JEE MAIN ONLINE PAPER 2021

Held on March 18, 2021 (Morning)

Instructions

1. This test will be a 3 hours Test.
2. This test consists of Physics, Chemistry and Mathematics questions with equal weightage of 100 marks.
3. Each question is of 4 marks.
4. In the question paper consisting of Physics (Q.no. 1 to 30), Chemistry (Q.no. 31 to 60) and Mathematics (Q.no. 61 to 90). There are two sections for each subject (Section-A : MCQ Type & Section-B : Numerical Response Type). Section-A consists of 20 multiple choice questions & Section-B consists of 10 Numerical Value type Questions. **Candidates have a choice to Answer 5 out of the 10 numerical value answer based questions per section.**
5. There will be only one correct choice in the given four choices in Section-A. For each question 4 marks will be awarded for correct choice, 1 mark will be deducted for incorrect choice and zero mark will be awarded for not attempted question. For Section-B questions 4 marks will be awarded for correct answer and zero for unattempted and incorrect answer.
6. Any textual, printed or written material, mobile phones, calculator etc. is not allowed for the students appearing for the test.
7. All calculations/written work should be done in the rough sheet provided.

PHYSICS

Section -A

Q.1 An oil drop of radius 2 mm with a density 3g cm^{-3} is held stationary under a constant electric field 3.55×10^5 V m^{-1} in the Millikan's oil drop experiment. What is the number of excess electrons that the oil drop will possess ?
[consider g = 9.81 m/s^2]
(1) 48.8×10^{11} (2) 1.73×10^{10}
(3) 17.3×10^{10} (4) 1.73×10^{12}

Q.2 Match List–I with List–II
List – I
(a) 10 km height over earth's surface
(b) 70 km height over earth's surface
(c) 180 km height over earth's surface
(d) 270 km height over earth's surface
List–II
(i) Thermosphere (ii) Mesosphere
(iii) Stratosphere (iv) Troposphere
(1) (a)–(iv), (b)–(iii), (c)–(ii), (d)–(i)
(2) (a)–(i), (b)–(iv), (c)–(iii), (d)–(ii)
(3) (a)–(iii), (b)–(ii), (c)–(i), (d)–(iv)
(4) (a)–(ii), (b)–(i), (c)–(iv), (d)–(iii)

Q.3 Imagine that the electron in a hydrogen atom is replaced by a muon (μ). The mass of muon particle is 207 times that of an electron. The ionization potential of this hydrogen atom will be :-
(1) 13.6 eV (2) 2815.2 eV
(3) 331.2 eV (4) 27.2 eV

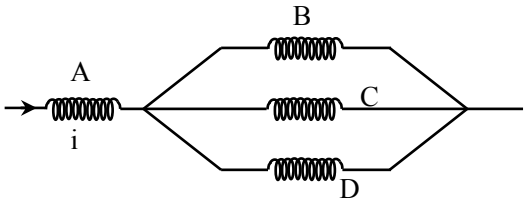
Q.4 A plane electromagnetic wave of frequency 100 MHz is travelling in vacuum along the x-direction. At a particular point in space and time, $\vec{B} = 2.0 \times 10^{-8}\ \hat{k}T$ (where, $\hat{k}$ is unit vector along z-direction) What is $\vec{E}$ at this point ?
(1) 0.6 $\hat{j}$ V/M (2) 6.0 $\hat{k}$ V/M
(3) 6.0 $\hat{j}$ V/m (4) 0.6 $\hat{k}$ V/m

Q.5 A thin circular ring of mass M and radius r is rotating about its axis with an angular speed ω. Two particles having mass m each are now attached at diametrically opposite points. The angular speed of the ring will become :

(1) $\omega\frac{M}{M+m}$ (2) $\omega\frac{M+2m}{M}$

(3) $\omega\frac{M}{M+2m}$ (4) $\omega\frac{M-2m}{M+2m}$

Q.6 Four identical long solenoids A, B, C and D are connected to each other as shown in the figure. If the magnetic field at the center of A is 3T. the field at the center of C would be : (Assume that the magnetic field is confined with in the volume of respective solenoid).



(1) 12T (2) 6T (3) 9T (4) 1T

Q.7 The time period of a simple pendulum is given by $T = 2\pi\sqrt{\frac{\ell}{g}}$. The measured value of the length of pendulum is 10 cm known to a 1 mm accuracy. The time for 200 oscillations of the pendulum is found to be 100 second using a clock of 1s resolution. The percentage accuracy in the determination of 'g' using this pendulum is 'x'. The value of 'x' to the nearest integer is :

(1) 2 % (2) 3 %

(3) 5 % (4) 4 %

Q.8 A constant power delivering machine has towed a box, which was initially at rest, along a horizontal straight line. The distance moved by the box in time 't' is proportional to

(1) $t^{2/3}$ (2) $t^{3/2}$ (3) t (4) $t^{1/2}$

Q.9 What will be the average value of energy along one degree of freedom for an ideal gas in thermal equilibrium at a temperature T? (k_B is Boltzmann constant)

(1) $\frac{1}{2}k_BT$ (2) $\frac{2}{3}k_BT$

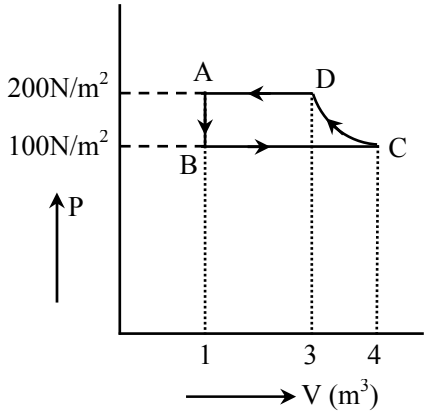
(3) $\frac{3}{2}k_BT$ (4) k_BT

Q.10 A radioactive sample disintegrates via two independent decay processes having half lives $T_{1/2}^{(1)}$ and $T_{1/2}^{(2)}$ respectively. The effective half-life $T_{1/2}$ of the nuclei is :

(1) None of the above (2) $T_{1/2} = T_{1/2}^{(1)} + T_{1/2}^{(2)}$

(3) $T_{1/2} = \frac{T_{1/2}^{(1)}T_{1/2}^{(2)}}{T_{1/2}^{(1)} + T_{1/2}^{(2)}}$ (4) $T_{1/2} = \frac{T_{1/2}^{(1)} + T_{1/2}^{(2)}}{T_{1/2}^{(1)} - T_{1/2}^{(2)}}$

Q.11 The P–V diagram of a diatomic ideal gas system going under cyclic process as shown in figure. The work done during an adiabatic process CD is (use $\gamma = 1.4$) :



(1) –500 J (2) –400 J

(3) 400 J (4) 200 J

Q.12 In Young's double slit arrangement, slits are separated by a gap of 0.5 mm, and the screen is placed at a distance of 0.5 m from them. The distance between the first and the third bright fringe formed when the slits illuminated by a monochromatic light of 5890 Å is :-

(1) 1178×10^{-9} m (2) 1178×10^{-6} m

(3) 1178×10^{-12} m (4) 5890×10^{-7} m

Q.13 A particle is travelling 4 times as fast as an electron. Assuming the ratio of de-Broglie wavelength of a particle to that of electron is 2 : 1, The mass of the particle is :

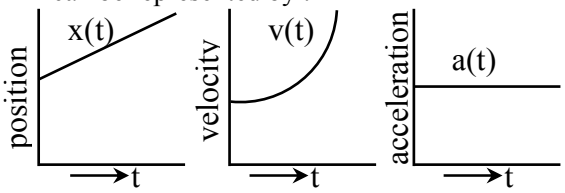
(1) $\frac{1}{16}$ times the mass of e^-

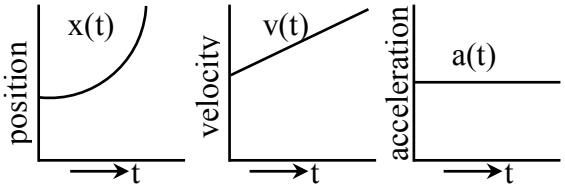
(2) 8 times the mass of e^-

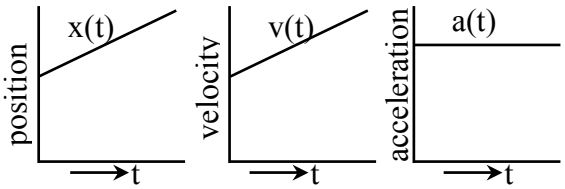
(3) 16 times the mass of e^-

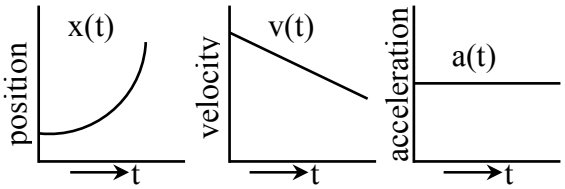
(4) $\frac{1}{8}$ times the mass of e^-

Q.14 The position, velocity and acceleration of a particle moving with a constant acceleration can be represented by :

(1) 

(2) 

(3) 

(4) 

Q.15 In the experiment of Ohm's law, a potential difference of 5.0 V is applied across the end of a conductor of length 10.0 cm and diameter of 5.00 mm. The measured current in the conductor is 2.00 A. The maximum permissible percentage error in the resistivity of the conductor is :-

(1) 3.9 (2) 8.4 (3) 7.5 (4) 3.0

Q.16 In a series LCR resonance circuit, if we change the resistance only, from a lower to higher value :

(1) The bandwidth of resonance circuit will in crease.
(2) The resonance frequency will increase.
(3) The quality factor will increase.
(4) The quality factor and the resonance frequency will remain constant.

Q.17 An AC source rated 220 V, 50 Hz is connected to a resistor. The time taken by the current to change from its maximum to the rms value is :

(1) 2.5 ms (2) 25 ms
(3) 2.5 s (4) 0.25 ms

Q.18 Your friend is having eye sight problem. She is not able to see clearly a distant uniform window mesh and it appears to her as non-uniform and distorted. The doctor diagnosed the problem as :

(1) Astigmatism
(2) Myopia with Astigmatism
(3) Presbyopia with Astigmatism
(4) Myopia and hypermetropia

Q.19 A loop of flexible wire of irregular shape carrying current is placed in an external magnetic field. Identify the effect of the field on the wire.

(1) Loop assumes circular shape with its plane normal to the field.
(2) Loop assumes circular shape with its plane parallel to the field.
(3) Wire gets stretched to become straight.
(4) Shape of the loop remains unchanged.

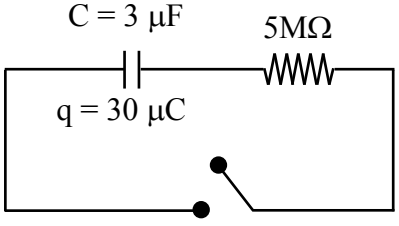
Q.20 The time period of a satellite in a circular orbit of radius R is T. The period of another satellite in a circular orbit of radius 9R is :

(1) 9 T (2) 27 T
(3) 12 T (4) 3 T

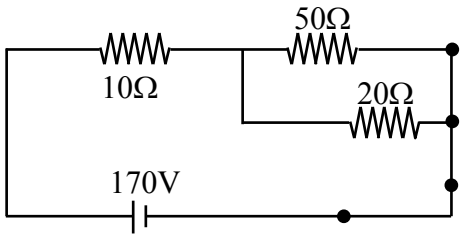
Section –B

Q.21 A particle performs simple harmonic motion with a period of 2 second. The time taken by the particle to cover a displacement equal to half of its amplitude from the mean position is $\frac{1}{a}$s. The value of 'a' to the nearest integer is ---.

Q.22 The circuit shown in the figure consists of a charged capacitor of capacity 3 μF and a charge of 30 μC. At time t = 0, when the key is closed, the value of current flowing through the 5 MΩ resistor is 'x' μ-A. The value of 'x to the nearest integer is ----------.



Q.23 The voltage across the 10Ω resistor in the given circuit is x volt.



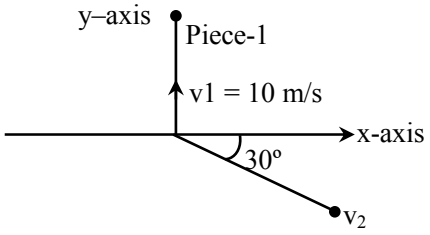
The value of 'x' to the nearest integer is —

Q.24 Two separate wires A and B are stretched by 2 mm and 4 mm respectively, when they are subjected to a force of 2 N. Assume that both the wires are made up of same material and the radius of wire B is 4 times that of the radius of wire A. The length of the wires A and B are in the ratio of a : b. Then a/b can be expressed as 1/x where x is --------.

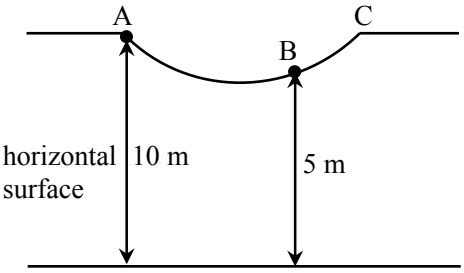
Q.25 A person is swimming with a speed of 10 m/s at an angle of 120º with the flow and reaches to a point directly opposite on the other side of the river. The speed of the flow is 'x' m/s. The value of 'x' to the nearest integer is ——

Q.26 A parallel plate capacitor has plate area 100 m^2 and plate separation of 10 m. The space between the plates is filled up to a thickness 5 m with a material of dielectric constant of 10. The resultant capacitance of the system is 'x' pF. The value of $\varepsilon_0 = 8.85 \times 10^{-12}$ $F.m^{-1}$
The value of 'x' to the nearest integer is ——.

Q.27 A ball of mass 10 kg moving with a velocity $10\sqrt{3}$ m/s along the x-axis, hits another ball of mass 20 kg which is at rest. After the collision. First ball comes to rest while the second ball disintegrates into two equal pieces. One piece starts moving along y-axis with a speed of 10 m/s. The second piece starts moving at an angle of 30º with respect to the x-axis. The velocity of the ball moving at 30º with x-axis is x m/s. The configuration of pieces after collision is shown in the figure below. The value of x to the nearest integer is ——— .



Q.28 As shown in the figure, a particle of mass 10 kg is placed at a point A. When the particle is slightly displaced to its right, it starts moving and reaches the point B. The speed of the particle at B is x m/s. (Take g = 10 m/s^2)
The value of 'x' to the nearest integer is ——.



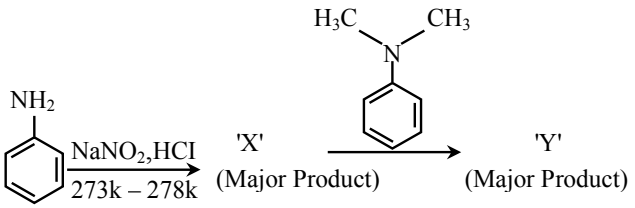
Q.29 An npn transistor operates as a common emitter amplifier with a power gain of 10^6. The input circuit resistance is 100Ω and the output load resistance is 10 KΩ. The common emitter current gain 'β' will be ——. (Round off to the Nearest Integer)

Q.30 A bullet of mass 0.1 kg is fired on a wooden block to pierce through it, but it stops after moving a distance of 50 cm into it. If the velocity of bullet before hitting the wood is 10 m/s and if slows down with uniform deceleration, then the magnitude of effective retarding force on the bullet is 'x' N. The value of 'x' to the nearest integer is ——.

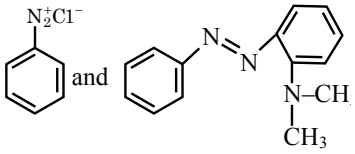
CHEMISTRY

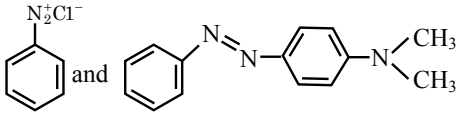
Section -A

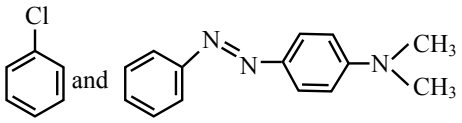
Q.31

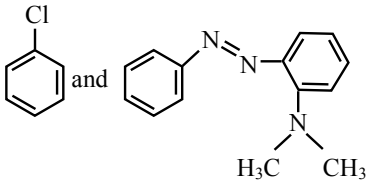


Considering the above reaction, X and Y respectively are :

(1) 

(2) 

(3) 

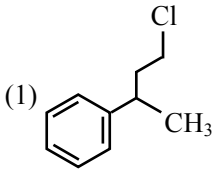
(4) 

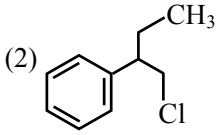
Q.32 The ionic radius of Na^+ ions is 1.02 Å. The ionic radii (in Å) of Mg^{2+} and $A1^{3+}$, respectively, are-

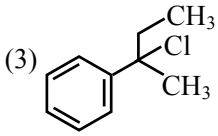
(1) 1.05 and 0.99
(2) 0.72 and 0.54
(3) 0.85 and 0.99
(4) 0.68 and 0.72

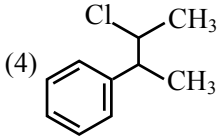
Q.33 Reaction of Grignard reagent, C_2H_5MgBr with C_8H_8O followed by hydrolysis gives compound "A" which reacts instantly with Lucas reagent to give compound B, $C_{10}H_{13}C1$.

The Compound B is :

(1) 

(2) 

(3) 

(4) 

Q.34 Reagent, 1-naphthylamine and sulphanilic acid in acetic acid is used for the detection of

(1) N_2O (2) NO_3^-
(3) NO (4) NO_2^-

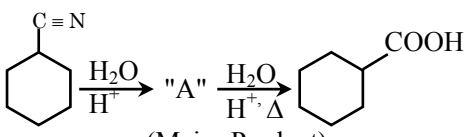
Q.35 A non-reducing sugar "A" hydrolyses to give two reducing mono saccharides. Sugar A is –

(1) Fructose
(2) Galactose
(3) Glucose
(4) Sucrose

Q.36 Match the list –I with list – II

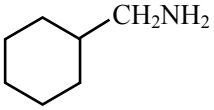
List –I (Class of Drug)	List –II (Example)
(a) Antacid	(i) Novestrol
(b) Artificial sweetener	(ii) Cimetidine
(c) Antifertility	(iii) Valium
(d) Tranquilizers	(iv) Alitame

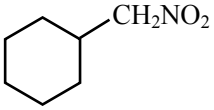
(1) (a) – (ii), (b) – (iv),(c) – (i), (d) – (iii)
(2) (a) – (iv), (b) – (i),(c) – (ii), (d) – (iii)
(3) (a) – (iv), (b) – (iii),(c) – (i), (d) – (ii)
(4) (a) – (ii), (b) – (iv),(c) – (iii), (d) – (i)

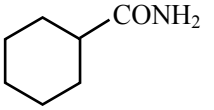
Q.37 

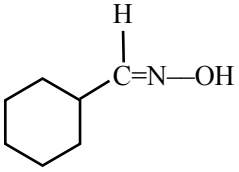
(Major Product)

Consider the above chemical reaction and identify product "A"

(1) 

(2) 

(3) 

(4) 

Q.38 Match List-I with List-II

List-I	List-II
(a) Chlorophyll	(i) Ruthenium
(b) Vitamin-B_{12}	(ii) Platinum
(c) Anticancer drug	(iii) Cobalt
(d) Grubbs catalyst	(iv) Magnesium

Choose the most appropriate answer from the options given below :

(1) a-iii, b-ii, c-iv, d-i (2) a-iv, b-iii), c-ii, d-i
(3) a-iv, b-iii, c-i, d-ii (4) a-iv, b-ii, c-iii, d-i

Q.39 Match List-I with List-II :

List-I
(Chemicals)
(a) Alcoholic potassium hydroxide
(b) Pd/ $BaSO_4$
(c) BHC (Benzene hexachloride)
(d) Polyacetylene
LIst-II
(Use / Preparation / Constituent)
(i) Electrodes in batteries
(ii) Obtained by addition reaction
(iii) Used for β - elimination reaction
(iv) Lindlar's catalyst
Choose the most appropriate match:
(1) a-ii, b-i, c-iv, d-iii (2) a-iii, b-iv, c-ii, d-i
(3) a-iii, b-i, c-iv, d-ii (4) a-ii, b-iv, c-i, d-iii

Q.40 The satements that are TRUE :
(A) Methane leads to both global warming and photochemical smog
(B) Methane is generated from paddy fields
(C) Methane is a stronger global warming gas than CO_2
(D) Methane is a part of reducing smog
Choose the most appropriate answer from the options given below :
(1) (A), (B), (C) only
(2) (A) and (B) only
(3) (B), (C), (D) only
(4) (A), (B), (D) only

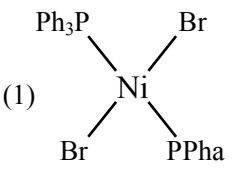
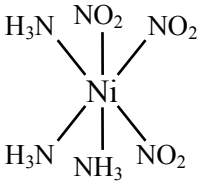
Q.41 Match List-I with List-II

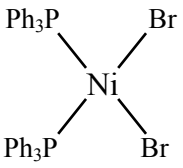
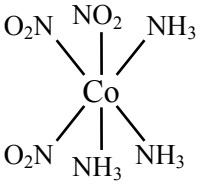
List-I	List-II
(a) $Ca(OCl)_2$	(i) Antacid
(b) $CaSO_4 \frac{1}{2} H_2O$	(ii) Cement
(c) CaO	(iii) Bleach
(d) $CaCO_3$	(iv) Plaster of paris

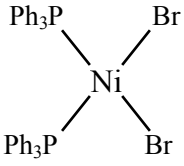
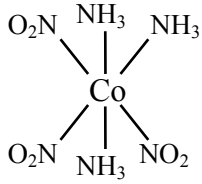
Choose the most appropriate answer form the options given below :
(1) a-i, b-iv, c-iii, d-ii (2) a-iii, b-ii, c-iv, d-i
(3) a-iii, b-iv, c-ii, d-i (4) a-iii, b-ii, c-i, d-iv

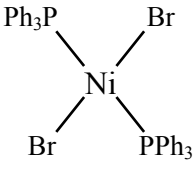
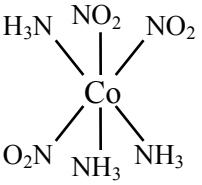
Q.42 Compound with molecular formula C_3H_6O can show :
(1) Positional isomerism
(2) Both positional isomerism and metamerism
(3) Metamerism
(4) Functional group isomerism

Q.43 The correct structures of trans-$[NiBr_2(PPh_3)_2]$ and meridonial-$[Co(NH_3)_3(NO_2)_3]$, respectively, are

(1)  and 

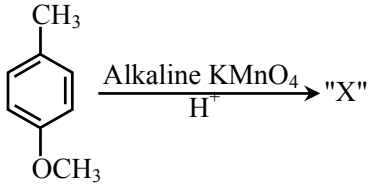
(2)  and 

(3)  and 

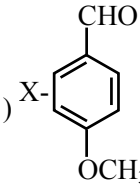
(4)  and 

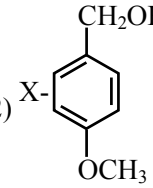
Q.44 A certain orbital has no angular nodes and two radial nodes. The orbital is :
(1) 2s (2) 3s (3) 3p (4) 2p

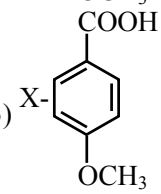
Q.45

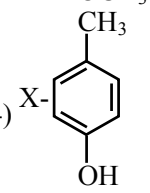


Considering the above chemical reaction, identify the product "X" :

(1) 

(2) 

(3) 

(4) 

Q.46 Match List-I with List-II

List-(process)	List-II (catalyst)
(a) Deacron's process	(i) ZSM-5
(b) Contact process	(ii) $CuCl_2$
(c) Cracking of hydrocarbons	(iii) Particles 'Ni'
(d) Hydrogenation of vegetable oils	(iv) V_2O_5

Choose the most appropriate answer from the options given below -
(1) a-ii, b-iv, c-i, d-iii (2) a-i, b-iii, c-ii, d-iv
(3) a-iii, b-i, c-iv, d-ii (4) a-iv, b-ii, c-i, d-iii

Q.47 Given below are two statements : One is labelled as Assertion A and the other labelled as reason R
Assertion A : During the boiling of water having temporary hardness, $Mg(HCO_3)_2$ is converted to $MgCO_3$
Reason R : The solubility product of $Mg(OH)_2$ is greater than that of $MgCO_3$.
In the light of the above statements, choose the most appropriate answer from the options given below :
(1) Both A and R are true but R is not the correct explanation of A
(2) A is true but R is false
(3) Both A and R are true and R is the correct explanation of A
(4) A is false but R is true

Q.48 The number of ionisable hydrogens present in the product obtained from a reaction of phosphorus trichloride and phosphonic acid is :
(1) 3 (2) 0 (3) 2 (4) 1

Q.49 In a binary compound, atoms of element A form a chp structure and those of element M occupy 2/3 of the tetrahedral voids of the hcp structure. The formula of the binary compound is :
(1) M_2A_3 (2) M_4A_3
(3) M_4A (4) MA_3

Q.50 The chemical that is added to reduce the melting point of the reaction mixture during the extraction of aluminium is :
(1) Cryolite (2) Bauxite
(3) Calamine (4) Kaolite

Section –B

Q.51 AX is a covalent diatomic molecule where A and X are second row elements of periodic table. Based on Molecular orbital theory, the bond order of AX is 25. The total number of electrons in AX is ——. (Round off to the Nearest Integer).

Q.52 In order to prepare a buffer solution of pH 5.74 sodium acetate is added to acetic acid. If the concentaration of acetic acid in the buffer is 1.0 M, the concentration of sodium acetate in the buffer is - - - - - M. (Round off to the Nearest Integer).
[Given : pKa (acetic acid) = 4.74]

Q.53 $2\ NO(g) + Cl_2(g) \rightleftharpoons 2\ NOCl\ (s)$
This reaction was studied at – 10ºC and the following data was obtained

Run	$[NO]_0$	$[Cl_2]_0$	r_0
1	0.10	0.10	0.18
2	0.10	0.20	0.35
3	0.20	0.20	1.40

$[NO]_0$ and $[Cl_2]_0$ are the initial concentrations and r_0 is the initial reaction rate.
The overall order of the reaction is ———.
(Round off to the Nearest Integer).

Q.54 For the reaction
$C_2H_6 \rightarrow C_2H_4 + H_2$
the reaction enthalpy ΔrH = ———— kJ mol^{-1}
(Round off to the Nearest Integer).
[Given : Bond enthalpies in kJ mol^{-1} : C–C : 347, C=C : 611; C–H : 414, H–H : 436]

Q.55 ——— grams of 3-Hydroxy propanal (MW=74) must be dehydrated to produce 7.8 g of acrolein (MW = 56) (C_3H_4O) if the percentage yield is 64. (Round off to the Nearest Integer).
[Given : Atomic masses : C : 12.0 u, H : 1.0 u, O : 16.0 u]

Q.56 A reaction of 0.1 mole of Benzylamine with bromomethane gave 23 g of Benzyl trimethyl ammonium bromide. The number of moles of bromomethane consumed in this reaction are $n \times 10^{-1}$, when n = ———. (Round off to the Nearest Integer).
(Given : Atomic masses : C : 12.0 u, H : 1.0 u, N : 14.0 u, Br : 80.0 u)

Q.57 The total number of unpaired electrons present in the complex $K_3[Cr(oxalate)_3]$ is ——.

Q.58 2 molal solution of a weak acid HA has a freezing point of 3.885ºC. The degree of dissociation of this acid is —— x 10^{-3}. (Round off to the Nearest Integer).
[Given : Molal depression constant of water = 1.85 K kg mol^{-1} Freezing point of pure water = 0ºC]

Q.59 For the reaction
$2Fe^{3+}(aq) + 2I^-(aq) \rightarrow 2Fe^{2+}(aq) + I_2(s)$
the magnitude of standard molar free energy change, $\Delta_r G^o_m$ = – ——— kJ (Round off to the Nearest Integer).

$$\left[\begin{array}{l} E^o_{Fe^{2+}/Fe(s)} = -0.440\,V; E^o_{Fe^{3+}/Fe(s)} = -0.036\,V \\ E^o_{I2/2I^-} = 0.539\,V; \qquad F = 96500\,C \end{array}\right]$$

Q.60 Complete combustion of 3 g of ethane gives $x \times 10^{22}$ molecules of water. The value of x is ––. (Round off to the Nearest Integer).
[Use : $N_A = 6.023 \times 10^{23}$; Atomic masses in u : C : 12.0 ; 0 : 16.0 ; H : 1.0]

MATHEMATICS

Section –A

Q.61 The differential equation satisfied by the system of parabolas $y^2 = 4a(x + a)$ is :

(1) $y\left(\frac{dy}{dx}\right)^2 - 2x\left(\frac{dy}{dx}\right) - y = 0$

(2) $y\left(\frac{dy}{dx}\right)^2 - 2x\left(\frac{dy}{dx}\right) + y = 0$

(3) $y\left(\frac{dy}{dx}\right)^2 + 2x\left(\frac{dy}{dx}\right) - y = 0$

(4) $y\left(\frac{dy}{dx}\right) + 2x\left(\frac{dy}{dx}\right) - y = 0$

Q.62 The number of integral values of m so that the abscissa of point of intersection of lines $3x + 4y = 9$ and $y = mx + 1$ is also an integer, is :
(1) 1 (2) 2
(3) 3 (4) 0

Q.63 Let $(1 + x + 2x^2)^{20} = a_0 + a_1x + a_2x^2 + ... + a_{40}x^{40}$
Then $a_1 + a_3 + a_5 + ... + a_{37}$ is equal to
(1) $2^{20}(2^{20} - 21)$ (2) $2^{19}(2^{20} - 21)$
(3) $2^{19}(2^{20} + 21)$ (4) $2^{20}(2^{20} + 21)$

Q.64 The solutions of the equation

$$\begin{vmatrix} 1+\sin^2 x & \sin^2 x & \sin^2 x \\ \cos^2 x & 1+\cos^2 x & \cos^2 x \\ 4\sin 2x & 4\sin 2x & 1+4\sin 2x \end{vmatrix} = 0, (0 < x < \pi)$$

Are
(1) $\frac{\pi}{12}, \frac{\pi}{6}$ (2) $\frac{\pi}{6}, \frac{5\pi}{6}$
(3) $\frac{5\pi}{12}, \frac{7\pi}{12}$ (4) $\frac{7\pi}{12}, \frac{11\pi}{12}$

Q.65 Choose the correct statement about two circles whose equations are given below :
$x^{2+} y^2 - 10x - 10y + 41 = 0$
$x^2 + y^2 - 22x - 10y + 137 = 0$
(1) circles have same centre
(2) circles have no meeting point
(3) circles have only one meeting point
(4) circles have two meeting points

Q.66 Let α, β, γ be the real roots of the equation, $x^3 + ax^2 + bx + c = 0$, $(a,b,c \in R$ and $a,b \neq 0)$. If the system of equations (in, u, v, w) given by $\alpha u + \beta v + \gamma w = 0$, $\beta u + \gamma v + \alpha w = 0$; $\gamma u + \alpha v + \beta w = 0$ has non-trivial solution, then the value of $\frac{a^2}{b}$ is
(1) 5 (2) 3 (3) 1 (4) 0

Q.67 The integral $\int \frac{(2x-1\cos)\sqrt{2x-1^2+5}}{\sqrt{4x^2-4x+6}}dx$ is equal to
(where c is a constant of integration)

(1) $\frac{1}{2}\sin\sqrt{(2x-1^2)+5}+c$

(2) $\frac{1}{2}\cos\sqrt{(2x+1^2)+5}+c$

(3) $\frac{1}{2}\cos\sqrt{(2x-1^2)+5}+c$

(4) $\frac{1}{2}\sin\sqrt{(2x+1^2)+5}+c$

Q.68 The equation of one of the straight lines which passes through the point (1,3) and makes an angles $\tan^{-1}(\sqrt{2})$ with the straight line, $y+1=3\sqrt{2}x$ is

(1) $4\sqrt{2x}+5y-(15+4\sqrt{2})=0$

(2) $5\sqrt{2x}+4y-(15+4\sqrt{2})=0$

(3) $4\sqrt{2x}+5y-4\sqrt{2}=0$

(4) $4\sqrt{2x}-5y-(5+4\sqrt{2})=0$

Q.69 If $\lim\limits_{x\to 0}\frac{\sin^{-1}x-\tan^{-1}x}{3x^3}$ is equal to L, then the value of (6L + 1) is

(1) $\frac{1}{6}$ (2) $\frac{1}{2}$ (3) 6 (4) 2

Q.70 A vector $\vec{a}$ has components 3p and 1 with respect to a rectangular cartesian system. This system is rotated through a certain angle about the origin in the counter clockwise sense. If, with respect to new system, $\vec{a}$ has components p + 1 and $\sqrt{10}$, then a value of p is equal to :

(1) 1 (2) $-\frac{5}{4}$ (3) $\frac{4}{5}$ (4) – 1

Q.71 If the equation $a|z|^2+\overline{\bar{\alpha}z+\alpha\bar{z}}+d=0$ represents a circle where a,d are real constants then which of the following condition is correct ?

(1) $|\alpha|^2-ad\neq 0$

(2) $|\alpha|^2-ad>0$ and $a\in R-\{0\}$

(3) $|\alpha|^2-ad\geq 0$ and $a\in R$

(4) $\alpha=0$, $a,d\in R^+$

Q.72 For the four circles M, N, O and P, following four equations are given :
Circle M : $x^2+y^2=1$
Circle N : $x^2+y^2-2x=0$
Circle O : $x^2+y^2-2x-2y+1=0$
Circle P : $x^2+y^2-2y=0$
If the centre of circle M is joined with centre of the circle N, further centre of circle N is joined with centre of the circle O, centre of circle O is joined with the centre of circle P and lastly, centre of circle P is joined with centre of circle M, then these lines from the sides of a :

(1) Rhombus (2) Square
(3) Rectangle (4) Parallelogram

Q.73 If α, β are natural numbers such that $100^{\alpha}-99\beta=(100)(100)+(99)(101)+(98)(102)+...+(1)(199)$, then the slope of the line passing through (α,β) and origin is :

(1) 540 (2) 550 (3) 530 (4) 510

Q.74 The real valued function $f(x)=\frac{\text{cosec}^{-1}x}{\sqrt{x-[x]}}$, where [x] denotes the greatest integer less than or equal to x, is defined for all x belonging to :

(1) all reals except integers
(2) all non-integers except the interval [–1,1]
(3) all integers except 0,–1,1
(4) all reals except the Interval [–1,1]

Q.75 $\frac{1}{3^2-1}+\frac{1}{5^2-1}+\frac{1}{7^2-1}+....+\frac{1}{(201)^2-1}$ is equal to

(1) $\frac{101}{404}$ (2) $\frac{25}{101}$
(3) $\frac{101}{408}$ (4) $\frac{99}{400}$

Q.76 If the functions are defined as $f(x)=\sqrt{x}$ and $g(x)=\sqrt{1-x}$, then what is the common domain of the following functions :
$f+g.f-g.f/g,g/f$, $g-f$ where $(f\pm g)(x)=f(x)\pm g(x)$, $(f/g)(x)=\frac{f(x)}{g(x)}$

(1) $0\leq x\leq 1$ (2) $0\leq x<1$
(3) $0<x<1$ (4) $0<x\leq 1$

Q.77 If $f(x) = \begin{cases} \frac{1}{|x|} & |x| \geq 1 \\ ax^2 + b, & |x| < 1 \end{cases}$ is differentiable at every point of the domain, then the values of a and b are respectively :

(1) $\frac{1}{2}, \frac{1}{2}$ (2) $\frac{1}{2}, -\frac{3}{2}$

(3) $\frac{5}{2}, -\frac{3}{2}$ (4) $-\frac{1}{2}, \frac{3}{2}$

Q.78 Let $A + 2B = \begin{bmatrix} 1 & 2 & 0 \\ 6 & -3 & 3 \\ -5 & 3 & 1 \end{bmatrix}$

And $2A - B = \begin{bmatrix} 2 & -1 & 5 \\ 2 & -1 & 6 \\ 0 & 1 & 2 \end{bmatrix}$. If Tr(A) denotes the sum of all diagonal elements of the matrix A, then Tr(A) – Tr(B) has value equal to

(1) 1 (2) 2 (3) 0 (4) 3

Q.79 The sum of all the 4-digit distinct numbers that can be formed with the digits 1,2,2 and 3 is :

(1) 26664 (2) 122664

(3) 122234 (4) 22264

Q.80 The value of $3 + \cfrac{1}{4 + \cfrac{1}{3 + \cfrac{1}{4 + \cfrac{1}{3 + \ldots \infty}}}}$ is equal to

(1) $1.5 + \sqrt{3}$ (2) $2 + \sqrt{3}$

(3) $3 + 2\sqrt{3}$ (4) $4 + \sqrt{3}$

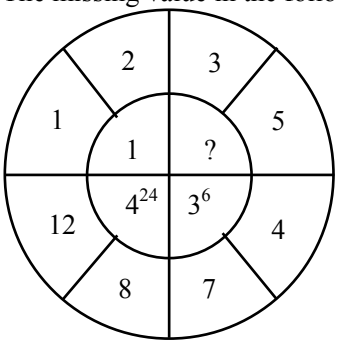
Section –B

Q.81 The number of times the digit 3 will be written when listing the integers from 1 to 1000 is

Q.82 Let the plane ax + by + cz + d = 0 bisect the line joining the points (4, –3, 1) and (2, 3, –5) at the right angles. If a, b, c, d are integers, then the minimum value of $(a^2 + b^2 + c^2 + d^2)$ is

Q.83 Let $f(x)$ and g(x) be two functions satisfying $f(x^2) + g(4 - x) = 4x^3$ and $g(4 - x) + g(x) = 0$, then the value of $\int_{-4}^{4} f(x)^2 dx$ is

Q.84 The missing value in the following figure is



Q.85 Let z_1, z_2 be the roots of the equation $z^2 + az + 12 = 0$ and z_1, z_2 form an equilateral triangle with origin. Then, the value of |a| is

Q.86 The equation of the planes parallel to the plane x – 2y + 2z – 3 = 0 which are at unit distance from the point (1, 2, 3) is ax + by + cz + d = 0. If (b – d) = K(c – a), then the positive value of K is

Q.87 The mean age of 25 teachers in a school is 40 years. A teacher retires at the age of 60 years and a new teacher is appointed in his place. If the mean age of the teachers in his school now is 39 years, then the age (in years) of the newly appointed teacher is .

Q.88 If $f(x) = \int \frac{5x^8 + 7x^6}{(x^2 + 1 + 2x^7)} dx, (x \geq 0), f(0) = 0$

And $f(1) = \frac{1}{K}$, then the value of K is

Q.89 A square ABCD has all its vertices on the curve $x^2y^2 = 1$. The midpoints of its sides also lie on the same curve. Then, the square of area of ABCD is

Q.90 The number of solutions of the equation $|\cot x| = \cot x + \frac{1}{\sin x}$ in the interval $[0, 2\pi]$ is

JEE MAIN ONLINE PAPER 2021

Held on March 18, 2021 (Morning)

Hints & Solutions

PHYSICS

SECTION-A

1.[2] $qE = Mg$

$neE = \rho\left(\frac{4}{3}\pi r^3\right) \times g$

$n \times 1.6 \times 10^{-19} \times 3.55 \times 10^5$

$= 3 \times 10^3 \times \frac{4}{3} \times \pi \times (2 \times 10^{-3})^3 \times 9.81$

$n = 173 \times 10^{(3-9-5+19)}$

$n = 1.73 \times 10^{10}$

2.[1] Order of atmosphere stratification form bottom Troposphere, stratosphere, Mesosphere, Thermosphere

(a) $\rightarrow$ (iv)

(b) $\rightarrow$ (iii)

(c) $\rightarrow$ (ii)

(d) $\rightarrow$ (i)

3.[2] $E \propto \frac{1}{r}$ $\qquad r \propto \frac{1}{m}$

$E \propto m$

Ionization potential $= 13.6 \times \frac{(\text{Mass}_\mu)eV}{(\text{Mass}_e)}$

$= 13.6 \times 207$ eV $= 2815.2$ eV

4.[3] $E = BC = 6$

(Dir. Of wave) $\| (\vec{E} \times \vec{B})$

$\hat{i} = \hat{j} \times \hat{k}$

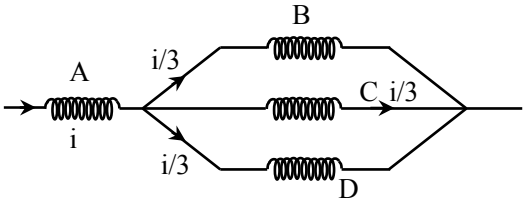
$\vec{E} = 6\hat{j}$ V/m

5.[3] Using conservation of angular momentum

$(Mr^2)\omega = (Mr^2 + 2mr^2)\omega'$

$\omega' = \frac{M\omega}{M + 2m}$

6.[4]



$\phi \propto i$

$\Rightarrow B \propto i$

So, field at centre of C $= \frac{3}{3} = 1$T

7.[2] $g = \frac{4\pi^2 \ell}{T^2}$

$\frac{\Delta g}{g} = \frac{\Delta \ell}{\ell} + 2\frac{\Delta T}{T} = \frac{0.1}{10} + 2\left(\frac{\frac{1}{200}}{0.5}\right)$

$\frac{\Delta g}{g} = \frac{1}{100} + \frac{1}{50}$

$\frac{\Delta g}{g} \times 100 = 3\%$

8.[2] $P = C$

$FV = C$

$M\frac{dV}{dt}V = C$

$\frac{V^2}{2} \propto t$

$V \propto t^{1/2}$

$\frac{dx}{dt} \propto t^{1/2}$

$x \propto t^{3/2}$

9.[1] Energy associated with each degree of freedom per molecule $= \frac{1}{2}k_B T$.

10.[3] $\lambda eq = \lambda_1 + \lambda_2$

$\frac{1}{T_{1/2}} = \frac{1}{T_{1/2}^{(1)}} + \frac{1}{T_{1/2}^{(2)}}$

$T_{1/2} = \frac{T_{1/2}^{(1)} T_{1/2}^{(2)}}{T_{1/2}^{(1)} + T_{1/2}^{(2)}}$

11.[1] Adiabatic process is from C to D

$$WD = \frac{P_2V_2 - P_1V_1}{1-\gamma}$$

$$= \frac{P_DV_D - P_CV_C}{1-\gamma}$$

$$= \frac{200(3) - (100)(4)}{1-1.4}$$

$= -500$ J Ans. (1)

12.[2] $\beta = \frac{\lambda D}{d} = \frac{5890\times10^{-10}\times0.5}{0.5\times10^{-3}}$

$= 589 \times 10^{-6}$ m

Distance between first and third bright fringe is

$2\beta = 2 \times 589 \times 10^{-6}$ m

$= 1178 \times 10^{-6}$ m

13.[4] $\lambda = \frac{h}{p}$

$$\frac{\lambda_p}{\lambda_e} = \frac{P_e}{P_p} = \frac{m_ev_e}{m_pv_p}$$

$$2 = \frac{m_e}{m_p} = \left(\frac{v_e}{4v_e}\right)$$

$$\therefore m_p = \frac{m_e}{8}$$

14.[2] Option (2) represent correct graph for particle moving with constant acceleration, as for constant acceleration velocity time graph is straight line with positive slope and x-t graph should be an opening upward parabola.

15.[1] $R = \frac{\rho\ell}{A} = \frac{V}{1}$

$$\rho = \frac{AV}{I\ell} = \frac{\pi d^2V}{4I\ell} \quad \left(A = \frac{\pi d^2}{4}\right)$$

$$\therefore \frac{\Delta\rho}{\rho} = \frac{2\Delta d}{d} + \frac{\Delta V}{V} + \frac{\Delta I}{I} + \frac{\Delta\ell}{\ell}$$

$$\frac{\Delta\rho}{\rho} = 2\left(\frac{1.01}{5.00}\right) + \frac{0.1}{5.0} + \frac{0.01}{2.00} + \frac{0.1}{10.0}$$

$$\frac{\Delta\rho}{\rho} = 0.004 + 0.02 + 0.005 + 0.01$$

$$\frac{\Delta\rho}{\rho} = 0.039$$

$$\% \text{ error} = \frac{\Delta\rho}{\rho} \times 100 = 0.039 \times 100 = 3.90\%$$

16.[1] Bandwidth = R/L

Bandwidth $\propto$ R

So bandwidth will increase

17.[1] $i = i_0 \cos(\omega t)$

$i = i_0$ at $t = 0$

$$i = \frac{i_0}{\sqrt{2}} \text{ at } \omega t = \frac{\pi}{4}$$

$$t = \frac{\pi}{4\omega} = \frac{\pi}{4(2\pi f)} = \frac{1}{8f}$$

$$t = \frac{1}{400} = 2.5\text{ms}$$

18.[2] If distant objects are blurry the problem is Myopia.

If objects are distorted then problem is Astigmatism

19.[1] Every part $(d\ell)$ of the wire is pulled by force $i\,(d\ell)\,B$ acting perpendicular to current & magnetic field giving it a shape of circle.

20.[2] $T^2 \propto R^3$

$$\left(\frac{T'}{T}\right)^2 = \left(\frac{9R}{R}\right)^3$$

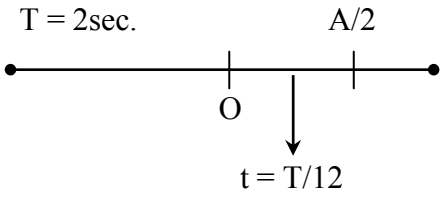
$T^2 = T^2 \times 9^3$

$T' = T \times 3^3$

$T' = 27\ T$

SECTION-B

21.[6]



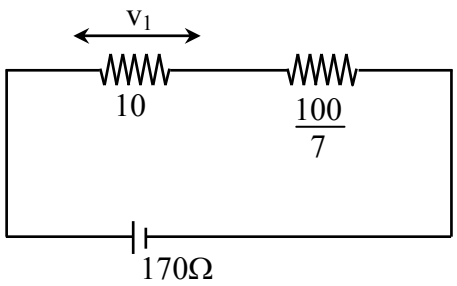
$$t = \frac{2}{12} = \frac{1}{6}$$

$\therefore$ Correct answer = 6.00

22.[2] $i_0 = \frac{V}{R} = \frac{30/3}{5\times10^6} = 2\times10^{-6}$

$\therefore$ Ans. = 2.00

23.[70] $R_{eq1} = \frac{50\times 20}{70} = \frac{100}{7}$



$R_{eq} = \frac{170}{7}$

$v_1 = \left[\frac{\frac{170}{7}}{\frac{170}{7}}\right]\times 10 = 70v$

Ans. = 70.00

24.[32] For A $\frac{E}{\pi r^2} = y\frac{2mm}{a}$ ..... (1)

For B $\frac{E}{\pi.16r^2} = y\frac{4mm}{b}$ ..... (2)

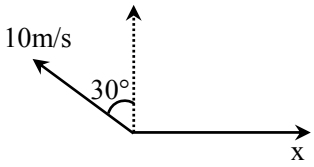
$\therefore$ (1)/(2)

$16 = \frac{2b}{4a}$

$\frac{a}{b} = \frac{1}{32}$

$\therefore$ Answer = 32

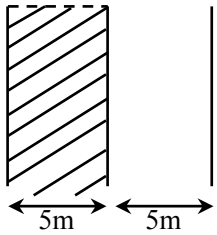
25.[5]



$10 \sin 30^\circ = x$

$x = 5$ m/s

26.[161]



$A = 100\ m^2$

Using $C = \frac{k\in_0 A}{d}$

$C_1 = \frac{10\in_0 (100)}{5}$

$= 200\in_0$

$C_2 = \frac{\in_0 (100)}{5} = 20\in_0$

C_1 & C_2 are in series so $C_{eqv} = \frac{C_1C_2}{C_1+C_2}$

$= \frac{4000\in_0}{220}$

$= 160.9 \times 10^{-12} = 161$ pF

27.[20] Let velocity of 2nd fragment is $\vec{v}$ then by conservation of linear momentum

$10\left(10\sqrt{3}\right)\hat{i} = (10)\left(10\hat{j}\right) + 10\vec{v}$

$\Rightarrow \vec{v} = 10\sqrt{3}\hat{i} - 10\hat{j}$

$\left|\vec{v}\right| = \sqrt{300+100} = \sqrt{400} = 20 m/s$

28.[10] Using work energy theorem,

$W_g = \Delta K.E.$

$(10)\ (g)\ (5) = \frac{1}{2}\ (10)v^2 - 0$

$v = 10$ m/s

29.[100] $10^6 = \beta^2 \times \frac{R_0}{R_i}$

$10^6 = \beta^2 \times \frac{10^4}{10^2}$

$\beta^2 = 10^4 \Rightarrow \beta = 100$

30.[10] $v^2 = u^2 + 2as$

$0 = (10)^2 + 2\ (-a)\left(\frac{1}{2}\right)$

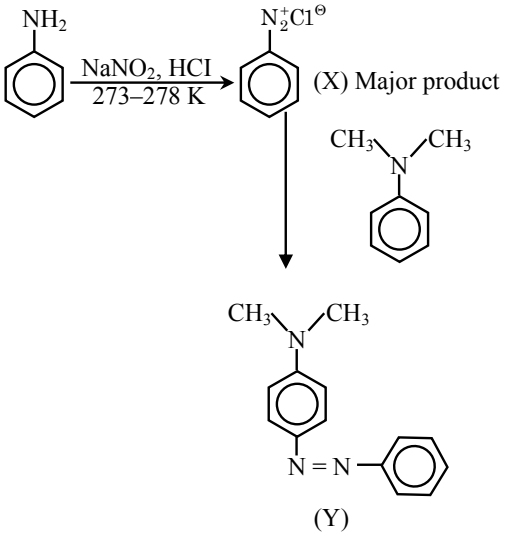
$A = 100\ m/s^2$

$F = ma = (0.1)\ (100) = 10$ N

CHEMISTRY

SECTION-A

31.[2]



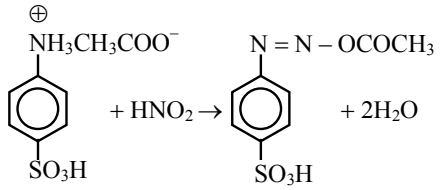
Major product

32.[2] The ionic radii order is
$Na^+ > Mg^{2+} > A1^{3+}$

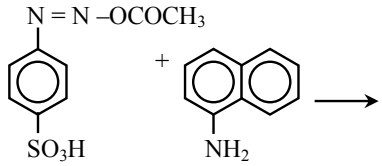
33.[3]

34.[4] For detection of NO^-_2, the following test is used.

$$NO_2^- + CH_3COOH \rightarrow HNO_2 + CH_3COO^-$$

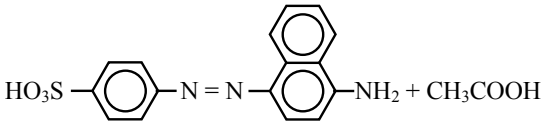


Sulphanilic acid solution



Diazotized acid

1-napthy1 amine

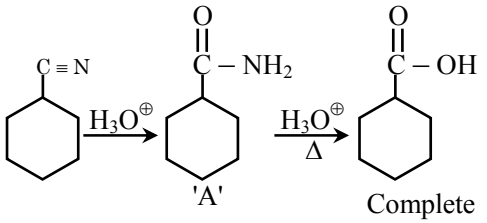


(Red azo dye)

35.[4] Sucrose $\xrightarrow{H_2O}$ glucose + Fructose
(Non reducing sugar) (Reducing sugar) (Reducing sugar)

36.[1] (a) Antacid : Cimetidine
(b) Artifical Sweetener : Alitame
(c) Antifertility : Novestrol
(d) Tranquilizers : Valium

37.[3]



38.[2] Chlorophyll is a coordination compound of magnesium.
Vitamin B – 12, cyanocobalamine is a coordination compound of cobalt.
Cisplatin is used as an anti-cancer drug and is a coordination compound of platinum.
Grubbs catalyst is a compound of Ruthenium.

39.[2] (a) Alcoholic potassium hydroxide → used for β-elimination
(b) pd/ $BaSO_4$ → Lindlar's catalyst
(c) BHC (Benzene hexachloride) → Obtained by addition reactions
(d) Polyacetylene → Electrodes in batteries

40.[1] Methane leads to both global warming & photochemical smog
Methane is generated in large amounts from paddy fields.
CO_2 can be absorbed by photosynthesis, or by formation of acid rain etc., while no such activities are there for methane.
Hence methane is stronger global warming gas than Ch_4 .
Methane is not a part of reducing smog.

41.[3] $Ca(OCl)_2$ is Bleach.

$CaSO_4 \frac{1}{2} H_2O$ is plaster of paris.

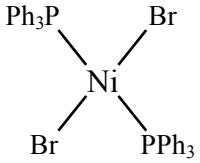
$CaSO_3$ is used as an antacid.

CaO is major component of cement.

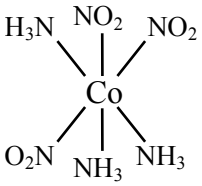
42.[4] $C_3H_6O \Rightarrow CH_3-CH_2-CH=O$

& $CH_3-\underset{\underset{O}{\|}}{C}-CH_3$

43.[4] trans-$[Ni\ Br_2\ (PPh_3)_2]$ is



meridional – $[Co\ (NH_3)_3(NO_2)_3]$ is



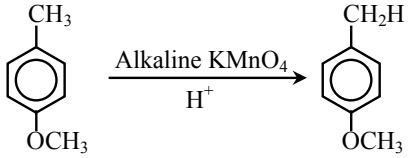
44.[2] $1 = 0 \Rightarrow$'s' orbital

n, $-l - 1 = 2$

$n - 1 = 2$

$n = 3$

45.[3]



(X)

46.[1] In manufacture of H_2SO_4 (contact process), V_2O_5 is used as a catalyst.

Ni catalysts enables the hydrogenation of fats.

$CuCl_2$ is used as catalyst in Deacon's process.

ZSM-5 used as catalyst in cracking of hydrocarbons.

47.[4] For temporary hardiness,

$Mg(HCO_3)_2 \xrightarrow{\text{heating}} Mg(OH)_2\downarrow + 2CO_2\uparrow$

Assertion is false.

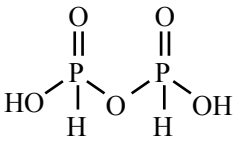
$MgCO_3$ has high solubility product than $Mg(OH)_2$.

According to data of NCERT table 7.9 (Equilibrium chapter), the solubility product of magnesium carbonate is 3.5×10^{-18} and solubility product of $Mg(OH)_2$ is 1.8×10^{-11}

Hence Reason is incorrect.

The question should be Bonus.

48.[3] $PCl_3 + H_3PO_3 \rightarrow H_4P_2O_5$



(Two ionisable H)

49.[2] $M_{12\times\frac{2}{3}}A_6$

M_8A_6

M_4A_3

50.[1] To reduce the melting point of reaction mixture, cryolite is added.

SECTION-B

51.[15] Ax a covalent diatomic molecule.

The molecule is NO.

Total no. of electrons is 15.

52.[10] $pH = pKa + 10g \frac{[CB]}{[WA]}$

$5.74 = 4.74 + 10g \frac{[CB]}{1}$

$\Rightarrow [CB] = 10\ M$

53.[3] $r = k[NO]^m [Cl_2]^n$

$= k(0.1)^m (0.1)^n$ ..... (1)

$= k(0.1)^m (0.2)^n$ ..... (2)

$= k(0.2)^m (0.2)^n$ ..... (3)

$n = 1$

$m = 2$

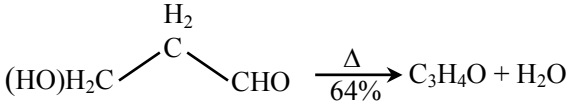
$m + n = 3$

54.[128] $\Delta_r H = [\in_{C-C} + 2 \in_{C-H}] - [\in_{C=C} + \in_{H-H}]$

$= [347 + 2 \times 414] - [611 + 436]$

$= 128$

55.[16]



$\frac{x}{74}$ mol $\qquad \frac{x}{74} \times 0.64 = \frac{7.8}{56}$

$x = 16.10$

$\simeq 16.00$

56.[3]

$$Ph–CH_2–NH_2 \xrightarrow[-HBr]{CH_3–Br} Ph–CH_2–\ddot{N}H–CH_3$$

$$\xrightarrow[-HBr]{CH_3–Br} Ph–CH_2–\overset{\oplus}{N}(CH_3)_3 \xrightarrow[-HBr]{CH_3–Br} Ph–CH_2–\ddot{N}(CH_3)_2$$

No of moles = 3

57.[3] $K_3[Cr(oxalate)_3]$
Chromium is in + 3 oxidation state.
Number of unpaired electrons in Cr^{+3} will be 3.

58.[50] $\Delta T_f = (1 + \alpha) K_f . m$
$\alpha = 0.05 = 50 \times 10^{-3}$

59.[45] **Official Ans. by NTA (46)**

$$Fe^{3+} \xrightarrow{E_1^0} Fe^{2+} \xrightarrow{E_2^0} Fe \qquad (E_3^0)$$

$E_1^0 + 2E_2^0 = 3E_3^0$
$E_1^0 + 3E_2^0 - 3E_2^0$
$= 3 (-0.036) - 2(-0.44)$
$= + 0.772$ V
$E^0_{cell} + E^0_{Fe^{3+}/Fe^{2+}} + E^0_{I^-/I_2} = 0.233$
$\Delta_r G^o = - 2 \times 96.5 \times 0.233 = - 45$ kJ

60.[18] $C_2H_6 \rightarrow 3H_2O$
0.1 mol $\quad 0.3 = 0.3 \times 6 \times 10^{23} = 18 \times 10^{22}$ mol
No. of molecules $= 0.3 \times 6.023 \times 10^{23}$
$= 18.069 \times 10^{22}$

MATHEMATICS

SECTION-A

61.[3] $y^2 = 4ax + 4a^2$
differentiate with respect to x

$$\Rightarrow 2y\frac{dy}{dx} = 4a$$

$$\Rightarrow a = \left(\frac{y}{2}\frac{dy}{dx}\right)$$

So, required differential equation is

$$y^2 = \left(4 \times \frac{y}{2}\frac{dy}{dx}\right)x + 4\left(\frac{y}{2}\frac{dy}{dx}\right)^2$$

$$\Rightarrow y^2\left(\frac{dy}{dx}\right)^2 + 2xy\left(\frac{dy}{dx}\right) - y^2 = 0$$

$$\Rightarrow y\left(\frac{dy}{dx}\right)^2 + 2x\left(\frac{dy}{dx}\right) - y = 0$$

62.[2] $3x + 4y = 9$
$y = mx + 1$
$\Rightarrow 3x + 4mx + 4 = 9$
$\Rightarrow (3 + 4m)\, x = 5$
$\Rightarrow$ x will be an integer when
$3 + 4m = 5, -5, 1, -1$

$$\Rightarrow m = \frac{1}{2}, -2, -\frac{1}{2}, -1$$

So, number of integral values of m is 2

63.[2] $(1 + x + 2x^2)^{20} = a_0 + a_1x + ... + a_{40}x^{40}$
put x = 1, – 1
$\Rightarrow a_0 + a_1 + a_2 + ... + a_{40} = 2^{20}$
$a_0 - a_1 + a_2 + ... + a_{40} = 2^{20}$

$$\Rightarrow a_1 + a_3 + ... + a_{39} = \frac{4^{20} - 2^{20}}{2}$$

$\Rightarrow a_1 + a_3 + ... + a_{37} = 2^{39} - 2^{19} - a_{39}$

Here $a_{39} = \frac{20!(2)^{19} \times 1}{19!} = 20 \times 2^{19}$

$\Rightarrow a_1 + a_3 + ... + a_{37} = 2^{19}(2^{20} - 1 - 20)$
$= 2^{19}(2^{20} - 21)$

64.[4]

$$\begin{vmatrix} 1+\sin^2 x & \sin^2 x & \sin^2 x \\ \cos^2 x & 1+\cos^2 x & \cos^2 x \\ 4\sin 2x & 4\sin 2x & 1+4\sin 2x \end{vmatrix} = 0$$

Use $R_1 \rightarrow R_1 + R_2 + R_3$
$\Rightarrow (2 + 4\sin 2x)$

$$\begin{vmatrix} 1 & 1 & 1 \\ \cos^2 x & 1+\cos^2 x & \cos^2 x \\ 4\sin 2x & 4\sin 2x & 1+4\sin 2x \end{vmatrix} = 0$$

$$\Rightarrow \sin 2x = -\frac{1}{2}$$

$$\Rightarrow 2x = \pi + \frac{\pi}{6}, \quad 2\pi - \frac{\pi}{6}$$

$$X = \frac{\pi}{2} + \frac{\pi}{12}, \quad \pi - \frac{\pi}{12}$$

65.[3] $x^2 + y^2 - 10x - 10y + 41 = 0$

A (5,5), $R_1 = 3$

$x^2 + y^2 - 22x - 10y + 137 = 0$

B(11,5), $R_2 = 3$

$AB = 6 = R_1 + R_2$

Touch each other externally

$\Rightarrow$ circles have only one meeting point.

66.[2] $\begin{vmatrix} \alpha & \beta & \gamma \\ \beta & \gamma & \alpha \\ \gamma & \alpha & \beta \end{vmatrix} = 0$

$\Rightarrow -(\alpha+\beta+\gamma)(\alpha^2+\beta^2+\gamma^2-\sum\alpha\beta) = 0$

$\Rightarrow -(-a)(a^2 - 2b - b) = 0$

$\Rightarrow a(a^2 - 3b) = 0$

$\Rightarrow a^2 = 3b \Rightarrow \dfrac{a^2}{b} = 3$

67.[1] $\displaystyle\int \frac{(2x-1)\cos\sqrt{(2x-1)^2+5}}{\sqrt{(2x-1)^2+5}}dx$

$(2x-1)^2 + 5 = t^2$

$2(2x-1)\,2dx = 2t\,dt$

$2\sqrt{t^2-5}dx = t\,dt$

So $\displaystyle\int \frac{\sqrt{t^2-5}\cos t}{2\sqrt{t^2-5}}dt = \frac{1}{2}\sin t + c$

$= \dfrac{1}{2}\sin\sqrt{(2x-1)^2+5} + c$

68.[1] $y = mx + c$

$3 = m + c$

$\sqrt{2} = \left|\dfrac{m - 3\sqrt{2}}{1 + 3\sqrt{2m}}\right|$

(i) $6m + \sqrt{2} = m - 3\sqrt{2}$

$\Rightarrow 5m = -4\sqrt{2} \Rightarrow m = -4\dfrac{\sqrt{2}}{5}$

(ii) $-\sqrt{2} - 6m = m - 3\sqrt{2}$

$\Rightarrow m = 2\dfrac{\sqrt{2}}{7}$

According to options take $m = \dfrac{-4\sqrt{2}}{5}$

So $y = \dfrac{-4\sqrt{2x}}{5} + \dfrac{3+4\sqrt{2}}{5}$

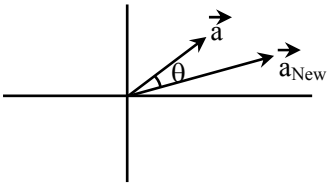
$4\sqrt{2}x + 5y - (15 + 4\sqrt{2}) = 0$

69.[4] $\displaystyle\lim_{x\to 0}\frac{\left(x + \frac{x^3}{3!}\ldots\right) - \left(x - \frac{x^3}{3}\ldots\right)}{3x^3} = \frac{1}{6}$

S,o 6L + 1 = 2

70.[4] $\vec{a}_{Old} = 3p\hat{i} + \hat{j}$

$\vec{a}_{New} = (p+1)\hat{i} + \sqrt{10}\hat{j}$



$\Rightarrow |\vec{a}_{Old}| = |\vec{a}_{New}|$

$\Rightarrow ap^2 + 1 = p^2 + 2p + 1 + 10$

$8p^2 - 2p - 10 = 0$

$4p^2 - p - 5 = 0$

$(4p - 5)(p + 1) = 0 \rightarrow p = \dfrac{5}{4}, -1$

71.[2] $az\bar{z} + \alpha\bar{z} + \bar{\alpha}z + d = 0 \rightarrow$ Circle

Centre $= \dfrac{-\alpha}{a}$ $\quad 2 = \sqrt{\dfrac{\alpha\bar{\alpha}}{a^2} - \dfrac{d}{a}} = \sqrt{\dfrac{\alpha\bar{\alpha} - ad}{a^2}}$

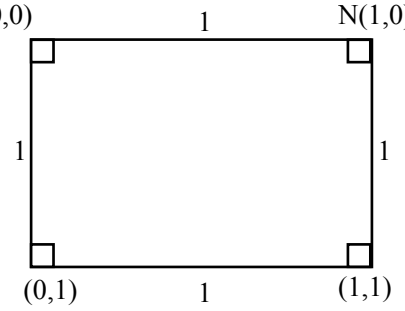
So $|\alpha|^2 - ad > 0$ & $a \in R - \{0\}$

72.[2] M : $x^2 + y^2 = 1$ (0,0)

N : $x^2 + y^2 - 2x = 0$ (1.0)

O : $x^2 + y^2 - 2x - 2y + 1 = 0$ (1,1)

P : $x^2 + y^2 - 2y = 0$ (0,1)



73.[2] $S = (100)(100) + (99)(101) + (98)(102)\ldots..$
$(2)(198) + (1)(199)$

$\displaystyle S = \sum_{x=0}^{99}(100-x)(100+x) = \sum 100^2 - x^2$

$= 100^3 - \dfrac{99\times100\times199}{6}$

$\alpha = 3 \qquad \beta = 1650$

Slope $= \dfrac{1650}{3} = 550$

74.[2] $f(x) = \dfrac{\text{cosec}^{-1}x}{\sqrt{\{x\}}}$

Domain $\in (-\infty,-1] \cup [1,\infty)$

$\{x\} \neq 0$ so $x \neq$ integers

75.[2] $T_n = \dfrac{1}{(2n+1)^2-1}\dfrac{1}{(2n+2)2n} = \dfrac{1}{4(n)(n+1)}$

$= \dfrac{(n+1)-n}{4n(n+1)} = \dfrac{1}{4}\left(\dfrac{1}{n} - \dfrac{1}{n+1}\right)$

$S = \dfrac{1}{4}\left(1 - \dfrac{1}{101}\right) = \dfrac{1}{4}\left(\dfrac{100}{101}\right) = \dfrac{25}{101}$

76.[3] $f(x) + g(x) = \sqrt{x} + \sqrt{1-x}$, domain [0, 1]

$F(x) - g(x) = \sqrt{x} - \sqrt{1-x}$, domain [0, 1]

$g(x) - f(x) = \sqrt{1-x} - \sqrt{x}$, domain [0,1]

$\dfrac{f(x)}{g(x)} = \dfrac{\sqrt{x}}{\sqrt{1-x}}$, domain [0,1)

$\dfrac{g(x)}{f(x)} = \dfrac{\sqrt{1-x}}{\sqrt{x}}$, domain (0,1]

So, common domain is (0,1)

77.[4] $f(x) = \begin{cases} \dfrac{1}{|x|} & |x| \geq 1 \\ ax^2 + b, & |x| < 1 \end{cases}$

at x = 1 function must be continuous

So, $1 = a + b$ ... (1)

Differentiability at x = 1

$\left(-\dfrac{1}{x^2}\right)_{x=1} = (2ax)_{x=1}$

$\Rightarrow -1 = 2a \Rightarrow a = -\dfrac{1}{2}$

$(1) \Rightarrow b = 1 + \dfrac{1}{2} = \dfrac{3}{2}$

78.[2] $A + 2B = \begin{pmatrix} 1 & 2 & 0 \\ 6 & -3 & 3 \\ -5 & 3 & 1 \end{pmatrix}$ ... (1)

$2A - B = \begin{pmatrix} 2 & -1 & 5 \\ 2 & -1 & 6 \\ 0 & 1 & 2 \end{pmatrix}$

$\Rightarrow 4A - 2B = \begin{pmatrix} 4 & -2 & 10 \\ 4 & -2 & 12 \\ 0 & 2 & 4 \end{pmatrix}$ ... (2)

$(1) + (2) \Rightarrow 5A = \begin{pmatrix} 5 & 0 & 10 \\ 10 & -5 & 15 \\ -5 & 5 & 5 \end{pmatrix}$

$A = \begin{pmatrix} 1 & 0 & 2 \\ 2 & -1 & 3 \\ -1 & 1 & 1 \end{pmatrix}$ and $2A = \begin{pmatrix} 2 & 0 & 4 \\ 4 & -2 & 6 \\ -2 & 2 & 2 \end{pmatrix}$

$\therefore B = \begin{pmatrix} 2 & 0 & 4 \\ 4 & -2 & 6 \\ -2 & 2 & 2 \end{pmatrix} - \begin{pmatrix} 2 & -1 & 5 \\ 2 & -1 & 6 \\ 0 & 1 & 2 \end{pmatrix}$

$B = \begin{pmatrix} 0 & 1 & -1 \\ 2 & -1 & 0 \\ -2 & 1 & 0 \end{pmatrix}$

tr (A) = 1 – 1 + 1 = 1

tr (B) = – 1

tr (A) = 1 and tr(B) = – 1

$\therefore$ tr(A) – tr(B) = 2

79.[1] Digits are 1,2, 2, 3

Total distinct numbers $\dfrac{4!}{2!} = 12.$

Total numbers when 1 at unit place is 3
2 at unit place is 6
3 at unit place is 3

So, sum = $(3 + 12 + 9)(10^3 + 10^2 + 10 + 1)$
= $(1111) \times 24$
= 26664

80.[1] Let $x = 3 + \cfrac{1}{4 + \cfrac{1}{3 + \cfrac{1}{4 + \cfrac{1}{3 + \ldots \infty}}}}$

So, $x = 3 + \dfrac{1}{4 + \dfrac{1}{x}} = 3 + \dfrac{1}{\dfrac{4x+1}{x}}$

$\Rightarrow (x-3) = \dfrac{x}{(4x+1)}$

$\Rightarrow (4x + 1)(x - 3) = x$

$\Rightarrow 4x^2 - 12x + x - 3 = x$

$\Rightarrow 4x^2 - 12x - 3 = 0$

$X = \dfrac{12 \pm \sqrt{(12)^2 + 12 \times 4}}{2 \times 4} = \dfrac{12 \pm \sqrt{12(16)}}{8}$

$= \dfrac{12 \pm 4 \times 2\sqrt{3}}{8} = \dfrac{3 \pm 2\sqrt{3}}{2}$

$x = \dfrac{3}{2} \pm \sqrt{3} = 1.5 \pm \sqrt{3}.$

But only positive value is accepted

So, $x = 1.5 + \sqrt{3}$

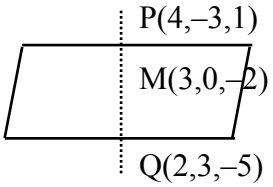
SECTION-B

81.[300] $3 - - = 10 \times 10 = 100$

$- 3 - = 10 \times 10 = 100$

$- - 3 = 10 \times 10 = \dfrac{100}{300}$

82.[28]



Plane is $1(x - 3) - 3(y - 0) + 3(z + 2) = 0$

$x - 3y + 3z + 3 = 0$

$(a^2 + b^2 + c^2 + d^2)_{min} = 28$

83.[512]

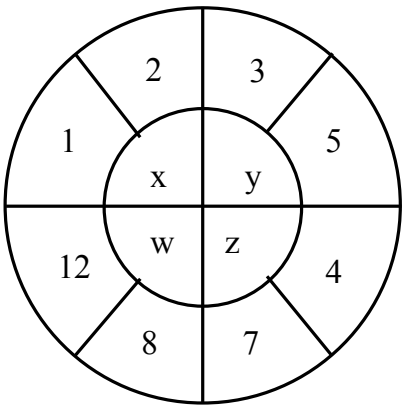
$I = 2\int_0^4 f(x^2)dx$ {Even funtion}

$= 2\int_0^4 (4x^3 - g(4 - x))dx$

$= 2\left(\dfrac{4x^4}{4}\Big|_0^4 - \int_0^4 g(4 - x)dx\right)$

$= 2(256 - 0) = 512$

84.[4] $x = (2 - 1)^{1!} = 1$



$W = (12 - 8)^{4!} = 4^{24}$

$Z = (7 - 4)^{3!} = 3^6$

Hence $y = (5 - 3)^{2!} = 2^2$

85.[6] If $0, z_1 z_2$ are vertices of equilateral triangles

$\Rightarrow a^2 + z_1^2 + z_2^2 = 0 (z_1 + z_2) + z_1 z_2$

$\Rightarrow (z_1 + z_2)^2 = 3z_1 z_2$

$\Rightarrow a^2 = 3 \times 12$

$\Rightarrow |a| = 6$

86.[4] Let plane is $x - 2y + 2z + \lambda = 0$

Distance from $(1,2,3) = 1$

$\Rightarrow \dfrac{|\lambda + 3|}{5} = 1 \Rightarrow \lambda = 0, -6$

$\Rightarrow a = 1, b = -2, c = 2, d = -6$ or 0

$b - d = 4$ or -2, $c - a = 1$

$\Rightarrow k = 4$ or -2

87.[35] $\dfrac{\sum x_i}{25} = 40$ & $\dfrac{\sum x_i - 60 + N}{25} = 39$

Let age of newly appointed teacher is N

$\Rightarrow 1000 - 60 + N = 975$

$\Rightarrow N = 35$ years

88.[4] $f(x) = \int \dfrac{(5x^8 + 7x^6)dx}{x^{14}(x^{-5} + x^{-7} + 2)^2}$

Let $x^{-5} + x^{-7} + 2 = t$

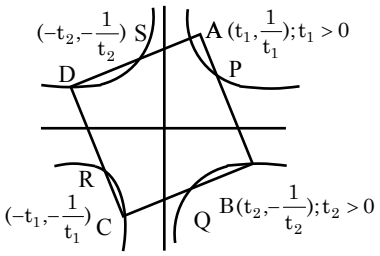
$(-5x^{-6} - 7x^{-8})dx = dt$

$\Rightarrow f(x) = \int -\dfrac{dt}{t^2} = \dfrac{1}{t} + c$

$f(x) = = \dfrac{x^7}{x^2 + 1 + 2x^7}$

$f(1) = \dfrac{1}{4}$

89.[80] $xy = 1, -1$



$\dfrac{t_1 + t_2}{2} \cdot \dfrac{\dfrac{1}{t_1} - \dfrac{1}{t_2}}{2} = 1$

$\Rightarrow t_1^2 - t_2^2 = 4t_1t_2$

$\frac{1}{t_1^2} \times \left(-\frac{1}{t_2^2}\right) = -1 \Rightarrow t_1t_2 = 1$

$t_1^2 - t_2^2 = 4$

$t_1^2 + t_2^2 = \sqrt{4^2+4} = 2\sqrt{5}$

$\Rightarrow t_1^2 = 2\sqrt{5} \Rightarrow \frac{1}{t_1^2} = \sqrt{5} - 2$

$AB^2 = (t_1 - t^2)^2 + \left(\frac{1}{t_1} + \frac{1}{t_2}\right)^2$

$= 2\left(t_1^2 + \frac{1}{t_1^2}\right) = 4\sqrt{5} \Rightarrow \text{Area}^2 = 80$

90.[1] If $\cot x > 0 \Rightarrow \frac{1}{\sin x} = 0$ (Not possible)

If $\cot x < 0 \Rightarrow 2\cot x + \frac{1}{\sin x} = 0$

$\Rightarrow 2\cos x = -1$

$\Rightarrow x = \frac{2\pi}{3} \text{ or } \frac{4\pi}{3} \text{(reject)}$