JEE MAIN ONLINE PAPER 2021

Held on March 17, 2021 (Morning)

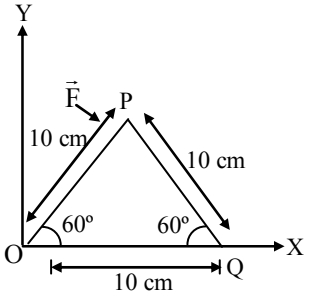
Instructions

1. This test will be a 3 hours Test.
2. This test consists of Physics, Chemistry and Mathematics questions with equal weightage of 100 marks.
3. Each question is of 4 marks.
4. In the question paper consisting of Physics (Q.no. 1 to 30), Chemistry (Q.no. 31 to 60) and Mathematics (Q.no. 61 to 90). There are two sections for each subject (Section-A : MCQ Type & Section-B : Numerical Response Type). Section-A consists of 20 multiple choice questions & Section-B consists of 10 Numerical Value type Questions. **Candidates have a choice to Answer 5 out of the 10 numerical value answer based questions per section.**
5. There will be only one correct choice in the given four choices in Section-A. For each question 4 marks will be awarded for correct choice, 1 mark will be deducted for incorrect choice and zero mark will be awarded for not attempted question. For Section-B questions 4 marks will be awarded for correct answer and zero for unattempted and incorrect answer.
6. Any textual, printed or written material, mobile phones, calculator etc. is not allowed for the students appearing for the test.
7. All calculations/written work should be done in the rough sheet provided.

PHYSICS

Section -A

Q.1 A triangular plate is shown. A force $\vec{F} = 4\ \hat{i} - 3\ \hat{j}$ is applied at point P. The torque at point P with respect to point 'O' and 'Q' are



(1) $-15 - 20\sqrt{3}$, $15 - 20\sqrt{3}$

(2) $15 + 20\sqrt{3}$, $15 - 20\sqrt{3}$

(3) $15 - 20\sqrt{3}$, $15 + 20\sqrt{3}$

(4) $-15 + 20\sqrt{3}$, $15 + 20\sqrt{3}$

Q.2 When two soap bubbles of radii a and b (b > a) coalesce, the radius of curvature of common surface is

(1) $\frac{ab}{b-a}$ (2) $\frac{a+b}{ab}$

(3) $\frac{b-a}{ab}$ (4) $\frac{ab}{a+b}$

Q.3 A polyatomic ideal gas has 24 vibrational modes. What is the value of γ ?

(1) 1.03 (2) 1.30

(3) 1.37 (4) 10.3

Q.4 If an electron is moving in the n^{th} orbit of the hydrogen atom, then its velocity (v_n) for the n^{th} orbit is given as :

(1) $v_n \propto n$

(2) $v_n \propto \frac{1}{n}$

(3) $v_n \propto n^2$

(4) $v_n \propto \frac{1}{n^2}$

Q.5 An electron of mass m and a photon have same energy E. The ratio of wavelength of electron to that of photon is : (c being the velocity of light)

(1) $\frac{1}{c}\left(\frac{2m}{E}\right)^{1/2}$ (2) $\frac{1}{c}\left(\frac{E}{2m}\right)^{1/2}$

(3) $\left(\frac{E}{2m}\right)^{1/2}$ (4) $c\,(2mE)^{1/2}$

Q.6 Two identical metal wires of thermal conductivities K_1 and K_2 respectively are connected in series. The effective thermal conductivity of the combination is

(1) $\frac{2K_1K_2}{K_1+K_2}$ (2) $\frac{K_1+K_2}{2K_1K_2}$

(3) $\frac{K_1+K_2}{K_1K_2}$ (4) $\frac{K_1K_2}{K_1+K_2}$

Q.7 The vernier scale used for measurement has a positive zero error of 0.2 mm. If while taking a measurement it was noted that '0' on the vernier scale lies between 8.5 cm and 8.6 cm, vernier concidence is 6, then the correct value of measurement is ——cm. (least count = 0.061 m)

(1) 8.36 cm (2) 8.54 cm

(3) 8.58 cm (4) 8.56 cm

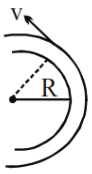
Q.8 An AC current is given by $I = I_1 \sin\omega t + I_2 \cos\omega t$. A hot wire ammeter will give a reading :

(1) $\sqrt{\frac{I_1^2 - I_2^2}{2}}$ (2) $\sqrt{\frac{I_1^2 + I_2^2}{2}}$

(3) $\frac{I_1+I_2}{\sqrt{2}}$ (4) $\frac{I_1+I_2}{2\sqrt{2}}$

Q.9 A modern grand –prix racing car of mass m is travelling on a flat track in a circular arc of radius R with a speed v. If the coefficient of static friction between the tyres and the track is μ_s, then the magnitude of negative lift F_L acting downwards on the car is

(Assume forces on the four tyres are identical and g = acceleration due to gravity)



(1) $m\left(\frac{v^2}{\mu_s R}+g\right)$ (2) $m\left(\frac{v^2}{\mu_s R}-g\right)$

(3) $m\left(g-\frac{v^2}{\mu_s R}\right)$ (4) $-m\left(g+\frac{v^2}{\mu_s R}\right)$

Q.10 A car accelerates from rest at a constant rate α for some time after which it decelerates at a constant rate β to come to rest. If the total time elapsed is t seconds, the total distance travelled is :

(1) $\frac{4\alpha\beta}{(\alpha+\beta)}t^2$ (2) $\frac{2\alpha\beta}{(\alpha+\beta)}t^2$

(3) $\frac{\alpha\beta}{2(\alpha+\beta)}t^2$ (4) $\frac{4\alpha\beta}{4(\alpha+\beta)}t^2$

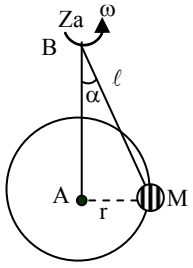
Q.11 A solenoid of 1000 turns per metre has a core with relative permeability 500. Insulated windings of the solenoid carry an electric current of 5A. The magnetic flux density produced by the solenoid is

(permeability of free space = $4\pi \times 10^{-7}$H/m)

(1) πT (2) $2 \times 10^{-3}\ \pi$T

(3) $\frac{\pi}{5}$ T (4) $10-4\pi$T

Q.12 A mass M hangs on a massless rod of length λ which rotates at a constant angular frequency. The mass M moves with steady speed in a circular path of constant radius. Assume that the system is in steady circular motion with constant angular velocity ω. The angular momentum of M about point A is L_A which lies in the positive z direction and the angular momentum of M about B is L_B. The correct statement for this system is



(1) L_A and L_B are both constant in magnitude and direction

(2) L_B is constant in direction with varying magnitude

(3) L_B is constant, both in magnitude and direction

(4) L_A is constant, both in magnitude and direction

Q.13 For what value of displacement the kinetic energy and potential energy of a simple harmonic oscillation become equal ?

(1) $x = 0$ (2) $x = \pm A$

(3) $x = \pm \frac{A}{\sqrt{2}}$ (4) $x = \frac{A}{2}$

Q.14 A carnot's engine working between 400 K and 800 K has a work output of 1200 J per cycle. The amount of heat energy supplied to the engine from the source in each cycle is

(1) 3200 J (2) 1800 J

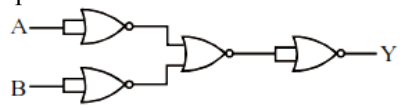
(3) 1600 J (4) 2400 J

Q.15 The thickness at the centre of a plano convex lens is 3 mm and the diameter is 6 cm. If the speed of light in the material of the lens is 2×10^8 ms^{-1}. The focal length of the lens is ——

(1) 0.30 cm (2) 15 cm

(3) 1.5 cm (4) 30 cm

Q.16 The output of the given combination gates represents :



(1) XOR Gate (2) NaND Gate

(3) AND Gate (4) NOR Gate

Q.17 A boy is rolling a 0.5 kg ball on the frictionless floor with the speed of 20 ms^{-1}. The ball gets deflected by an obstacle on the way. After deflection it moves with 5% of its initial kinetic energy. What is the speed of the ball now ?

(1) 19.0 ms^{-1} (2) 4.47 ms^{-1}

(3) 14.41 ms^{-1} (4) 1.00 ms^{-1}

Q.18 Which level of the single ionized carbon has the same energy as the ground state energy of hydrogen atom ?

(1) 1 (2) 6

(3) 4 (4) 8

Q.19 Two ideal polyatomic gases at temperatures T_1 and T_2 are mixed so that there is no loss of energy. If F_1 and F_2, m_1 and m_2, n_1 and n_2 be the degrees of freedom, masses, number of molecules of the first and second gas respectively, The temperature of mixture of these two gases is :

(1) $\frac{n_1T_1 + n_2T_2}{n_1 + n_2}$ (2) $\frac{n_1F_1T_1 + n_2F_2T_2}{n_1F_1 + n_2F_2}$

(3) $\frac{n_1F_1T_1 + n_2F_2T_2}{F_1 + F_2}$ (4) $\frac{n_1F_1T_1 + n_2F_2T_2}{n_1 + n_2}$

Q.20 A current of 10A exists in a wire of crosssectional area of 5 mm^2 with a drift velocity of 2×10^{-3} ms^{-1}. The number of free electrons in each cubic meter of the wire is ——

(1) 2×10^6 (2) 625×10^{25}

(3) 2×10^{25} (4) 1×10^{23}

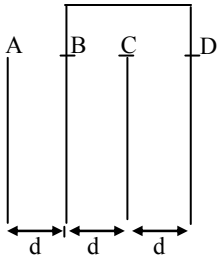
Section -B

Q.21 For VHF signal broadcasting, —— km^2 of maximum service area will be covered by an antenna tower of height 30m if the receiving antenna is placed at ground. Let radius of the earth be 6400 km. (Round off to the Nearest Integer) Take π as 3.14

Q.22 The angular speed of truck wheel is increased from 900 rpm to 2460 rpm in 26 seconds. The number of revolutions by the truck engine during this time is —
(Assuming th acceleration to be uniform)

Q.23 The equivalent resistance of series combination of two resistors is 's' When they are connected in parallel, the equivalent resistance is 'p' If $s = np$, then the minimum value for n is ——
(Round off to the Nearest Integer)

Q.24 Four identical rectangular plates with length, ℓ = 2 cm and breadth, b = $\frac{3}{2}$ cm are arranged as shown in figure. The equivalent capacitance between A and C is $\frac{x\varepsilon_0}{d}$. The value of x is —
(Round off to the Nearest Integer)



Q.25 The radius in kilometer to which the persent radius of earth (R = 6400 km) to be compressed so that the escape velocity is increased 10 time is ——

Q.26 Consider two identical springs each of spring constant k and negligible mass compared to the mass M as shown Fig. 1 shows one of them and Fig.2 shows their series combination. The ratios of time period of oscillation of the two SHM is

$\frac{T_b}{Ta} = \sqrt{x}$, where value of x is ------

(Round off to the Nearest Integer)

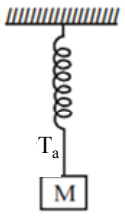


Fig. 1

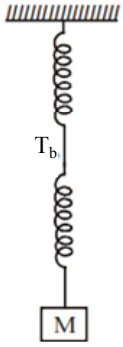


Fig. 2

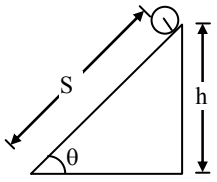
Q.27 The following bodies,

(1) a ring (2) a disc

(3) a solid cylinder (4) a solid sphere,

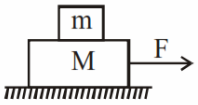
of same mass 'm' and radius 'R' are allowed to roll down without slipping simultaneously form the top of the inclined plane. The body which will reach first at the bottom of the inclined plane is ——

[Mark the body as per their respective numbering given in the question]



Q.28 A Parallel plate capacitor whose capacitance C is 14 pF is charged by a battery to a potential difference V = 12V between its plates. The charging battery is now disconnected and a procelin plate with k = 7 is inserted between the plates, then the plate would oscillate back and forth between the plates with a constant mechanical energy of ——pJ.

Q.29 Two blocks (m = 0.5 kg and M = 4.5 kg) are arranged on a horizontal frictionless table as shown in figure. The coefficient of static friction between the two blocks is $\frac{3}{7}$. Then the maximum horizontal force that can be applied on the larger block so that the blocks move together is ——N. (Round off to the Nearest Integer) [Take g as 9.8 ms^{-2}]



Q.30 If 2.5 × 10–6 N average force is exerted by a light wave on a non-reflecting surface of 30 cm^2 area during 40 minutes of time span, the energy flux of light just before it falls on the surface is ——W/ cm^2

(Round off to the Nearest Integer)

(Assume complete absorption and normal incidence conditions are there)

CHEMISTRY

Section -B

Q.31 With respect to drug-enzyme interaction, identify the wrong statement :

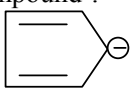
(1) Non-Competitive inhibitor binds to the allosteric site

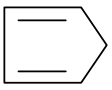
(2) Allosteric inhibitor changes the enzyme's active site

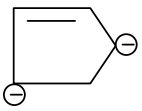
(3) Allosteric inhibitor competes with the enzyme's active site

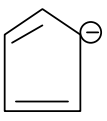
(4) Competitive inhibitor binds to the enzyme's active site

Q.32 Which of the following is an aromatic compound ?

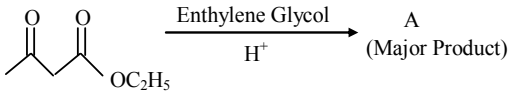
(1) 

(2) 

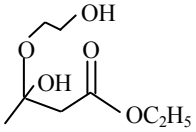
(3) 

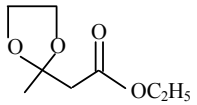
(4) 

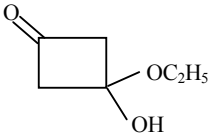
Q.33

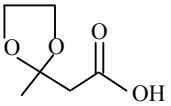


The product "A" in the above reaction is :

(1) 

(2) 

(3) 

(4) 

Q.34 A central atom in a molecule has two lone pairs of electrons and forms three single bonds. The shape of this molecule is :

(1) see-saw (2) planar triangular

(3) T-shaped (4) trigonal pyramidal

Q.35 Given below are two statements :

Statement I : Potassium permanganate on heating at 573 K forms potassium manganate.

Statement II : Both potassium permaganate and potassium manganate are tetrahedral and paramagnetic in nature.

In the light of the above statements, choose the most appropriate answer from the options given below:

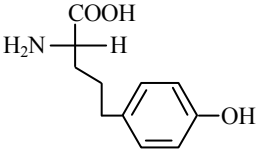
(1) Statement I is true but statement II is false

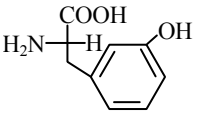
(2) Both statement I and statement II are true

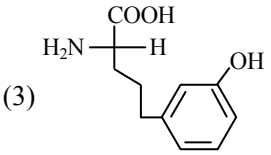
(3) Statement I is false but statement II is true

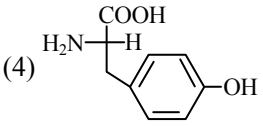
(4) Both statement I and statement II are false

Q.36 Which of the following is correct structure of tyrosine ?

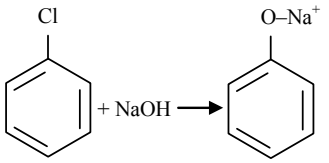
(1) 

(2) 

(3) 

(4) 

Q.37



The above reaction requires which of the following reaction conditions ?

(1) 573. K, Cu, 300 atm

(2) 623 K, Cu, 300 atm

(3) 573 K, 300 atm

(4) 623 K, 300 atm

Q.38 The absolute value of the electron gain enthalpy of halogens satisfies :

(1) I > Br > Cl > F

(2) Cl > Br > F > I

(3) Cl > F > Br > I

(4) F > Cl > Br > I

Q.39 Which of the following compound CANNOT act as a Lewis base ?

(1) NF_3 (2) PCl_5

(3) SF_4 (4) ClF_3

Q.40 Reducing somg is a mixture of :

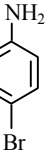
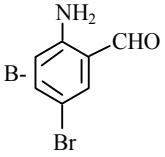
(1) Smoke, fog and O_3

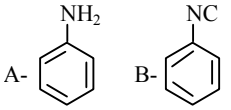
(2) Smoke, fog and SO_2

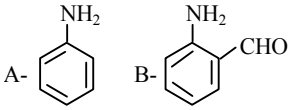
(3) Smoke, fog and $CH_2 = CH–CHO$

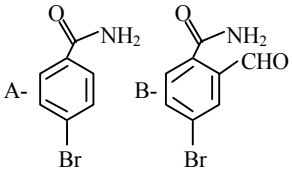
(4) Smoke, fog and N_2O_3

Q.41 Hoffmann bromomide degradation of benzamide gives product A, which upon heating with $CHCl_3$ and NaOH gives product B. The structures of A and B are

(1) A –  B- 

(2) 

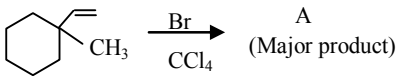
(3) 

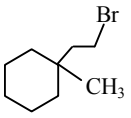
(4) 

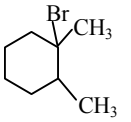
Q.42 Mesityl oxide is a common name of :
(1) 2,4–Dimethyl pentan -3one
(2) 3-Methyl cyclohexane carbaldenhyde
(3) 2-Methyl cyclohexanone
(4) 4-Methyl pent -3 –en-2one

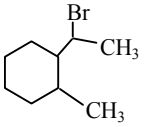
Q.43 Which of the following reaction is an example of ammonolysis ?
(1) $C_6H_5COCl + C_6H_5NH_2 \rightarrow C_6H_5CONHC_6H_5$
(2) $C_6H_5CH_2CN \xrightarrow{(H)} C_6H_5CH_2CH_2NH_2$
(3) $C_6H_5NH_2 \xrightarrow{HCl} C_6H_5\overset{+}{N}H_3Cl^-$
(4) $C_6H_5CH_2Cl + NH_3 \rightarrow C_6H_5CH_2NH_2$

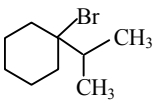
Q.44



(1) 

(2) 

(3) 

(4) 

Q.45 A colloidal system consisting of a gas dispersed in a solid is called a/an :
(1) solid sol (2) gel
(3) aerosol (4) foam

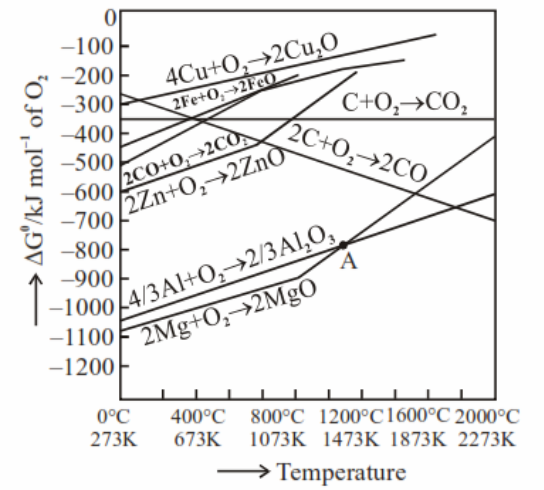
Q.46 The INCORRECT statement (s) about heavy water is (are)
(A) used as amoderator in nuclear reactor
(B) obtained as a by-product in fertilizer industry.
(C) used for the study of reaction mechanism
(D) has a higher dielectric constant than water
Choose the correct answer from the options given below.
(1) (B) only
(2) (C) only
(3) (D) only
(4) (B) and (D) only

Q.47 The correct order of conductivity of ions in water is :
(1) $Na^+ > K^+ > Rb^+ > Cs^+$
(2) $Cs^+ > Rb^+ > K^+ > Na^+$
(3) $K^+ > Na^+ > Cs^+ > Rb^+$
(4) $Rb^+ > Na^+ > K^+ > Li^+$

Q.48 What is the spin-only magnetic moment value (BM) of a divalent metal ion with atomic number 25, in it's aqueous solution ?
(1) 5.92 (2) 5.0
(3) zero (4) 5.26

Q.49 Given below are two statement :
Statement-I : Retardation factor (R_f) can be measured in meter/ centimeter.
Statement –II R_f value of a compound remains constant in all solvents.
Choose the most appropriate answer from the options given below :
(1) Statement –I is true but statement-II is false
(2) Both statement –I and statement-II are true
(3) Both statement -I and statement-II are false
(4) Statement-I is false but statement-II is true

Q.50 The point of intersection and sudden increase in the slope, in the diagram given below, respectively, indicates :



(1) $\Delta G = 0$ and melting or boiling point of the metal oxide

(2) $\Delta G > 0$ and decomposition of the metal oxide

(3) $\Delta G < 0$ and decomposition of the metal oxide

(4) $\Delta G = 0$ and reduction of the metal oxide

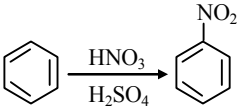
Section -B

Q.51 The reaction of white phosphorus on boiling with alkali in intert atomsphere resulted in the formation of product 'A'. The reaction 1 mol of 'A' with excess of $AgNO_3$ in aqueous medium gives ——mol(s) of Ag. (Round off to the Nearest Integer).

Q.52 0.01 moles of a weak acid HA(K_a) = 2.0×10^{-6}) is dissolved in 1.0 L of 0.1 M HCl solution. the degree of dissociation of HA is ——$\times 10^{-5}$
(Round off to the Nearest Integer).
[Neglect volume change on adding HA.
Assume degree of dissociation << 1]

Q.53 A certain orbital has n = 4 and $m_L = -3$.. The number of radial nodes in this orbital is ——
(Round off to the Nearest Integer) .

Q.54



In the above reaction, 3.9 g of benzene on nitration gives 4.92 g of nitrobenzene. The percentage yield of nitrobenzene in the above reaction is ——
(Given atomic mass : 12.0 u, H : 1 .0u, O : 16.0 u , N : 14.0 u)

Q.55 The mole fraction of a solute in a 100 molal aqueous solution ——— $\times 10^{-2}$
(Round off to the Nearest Integer) .
[Given : Atomic masses : H : 1.0 u, O : 16.0 u]

Q.56 For a certain first order reaction 32% of the reactant is left after 570s. The rate constant of this reaction is —— $\times 10^{-3}$ s^{-1} . (Round off to the Nearest Integer).
[Given : $\log_{10}2 = 0.301$, in 10 = 2.303]

Q.57 The standard enthalpies of formation of Al_2O_3 and CaO are – 1675 kJ mol^{-1} and – 635 kJ mol^{-1} respectively.
For the reaction
$3CaO + 2Al \rightarrow 3Ca + Al_2O_3$ the standard reaction enthalpy $\Delta_r H^0$ = —— kJ.
(Round off to the Nearest Integer).

Q.58 15 mL of aqueous solution of Fe^{2+} in acidic medium completely reacted with 20mL of 0.03 M aqueous $Cr_2O_7^{2-}$. The molarity of the Fe^{2+} solution is —— $\times 10^{-2}$ M (Round off the Nearest Integer).

Q.59 The oxygen dissolved in water exerts a partial pressure of 20 kPa in the vapour above water. The molar solubility of oxygen in water is —— $\times 10^{-5}$ mol dm^{-3}
(Round off to the Nearest Integer).
[Given : Henry's law constant]
= $K_H = 8.0 \times 10^4$ kPa for O_2.
Density of water with dissolved oxygen = 1.0 kg dm^{-3}

Q.60 The pressure exerted by a non-reactive gaseous mixture of 6.4 g of methane and 8.8 g of carbon dioxide in a 10L vessel at 27ºC is ——kPa.
(Round off to the Nearest Integer).
[Assume gases are ideal, R = 8.314 $mol^{-1}K^{-1}$]
Atomic masses : C : 12.0 u, H : 1.0u, O : 16.0 u]

MATHEMATICS

Section -A

Q.61 The inverse of $y = 5\,l^{ogx}$ is :
(1) $x = 5\log y$
(2) $x = y\log 5$
(3) $x = y^{\frac{1}{\log 5}}$
(4) $x = 5^{\frac{1}{\log y}}$

Q.62 Let $\vec{a} = 2\hat{i} - 3\hat{j} + 4\hat{k}$ and $\vec{b} = 7\hat{i} + \hat{j} - 6\hat{k}$
If $\vec{r} \times \vec{a} = \vec{r} \times \vec{b}, \vec{r}.(\hat{i} + 2\hat{j} + \hat{k}) = -3$, then $\vec{r}.(2\hat{i} - 3\hat{j} + \hat{k})$ is equal to :
(1) 12 (2) 8
(3) 13 (4) 10

Q.63 In a triangle PQR, the co-ordinates of the points P and Q are (–2, 4) and (4,–2) respectively. If the equation of the perpendicular bisector of PR is $2x - y + 2 = 0$, then the centre of the circumcircle of the ΔPQR is
(1) (–1, 0) (2) (–2, –2)
(3) (0, 2) (4) (1, 4)

Q.64 The system of equations $kx + y + z = 1$, $x + ky + z = k$ and $x + y + zk = k^2$ has no solution if k is equal to
(1) 0 (2) 1
(3) –1 (4) –2

Q.65 If $\cot^{-1}(\alpha) = \cot^{-1} 2 + \cot^{-1} 8 + \cot^{-1} 18 + \cot^{-1} 32 + \ldots$ upto 100 terms, then α is :
(1) 1.01 (2) 1.00
(3) 1.02 (4) 1.03

Q.66 The equation of the plane which contains the y-axis and passes through the point (1, 2, 3) is :
(1) $x + 3z = 10$ (2) $x + 3z = 0$
(3) $3x + z = 6$ (4) $3x - z = 0$

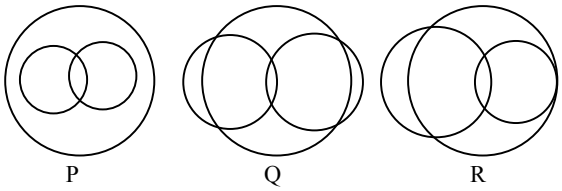
Q.67 If $A = \begin{pmatrix} 0 & \sin\alpha \\ \sin\alpha & 0 \end{pmatrix}$ and $\det\left(A^2 - \frac{1}{2}I\right) = 0$, then a possible value of α is
(1) $\frac{\pi}{2}$ (2) $\frac{\pi}{3}$
(3) $\frac{\pi}{4}$ (4) $\frac{\pi}{2}$

Q.68 If the Boolean expression $(p \Rightarrow q) \Leftrightarrow (q^* (\sim P))$ is a tautology, then the Boolean expression $p * (\sim q)$ is equivelent to :
(1) $q \Rightarrow p$ (2) $\sim q \Rightarrow p$
(3) $p \Rightarrow \sim q$ (4) $p \Rightarrow q$

Q.69 Two dices are rolled. If both dices have six faces numbered 1, 2, 3, 5, 7 and 11, then the probability that the sum of the numbers on the top faces is less than or equal to 8 is :
(1) $\frac{4}{9}$ (2) $\frac{17}{36}$
(3) $\frac{5}{12}$ (4) $\frac{1}{2}$

Q.70 If the fourth term in the expansion of $(x + x^{\log 2})^7$ is 4480, then the value of x where $x \in N$ is equal to
(1) 2 (2) 4
(3) 3 (4) 1

Q.71 In a school, there are three types of games to be played. Some of the students play two types of games, but none play all the three games. Which Venn diagrams can justify the above statement



(1) P and Q (2) P and R
(3) None of these (4) Q and R

Q.72 The sum of possible values of x for $\tan^{-1}(x + 1) + \cot^{-1}\left(\frac{1}{x - 1}\right) = \tan^{-1}\frac{8}{31}$ is :
(1) $-\frac{32}{4}$ (2) $-\frac{31}{4}$
(3) $-\frac{30}{4}$ (4) $-\frac{33}{4}$

Q.73 The area of the triangle with vertices A(z), B(iz) and C(z + iz) is :
(1) 1 (2) $\frac{1}{2}|z|^2$
(3) $\frac{1}{2}$ (4) $\frac{1}{2}|z + iz|^2$

Q.74 The line $2x - y + 1 = 0$ is a tangent to the circle at the point (2, 5) and the centre of the circle lies on $x - 2y = 4$. Then the radius of the circle is

(1) $3\sqrt{5}$ (2) $5\sqrt{3}$

(3) $5\sqrt{4}$ (4) $4\sqrt{5}$

Q.75 Team 'A' consists of 7 boys and n girls and Team 'B' has 4 boys and 6 girls. If a total of 52 single matches can be arranged between these two teams when a boy plays against a boy and a girl plays against a girl, then n is equal to :

(1) 5 (2) 2

(3) 4 (4) 6

Q.76 The value of $4+\cfrac{1}{5+\cfrac{1}{4+\cfrac{1}{5+\cfrac{1}{4+....\infty}}}}$ is :

(1) $2+\frac{2}{5}\sqrt{30}$ (2) $2+\frac{4}{\sqrt{5}}\sqrt{30}$

(3) $4+\frac{4}{\sqrt{5}}\sqrt{30}$ (4) $5+\frac{2}{5}\sqrt{30}$

Q.77 Choose the incorrect statement about the two circles whose equations are given below :

$x^2 + y^2 - 10x - 10y + 41 = 0$ and

$x^2 + y^2 - 16x - 10y + 80 = 0$

(1) Distance between two centres is the average of radii of both the circles.

(2) Both circles' centres lie inside region of one another.

(3) Both circles pass through the centre of each other.

(4) Circles have two intersection points.

Q.78 Which of the following statements is incorrect for the founction g (α) for $\alpha \in R$ such that

$$g(\alpha) = \int_{\pi/6}^{\pi/3} \frac{\sin^{\alpha} x}{\cos^{\alpha} x + \sin^{\alpha} x} dx$$

(1) g (α)is a stricitly increasing function

(2) g(α) has an inflection point at $\alpha = -\frac{1}{2}$

(3) g(α) is a strictly decreasing function

(4) g(α) is an even function

Q.79 Which of the following is true for y (x) that satisfies the differential equation

$\frac{dy}{dx} = xy - 1 + x - y$; y (0) = 0 :

(1) $y(1) = e^{\frac{1}{2}} - 1$ (2) $y(1) = e^{\frac{1}{2}} - e^{-\frac{1}{2}}$

(3) y (1) = 1 (4) $y(1) = e^{\frac{1}{2}} - 1$

Q.80 The value of

$$\lim_{x \to 0^+} \frac{\cos^{-1}(x-[x]^2).\sin^{-1}(x-[x]^2)}{x - x^3}$$

[x] denotes the greatest integer $\le$ x is :

(1) π (2) 0 (3) $\frac{\pi}{4}$ (4) $\frac{\pi}{2}$

Section -B

Q.81 The maximum value of z in the following equation $z = 6xy + y^2$, where $3x + 4y \le 100$ and $4x + 3y \le 75$ for $x \ge 0$ and $y \ge 0$ is ——

Q.82 If the function $f(x) = \frac{\cos(\sin x) - \cos x}{x^4}$ is continuous at each point in its domain and $f(0) = \frac{1}{k}$, then k is ——

Q.83 If $f(x) = \sin\left(\cos^{-1}\left(\frac{1-2^{2x}}{1+2^{2x}}\right)\right)$ and its first derivative with respect to x is $-\frac{b}{a}\log_e 2$ when x = 1, where a and b are integers, then the minimum value of $\left|a^2 - b^2\right|$ is ——

Q.84 Let there be three independent events E_1, E_2 and E_3. The probability that only E_1 occurs is α, only E_2 occurs is β and only E_3 occurs is γ. Let 'p' denote the probability of none of events occurs that satisfies the equations $(\alpha - 2\beta)p = \alpha\beta$ and $(\beta - 3\gamma) = 2\beta\gamma$. All the given probabilites are assumed to lie in the interval (0, 1).

Then, $\frac{\text{Probability of occurrence of } E_1}{\text{Probability of occurrence of } E_3}$ is equal to ——

Q.85 If $\vec{a} = a\hat{i} + \beta\hat{j} + 3\hat{k}$
$\vec{b} = -\beta\hat{i} - \alpha\hat{j} - \hat{k}$ and
$\vec{c} = \hat{i} - 2\hat{j} - \hat{k}$
such that $\vec{a} \cdot \vec{b} = 1$ and $\vec{b} \cdot \vec{c} = -3$, then
$\frac{1}{3}((\vec{a} \times \vec{b}) \cdot \vec{c})$ is equal to ——

Q.86 If $A = \begin{bmatrix} 2 & 3 \\ 0 & -1 \end{bmatrix}$, then the value of
$\det(A^4) + \det(A^{10} - (\text{Adj})(2A))^{10}$ is equal to ——

Q.87 If [.] represents the greatest integer function, then the value of
$\left| \int_0^{\sqrt{\frac{\pi}{2}}} \left[\left[x^2 \right] - \cos x \right] dx \right|$ is

Q.88 The minimum distance between any two points P_1 and P_2 while considering point P_1 on one circle and point P_2 on the other circle for the given circles' equations
$x^2 + y^2 - 10x - 10y + 41 = 0$
$x^2 + y^2 - 24x - 10y + 160 = 0$ is ——

Q.89 If the equation of the plane passing through the line of intersection of the planes $2x - 7y + 4z - 3 = 0$, $3x - 5y + 4z + 11 = 0$ and the point $(-2, 1, 3)$ is $ax + by + cz = 7$, then the value of $2a + b + c - 7$ is ——

Q.90 If $(2021)^{3762}$ is divided by 17, then the remainder is ——

JEE MAIN ONLINE PAPER 2021

Held on March 17, 2021 (Morning)

Hints & Solutions

PHYSICS

SECTION-A

1.[1] $\vec{F} = 4\,\hat{i} - 3\,\hat{j}$

$\vec{r}_1 = 5\,\hat{i} + 5\sqrt{3}\,\hat{j}$ & $\vec{r}_2 = -5\,\hat{i} + 5\sqrt{3}\,\hat{j}$

torque about 'O'

$\vec{\tau}_O = \vec{r}_1 \times \vec{F} = (-15 - 20\sqrt{3})\,\hat{k}$

$= (15 + 20\sqrt{3})\,(-\hat{k})$ Torque about 'Q'

$\vec{\tau}_Q = \vec{r}_2 \times \vec{F} = (-15 + 20\sqrt{3})\,\hat{k}$

$= (15 - 20\sqrt{3})\,(-\hat{k})$

2.[1] Excess pressure at common surface is given by

$$P_{ex} = 4T\left(\frac{1}{a} - \frac{1}{b}\right) = \frac{4T}{r}$$

$$\therefore \frac{1}{r} = \frac{1}{a} - \frac{1}{b}$$

$$r = \frac{ab}{b-a}$$

3.[1] Since each vibrational mode has 2 degrees of freedom hence total vibrational degrees of freedom = 48

$f = 3 + 3 + 48 = 54$

$$\gamma' = 1 + \frac{2}{f} = \frac{28}{27} = 1.03$$

4.[2] We know velocity of electron in n^{th} shell of hydrogen atom is given by

$$v = \frac{2\pi k Z e^2}{nh}$$

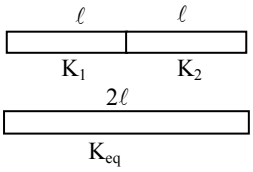
$$\therefore v \propto \frac{1}{n}$$

5.[2] $$\lambda_1 = \frac{h}{\sqrt{2mE}}$$

$$\lambda_2 = \frac{hc}{E}$$

$$\frac{\lambda_1}{\lambda_2} = \frac{1}{c}\left(\frac{E}{2m}\right)^{1/2}$$

6.[1]



$$R_{eff} = \frac{l}{K_1 A} + \frac{l}{K_2 A} = \frac{2l}{K_{aq} A}$$

$$K_{aq} = \frac{2K_1K_2}{K_1 + K_2}$$

7.[2] Positive zero error = 0.2 mm

Main scale reding = 8.5 cm

vernier scale reading = 6 × 0.01 = 0.06 cm

Final reading = 8.5 + 0.06 – 0.02 = 8.54 cm

8.[2] $I = I_1 \sin\omega t + I_2 \cos\omega t$

$$\therefore I_0 = \sqrt{I_1^2 + I_2^2}$$

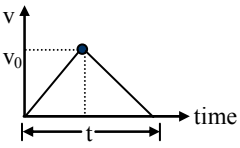
$$\therefore I_{rms} = \frac{I_0}{\sqrt{2}} = \sqrt{\frac{I_1^2 + I_2^2}{2}}$$

9.[2] $$\mu_s N = \frac{mv^2}{R}$$

$$N = \frac{mv^2}{\mu_s R} = mg + F_L$$

$$F_L = \frac{mv^2}{\mu_s R} - mg$$

10.[3]



$V_0 = at_1$ and $0 = v_0 - \beta t_2 \Rightarrow v_0 = \beta t_2$

$t_1 + t_2 = t$

$$v_0\left(\frac{1}{\alpha} + \frac{1}{\beta}\right) = t$$

$$v_0 = \frac{\alpha\beta}{\alpha + \beta}$$

Distance = area of v-t graph

$$= \frac{1}{2} \times t \times v_0 = \frac{1}{2} \times t \times \frac{1}{2}\frac{\alpha\beta t}{\alpha + \beta} = \frac{\alpha\beta t^2}{2(\alpha + \beta)}$$

11.[1] $B = unI = \mu_0 u_r nI$

$B = 4\pi \times 10^{-7} \times 500 \times 1000 \times 5$

$B = \pi$ Tesla

12.[4] We know, $\vec{L} = m(\vec{r} \times \vec{v})$

Now with respect to A, we always get direction of $\vec{L}$ along +ve z-axis and also constant magnitude as mvr. But with respect to B, we get constant magnitude but continuosuly changing direction.

13.[3] KE = PE

$$\frac{1}{2} m\omega^2 (A^2 - x^2) = \frac{1}{2} m\omega^2 x^2$$

$A^2 - x^2 = x$

$2x^2 = A^2$

$$x = \pm \frac{A}{\sqrt{2}}$$

14.[4] $$\eta = \frac{T_2}{T_1} = \frac{Q_2}{Q_1} = \frac{Q_1 - W}{Q_1} \quad (\because W = Q_1 - Q_2)$$

$$\frac{400}{800} = 1 - \frac{W}{Q_1}$$

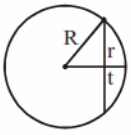
$$\frac{W}{Q_1} = 1 - \frac{1}{2} = \frac{1}{2}$$

$Q_1 = 2W = 2400$ J

15.[4] $R^2 = r^2 + (R - t)^2$

$R^2 = r^2 + R^2 + t^2 - 2Rt$

Neglecting t^2, we get



$$R = \frac{r^2}{2t}$$

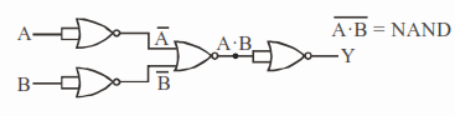
$$\therefore \frac{1}{f} = (\mu - 1)\left(\frac{1}{R} - \frac{1}{\infty}\right) = \frac{\mu - 1}{R}$$

$$f = \frac{R}{\mu - 1} = \frac{r^2}{2t(\mu - 1)} = \frac{(3 \times 10^{-2})^2}{2 \times 3 \times 10^{-3} \times \left(\frac{3}{2} - 1\right)}$$

$$= \frac{9 \times 10^{-4}}{6 \times 10^{-3} \times 1} \times 2$$

$f = 0.3$ m $= 30$ cm

16.[2] By De Morgan's theorem, we have



17.[2] Given, m = 0.5 kg and u = 20 m/s

Initial kinetic energy $(k_i) = \frac{1}{2} mu^2$

$$= \frac{1}{2} \times 0.5 \times 20 \times 20 = 100 \text{ J}$$

After deflection it moves with 5% of k_i

$$\therefore k_f = \frac{5}{100} \times k_i \Rightarrow \frac{5}{100} \times 100$$

$\Rightarrow k_f = 5$ J

Now, let the final speed be 'v' m/s, then :

$$k_f = 5 = \frac{1}{2} mv^2$$

$\Rightarrow v^2 = 20$

$\Rightarrow v = \sqrt{20} = 4.47$m/s

18.[2] Energy of H-atom is $E = -13.6\, Z^2/n^2$

for H-atom Z = 1 & for ground state, n = 1

$$\Rightarrow E = -13.6 \times \frac{1^2}{1^2} = -13.6\text{eV}$$

Now for carbon atom (single ionised), Z = 6

$$E = -13.6 \frac{Z^2}{n^2} = -13.6 \text{ (given)}$$

$\Rightarrow n^2 = 6^2 \Rightarrow n = 6$

19.[2] Let the final temperature of the mixture be T. Since, there is no loss in energy.

$AU = 0$

$$\Rightarrow \frac{F_1}{2} n_1 R\Delta T + \frac{F_2}{2} n_2 R\Delta T = 0$$

$$\Rightarrow \frac{F_1}{2} n_1 R (T_1 - T) + \frac{F_2}{2} n_2 R (T_2 - T) = 0$$

$$\Rightarrow T = \frac{F_1 n_1 R T_1 + F_2 n_2 R T_2}{F_1 n_1 R + F_2 n_2 R} \Rightarrow \frac{F_1 n_1 T_1 + F_2 n_2 T_2}{F_1 n_1 + F_2 n_2}$$

20.[2] i = 10A, A = 5 mm^2 = 5×10^{-6}m^2

and $v_d = 2 \times 10^{-3}$ m/s

We know, i = neAvd

$\therefore 10 = n \times 1.6 \times 10^{-19} \times 5 \times 10^{-6} \times 2 \times 10^{-3}$

$\Rightarrow n = 0.625 \times 10^{28} = 625 \times 10^{25}$

SECTION-B

21.[1206]

$d = \sqrt{2Rh}$

$A = \pi d^2$

$A' = \pi 2Rh$

$= 3.14 \times 2 \times 6400 \times \frac{30}{1000}$

$A = 1205.76\ km^2$

$A \simeq 1206\ km^2$

22.[728] We know, $\theta = \left(\frac{\omega_1 + \omega_2}{2}\right)t$

Let number of revolutions be N

$\therefore\ 2\pi N = 2\pi \left(\frac{900 + 2460}{60 \times 2}\right) \times 26$

$N = 728$

23.[4] $R_1 + R_2 = s$ .... (1)

$\frac{R_1 R_2}{R_1 + R_2} = p$ .....(2)

$R_1 R_2 = sp$

$R_1 R_2 = np^2$

$R_1 + R_2 = \frac{mR_1 R_2}{(R_1 + R_2)}$

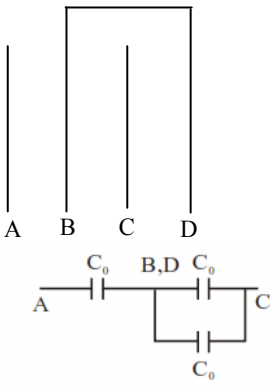
$\frac{(R_1 + R_2)^2}{R_1 R_2} = n$

for minimum value of n

$R_1 = R_2 = R$

$\therefore\ n = \frac{(2R)^2}{R^2} = 4$

24.[2]



$C^{eq} = \frac{2C_0}{3} = \frac{2}{3}\frac{\epsilon_0 A}{d}$

$C_{eq} = \frac{2\epsilon_0}{3d} \times \left(2 \times \frac{3}{2}\right) = 2\ (\because A = 1b = 2 \times \frac{3}{2})$

25.[64] $V_c = \sqrt{\frac{2Gm}{R}}$ .... (1)

$10V_e = \sqrt{\frac{2Gm}{R'}}$ ....(2)

$\therefore\ 10\sqrt{\frac{R}{R'}}$

$\Rightarrow R' = \frac{R}{100} = \frac{6400}{100} = 64\ km$

26.[2] $T_a = 2\pi\sqrt{\frac{M}{K}}$

$T_b = 2\pi\sqrt{\frac{M}{K/2}}$

$\frac{T_b}{T_a} = \sqrt{2} = \sqrt{x}$

$\Rightarrow x = 2$

27.[4] $Mg\sin\theta\ R = (mk^2 + mR^2)\alpha$

$\alpha = \frac{Rg\sin\theta}{k^2 + R^2} \Rightarrow a = \frac{g\sin\theta}{1 + \frac{k^2}{R^2}}$

$t = \sqrt{\frac{2s}{a}} = \sqrt{\frac{2s}{g\sin\theta\left(1 + \frac{k^2}{R^2}\right)}}$

for least time, k should be least & we know k is least for solid sphere.

28.[864] $U_i = \frac{1}{2} \times 14 \times 12 \times 12\ pJ\ (\because = \frac{1}{2}CV^2)$

$= 1008\ pJ$

$U_f = \frac{1008}{7}\ pJ = 144\ pJ\quad (\because C_m = kC_0)$

Mechanical energy $= \Delta U$

$= 1008 - 144$

$= 864\ pJ$

29.[21] $a_{max} = \mu g = \frac{3}{7} \times 9.8$

$F = (M + m)\ a_{max} = 5\ a_{max}$

$= 21$ Newton

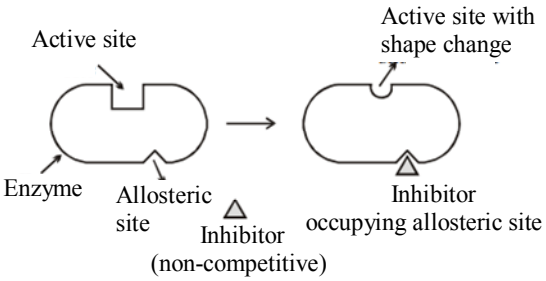
30.[25] $F = \frac{IA}{C}$

$I = \frac{FC}{A} = \frac{2.5 \times 10^{-6} \times 3 \times 10^8}{30} = 25\ W/cm^2$

CHEMISTRY

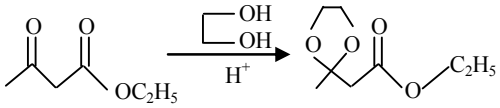
SECTION-A

31.[3] Some drug do not bind to the Enzyme's active site. These bind to a different site of enzyme which called allosteric site.
This binding of inhibitor at allosteric site changes the shape of the active site in such a way that substrate can not recognise it. Such inhibitor is known as Non-competitive inhibitor.

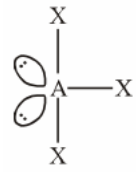


32.[1]  → Aromatic compound

33.[2]



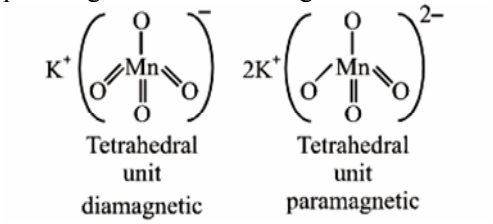
34.[3]



sp^3 d hybridised
T –shaped

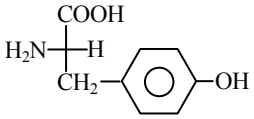
35.[1] $2KMnO_4 \xrightarrow{573K} K_2MnO_4 + MnO_2 + O_2$

Potassium permanganate — Potassium manganate

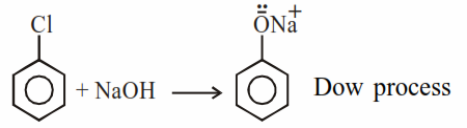


Statement –I is correct.
Statement –II is incorrect.

36.[4] The structure of Tyrosine amino acid is



37.[4]



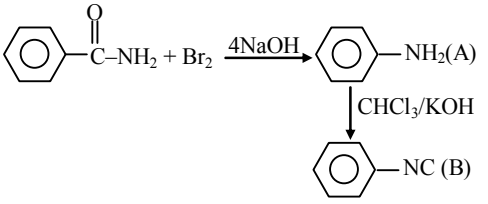
Temperature = 623 K
Pressure = 300 atm

38.[3] Order of electron gain enthalpy
(Absolute value)
Cl > F > Br > I

39.[2] Lewis base : Chemical species which has capability to donate electron pair.
In N_F, SF_4, CIF_3 central atom (i.e. N,S,Cl) having lone pair therefore act as lewis base.
In PCl_5 central atom (P) does not have lone pair therefore does not act as lewis base.

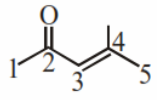
40.[2] Reducing or classical smog is the combination of smoke, fog and SO_2 .

41.[2] Hoffmann bromamide degradation reaction :



Carbylamine reaction

42.[4]



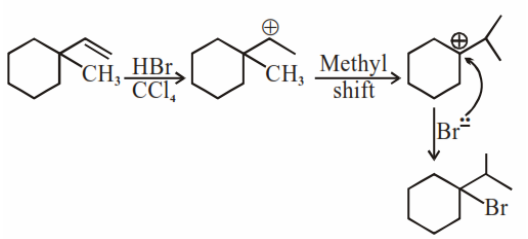
Mesityloxide

IUPAC [4-Methylpent-3-en-2-one]

43.[4] The process of cleavage of the C-X bond by Ammonia molecule is known as ammonolysis.

Ex : $R\text{-}CH_2\text{–}Cl + \ddot{N}H_3 \rightarrow R\text{–}CH_2\text{–}NH_2$

44.[4]



45.[1] Colloid of gas dispersed in solid is called solid sol.

46.[3] The dielectric constant of H_2O is greater than heavy water.

47.[2]

$Li^+ \; Na^+ \; K^+ \; Rb^+ \; Cs^+$

$\longrightarrow$ Hydration energy ↑

$\longrightarrow$ Ionic mobility ↓

$\longrightarrow$ Conductivity ↓

$\therefore$ Correct option is $Na^+ > K^+ > Rb^+ > Cs^+$

OR

As the size of gaseous ion decreases, it get more hydrated in water and hence, the size of aqueous ion increases. When this bulky ion move in solution, it experience greater resistance and hence lower conductivity.

Size of gasesous ion : $Cs^+ > Rb^+ > K^+ > Na^+$

Size of aqueous : $Cs^+ < Rb^+ < K^+ < Na^+$

Conductivity : $Cs^+ > Rb^+ > K^+ > Na^+$

48.[1] Electronic configuration of divalent metal ion having atomic number 25 is

$$Mn^{+2}_{(aq)} \Rightarrow \underset{Is^2}{\uparrow\downarrow} \; \underset{2s^2}{\uparrow\downarrow} \; \underset{2p^6}{\uparrow\downarrow\uparrow\downarrow\uparrow\downarrow} \; \underset{3s^2}{\uparrow\downarrow} \; \underset{3p^6}{\uparrow\downarrow\uparrow\downarrow\uparrow\downarrow} \; \underset{3d^5}{\uparrow\uparrow\uparrow\uparrow\uparrow}$$

Total number of unpaired electrons = 5

μ (Magnetic moment) = $\sqrt{n(n+2)}$ BM

where n = number of unpaired e^-

$\therefore \; \mu = \sqrt{5(5+2)} = \sqrt{35}$ BM = 5.92 BM

49.[3] R_f = retardation factor

$$R_f = \frac{\text{Distance travelled by the substance from reference line (c.m)}}{\text{Distance travelled by the solvent from reference line (c.m)}}$$

Note : R_f value of different compounds are different.

50.[1] At intersection point $\Delta G = 0$ and sudden increase in slope is due to melting or boiling point of the metal.

SECTION-B

51.[4] $P_4 + 3OH^- + 3H_2O \rightarrow PH_3 + \underset{(A)}{3H_2PO_2^-}$

$$\underset{1\,mole}{H_2PO_2^-} + \underset{excess}{4Ag^+} + 2H_2O \rightarrow \underset{4\,mole}{4Ag} + H_3PO_4 + 3H^+$$

52.[2]

$$HA \rightleftharpoons H^+ + A^-$$

Initial conc. 0.01M 0.1M 0

Equ. conc. (0.01–x) (0.1 + x) xM

$\approx 0.01M \;\; \approx 0.1$ M

Now, $K_a = \frac{[x^+][A^-]}{[HA]} \Rightarrow 2 \times 10^{-6} = \frac{0.1 \times x}{0.01}$

$\therefore \; x = 2 \times 10^{-7}$

Now , $\alpha = \frac{x}{0.01} = \frac{2 \times 10^{-7}}{0.01} = 2 \times 10^{-5}$

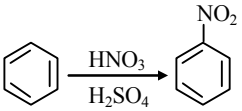
53.[4] $n = 4$ and $m_\ell = -3$

Hence, ℓ value must be 3.

Now, number of radial nodes = $n - \ell - 1$

$= 4 - 3 - 1 = 0$

54.[80]



1' mole 1 mole

78 gm 123 gm

3.9 gm $\frac{123}{78} \times 3.9 = 6.15$ gm

But actual amount of introbenzene formed is 4.92 gm and hence.

Percentage yield = $\frac{4.92}{6.15} \times 100 = 80\%$

55.[64] 100 moal aqueous solution means there is 100 mole solute in 1 kg = 1000 gm water.

Now

mole-fraction of solute = $\frac{n_{solute}}{n_{solute} + n_{solvent}}$

$= \frac{100}{100 + \frac{1000}{18}} = \frac{1800}{2800} = 0.6428$

$= 64.28 \times 10^{-2}$

56.[2] For 1st order reaction,

$$K = \frac{2.303}{t} \cdot \log \frac{[A_0]}{[A_t]} = \frac{2.303}{570\,sec} \cdot \log\left(\frac{100}{32}\right)$$

$= 1.999 \times 10^{-3}\ sec^{-1} \approx 2 \times 10^{-3}\ sec^{-1}$

57.[230] Given reaction :

$3CaO + Al \rightarrow Al_2O_3 + 3Ca$

Now, $\Delta_f H^o = \Sigma \Delta_f H^o_{\text{products}} - \Sigma \Delta_f H^o$ Reactants

$= [1 \times (-1675) + 3 \times 0] - [3 \times (-635) + 2 \times 0]$

$= +230\ kJ\ mol^{-1}$

58.[24] $n_{eq}\ Fe^{2+} = n_{eq}\ Cr_2O_7^{2-}$

$$\text{or, } \left(\frac{15 \times M_{Fe2+}}{1000}\right) \times 1 = \left(\frac{20 \times 0.03}{1000}\right) \times 6$$

$\therefore\ M_{Fe^{2+}} = 0.24\ M = 24 \times 10^{-2} M$

59.[1389] $P = K_H . X$

$$\text{or, } 20 \times 10^3 = (8 \times 10^4 \times 10^3) \times \frac{n_{O_2}}{n_{O_2} + n_{water}}$$

$$\frac{1}{4000} = \frac{n_{O_2}}{n_{O_2} + n_{water}} = \frac{n_{O_2}}{n_{water}}$$

Means 1 mole water (= 18 gm = 18ml) dissolves $\frac{1}{4000}$ moles O_2. Hence molar solubility

$$\frac{\left(\frac{1}{4000}\right)}{18} \times 1000 = \frac{1}{72}\ mol\ dm^{-3}$$

$= 1388.89 \times 10^{-5}\ mol\ dm^{-3} \approx 1389\ mol\ dm^{-3}$

$= 1388.89 \times 10^{-5}\ mol\ dm^{-3} \approx 1389\ mol\ dm^{-3}$

60.[150] Total moles of gases, $n = nCH_4 + n_{CO_2}$

$$= \frac{6.4}{16} + \frac{8.8}{44} = 0.6$$

$$\text{Now, } P = \frac{nRT}{V} = \frac{0.6 \times 8.314 \times 300}{10 \times 10^{-3}}$$

$= 1.49652 \times 10^5\ Pa = 149.652\ kPa$

$\approx 150\ kPa$

MATHEMATICS

SECTION-A

61.[3] Given $y = 5^{(\log_a x)} = f(x)$

Interchanging x & y for inverse

$x = (\log_a y) = y\ (\log_a 5)$

option (1) or option (2)

Further, from given relation

$\log_5 y = \log_a x$

$\Rightarrow x = a^{(\log_5 y)} = y^{(\log_5 a)}$

$\Rightarrow\ x = y^{\left(\frac{1}{\log_a 5}\right)} = f^{-1}(y)$

option (3)

62.[1] $\vec{r} \times \vec{a} - \vec{r} \times \vec{b} = 0$

$\Rightarrow \vec{r}\,(\vec{a} - \vec{b}) = 0$

$\Rightarrow \vec{r} = \lambda\,(\vec{a} - \vec{b})$

$\Rightarrow \vec{r} = \lambda\,(-5\hat{i} - 4\hat{j} + 10\hat{k})$

Also $\vec{r} \cdot (\hat{i} + 2\hat{j} + \hat{k}) = -3$

$\Rightarrow \lambda(-5 - 8 + 10) = -3$

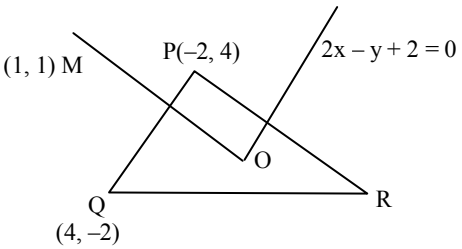
$\lambda = 1$

Now $\vec{r} = -5\hat{i} - 4\hat{j} + 10\hat{k}$

$= \vec{r} \cdot (2\hat{i} - 3\hat{j} + \hat{k})$

$= -10 + 12 + 10 = 12$

63.[2]



Equation of perpendicular bisector of PR is $y = x$

Solving with $2x - y + 2 = 0$ will give

$(-2, 2)$

64.[4] $kx + y + z = 1$

$x + ky + z = k$

$x' + y + zk = k^2$

$$\Delta = \begin{vmatrix} K & 1 & 1 \\ 1 & K & 1 \\ 1 & 1 & K \end{vmatrix}$$

$= K(K^2 - 1) - 1(K - 1) + 1(1 - K)$

$= K^3 - K - K + 1 + 1 - K$

$= K^3 - 3K + 2$

$= (K - 1)^2 (K + 2)$

For $K = 1$

$\Delta = \Delta_1 = \Delta_2 = \Delta_3 = 0$

But for $K = -2$, at least one out of $\Delta_1, \Delta_2, \Delta_3$ are not zero

Hence for no sol^n, $K = -2$

65.[1] $\text{Cot}^{-1}(\alpha) = \cot^{-1}(2) + \cot^{-1}(8) + \cot^{-1}(18 + ...)$

$= \sum_{n=1}^{100} \tan^{-1}\left(\frac{2}{4n^2}\right)$

$= \sum_{n=1}^{100} \tan^{-1}\left(\frac{(2n+1)-(2n-1)}{1+(2n+1)(2n-1)}\right)$

$= \sum_{n=1}^{100} \tan^{-1}(2n+1) - \tan^{-1}(2n-1)$

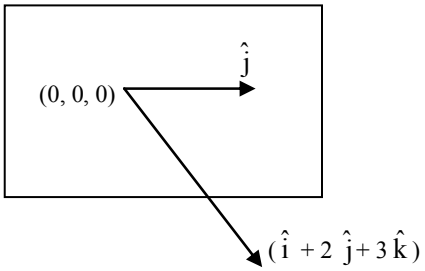
$= \tan^{-1} 201 - \tan^{-1} 1$

$= \tan^{-1}\left(\frac{200}{202}\right)$

$\therefore \cot^{-1}(\alpha) = \cot^{-1}\left(\frac{200}{202}\right)$

$\alpha = 1.01$

66.[4]



$\vec{n} = \hat{j} \times (\hat{i} + 2\hat{j} + 3\hat{k})$

$= -3\hat{i} + 0\hat{j} + \hat{k}$

So, $(-3)(x-1) + 0(y-2) + (1)(z-3) = 0$

$\Rightarrow 3x + z = 0$

option 4

Alternate :

Required plane is

$$\begin{vmatrix} x & y & z \\ 0 & 1 & 0 \\ 1 & 2 & 3 \end{vmatrix} = 0$$

$\Rightarrow 3x - z = 0$

67.[3] $A^2 = \sin^2\alpha\, I$

So, $\left|A^2 - \frac{I}{2}\right| = \left(\sin^2\alpha - \frac{1}{2}\right)^2 = 0$

$\Rightarrow \sin\alpha = \frac{1}{\sqrt{2}}$

68.[1] $\because p \to q \equiv \sim p \vee q$

so, $* \equiv \vee$

Thus, $p^* (\sim q) \equiv p \vee (\sim q)$

$\equiv q \to p$

69.[2] $n(E) = 5 + 4 + 4 + 3 + 1 = 17$

So, $P(E) = \frac{17}{36}$

70.[1] ${}^7C_3 x^4 \; x^{(3\log_2^x)} = 4480$

$\Rightarrow x^{(4+3\log_2^x)} = 2^7$

$\Rightarrow (4+3t)\, t = 7;\ t = \log_2^x$

$\Rightarrow t = 1, \frac{-7}{3} \Rightarrow x = 2$

71.[3] $A \cap B \cap C$ is visible in all three venn diagram

Hence, Option (3)

72.[1] $\tan^{-1}(x+1) + \cot^{-1}\left(\frac{1}{x-1}\right) = \tan^{-1}\frac{8}{31}$

Taking tangent both sides –

$\frac{(x+1)+(x-1)}{1-(x^2-1)} = \frac{8}{31}$

$\frac{2x}{2-x^2} = \frac{8}{31}$

$\Rightarrow 4x^2 + 31x - 8 = 0$

$\Rightarrow x = -8, \frac{1}{4}$

But, if $x = \frac{1}{4}$

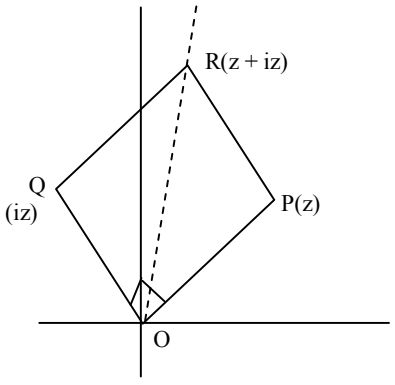
$\tan^{-1}(x+1) \in \left(0, \frac{\pi}{2}\right)$

& $\cot^{-1}\left(\frac{1}{x-1}\right) \in \left(\frac{\pi}{2}, \pi\right)$

$\Rightarrow$ LHS $> \frac{\pi}{2}$ & RHS $< \frac{\pi}{2}$ (Not possible)

Hence, $x = -8$

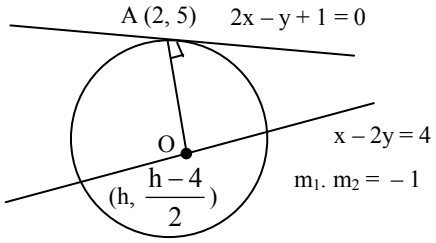
73.[2]



$A = \frac{1}{2}|z|\,|iz|$

$= \frac{|z|^2}{2}$

74.[1]



$$\left(\frac{h-\frac{(h-4)}{2}}{2-h}\right)(2) = -1$$

$h = 8$

centre (8, 2)

Radius = $\sqrt{(8-2)^2+(2-5)^2} = 3\sqrt{5}$)

75.[3] Total matches between boys of both team $= {}^7C_1 \times {}^4C_1 = 28$

Total matches between girls of both team $= {}^nC_1\ {}^6C_1 = 6_n$

Now, $28 + 6n = 52$

$\Rightarrow n = 4$

76.[1] $y = 4 + \dfrac{1}{\left(5+\frac{1}{y}\right)}$

$$y - 4 = \frac{y}{(5y+1)}$$

$5y^2 - 20y - 4 = 0$

$$y = \frac{20+\sqrt{480}}{10}$$

$$y = \frac{20-\sqrt{480}}{10} \rightarrow \text{rejected}$$

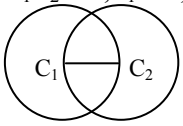
$$y = 2 + \sqrt{\frac{480}{100}}$$

Correct with Option (A)

77.[2] $r_1 = 3, c_1 (5, 5)$

$r_2 = 3, c_2 (8, 5)$

$C_1C_2 = 3, r_1 = 3, r_2 = 3$



78.[4] $\displaystyle\int_{\pi/6}^{\pi/3} \frac{\sin^\alpha x}{(\sin^\alpha x + \cos^\alpha x)} dx$ .....(i)

$$g(\alpha) = \int_{\pi/6}^{\pi/3} \frac{\sin^\alpha x}{(\sin^\alpha x + \cos^\alpha x)} \text{.....(ii)}$$

(1) + (2)

$$2g(\alpha) = \frac{\pi}{6}$$

$$g(\alpha) = \frac{\pi}{12}$$

Constant and even function

Due to typing mistake it must be bonus.

79.[1] $\dfrac{dy}{dx} = (1 + y)(x - 1)$

$$\frac{dy}{(y+1)} = (x - 1)\, dx$$

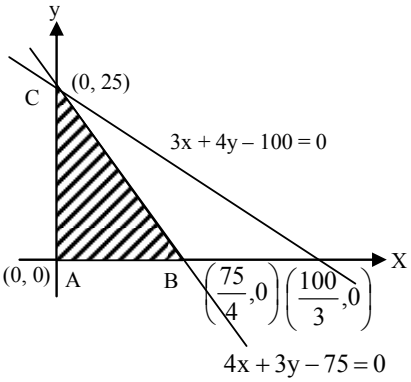
Integrate $\ln (y + 1) = \dfrac{x^2}{2} - x + c$

$(0, 0) \Rightarrow c = 0 \Rightarrow y = e^{\left(\frac{x^2}{2}-x\right)} - 1$

80.[4] $\displaystyle\lim_{x\to 0^+} \frac{\cos^{-1} x}{(1-x^2)} \times \frac{\sin^{-1} x}{x} = \frac{\pi}{2}$

SECTION-B

81.[904]



$z = 6xy + y^2 = y(6x + y)$

$3x + 4y \le 100$ ....(i)

$4x + 3y \le 75$ ...(ii)

$x \ge 0$

$y \ge 0$

$$x \le \frac{75-3y}{4}$$

$Z = y(6x + y)$

$$Z \le y\left(6.\left(\frac{75-3y}{4}\right)+y\right)$$

$$Z \le \frac{1}{2}(225y - 7y^2) \le \frac{(225)^2}{2\times 4\times 7}$$

$= \frac{50625}{56}$

≈ 904.0178

≈ 904.02

It will be attained at $y = \frac{225}{14}$

82.[6] $\lim\limits_{x\to 0} \frac{\cos(\sin x) - \cos x}{x^4} = f(0)$

$\Rightarrow \lim\limits_{x\to 0} \frac{2\sin\left(\frac{\sin x + x}{2}\right)\sin\left(\frac{x - \sin x}{2}\right)}{x^4} = \frac{1}{K}$

$\Rightarrow \lim\limits_{x\to 0} 2\left(\frac{\sin x + x}{2x}\right)\left(\frac{x - \sin x}{2x^3}\right) = \frac{1}{K}$

$\Rightarrow 2 \times \frac{(1+1)}{2} \times \frac{1}{2} \times \frac{1}{6} = \frac{1}{K}$

$\Rightarrow K = 6$

83.[481] $f(x) = \sin\left(\cos^{-1}\left(\frac{1-2^{2x}}{1+2^{2x}}\right)\right)$ at $x = 1$; $2^{2x} = 4$

for $\sin\left(\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)\right)$

Let $\tan^{-1} x = \theta$; $\theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$\therefore \sin(\cos^{-1}\cos 2\theta) = \sin 2\theta$

$\left\{\begin{array}{l} \text{If} \quad x > 1 \Rightarrow \frac{\pi}{2} > \theta > \frac{\pi}{4} \\ \therefore \qquad \pi > 2\theta > \frac{\pi}{2} \end{array}\right\}$

$= 2\sin\theta\cos\theta = \frac{2\tan\theta}{1+\tan^2\theta}$

$= \frac{2x}{1+x^2}$

Hence, $f(x) = \frac{2\cdot 2^x}{1+2^{2x}}$

$\therefore f'(x) = \frac{(1+2^{2x})(2.2^x \ln 2) - 2^{2x}\cdot 2\cdot \ln 2\cdot 2\cdot 2^x}{(1+2^{2x})}$

$\therefore f'(1) = \frac{20\ln 2 - 32\ln 2}{25} = -\frac{12}{25}\ln 2$

So, $a = 25$, $b = 12 \Rightarrow |a^2 - b^2| = 25^2 - 12^2$

$= 625 - 144$

$= 481$

84.[6] Let $P(E_1) = P_1$; $P(E_2) = P_2$; $P(E_3) = P_3$

$P(E_1 \cap \overline{E}_2 \cap \overline{E}_3) = \alpha = P_1(1-P_2)(1-P_3)$ ....(1)

$P(\overline{E}_1 \cap E_2 \cap \overline{E}_3) = \beta = (1-P_1)P_2(1-P_3)$ ....(2)

$P(\overline{E}_1 \cap \overline{E}_2 \cap E_3) = \gamma = (1-P_1)(1-P_2)P_3$ ....(3)

$P(\overline{E}_1 \cap \overline{E}_2 \cap \overline{E}_3) = P = (1-P_1)(1-P_2)(1-P_3)$ ...(4)

Given that, $(\alpha - 2\beta)P = \alpha\beta$

$\Rightarrow (P_1(1-P_2)(1-P_3) - 2(1-P_1)P_2(1-P_3))P = P_1P_2(1-P_1)(1-P_2)(1-P_3)^2$

$\Rightarrow (P_1(1-P_2) - 2(1-P_1)P_2) = P_1P_2$

$\Rightarrow (P_1 - P_1P_2 - 2P_2 + 2P_1P_2) = P_1P_2$

$\Rightarrow P_1 = 2P_2$ ....(1)

and similarly, $(\beta - 3\gamma)P = 3B\gamma$

$P_2 = 3P_3$ .....(2)

So, $P_1 = 6P_3 \Rightarrow \boxed{\frac{P_1}{P_3} = 6}$

85.[2] $\vec{a}\cdot\vec{b} = 1 \Rightarrow -\alpha\beta - \alpha\beta - 3 = 1$

$\Rightarrow -2\alpha\beta = 4 \Rightarrow \boxed{\alpha\beta = -2}$ ...(1)

$\vec{b}\cdot\vec{c} = -3 \Rightarrow -\beta + 2\alpha + 1 = -3$

$\boxed{\beta - 2\alpha = 4}$

Solving (1) & (2), $(\alpha, \beta) = (-1, 2)$

$\frac{1}{3}[\vec{a}\ \vec{b}\ \vec{c}] = \frac{1}{3}\begin{vmatrix} \alpha & \beta & 3 \\ -\beta & -\alpha & -1 \\ 1 & -2 & -1 \end{vmatrix}$

$= \frac{1}{3}\begin{vmatrix} -1 & 2 & 3 \\ -2 & 1 & -1 \\ 1 & -2 & -1 \end{vmatrix}$

$= \frac{1}{3}\begin{vmatrix} 0 & 0 & 2 \\ -2 & 1 & -1 \\ 1 & -2 & -1 \end{vmatrix} = \frac{1}{3}[2(4-1)] = 2$

86.[16] $2A\ \text{adj}(2A) = |2A|I$

$\Rightarrow A\ \text{adj}(2A) = -4I$ ....(i)

Now, $E = |A^4| + A^{10} - (\text{adj}(2A))10|$

$= (-2)^4 + \frac{\left|A^{20} - A^{10}(\text{adj}2A)^{10}\right|}{|A|^{10}}$

$= 16 + \frac{|A^{20} - (A\text{adj}(2A))^{10}|}{|A|^{10}}$

$= 16 + \frac{|A^{20} - 2^{10}I|}{2^{10}}$ (from (1))

Now, characteristic roots of A are 2 and – 1 So, characteristic roots of A^{20} are 2^{10} and 1.

Hence, $(A^{20} - 2^{10}I)(A^{20} - I) = 0$

$\Rightarrow |A^{20} - 2^{10}I| = 0$ (as $A^{20} \neq I$)

$\Rightarrow E = 16$ Ans.

87.[1] $I = \int_0^{\sqrt{\pi/2}} \left([x^2] + [-\cos x]\right) dx$

$= \int_0^1 0\, dx + \int_0^{\sqrt{\pi/2}} dx + \int_0^{\sqrt{\pi/2}} (-1)dx$

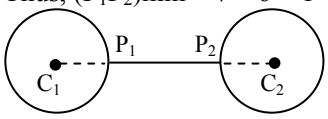
$= \sqrt{\frac{\pi}{2}} - 1 - \sqrt{\frac{\pi}{2}} = -1$

$\Rightarrow |I| = 1$

88.[1] Given C_1 (5, 5), $r_1 = 3$ and C_2 (12, 5), $r_2 = 3$

Now $C_1C_2 > r_1 + r_2$

Thus, $(P_1P_2)\min = 7 - 6 = 1$



89.[4] Required plane is

$p_1 + \lambda p_2 = (2 + 3\lambda)\, x - (7 + 5\lambda)\, y$

$+ (4 + 4\lambda)z - 3 + 11\lambda = 0$;

which is satisfied by (–2, 1, 3)

Hence, $\lambda = \frac{1}{6}$

Thus, plane is $15x - 47y + 28z - 7 = 0$

So, $2a + b + c - 7 = 4$

90.[4] $(2023 - 2)^{3762} = 2023\, K_1 + 23762$

$= 17k_2 + 2^{3762}$ (as $2023 = 17 \times 17 \times 9$)

$= 17k_2 + 4 \times 16^{940}$

$= 17k_2 + 4 \times (17 - 1)^{940}$

$= 17k_2 + 4\,(17k_3 + 1)$

$= 17k + 4 \Rightarrow$ remainder = 4