2-\frac{1}{e^2}$ diffentiate equation (i) & put x = 1

we get $\left(\frac{dy}{dx}\right)_{x=1}=-\frac{e^{3/2}}{(1+e^2)^2}$

62.[2]



$|\vec{a}|=8, |\vec{b}|=7, |\vec{c}|=10$

$\cos\theta=\frac{|\vec{b}|^2+|\vec{c}|^2-|\vec{a}|^2}{2|\vec{b}||\vec{c}|}=\frac{17}{28}$

Projection of $\vec{c}$ on $\vec{b}$

$=|\vec{c}|\cos\theta$

$=10\times\frac{17}{28}=\frac{85}{14}$

63.[1] For non-trivial solution

$$\begin{vmatrix}4 & \lambda & 2\\ 2 & -1 & 1\\ \mu & 2 & 3\end{vmatrix}=0$$

$\Rightarrow 2\mu - 6\lambda + \lambda\mu = 12$

when $\mu = 6$, $12 - 6\lambda + 6\lambda = 12$

which is satisfied by all λ.

64.[3] $f(x)=y=\frac{x-2}{x-3}$

$\therefore\ x=\frac{3y-2}{y-1}$

$\therefore\ f^{-1}(x)=\frac{3x-2}{x-1}$

& $g(x) = y = 2x - 3$

$\therefore\ x=\frac{y+3}{2}$

$\therefore\ g^{-1}(x)=\frac{x+3}{2}$

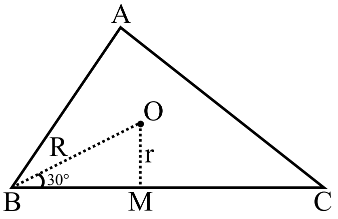
$\therefore f^{-1}(x)+g^{-1}(x)=\frac{13}{2}$

$\therefore x^2 - 5x + 6 = 0 \begin{cases} x_1 \\ x_2 \end{cases}$

$\therefore$ sum of roots

$x_1 + x_2 = 5$

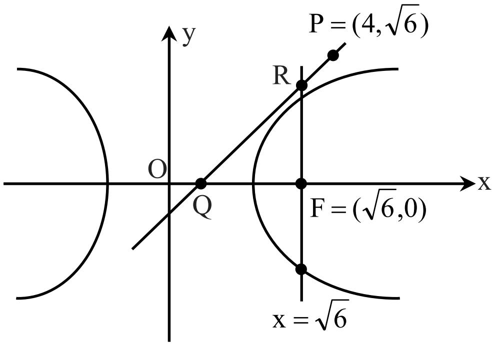
65.[1]



$r=OM=\frac{3}{\sqrt{2}}$ & $\sin 30^\circ=\frac{1}{2}=\frac{r}{R}\Rightarrow R=\frac{6}{\sqrt{2}}$

$\therefore\ r+R=\frac{9}{\sqrt{2}}$

66.[3]



$\frac{x^2}{4}-\frac{y^2}{2}=1$

$e=\sqrt{1+\frac{b^2}{a^2}}=\sqrt{\frac{3}{2}}$

$\therefore$ Focus F(ae, 0) $\Rightarrow$ $F(\sqrt{6},0)$

equation of tangent at P to the hyperbola is $2x-y\sqrt{6}=2$

tangent meet x-axis at Q(1, 0)

& latus rectum $x=\sqrt{6}$ at $R\left(\sqrt{6},\frac{2}{\sqrt{6}}(\sqrt{6}-1)\right)$

$$\therefore \text{ Area of } \Delta_{QFR} = \frac{1}{2}(\sqrt{6}-1)\cdot\frac{2}{\sqrt{6}}(\sqrt{6}-1)$$

$$= \frac{7}{\sqrt{6}} - 2$$

67.[2] LHS of all the options are some i.e.

$((P \rightarrow Q) \wedge \sim Q)$

$\equiv (\sim P \vee Q) \wedge \sim Q$

$\equiv (\sim P \wedge \sim Q) \vee (Q \wedge \sim Q)$

$\equiv \sim P \wedge \sim Q$

(A) $(\sim P \wedge \sim Q) \rightarrow Q$

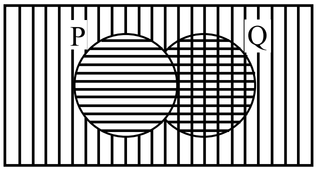
$\equiv \sim(\sim P \wedge \sim Q) \vee Q$

$\equiv (P \vee Q) \vee Q \neq$ tautology

(B) $(\sim P \wedge \sim Q) \rightarrow \sim P$

$\equiv \sim(\sim P \wedge \sim Q) \vee \sim P$

$\equiv (P \vee Q) \vee \sim P$



$\Rightarrow$ Tautology

(C) $(\sim P \wedge \sim Q) \rightarrow P$

$\equiv (P \vee Q) \vee P \neq$ Tautology

(D) $(\sim P \wedge \sim Q) \rightarrow (P \wedge Q)$

$\equiv (P \vee Q) \vee (P \wedge Q) \neq$ Tautology

Aliter :

P	Q	$P \vee Q$	$P \vee Q$	$\sim P$	$(P \vee Q) \vee \sim P$
T	T	T	T	F	T
T	F	T	F	F	T
F	T	T	F	T	T
F	F	F	F	T	T

68.[3] $\frac{1}{3} \le f(t) \le 1 \ \forall\ t \in [0, 1]$

$0 \le f(t) \le \frac{1}{2} \ \forall\ t \in (1, 3]$

Now, $g(3) = \int_0^3 f(t)dt = \int_0^1 f(t)dt + \int_1^3 f(t)dt$

$\because \int_0^1 \frac{1}{3}dt \le \int_0^1 f(t)dt \le \int_0^1 1\cdot dt$ ...(1)

and $\int_1^3 0\ dt \le \int_1^3 f(t)dt \le \int_1^3 \frac{1}{2}dt$ ...(2)

Adding, we get

$\frac{1}{3} + 0 \le g(3) \le 1 + \frac{1}{2}(3-1)$

$\frac{1}{3} \le g(3) \le 2$

69.[4] $S_{2n} = \frac{2n}{2}[2a + (2n-1)d]$,

$S_{4n} = \frac{4n}{2}[2a + (4n-1)d]$

$\Rightarrow S_2 - S_1 = \frac{4n}{2}[2a + (4n-1)d]$

$- \frac{2n}{2}[2a + (2n-1)d]$

$= 4an + (4n - 1)2nd - 2na - (2n - 1)dn$

$= 2na + nd[8n - 2 - 2n + 1]$

$\Rightarrow 2na + nd[6n - 1] = 1000$

$2a + (6n-1)d = \frac{1000}{n}$

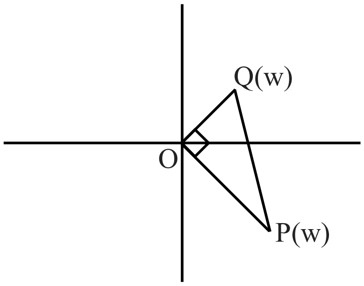
Now, $S_{6n} = \frac{6n}{2}[2a + (6n-1)d]$

$= 3n \cdot \frac{1000}{n} = 3000$

70.[2] $w = 1 - \sqrt{3}\cdot i \Rightarrow |w| = 2$

Now, $|z| = \frac{1}{|w|} \Rightarrow |z| = \frac{1}{2}$

and amp(z) $= \frac{\pi}{2} + \text{amp}(w)$



$\Rightarrow$ Area of triangle $= \frac{1}{2}\cdot OP \cdot OQ$

$= \frac{1}{2}\cdot 2 \cdot \frac{1}{2} = \frac{1}{2}$

71.[1] Let observations are denoted by x_i for $1 \le i < 2n$

$\bar{x} = \frac{\sum x_i}{2n} = \frac{(a + a + ... + a) - (a + a + ... + a)}{2n}$

$\Rightarrow \bar{x} = 0$

$$\& \ \sigma_x^2 = \frac{\sum x_i^2}{2n} - (\overline{x})^2 = \frac{a^2 + a^2 + ... + a^2}{2n} - 0 = a^2$$

$\Rightarrow \sigma_x = a$

Now, adding a constant b then $\overline{y} = \overline{x} + b = 5$

$\Rightarrow b = 5$

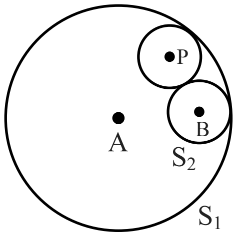
and $\sigma_y = \sigma_x$ (No change is S.D.) $\Rightarrow a = 20$

$\Rightarrow a^2 + b^2 = 425$

72.[3] $S_1 : x^2 + y^2 = 9$ ⟶ $r_1 = 3$, $A(0,0)$

$S_2 : (x-2)^2 + y^2 = 1$ ⟶ $r_2 = 1$, $B(2,0)$

$\because c_1c_2 = r_1 - r_2$



$\therefore$ given circle are touching internally

Let a veriable circle with centre P and radius r

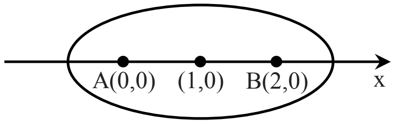
$\Rightarrow PA = r_1 - r$ and $PB = r_2 + r$

$\Rightarrow PA + PB = r_1 + r_2$

$\Rightarrow PA + PB = 4$ (> AB)

$\Rightarrow$ Locus of P is an ellipse with foci at A(0, 0) and B(2, 0) and length of major axis is 2a = 4, $e = \frac{1}{2}$

$\Rightarrow$ centre is at (1, 0) and $b^2 = a^2(1 - e^2) = 3$ if x-ellipse



$$\Rightarrow \ E : \frac{(x-1)^2}{4} + \frac{y^2}{3} = 1$$

which is satisfied by $\left(2, \pm\frac{3}{2}\right)$

73.[2] $|\vec{a}| = |\vec{b}|, |\vec{a} \times \vec{b}| = |\vec{a}|, \vec{a} \perp \vec{b}$

$|\vec{a} \times \vec{b}| = |\vec{a}|$

$\Rightarrow |\vec{a}||\vec{b}|\sin 90° = |\vec{a}|$

$\Rightarrow |\vec{b}| = 1 = |\vec{a}|$

$\vec{a}$ & $\vec{b}$ are mutually perpendicular unit vectors.

Let $\vec{a} = \hat{i}, \vec{b} = \hat{j} \Rightarrow \vec{a} \times \vec{b} = \hat{k}$

$$\cos\theta = \frac{(\hat{i} + \hat{j} + \hat{k}) \cdot \hat{i}}{\sqrt{3}\sqrt{1}} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \theta = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

74.[1] $P(X = 1) = {}^5C_1.p.q^4 = 0.4096$

$P(X = 2) = {}^5C_2.p^2.q^3 = 0.2048$

$\Rightarrow \frac{q}{2p} = 2$

$\Rightarrow q = 4p$ and $p + q = 1$

$\Rightarrow p = \frac{1}{5}$ and $q = \frac{4}{5}$

Now

$$P(X = 3) = {}^5C_3 \cdot \left(\frac{1}{5}\right)^3 \cdot \left(\frac{4}{5}\right)^3 = \frac{10 \times 16}{125 \times 25} = \frac{32}{625}$$

75.[3] Equation of tangent be

$$\frac{x\cos\theta}{3\sqrt{3}} + \frac{y.\sin\theta}{1} = 1, \quad \theta \in \left(0, \frac{\pi}{2}\right)$$

intercept on x-axis

$OA = 3\sqrt{3}\sec\theta$

intercept on y-axis

$OB = \operatorname{cosec}\theta$

Now, sum of intercept

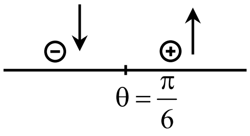
$= 3\sqrt{3}\sec\theta + \operatorname{cosec}\theta = f(\theta)$ let

$$f'(\theta) = 3\sqrt{3}\sec\theta\tan\theta - \operatorname{cosec}\theta\cot\theta$$

$$= 3\sqrt{3}\frac{\sin\theta}{\cos^2\theta} - \frac{\cos\theta}{\sin^2\theta}$$

$$= \underbrace{\frac{\cos\theta}{\sin^2\theta}}_{\oplus} \cdot 3\sqrt{3}\left[\tan^3\theta - \frac{1}{3\sqrt{3}}\right] = 0$$

$\Rightarrow \theta = \frac{\pi}{6}$



$\Rightarrow$ at $\theta = \frac{\pi}{6}$, $f(\theta)$ is minimum

76.[3] A and B are matrices of n × n order & ARB iff there exists a non singular matrix $P(\det(P) \neq 0)$ such that $PAP^{-1} = B$

For reflexive

$ARA \Rightarrow PAP^{-1} = A$ ...(1) must be true

for $P = I$, Eq.(1) is true so 'R' is reflexive

For symmetric

$ARB \Leftrightarrow PAP^{-1} = B$ ...(1) is true

for BRA iff $PBP^{-1} = A$ ...(2) must be true

$\because PAP^{-1} = B$

$P^{-1}PAP–1 = P–1B$

$IAP^{-1}P = P^{-1}BP$

$A = P^{-1}BP$ ...(3)

from (2) & (3) $PBP^{-1} = P^{-1}BP$

can be true some $P = P^{-1} \Rightarrow P^2 = I$ ($\det(P) \neq 0$)

So 'R' is symmetric

For transitive

$ARB \Leftrightarrow PAP^{-1} = B$... is true

$BRC \Leftrightarrow PBP^{-1} = C$... is true

now $PPAP^{-1}P^{-1} = C$

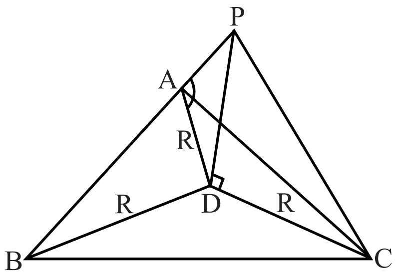
$P^2A(P^2)^{-1} = C \Rightarrow ARC$

So 'R' is transitive relation

$\Rightarrow$ Hence R is equivalence

77.[2] Let PD = h, R = 2

As angle of elevation of top of pole from A, B, C are equal So D must be circumcentre of ΔABC



$$\tan\left(\frac{\pi}{3}\right) = \frac{PD}{R} = \frac{h}{R}$$

$$h = R\tan\left(\frac{\pi}{3}\right) = 2\sqrt{3}$$

78.[4] $15\sin^4\alpha + 10\cos^4\alpha = 6$

$15\sin^4\alpha + 10\cos^4\alpha = 6(\sin^2\alpha + \cos^2\alpha)^2$

$(3\sin^2\alpha - 2\cos^2\alpha)^2 = 0$

$\tan^2\alpha = \frac{2}{3}. \quad \cot^2\alpha = \frac{3}{2}$

$\Rightarrow 27\sec^6\alpha + 8\mathrm{cosec}^6\alpha$

$= 27(\sec^2\alpha)^3 + 8(\mathrm{cosec}^2\alpha)^3$

$= 27(1 + \tan^2\alpha)^3 + 8(1 + \cot^2\alpha)^3$

$= 250$

79.[3] $4y^2 = x^2(4 - x)(x - 2)$

$$|y| = \frac{|x|}{2}\sqrt{(4-x)(x-2)}$$

$$\Rightarrow y_1 = \frac{x}{2}\sqrt{(4-x)(x-2)}$$

$$\text{and } y_2 = \frac{-x}{2}\sqrt{(4-x)(x-2)}$$

$D : x \in [2, 4]$

Required Area

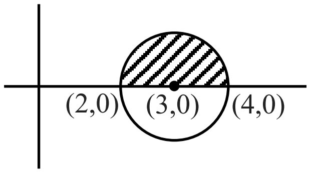
$$= \int_2^4 (y_1 - y_2)dx = \int_2^4 x\sqrt{(4-x)(x-2)}\,dx \quad ...(1)$$

$$\text{Applying } \int_a^b f(x)dx = \int_a^b f(a+b-x)dx$$

$$\text{Area} = \int_2^4 (6-x)\sqrt{(4-x)(x-2)}\,dx \quad ...(2)$$

(1) + (2)

$$2A = 6\int_2^4 \sqrt{(4-x)(x-2)}\,dx$$



$$A = 3\int_2^4 \sqrt{1-(x-3)^2}\,dx$$

$$A = 3\cdot\frac{\pi}{2}\cdot 1^2 = \frac{3\pi}{2}$$

80.[4] f(x) is continuous at x = 0

$$\lim_{x\to 0^+} f(x) = f(0) = \lim_{x\to 0^-} f(x) \quad ...(1)$$

$f(0) = b$ ...(2)

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^-}\left(\frac{\sin(a+1)x}{2x} + \frac{\sin 2x}{2x}\right)$$

$$= \frac{a+1}{2} + 1 \quad ...(3)$$

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} \frac{\sqrt{x+bx^3} - \sqrt{x}}{bx^{5/2}}$$

$$= \lim_{x\to 0^+} \frac{(x + bx^3 - x)}{bx^{5/2}(\sqrt{x+bx^3} + \sqrt{x})}$$

$$= \lim_{x\to 0^+} \frac{\sqrt{x}}{\sqrt{x}\,(\sqrt{1+bx^2}+1} = \frac{1}{2} \qquad ...(4)$$

Use (2), (3) & (4) in (1)

$$\frac{1}{2} = b = \frac{a+1}{2} + 1$$

$$\Rightarrow\ b = \frac{1}{2},\ a = -2$$

$$a + b = \frac{-3}{2}$$

SECTION-B

81.[0] $P(x) = f(x^3) + xg(x^3)$

$P(1) = f(1) + g(1)$ ...(1)

Now P(x) is divisible by $x^2 + x + 1$

$\Rightarrow P(x) = Q(x)(x^2 + x + 1)$

$P(w) = 0 = P(w^2)$ where w, w^2 are non-real cube roots of units

$P(x) = f(x^3) + xg(x^3)$

$P(w) = f(w^3) + wg(w^3) = 0$

$f(1) + wg(1) = 2$ ...(2)

$P(w^2) = f(w^6) + w^2g(w^6) = 0$

$f(1) + w^2g(1) = 0$ ...(3)

(2) + (3)

$\Rightarrow 2f(1) + (w + w^2)g(1) = 0$

$2f(1) = g(1)$ ...(4)

(2) – (3)

$\Rightarrow (w - w^2)g(1) = 0$

$g(1) = 0 = f(1)$ from (4)

from (1) $P(1) = f(1) + g(1) = 0$

82.[6] $P = \begin{bmatrix} 2 & -1 \\ 5 & -3 \end{bmatrix}$

$$5I - 8P = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} - \begin{bmatrix} 16 & -8 \\ 40 & -24 \end{bmatrix} = \begin{bmatrix} -11 & 8 \\ -40 & 29 \end{bmatrix}$$

$$P^2 = \begin{bmatrix} -1 & 1 \\ -5 & 4 \end{bmatrix}$$

$$P^3 = \begin{bmatrix} 3 & -2 \\ 10 & -7 \end{bmatrix} \Rightarrow P^6 = \begin{bmatrix} -11 & 8 \\ -40 & 29 \end{bmatrix} = P^n$$

$\Rightarrow n = 6$

83.[160] $\sum_{r=1}^{10} r!\{(r+1)(r+2)(r+3) - 9(r+1) + 8\}$

$$= \sum_{r=1}^{10} \left[\{(r+3)! - (r+2)!\} - 8\{(r+1)! - r!\}\right]$$

$= (13! + 12! - 2! - 3!) - 8(11! - 1)$

$= (12.13 + 12 - 8).11! - 8 + 8$

$= (160)(11)!$

Hence $\alpha = 160$

84.[210] $\left((x^{1/3}+1) - \left(\frac{\sqrt{x}+1}{\sqrt{x}}\right)\right)^{10}$

$(x^{1/3} - x^{-1/2})^{10}$

$T_{r+1} = {}^{10}C_r(x^{1/3})^{10-r}(-x^{-1/2})r$

$\frac{10-r}{3} - \frac{r}{2} = 0 \Rightarrow 20 - 2r - 3r = 0$

$\Rightarrow r = 4$

$$T_5 = {}^{10}C_4 = \frac{10\times 9\times 8\times 7}{4\times 3\times 2\times 1} = 210$$

85.[8] Let $p'(x) = a(x - 1)(x + 1) = a(x^2 - 1)$

$$p(x) = a\int (x^2 - 1)\,dx + c$$

$$= a\left(\frac{x^3}{3} - x\right) + c$$

Now $p(-3) = 0$

$$\Rightarrow\ a\left(-\frac{27}{3} + x\right) + c = 0$$

$\Rightarrow -6a + c = 0$ ...(1)

$$\text{Now } \int_{-1}^{1}\left(a\left(\frac{x^3}{3} - x\right) + c\right)dx = 18$$

$= 2c = 18 \Rightarrow c = 9$ ...(2)

$\Rightarrow$ from (1) & (2)

$\Rightarrow -6a + 9 = 0$

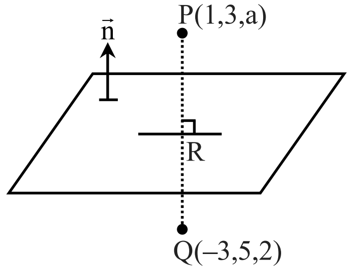
$$\Rightarrow\ a = \frac{3}{2}$$

$$\Rightarrow\ p(x) = \frac{3}{2}\left(\frac{x^3}{3} - x\right) + 9$$

sum of coefficient

$$= \frac{1}{2} - \frac{3}{2} + 9 = 8$$

86.[1]



plane = 2x – y + z = b

$R \equiv \left(-1, 4, \frac{a+2}{2}\right) \rightarrow$ on plane

$\therefore\ -2-4+\frac{a+2}{2} = b$

$\Rightarrow a + 2 = 2b + 12$

$\Rightarrow a = 2b + 10$ ...(i)

$< PQ > = < 4, –2, a – 2 >$

$\therefore\ \frac{2}{4} = \frac{-1}{-2} = \frac{1}{a-2}$

$\Rightarrow a – 2 = 2$

$\Rightarrow a = 4, b = –3$

$\therefore\ |a + b| = 1$

87.[3] If $f(x + y) = f(x).f(y)$ & $f'(0) = 3$ then

$f(x) = a^x \Rightarrow f'(x) = a^x.\ell na$

$\Rightarrow f'(0) = \ell na = 3$

$\Rightarrow a = e^3$

$\Rightarrow f(x) = (e^3)x = e^3x$

$$\lim_{x\to 0}\frac{f(x)-1}{x} = \lim_{x\to 0}\left(\frac{e^{3x}-1}{3x}\times 3\right) = 1\times 3 = 3$$

88.[Bonus]

Official Ans. by NTA (19)

Instead of ${}^nC_k$ it must be ${}^{10}C_k$ i.e.

$$\sum_{k=0}^{10}(2^2+3k)\,{}^{10}C_k = \alpha.3^{10} + \beta.2^{10}$$

$$LHS = 4\sum_{k=0}^{10}{}^{10}C_k + 3\sum_{k=0}^{10}k\cdot\frac{10}{k}\cdot{}^9C_{k-1}$$

$= 4.2^{10} + 3.10.2^9$

$= 19.2^{10} = \alpha.3^{10} + \beta.2^{10}$

$\Rightarrow \alpha = 0, \beta = 19$

$\Rightarrow \alpha + \beta = 19$

89.[38] Equation of plane is $\begin{vmatrix} x-1 & y+6 & z+5 \\ 3 & 4 & 2 \\ 4 & -3 & 7 \end{vmatrix} = 0$

Now $(1, –1, \alpha)$ lies on it so

$$\begin{vmatrix} 0 & 5 & \alpha+5 \\ 3 & 4 & 2 \\ 4 & -3 & 7 \end{vmatrix} = 0$$

$\Rightarrow 5\alpha + 38 = 0$

$\Rightarrow |5\alpha| = 38$

90.[4] $xdy - ydx = \sqrt{x^2-y^2}\,dx$

$$\Rightarrow \frac{xdy-ydx}{x^2} = \frac{1}{x}\sqrt{1-\frac{y^2}{x^2}}dx$$

$$\Rightarrow \int\frac{d\left(\frac{y}{x}\right)}{1-\left(\frac{y}{x}\right)^2} = \int\frac{dx}{x}$$

$$\Rightarrow \sin^{-1}\left(\frac{y}{x}\right) = \ell n\,|x| + c$$

at $x = 1, y = 0 \Rightarrow c = 0$

$y = x \sin(\ell nx)$

$$A = \int_1^{e^\pi} x\sin(\ell n\,x)\,dx$$

$x = e^t, dx = e^t dt$

$$\Rightarrow \int_0^\pi e^{2t}\sin(t)\,dt = A$$

$$\alpha e^{2\pi} + \beta = \left(\frac{e^{2t}}{5}(2\sin t - \cos t)\right)_0^\pi = \frac{1+e^{2\pi}}{5}$$

$\alpha = \frac{1}{5}, \beta = \frac{1}{5}$ so $10(\alpha + \beta) = 4$